Optimal Relay Station Placement in IEEE 802.16j Networks

Bin Lin, Pin-Han Ho, Liang-Liang Xie, and Xuemin (Sherman) Shen
Dept. of Electrical and Computer Engineering, University of Waterloo
Waterloo, Ontario, Canada
{b2lin, pinhan, xshen}@bbcr.uwaterloo.ca, llxie@ece.uwaterloo.ca

ABSTRACT

To make the WiMAX Point-to-Multi-Point (PMP) systems more competitive and applicable to the future metropolitan area networking scenarios, deploying relay stations (RSs) as defined in IEEE 802.16j has been considered a promising solution that can replace the 802.16e mesh mode for coverage extension and throughput enhancement. In this paper, we are committed to tackle the task of RS placement and relay time allocation in IEEE 802.16j Mobile Multi-hop Relay (MMR) networks, in order to meet the uneven distributed traffic demand of each subscriber station (SS) as well as the thirst for system capacity. By incorporating advanced cooperative relaying technology such as Decode-Forward (D-F) or Compress-Forward (C-F), the task of RS placement and relay time allocation is formulated into an optimization problem, aiming at finding the optimal location of a single RS and the resource allocation for all the SSs. Numerical analysis is conducted through a number of case studies to demonstrate the performance gain by using the proposed approach for relay placement and relay time allocation.

Categories and Subject Descriptors

C.2 [**Computer-Communication Networks**]: Network Architecture and Design—*Network topology, Wireless communication*

General Terms

Design

Keywords

Mobile Multi-hop Relay network, Decode-Forward, Compress-Forward, placement problem.

1. INTRODUCTION

Following the success of the IEEE 802.11 Wireless Local Area Networks (WLANs), the Wireless Metropolitan Area Network (WMAN) technology by way of IEEE 802.16 has been well recognized to serve as the backhaul of broadband wireless access in the emerging fourth-generation telecommunication system. However, the current deployment of fixed-infrastructure and mobile 802.16 networks exhibits certain inherent problems in practice, such as low signal-to-noise-ratio (SNR) at the cell edge, coverage holes that exist due to shadowing and non-light-of-sight (NLOS) connections, the access requirement of non-uniform distributed traffic in densely populated areas (e.g. hotspots). In order to meet the growing demand and stringent design requirements for coverage extension, throughput and capacity enhancement, deploying relay stations (RSs) has been considered as a promising solution to IEEE 802.16 Point-to-Multi-Point (PMP) networks, which are thus amended in standardization process of IEEE 802.16j Mobile Multi-hop Relay (MMR) by taking advantages of the less complexity and lower cost of RSs. Compared with IEEE 802.16e mesh mode, 802.16j has been identified with a better feasibility and efficiency due to the similarities in the MAC and PHY layers and the support of fast route change [1].

A network operator always desires the most cost-effective solution with the minimal deployment expenditure to provide a satisfactory service. The RS location for IEEE 802.16j MMR networks in the network planning stage is critical and will address fundamental impacts on the subsequent service provisioning scenario. In this study, we consider a four-tiered MMR network architecture, where in each cell a Base Station (BS) and a number of fixedly located Subscriber Stations (SSs) are presumably located. Based on hybrid Time Division Multiple Access /Code Division Multiple Access (TDMA/CDMA), the RSs are responsible for relaying and regulating the data transmission between the BS and SSs within the micro-RS-cell such that the SSs do not feel the existence of the RSs. In this case, the system capacity is determined not only by the location of each RS, but by the resource allocation at each RS in terms of the reserved relay time for each corresponding SS.

In this paper, we consider a practical deployment scenario where each SS represents a hotspot and imposes some amount of traffic demand during a specific time window. Without loss of generality, the RSs are nomadic, and are subject to repositioning in each time window. In a metropolitan area, a time window could be a few hours, and the traffic load of each SS may follow a time-of-a-day basis. Thus, our objective is to provide a general framework to maximize the system capacity by deploying the set of available nomadic RSs (NRSs) in an MMR cell according the location and traf-

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fic demand of each SS during each time window. We formulate and solve the single RS placement problem in multi-SS model in order to yield the optimal deployment and resource allocation for each single RS regarding a given set of SSs. We further incorporate advanced cooperative relay technology at physical layer, including Decode-Forward (D-F) and Compress-Forward (C-F), respectively, to enhance the overall system capacity in terms of the weighted sum of achievable rate at each SS. We expect that the proposed approach and results will provide a guideline for the infrastructure provider for the RS deployment and resource allocation in practice. To our best knowledge, this is the first study that specifically handles the RS placement and relay time allocation problem by employing a cooperative relay technology (e.g., D-F and C-F).

The remainder of the paper is organized as follows. We first review the related work in Section II. In Section III, the system model, including the network architecture, the channel propagation model, and the cooperative relay strategies, is presented. In Section IV, the single RS placement problem is formulated, which starts with the simplest 3-node relay model, followed by the formulation of single RS placement and relay time allocation in the presence of multiple SSs. Numerical analysis and case study are given in Section V. Section VI concludes this paper.

2. RELATED WORK

The RS placement in wireless communications plays an important role in the initial system design. Although to our best knowledge, there has not been any study reported for RS replacement exclusively for MMR networks, the related and similar problems have been extensively tackled in various application scenarios, such as wireless sensor networks(WSNs)[2][3][4][5][8], WLANs[6][7], and wireless relay /mesh networks (WMNs)[9][10][11], etc. The general objectives involve the efforts in optimizing deployment cost, network lifetime, system throughput or end-user throughput, in order to create a cost-effective carrier environment with on-demand resource allocation.

In WSNs, the coverage of each sensor is a key factor when node placement is considered. Some studies formulated the node placement problem by making analog to the *illumination* or *art gallery* problems [2]. In [3], sensor nodes (SNs) are assumed to be energy constrained while the relay nodes (RNs) are not. The optimal RN placement is modeled as a minimum set cover problem that was solved by a recursive algorithm. In [4], the RNs function as Cluster Heads, each governing a number of SNs. Based on a Far-Near Max-Min principle, an enhanced localized heuristic scheme was proposed to yield an optimal or near optimal solution. In [5], a number of deployment strategies for determining optimal placement of SNs, RNs, and Sink, were introduced, in order to achieve the required coverage, connectivity, bandwidth, and robustness. The placement problems for both reliable as well as unreliable/probabilistic detection models were formulated as Integer Linear Programs (ILPs).

In WLANs, the study in [6] introduced an extension point (EP) placement algorithm which aims at improving the network layer throughput of a rectilinear network by way of a divide-and-conquer searching algorithm. Similarly, to achieve optimal placement for a fixed number of RNs for relaying traffic in a two-hop fashion in a WLAN, the optimization problem was converted to a *p-median* problem in [7]. A solution based on Lagrange relaxation and sub-gradient optimization algorithms was presented.

In the aspect of WMNs, Internet Transit Access Points (ITAPs) was applied in [9] to serve as gateways to the Internet to form a Wireless Neighborhood Network. The algorithms were proposed to make informed placement decisions based on the neighborhood layouts, where the fault tolerance and workload variation were taken into account. In [10], the placement of relay infrastructure in heterogeneous WMNs was formulated as an integer programming which was solved using Bender's Decomposition algorithm.

3. SYSTEM MODEL

3.1 Network Architecture

The logic view of the four-tiered MMR network architecture, which consists of four network entities: the MMR-BS, the nomadic RSs, the fixed SSs, and the user terminals, is shown in Fig. 1.

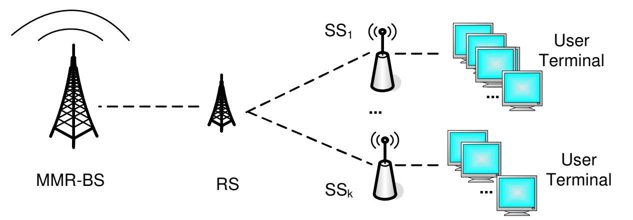


Figure 1: The logic view of the four-tier MMR architecture

The MMR-BS is equipped with sufficient intelligence that handles all the routing and signaling issues in the cell. The RSs are responsible for relaying data between the SSs and the BS. Note that the RSs considered in the study have sufficient power and are not directly connected to the wired backhaul. The links between the BS and RSs, and between the RSs and SSs are assumed relatively static, where a deterministic TDMA/CDMA scheme is utilized as the medium access technique in our system model. In other words, the long-term resources allocated to each SS in a time window will be pre-determined and jointly formulated in the RS placement problem (which will be described in Section III). Based on such a principle, the RSs relay data to/from a SS at each time slot by way of a cooperative relaying technology (like D-F or C-F). Here, we only consider the single RS placement problem, which does not consume extra bandwidth resource.

3.2 Channel Propagation Model

We adopt the Two-Ray Ground path loss model in [12], which is generally for modeling the large scale signal strength over the distance of several kilometers that use tall towers as well as for line-of-sight micro-cell channels in urban environments. Given the transmission power P_t and the Euclidean distance d between the transmitter and the receiver, the receiving power P_r is given by:

$$P_r = P_t G_t G_r h_t^2 h_r^2 d^{-\alpha} \quad (1)$$

where G_t (h_t) and G_r (h_r) are the gains (heights) of transmitter antenna and receiver antenna, respectively, α is the attenuation factor, which is dependent on the environment and typically vary in a range of 2-4 for terrestrial propagation. Obviously, the received power decreases as the distance between the transmitter and the receiver increases.

3.3 Cooperative Relay Strategy

Our approach is based on either a D-F or C-F technology for taking advantages of the spatial diversity. With the D-F scheme, the relay cooperates with the source by demodulating and decoding the received data packets, and forwarding them to the destination possibly using a different code. On the other hand, with the C-F scheme, the relay realizes the cooperation by sending a Wyner-Ziv compressed version of its received signal to the destination. The C-F scheme is especially suitable for the situation where the channel between the source and relay is worse than that between the source and destination.

By cooperating with D-F or C-F relaying, as shown in Fig. 2, an alternative connection can be established through two shorter-ranged links with better reliability and larger data rates [13]. The simplest relay channel models a class of three node communication networks. Cover and El Gamal [14] derived the following achievable rate for the Gaussian relay channel:

$$R = \min\{C(\frac{\theta P_s}{N_r}), C(\frac{P_s + P_r + 2\sqrt{\overline{\theta} P_s P_r}}{N_d})\} \tag{2}$$

where θ is the power allocation ratio at source node, $\overline{\theta} = 1-\theta$ $(0 < \theta < 1)$, the source and the relay have power constraint P_s and P_r, respectively. N_r and N_d are the noise power at the relay and destination node, and $C(.)$ is a function such that $C(x) = \frac{1}{2}\log(1+x)(x \geq 0)$, where B is the channel bandwidth.

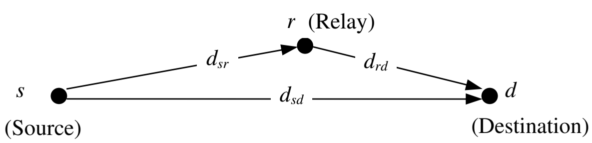


Figure 2: The 3-node relay channel model

The benefits of cooperative relaying are mainly due to the potential spatial diversity of the virtually formed antenna arrays at the source and the relay. Performance gain can be obtained and demonstrated via either a higher data rate or an increase in cell coverage at the same power level, or a reduced transmit power at the same data rate, or a macro-diversity gain that allows combating shadowing.

4. PROPOSED FORMULATION

The optimization problem of single RS placement for multiple stationary SSs is formulated in this section. We start with the formulation in a simple 3-node relay model as shown in Fig. 2, with a single source and destination.

4.1 Single RS Placement Formulation in 3-Node Relay Model

Given the location of source and destination along with the transmission power of the source and the relay, the objective is to find the optimal location of the relay such that the maximum data rate at the destination can be achieved by using a specific cooperative relay scheme (i.e., either D-F or C-F in the study). Since the received power decreases as the distance between the transmitter and the receiver increases, the optimal location of the relay must be on the line passing through the source and destination as shown in Fig. 3. Without loss of generality, we assume the reference distance between the source and destination is of unit length, i.e., $d_{sd} = 1, d_{sr} = x$, and thus $d_{sr} = 1 - x$. Our goal is to find the value of x such that the achievable rate is the maximal. In the following, we will investigate the cases with full duplex and half duplex modes, respectively.

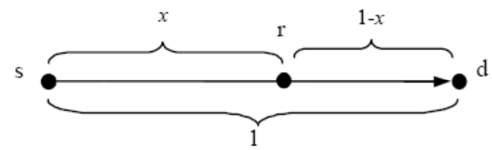


Figure 3: The simplified 3-node relay model.

- **Full Duplex**

In the full duplex mode, the relay can transmit and receive simultaneously. The problem formulation of the single relay placement in 3-node relay model in the full duplex mode is as follows.

Objective:

$$\max_x R^{full}_{relay} = \max_x\{\max(R^{full}_{DF}, R^{full}_{CF})\} \tag{3}$$

where R^{full}_{relay} is the maximal achievable rate by using a specific cooperative relay method, and R^{full}_{DF}, R^{full}_{CF}, is the achievable data rate with the D-F and C-F scheme in full duplex, respectively [15]. R^{full}_{DF} can be expressed as:

$$R^{full}_{DF} = \min(C(\frac{\theta P_s}{x^\alpha}), C(P_s + \frac{P_r}{(1-x)^\alpha} + 2\sqrt{\frac{\overline{\theta} P_s P_r}{(1-x)^\alpha}})), \tag{4}$$

and R^{full}_{CF} can be expressed as:

$$R^{full}_{CF} = C(\frac{P_s}{x^\alpha(1+\widehat{N}_2)} + P_s) \tag{5}$$

where $\widehat{N}_2 = \frac{P_s(x^{-\alpha}+1)+1}{P_r(1-x)^\alpha}$.

- **Half Duplex**

In the half-duplex mode with a time window T, let the relay be in the receiving mode for a fraction of the time yT (or termed the relay-receive period), and be in the transmitting mode for $(1-y)T$ (or termed the relay-transmit period). Similarly, the formulation of the relay placement in 3-node relay model in the half-duplex mode is as follows.

Objective:

$$\max_x R^{half}_{relay} = \max_x\{\max(R^{half}_{DF}, R^{half}_{CF})\} \tag{6}$$

where R^{half}_{relay} is the maximal achievable rate by using a specific cooperative relay method, and R^{half}_{DF} and R^{half}_{CF} is the achievable data rate with the D-F and C-F scheme in half duplex, respectively [15]. Thus, R^{half}_{DF} can be expressed as:

$$R^{half}_{DF} = B\min(R^{half}_{DF}(1), R^{half}_{DF}(2)), \tag{7}$$

where

$$R^{half}_{DF}(1) = yC(\frac{P_s}{x^\alpha}) + (1-y)C(\theta P_s), \tag{8}$$

$$R^{half}_{DF}(2) = yC(P_s) + (1-y)C(P_s + \frac{P_r}{(1-x)^\alpha} + 2\sqrt{\frac{\overline{\theta} P_s P_r}{(1-x)^\alpha}}), \tag{9}$$

and R_{CF}^{half} can be expressed as:

$$R_{CF}^{half} = yC(P_s + \frac{P_s x^{-\alpha}}{(1+\sigma)}) + (1-y)C(P_s), \quad (10)$$

where σ is the "compression noise" given by

$$\sigma = \frac{P_s x^{-\alpha} + P_s + 1}{((1 + \frac{P_r(1-x)^{-\alpha}}{1+P_s})^{(1-y)/y} - 1)(P_s + 1)} \quad (11)$$

4.2 Single RS Placement Formulation in Multi-SS Model

In the multi-SS model, a geographic service area is mapped into a unit square planar interpretation. The MMR-BS is located at the origin (0, 0). The RS employs a deterministic hybrid TDMA/CDMA scheme multiple-accessed by all the SSs within the micro-RS-cell. Let $I = \{i|1 \le i \le K\}$ denote the index of all the K SSs within the micro-RS-cell, $A = \{(x_1, y_1), \ldots, (x_K, y_K)\}$ and $P = \{\rho_1, \ldots, \rho_K\}$ denote the location coordinate and statistic traffic demand of each SS, respectively, $T = \{\tau_1, \ldots, \tau_K\}$ denote the reserved relay time fraction for each SS.

Given the statistic traffic demand P and location A of each SS, the infeasible design region Ω for RS, the objective is to find the optimal location (x, y) of the RS and the relay time allocation vector T such that each SS's achievable rate can satisfy its traffic load demand, meanwhile the overall system capacity C can be as large as possible.

Objective:

$$\max_{(x,y),T} C = \max_{(x,y),T} \sum_{i=1}^{k} R_{SS_i} = \max_{(x,y),T} \sum_{i=1}^{k} \tau_i r_i + (1-\tau_i) r_i^0 \quad (12)$$

Subject to:

$$R_{SS_i} \ge \rho_i, \forall i \quad (13)$$

$$\sum_{i=1}^{k} \tau_i = 1 \quad (14)$$

$$\tau_i \in [0, 1], \forall i \quad (15)$$

$$(x, y) \in \Omega \quad (16)$$

where R_{SS_i} is SS_i 's throughput, r_i is SS_i's relay rate (or the achievable rate with relaying),$r_i = \max\{R_{DF}^{full}, R_{CF}^{full}\}$ in the full duplex mode, or $r_i = \max\{R_{DF}^{half}, R_{CF}^{half}\}$ in the half duplex mode, and r_i^0 is the SS_i's rate without relaying (or the SS_i's maximum achievable rate through direct transmission with the BS).

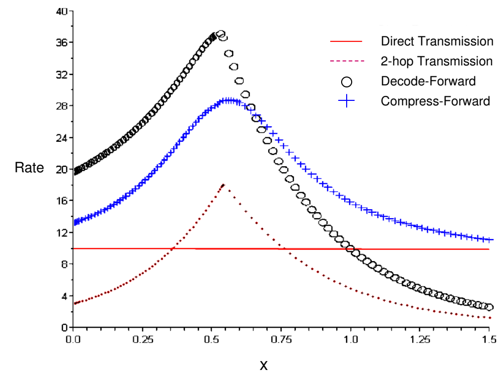
The objective function of (12) maximizes the overall system capacity. The constraint of (13) ensures that the throughput for each SS is larger than its traffic load demand. The constraint of (14) and (15) ensure the relay serving time for each SS. The constraint of (16) ensures that the location of the RS is not in an infeasible area, such as the mid of a river.

5. NUMERICAL STUDY

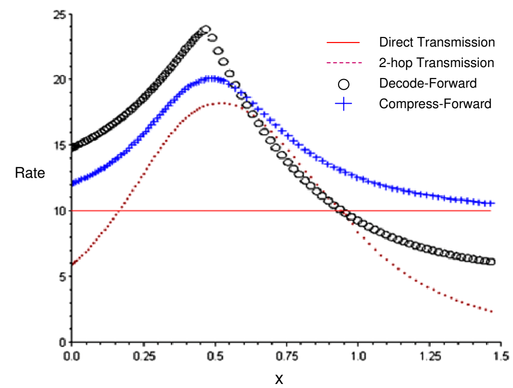
In this section, numerical results through a case study are presented to demonstrate the performance gain using the proposed approach.

5.1 Single RS Placement in 3-Node Relay Model

The transmission power of the BS and RS is set as 1w and 0.5w, respectively. The attenuation factor α is 3. The channel bandwidth is 20MHz. It is assumed all the receivers are subject to Additive White Gaussian Noise (AWGN) with zero mean and unit variance.



(a)



(b)

Figure 4: Achievable rate comparison in 3-node relay model. (a) Full-duplex, (b) Half-duplex

Fig. 4 shows the achievable rate vs. locations of RS in (a) the full duplex and (b) the half duplex mode. It is observed that the 2-hop transmission rate by introducing a relay node is not always larger than the direct transmission rate without the relay. In other words, the relay will not definitely increase the rate since it introduces extra hop which consume extra resources. However, as long as cooperative relaying (D-F or C-F) is employed, the achievable rate is always better than the 2-hop non-cooperative transmission rate and the direct transmission rate with extra relay power. Moreover, the upper bound of the rate gain in the D-F scheme can be obtained provided with the power constraint of the transmitter and the relay, respectively. On the other hand, the rate of the C-F scheme is larger than the D-F rate when the path between the source and relay is larger than that between the relay and destination. Thus, the C-F scheme can be used for receiver cooperation while the D-F scheme is more suitable for transmitter cooperation with the relay.

We define two metrics for performance gain: the relative throughput gain G_r^{full} (or G_r^{half}), and the absolute throughput gain G_a^{full} (or G_a^{half}) in the full duplex (or half duplex) mode, which aim to reflect the fairly throughput increase and absolute throughput increase with relay, respectively. $G_r^{full} = \frac{R_{relay}^{full}}{R_0}, G_r^{half} = \frac{R_{relay}^{half}}{R_0}, G_a^{full} = R_{relay}^{full} - R_0$, and

$G_a^{half} = R_{relay}^{half} - R_0$, where R_0 is the direct transmission rate without relaying.

By observing Fig. 5, the relative throughput gain is increased with the distance between BS and SS; while the absolute throughput gain is decreased with the distance between BS and SS. This observation gives insights for the single RS placement in the multiple SS scenario. For example, we consider the following situation where there are two SSs (say, nodes 1 and 2) within the micro-RS-cell, and the distance between node 1 and the BS is larger than that between node 2 and the BS, i.e., $d_{1-BS} > d_{2-BS}$. Let point P_1 and point P_2 denote the best points for node 1 and node 2 to achieve the maximum achievable rate, respectively. If the RS is deployed close to P_1, better max-min fairness can be achieved since the deployment aims to increasing the minimum data rate within the micro-RS-cell which results in an increase in the sum of relative throughput gain. If the RS is placed close to P_2, an immediate increase in the sum of absolute throughput gain and, consequently, a significant growth of overall throughput in the micro-RS-cell can be obtained. In the application scenario of interest, since each SS is subject to a given traffic demand instead of saturated traffic, fairness is not an issue to be considered in this study. Therefore, the proposed mathematic formulation for the problem simply takes the sum of the maximum achievable rate of all the SSs as the target, while taking the minimum required rate for each SS as the lower bound on the maximum achievable rate of each SS, as shown in Ineq. (13). With this, each SS will be provisioned on-demand in the presence of the RS.

5.2 A Case Study of Single RS Placement in Multi-SS Model

In the case study, we consider the scenario of eight SSs (SS1-SS8) randomly located in the area coordinated from 0.0 - 1.0. The coordinate of each SS is shown in Fig. 6, where the amount of traffic demand is proportional to its radius. In addition, the external interference is assumed to be constant and embedded into the thermal noise. The code cross correlation is assumed to be 10^{-4}. We still take the transmitting power of the BS and RS as 1w and 0.5w, respectively. The attenuation factor α is set to 3. The channel bandwidth is 20MHz.

All the simulations were conducted with the half duplex mode, where the RS is in the receiving mode for 50% fraction of time. By solving the optimization problem, the result of the optimal coordinate of the RS is (0.275, 0.221), which is also shown in Fig. 6. Fig. 7 illustrates the allocated relay time fraction for each SS, which shows a great variance with the normalized traffic demand of each SS. We also found that the effective throughput of each SS can meet the corresponding traffic demand with a proper RS placement and relay time allocation. To illustrate the importance of the relay time allocation, we compare our result with another allocation scheme, in which each reserved relay time fraction is proportional to their traffic demand. Although the overall capacity of the latter is slightly larger than that of the former, the throughput of the latter cannot meet the traffic load demand for all the SSs.

We further demonstrate the performance improvement in Fig. 8. Each SS's relay rate (i.e., the achievable rate with relaying) increases significantly, which contributes to the overall system performance gain. The system capacity due to the

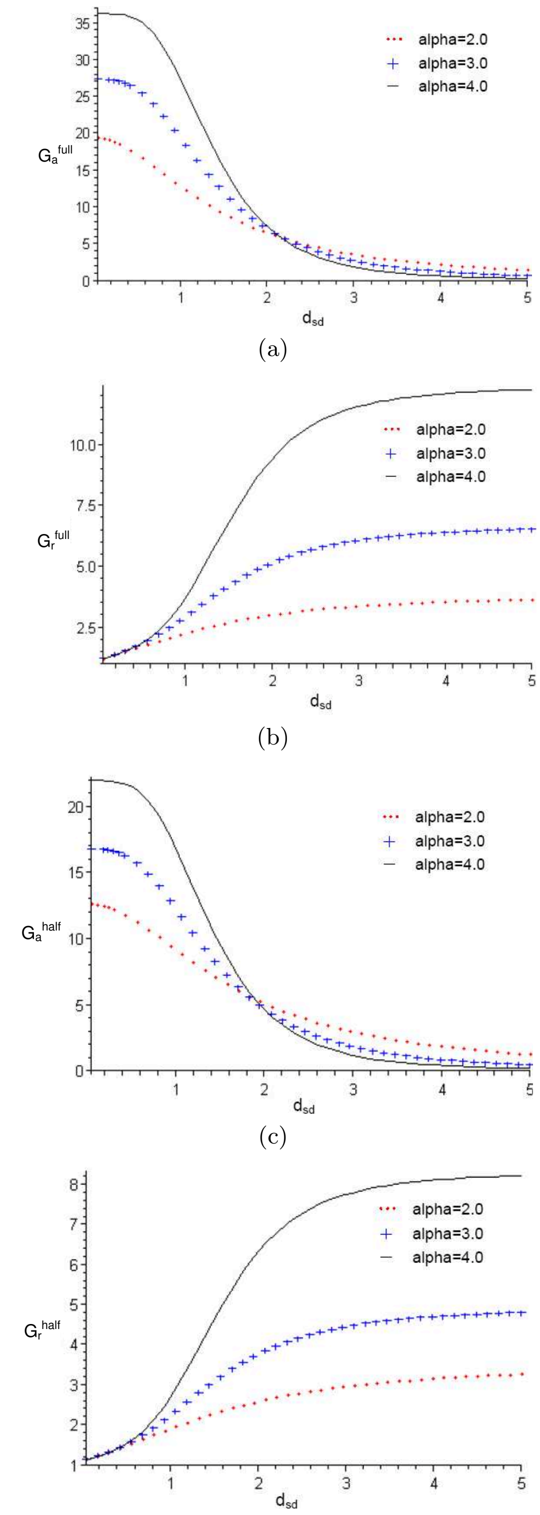


Figure 5: Maximum relay rate gain in the 3-node relay model: (a) absolute and (b) relative throughput gain in the full-duplex mode, and (c) absolute and (d) relative throughput gain in the half-duplex mode.

deployment of the RS increases at least 17% over the upper bound of that in the direct transmission case.

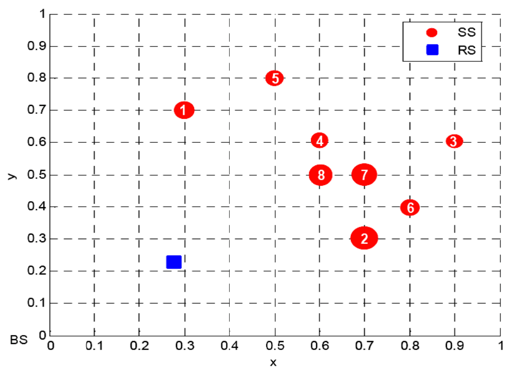


Figure 6: Topology and traffic demand distribution of all the SSs.

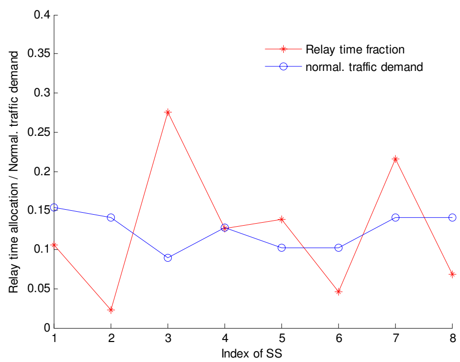


Figure 7: Relay time allocation for each SS.

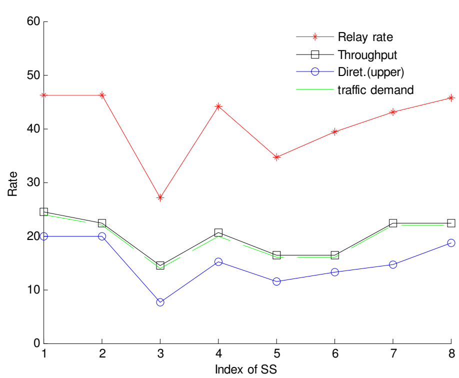


Figure 8: Performance improvement by optimal RS placement and relay time allocation.

6. CONCLUSIONS

In this paper, we have explored the single RS placement problem in IEEE 802.16j networks. Most notably, we have considered both the optimal RS location and relay time allocation in a single stage by incorporating an cooperative relay strategy of Decode-Forward (D-F) or Compress-Forward (C-F), which aims at maximizing the overall system capacity as well as meeting the traffic demand of each SS. Numerical analysis has been conducted by way of a number of case studies, which has demonstrated the performance gain due to relay placement and relay time allocation.

7. ACKNOWLEDGMENTS

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