Adaptive Multicode CDMA for Uplink Throughput Maximization

Syed Ali Jafar and Andrea Goldsmith
Stanford University, USA

Abstract

We determine the optimal adaptive rate and power control strategies to maximize the total average throughput in a multicode CDMA system. Peak power and instantaneous bit error rate (BER) constraints are assumed at the transmitter with matched filter detection at the receiver. We first obtain results for the case where the codes available to each user are unrestricted, and we then consider the more practical scenario where each user has a finite discrete set of codes. An upper bound for the maximum average throughput is obtained and evaluated for Rayleigh fading. Suboptimal low-complexity schemes are considered to illustrate the performance tradeoffs between optimality and complexity. We also show that the optimum rate and power adaptation scheme with unconstrained rates is in fact just a rate adaptation scheme with fixed transmit powers, and it performs significantly better than adapting just power alone.

1. Introduction

Although adaptive modulation and multirate CDMA form the foundations for the third generation of wireless communication systems, adaptive CDMA remains a relatively unexplored area of research. In this work we investigate the maximum throughput that can be achieved through joint rate and power adaptation in a multirate CDMA system. We assume conventional matched filter detection with perfect channel information and an instantaneous BER constraint. We restrict our attention to multicode or multiple processing gain schemes, which have been shown to have almost the same performance [1][2]. Other multirate CDMA schemes have also been proposed but are not as viable as multicode or multiple processing gain schemes. The conventional matched filter receiver, although suboptimal for multiuser detection, remains popular because of its simplicity. We use a maximum power constraint since any transmitter will in practice necessarily have an upper limit on its transmit power, especially on the reverse link. Our QOS measure is the maximum instantaneous bit error rate or, equivalently, the minimum $(E_b/N_o)_{eff}$ necessary for each of the transmitted spreading codes of a user.

2. System Model

We consider a single cell, variable rate multicode (or multiple processing gain) CDMA system with K users, each having a specific set of M code sequences (or spreading gain values) to choose from. The code sequences assigned to a user are orthogonal so that a user does not interfere with himself. However, users do interfere with each other, ie. sequences transmitted by different users are not orthogonal to each other. The system uses BPSK with coherent demodulation. A maximum instantaneous bit error rate constraint must be met for a user to transmit on the channel. The channel is affected by slow fading (assumed constant over a bit time), additive white Gaussian noise (AWGN), and multiple access interference (MAI) due to other users. The user's channel access is assumed to be asynchronous.

Under the standard Gaussian approximation [2] the instantaneous bit energy to noise spectral density ratio for the i^{th} user can be expressed as [3]

$$\left(\frac{E_b}{N_o}\right)_{eff} = \frac{\frac{P_i(\overline{\chi})}{n_i(\overline{\chi})}}{\frac{1}{3N_s}\sum_{k\in I-\{i\}} P_k(\overline{\chi}) + \frac{N_o}{2N_sT_c}}, \quad (1)$$

where $I = \{1, 2, \cdots, K\}$ is the index set of users and $\overline{\chi} = \{\chi_1, \chi_2, \cdots, \chi_K\}$ is the vector of channel power fade levels experienced by each user. $P_i(\overline{\chi}) = S_i(\overline{\chi})g_i(r_i)\chi_i$ is the received power, where $S_i(\overline{\chi})$ is the transmitted power, r_i is the user's distance from the base station and $g_i(r_i)$ is the propagation path loss. N_o is the AWGN power spectral density. For multicode CDMA, N_s is the spreading gain and $n_i(\overline{\chi})$ is the number of orthogonal, synchronous codes transmitted in parallel by user i. For variable spreading, N_s is the maximum spreading gain (corresponding to the minimum or unit rate) of the system and rates $n_i(\overline{\chi})$ are achieved by reducing the spreading factor for user i to $\frac{N_s}{n_i(\overline{\chi})}$. The chip duration T_c remains fixed in all schemes.

We assume that a perfect estimate of the channel state χ_i and the propagation path loss $g_i(r_i)$ of each user is available at the base station. Also, a reliable feedback channel exists to send power and rate control information from the base station to the mobiles with no errors and negligible delay.

Henceforth, for brevity we will limit our discussion to multicode systems. However, note that the system model as defined earlier applies to both multicode and multiple spreading gain systems, and therefore our results can be reformulated to apply to multiple spreading gain systems [4].

3. Problem Definition and Constraints

Our goal is to maximize the total throughput of the system averaged over the fading distributions of the users, subject to a peak transmit power constraint and an instantaneous BER constraint. The total throughput is defined as the sum of the data rates of all users. The instantaneous BER constraint implies that a user can transmit on the channel only if his instantaneous $\left(\frac{E_b}{N_o}\right)_{eff}$ is above a specified target level. We assume the same BER constraint for all users. The peak transmit power is usually determined by the transmitter hardware. However, depending on the path loss and channel fade different users may have different peak received power constraints. Subject to these constraints, we wish to find the optimal rate and power adaptation on the fading channel that maximizes the average total throughput. Note that since our power and BER constraints are instantaneous (rather than average), average throughput maximization is the same as instantaneous throughput maximization for each fade vector. The general optimization problem is therefore as follows: Find the optimal rate and power adaptation to maximize the instantaneous throughput

$$T_{opt}(\overline{\chi}|c_1, c_2 \ldots c_n) = \max_{c_1, c_2 \ldots c_n} \sum_{k \in I} n_k(\overline{\chi})$$

subject to the constraints $c_1, c_2, \ldots, c_n$.

The constraints we use in different sections, abbreviated as c_k or c'_k are provided here for reference. For all $i \in I$, the constraints are

Peak Power (c_1): $0 \le P_i(\overline{\chi}) \le P_{i,max}(\chi_i)$

Max BER (c_2): when $P_i(\overline{\chi}) > 0$ then

$$\frac{\frac{P_i(\overline{\chi})}{n_i(\overline{\chi})}}{\frac{1}{3N_s}\sum_{k \in I-\{i\}} P_k(\overline{\chi}) + \frac{N_o}{2N_s T_c}} \ge (Q^{-1}(\text{BER}))^2$$

Unlimited Rates (c_3): $0 \le n_i(\overline{\chi}) \le \infty$

Continuous Rates (c_4): $n_i(\overline{\chi}) \in R^+$

Limited Rates (c'_3): $0 \le n_i(\overline{\chi}) \le M$

Discrete Rates (c'_4): $n_i(\overline{\chi}) \in Z^+$

4. Optimal Unlimited Continuous Rate and Power Adaptation

In practical multicode CDMA systems the number of codes transmitted by a user at any instant can only be an integer between 0 and M. In this section we relax this constraint and treat $n_i(\overline{\chi})$ as a continuous variable that takes values over the entire range of positive real numbers. The resulting maximum throughput gives an upper bound on the performance of practical systems. Under this assumption we present the following proposition:

Theorem 1 *The optimal solution that maximizes the average total throughput is such that*

$$P_k(\overline{\chi}) \in \{0, P_{k,max}(\chi_k)\} \qquad \forall k \in I.$$

That is, either a user does not transmit, or he transmits at full power.

Proof From $(c2)$ we get that

$$n_i(\overline{\chi}) = \frac{P_i(\overline{\chi})}{\sum_{k \in I-\{i\}} P_k(\overline{\chi}) + \frac{3NN_o}{2T_b}} \frac{3N}{(Q^{-1}(\text{BER}))^2}. \quad (2)$$

Note that we replaced the inequality in (c_2) by an equality. This is because in a CDMA system, transmit power needs to be just enough to meet the BER constraint. Differentiating the total instantaneous throughput $T(\overline{\chi}) = \sum_{i \in I} n_i(\overline{\chi})$ twice with respect to $P_i(\overline{\chi})$ we can easily verify that $\frac{\partial^2 T(\overline{\chi})}{\partial P_i(\overline{\chi})^2} \ge 0$. Hence $T(\overline{\chi})$ is a convex function of $P_i(\overline{\chi})$ and the maximum value will always lie at the boundary. This completes the proof.

Without loss of generality we assume that

$$P_{i,max}(\chi_i, r_i) \ge P_{j,max}(\chi_j, r_j) \quad \forall i \le j \quad i, j \in I. \quad (3)$$

Under this assumption we define the *best* k users as the *first* k users in the index set I. Note that these are the *best* users since they have the greatest *received* powers at the base station when transmitting at their maximum power.

In light of Theorem 1, we can write the optimum instantaneous throughput subject to our constraints as

$$T_{opt}(\overline{\chi}|c_1, c_2, c_3, c_4) = \sum_{i \in I_{opt}} \frac{DP_{i,max}(\chi_i)}{\sum_{k \in I_{opt}-\{i\}} P_{k,max}(\chi_k) + C},$$

where, for notational convenience, we define $D = \frac{3N}{(Q^{-1}(\text{BER}))^2}$ and $C = \frac{3NN_o}{2T_b}$. $I_{opt} \subset I$ is the set of users transmitting at their peak powers for maximum throughput. We still need to find I_{opt} for the optimum solution. There are $2^K - 1$ non-empty subsets of I, and the throughput for each can be found according to equation given above. However, it is easy to see that if k_{opt} users need to transmit according to the optimal adaptation then without loss of generality they can be the k_{opt} *best* users. Thus, to obtain I_{opt} we only need to find k_{opt} for which there are only K possibilities.

5. Optimum Limited Continuous Rate and Power Adaptation Scheme

In the previous section we allowed an unlimited number of codes for every user. However, since a user's codes are orthogonal, the number of codes available to a user cannot exceed the processing gain of the system. Another factor that further limits the number of codes in a practical system is the constraint on the Peak-to-Mean Envelope Power Ratio (PMEPR) for linear amplification. It has been shown that for random codes PMEPR increases linearly with the number of parallel codes [2]. It is therefore interesting to see how throughput is affected when we limit the number of codes available to a user.

We wish to find $T_{opt}(\overline{\chi}, \overline{r}|c_1, c_2, c_3', c_4)$. Note that we still assume that the number of codes is a continuous variable (c_4). Let the optimum throughput be achieved with the k_{opt} best users transmitting. Let us partition the set I_{opt} into three mutually disjoint and collectively exhaustive subsets I_M, I_P and I_R, such that I_M is the index set of users operating at the rate boundary (transmitting M codes), I_P is the index set of users at the power boundary (transmitting at peak power $S_{i,max}$) that are not at the rate boundary, and I_R is the index set of the remaining users that need to transmit to achieve the maximum throughput. Let the number of users in each set be k_M, k_P, and k_R respectively. The following propositions characterize the optimal powers and rates of users in these sets.

Theorem 2 *The optimal solution is such that* $\forall i, j \in I_{opt}$, *if* $P_i(\overline{\chi}, \overline{r}) > P_j(\overline{\chi}, \overline{r}) > 0$, *then* $i \in I_M \cup I_P$.

Proof It can be easily verified that for two users, i and j, such that $P_i(\overline{\chi}, \overline{r}) > P_j(\overline{\chi}, \overline{r}) > 0$, and $i \notin I_M \cup I_P$, the total throughput increases if we decrease P_i and increase P_j so that the total power $P_i + P_j$ is fixed. Thus, the proof follows by contradiction. For details see [4].

Theorem 3 *The optimum solution is such that*

$$\begin{aligned}
\forall i,j \in I_M \quad & P_i(\overline{\chi}, \overline{r}) && = P_j(\overline{\chi}, \overline{r}) = P_M(\overline{\chi}, \overline{r}), && (4)\\
\forall i \in I_P \quad & P_i(\overline{\chi}, \overline{r}) && = P_{i,max}(\chi_i, r_i), && (5)\\
& k_R && \leq 1, && (6)\\
\forall i \in I_P \quad & P_M(\overline{\chi}, \overline{r}) && \geq P_i(\overline{\chi}, \overline{r}) \geq P_R, && (7)
\end{aligned}$$

where P_R *is the received power of the user in* I_R *(i.e. not at his rate or power boundary). By (6) there can be at most one such user. If there is no such user we define* $P_R = 0$.

Proof First, consider two users on the rate boundary $i, j \in I_M$. This implies that $n_i(\overline{\chi}, \overline{r}) = n_j(\overline{\chi}, \overline{r})$. Substituting from (2) proves (4). (5) follows from the definition of I_P. (6) follows from Proposition 2 and the fact that equal received powers correspond to a minimum in the total throughput as mentioned earlier. Since higher rates require higher powers, (7) is trivial.

Propositions 2 and 3 tell us that the maximum throughput with finite rates is achieved with k_M *best* users at the rate boundary, each with the same received power $P_M(\overline{\chi})$, the next k_P *best* users at the power boundary (transmitting at $S_{i,max}$), and at most one user with received power P_R that is not at his power or rate boundary. Assuming we know k_M and k_P we now want to find the optimum values of $P_M(\overline{\chi}, \overline{r})$ and $P_R(\overline{\chi}, \overline{r})$.

Using (2) to obtain P_R in terms of $P_M(\overline{\chi})$ and $P_{i,max}$, the total throughput can be written as

$$\begin{aligned}
&T_{k_M,k_P}(\overline{\chi}|c_1, c_2, c_3', c_4) = \\
&\quad k_M M + \tfrac{D\left(P_M(\overline{\chi},\overline{r})(\frac{D}{M}+1-k_M)-P_p-C\right)}{P_p+k_M P_M(\overline{\chi},\overline{r})+C} \\
&\quad + \textstyle\sum_{i\in I_P} \tfrac{DP_{i,max}(\chi_i,r_i)}{P_M(\overline{\chi},\overline{r})(1+\frac{D}{M})-P_{i,max}(\chi_i,r_i)}. \qquad (8)
\end{aligned}$$

Differentiating the total throughput with respect to $P_M(\overline{\chi}, \overline{r})$ and equating the derivative to zero gives us the equation that can be solved to obtain $P_M(\overline{\chi})$ and $P_R(\overline{\chi})$. A more detailed description of the derivation is given in [4]. We use this scheme to find the optimum average throughput for our system in Section 9.

6. Analytical Upper bound for Unlimited Continuous Rate and Power Adaptation

The optimal scheme considered earlier gives us a way to find the maximum *instantaneous* throughput achievable for a given channel fade-vector and a given user location vector. However, an analytical expression for the maximum *average* (averaged out over the probability distribution of the channel fade vector) throughput is difficult to achieve. We can upper bound this optimal average throughput by the throughput achieved when every user is as good as the *best* user. Mathematically,

$$T_{opt}(\overline{\chi}|c_1, c_2) \leq T_{opt}(\overline{\chi}|c_1', c_2),$$

where

$$c_1{}' : 0 \leq P_i(\overline{\chi}) \leq P_{max}(\overline{\chi}) = \max_{j \in I} P_{j,max}(\chi_j).$$

Now, the optimum rate and power allocation strategy gives us

$$\begin{aligned}
T_{opt}(\overline{\chi}|c_1', c_2) &= \max_{0<k_{trans}\leq K} \frac{Dk_{trans}P_{max}(\overline{\chi})}{(k_{trans}-1)P_{max}(\overline{\chi})+C} \\
&= \begin{cases} \frac{DKP_{max}(\overline{\chi})}{(K-1)P_{max}(\overline{\chi})+C} & \text{if } P_{max}(\overline{\chi}) < C \\ \frac{DP_{max}(\overline{\chi})}{C} & \text{otherwise.} \end{cases}
\end{aligned}$$

For the symmetric case, $S_{i,max}g_i(r_i) = S_{max}, \ \forall i \in I$ and assuming Rayleigh fading, the χ_is are exponential distributed as $p_{\chi_i}(x) = \frac{1}{\Omega}\exp(-\frac{x}{\Omega})$ and the probability distribution of P_{max} is found to be [4]

$$p_{P_{max}}(x) = \frac{K}{\Omega}(1-\exp(-\frac{x}{\Omega}))^{K-1}\exp(-\frac{x}{\Omega}). \qquad (9)$$

Straightforward integration then yields the average throughput upperbound as

$$\overline{T} = \sum_{i=0}^{K-1} \binom{K-1}{i} \frac{K}{\Omega}(-1)^i \left\{ \frac{D}{C} \left(\frac{\Omega}{i+1}\right)^2 \Gamma\left[2, \frac{C(i+1)}{\Omega}\right] + \frac{DK}{K-1} \left\{ \frac{\Omega}{i+1} \left(1 - \exp[-\frac{C(i+1)}{\Omega}]\right) - \frac{C}{K-1} \exp[\frac{(i+1)C}{(K-1)\Omega}] \left(\Gamma\left[0, \frac{C(i+1)}{\Omega(K-1)}\right] - \Gamma\left[0, \frac{KC(i+1)}{(K-1)\Omega}\right]\right)\right\}\right\}$$

In Section 9 we compare the average throughput given by this upper bound with the optimal average throughput found through Monte Carlo simulations with the optimal schemes described in the other sections.

7. Optimum Limited Discrete Rate and Power Adaptation

We now consider the throughput in a more practical system with limited discrete rates $T_{opt}(\overline{\chi}, \overline{r}|c_1, c_2, c_3', c_4')$. We need the following proposition:

Theorem 4 *The received powers required to achieve a rate vector* $\overline{n} = \{n_1, n_2, \cdots, n_k\}$, $n_i \in Z^+$, *can be expressed as*

$$P_i(\overline{\chi}) = C\left(\frac{n_i}{n_i + D}\right)\frac{1}{1-\gamma} \quad \forall i \in I, \tag{10}$$

where $\gamma = \sum_{j \in I} \frac{n_j}{n_j + D}$. *Moreover, for a given rate vector to be achievable, we must have* $\gamma < 1$.

Proof The expression for $P_i(\overline{\chi})$ is obtained by solving (2) for a given rate vector. The achievability condition $\gamma < 1$ is required because the powers have to be positive.

The achievability constraint $\gamma < 1$ gives us the maximum number of users that can transmit simultaneously as

$$K_{max} < 1 + D. \tag{11}$$

Since we only have discrete limited rates, we can construct a table of all possible rate vectors (that satisfy $\gamma < 1$) and the received power vectors required to achieve the corresponding rate vectors obtained using Proposition 4. Note that this table does not depend on the channel state. However, for a given channel state the peak recieved power constraint depends on the channel state. Thus the optimal adaptation policy is to find the maximum throughput rate vector in this table for which the required received power vector satisfies the peak received power constraint for that channel state.

Although the size of the table can be large, note that it is much less than M^K since the maximum achievable throughput is much less than MK for large K. Moreover, the size of the table does not increase as the number of users increases beyond K_{max} (defined in (11)).

A more detailed discussion on how the search can be made more efficient is included in [4]

8. Optimal Power Control vs Optimal Rate Control

In this section we wish to find the maximum average throughputs achieved with rate or power control alone. First we consider optimum power control, so that whenever a user transmits, he uses a fixed number of codes M, while the transmit power is adapted to the channel fade vector. The average throughput for such a system is $\overline{T} = M\mathrm{E}[k(\overline{\chi})]$, where $k(\overline{\chi})$ is the number of users transmitting M codes for a given channel fade vector $\overline{\chi}$. Since we are interested in the average over the fading distribution, we consider the symmetric case, $P_i(\overline{\chi}) \leq S_{max}\chi_i$. Now, if $k(\overline{\chi})$ users are transmitting the same number of codes simultaneously, they must all have the same received power $P_k(\overline{\chi})$. Substituting into (2) we get $P_k(\overline{\chi}) = \frac{C}{\frac{D}{M}+1-k(\overline{\chi})}$ which implies that $\chi_i > \frac{C}{S_{max}(\frac{D}{M}+1-k(\overline{\chi}))} = \chi_{min}(k) \;\; \forall i \in I_{opt}(\overline{\chi})$. The optimum power adaptation in this case is to choose the maximum k such that at least k users have channel fades better than $\chi_{min}(k)$. The distribution of k is found as $\text{Prob}[k(\overline{\chi}) \geq k] = \text{Prob}\,[\text{k or more users have fades } \chi_i \geq \chi_{min}(k)] = \sum_{i=k}^{K} \binom{K}{i} p_k^i (1-p_k)^{K-i}$, where $p_k = \text{Prob}[\chi_i > \chi_{min}(k)]$ and the $\{\chi_i\}$ are independent, identically distributed. Using this we obtain the average throughput achievable with power adaptation alone as

$$\overline{T} = M \sum_{k=1}^{\min(K,\lfloor D/M+1 \rfloor)} k \left(\sum_{i=k}^{K} \binom{K}{i} p_k^i (1-p_k)^{K-i} - \sum_{i=k+1}^{K} \binom{K}{i} p_{k+1}^i (1-p_{k+1})^{K-i} \right). \tag{12}$$

Next we look at optimum rate adaptation. We restrict our system so that whenever a user transmits, he uses a fixed power P, while the number of codes is adapted to the channel fade vector. We assume the number of codes to be continuous and unlimited. But this gives us our optimum power and rate adaptation scheme considered in Section 4. Hence, the optimum rate and power adaptation is actually just the optimum rate adaptation.

9. Numerical Results

The average throughput under different constraints is found and plotted in Figure 1. The parameters common to the curves in Figure 1 are : Spreading Gain $N = 63, D = \frac{3N}{(Q^{-1}(\text{BER}))^2} = 20$, $\left(\frac{S_{max}g(r)T_b}{N_o}\right) = 12$db. All users are assumed to have the same peak transmit power and the same propagation path loss. Channel power fades are assumed to be exponential distributed (flat Rayleigh fading) with unit variance. M=10 for limited rates schemes. With these parameters, the maximum number of users that can transmit simultaneously is $K_{max} = 20$. Average throughput for the various schemes is found using Monte Carlo simulations. The analytical upper bound curve is also plotted in the same figure. As expected, for a given number of users in the system, the average throughputs for various schemes are in the order:

Analytical Upper bound (AU)$>$ Optimum Unlimited Continuous Rate and Power Adaptation (OUCRPA) $>$ Optimum Limited Continuous Rate and Power Adaptation (OLCRPA) $>$ Optimum Limited Discrete Rate and Power Adaptation (OLDRPA) $>$ Optimum Power Adaptation (OPA).

The *Quantized Unlimited Rates and Power Adaptation*(QURPA) curve represents a direct quantization scheme where the OUCRPA rate vector is quantized by componentwise truncation. Obviously as the number of transmitting users increases, more quantizations have to be performed and the throughput actually *decreases*. Thus a "direct" quantization of OUCRPA fails miserably. On the other hand a slightly smarter adaptation of OUCRPA to discrete rates is given by the *Discrete Unlimited Rate and Power Adaptation*(DURPA) curve. The adaptation is such that

$$T(\overline{\chi}, \overline{r}) = \max_{0 \le n \le K} D \sum_{i \in I_{opt}} \lfloor \frac{P_{i,max}(\chi_i, r_i)}{\sum_{k \in I_{opt}-\{i\}} P_{k,max}(\chi_k, r_k) + C} \rfloor$$

where $I_{opt} = \{1, 2, \cdots, n\}$. As apparent from Figure 1, this scheme provides a good approximation to ODLRPA.

The OUCRPA and OLCRPA curves in Figure 1 suggest that increasing the total number of codes assigned to each user above a sufficiently large value does not significantly increase throughput.

10. Conclusion

We derive the maximum throughput achievable in a variable rate variable power CDMA system using a matched filter receiver. We find the optimum rate and power adaptation schemes for limited/unlimited and continuous/discrete rates. We present an analytical upper bound to estimate the maximum achievable average throughput. We also compare optimal rate adaptation to optimal power adaptation. Our numerical results compare the performances of these optimal schemes under various assumptions (limited/unlimited, continuous/discrete) on the rates available to each user. We also simulate some sub-optimal schemes with lower complexity to determine the performance tradeoffs between complexity and optimality.

We find that increasing the maximum rate (number of codes) available to each user beyond a given value does not significantly increase throughput. We also find that a peak power constraint limits the maximum achievable average throughput even with unlimited rates (codes) available to every user. This is in contrast to the case where there is an average power constraint: in this case it can be shown that the unlimited rates (codes) assumption leads to unbounded average throughput. Thus, a maximum power constraint is a more fundamental limitation than an average power constraint.

The optimal rate and power adaptation scheme with unconstrained rates is in fact just a rate adaptation scheme, with fixed transmit powers. On the other hand, the optimum power adaptation scheme with fixed rates yields significantly lower average throughput. Thus although power adaptation is much simpler to implement than rate adaptation, it does not achieve the average throughputs possible with rate adaptation.

In practice any system will have additional constraints arising out of the need to be "fair" to users in deep fades, other concerns like prolonging battery life (average power constraint), or additional QOS requirements (delay constraints) etc. which will reduce the maximum achievable throughput.

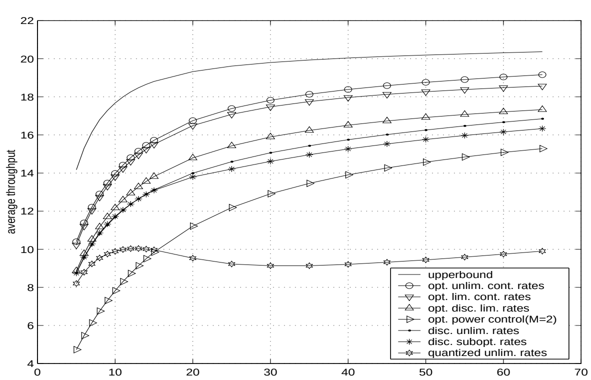

Figure 1. Average Throughputs

References

[1] T. Ottosson and A. Svensson, "Multi-Rate Schemes in DS-CDMA Systems", VTC'95, Chicago, 1995.

[2] T. Ottosson and A. Svensson, "On schemes for multirate support in DS-CDMA systems", Wireless Personal Communications, Kluwer Academic Publishers, Vol. 6, no. 3, pp. 265-287, March 1998.

[3] S. W. Kim and Y. H. Lee, "Combined Rate and Power Adaptation in DS/CDMA Communications over Nakagami Fading Channels", IEEE Transactions on Communications, Vol. 48, Jan 2000.

[4] A. J. Syed, A. J. Goldsmith, "Adaptive Multicode CDMA for Uplink Throughput Maximization", http://wsl.stanford.edu/Publications/Syed/j1.ps, submitted.