A Quaternion-Based Adaptive Attitude Tracking Controller Without Velocity Measurements[1]

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Abstract: The main problem addressed in this paper is the quaternion-based, attitude tracking control of rigid spacecraft without angular velocity measurements and in the presence of an unknown inertia matrix. As a stepping-stone, we first design an adaptive, full-state feedback controller that compensates for parametric uncertainty while ensuring asymptotic attitude tracking errors. The adaptive, full-state feedback controller is then redesigned such that the need for angular velocity measurements is eliminated. The proposed adaptive, output feedback controller ensures asymptotic attitude tracking.

1 Introduction

The attitude control of rigid bodies has important applications ranging from rigid aircraft and spacecraft systems to coordinated robot manipulators (see [18] for a literature review of the many different types of applications). For example, rigid spacecraft applications in particular (*e.g.*, satellite surveillance and communication) often have need of highly accurate slewing and/or pointing maneuvers that require the spacecraft to rotate along a relatively large angle amplitude trajectory. As noted in [1], these requirements necessitate the use of a nonlinear dynamic spacecraft model for control system synthesis. The control problem is further complicated by the uncertainty of the spacecraft mass and inertia properties due to fuel consumption, payload variation, appendage deployment, *etc.*

The attitude motion of a rigid body is basically represented by a set of two equations [7, 8, 18]: (i) Euler's dynamic equation, which describes the time evolution of the angular velocity vector, and (ii) the kinematic equation, which relates the time derivatives of the orientation angles to the angular velocity vector. Several kinematic parametrizations exist to represent the orientation angles, including singular, three-parameter representations (*e.g.*, the Euler angles, Gibbs vector, Cayley-Rodrigues parameters, and modified Rodrigues parameters) and the nonsingular, four-parameter representation given by the unit quaternion (*i.e.*, the Euler parameters). Whereas the three-parameter representations always exhibit singular orientations (*i.e.*, the Jacobian matrix in the kinematic equation is singular for some orientations), the unit quaternion globally represents the spacecraft attitude without singularities; however, an additional constraint equation is introduced through the use of the four-parameter representation.

Several solutions to the attitude control problem have been presented in the literature since the early 1970's [12]. See [18] for a comprehensive literature review of earlier work. In [18], the authors presented a general attitude control design framework which includes PD, model-based, and adaptive set-point controllers. Adaptive tracking control schemes based on three-parameter, kinematic representations were presented in [14, 16] to compensate for the unknown, spacecraft inertia matrix. In [1], an adaptive attitude tracking controller based on the unit quaternion was proposed that identified the inertia matrix via periodic command signals. The work of [1] was later applied to the angular velocity tracking problem in [2]. An $\mathcal{H}_\infty$-suboptimal state feedback controller was developed for the quaternion representation in [5]. In [9], the authors designed an inverse optimal control law for attitude regulation using the backstepping method for a three-parameter representation. Recently, [3] presented a variable structure tracking controller using quaternions in the presence of spacecraft inertia uncertainties and external disturbances.

A typical feature in all the above-mentioned attitude control schemes is that angular velocity measurements are required. Unfortunately, this requirement is not always satisfied in reality. Thus, a common practice is to approximate the angular velocity signal through an ad-hoc numerical differentiation of the attitude angles, and directly use this surrogate signal for control design with no guarantee of closed-loop stability. With this in mind, an angular velocity observer was developed in [13] for the quaternion representation; however, the observer was based on an unproven separation principle argument. In [11], a passivity approach was used to develop an asymptotically stabilizing *setpoint* controller that eliminated velocity measurements via the filtering of the unit quaternion. The passivity-based, velocity-free setpoint controller of [11] was later applied to the simpler, three-parameter problem in [17]. Recently in Wong *et al.* [19], an adaptive attitude tracking controller without angular velocity measurements was proposed using the modified Rodrigues parameters.

In this paper, we provide an adaptive control solution to the quaternion-based, attitude *tracking* control problem that

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eliminates angular velocity measurements and compensates for parametric uncertainty. Specifically, we first apply a novel transformation to the open-loop, quaternion tracking error dynamics developed in [1]. The transformed tracking error dynamics are then used to design a new adaptive, full-state feedback controller that compensates for uncertainties in the inertia matrix. A non-standard, Lyapunov-like function, which exploits the quaternion constraint equation, is used to prove asymptotic attitude tracking. To achieve the goal of elimination of velocity measurements, we then exploit the structure of the adaptive, full-state feedback controller and its corresponding stability argument. Specifically, we utilize a filter, whose structure is motivated by the Lyapunov-like stability analysis, to generate a velocity-related signal from attitude measurements. The proposed output feedback controller is shown to guarantee asymptotic attitude tracking. To the best of our knowledge, this represents the first solution to the adaptive, output feedback, attitude tracking control problem *for the quaternion representation* (note that the adaptive output feedback result of Wong was done for the simpler 3-parameter case). Note that this is an intricate problem due to the non-square (*i.e.*, 4×3) nature of the original Jacobian matrix in the kinematic equation. As a result, a judicious error system development, control design, and Lyapunov-like stability analysis, which make appropriate use of the quaternion constraint equation, are crucial to the solution of the problem.

The paper is organized as follows. Section 2 contains the derivation of the spacecraft model. The adaptive, full-state feedback controller is presented in Section 3 while the output feedback controller is developed in Section 4. Section 5 presents some concluding remarks.

2 Model Formulation

2.1 Spacecraft Dynamics

We consider the problem of a rigid spacecraft with actuators that provide body-fixed torques about a body-fixed reference frame $\mathcal{B}$ located at some point on the spacecraft [1]. The body-fixed torques can be applied to each axis by a pair of equal but opposite forces that act in a direction perpendicular to the line joining the actuators. We then translate this body-fixed frame $\mathcal{B}$ to another body-fixed frame $\mathcal{F}$ with the same orientation, but located at the center of mass of the spacecraft. The dynamic model for the described rigid spacecraft can be expressed as follows [7, 8]

$$\begin{aligned} J\dot{\omega} &= -\omega^{\times} J\omega + u \quad (1)\\ \dot{q} &= \frac{1}{2}\left(q^{\times}\omega + q_0\omega\right) \quad (2)\\ \dot{q}_0 &= -\frac{1}{2}q^{\mathsf{T}}\omega \quad (3)\end{aligned}$$

where $J \in \Re^{3\times 3}$ represents the constant, positive-definite, symmetric inertia matrix, $\omega(t) \in \Re^3$ is the angular velocity of the body-fixed reference frame $\mathcal{F}$ with respect to an inertial reference frame $\mathcal{I}$, $u(t) \in \Re^3$ is a vector of control torques, and the notation $\zeta^{\times}$, $\forall \zeta = \begin{bmatrix} \zeta_1 & \zeta_2 & \zeta_3 \end{bmatrix}^{\mathsf{T}}$, denotes the following skew-symmetric matrix

$$\zeta^{\times} \triangleq \begin{bmatrix} 0 & -\zeta_3 & \zeta_2 \\ \zeta_3 & 0 & -\zeta_1 \\ -\zeta_2 & \zeta_1 & 0 \end{bmatrix}. \quad (4)$$

In (2) and (3), $\mathsf{q}(t) \triangleq \{q_0(t), q(t)\} \in \Re \times \Re^3$ represents the unit quaternion [7] describing the orientation of the body-fixed frame $\mathcal{F}$ (see Figure 1) with respect to the inertial frame $\mathcal{I}$, which are subject to the constraint

$$q^{\mathsf{T}} q + q_0^2 = 1. \quad (5)$$

The rotation matrix that brings $\mathcal{I}$ onto $\mathcal{F}$, denoted by

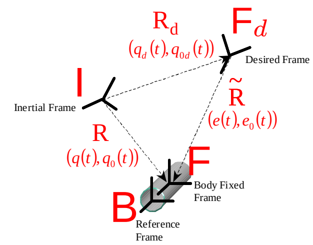

Figure 1: Relationship between coordinate frames.

$R(q, q_0) \in \Re^{3\times 3}$, is defined as follows

$$R \triangleq \left(q_0^2 - q^{\mathsf{T}} q\right) I_3 + 2qq^{\mathsf{T}} - 2q_0 q^{\times} \quad (6)$$

where I_3 denotes the 3×3 identity matrix, and the angular velocity of $\mathcal{F}$ with respect to $\mathcal{I}$ expressed in $\mathcal{F}$, denoted by $\omega(t)$, can be computed from (2) and (3) as follows

$$\omega = 2\left(q_0\dot{q} - q\dot{q}_0\right) - 2q^{\times}\dot{q}. \quad (7)$$

2.2 Open-Loop Tracking Error System Development

Similar to [1], we assume that the desired attitude of the spacecraft can be described by a desired, body-fixed reference frame $\mathcal{F}_{\mathsf{d}}$ whose orientation with respect to the inertial frame $\mathcal{I}$ is specified by the desired unit quaternion $\mathsf{q}_{\mathsf{d}}(t) \triangleq \{q_{0\mathsf{d}}(t), q_{\mathsf{d}}(t)\} \in \Re \times \Re^3$ that is constructed to satisfy

$$q_{\mathsf{d}}^{\mathsf{T}} q_{\mathsf{d}} + q_{0\mathsf{d}}^2 = 1. \quad (8)$$

The corresponding rotation matrix, denoted by $R_{\mathsf{d}}(q_{\mathsf{d}}, q_{0\mathsf{d}}) \in \Re^{3\times 3}$, that brings $\mathcal{I}$ onto $\mathcal{F}_{\mathsf{d}}$ is then defined as follows

$$R_{\mathsf{d}} \triangleq \left(q_{0\mathsf{d}}^2 - q_{\mathsf{d}}^{\mathsf{T}} q_{\mathsf{d}}\right) I_3 + 2q_{\mathsf{d}}q_{\mathsf{d}}^{\mathsf{T}} - 2q_{0\mathsf{d}}q_{\mathsf{d}}^{\times}. \quad (9)$$

The desired quaternion is related to the desired angular velocity of $\mathcal{F}_{\mathsf{d}}$ with respect to $\mathcal{I}$ expressed in $\mathcal{F}_{\mathsf{d}}$, denoted by $\omega_{\mathsf{d}}(t) \in \Re^3$, through the following dynamic equations

$$\begin{aligned} \dot{q}_{\mathsf{d}} &= \frac{1}{2}\left(q_{\mathsf{d}}^{\times}\omega_{\mathsf{d}} + q_{0\mathsf{d}}\omega_{\mathsf{d}}\right) \quad (10)\\ \dot{q}_{0\mathsf{d}} &= -\frac{1}{2}q_{\mathsf{d}}^{\mathsf{T}}\omega_{\mathsf{d}}. \quad (11)\end{aligned}$$

Note that (10) and (11) can be used to explicitly compute an expression for ω_{d} as shown below

$$\omega_{\mathsf{d}} = 2\left(q_{0\mathsf{d}}\dot{q}_{\mathsf{d}} - q_{\mathsf{d}}\dot{q}_{0\mathsf{d}}\right) - 2q_{\mathsf{d}}^{\times}\dot{q}_{\mathsf{d}}. \quad (12)$$

To quantify the mismatch between the actual and desired spacecraft attitudes, we define the rotation matrix $\tilde{R}(e, e_0) \in \Re^{3\times 3}$ that brings $\mathcal{F}_{\mathsf{d}}$ onto $\mathcal{F}$ as follows

$$\tilde{R} \triangleq RR_{\mathsf{d}}^{\mathsf{T}} = \left(e_0^2 - e^{\mathsf{T}} e\right) I_3 + 2ee^{\mathsf{T}} - 2e_0 e^{\times} \quad (13)$$

where $R(q, q_0)$ and $R_{\mathsf{d}}(q_{\mathsf{d}}, q_{0\mathsf{d}})$ were defined in (6) and (9), respectively, and the quaternion tracking error $\mathsf{e}(t) \triangleq \{e_0(t), e(t)\} \in \Re \times \Re^3$ is defined as shown below

$$\begin{aligned} e_0 &\triangleq q_0 q_{0\mathsf{d}} + q^{\mathsf{T}} q_{\mathsf{d}} \quad (14)\\ e &\triangleq q_{0\mathsf{d}} q - q_0 q_{\mathsf{d}} + q^{\times} q_{\mathsf{d}}. \quad (15)\end{aligned}$$

Note that, based on the definition given by (13), the attitude control objective can be stated as follows

$$\lim_{t\to\infty} \tilde{R}(e(t), e_0(t)) = I_3. \tag{16}$$

Based on the above tracking error formulation, we define the angular velocity of $\mathcal{F}$ with respect to $\mathcal{F}_d$ expressed in $\mathcal{F}$, denoted by $\tilde{\omega}(t) \in \Re^3$, as follows

$$\tilde{\omega} \;,\; \omega - \tilde{R}\omega_d. \tag{17}$$

We can now use (1), (2), (3), (10), (11), (14), (15), and (17) to compute the open-loop tracking error dynamics as follows

$$\begin{aligned} J\,\dot{\tilde{\omega}} = & -\left(\tilde{\omega} + \tilde{R}\omega_d\right)^{\times} J\left(\tilde{\omega} + \tilde{R}\omega_d\right) \\ & + J\left(\tilde{\omega}^{\times}\tilde{R}\omega_d - \tilde{R}\dot{\omega}_d\right) + u \end{aligned} \tag{18}$$

$$\dot{e} = \tfrac{1}{2}\left(e^{\times} + e_0 I_3\right)\tilde{\omega} \tag{19}$$

$$\dot{e}_0 = -\tfrac{1}{2}e^{\mathsf{T}}\tilde{\omega} \tag{20}$$

where we have used the fact that $\dot{\tilde{R}} = -\tilde{\omega}^{\times}\tilde{R}$.

Remark 1 *We will assume that $q_{0d}(t)$, $q_d(t)$, and their first three time derivatives are bounded for all time. Note that this assumption ensures that $\omega_d(t)$ of (12) and its first two time derivatives are bounded for all time.*

Remark 2 *The relations given in (14) and (15) can be explicitly calculated via quaternion algebra by noticing that the quaternion equivalent of (13) is the quaternion product (see [20] and Theorem 5.3 of [10])*

$$\mathsf{e} = \mathsf{q}_d^* \mathsf{q} \tag{21}$$

where the unit quaternions e and q were defined above, and $\mathsf{q}_d^*(t)$, $\{q_{0d}(t), -q_d(t)\} \in \Re \times \Re^3$ is the unit quaternion representing the rotation matrix R_d^{T}.

Remark 3 *After utilizing (5), (8), (14), and (15), it is not difficult to show that the quaternion tracking error variables satisfy the following constraint*

$$e^{\mathsf{T}}e + e_0^2 = 1. \tag{22}$$

Based on the constraint given by (22), we can see that

$$0 \le \|e(t)\| \le 1 \qquad 0 \le |e_0(t)| \le 1 \tag{23}$$

for all time, where $\|\cdot\|$ represents the standard Euclidean norm. It is also easy to see from (22) that

$$\text{if} \quad \lim_{t\to\infty} e(t) = 0, \quad \text{then} \quad \lim_{t\to\infty} |e_0(t)| = 1, \tag{24}$$

and hence, we can see from (13) that if $\lim_{t\to\infty} e(t) = 0$ then the control objective defined by (16) will be achieved.

2.3 Transformed Open-Loop Tracking Error System

In order to express the open-loop tracking error dynamics given in (18)-(20) in a more convenient manner, we first rewrite (19) as follows

$$\dot{e} = \frac{1}{2}T\tilde{\omega} \tag{25}$$

where the Jacobian-type matrix $T(e, e_0) \in \Re^{3\times3}$ is defined as follows

$$T \;,\; e^{\times} + e_0 I. \tag{26}$$

After taking the time derivative of (25) and premultiplying both sides of the resulting expression by $T^{-\mathsf{T}}JT^{-1}$, we obtain the following

$$J^*\ddot{e} = \frac{1}{2}J^*\dot{T}\tilde{\omega} + \frac{1}{2}P^{\mathsf{T}}J\,\dot{\tilde{\omega}} \tag{27}$$

where $J^*(e, e_0) \in \Re^{3\times3}$ is an auxiliary matrix defined as

$$J^* \;,\; P^{\mathsf{T}}JP \tag{28}$$

and $P(e, e_0) \in \Re^{3\times3}$ is defined as

$$P \;,\; T^{-1}. \tag{29}$$

After substituting (18) into the right-hand side of (27), we can obtain the following expression for the open-loop tracking error dynamics

$$J^*(e, e_0)\ddot{e} + C^*(e, e_0, \dot{e})\,\dot{e} + N^*(e, e_0, \dot{e}, \omega_d, \dot{\omega}_d) = u^* \tag{30}$$

where the new control input $u^*(t) \in \Re^3$ is defined as

$$u^* \;,\; \frac{1}{2}P^{\mathsf{T}}u, \tag{31}$$

and the auxiliary dynamic terms $C^*(e, e_0, \dot{e}) \in \Re^{3\times3}$, $N^*(e, e_0, \dot{e}, \omega_d, \dot{\omega}_d) \in \Re^3$ are defined as follows

$$C^* \;,\; -J^*\dot{P}^{-1}P - 2P^{\mathsf{T}}\left(JP\dot{e}\right)^{\times}P \tag{32}$$

$$\begin{aligned} N^* \;,\; & P^{\mathsf{T}}\left((P\dot{e})^{\times}J\tilde{R}\omega_d\right) + P^{\mathsf{T}}\left(\left(\tilde{R}\omega_d\right)^{\times}JP\dot{e}\right) \\ & + \frac{1}{2}P^{\mathsf{T}}\left(\left(\tilde{R}\omega_d\right)^{\times}J\tilde{R}\omega_d\right) \\ & - \frac{1}{2}P^{\mathsf{T}}J\left((2P\dot{e})^{\times}\tilde{R}\omega_d - \tilde{R}\dot{\omega}_d\right). \end{aligned} \tag{33}$$

The dynamic model given in (30) is characterized by the following two properties that will be utilized in the subsequent control development and analysis.

Property 1: The inertia and Centripetal-Coriolis matrices satisfy the following skew-symmetric relationship

$$\xi^{\mathsf{T}}\left(\frac{1}{2}\dot{J}^* - C^*\right)\xi = 0 \quad \forall\xi \in \Re^3. \tag{34}$$

Property 2: The inertia matrix can be lower and upper bounded as follows

$$j_1\|\xi\|^2 \le \xi^{\mathsf{T}}J\xi \le j_2\|\xi\|^2 \quad \forall\xi \in \Re^n \tag{35}$$

where $j_1, j_2 \in \Re$ are some positive constants.

Remark 4 *In order to ensure that $T(e, e_0)$ defined in (26) is invertible, it is a straightforward matter to show that we must guarantee that*

$$\det(T) = e_0(t) \neq 0, \quad \forall t \in [0, \infty). \tag{36}$$

To ensure that (36) remains valid, we will require that the initial conditions be restricted such that $e_0(0) \neq 0$, and that the subsequent control strategies be designed to guarantee that $e_0(t) \neq 0$ for all time. With regard to the restriction on the initial conditions, it is easy to see from (24) and (14) that the desired trajectory can always be initialized to guarantee that $e_0(0) \neq 0$; hence, the initial conditions restriction is actually a very mild restriction on the desired trajectory signals.

3 Adaptive Full-State Feedback Control Development

In this section, our control objective is to design an adaptive attitude controller for the open-loop tracking error dynamics given by (30) under the constraint that the spacecraft inertia matrix, J, is unknown. In order to quantify the parametric mismatch, we define the parameter estimation error, $\tilde{\theta}(t) \in \Re^6$, as follows

$$\tilde{\theta}(t) \;,\; \theta - \hat{\theta}(t) \tag{37}$$

where $\theta \in \Re^6$ is a constant, unknown vector of inertia parameters defined as follows

$$\theta \triangleq \left[\begin{array}{cccccc} J_{11} & J_{12} & J_{13} & J_{22} & J_{23} & J_{33} \end{array}\right]^T \quad (38)$$

with J_{ij} being the elements of J, and $\hat{\theta}(t) \in \Re^6$ being a dynamic estimate for θ which is yet to be defined. To facilitate the controller design, we also define the filtered tracking error, denoted by $r(t) \in \Re^3$, as follows

$$r \triangleq \dot{e} + \alpha e \quad (39)$$

where $e(t)$ and $\dot{e}(t)$ were defined in (15) and (19), respectively, and $\alpha \in \Re^{3\times 3}$ is a constant, positive-definite, diagonal, control gain matrix.

3.1 Control Torque Input Design

Based on the open-loop tracking error system given by (30) and the subsequent stability analysis, we design the control input $u^*(t)$ as follows

$$u^* = -Y\left(e, e_0, \dot{e}, \omega_d, \dot{\omega}_d\right)\hat{\theta} - Kr - \frac{e}{\left(1 - e^T e\right)^2} \quad (40)$$

where $Y\left(e, e_0, \dot{e}, \omega_d, \dot{\omega}_d\right) \in \Re^{3\times 6}$ is a known regression matrix constructed according to the following parametrization

$$Y(\cdot)\,\theta = J^*\alpha\dot{e} + C^*\alpha e - N^*, \quad (41)$$

θ was defined in (38), $K \in \Re^{3\times 3}$ is a constant, positive-definite, diagonal, control gain matrix, $\hat{\theta}(t)$ is generated via the following dynamic update law

$$\dot{\hat{\theta}} = \Gamma Y^T(\cdot)\, r, \quad (42)$$

and $\Gamma \in \Re^{6\times 6}$ is a constant, positive-definite, diagonal, adaptation gain matrix. To develop the closed-loop tracking error system, we take the time derivative of (39) and then premultiply both sides of the resulting equation by J^* to obtain the following expression

$$J^*\dot{r} = J^*\ddot{e} + J^*\alpha\dot{e}. \quad (43)$$

After substituting (30) into (43), we obtain

$$J^*\dot{r} = -C^*r + Y\theta + u^* \quad (44)$$

where (39) and (41) were utilized. After substituting (40) for $u^*(t)$, we obtain the final expression for the closed-loop tracking error system

$$J^*\dot{r} = -C^*r + Y\tilde{\theta} - Kr - \frac{e}{\left(1 - e^T e\right)^2} \quad (45)$$

where $\tilde{\theta}(t)$ was defined in (37).

3.2 Stability Analysis

Theorem 1 *Given the closed-loop dynamics given in (39) and (45), the adaptive controller of (40) and (42) ensures asymptotic attitude tracking in the sense that*

$$\lim_{t\to\infty} e(t) = 0 \quad \textit{and} \quad \lim_{t\to\infty} \tilde{\omega}(t) = 0, \quad (46)$$

provided that the initial conditions are selected such that

$$\|e_0(0)\| \neq 0 \quad (47)$$

Proof: In order to prove Theorem 1, we define the non-negative function

$$V(t) \triangleq \frac{1}{2}\left(\frac{e^T e}{1 - e^T e}\right) + \frac{1}{2}y^T J y + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \quad (48)$$

where $y(t) \in \Re^3$ is defined as

$$y \triangleq Pr \quad (49)$$

and $P(e, e_0)$ was defined in (29). After taking the time derivative of (48), and then making the appropriate substitutions from (28), (42), (45), and (49), we obtain

$$\begin{aligned}\dot{V}(t) = & \frac{e^T\dot{e} - e^T e e^T\dot{e} + e^T e e^T\dot{e}}{\left(1 - e^T e\right)^2} - r^T C^* r + r^T Y\tilde{\theta} \\ & - r^T K r - \frac{r^T e}{\left(1 - e^T e\right)^2} + \frac{1}{2}r^T\dot{J}^* r - \tilde{\theta}^T Y^T(\cdot)\, r.\end{aligned} \quad (50)$$

After substituting (39) into (50) for $\dot{e}(t)$ and then canceling common terms, we have that

$$\dot{V}(t) = \frac{-e^T\alpha e}{\left(1 - e^T e\right)^2} - r^T K r \quad (51)$$

where Property 1 has been utilized.

Given (22), (23), and (47), it is clear that $\|e(0)\| < 1$. From (48) and (51), it is straightforward to see that

$$0 \le V(t) \le V(0) < \infty; \quad (52)$$

hence, we can see from (48) that $y(t), \hat{\theta}(t) \in \mathcal{L}_\infty$ and $\|e(t)\| < 1$ for all time. Since $\|e(t)\| < 1$ for all time, we can conclude from (22) that $\|e_0(t)\| \neq 0$ for all time; hence, we know from (36) that P defined in (29) has full rank for all time. Since $y(t) \in \mathcal{L}_\infty$ and P has full rank for all time, we can use (49) to show that $r(t) \in \mathcal{L}_\infty$. Since $r(t) \in \mathcal{L}_\infty$, we can use (39) to show that $e(t), \dot{e}(t) \in \mathcal{L}_\infty$ [6]. Standard signal chasing arguments can now be employed to show that all other signals remain bounded. From (51) and (52), we know that $r(t) \in \mathcal{L}_2$ while from (45) it is easy to show that $\dot{r}(t) \in \mathcal{L}_\infty$. We can now utilize Barbalat's Lemma [16, 6] to prove that

$$\lim_{t\to\infty} r(t) = 0. \quad (53)$$

Given the result of (53), we can use (39) to obtain the first result of (46), and also show that $\lim_{t\to\infty} \dot{e}(t) = 0$ [6]. Hence, from (25) and the fact that P has full rank, we can prove the second result of (46) . ¤

4 Adaptive Output Feedback Controller

In this section, we redesign the adaptive controller under the constraint that velocity measurements are not available. In order to facilitate the subsequent stability analysis, we define an auxiliary error signal, denoted by $\eta(t) \in \Re^3$, as follows

$$\eta \triangleq \dot{e} + e + e_f \quad (54)$$

where $e_f(t) \in \Re^3$ is a filter signal which is yet to be designed.

4.1 Control Torque Input Design

Motivated by the desire to design a control torque input that is independent of velocity measurements, we construct the filter signal, $e_f(t)$, as follows

$$e_f = -ke + p \quad (55)$$

where $k \in \Re$ is a positive, constant control gain, and $p(t) \in \Re^3$ is generated via the following dynamic expression

$$\dot{p} = -(k+1)\,p + k^2 e + \frac{e}{\left(1 - e^T e\right)^2}, \quad p(0) = ke(0). \quad (56)$$

Based on the subsequent stability analysis and structure of (30), we design the control input as follows

$$u^* = -W_{\mathsf{d}}\hat{\theta} + ke_{\mathsf{f}} - \frac{e}{\left(1 - e^{\mathsf{T}} e\right)^2} \qquad (57)$$

where $W_{\mathsf{d}}(\omega_{\mathsf{d}}, \dot{\omega}_{\mathsf{d}}) \in \Re^{3\times 6}$ is a known regression matrix constructed according to the following parametrization

$$W_{\mathsf{d}}\theta = -J\dot{\omega}_{\mathsf{d}} - \frac{1}{2}\omega_{\mathsf{d}}^{\times} J\omega_{\mathsf{d}}, \qquad (58)$$

$\hat{\theta}(t)$ is the dynamic update law now designed as

$$\dot{\hat{\theta}} = \Gamma W_{\mathsf{d}}^{\mathsf{T}}\eta, \qquad (59)$$

and k is the same control gain defined in (55), which is selected as follows

$$k = \frac{1}{j_1}\left(k_{\mathsf{N}} + 1\right) \qquad (60)$$

where j_1 was defined in (35), and $k_{\mathsf{N}} \in \Re$ is an additional, positive, constant control gain.

Remark 5 *In order to illustrate that $\hat{\theta}(t)$ can be calculated using only measurable signals, we first rewrite (59) as the following integral expression*

$$\begin{aligned}\hat{\theta}\left(t\right) &= \hat{\theta}(0)\\ &+\Gamma\int_0^t W_{\mathsf{d}}^{\mathsf{T}}\left(\omega_{\mathsf{d}}\left(\sigma\right), \dot{\omega}_{\mathsf{d}}\left(\sigma\right)\right)\left(\dot{e}\left(\sigma\right) + e\left(\sigma\right) + e_{\mathsf{f}}\left(\sigma\right)\right)d\sigma \end{aligned} \qquad (61)$$

After performing integration by parts, (61) can be written in the following velocity-independent form

$$\begin{aligned}\hat{\theta}\left(t\right) &= \Gamma W_{\mathsf{d}}^{\mathsf{T}} e + \hat{\theta}(0)\\ &+\Gamma\int_0^t \left[W_{\mathsf{d}}^{\mathsf{T}}\left(\cdot\right)\left(e\left(\sigma\right) + e_{\mathsf{f}}\left(\sigma\right)\right) - \dot{W}_{\mathsf{d}}^{\mathsf{T}}\left(\cdot\right)e\left(\sigma\right)\right]d\sigma. \end{aligned} \qquad (62)$$

To determine the dynamics for $e_{\mathsf{f}}(t)$, we take the time derivative of (55) and then substitute (56) into the resulting expression to obtain

$$\dot{e}_{\mathsf{f}} = -k\dot{e} - (k+1)\,p + k^2 e + \frac{e}{\left(1 - e^{\mathsf{T}} e\right)^2}. \qquad (63)$$

After rearranging (55), we can substitute for $p(t)$ in (63) and then simplify the resulting expression to obtain the following

$$\dot{e}_{\mathsf{f}} = -k\eta - e_{\mathsf{f}} + \frac{e}{\left(1 - e^{\mathsf{T}} e\right)^2} \qquad (64)$$

where (54) was utilized. To develop the open-loop expression for $\eta(t)$, we take the time derivative of (54), premultiply the resulting expression by J^*, and then substitute (30) for $J^*\ddot{e}$ to obtain

$$J^*\dot{\eta} = u^* - C^*\dot{e} - N^* + J^*\dot{e} + J^*\dot{e}_{\mathsf{f}}. \qquad (65)$$

After utilizing (54) and (64), we can rewrite (65) as follows

$$\begin{aligned}J^*\dot{\eta} = \;& u^* + (1-k)\,J^*\eta - 2J^*e_{\mathsf{f}} - J^*e - C^*\eta\\ &+J^*\frac{e}{\left(1-e^{\mathsf{T}}e\right)^2} + C^*e_{\mathsf{f}} + C^*e - N^*. \end{aligned} \qquad (66)$$

After adding and subtracting $W_{\mathsf{d}}\theta$ to the right-side of (66) and substituting (57) for $u^*(t)$, we can obtain the following expression for the closed-loop error system for $\eta(t)$

$$J^*\dot{\eta} = \chi + W_{\mathsf{d}}\tilde{\theta} + ke_{\mathsf{f}} - \frac{e}{\left(1 - e^{\mathsf{T}} e\right)^2} - kJ^*\eta - C^*\eta \qquad (67)$$

where $\tilde{\theta}(t)$ was defined in (37), and the auxiliary signal $\chi(e, e_0, e_f, \eta, \omega_{\mathsf{d}}, \dot{\omega}_{\mathsf{d}}) \in \Re^3$ is defined as follows

$$\begin{aligned}\chi \;,\;\; & C^*\left(e_{\mathsf{f}} + e\right) + J^*\left(\eta - e + \frac{e}{\left(1-e^{\mathsf{T}}e\right)^2}\right)\\ & -2J^*e_{\mathsf{f}} - W_{\mathsf{d}}\theta - N^*. \end{aligned} \qquad (68)$$

Remark 6 *To facilitate the subsequent stability analysis, we utilize (68) and (29) to construct the following auxiliary variable*

$$\overline{\chi} \;,\; P^{-\mathsf{T}}\chi. \qquad (69)$$

Based on the assumptions on the boundedness of the desired trajectory and the structure of (69), we can show that $\overline{\chi}$ can be upper bounded as follows

$$\overline{\chi} \leq \rho\left(\|z\|\right)\|z\| \qquad (70)$$

where $\rho(\cdot)$ is a positive, nondecreasing function, and the auxiliary signals $z(t) \in \Re^9$ and $y(t) \in \Re^3$ are now defined as follows

$$z \;,\; \left[\begin{array}{ccc}\dfrac{e^{\mathsf{T}}}{\sqrt{1-e^{\mathsf{T}}e}} & e_{\mathsf{f}}^{\mathsf{T}} & y^{\mathsf{T}}\end{array}\right]^{\mathsf{T}} \qquad (71)$$

$$y \;,\; P\eta. \qquad (72)$$

4.2 Stability Analysis

Theorem 2 *Given closed-loop error systems of (54), (59), (64), and (67), the adaptive controller of (55), (56), (57), and (62) ensures asymptotic attitude tracking in the sense that*

$$\lim_{t\to\infty} e\left(t\right) = 0 \qquad \textit{and} \qquad \lim_{t\to\infty}\tilde{\omega}(t) = 0, \qquad (73)$$

provided that the initial conditions are selected such that (47) is satisfied, and the control gain k_{N} introduced in (60) is selected according to the following inequality

$$k_{\mathsf{N}} > \rho^2\left(\sqrt{\frac{\lambda_2}{\lambda_1}}\left\|x\left(0\right)\right\|\right) \qquad (74)$$

where $\rho(\cdot)$ was defined in (70), $x(t) \in \Re^{12}$ is defined as follows

$$x \;,\; \left[\begin{array}{cc} z^{\mathsf{T}} & \tilde{\theta}^{\mathsf{T}}\end{array}\right]^{\mathsf{T}}, \qquad (75)$$

λ_1, λ_2 are positive constants defined as

$$\begin{aligned}\lambda_1 \;&,\; \tfrac{1}{2}\min\left\{1, j_1, \lambda_{\min}\left\{\Gamma^{-1}\right\}\right\}\\ \lambda_2 \;&,\; \tfrac{1}{2}\max\left\{1, j_2, \lambda_{\max}\left\{\Gamma^{-1}\right\}\right\},\end{aligned} \qquad (76)$$

and $\lambda_{\min}\{\cdot\}$ and $\lambda_{\max}\{\cdot\}$ represent the minimum and maximum eigenvalue of a matrix, respectively.

Proof: To prove the above theorem, we define the nonnegative function

$$V \;,\; \frac{1}{2}\left(\frac{e^{\mathsf{T}}e}{1-e^{\mathsf{T}}e}\right) + \frac{1}{2}e_{\mathsf{f}}^{\mathsf{T}}e_{\mathsf{f}} + \frac{1}{2}y^{\mathsf{T}}Jy + \frac{1}{2}\tilde{\theta}^{\mathsf{T}}\Gamma^{-1}\tilde{\theta}. \qquad (77)$$

Based upon the structure of (77) and Property 2, we can lower and upper bound $V(t)$ as follows

$$\lambda_1\|x\|^2 \leq V \leq \lambda_2\|x\|^2 \qquad (78)$$

where $x(t)$ was defined in (75). After taking the time derivative of (77), substituting (72) for $y(t)$, using the definition of J^* from (28), substituting (67), and then simplifying the resulting expression, we obtain

$$\begin{aligned}\dot{V} = \;& \frac{e^{\mathsf{T}}\dot{e}}{\left(1-e^{\mathsf{T}}e\right)^2} + e_{\mathsf{f}}^{\mathsf{T}}\dot{e}_{\mathsf{f}} + \frac{1}{2}\eta^{\mathsf{T}}\dot{J}^*\eta + \tilde{\theta}^{\mathsf{T}}\Gamma^{-1}\dot{\tilde{\theta}}\\ &+\eta^{\mathsf{T}}\left(\chi + W_{\mathsf{d}}\tilde{\theta} + ke_{\mathsf{f}} - \frac{e}{\left(1-e^{\mathsf{T}}e\right)^2} - kJ^*\eta - C^*\eta\right). \end{aligned} \qquad (79)$$

After utilizing (54), (64), (69), (72), and Property 1, we can rewrite (79) as follows

$$\begin{aligned}\dot{V} = \;& -\frac{e^{\mathsf{T}}e}{\left(1-e^{\mathsf{T}}e\right)^2} - e_{\mathsf{f}}^{\mathsf{T}}e_{\mathsf{f}} + y^{\mathsf{T}}\overline{\chi}\\ & -\eta^{\mathsf{T}}kJ^*\eta + \tilde{\theta}^{\mathsf{T}}\left(W_{\mathsf{d}}^{\mathsf{T}}\eta + \Gamma^{-1}\dot{\tilde{\theta}}\right). \end{aligned} \qquad (80)$$

After utilizing (28), (59), and (72), the expression in (80) can be upper bounded as follows

$$\dot{V} \leq -\frac{e^{\mathsf{T}}e}{\left(1-e^{\mathsf{T}}e\right)^2} - e_{\mathsf{f}}^{\mathsf{T}}e_{\mathsf{f}} - kj_1\|y\|^2 + \|y\|\,\|\overline{\chi}\|. \qquad (81)$$

After substituting (60), (70), and (71) into (81), we obtain the following expression

$$\dot{V} \leq -\|z\|^2 + \left[\|y\|\,\|z\|\,\rho\left(\|z\|\right) - k_{\mathsf{N}}\|y\|^2\right]. \qquad (82)$$

After applying the nonlinear damping tool [6] to the bracketed term in (82), we obtain

$$\dot{V} \leq -\left(1 - \frac{\rho^2\left(\|z\|\right)}{k_{\mathsf{N}}}\right)\|z\|^2. \qquad (83)$$

Note that from (83) we can write

$$\dot{V} \leq -\beta\|z\|^2 \quad \text{for} \quad k_{\mathsf{N}} > \rho^2\left(\|x\|\right) \qquad (84)$$

where β is some positive constant, and we have used the fact that $\|x\| \geq \|z\|$, as indicated by (75). Upon utilization of (78), we can develop a sufficient condition for (84) as follows

$$\dot{V} \leq -\beta \|z\|^2 \quad \text{for} \quad k_N > \rho^2 \left(\sqrt{\frac{V(t)}{\lambda_1}} \right). \tag{85}$$

From (78) and (85), we can see that $V(t)$ is nonnegative and $\dot{V}(t) \leq 0$; hence, we can conclude that

$$0 \leq V(t) \leq V(0) < \infty. \tag{86}$$

We now use (78) and (86) to develop a sufficient condition for (85) as follows

$$\dot{V} \leq -\beta \|z\|^2 \quad \text{for} \quad k_N > \rho^2 \left(\sqrt{\frac{\lambda_2}{\lambda_1}} \|x(0)\| \right). \tag{87}$$

Given (22), (23), and (47), it is clear that $\|e(0)\| < 1$. From (86) and (51), it is straightforward to see from (77) that $y(t)$, $\hat{\theta}(t)$, $e_f(t) \in \mathcal{L}_\infty$ and $\|e(t)\| < 1$ for all time. Since $\|e(t)\| < 1$ for all time, we can conclude from (22) that $\|e_0(t)\| \neq 0$ for all time; hence, we know from (36) that P defined in (29) has full rank for all time. Since $y(t) \in \mathcal{L}_\infty$ and P has full rank for all time, we can use (72) to show that $\eta(t) \in \mathcal{L}_\infty$. We can now utilize the above information and (54) to show that $\dot{e}(t) \in \mathcal{L}_\infty$. Standard signal chasing arguments can now be employed to show that all other signals remain bounded.

From (84) and (86), we know that $z(t) \in \mathcal{L}_2$. Given that all signals are bounded and the fact that $\|e(t)\| < 1$ for all time, we can use (71), (54), (64), and (67) to show that $\dot{z}(t) \in \mathcal{L}_\infty$. We can now utilize Barbalat's Lemma to prove that

$$\lim_{t \to \infty} z(t) = 0 \quad \text{for} \quad k_N > \rho^2 \left(\sqrt{\frac{\lambda_2}{\lambda_1}} \|x(0)\| \right). \tag{88}$$

Given the result of (88), we can use (25), (54), (71), (72), and the fact that P has full rank to obtain (73). ¤

5 Conclusion

In this paper, we have presented two adaptive controllers which address the attitude tracking problem for rigid spacecraft based on the unit quaternion, kinematic representation. The first controller is a full-state feedback controller that adapts for the unknown spacecraft inertia matrix and achieves asymptotic attitude tracking. The adaptive controller is then redesigned to eliminate the need for angular velocity measurements and still obtains asymptotic attitude tracking. Since the proposed controller is based on Lyapunov stability analysis, several extensions to the proposed work are straightforward. For example, one could easily use the full-state feedback controller structure to develop variable structure or high-gain/high-frequency robust controllers that compensate for parametric uncertainty and additive bounded disturbances, while producing exponential tracking and uniform ultimate boundedness tracking, respectively. In addition, one could easily use the output feedback controller structure to develop model-based or high-gain robust controllers that compensate for parametric uncertainty and additive bounded disturbances while producing exponential tracking and uniform ultimate boundedness tracking, respectively. Future plans for this research will include experimental verification on a gyroscopic testbed.

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