

Optimal Strategy Design for Enabling the Coexistence of Heterogeneous Networks in TV White Space

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Abstract—Very recently, regulatory bodies worldwide have started to approve the dynamic access of unlicensed networks to the TV White Space spectrum. Hence, in the near future, multiple heterogeneous and independently-operated unlicensed networks will coexist within the same geographical area over shared TV White Space. Although heterogeneity and coexistence are not unique to TV White Space scenarios, their distinctive characteristics pose new and challenging issues. In this paper, the problem of the coexistence interference among multiple heterogeneous and independently-operated secondary networks in absence of secondary cooperation is addressed. Specifically, the optimal coexistence strategy, which adaptively and autonomously selects the channel maximizing the expected throughput in presence of coexistence interference, is designed. More in detail, at first, an analytical framework is developed to model the channel selection process for an arbitrary secondary network as a decision process. Then, the problem of the optimal channel selection, i.e., the channel maximizing the expected throughput, is proved to be computational prohibitive (NP-hard). Finally, under the reasonable assumption of identically distributed interference on the available channels, the optimal channel selection problem is proved not to be NP-hard, and a computational-efficient (polynomial-time) algorithm for finding the optimal strategy is designed. Numerical simulations validate the theoretical analysis.

Index Terms—Spectrum Sharing, TV White Space, Cognitive Radio, Coexistence, Interference, Optimality, Throughput.

I. INTRODUCTION

Very recently, regulatory bodies worldwide have approved the dynamic access of Secondary Networks¹ (SNs) to the TV White Space (TVWS) spectrum. The existing rulings [2], [3], [4] obviate the spectrum sensing as the mechanism for the SNs to determine the TVWS availability at their respective locations. Instead, they require the SNs to periodically access to a geolocated database [5] for acquiring the list of TVWS channels free from incumbents.

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¹In the following, we refer to the unlicensed networks aiming at opportunistically exploiting the TV White Space spectrum when it is not used by the licensed users as *secondary networks*.

Clearly, the introduction of a database for incumbent protection significantly simplifies the secondary access to the TVWS spectrum, and the research community is actively working on defining several new standards aiming at enabling TVWS communications, such as IEEE 802.22 [6], IEEE 802.11af [7], IEEE 802.15m [8], ECMA 392 [9], and IEEE SCC41 [10]. Hence, in the near future, multiple heterogeneous and independently-operated SNs will coexist within the same geographical area as shown in Fig. 1.

Although heterogeneity and coexistence are not unique to TVWS scenarios, the distinctive characteristics of the TVWS scenarios pose new and challenging issues [11], [12]. At first, the excellent propagation characteristics of the TVWS spectrum cause severe interference among the coexisting SNs sharing the same spectrum band. In addition, the heterogeneity among the TVWS standards can prevent the adoption of interference-avoidance schemes based on cooperation. Finally, experimental studies have shown that the TVWS spectrum is significantly scarce in densely populated areas [13], [14]. As a consequence, it is likely to expect several SNs sharing the same spectrum band.

Hence, the research community is focusing on designing solutions for the secondary coexistence in TVWS [15], [16], as we discuss in detail in Sec. II-C.

In this paper, we investigate the problem of the interference avoidance among multiple heterogeneous and independently-operated SNs coexisting within the same geographical area in absence of secondary cooperation.

Specifically, we design the *optimal coexistence strategy*, which *adaptively* and *autonomously* selects the TVWS channel allowing the SN to maximize the expected throughput in presence of coexistence interference.

More in detail, at first, we develop an analytical framework to model the TVWS channel selection process for an arbitrary SN as a decision process, where the reward models the data rate achievable on a channel and the cost models the communication overhead for assessing the coexistence interference. Then, we prove that the problem of the optimal channel selection, i.e., the channel maximizing the expected throughput, is computational prohibitive (NP-hard). Finally, we prove that, under the reasonable assumption of identically distributed interference on the available TVWS channels, the problem of the optimal channel selection is not anymore NP-hard. Specifically, we prove that the optimal strategy exhibits a threshold behavior, and we exploit this threshold structure to design a computational-efficient (polynomial-time) algorithm

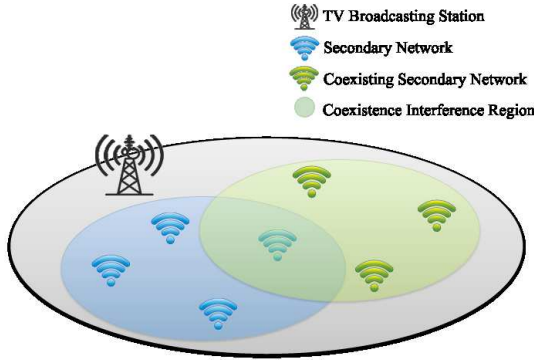


Fig. 1. Example of TVWS Scenario: two secondary networks coexist with a TV broadcast station within the same geographical area.

for finding the optimal strategy.

The rest of the paper is organized as follows. In Sec. II, we present the problem statement and we highlight the contributions of the paper. In Sec. III, we describe the network model along with some preliminaries. In Sec. IV, we design the optimal strategy. In Sec. V, we validate the theoretical framework through a case study. In Sec. VI, we conclude the paper, and, finally, some proofs are gathered in the appendix.

II. PROBLEM STATEMENT

Let us consider the typical TVWS scenario shown in Fig 1, in which a SN coexists with multiple heterogenous and independently-operated SNs within the same geographical area by sharing a certain number of TVWS channels declared available from the TVWS database.

As shown in Fig. 2, there exist three different classes of coexistence interference: i) the *self-coexistence interference*, caused by SNs operating according to the same standard and experienced mainly in dense scenarios; ii) the *heterogenous coexistence interference*, caused by SNs operating according to dissimilar standards or technologies; iii) the *vertical coexistence interference*, caused by the incumbents.

While the self-coexistence interference is handled within the TVWS standards [6] and the vertical coexistence interference is handled by a centralized database as mentioned in Sec. I, the mitigation of heterogenous coexistence interference represents an open problem as pointed out in Sec. II-A.

A. Challenges

The design of a strategy for the coexistence of heterogenous and independently-operated SNs arises several challenges.

- *Dynamic Interference.*

In TVWS scenarios, the heterogenous coexistence interference is both time and spatial-variant. In fact, as recently proved in [16], such dynamics depend on several factors, such as the number of SNs roaming within a certain geographic area, the number of Secondary Users (SUs) belonging to each SN, the interference range and the traffic/mobility patterns of each SN, as well as the SN

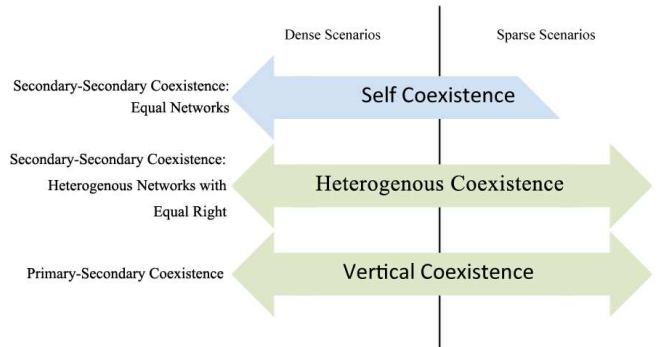


Fig. 2. Coexistence Taxonomy in TVWS Scenarios.

interference ranges and the changes in wireless propagation conditions. Hence, any coexistence strategy should be adaptive to such dynamics.

- *Heterogeneity.*

As mentioned in Section I, several TVWS standards have been proposed in the last years. Although significant work is currently ongoing [17], a complete interoperability based on over-the-air communications among heterogeneous TVWS standards is still missing [11]. Hence, an appealing characteristic of any heterogenous coexistence strategy is to be *autonomous*, i.e., to be independent of any form of coordination with the coexisting SNs.

- *Harmless-to-Incumbents Interference.*

In classical Cognitive Radio scenarios, any SN is required to adopt the *sense-before-talk* strategy as mechanism to protect the incumbents from harmful interference. In TVWS scenarios, such a requirement does not hold necessarily. Specifically, since the vertical coexistence interference is managed through a database-based mechanism, any interference level on a channel granted by the database is harmless against the incumbents. As a consequence, a SN must handle only the *heterogenous coexistence interference* caused by the peer coexisting SNs. Hence, any strategy aiming at mitigating the coexistence interference is *discretionary*, i.e., it should be performed only when it is convenient. As an example, let us consider Figure 3, in which only one TVWS channel is available for secondary communications. As we will prove in Sec. IV-B with Corollary 1, a SN aiming at maximizing the expected throughput should use the channel independently of any interference level. Hence, any coexistence strategy should allow *discretionary* interference-avoidance schemes.

B. Optimal Autonomous Coexistent Strategy Design

By taking into account the aforementioned challenges, in this paper we design a coexistence strategy for TVWS scenarios exhibiting the following attractive features.

- 1) The strategy is *optimal*, i.e., it allows the SN to maximize the expected throughput achievable by the SUs.

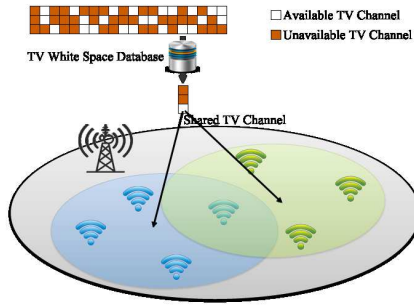


Fig. 3. Harmless-to-Incumbents Interference: the SNs do not need a *sense-before-talk* strategy as mechanism to protect the incumbents from harmful interference.

- 2) The strategy is *feasible*, since it requires a reasonable amount of a-priori knowledge, i.e., the first-order distribution of the interference levels.
- 3) The strategy is iterative and *adaptive* to interference dynamics.
- 4) The strategy is *autonomous*, since it allows the SN to make its decisions independently of any other coexisting network. Hence, it is low in complexity, and it can be easily integrated with centralized or distributed mechanisms to build hybrid strategies.
- 5) The strategy implements a *discretionary* interference avoidance scheme, i.e., it accounts for the harmless-to-incumbents property of interference in TVWS.

More in detail, we model the problem of choosing the TVWS channel maximizing the expected throughput as a decision process, where the reward models the data rate provided by a channel and the cost models the communication overhead (i.e., sensing times) for assessing the interference caused by coexisting secondary networks.

Then, we prove that the problem of the optimal channel selection is NP-hard by reducing it to the widely-known NP-hard Traveling Salesman Problem (TSP) [18]. This is an important result since it follows that: i) the optimal strategy can be found only through exhaustive search, i.e., there is no *smart* (computational efficient) way of searching the optimal solution; ii) the wide literature on exact and approximate algorithms for the TSP can be efficiently adopted for searching the optimal solution.

Furthermore, we prove that, under the reasonable hypothesis of identically distributed interference, the problem of the optimal channel selection is not anymore NP-hard. Specifically, we prove that the optimal strategy exhibits a threshold behavior. This result is valuable, since it allows us to design a computational-efficient algorithm for searching the optimal strategy by exploiting the threshold structure.

C. Related Work

In the last ten years, the primary-secondary coexistence problem has been extensively studied in classical Cognitive Radio networks, and several solutions for mitigating the vertical coexistence interference have been proposed [19], [20], [21], [22], [23], [24], [25], [26]. However, as pointed out in

Sec. II-A, the existing results cannot be applied in TVWS scenarios, since they are based on the assumption that the *sense-before-talk* mechanism is mandatory. Hence, a more general model that accounts for the unique TVWS characteristics is required and we address this issue in Sections III and IV.

Very recently, the problem of secondary-secondary coexistence is gaining attention [15] and in [16], the intrinsic relationship between environmental and system parameters in affecting the secondary coexistence has been disclosed. Several TVWS standards, such as IEEE 802.22 [6], IEEE 802.11af [7] and ECMA 392 [9], define self-coexistence mechanisms to mitigate the mutual interference among similar networks. Nevertheless, none of these mechanisms can be applied to mitigate the interference among heterogeneous networks.

Finally, few works address the heterogeneous coexistence problem. Some works address the coexistence among low vs high-power [27] or contention vs reserved-based [28] networks. However, the proposed strategies are targeted to couples of specific technologies and, hence, are not suitable for heterogeneous scenarios such as the TVWS ones. Differently, in this paper we design a general strategy allowing an arbitrary SN to make its decisions independently of the coexisting technologies. IEEE 802.16m [29] defines uncoordinated mechanisms for heterogeneous coexistence. However, the standard focuses on license-exempt spectrum and the proposed strategy simply aims at selecting a channel with tolerable interference. Differently, in this paper, we focus on licensed spectrum and we propose an optimal strategy, i.e., a strategy maximizing the expected throughput achievable by the SN. IEEE 802.19.1 [17] aims at providing general solutions to the heterogeneous coexistence by envisioning a coexistence manager, acting as a centralized resource allocator, and a coexistence enabler, aiming at maintaining interfaces between the coexistence enabler and coexisting CR networks. However, the proposed strategies either focus on selecting non-overlapping channels or require a certain degree of collaboration among the coexisting networks. Differently, in this paper we design an autonomous strategy allowing the SN to make its decisions independently of any other coexisting network.

III. MODEL AND PRELIMINARIES

In this section, we first describe the system model in Sec. III-A. Then, in Sec. III-B, we collect several definitions that will be used through the paper.

A. System Model

We consider a SN operating within the TVWS Spectrum according to the existing regulations and standards. Time is organized into fixed-size slots of duration T , and by accessing to the TVWS database, the SN obtains the list of channels free from primary incumbents within an arbitrary time slot. In the following, we denote the set of incumbent-free channels in a given time slot with $\Omega = \{1, 2, \dots, M\}$.

Since multiple SNs are allowed to operate within the same geographical region, a channel $m \in \Omega$ free from primary incumbents can be affected by coexistence interference caused by other heterogeneous secondary networks coexisting within

the same geographical area. Hence, a SU aiming at maximizing the available data rate assesses the strength of such an interference² by sensing the m -th channel for a certain amount of time, say τ_s . Depending on the measured strength, the SU can transmit over the m -th channel with a certain data rate, whose value belongs to an ordered set of K discrete rates³ $\{\tilde{r}_0, \tilde{r}_1, \dots, \tilde{r}_K\}$, with \tilde{r}_k increasing with k and $\tilde{r}_0 \triangleq 0$ denoting a channel sensed as unusable due to excessive interference. By denoting with R_m the r.v. characterizing the data rate achievable on the m -th channel during an arbitrary time slot⁴, $p_{m,k} = P(R_m = \tilde{r}_k)$ represents the probability of the data rate achievable on channel m at time slot n being \tilde{r}_k . The SUs can estimate the first-order distribution of the achievable data rates through the past-channel throughput histories.

After assessing⁵ the admissible data rate on channel m , say rate r_m , the SU decides whether to use or not to use channel m by comparing r_m with a certain threshold, say y_m . Whenever $r_m \geq y_m$, the SU transmits over channel m for the remaining of the time slot, whereas whenever $r_m < y_m$, the SU skips channel m to sense another channel looking for better communication opportunities. Clearly, the SU can decide to use channel m without assessing the coexistence interference by setting the threshold as⁶ $y_m = 0$.

Both the sequence of channels to be sensed and the corresponding rate thresholds, referred to in the following as *stopping rules*, deeply affect the performance of any secondary network. Hence, the goal can be summarized as *to find the channel providing the highest data rate as quickly as possible*. In the next subsection, we rigorously formulate the problem.

B. Problem Formulation

Here, we formulate the problem of choosing a coexistence strategy maximizing the expected data rate achievable by the SU during an arbitrary time slot n . Without loss of generality, in the following we omit the time dependence to simplify the adopted notation, shown in Table I.

²We note that, although the transmitting SU should be concerned with the interference levels present at the receiver side, it is likely that both the transmitter and the receiver suffer similar interference, due to the excellent propagation conditions of TVWS [30]. In any case, the conducted analysis continues to hold since the interference can be estimated at the receiver based on the previous transmissions. In fact, since we develop a probabilistic model aiming at maximizing the expected reward, the instantaneous values of the interference are not of interest, and the average interference levels can be easily estimated based on the through the past channel-interference history [31], [32].

³The data rate set depends on the communication standard of the arbitrary SN. As an example, IEEE 802.11af defines eleven different data rates for 6MHz wide TVWS channels, as detailed in Sec. V-A.

⁴It is well-known [33] that: i) the achievable data rate over a channel is deeply affected by the Signal-to-Interference-plus-Noise Ratio (SINR); ii) the SINR range can be partitioned in different regions, each of one associated with a certain data rate.

⁵The proposed framework can be easily extended to the case of imperfect sensing by setting the k -th data rate for the m -th channel to $p_{m,0}(1-p_{fa})\tilde{r}_k$ as in [34], with p_{fa} denoting the false-alarm probability of the adopted sensing mechanism.

⁶Through the null threshold concept, we are able to design a *discretionary* coexistence strategy as detailed in Sec. II-A.

TABLE I
ADOPTED NOTATION

Symbol	Definition
M	number of available channels
T	duration of a time slot
$\{\tilde{r}_k\}_{k=0}^K$	set of admissible data rates
$p_{m,k}$	probability of \tilde{r}_k being the data rate achievable on channel m
x_m	m -th channel to be sensed
\tilde{r}_{y_m}	data rate threshold for channel x_m
$V_{\mathbf{x},\mathbf{y}}$	expected average throughput
$\mathbf{x}^*, \mathbf{y}^*$	sensing sequence and stopping rule maximizing $V_{\mathbf{x},\mathbf{y}}$
$p_{\mathbf{x},\mathbf{y}}(m)$	probability of using the m -th sensed channel
$\tilde{r}_{\mathbf{x},\mathbf{y}}(m)$	expected data rate through channel x_m
$c_{\mathbf{y}}(m)$	portion of the time slot devoted to packet transmission by using channel x_m

Definition 1. (Sensing Sequence) The *sensing sequence* \mathbf{x} is the ordered sequence of channels to be sensed:

$$\mathbf{x} = (x_1, x_2, \dots, x_M) \quad (1)$$

with $x_m \in \Omega$ for any m , and $x_m \neq x_l$ for any $m \neq l$. In the following, we denote with \mathbf{X} the set of all possible sensing sequences and, by recognizing that a sensing sequence is a *permutation without repetition* over the set Ω , it results $|\mathbf{X}| = M!$.

Definition 2. (Stopping Rule) The *stopping rule* \mathbf{y} is the ordered sequence of data rate threshold indices:

$$\mathbf{y} = (y_1, y_2, \dots, y_M), \quad y_m = 0, \dots, K \quad (2)$$

where $\tilde{r}_{y_m} \in \{\tilde{r}_0, \dots, \tilde{r}_K\}$ denotes the channel reward threshold for the m -th sensed channel $x_m \in \Omega$. In the following, we denote with \mathbf{Y} the set of stopping rules and, by recognizing that a stopping rule is a *permutation with repetition* over the set of admissible discrete rates indices, it results $|\mathbf{Y}| = (K+1)^M$.

Remark. A stopping rule \mathbf{y} is a M -tuple of integers in $[0, K]$, with the m -th integer y_m denoting the minimum data rate, i.e., the data rate threshold, required to use the m -th sensed channel x_m . As an example, by assuming $\mathbf{y} = (2, 1, 4)$ with $M = 3$, it results that: i) \tilde{r}_2 is the threshold for the first sensed channel x_1 ; ii) \tilde{r}_1 is the threshold for the second sensed channel x_2 ; iii) \tilde{r}_4 is the threshold for the third sensed channel x_3 . Hence, the first sensed channel x_1 will be used if and only if it admits a data rate equal or greater than \tilde{r}_2 . Clearly, channel x_2 will be sensed if and only if the first sensed channel admits a data rate lower than \tilde{r}_2 , and it will be used if and only if it admits a data rate equal or greater than \tilde{r}_1 .

Remark. At the beginning of an arbitrary time slot, the SU can either: i) transmit over channel x_1 regardless of the coexistence interference; ii) sense channel x_1 and, based on the sensed interference, decide whether to use or not to use such a channel. According to the adopted notation, the former case is denoted with $y_1 = 0$, whereas the latter is denoted with $y_1 = i$ with $i \neq 0$. If the SU decides to skip the first sensed channel, then it can either transmit or sense channel x_2 , based on the value of y_2 . By iterating the above mentioned process, the SU eventually selects one of the M incumbent-free channels to be used for secondary communications.

Remark. To simplify the notation, we assume in Definition 2 the thresholds belong to the set of admissible data rates $\{\tilde{r}_0, \dots, \tilde{r}_M\}$. It is easy to recognize that such an assumption is not restrictive by noting that for any threshold value $r \in (\tilde{r}_{m-1}, \tilde{r}_m]$ the SU uses the channel x_m if $R_{x_m} \geq \tilde{r}_m$, whereas it skips such a channel if $R_{x_m} < \tilde{r}_m$. Hence, any $r \in (\tilde{r}_{m-1}, \tilde{r}_m]$ can be replaced by \tilde{r}_m without loss of generality.

Definition 3. (Expected Reward) The *expected reward* $V_{\mathbf{x}, \mathbf{y}}$ denotes the expected throughput achievable by the SU in a given time slot by following the sensing sequence $\mathbf{x} \in \mathbf{X}$ and the stopping rule $\mathbf{y} \in \mathbf{Y}$.

Remark. The *expected reward* represents a trade-off between: i) the data rate achievable on the selected channel; ii) the time spent for selecting the channel. In general, longer search times assure higher data rates at the price of a shorter portion of the time slot T devoted to packet transmission.

Problem 1. (Optimal Coexistence Strategy) The goal is to jointly choose the optimal sensing sequence \mathbf{x}^* and the optimal stopping rule \mathbf{y}^* maximizing the expected reward:

$$V_{\mathbf{x}^*, \mathbf{y}^*} = \arg \max_{\mathbf{x} \in \mathbf{X}, \mathbf{y} \in \mathbf{Y}} \{V_{\mathbf{x}, \mathbf{y}}\} \quad (3)$$

Insight 1. We note that jointly finding the *optimal sensing sequence* and the *optimal stopping rule* through brute-force searching is computational unfeasible. In fact, for each of the $M!$ sensing sequences, $(K+1)^M$ stopping rules need to be evaluated.

Remark. Through the general notion of reward, we abstract the derived results from the particulars, making the conducted analysis general. In fact, it can be easily applied to a variety of real-world scenarios, by choosing the proper reward measure. Within the manuscript, we adopted as performance metric the data rate, hence the reward \tilde{r}_{y_m} models the data rate achievable on channel y_m . Clearly, depending on the scenario, a different performance metric can be more suitable [35], [36], e.g., the channel reliability. In such a case, by simply modeling with the reward \tilde{r}_{y_m} the reliability of channel y_m , all the results derived within the paper continue to hold.

IV. OPTIMAL COEXISTENCE INTERFERENCE AVOIDANCE STRATEGY

At first, in Sec. IV-A, we derive in Theorem 1 the closed-form expression of the *expected reward*. Then, in Sec. IV-B, we efficiently (polynomial-time complexity) compute the stopping rule maximizing the expected reward for a given sensing sequence with Algorithm 1. Stemming from this, we first prove that the problem of computing the optimal sensing sequence is NP-hard in Theorem 3, and then we compute both the optimal sensing sequence and the optimal stopping rule with Algorithm 2. Finally, in Sec. IV-C, we efficiently (polynomial-time complexity) compute both the optimal sensing sequence and the optimal stopping rule with Algorithm 3 under the reasonable hypothesis of identically distributed coexistence interference levels.

A. Preliminaries

Here, we derive in Theorem 1 the closed-form expression of the *expected reward*. The proof of Theorem 1 requires the following preliminary lemmas.

Lemma 1. (Conditional Stopping Probability) The *conditional stopping probability* $p_{\mathbf{x}, \mathbf{y}}(m)$, i.e., the probability of SU using the m -th sensed channel given that it skipped the first $m-1$ sensed channels, is equal to:

$$p_{\mathbf{x}, \mathbf{y}}(m) = \sum_{k=y_m}^K p_{x_m, k} \triangleq 1 - \bar{p}_{\mathbf{x}, \mathbf{y}}(m) \quad (4)$$

with $\mathbf{x} \in \mathbf{X}$ denoting the adopted sensing sequence and $\mathbf{y} \in \mathbf{Y}$ denoting the adopted stopping rule.

Proof: See Appendix A. ■

Remark. The SU uses channel x_1 with probability $p_{\mathbf{x}, \mathbf{y}}(1)$ whereas it skips x_1 with probability $\bar{p}_{\mathbf{x}, \mathbf{y}}(1)$. Given that it skipped channel x_1 , it uses channel x_2 with probability $p_{\mathbf{x}, \mathbf{y}}(2)$. Clearly, the lower is y_1 , the more likely the SU uses channel x_1 and, from Def. 2, we have $y_1 = 0 \implies p_{\mathbf{x}, \mathbf{y}}(1) = 1$.

Lemma 2. (Rate Expectation) The *rate expectation* $\bar{r}_{\mathbf{x}, \mathbf{y}}(m)$, i.e., the expected data rate achievable by the SU through the m -th sensed channel, is equal to:

$$\bar{r}_{\mathbf{x}, \mathbf{y}}(m) = \sum_{k=y_m}^K \frac{p_{x_m, k}}{p_{\mathbf{x}, \mathbf{y}}(m)} \tilde{r}_k \quad (5)$$

with $\mathbf{x} \in \mathbf{X}$ denoting the adopted sensing sequence and $\mathbf{y} \in \mathbf{Y}$ denoting the adopted stopping rule.

Proof: See Appendix B. ■

Remark. The lower is the threshold index y_m , the lower is the data rate threshold \tilde{r}_{y_m} , the more likely the data rate on channel x_m exceeds the threshold. Hence, the more likely the SU uses channel x_m , but the lower is the expected data rate $\bar{r}_{\mathbf{x}, \mathbf{y}}(m)$ through channel x_m .

Theorem 1. (Expected Reward) The *expected reward* $V_{\mathbf{x}, \mathbf{y}}$ achievable by the SU following the sensing sequence $\mathbf{x} \in \mathbf{X}$ and the stopping rule $\mathbf{y} \in \mathbf{Y}$ is equal to:

$$V_{\mathbf{x}, \mathbf{y}} = \sum_{m=1}^M p_{\mathbf{x}, \mathbf{y}}(m) q_{\mathbf{x}, \mathbf{y}}(m) \bar{r}_{\mathbf{x}, \mathbf{y}}(m) c_{\mathbf{y}}(m) \quad (6)$$

where the probability $q_{\mathbf{x}, \mathbf{y}}(m)$ of skipping the first $m-1$ sensed channels is given by:

$$q_{\mathbf{x}, \mathbf{y}}(m) = \begin{cases} 1 & \text{if } m = 1 \\ \prod_{l=1}^{m-1} \bar{p}_{\mathbf{x}, \mathbf{y}}(l) & \text{otherwise} \end{cases} \quad (7)$$

and the scaling factor $c_{\mathbf{y}}(m)$ is given by:

$$c_{\mathbf{y}}(m) = \begin{cases} (1 - (m-1)\tau_s/T) & \text{if } y_m = \tilde{r}_0 \\ (1 - m\tau_s/T) & \text{otherwise} \end{cases} \quad (8)$$

Proof: See Appendix C. ■

Remark. The *expected reward* $V_{\mathbf{x}, \mathbf{y}}$ allows us to estimate the expected throughput achievable by the SU during the time slot.

Specifically, $V_{\mathbf{x},\mathbf{y}}$ is the sum of the rate expectation $\bar{r}_{\mathbf{x},\mathbf{y}}(m)$ on channel x_m , weighted by the probability of using such a channel. Since the more channels are sensed by the SU the less time can be devoted to packet transmission, the rate expectation for channel x_m is weighted by the scaling factor $c_{y_m}(m)$, which accounts for the portion of the time slot T devoted to packet transmission.

B. Optimal Coexistence Strategy

Here, we derive in Theorem 2 the optimal stopping rule for a given sensing sequence. Stemming from this, we prove with Corollary 2 that Problem 1 can be polynomial-time reduced to another problem, referred to in the following as Problem 2. Intuitively, a polynomial-time reduction proves that the first problem is no more difficult than the second one, because whenever an efficient algorithm exists for the second problem, one exists for the first problem as well. Furthermore, in Theorem 3 we prove that Problem 2 is NP-hard. Hence, due to the reduction property, we can conclude that it does not exist an efficient algorithm for the considered problem, i.e., Problem 1.

The proof of Theorem 2 requires the following preliminary lemmas.

Lemma 3. (Expected Remaining Reward) *The expected remaining reward $v_{\mathbf{x},\mathbf{y}}(m)$, i.e., the expected reward achievable by the SU through channels x_{m+1}, \dots, x_M given that it skipped the first m sensed channels, is given by:*

$$v_{\mathbf{x},\mathbf{y}}(m) = \sum_{l=m+1}^M p_{\mathbf{x},\mathbf{y}}(l) \prod_{i=m+1}^{l-1} \bar{p}_{\mathbf{x},\mathbf{y}}(i) \bar{r}_{\mathbf{x},\mathbf{y}}(l) c_{\mathbf{y}}(l) \quad (9)$$

Proof: See Appendix D. ■

Remark. The expected remaining reward $v_{\mathbf{x},\mathbf{y}}(m)$ allows us to estimate the expected throughput given that the first m sensed channels are skipped, i.e., $q_{\mathbf{x},\mathbf{y}}(m+1) = 1$.

Algorithm 1 Optimal Stopping Rule Given Sensing Sequence

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1: // input:  $\mathbf{x} = \{x_1, \dots, x_M\}$ 
2: // output:  $v, \check{\mathbf{y}} = \{\check{y}_1, \dots, \check{y}_M\}$ 
3: // base step
4:  $\check{y}_M = 0$ 
5:  $v = (1 - (M - 1)\tau_s/T) \sum_{k=1}^K p_{x_M, k} r_k$ 
6: // recursive step
7: for  $m = M - 1 : 1$  do
8:    $\check{y}_m = \min_k \{\tilde{r}_k(1 - m\tau_s/T) \geq v\}$ 
9:    $t_0 = (1 - (m - 1)\tau_s/T) \sum_{k=1}^K p_{x_m, k} r_k$ 
10:   $t_1 = (1 - m\tau_s/T) \sum_{k=\check{y}_m}^K p_{x_m, k} r_k + \sum_{k=0}^{\check{y}_m-1} v$ 
11:  if  $t_0 > t_1$  then
12:     $\check{y}_m = 0, v = t_0$ 
13:  else
14:     $\check{y}_m = \check{y}_m, v = t_1$ 
15:  end if
16: end for

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Lemma 4. *Given the sensing sequence $\mathbf{x} \in \mathbf{X}$ and the stopping rule $\mathbf{y} \in \mathbf{Y}$ with $y_m > 0$, it results:*

$$V_{\mathbf{x},\mathbf{y}} \leq V_{\mathbf{x},\check{\mathbf{y}}} \quad (10)$$

with

$$\check{y}_l = y_l \forall l \neq m \wedge \check{y}_m = \min_k \{\tilde{r}_k(1 - m\tau_s/T) \geq v_{\mathbf{x},\mathbf{y}}(m)\} \quad (11)$$

Proof: See Appendix E. ■

Remark. Lemma 4 allows us to establish, for an arbitrary sensing sequence, the m -th stopping rule maximizing the expected reward given that channel x_m is sensed. Stemming from this, in Theorem 2 we derive the optimal m -th stopping rule for an arbitrary sensing sequence.

Theorem 2. (Stopping Rule Given Sensing Sequence) *Given the sensing sequence $\mathbf{x} \in \mathbf{X}$ and the stopping rule $\mathbf{y} \in \mathbf{Y}$, it results:*

$$V_{\mathbf{x},\mathbf{y}} \leq V_{\mathbf{x},\check{\mathbf{y}}} \quad (12)$$

with

$$\check{y}_l = y_l \forall l \neq m \quad (13)$$

$$\check{y}_m = \begin{cases} 0 & \text{if } E[R_{x_m}] \geq \frac{v_{\mathbf{x},\mathbf{y}}(m-1)}{1 - (m-1)\tau_s/T} \\ \check{y}_m & \text{otherwise} \end{cases} \quad (14)$$

with \check{y}_m given in (11).

Proof: See Appendix F. ■

Corollary 1. (M-th Stopping Rule Given Sensing Sequence) *For any sensing sequence $\mathbf{x} \in \mathbf{X}$, the M -th component of the stopping rule $\mathbf{y} \in \mathbf{Y}$ maximizing the expected reward $V_{\mathbf{x},\mathbf{y}}$ is given by:*

$$y_M = 0 \quad (15)$$

Proof: The proof follows directly from Theorem 2 since $v_{\mathbf{x},\mathbf{y}}(M) = 0$ for any \mathbf{x}, \mathbf{y} . ■

Remark. From Corollary 1, it follows that, when only one channel is left, the SU reduces the achievable expected reward by sensing such a channel instead of simply using it. This is reasonable since, even if the SU senses the channel as unavailable, i.e., $R_M = r_0$, there is no other option (channel) left.

Remark. By iteratively applying Theorem 1, it follows that Algorithm 1 effectively finds the stopping rule $\check{\mathbf{y}} \in \mathbf{Y}$ maximizing the expected reward for any given sensing sequence $\mathbf{x} \in \mathbf{X}$. We note that the time complexity of Algorithm 1 is linear with respect to both M and K , i.e., $\mathcal{O}(NK)$.

Stemming from Theorem 2, we can now reformulate Problem 1 as follows.

Problem 2. (Optimal Sensing Sequence) The goal is to choose the optimal sensing sequence \mathbf{x}^* maximizing the expected reward:

$$V_{\mathbf{x}^*,\mathbf{y}(\mathbf{x}^*)} = \arg \max_{\mathbf{x} \in \mathbf{X}} \{V_{\mathbf{x},\mathbf{y}(\mathbf{x})}\} \quad (16)$$

where $\mathbf{y}(\mathbf{x})$ is given by Algorithm 1 for any \mathbf{x} .

Corollary 2. (Problem Equivalence) Problem 1 can be polynomial-time reduced to Problem 2.

Proof: The proof follows from Theorem 2 by accounting for the time complexity of Algorithm 1. ■

Theorem 3. (Problem Complexity) Problem 2 is NP-hard.

Proof: See Appendix G. ■

Remark. As proved in Appendix G, the *Optimal Coexistence Strategy* problem can be polynomial-time reduced to the Traveling Salesman Problem (TSP). Hence, the existing literature on exact/approximate algorithms for solving the TSP can be efficiently adopted for searching the optimal/sub-optimal strategy.

Insight 2. In scenarios of practical interest such as the urban ones, it has been shown that the TVWS spectrum is scarce with roughly five channels available to secondary access [13]. Hence, as we show in Sec. V-B, the optimal solution can be found in almost real time with commercial hardware through Algorithm 2. More in detail, with Algorithm 2: i) the sensing sequence maximizing the expected reward is found through exhaustive search; ii) the stopping rule maximizing the expected reward for any given sensing sequence is found through Algorithm 1. Furthermore, in Section IV-C, by considering identically distributed coexistence interference levels, we derive an efficient (polynomial-time) algorithm for searching the optimal strategy.

C. Optimal Coexistence Strategy under Identically Distributed Interference

Here, we derive in Theorem 4 the optimal sensing strategy when the coexistence interference levels are identically distributed among the available channels. Hence, in the following, we denote with p_k the probability of \tilde{r}_k being the admissible data rate for channel x_m for any $m \in \Omega$.

The proof of Theorem 4 requires the following intermediate results.

Lemma 5. (Conditional Stopping Probability) For any sensing sequence $\mathbf{x} \in \mathbf{X}$, it results:

$$p_{\mathbf{x},\mathbf{y}}(m) = \sum_{k=y_m}^K p_k \triangleq p_{\mathbf{y}}(m) \triangleq 1 - \bar{p}_{\mathbf{y}}(m) \quad (17)$$

with $\mathbf{y} \in \mathbf{Y}$ denoting the adopted stopping rule.

Algorithm 2 Optimal Sensing Strategy

```

1: // input:  $\mathbf{X}$  = permutations ( $\{1, \dots, M\}$ )
2: // output:  $\mathbf{x}^*, \mathbf{y}^*$ 
3: // base step
4:  $v^* = 0, \mathbf{x}^* = \{0, \dots, 0\}, \mathbf{y}^* = \{0, \dots, 0\}$ 
5: // iterative step
6: for  $\mathbf{x} \in \mathbf{X}$  do
7:    $v, \mathbf{y}$  computed with Algorithm 1
8:   if  $v^* > v$  then
9:      $v^* = v, \mathbf{x}^* = \mathbf{x}, \mathbf{y}^* = \mathbf{y}$ 
10:  end if
11: end for

```

Proof: See Appendix H. ■

Remark. In the following, we adopt the notation $p_{\mathbf{y}}(m)$ to highlight the independence of the conditional stopping probability from the sensing sequence due to the identical distribution hypothesis.

Lemma 6. (Rate Expectation) For any sensing sequence $\mathbf{x} \in \mathbf{X}$, it results:

$$\bar{r}_{\mathbf{x},\mathbf{y}}(m) = \sum_{k=y_m}^K \frac{p_k}{p_{\mathbf{y}}(m)} \tilde{r}_k \triangleq \bar{r}_{\mathbf{y}}(m) \quad (18)$$

with $\mathbf{y} \in \mathbf{Y}$ denoting the adopted stopping rule.

Proof: The proof follows by reasoning as in Appendix H. ■

Corollary 3. (Expected Reward) For any sensing sequence $\mathbf{x} \in \mathbf{X}$, it results:

$$V_{\mathbf{x},\mathbf{y}} = \sum_{m=1}^M p_{\mathbf{y}}(m) q_{\mathbf{y}}(m) \bar{r}_{\mathbf{y}}(m) c_{\mathbf{y}}(m) \triangleq V_{\mathbf{y}} \quad (19)$$

with $\mathbf{y} \in \mathbf{Y}$ denoting the adopted stopping rule, $c_{\mathbf{y}}(m)$ given in (8) and $q_{\mathbf{y}}(m)$ equal to:

$$q_{\mathbf{y}}(m) = \begin{cases} 1 & \text{if } m = 1 \\ \prod_{l=1}^{m-1} \bar{p}_{\mathbf{y}}(l) \triangleq \prod_{l=1}^{m-1} (1 - p_{\mathbf{y}}(l)) & \text{otherwise} \end{cases} \quad (20)$$

Proof: The proof follows from Lemmas 5 and 6. ■

Remark. The result of Corollary 3 is reasonable: since the coexistence interference is identically distributed over the available channels, the optimal sensing strategy: i) depends on the stopping rule \mathbf{y} ; ii) does not depend of the sensing sequence \mathbf{x} . Stemming from this, we can now derive in Theorem 4 the optimal sensing strategy.

Algorithm 3 Optimal Sensing Strategy for Identically Distributed Channel Interference

```

1: // output:  $\mathbf{y}^*$ 
2: // base step
3:  $y_M = 0$ 
4:  $v = (1 - (M - 1)\tau_s/T) \sum_{k=1}^K p_k r_k$ 
5: // recursive step
6: for  $m = M - 1 : 1$  do
7:    $\tilde{y}_m = \min_k \{\tilde{r}_k (1 - m\tau_s/T)\} \geq v$ 
8:    $t_0 = (1 - (m - 1)\tau_s/T) \sum_{k=1}^K p_k r_k$ 
9:    $t_1 = (1 - m\tau_s/T) \sum_{k=\tilde{y}_m}^K p_k r_k + \sum_{k=0}^{\tilde{y}_m-1} v$ 
10:  if  $t_0 > t_1$  then
11:     $y_m = 0, v = t_0$ 
12:  else
13:     $y_m = \tilde{y}_m, v = t_1$ 
14:  end if
15: end for

```

TABLE II
PERFORMANCE EVALUATION PARAMETER SETTING

Symbol	Definition	Fig. 4	Fig. 5	Fig. 6	Fig. 7	Fig. 8
M	number of TVWS channels	4	4	2-8	4	2-8
\tilde{r}_k	admissible data rate	{0,1.8,3.6,5.4,7.2,10.8,14.4,16.2,18,21.6,24}Mbit/s				
τ	normalized sensing time	0.01	0.01	0.01	0.01	0.01-0.5
$p_{m,k}$	probability of \tilde{r}_k being the data rate achievable on channel m	uniformly distributed in $[0, 1]$				

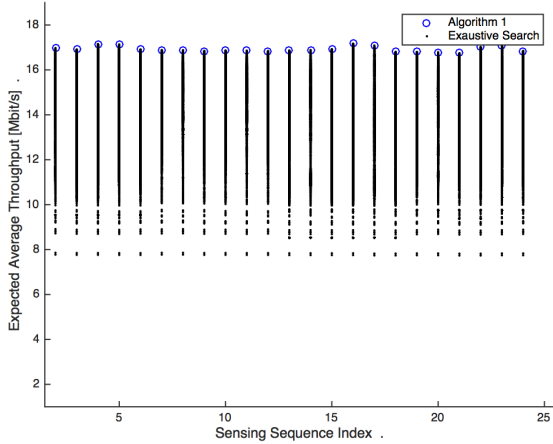


Fig. 4. Optimality: Algorithm 1 vs Exhaustive Search. Each dot refers to a pair (\mathbf{x}, \mathbf{y}) , where coordinate (x) denotes the sensing sequence index and coordinate (y) denotes the expected reward $V_{\mathbf{x}, \mathbf{y}}$. Each circle refers to a pair $(\mathbf{x}, \tilde{\mathbf{y}})$ with $\tilde{\mathbf{y}}$ given by Algorithm 1, where coordinate (x) denotes the sensing sequence index and coordinate (y) denotes the expected reward $V_{\mathbf{x}, \tilde{\mathbf{y}}}$.

Theorem 4. (Optimal Sensing Strategy) The optimal stopping rule $\mathbf{y}^* \in \mathbf{Y}$ is recursively defined as:

$$y_m^* = \begin{cases} 0 & m = M \\ \begin{cases} 0 & \text{if } \sum_{k=0}^K p_k \tilde{r}_k \geq \frac{v_{\mathbf{y}}(m)}{1 - (m-1)\tau_s/T} \\ \tilde{y}_m & \text{otherwise} \end{cases} & m < M \end{cases} \quad (21)$$

with $\tilde{y}_m = \min_k \{ \tilde{r}_k (1 - m\tau_s/T) \geq v_{\mathbf{y}}(m) \}$ and $v_{\mathbf{y}}(m)$ equal to:

$$v_{\mathbf{y}}(m) = \sum_{l=m+1}^M p_{\mathbf{y}}(l) \prod_{i=m+1}^{l-1} \bar{p}_{\mathbf{y}}(i) \bar{r}_{\mathbf{y}}(l) c_{\mathbf{y}}(l) \quad (22)$$

Proof: See Appendix I. ■

Remark. From Theorem 4, it follows that Algorithm 3 efficiently (polynomial-time) finds the optimal sensing strategy in presence of identically distributed interference levels. Furthermore, in Sec. V-B, we evaluate the feasibility of Algorithm 3 in presence of non identically distributed interference levels.

Remark. By assuming a negligible sensing overhead, i.e., $\tau_s \ll T$, it is straightforward to prove that the optimal stopping rule y_m^* is equal to \tilde{y}_m for any $m < M$.

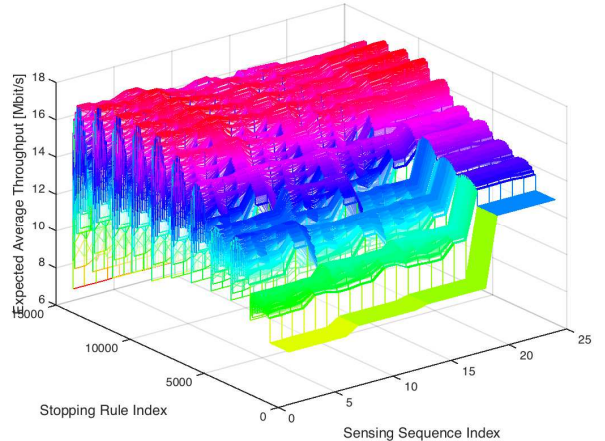


Fig. 5. Optimality: expected reward vs. sensing sequence-stopping rule.

V. PERFORMANCE EVALUATION

In this section, we evaluate the performance of the proposed coexistence strategy by adopting, as case study, an IEEE 802.11af networks operating in the TVWS spectrum.

A. Optimality

Here, we validate the optimality property of the proposed coexistence strategy by showing that the sensing rule derived in Algorithm 1 assures, for any sensing sequence, the highest expected reward.

The simulation set, summarized in Table II, is as follows: $M=4$ channels are available to secondary access and, by adopting 6MHz wide channels, the admissible data rates in IEEE 802.11af are $\{0, 1.8, 3.6, 5.4, 7.2, 10.8, 14.4, 16.2, 18, 21.6, 24\}$ Mbit/s. The channel interference levels are independent of each others and uniformly distributed within the corresponding SINR regions, and the sensing process is characterized by a normalized sensing time⁷ $\tau_s/T = 0.01$.

In Fig. 4, for each of the $M! = 24$ sensing sequences we report: i) the expected reward achievable by using the stopping rule $\tilde{\mathbf{y}}$ derived in Algorithm 1; ii) the expected rewards achievable by using any other stopping rule $\mathbf{y} \in \mathbf{Y}$, with $|\mathbf{Y}| = (K+1)^M = 14641$, found through exhaustive

⁷By abstracting from the sensing particulars, the notion of normalized sensing time allows us to focus on the effects of the coexistence strategy in terms of expected reward.

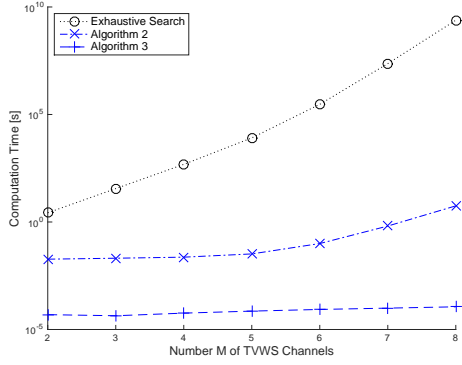


Fig. 6. Time Complexity: running times vs number M of available TVWS channels. Logarithmic scale for (y) axis.

enumeration. First, we note that Algorithm 1 effectively finds the stopping rule maximizing the expected reward for any given sensing sequence. Hence, the proposed coexistence strategy is *optimal*. We further observe that there is a significant variability among the different stopping rules in terms of expected reward, ranging from roughly 8Mbit/s to over 16Mbit/s. This result highlights that, even when the TVWS spectrum is scarce ($M = 4$), there exists a significant diversity among the strategies in terms of expected throughput. Hence, an optimal strategy design is crucial to take advantage of such a diversity.

To better understand the effects of the strategy design in terms of expected reward, in Fig. 5 we report the expected reward as a function of the couple *sensing sequence-stopping rule*. Specifically, each three-dimensional point refers to a pair (\mathbf{x}, \mathbf{y}) , where coordinate (x) denotes the sensing sequence index, coordinate (y) denotes the stopping rule index, and coordinate (z) denotes the expected reward $V_{\mathbf{x}, \mathbf{y}}$. We observe that the expected reward achievable significantly changes not only with the stopping rule, but also with the sensing sequence, i.e., there exists two degree of freedom that must be explored to design an optimal strategy. This agrees with the theoretical results derived in Sec. IV. Furthermore, we note that, for a given stopping rule, by changing the sensing sequence the expected reward can vary up to a notable 40%, confirming the considerations made for Fig. 4.

B. Feasibility

Here, we assess the feasibility of the proposed coexistence strategy in terms of computational complexity. Specifically, we compare the running times for computing the optimal coexistence strategy with three different algorithms: i) Algorithm 2; ii) Algorithm 3; iii) exhaustive search.

Fig. 6 presents the running times⁸ of the considered algorithms as the number M of available TVWS channels increases, with a logarithmic scale for the (y) axis. The simulation set is as in Sec. V-A. First, we observe that the

⁸We note that the times have been obtained by running the algorithms on a general purpose architecture (MacBook Pro). Hence, it is reasonable to believe that a reduction of one or two orders of magnitude can be easily obtained by adopting dedicated hardware.

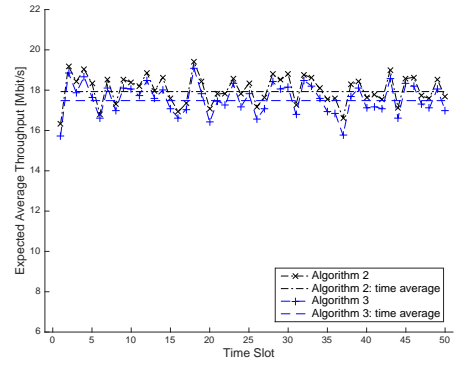


Fig. 7. Fast Computation: expected reward vs time in presence of independent interference.

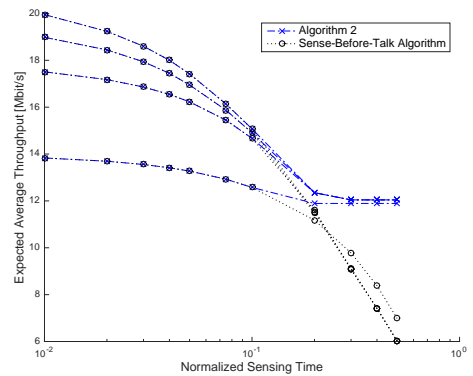


Fig. 8. Discretionary Interference Sensing: expected reward vs normalized sensing time τ_s/T for different values of the number M of available TVWS channels. Logarithmic scale for (x) axis.

time complexity of the exhaustive search makes this choice unfeasible even in urban scenarios, with running times within the order of magnitude of minutes and hours for $M = 3$ and $M = 4$, respectively. On the other hand, we observe that Algorithm 2 performs satisfactory in urban scenarios, with running times within the order of magnitude of seconds or less up to $M = 8$. Finally, we note that Algorithm 3 performs considerably well in both urban and rural scenarios, with running times in the order of 10^{-4} seconds also for the larger values of M .

One question arises spontaneously: what if the considered scenario is characterized by TVWS abundance in presence of independent but non identically distributed interference? In other words, what if we resort to Algorithm 3 when the interference is not identically distributed?

We focus on such a scenario in Fig. 7. Specifically, the figure presents the expected reward as a function of the time for both Algorithm 2 and 3, along with the corresponding time averages of the expected rewards. The simulation set is as in Sec. V-A, with Algorithm 3 rate probability p_k set to the average value of the channel data rate probabilities $p_{m,k}$, i.e., $p_k = 1/M \sum_{m=1}^M p_{m,k}$. Clearly, Algorithm 2, i.e., the algorithm designed for the considered scenario, outperforms

Algorithm 3 both instantaneously and in average. Nevertheless, the differences in terms of expected reward between the optimal and the approximate algorithms are moderate. Hence, Algorithm 3 represents a sub-optimal but efficient solution when the running times of Algorithm 2 are unfeasible.

C. Discretionary Interference Sensing

Here, we analyze the benefits provided by a discretionary interference sensing in terms of expected throughput. More in detail, we compare the expected throughputs achievable with the proposed coexistence strategy (Algorithm 2) with those achievable by an algorithm that implements a mandatory interference sensing, referred to as *Sense-Before-Talk Algorithm*. As pointed out in Section II-A, the mandatory interference sensing represents a requirement of the existing literature on channel selection for CR networks. Hence, by comparing the proposed coexistence strategy with the *Sense-Before-Talk Algorithm* we aim at assessing the performance improvement over the state of the art.

Fig. 8 presents the expected reward as the normalized sensing time τ_s/T increases for different values of the number M of available TVWS channels. The simulation set is as in Sec. V-A. At first, we observe that the higher is the normalized sensing time, the lower is the expected reward. This agrees with both the intuition and Theorem 1. Furthermore, we observe that the differences between the optimal and the suboptimal strategies in terms of rewards increase as τ_s/T . This result is reasonable since the larger are the sensing times, the higher are the sensing overheads and hence the higher is the positive impact of the discretionary interference sensing in terms of reward. Finally, we observe that the differences between the optimal and the suboptimal strategies in terms of rewards increase as M decreases for a fixed normalized sensing time. This is reasonable: the lower are the available channels, the less are the sensing sequences, i.e., the less significant are the effects of the channel diversity on the expected reward. Consequently, the lower are the available channels, the more significant are the stopping rules in terms of expected reward.

VI. CONCLUSIONS

In this paper, we addressed the problem of the coexistence interference among multiple heterogeneous and independently-operated secondary networks coexisting within the same geographical area over shared TV white space in absence of secondary cooperation. Specifically, we designed the optimal coexistence strategy, i.e., the strategy that maximizes the throughput achievable by an arbitrary secondary network. Such a strategy exhibits several attractive features: i) feasibility, since it requires a reasonable amount of a-priori knowledge; ii) adaptivity to interference dynamics; iii) autonomous, since it allows the secondary network to make its decisions independently of any other coexisting networks; iv) discretionary interference avoidance, since it accounts for the harmless-to-incumbents property of interference in TVWS. More in detail, we proved that the problem of the optimal channel selection, i.e., the channel maximizing the expected throughput, is

computational prohibitive (NP-hard). Nevertheless, under the reasonable assumption of identically distributed interference on the available TVWS channels, we prove that the optimal channel selection problem is not anymore NP-hard. Specifically, we proved that the optimal strategy exhibits a threshold behavior, and by exploiting this threshold structure, we designed a computational-efficient (polynomial-time) algorithm. The performance evaluation validated the proposed theoretical analysis.

APPENDIX

A. Proof of Lemma 1

Proof: By observing that the SU uses the m -th sensed channel, given that it skipped the first $m-1$ sensed channels, if and only if $R_{x_m} \geq \tilde{r}_{y_m}$, it follows:

$$\begin{aligned} p_{\mathbf{x},\mathbf{y}}(m) &= P(R_{x_m} \geq \tilde{r}_{y_m}) = \sum_{k=y_m}^K P(R_{x_m} = \tilde{r}_k) = \\ &= \sum_{k=y_m}^K p_{x_m,k} \end{aligned} \quad (23)$$

■

B. Proof of Lemma 2

Proof: By noting that the rate expectation $\bar{r}_{\mathbf{x},\mathbf{y}}(m)$ represents the expectation of R_{x_m} given that channel x_m is used, i.e., given that $R_{x_m} \geq \tilde{r}_{y_m}$, it follows:

$$\bar{r}_{\mathbf{x},\mathbf{y}}(m) = E[R_{x_m} | R_{x_m} \geq \tilde{r}_{y_m}] = \frac{\sum_{k=y_m}^K p_{x_m,k} \tilde{r}_k}{p_{\mathbf{x},\mathbf{y}}(m)} \quad (24)$$

where the last equality accounts for Lemma 1 and for the definition of expectation of a truncated random variable. ■

C. Proof of Theorem 1

Proof: First, we observe that, when the m -th sensed channel is used, the portion of the time slot devoted to packet transmission is not greater than $1 - (m-1)\tau_s/T$. In fact, any channel in $\{x_1, \dots, x_{m-1}\}$ has been sensed, i.e., $y_k > 0$ for any $k < m$. Specifically, such a time slot fraction is equal to $1 - (m-1)\tau_s/T$ when $y_m = 0$, whereas it is equal to $1 - m\tau_s/T$ when $y_m > 0$. Hence, the expected reward achievable on channel x_m is equal to $\bar{r}_{\mathbf{x},\mathbf{y}}(m)c_{\mathbf{y}}(m)$. Since channel x_m is used with probability $p_{\mathbf{x},\mathbf{y}}(m)$ if and only if the previous $m-1$ sensed channels were skipped, and since such a probability is equal to $q_{\mathbf{x},\mathbf{y}}(m)$ given in (7), the thesis follows. ■

D. Proof of Lemma 3

Proof: Similarly to the proof of Theorem 1, the expected reward achievable through a channel x_l in x_{m+1}, \dots, x_K is equal to $\bar{r}_{\mathbf{x},\mathbf{y}}(l)c_{\mathbf{y}}(l)$. Since channel x_l is used with probability $p_{\mathbf{x},\mathbf{y}}(l)$ if and only if the channels x_{m+1}, \dots, x_{l-1} were skipped, and since such a probability is equal to $\prod_{i=m+1}^{l-1} \bar{p}_{\mathbf{x},\mathbf{y}}(i)$, the thesis follows. ■

E. Proof of Lemma 4

Proof: We prove the thesis with a *reductio ad absurdum* by supposing there exists $\mathbf{x} \in \mathbf{X}$ and $\mathbf{y} \in \mathbf{Y}$ so that:

$$V_{\mathbf{x},\mathbf{y}} > V_{\mathbf{x},\check{\mathbf{y}}} \quad (25)$$

with $\check{\mathbf{y}}$ given in (11). We have two cases.

i) Case $y_m < \check{y}_m$. Let us assume, without loss of generality, $y_m = \check{y}_m - 1$. Hence, by accounting for (11), we have (26) shown at the top of the next page.

By substituting (26) in (25) and by accounting for (4) and (5), we obtain:

$$\begin{aligned} & \sum_{k=\check{y}_m}^K p_{x_m,k} \tilde{r}_k c_{\mathbf{y}}(m) + p_{x_m,\check{y}_m-1} \tilde{r}_{\check{y}_m-1} c_{\mathbf{y}}(m) + \\ & + \sum_{k=0}^{\check{y}_m-2} p_{x_m,k} v_{\mathbf{x},\mathbf{y}}(m) > \sum_{k=\check{y}_m}^K p_{x_m,k} \tilde{r}_k c_{\mathbf{y}}(m) + \\ & + p_{x_m,\check{y}_m-1} v_{\mathbf{x},\mathbf{y}}(m) + \sum_{k=0}^{\check{y}_m-2} p_{x_m,k} v_{\mathbf{x},\mathbf{y}}(m) \end{aligned} \quad (27)$$

and, since $\tilde{r}_{\check{y}_m-1} c_{\mathbf{y}}(m) = \tilde{r}_{\check{y}_m-1} (1 - m\tau_s/T) < v_{\mathbf{x},\mathbf{y}}(m)$ from (11), (27) constitutes a *reductio ad absurdum*.

ii) Case $y_m > \check{y}_m$. Let us assume, without loss of generality, $y_m = \check{y}_m + 1$. By substituting (26) in (25) and by accounting for (4) and (5), we obtain:

$$\begin{aligned} & \sum_{k=\check{y}_m+1}^K p_{x_m,k} \tilde{r}_k c_{\mathbf{y}}(m) + p_{x_m,\check{y}_m} v_{\mathbf{x},\mathbf{y}}(m) + \\ & + \sum_{k=0}^{\check{y}_m-1} p_{x_m,k} v_{\mathbf{x},\mathbf{y}}(m) > \sum_{k=\check{y}_m+1}^K p_{x_m,k} \tilde{r}_k c_{\mathbf{y}}(m) + \\ & + p_{x_m,\check{y}_m} \tilde{r}_{\check{y}_m} c_{\mathbf{y}}(m) + \sum_{k=0}^{\check{y}_m-1} p_{x_m,k} v_{\mathbf{x},\mathbf{y}}(m) \end{aligned} \quad (28)$$

and, since $\tilde{r}_{\check{y}_m} c_{\mathbf{y}}(m) = \tilde{r}_{\check{y}_m} (1 - m\tau_s/T) \geq v_{\mathbf{x},\mathbf{y}}(m)$ from (11), (28) constitutes a *reductio ad absurdum*. \blacksquare

F. Proof of Theorem 2

Proof: *i) Case $E[R_{x_m}] \geq v_{\mathbf{x},\mathbf{y}}(m-1)/(1-(m-1)\tau_s/T)$.* We prove the thesis with a *reductio ad absurdum* by supposing there exists $\mathbf{x} \in \mathbf{X}$ and $\mathbf{y} \in \mathbf{Y}$ so that:

$$V_{\mathbf{x},\mathbf{y}} > V_{\mathbf{x},\check{\mathbf{y}}} \quad (29)$$

with $\check{\mathbf{y}}$ given in (13). Hence, we have:

$$\begin{aligned} V_{\mathbf{x},\check{\mathbf{y}}} &= \sum_{l=1}^M p_{\mathbf{x},\check{\mathbf{y}}}(l) q_{\mathbf{x},\check{\mathbf{y}}}(l) \bar{r}_{\mathbf{x},\check{\mathbf{y}}}(l) c_{\check{\mathbf{y}}}(l) = \\ &= \sum_{l=1}^{m-1} p_{\mathbf{x},\mathbf{y}}(l) q_{\mathbf{x},\mathbf{y}}(l) \bar{r}_{\mathbf{x},\mathbf{y}}(l) c_{\mathbf{y}}(l) + \\ &+ \bar{p}_{\mathbf{x},\mathbf{y}}(m-1) q_{\mathbf{x},\mathbf{y}}(m-1) v_{\mathbf{x},\check{\mathbf{y}}}(m-1) \end{aligned} \quad (30)$$

By substituting (30) in (29), we obtain:

$$\begin{aligned} v_{\mathbf{x},\mathbf{y}}(m-1) &= \sum_{k=y_m}^K p_{x_m,k} \tilde{r}_k c_{\mathbf{y}}(m) + \sum_{k=0}^{y_m-1} p_{x_m,k} v_{\mathbf{x},\mathbf{y}}(m) \\ &> \sum_{k=0}^K p_{x_m,k} \tilde{r}_k c_{\check{\mathbf{y}}}(m) \end{aligned} \quad (31)$$

By accounting for the hypothesis and by noting that $c_{\check{\mathbf{y}}}(m) = 1 - (m-1)\tau_s/T$, (31) constitutes a *reductio ad absurdum*.

ii) Case $E[R_{x_m}] < v_{\mathbf{x},\mathbf{y}}(m-1)/(1-(m-1)\tau_s/T)$. By following the same reasoning of *Case i)*, the thesis follows. \blacksquare

G. Proof of Theorem 3

Proof: We prove the Theorem by adopting a typical tool of computational complexity theory, i.e., reduction [18]. A reduction is a procedure for transforming one problem into another problem, and it can be used to show that the second problem is at least as difficult as the first. Specifically, we reduce Problem 2 to a notable NP-hard problem, the Traveling Salesman Problem (TSP), by showing that there exists a one-to-one mapping between Problem 2 and the single-machine job-scheduling problem with sequence-dependent setup times. Since such a problem can be polynomial-time reduced to the TSP, we have the thesis.

Let us focus on the contribute of the m -th sensed channel to the expected reward $V_{\mathbf{x},\mathbf{y}(\mathbf{x})}$:

$$p_{\mathbf{x},\mathbf{y}(\mathbf{x})}(m) q_{\mathbf{x},\mathbf{y}(\mathbf{x})}(m) \bar{r}_{\mathbf{x},\mathbf{y}(\mathbf{x})}(m) c_{\mathbf{y}(\mathbf{x})}(m) \quad (32)$$

From Algorithm 1, it follows that the m -th component $y_m(\mathbf{x}) \in \mathbf{y}(\mathbf{x})$ is a function of the last $M-m+1$ components of \mathbf{x} . Hence, by accounting for (4), (5) and (8), it follows that (32) depends on (x_m, \dots, x_M) . Furthermore, by accounting for (7), it follows that (32) depends on (x_1, \dots, x_{m-1}) . Hence, by denoting $y_m(\mathbf{x})$ as $f(x_m, \dots, x_M)$, it is easy to recognize that the expected reward $V_{\mathbf{x},\mathbf{y}(\mathbf{x})}$ achievable by using the sensing sequence $\mathbf{x} = (x_1, \dots, x_M)$ is equivalent to:

$$V_{\mathbf{x},\mathbf{y}(\mathbf{x})} = \sum_{m=1}^M g_{(x_1, \dots, x_M)}(m) \quad (33)$$

with $g_{(x_1, \dots, x_M)}(m)$ recursively defined as in (34), shown at the top of the next page.

By denoting with $s_{(x_1, \dots, x_M)}(m) = -g_{(x_1, \dots, x_M)}(m)$, we have:

$$\begin{aligned} \max_{\mathbf{x} \in \mathbf{X}} \{V_{\mathbf{x},\mathbf{y}(\mathbf{x})}\} &= \min_{\mathbf{x} \in \mathbf{X}} \left\{ \sum_{m=1}^M s_{(x_1, \dots, x_M)}(m) \right\} \\ &= \min_{\mathbf{x} \in \mathbf{X}} \left\{ \sum_{m=1}^M p_m + \sum_{m=1}^M s_{(x_1, \dots, x_M)}(m) \right\} \end{aligned} \quad (35)$$

Hence, solving Problem 2 is equivalent to solve the single-machine job-scheduling problem with: *i)* equal-release times $p_m = 0$; *ii)* sequence-dependent setup times $s_{(x_1, \dots, x_M)}(m)$. Since such a problem can be polynomial-time reduced [37], [38] to the TSP, we have the thesis. \blacksquare

$$\begin{aligned}
V_{\mathbf{x},\tilde{\mathbf{y}}} &= \sum_{l=1}^M p_{\mathbf{x},\tilde{\mathbf{y}}}(l) q_{\mathbf{x},\tilde{\mathbf{y}}}(l) \bar{r}_{\mathbf{x},\tilde{\mathbf{y}}}(l) c_{\tilde{\mathbf{y}}}(l) = \\
&= \sum_{l=1}^{m-1} p_{\mathbf{x},\mathbf{y}}(l) q_{\mathbf{x},\mathbf{y}}(l) \bar{r}_{\mathbf{x},\mathbf{y}}(l) c_{\mathbf{y}}(l) + p_{\mathbf{x},\tilde{\mathbf{y}}}(m) q_{\mathbf{x},\mathbf{y}}(m) \bar{r}_{\mathbf{x},\tilde{\mathbf{y}}}(m) c_{\mathbf{y}}(m) + \bar{p}_{\mathbf{x},\tilde{\mathbf{y}}}(m) q_{\mathbf{x},\mathbf{y}}(m) v_{\mathbf{x},\mathbf{y}}(m)
\end{aligned} \tag{26}$$

$$g(x_1, \dots, x_M)(m) = \begin{cases} \left(\sum_{l=1}^{M-1} \prod_{k < f(x_l, \dots, x_M)} p_{x_l, k} \right) \sum_{k \geq 0} p_{x_M, k} \bar{r}_k (1 - (M-1)\tau_s/T) & \text{if } m = M \\ \left(\sum_{l=1}^{m-1} \prod_{k < f(x_l, \dots, x_M)} p_{x_l, k} \right) \max \left\{ \begin{array}{l} \sum_{k \geq 0} p_{x_m, k} \bar{r}_k (1 - (m-1)\tau_s/T) \\ \sum_{k \geq f(x_m, \dots, x_M)} p_{x_m, k} \bar{r}_k (1 - m\tau_s/T) \end{array} \right\} & \text{if } 1 < m < M \\ \max \left\{ \begin{array}{l} \sum_{k \geq 0} p_{x_1, k} \bar{r}_k T \\ \sum_{k \geq f(x_1, \dots, x_M)} p_{x_1, k} \bar{r}_k (1 - \tau_s/T) \end{array} \right\} & \text{if } m = 1 \end{cases} \tag{34}$$

H. Proof of Lemma 5

Proof: By hypothesis, it results

$$p_{x_m, k} = P(R_{x_m} = \tilde{r}_k) = p_k \quad \forall x_m \in \Omega \tag{36}$$

Hence, by substituting (36) in (23), we have the thesis. \blacksquare

I. Proof of Theorem 4

Proof: We prove the thesis through backward induction.

i) *Case* $m = M$. The thesis follows by noting that, for any $y_m \neq 0$ and for any $\tau_s > 0$, it results:

$$\sum_{k=0}^K p_k \tilde{r}_k (1 - (m-1)\tau_s/T) > \sum_{k=y_m}^K p_k \tilde{r}_k (1 - m\tau_s/T) \tag{37}$$

ii) *Case* $m < M$. It is straightforward to prove the thesis by accounting for the results derived in Lemma 4 and Theorem 2. \blacksquare

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