LINEAR RF POWER AMPLIFIER DESIGN FOR TDMA SIGNALS: A SPECTRUM ANALYSIS APPROACH

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ABSTRACT

One of the critical and costly components in digital cellular communication systems is the RF power amplifier. Theoretically, one of the main concerns in an RF power amplifier design is the nonlinear effect of the amplifier. Quantitatively, so far, no explicit relationship or expression currently exists between the out-of-band emission level and the nonlinearity description related to the third-order intercept point (IP_3). Further, in experiments and analysis, it was discovered that, in some situations, using IP_3 only is not accurate enough to describe the spectrum regrowth, especially when the fifth-order intercept point (IP_5) is relatively significant compared to the third-order intermodulation. In this article, we analyze the nonlinear effect of an RF power amplifier in TDMA (IS-54 Standard) system, give an expression of the estimation of the out-of-band emission levels for a TDMA power spectrum in terms of the IP_3 and the IP_5, as well as the power level of the signal. This result will be useful in the design of RF power amplifier for a TDMA wireless system.

1. INTRODUCTION

In the recent years, *Time Division Multiple Access (TDMA)* has been recognized as one of the most efficient and reliable schemes for cellular radio communications [1]. As in other communication systems, one of the critical and costly components in TDMA system is the RF power amplifier. One of the main concerns in RF power amplifier design is the nonlinearity of an RF amplifier can degrade the quality of the TDMA signal, increasing bit error rate and interference to adjacent channels. The nonlinearity control is specified by the out-of-band power emission levels in the IS-54 standard [2]. The nonlinearity control is also called *spectrum regrowth*. Traditionally, the nonlinearity of an RF amplifier is described by using the *third-order interception point* (IP_3) [3]. In experiments and analyses, it was discovered that, in some cases, using IP_3 only is not accurate enough to describe the spectrum regrowth, especially when the fifth-order intermodulation is relatively significant compared the third-order intermodulation. Quantitatively, to the best of our knowledge, there is no explicit relationship or expression between the out-of-band emission level and the traditional amplifier nonlinearity description for the TDMA signal amplification. The lack of such a relationship makes difficulties for RF power amplifier designers choosing components. In our early effort, we analyzed the nonlinear effect of an RF power amplifier on CDMA systems [4, 5]. Continuing this effort into developing the spectrum analysis approach for TDMA signals, we derive the expressions of the estimated out-of-band emission levels for TDMA signal, present the relationship between an amplifier's out-of-band power emission levels and its nonlinearity parameters, IP_3 and the *fifth-order interception point* (IP_5). The results presented in this article allow RF Amplifier designers to specify and measure the TDMA signal amplifiers using simple IP_3 and IP_5 descriptions. The expression turn out to be simpler and easier to use for the case where IP_5 may be ignored. In addition, a spectrum comparison between the simulated and predicted results is presented.

2. MODEL DESCRIPTION

2.1. The TDMA Signal Mathematical Model

Generally, the mathematical model of the IS-54 TDMA signal can be presented as [2]

$$\begin{aligned} s(t) &= \sum_{n=-\infty}^{\infty} A\,h(t-nT_s)\cos\left[2\pi f_c t+\theta_0+\Phi_n\right] \\ &= \mathrm{Re}\left\{\left[\sum_{n=-\infty}^{\infty} Ah(t-nT_s)e^{j(\theta_0+\Phi_n)}\right]\cdot e^{j\omega_c t}\right\} \\ &= \mathrm{Re}\left\{g(t)\cdot e^{j\omega_c t}\right\} \end{aligned} \tag{1}$$

where $h(t)$ is the baseband filter which have linear phase and square root raised cosine frequency response, A is a constant, depending only on the minimum TDMA symbol energy, Φ_n is the absolute phase corresponding to the *n-th* symbol interval, T_s is the symbol period, equal to $41.15\mu s$ for IS-54 standard, f_c is the carrier center frequency, θ_0 is an initial phase, $\mathrm{Re}\{\}$ denotes the real part of $\{\}$, and $g(t)=\sum_{n=-\infty}^{\infty} Ah(t-nT_s)e^{j(\theta_0+\Phi_n)}$, which is a pulse shaped *Nonreturn-to-Zero (NRZ)* function. Its *Power Spectrum Density (PSD)* can be obtained through a lengthy derivation:

$$P_g = A^2 R_s \left|H(f)\right|^2 \tag{2}$$

where $R_s = 1/T_s$ is the symbol rate, and $H(f)$ is the *Frequency Response* of the baseband filter.

Since the spectrum of a band-pass signal is directly related to the spectrum of its baseband envelope, the PSD of a TDMA signal, $s(t)$, can be expressed as [3]

$$P_s(f)=\frac{A^2 R_s}{4}\left[\left|H(f-f_c)\right|^2+\left|H(-f-f_c)\right|^2\right] \quad (3)$$

where $s(t)=\mathrm{Re}\left\{g(t)\cdot e^{j\omega_c t}\right\}$ was presented in Equation 1.

Equivalently, the mathematical model of $s(t)$ can also be described as

$$\begin{aligned} s(t) &= r(t)\cos\theta(t)\cos(2\pi f_c t+\theta_0)-r(t)\sin\theta(t)\sin(2\pi f_c t+\theta_0) \\ &= r(t)\cos\left[2\pi f_c t+\theta(t)+\theta_0\right] \\ &= r(t)\cos\left[2\pi f_c t+\theta\right] \end{aligned} \quad (4)$$

where

$$r(t)\cos\theta(t)=I(t)=\sum_{n=-\infty}^{\infty} A\,h(t-nT_s)\cos\Phi_n$$

$$r(t)\sin\theta(t)=Q(t)=\sum_{n=-\infty}^{\infty} A\,h(t-nT_s)\sin\Phi_n$$

$$\theta(t)=\tan^{-1}\{Q(t)/I(t)\}$$

$$\theta=\theta(t)+\theta_0$$

and $r(t)=\sqrt{I^2(t)+Q^2(t)}$ is the baseband envelope of $s(t)$. Its *Fourier Transform* can be derived from the PSD of $g(t)$, P_g through a lengthy derivation:

$$F\{r(t)\}=A\sqrt{R_s}\cdot\left|H(f)\right| \quad (5)$$

where $F\{\cdot\}$ is the *Fourier Transform* of $\{\cdot\}$.

2.2. A Power Amplifier's Mathematical Model

Generally speaking, a practical amplifier is only a linear device in its linear region, meaning that the output of the amplifier will not exactly a scaled copy of the input signal when the amplifier works beyond the linear region. Considering an amplifier as a functional box, it can be modeled by a Taylor series [3]. The Taylor series model is only valid for a memoryless nonlinearity function. For a memoryless amplifier with only a few stages, a Taylor series model is fairly good for predicting the nonlinearity.Therefore, the Taylor series is adopted for modeling RF power amplifiers. Using the TDMA signal equivalent mathematical model $s(t)$ in Equation 4, the output of an amplifier generally can be written as

$$y(t)=O\{s(t)\}=F[r(t)]\cdot\cos\{2\pi f_c t+\theta+\Phi[r(t)]\} \quad (6)$$

where $O\{\cdot\}$ denotes the operation of amplifier, $F[\cdot]$ is the amplitude to amplitude conversion (AM/AM), and $\Phi[\cdot]$ is amplitude to phase conversion (AM/PM). The functions $F[\cdot]$ and $\Phi[\cdot]$ are dependent on the nonlinearity of the amplifier and modeling type.

Since we are generally interested only in the output band near the carrier frequency f_c, the phase distortion in the band is negligible using a Taylor series model, i.e., $\Phi[r(t)]=0$ [6]. Therefore, Equation 6 becomes

$$y(t)=O\{s(t)\}=F[r(t)]\cdot\cos(2\pi f_c t+\theta)\ . \quad (7)$$

Let $\tilde{y}(t)=F[r(t)]$, the Taylor expansion of $O\{s(t)\}$ can be used to determine $\tilde{y}(t)$. Generally, the Taylor model of an RF amplifier can be written as

$$y(t)=\sum_{i=0}^{\infty} a_{2i+1}s^{2i+1}(t). \quad (8)$$

Here, only the odd-order terms in the Taylor series are considered, since the spectra generated by the even-order terms are at least f_c away from the center of the passband, the effects from these terms on the passband are negligible. Furthermore, as a linear amplifier, the third- and fifth- order terms dominate in Equation 8 for distortion. Therefore, in this analysis, the following model is used for an RF amplifier:

$$y(t)=a_1 s(t)+a_3 s^3(t)+a_5 s^5(t)\ . \quad (9)$$

Substituting the input passband signal $s(t)=r(t)\cos(2\pi f_c t+\theta)$ into $y(t)$ of Equation 9 (after manipulation) produces

$$y(t)=\tilde{y}(t)\cos(2\pi f_c t+\theta) \quad (10)$$

where

$$\tilde{y}(t)=\tilde{a}_1 r(t)+\tilde{a}_3 r^3(t)+\tilde{a}_5 r^5(t) \quad (11)$$

with

$$\tilde{a}_1=a_1,\quad \tilde{a}_3=\frac{3}{4}a_3,\quad \tilde{a}_5=\frac{5}{8}a_5\ . \quad (12)$$

Here, the coefficient a_1 is related to the linear gain G of the amplifier, and the coefficients a_3 and a_5 are directly related to IP_3 and IP_5 respectively. It can be proven after a lengthy derivation that the expression for these coefficients becomes

$$a_1=10^{\frac{G}{20}},\quad a_3=-\frac{2}{3}10^{\left(-\frac{IP_3}{10}+\frac{3G}{20}\right)},\quad a_5=-\frac{2}{5}10^{\left(-\frac{IP_5}{5}+\frac{G}{4}\right)}. \quad (13)$$

From Equations 10-13, it can be seen that an amplifier's output $y(t)$ is a function of G, IP_3, IP_5 and the input signal $s(t)$. Consequently, using Equation 10 and the PSD of $s(t)$ in Equation 3, the PSD of $y(t)$ can be calculated and the power emission levels can be determined. Therefore, all of the nonlinear effects of the amplifier with the TDMA signals can be evaluated.

3. THE POWER SPECTRUM DENSITY (PSD) OF THE AMPLIFIED TDMA SIGNAL

Now, the PSD of $y(t)$ can be calculated. Since $y(t)=\tilde{y}(t)\cdot\cos(2\pi f_c t+\theta)$, the PSD of $y(t)$ can be determined by the PSD of $\tilde{y}(t)$ as [3]

$$P_y(f)=\frac{1}{4}\left[P_{\tilde{y}}(f-f_c)+P_{\tilde{y}}(f+f_c)\right] \quad (14)$$

and then, the PSD of $\tilde{y}(t)$ can be derived by *Wiener-Khintchine Theorem* as [3]

$$P_{\tilde{y}}(f)=\int_{-\infty}^{\infty} R_{\tilde{y}}(\tau)e^{-j2\pi f\tau}\,d\tau=F\left\{R_{\tilde{y}}(\tau)\right\}. \quad (15)$$

By definition, $R_{\tilde{y}}(\tau)$ is expressed as

$$R_{\tilde{y}}(\tau)=E\{\tilde{y}(t)\cdot\tilde{y}(t+\tau)\} \quad (16)$$

where $E\{\cdot\}$ is the *mathematical expectation* of $\{\}$.

Since $\tilde{y}(t)=\tilde{a}_1 r(t)+\tilde{a}_3 r^3(t)+\tilde{a}_5 r^5(t)$, $P_{\tilde{y}}(f)$ can be expressed in terms of the Fourier Transform of $r(t)$ through a lengthy derivation as

$$\begin{aligned}P_{\tilde{y}}(f) &= F\{R_{\tilde{y}}(\tau)\} = F\{E\{\tilde{y}(t)\cdot \tilde{y}(t+\tau)\}\} \\ &= \tilde{a}_1^2 \cdot F\{r(t)\}\cdot F\{r(t)\} + 2\tilde{a}_1\tilde{a}_3 \cdot F\{r(t)\}\cdot F\{r^3(t)\} \\ &\quad + 2\tilde{a}_1\tilde{a}_5 \cdot F\{r(t)\}\cdot F\{r^5(t)\} + \tilde{a}_3^2 \cdot F\{r^3(t)\}\cdot F\{r^3(t)\} \\ &\quad + 2\tilde{a}_3\tilde{a}_5 \cdot F\{r^3(t)\}\cdot F\{r^5(t)\} + \tilde{a}_5^2 \cdot F\{r^5(t)\}\cdot F\{r^5(t)\}\end{aligned} \tag{17}$$

where $F\{r(t)\}=A\sqrt{R_s}\cdot|H(f)|$ was described in Equation 5.

Let $P_1=|H(f)|=F\{r(t)\}/(A\sqrt{R_s})$, we can get $F\{r(t)\}=A\sqrt{R_s}\cdot P_1$, $F\{r^3(t)\}=(A\sqrt{R_s})^3\cdot P_3$ and $F\{r^5(t)\}=(A\sqrt{R_s})^5\cdot P_5$, where $P_3=P_1\otimes P_1\otimes P_1$, $P_5=P_1\otimes P_1\otimes P_1\otimes P_1\otimes P_1$, in which $\otimes$ denotes *convolution operator*. Also, the linear portion of the amplifier output power P_o can be described as [3]

$$P_o = a_1^2\cdot P_{in} = a_1^2\cdot A^2 R_s^2/2\ . \tag{18}$$

Substituting P_1, P_3, P_5 and P_o into Equation 17, and by the relationship between $P_y(f)$ and $P_{\tilde{y}}(f)$ in Equation 14, we can obtain the final result of the power spectrum $P_y(f)$ of $y(t)$ in terms of G, IP_3, IP_5 and P_o:

$$\begin{aligned}P_y(f) &= \frac{P_o}{R_s}\cdot P_1^2(f-f_c) - \frac{2P_o^2}{R_s^2}10^{-\frac{IP_3}{10}}\cdot P_1(f-f_c)\cdot P_3(f-f_c) \\ &\quad - \frac{2P_o^3}{R_s^3}10^{-\frac{IP_5}{5}}\cdot P_1(f-f_c)\cdot P_5(f-f_c) \\ &\quad + \frac{P_o^3}{R_s^3}10^{-\frac{IP_3}{5}}\cdot P_3^2(f-f_c) \\ &\quad + \frac{2P_o^4}{R_s^4}10^{-\frac{IP_3}{10}}\cdot 10^{-\frac{IP_5}{5}}\cdot P_3(f-f_c)\cdot P_5(f-f_c) \\ &\quad + \frac{P_o^5}{R_s^5}10^{-\frac{2IP_5}{5}}\cdot P_5^2(f-f_c).\end{aligned} \tag{19}$$

where f_c is the carrier center frequency.

If IP_5 is ignored, Equation 19 will become

$$\begin{aligned}P_y(f) &= \frac{P_o}{R_s}\cdot P_1^2(f-f_c) \\ &\quad - \frac{2P_o^2}{R_s^2}10^{-\frac{IP_3}{10}}\cdot P_1(f-f_c)\cdot P_3(f-f_c) \\ &\quad + \frac{P_o^3}{R_s^3}10^{-\frac{IP_3}{5}}\cdot P_3^2(f-f_c).\end{aligned} \tag{20}$$

Several observations are made by inspecting Equation 20: the first term $\frac{P_o}{R_s}\cdot P_1^2$ corresponds to the linear output power density; the remaining terms in Equation 20 are caused by the nonlinearity. In other words, these remaining terms are due to the intermodulation. For a linear amplifier, the intermodulation is usually much lower than the linear output power. Therefore, the intermodulation does not affect the passband spectrum significantly.

With the explicit power spectrum of the output TDMA signal, the out-of-band spurious emission power may be calculated in a particular frequency band. It is this power that used in IS-54 to specify the limit for the out-of-band control. To keep the result easy to use, only IP_3 is considered here.

Let a frequency band be defined by f_1 and f_2 outside the passband. Using the results from $P_y(f)$ of Equation 20, the emission power level within the band (f_1, f_2), denoted as $P_{IM_3}(f_1, f_2)$, can be determined easily by

$$\begin{aligned}P_{IM_3}(f_1, f_2) &= \int_{f_1}^{f_2} P_y(f)df = \frac{P_o}{R_s}\cdot\int_{f_1}^{f_2} P_1^2(f-f_c)df \\ &\quad - \frac{2P_o^2}{R_s^2}10^{-\frac{IP_3}{10}}\cdot\int_{f_1}^{f_2} P_1(f-f_c)\cdot P_3(f-f_c)df \\ &\quad + \frac{P_o^3}{R_s^3}10^{-\frac{IP_3}{5}}\cdot\int_{f_1}^{f_2} P_3^2(f-f_c)df.\end{aligned} \tag{21}$$

Equation 21 can be also expressed as

$$C_1\cdot 10^{-\frac{IP_3}{5}} + C_2\cdot 10^{-\frac{IP_3}{10}} + C_3 = 0 \tag{22}$$

where

$$\begin{aligned}C_1 &= \frac{P_o^3}{R_s^3}\cdot\int_{f_1}^{f_2} P_3^2(f-f_c)df \\ C_2 &= -\frac{2P_o^2}{R_s^2}\cdot\int_{f_1}^{f_2} P_1(f-f_c)\cdot P_3(f-f_c)df \\ C_3 &= \frac{P_o}{R_s}\cdot\int_{f_1}^{f_2} P_1^2(f-f_c)df - P_{IM_3}(f_1, f_2).\end{aligned} \tag{23}$$

In most design procedures, a designer is concerned with the required IP_3 for a given out-of-band emission level. To obtain the desired IP_3, Equation 22 is solved for IP_3 with given $P_{IM_3}(f_1, f_2)$, which yields

$$IP_3 = -10\cdot\log_{10}\left(\frac{-C_2+\sqrt{C_2^2-4C_1C_3}}{2C_1}\right) \tag{24}$$

where C_1, C_2 and C_3 are described in Equation 23.

This result provides a direct relationship between the out-of-band emission power of a TDMA signal power amplifier and its IP_3. With a given required IP_3, the power amplifier design for a TDMA signal becomes a conventional RF power amplifier design.

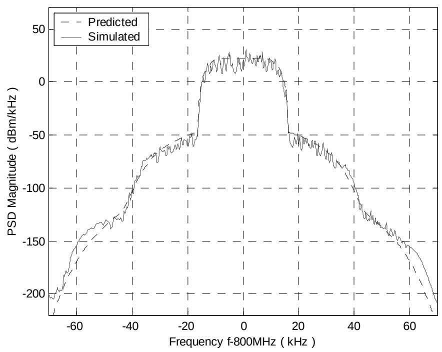

Figure 1. Power spectrum of amplified TDMA signal

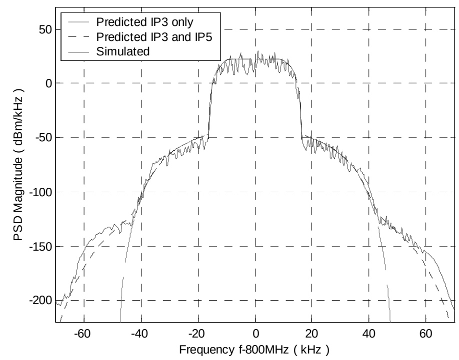

Figure 2. Power spectrum of amplified TDMA signal

4. DESIGN EXAMPLE AND COMPARISON WITH SIMULATIONS

In this example, the result shown in Equation 24 is used to design a $4\,Watt$ amplifier, which complies with the out-of-band emission level control requirement proposed for TDMA amplifiers. The out-of-band emission level controls required in IS-54 are given as followings: The total TDMA signal bandwidth is 30 kHz. In the band of $(f_c + 18\,kHz)$ to $(f_c + 47.5\,kHz)$, the suppression level between the output power and emission power at $0.72\,kHz$ bandwidth must be larger than 45 dB.

For this amplifier, $P_o = 4\,W$ and for the $(f_c + 18\,kHz)$ to $(f_c + 47.5\,kHz)$ band, the corresponding maximum $P_{IM_3}(f_1,\ f_2)$ is expressed as

$$P_{IM_3}(f_1,\ f_2) = 4 \times 10^{-\frac{46}{10}} = 0.1 \times 10^{-3}\,W \qquad (25)$$

For the worst case, f_1 and f_2 are assumed at the lower edge of $[f_c + 18\,kHz,\ f_c + 47.5\,kHz]$, that is $f_1 = f_c + 18\,kHz = f_c + 0.018\,MHz$ and $f_1 = f_c + 18\,kHz + 0.72kHz = f_c + 0.01872\,MHz$.

Then, from Equation 24, the required IP_3 becomes $IP_3 = 48.6\,dBm$. For the band described above, in order to meet the IS-54 requirement, the TDMA amplifier must have an IP_3 of at least 48.6 dBm.

As we mentioned before, IP_5 is not given in the data book. Fortunately, IP_5 could be measured through a *two-tone test* [7]. Therefore, without loss of generality, IP_5 can be assumed as 45 dBm at the same output power level. *Figure 1* shows the power spectrum predicted from this example compared to the spectrum given by simulation. The simulated RF amplifier spectrum agrees with the analytically predicted spectrum in both the passband and shoulder area.

The predicted result using only IP_3 vs. both IP_3 and IP_5 is shown in *Figure 2.* It can be seen clearly that a better fit exits when both IP_3 and IP_5 are used vs. IP_3 only.

5. CONCLUSION

It was assumed traditionally that the effects of the fifth- or higher order intermodulation could be ignored. However, if the fifth-order intermodulation is relatively high compared the third-order intermodulation, the out-of-band emission power levels caused by fifth-order intermodulation could be significant.

In this article, we propose a theoretical method to predict the output power spectrum of a TDMA standard RF power amplifier so that the traditional nonlinearity parameter IP_3 and additional parameter IP_5 are linked directly with out-of-band emission levels. This analysis makes it possible for RF power amplifier designers to use a conventional approach to design RF power amplifiers for TDMA signals. In addition to the results presented in this article, this derivation approach can be applied to out-of-band emission level analysis for other communication standards.

6. REFERENCES

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