Decentralized Feedback Stabilization of Multiple Nonholonomic Agents

(Invited Paper)

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***Abstract*— This paper represents an extension of our previous work [1], [2] on multiagent navigation to the case of decentralized control of multiple nonholonomic vehicles. Our main motivation comes from the field of air traffic management systems and from the field of micro robotic multiagent systems. A discontinuous feedback control scheme, based on dipolar navigation fields, is implemented and integrator backstepping is applied to suppress chattering behavior. The methodology has guaranteed global convergence and collision avoidance properties, which are verified by nontrivial computer simulations.**

INTRODUCTION

When it comes to multiagent coordination, decentralized approaches are more appealing to centralized ones, due to their reduced computational complexity and increased robustness wrt to agent failures. In the proposed methodology, each agent runs it's own controller which has knowledge of the agent's assigned target but ignores the targets of others. Through sensor measurements, each agent is capable of estimating other agents positions and velocities.

Nonholonomic feedback stabilization has attracted the attention of the control community over the years, due to the fact that nonholonomic systems do not satisfy the Brockett's necessary smooth feedback stabilization condition. Collision avoidance issues for nonholonomic systems have been addressed in [3] using a game-theoretic perspective, while in [4] the authors apply a cooperative suboptimal scheme to the problem.

In the proposed methodology we incorporate a discontinuous feedback control scheme based on dipolar navigation fields [5], [2]. Inherent in discontinuous control systems is the presence of high frequency switching known as chattering. This is very unfavorable when it comes to implementation on actual systems, since it causes excessive wear on the actuating system. We tackle this issue using integrator backstepping [6] which acts as a low pass filter and we provide theoretical guarantees that the resulting system will have the sought navigation properties.

Eventually nontrivial scenarios are simulated to verify the feasibility of the proposed scheme.

The rest of this paper is organized as follows: Section I states the problem and the related assumptions. Section II summarizes previous work and presents the decentralized multiagent navigation function for non-holonomic vehicles. In section III a two stage control scheme is presented which is formulated as a hybrid automaton in section IV. Section V presents simulation results while in section VI conclusions and issues for further research are discussed.

I. PROBLEM STATEMENT

Consider the following system of m nonholonomic vehicles:

$$\begin{aligned} \dot{x}_i &= u_i \cdot \cos(\theta_i) \\ \dot{y}_i &= u_i \cdot \sin(\theta_i) \\ \dot{\theta}_i &= w_i \end{aligned} \tag{1}$$

with $i \in \{1 \ldots m\}$. (x_i, y_i, θ_i) are the position and orientation of each agent, u_i and w_i are the translational and rotational velocities respectively.

The problem can be now stated as follows: *"Given the m nonholonomic systems (1), derive a control law that steers every system from any feasible initial configuration to its goal configuration avoiding collisions. The control law must be decentralized in the sense that each system has no knowledge of the targets of other systems."*

Assumptions:

- Environment is assumed perfectly known and stationary
- Each agent acts as a potential obstacle to the others.
- Agents have no information about other agents targets.
- Around the target of each agent $\mathcal{A}$ there is a region called the agent's $\mathcal{A}$ safe region
- Agent's $\mathcal{A}$ safe region is only accessible by agent $\mathcal{A}$, while regarded as an obstacle by other agents.
- The minimum distance between any two safe regions of any two agents is greater than the diameter of the largest agent.

II. MULTI-AGENT NAVIGATION FUNCTIONS

In previous work [1], [7] the authors presented an extension to the navigation function methodology with applications to multiple agent navigation. In this section we present how this novel class of potential functions can be enhanced with a dipolar structure [5] to provide trajectories suitable for nonholonomic navigation.

As it was shown in [1] the function: $\varphi_i = \frac{\gamma_{di}}{\left(\gamma_{di}^k + G_i\right)^{1/k}}$ with a proper selection of G_i can be used for decentralized motion planning of multiple holonomic agents and can be made a navigation function by an appropriate choice of k. Our assumption that we have spherical agents and spherical obstacles does not constrain the generality of this work since it has been proven [8] that navigation properties are invariant under diffeomorphisms. Methods for constructing analytic diffeomorphisms are discussed in [9], [10] for point agents and in [11], [12]for rigid body agents.

Let us assume the following situation: We have m mobile agents, and their workspace $W \subset R^2$. Each agent R_i, $i = 1 \ldots m$ occupies a disk in the workspace: $R_i = \left\{q \in R^2 : \|q - q_i\| \le r_i\right\}$ where $q_i \in R^2$ is the center of the disk and r_i is the radius of the agent. The position vector of the agents is represented by $q = [q_1 \ldots q_m]$. The orientation vector of the agents is represented by $\theta = [\theta_1 \ldots \theta_m]$ where θ_i represents the orientation of each agent . Let $\mathcal{W}_i \subseteq R^2 \times (-\pi, \pi]$ represent each agent's workspace. The configuration of each agent is represented by $p_i = \left[\begin{array}{cc} q_i & \theta_i \end{array}\right] \in \mathcal{W}_i$ and it's target by $p_{d_i} = \left[\begin{array}{cc} q_{d_i} & \theta_{d_i} \end{array}\right] \in \mathcal{W}_i$.

Dipolar Navigation Functions

To be able to produce a dipolar potential field, φ_i must be modified as follows:

$$\varphi_i = \frac{\gamma_{di}}{\left(\gamma_{di}^k + H_{nh_i} \cdot G_i \cdot \beta_{0_i}\right)^{1/k}} \tag{2}$$

where H_{nh_i} has the form of a pseudo - obstacle. A possible selection of H_{nh_i} would be:

$$H_{nh_i} = \varepsilon_{nh} + \eta_{nh_i}$$

with $\eta_{nh_i} = \|(q_i - q_{di}) \cdot \mathbf{n}_{d_i}\|^2$, where $\mathbf{n}_{d_i} = [\cos(\theta_{d_i}) \ \sin(\theta_{d_i})]^T$. Subscript d denotes destination. Moreover $\gamma_{di} = \|q_i - q_{d_i}\|^2$, i.e. the angle is not incorporated in the distance to the destination metric. $\beta_{0_i} = r_{world}^2 - \|q_i - q_{d_i}\|^2$ is the workspace bounding obstacle and r_{world} is the workspace radius. Figure 1 shows a 2D dipolar navigation function. As

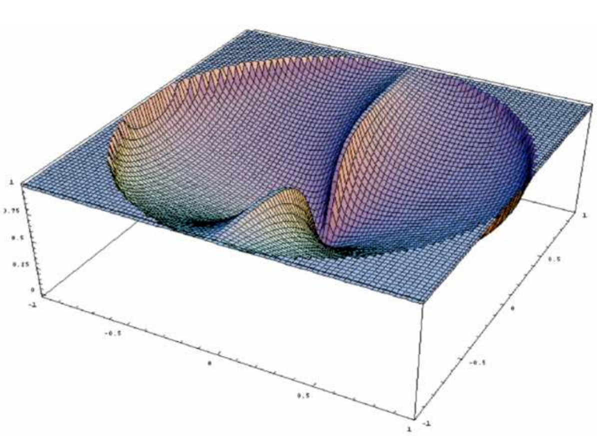

Fig. 1. 2D dipolar navigation function

is shown in [13], the proposed modifications of the potential function do not affect its navigation properties, as long as the workspace is bounded and $\varepsilon_{nh} > \varepsilon(k)$. An important feature that should be noticed is the fact that in the decentralized setup, the sense of the term "critical point" is slightly different than that of the centralized case [1]. The set of critical points of φ_i is defined as $C_{\varphi_i} = \{q : \partial\varphi_i/\partial q_i\} = 0$. A critical point is *non-degenerate* if $\partial^2\varphi_i/\partial^2 q_i$ has full rank at that point.

III. Nonholonomic Control

Thus far we have established that the dipolar function φ_i has navigation properties. We consider convergence of the multiagent system as a two stage process: In the first stage agents converge to a ball of radius ε called safe region, containing the desired destination of each agent. Each agent can get in its own safe region but not in others. The safe region of one agent is regarded as an obstacle from the other agents. Once an agent gets in its own safe region, it remains in the set and asymptotically converges to the origin.

Before defining the control we need some preliminary definitions:

We define by ${}^i\nabla^2\varphi_i(q_i, t)$ the hessian of φ_i. Let $\lambda_{\min}, \lambda_{\max}$ be the minimum and maximum eigenvalues of the hessian and $\hat{v}_{\lambda_{\min}}, \hat{v}_{\lambda_{\max}}$ the unit eigenvectors corresponding to the minimum and maximum eigenvalues of the hessian. Since navigation functions are Morse functions [8], [14], their Hessian at critical points is never degenerate, i.e. their eigenvalues have always nonzero values.

As discussed before, φ_i is a dipolar navigation function. The flows of the dipolar navigation field, provide feasible directions for nonholonomic navigation. What we need now is to extract this information from the dipolar function. To this extend we define the "nonholonomic angle":

$$\theta_{nh_i} = \begin{cases} \arg\left(\frac{\partial\varphi_i}{\partial x_i} \cdot s_i + \mathbf{i}\frac{\partial\varphi_i}{\partial y_i} \cdot s_i\right), & \neg P_1 \\ \arg\left(d_i \cdot s_i \left(v_{\lambda_{\min}}^x + \mathbf{i} v_{\lambda_{\min}}^y\right)\right), & P_1 \end{cases}$$

where proposition P_1 is used to identify sets of points that contain measure zero sets whose positive limit sets are saddle points:

$$P_1 = (\lambda_{\min} < 0) \wedge (\lambda_{\max} > 0) \wedge \left(\left|\hat{v}_{\lambda_{\min}} \cdot {}^i\nabla\varphi_i\right| < \varepsilon_1\right)$$

where

$$\begin{aligned} \varepsilon_1 &< \min_{C=\left\{q_i:\left\|q_i - q_{d_i}\right\|=\varepsilon\right\}} \left(\left\|{}^i\nabla\varphi_i(C)\right\|\right) \\ s_i &= \operatorname{sgn}\left(\left(q_i - q_{d_i}\right) \cdot \eta_{d_i}\right) \\ d_i &= \operatorname{sgn}\left(v_{\lambda_{\min_i}} \cdot {}^i\nabla\varphi_i\right) \\ \eta_{d_i} &= \left[\begin{array}{cc} \cos(\theta_{d_i}) & \sin(\theta_{d_i}) \end{array}\right]^T \\ \eta_i &= \left[\begin{array}{cc} \cos(\theta_i) & \sin(\theta_i) \end{array}\right]^T \end{aligned}$$

Before proceeding we need the following:

Lemma 1: If $\left|\hat{v}_{\lambda_{min}} \cdot {}^i\nabla\varphi_i\right| = 0$ then the set P_1 consists of the measure zero set of initial conditions that lead to saddle points

Proof: For $\left|\hat{v}_{\lambda_{min}} \cdot {}^i\nabla\varphi_i\right|$ to be zero we must either have

- ${}^i\nabla\varphi_i = \mathbf{0}$ which is the set of saddle points and the target. The target is excluded, since at the target $\lambda_{max} = \lambda_{min} > 0$ [13].

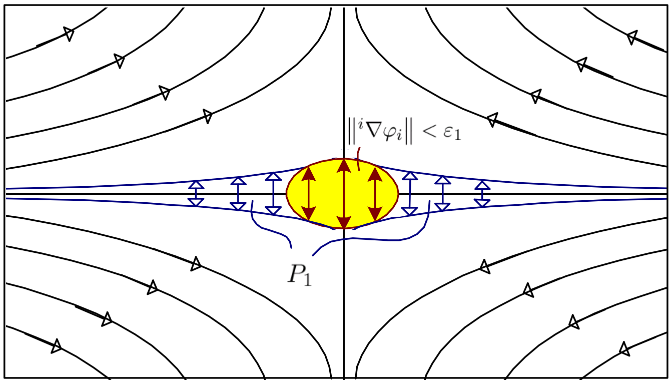

Fig. 2. Sets P_1 and $\|^i\nabla\varphi_i\| < \varepsilon_1$

- Or $\hat{v}_{\lambda_{min}} \cdot {}^i\nabla\varphi_i = 0$ with ${}^i\nabla\varphi_i \neq \mathbf{0}$. Since $\lambda_{\min} < 0 < \lambda\max$ the eigen-spaces $U_{\min} = span\{v_{\lambda_{\min}}\}$ and $U_{\max} = span\{v_{\lambda_{\max}}\}$ are linearly independent and $R^2 = span\{v_{\lambda_{\min}}, v_{\lambda_{\max}}\}$. But since ${}^i\nabla\varphi_i \subseteq R^2$ and $\hat{v}_{\lambda_{min}} \cdot {}^i\nabla\varphi_i = 0$, this means that ${}^i\nabla\varphi_i$ lies completely in $U_{\max}$. This can only occur at an extremal point of ${}^i\varphi_i(U_{\lambda_{min}})$ which means that we are at a "ridge" point of ${}^i\varphi_i$. For navigation functions, there can be no segment-type ridges, since at the end of such a ridge there would be a non-quadratic critical point [15] and since the navigation function is a Morse function, all critical points are non-degenerate and hence quadratic. Hence for the specified class of navigation functions, "ridge" points are sets of initial conditions of measure zero that lead to saddle points.

■

Figure 2 shows the sets $\|^i\nabla\varphi_i\| < \varepsilon_1$ and P_1 as defined above and the flows of $-\nabla\varphi_i$ near a saddle point in the set $\neg P_1$ and the flow directions imposed by the nonholonomic controller (3) in the set P_1.

In view of Lemma 1, ε_1 in P_1 can be chosen to be arbitrarily small so the sets defined by P_1 eventually consist of thin sets containing sets of initial conditions that lead to saddle points.

A suitable nonholonomic controller for the first stage is provided by the following:

Proposition 1: System (1) under the control law

$$\begin{aligned} u_i &= -\mathrm{sgn}\left({}^i\nabla\varphi_i \cdot \eta_i\right) \cdot \\ &\quad \cdot \left(K_{u_i} \cdot K_{z_i} + c_i \frac{\left|{\partial\varphi_i}/{\partial t}\right|}{\left|{}^i\nabla\varphi_i \cdot \eta_i\right|} \tanh\left(\left|{}^i\nabla\varphi_i \cdot \eta_i\right|^2\right)\right) \\ w_i &= \frac{\partial\theta_{nh_i}}{\partial t} + u_i\eta_i \cdot {}^i\nabla\theta_{nh_i} + (\theta_{nh_i} - \theta_i) \cdot \\ &\quad \cdot \left(K_{\theta_i} + c_i \frac{\left|{\partial\varphi_i}/{\partial t}\right|}{2\left(\theta_{nh_i} - \theta_i\right)^2} \tanh\left(\left|\theta_{nh_i} - \theta_i\right|^3\right)\right) \end{aligned} \tag{3}$$

converges to the set

$$B_i = \{p_i : \|q_i - q_{d_i}\| \leq \varepsilon - \delta, \quad \theta_i \in (-\pi, \pi]\}$$

, $i \in \{1 \ldots m\}$, almost everywhere[1] in $\mathcal{W}_i$. $K_{z_i} = \|^i\nabla\varphi_i\|^2 + \|q_i - q_{d_i}\|^2$ and K_{u_i}, K_{θ_i} are positive constants and $0 < \delta < \varepsilon$. c_i must be chosen such that $c_i > \frac{\varepsilon_2 + 1}{\varepsilon_2}$ where $\varepsilon_2 = 2\varepsilon_1^2\pi^3\left(4\varepsilon_1 + \sqrt{2}\cdot\pi^{3/2}\right)^{-2}$.

Proof: See Appendix A. ■

For the second stage each agent is isolated from the rest of the system. The dipolar navigation function (2) for this case becomes:

$$\varphi_{int_i}(x_i, y_i, \theta_i) = \frac{\gamma_{d,\theta_i}}{\left(\gamma_{d,\theta_i}^k + H_{nh_i} \cdot \beta_{int_i}\right)^{1/k}} \tag{4}$$

where $\beta_{int_i} = \varepsilon^2 - \|q_i - q_{d_i}\|^2$, and $\gamma_{d,\theta_i} = \|q_i - q_{d_i}\|^2 + (\theta - \theta_{d_i})^2$. Define

$$\Delta_i = K_{\theta_i} \cdot {\partial\varphi_{\mathrm{int}_i}}/{\partial\theta_i} \cdot (\theta_{inh_i} - \theta_i) - K_{u_i} \cdot K_{z_i} \cdot \left|{}^i\nabla\varphi_{\mathrm{int}_i} \cdot \eta_i\right|$$

and

$$\theta_{inh_i} = \arg\left(\frac{\partial\varphi_{\mathrm{int}_i}}{\partial x_i} \cdot s_i + \mathbf{i}\frac{\partial\varphi_{\mathrm{int}_i}}{\partial y_i} \cdot s_i\right)$$

Then, for each agent in isolation we have the following:

Proposition 2: Subsystem i of system (1) under the control law:

$$\begin{aligned} u_i &= -K_{u_i} \cdot K_{z_i} \cdot \mathrm{sgn}\left({}^i\nabla\varphi_{\mathrm{int}_i} \cdot \eta_i\right) \\ w_i &= K_{\theta_i}\left(\theta_{inh_i} - \theta_i\right), \quad \Delta_i < 0 \\ w_i &= -K_{\theta_i}\frac{\partial\varphi_{\mathrm{int}_i}}{\partial\theta_i}, \quad \Delta_i \geq 0 \end{aligned} \tag{5}$$

converges globally asymptotically to p_{d_i}.

Proof: See Appendix B. ■

Lemma 2: For the subsystem i of system (1) under the control law (5), the set

$$B_{\mathrm{int}_i} = \{p_i : \|q_i - q_{d_i}\| \leq \varepsilon, \quad \theta_i \in (-\pi, \pi]\}$$

is positive invariant.

Proof: The boundary of (4) is the set $\mathcal{B}_{int_i} = \{p_i : \beta_{int_i}(q_i) = 0\} = \{p_i : \|q_i - q_{d_i}\| = \varepsilon\} = \partial B_{\mathrm{int}_i}$, i.e. the workspace boundary, which is positive invariant for a navigation function [8],[13]. ■

Implementation Issues

Inherent in discontinuous controllers is the fast switching behavior which occurs over the sliding surfaces. According to Filippov [16], the solutions over the sliding surfaces are absolutely continuous and flow tangentially to the sliding surface. Unfortunately, due to the discrete time integration of the control signal, a behavior known as chattering emerges when it comes to discontinuous controllers. We tackle this issue by using integrator backstepping [17], [6], which acts as a low pass filter, smoothing out the system's behavior over the sliding surfaces. Consider the initial system (1) augmented with virtual states, as follows:

$$\begin{aligned} \dot{x}_i &= (k_{1_i} z_{1_i} + u_i) \cdot \cos(\theta_i) \\ \dot{y}_i &= (k_{1_i} z_{1_i} + u_i) \cdot \sin(\theta_i) \\ \dot{\theta}_i &= k_{2_i} z_{2_i} + w_i \\ \dot{z}_{1_i} &= -c_{1_i} z_{1_i} - v_i \\ \dot{z}_{2_i} &= -c_{2_i} z_{2_i} - \omega_i \end{aligned} \tag{6}$$

[1] i.e. everywhere except a set of initial conditions of measure zero

We can then state the following:

Proposition 3: If u_i and w_i are stabilizing controllers of system (1) and V_i a Lyapunov function, then system (6) under the control law:

$$\begin{aligned} v_i &= \frac{\partial V_i}{\partial x_i}\cos(\theta_i) + \frac{\partial V_i}{\partial y_i}\sin(\theta_i) \\ \omega_i &= \frac{\partial V_i}{\partial \theta_i} \end{aligned} \tag{7}$$

where k_1, k_2, c_1, c_2 positive constants, is globally asymptotically stable.

Proof: Consider the control Lyapunov function $V_a = V + \frac{k_1}{2}{z_1}^2 + \frac{k_2}{2}{z_2}^2$, where V is the Lyapunov function used to prove stability of controllers u and w (indices are dropped for notational brevity) . Taking the time derivative of V_a we get: $\dot{V}_a = \dot{V} + k_1 \cdot z_1 \cdot \dot{z}_1 + k_2 \cdot z_2 \cdot \dot{z}_2 = \frac{\partial V}{\partial t} + \dot{\mathbf{x}} \cdot \nabla V + k_1 z_1(-c_1 z_1 - v) + k_2 z_2(-c_2 z_2 - \omega) = {}^0\dot{V} + k_1 z_1 \left(\frac{\partial V}{\partial x}\cos(\theta) + \frac{\partial V}{\partial y}\sin(\theta)\right) + k_2 z_2 \frac{\partial V}{\partial \theta} + k_1 z_1(-c_1 z_1 - v) + k_2 z_2(-c_2 z_2 - \omega) = {}^0\dot{V} - k_1 c_1 z_1^2 - k_2 c_2 z_2^2 \leq 0$ where ${}^0\dot{V} \leq 0$ is the time derivative of the Lyapunov function of the initial system with the initial controllers, with the equalities holding only at the origin. ■

Of course system (1) can always perform the trajectories of (6).

IV. Controller Synthesis

As stated before convergence to the target is regarded as a two stage process. The system can be conveniently described in the framework of Hybrid Automata [18](see Figure 3). Let us define two discrete locations for each agent i, location $\mathcal{A}$ and location $\mathcal{B}$. Let ${}^iX_{0_A} = \mathcal{W}_i \backslash B_{\text{int}_i}$ and ${}^iX_{0_B} = B_{\text{int}_i}$ be the set of initial states of the two locations, ${}^iX_{F_A} = \partial B_{\text{int}_i}$ and ${}^iX_{F_B} = \{p_{d_i}\}$ be the set of final states of the two locations resp. To location $\mathcal{A}$ we assign the control law (3) augmented with the backstepping controller as in system (6) and denote it by f_1 and to location $\mathcal{B}$ the control law (5) augmented with the backstepping controller as in system (6) and denote it by f_2. We define the transition $\mathcal{A} \to \mathcal{B}$ to be the only feasible transition. We define $I(\mathcal{A}) = \mathcal{W}_i \backslash B_i$ and $I(\mathcal{B}) = B_{\text{int}_i}$ to be the invariants of locations $\mathcal{A}$ and $\mathcal{B}$ resp. We assign to the transition $\mathcal{A} \to \mathcal{B}$ the guard $\mathcal{A} \times \left\{\bar{B}_{\text{int}_i} \backslash \bar{B}_i\right\}$ where the bar denotes the internal of a set. By Proposition 1 we establish that transition $\mathcal{A} \to \mathcal{B}$ will eventually happen if our initial conditions are in ${}^iX_{0_A}$ and Lemma 2 establishes that no transition is possible from location $\mathcal{B}$.

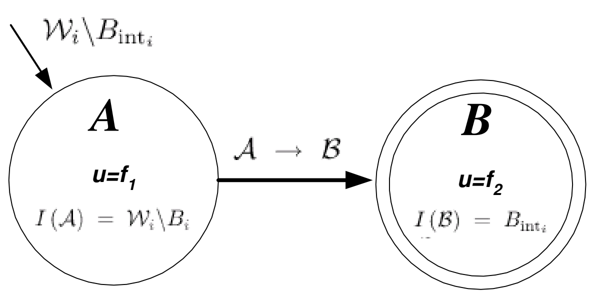

Fig. 3. Hybrid Automaton representation

V. Simulation Results

To demonstrate the navigation properties of our decentralized approach, we present three simulations of four nonholonomic agents that have to navigate from an initial to a final configuration, avoiding collisions. The chosen configurations constitute non-trivial setups since the straight-line paths connecting initial and final positions of each agent are obstructed by other agents. The following sequence of figures verifies the collision avoidance and global convergence properties of our algorithm. In each figure the circles denote the targets of each agent while the ring around each target represents the corresponding transition guard.

In the first simulation Fig. (4), the agents were placed at $p_1(0) = \begin{bmatrix} -.3 & -.3 & 0 \end{bmatrix}$ (red), $p_2(0) = \begin{bmatrix} 0 & -.3 & \pi/2 \end{bmatrix}$ (blue), $p_3(0) = \begin{bmatrix} .3 & -.3 & \pi \end{bmatrix}$ (pink), $p_4(0) = \begin{bmatrix} .3 & .3 & 0 \end{bmatrix}$ (green) and their targets were set at $p_{d1} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$, $p_{d2} = \begin{bmatrix} 0 & .3 & 0 \end{bmatrix}$, $p_{d3} = \begin{bmatrix} -.3 & 0 & 0 \end{bmatrix}$, $p_{d4} = \begin{bmatrix} .3 & 0 & 0 \end{bmatrix}$. The agents successfully navigate to their targets without collisions. Observe the change of behavior when entering the second mode of operation.

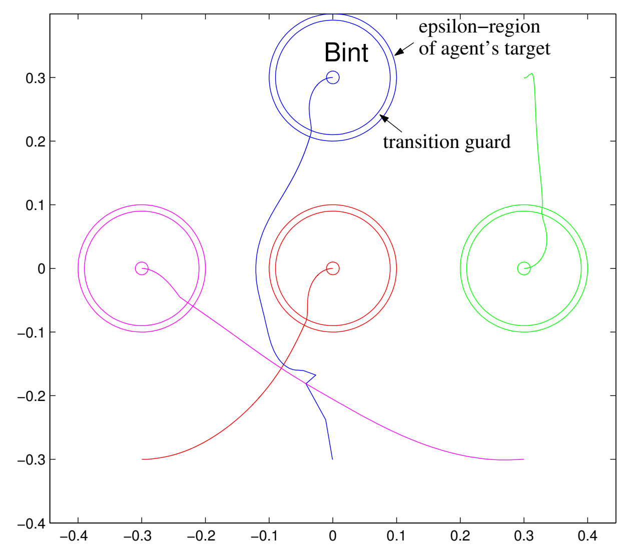

Fig. 4. Trajectories of Simulation 1

In the second simulation, Fig. (5), agents were placed at $p_1(0) = [\ 0 \quad .5 \quad -\pi/2\ ]$, $p_2(0) = [\ 0 \quad -.5 \quad \pi/2\ ]$, $p_3(0) = [\ -.5 \quad 0 \quad 0\ ]$, $p_4(0) = [\ .5 \quad 0 \quad \pi\ ]$, and their targets at: $p_{d1} = [\ 0 \quad -.3 \quad \pi\ ]$, $p_{d2} = [\ 0 \quad .25 \quad -\pi/4\ ]$, $p_{d3} = [\ .3 \quad 0 \quad 0\ ]$, $p_{d4} = [\ .25 \quad 0 \quad \pi\ ]$. And in this simulation, the proposed methodology achieves to navigate the agents avoiding collisions and stabilizes them at their targets.

In the third simulation Fig. (6), the initial conditions were set at: $p_1(0) = [\ -.4 \quad -.3 \quad 0\ ]$, $p_2(0) = [\ .25 \quad -.3 \quad \pi\ ]$, $p_3(0) = [\ 0 \quad .4 \quad 0\ ]$, $p_4(0) = [-\ .3 \quad 0 \quad 0\ ]$, and the targets were set at: $p_{d1} = [\ .4 \quad .2 \quad 0\ ]$, $p_{d2} = [\ -.1 \quad .3 \quad \pi\ ]$, $p_{d3} = [\ .1 \quad -.4 \quad -\pi/3\ ]$, $p_{d4} = [\ .3 \quad 0 \quad 0\ ]$. And in this situation, the algorithm drives the agents at their targets avoiding collisions.

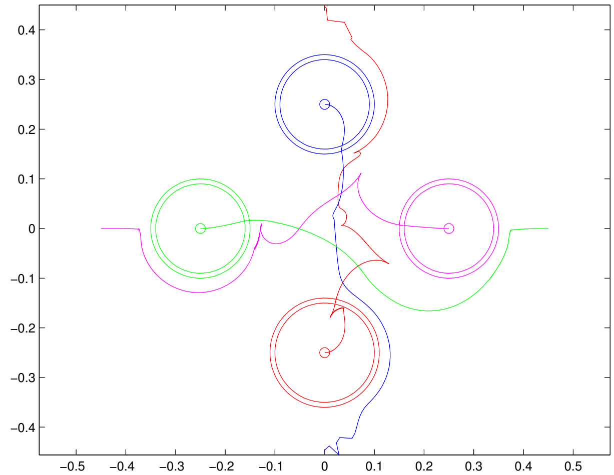

Fig. 5. Trajectories of Simulation 2

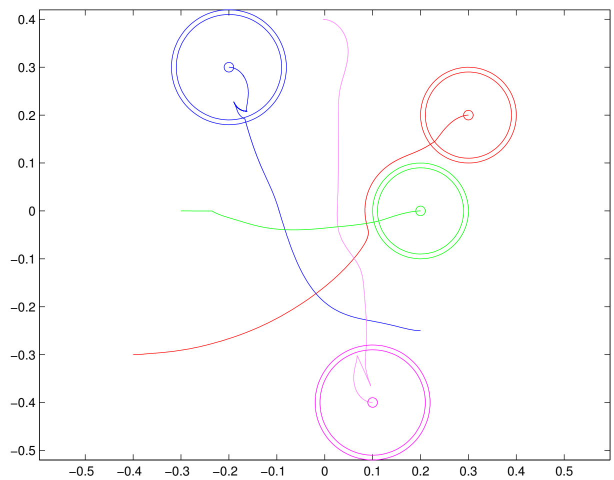

Fig. 6. Trajectories of Simulation 3

VI. Conclusions

In this paper a decentralized approach for navigation of multiple non-holonomic vehicles was presented. The convergence process is realized in two stages. Fist the agents converge to a safe region that contains their target. This region is inaccessible to all other agents. When there, each agent is stabilized at its target configuration. The control algorithms exploited the navigation properties of dipolar navigation functions and of decentralized multiagent navigation functions. A backstepping controller was implemented to suppress chattering, an inherent behavior in discontinuous controllers. The proposed methodology has guaranteed convergence and collision avoidance properties.

Further research issues include task based automated controller synthesis for multi-agent motion tasks, extending the multi-agent navigation schemes to air traffic management, and to apply input constraints to the system.

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Appendix A
Proof of Proposition 1

We form the following Lyapunov function:

$$V_i = \varphi_i\left(x_i, y_i, t\right) + \left(\theta_{nh_i}\left(x_i, y_i, t\right) - \theta_i\right)^2$$

and take it's time derivative:

$$\dot{V}_i = \frac{\partial\varphi_i}{\partial t} + u_i \eta_i{}^i\nabla\varphi_i + \\ +2\left(\theta_{nh_i} - \theta_i\right)\left(-w_i + \frac{\partial\theta_{nh_i}}{\partial t} + u_i\eta_i \cdot {}^i\nabla\theta_{nh_i}\right)$$

After substituting the control law u_i and w_i, we get: $\dot{V}_i = \frac{\partial\varphi_i}{\partial t} - \left({}^i\nabla\varphi_i \cdot \eta_i\right) \cdot \operatorname{sgn}\left({}^i\nabla\varphi_i \cdot \eta_i\right) \cdot \left(K_{z_i} + c_i \frac{\left|\partial\varphi_i/\partial t\right|}{\left|{}^i\nabla\varphi_i\cdot\eta_i\right|} \tanh\left(\left|{}^i\nabla\varphi_i \cdot \eta_i\right|^2\right)\right) - 2\left(\theta_{nh_i} - \theta_i\right)^2 \cdot$

$\left(K_{\theta_i} + c_i \frac{\left|\partial\varphi_i/\partial t\right|}{2\left(\theta_{nh_i}-\theta_i\right)^2} \tanh\left(\left|\theta_{nh_i}-\theta_i\right|^3\right)\right) \leq \frac{\partial\varphi_i}{\partial t} - c_i \left|\partial\varphi_i/\partial t\right| \left(\tanh\left(\left|{}^i\nabla\varphi_i\cdot\eta_i\right|^2\right) + \tanh\left(\left|\theta_{nh_i}-\theta_i\right|^3\right)\right)$. Since the set P_1 is by construction repulsive for ε_1 sufficiently small, we only need to consider the set $\neg P_1$. Then: $\left|{}^i\nabla\varphi_i\cdot\eta_i\right|^2 = \left\|{}^i\nabla\varphi_i\right\|^2\cos^2\left(\theta_{nh_i}-\theta_i\right)$. Let $\Delta\theta = \left|\theta_{nh_i}-\theta_i\right|$. After substituting we get: $\dot{V}_i \leq \frac{\partial\varphi_i}{\partial t} - \left|\partial\varphi_i/\partial t\right| \left(c_i\tanh\left(\left\|{}^i\nabla\varphi_i\right\|^2\cos^2\Delta\theta\right) + c_i\tanh\left(\Delta\theta^3\right)\right)$. Before proceeding we need the following:

Lemma 3: The following inequalities hold:

1) $\tanh(x) \geq \frac{x}{x+1}\ ,\quad x \geq 0$
2) $\frac{x}{x+1} + \frac{y}{y+1} \geq \frac{x+y}{x+y+1}\ ,\quad x, y \geq 0$
3) $\cos^2\Delta\theta \geq \frac{8}{\pi^3}\left(\left|\Delta\theta - \frac{\pi}{2}\right|\right)^3 \qquad \Delta\theta \in \left[0, \frac{\pi}{2}\right]$

Proof:

1) For $x \geq 0$ we have that $e^{2x} - 1 - 2x \geq 0$. Hence $(x+1)\left(e^x - e^{-x}\right) \geq x\left(e^x + e^{-x}\right)$ and we get the result: $\tanh(x) \geq \frac{x}{x+1}$. The equality holds at $x = 0$.
2) With $x, y \geq 0$ we have $\frac{x}{x+1} + \frac{y}{y+1} = \frac{2xy+x+y}{xy+x+y+1} \geq \frac{xy+x+y}{xy+x+y+1} \geq \frac{x+y}{x+y+1}$ and the equality holds at $x = y = 0$
3) Denote $A(\Delta\theta) = \cos^2\Delta\theta$ and $B(\Delta\theta) = \frac{8}{\pi^3}\left(\left|\Delta\theta - \frac{\pi}{2}\right|\right)^3$. Solving $A(\Delta\theta) = B(\Delta\theta)$, for $\Delta\theta \in \left[0, \frac{\pi}{2}\right]$ we get $\Delta\theta = 0$ for $A = B = 1$ and $\Delta\theta = \frac{\pi}{2}$ for $A = B = 0$. But at $\frac{\partial A}{\Delta\theta}_{|\Delta\theta=0} = 0 > -\frac{6}{\pi} = \frac{\partial \mathrm{B}}{\Delta\theta}_{|\Delta\theta=0}$ and since A and B have no other intersection for $\Delta\theta \in \left[0, \frac{\pi}{2}\right]$ it follows that $A(\Delta\theta) \geq B(\Delta\theta)$, for $\Delta\theta \in \left[0, \frac{\pi}{2}\right]$.

□

By use of Lemma 3.1 we get: $\dot{V}_i \leq \frac{\partial\varphi_i}{\partial t} - \left|\partial\varphi_i/\partial t\right| \left(c_i \frac{\left\|{}^i\nabla\varphi_i\right\|^2\cos^2\Delta\theta}{\left\|{}^i\nabla\varphi_i\right\|^2\cos^2\Delta\theta+1} + c_i\frac{\Delta\theta^3}{\Delta\theta^3+1}\right)$. By use of Lemma 3.2 we get: $\dot{V}_i \leq \frac{\partial\varphi_i}{\partial t} - \left|\partial\varphi_i/\partial t\right| c_i \left(\frac{\left\|{}^i\nabla\varphi_i\right\|^2\cos^2\Delta\theta+\Delta\theta^3}{\left\|{}^i\nabla\varphi_i\right\|^2\cos^2\Delta\theta+\Delta\theta^3+1}\right)$ and from Lemma 3.3 we get: $\dot{V}_i \leq \frac{\partial\varphi_i}{\partial t} - \left|\partial\varphi_i/\partial t\right| c_i \frac{\left\|{}^i\nabla\varphi_i\right\|^2\frac{8}{\pi^3}\left(\left|\Delta\theta-\frac{\pi}{2}\right|\right)^3+\Delta\theta^3}{\left\|{}^i\nabla\varphi_i\right\|^2\frac{8}{\pi^3}\left(\left|\Delta\theta-\frac{\pi}{2}\right|\right)^3+\Delta\theta^3+1}$. In view of the fact that the function $\frac{f(x)}{f(x)+1}$ has the same extremal points with $f(x) \geq 0$ (see [8] for a proof), the minimum of $\left[\frac{\left\|{}^i\nabla\varphi_i\right\|^2\frac{8}{\pi^3}\left(\left|\Delta\theta-\frac{\pi}{2}\right|\right)^3+\Delta\theta^3}{\left\|{}^i\nabla\varphi_i\right\|^2\frac{8}{\pi^3}\left(\left|\Delta\theta-\frac{\pi}{2}\right|\right)^3+\Delta\theta^3+1}\right.$ coincides with the minimum of $m = \left\|{}^i\nabla\varphi_i\right\|^2\frac{8}{\pi^3}\left(\left|\Delta\theta - \frac{\pi}{2}\right|\right)^3 + \Delta\theta^3$. Trying to minimize m, we get: $\frac{\partial m}{\partial\left\|{}^i\nabla\varphi_i\right\|} = \frac{16}{\pi^3}\left\|{}^i\nabla\varphi_i\right\|\left(\left|\Delta\theta - \frac{\pi}{2}\right|\right)^3 \geq 0$ which means that m is strictly increasing in the direction of $\left\|{}^i\nabla\varphi_i\right\|$. Examining $\frac{\partial m}{\partial\Delta\theta} = 3\cdot\Delta\theta^2 + \frac{24}{\pi^3}\left\|{}^i\nabla\varphi_i\right\|^2\cdot\left(\Delta\theta - \frac{\pi}{2}\right)^2\cdot sign\left(\Delta\theta - \frac{\pi}{2}\right)$ and requiring $\frac{\partial m}{\partial\Delta\theta} = 0$ for an extremum in the direction of $\Delta\theta$, we get:

$$\Delta\theta = \begin{cases} \dfrac{2\left\|{}^i\nabla\varphi_i\right\|\pi}{4\left\|{}^i\nabla\varphi_i\right\| \pm \sqrt{2}\cdot\pi^{3/2}} & \Delta\theta \leq \pi/2 \\[2ex] \dfrac{2\left\|{}^i\nabla\varphi_i\right\|\pi}{4\left\|{}^i\nabla\varphi_i\right\| \pm \mathbf{i}\sqrt{2}\cdot\pi^{3/2}} & \Delta\theta > \pi/2 \end{cases}$$

The only feasible solution is: $\Delta\theta = \frac{2\left\|{}^i\nabla\varphi_i\right\|\pi}{4\left\|{}^i\nabla\varphi_i\right\|+\sqrt{2}\cdot\pi^{3/2}}$. Substituting the solution in m we get: $\min_{\Delta\theta}(m) = \frac{2\left\|{}^i\nabla\varphi_i\right\|^2\pi^3}{\left(4\left\|{}^i\nabla\varphi_i\right\|+\sqrt{2}\cdot\pi^{3/2}\right)^2}$. Minimizing the last we get: $\frac{\partial \min_{\Delta\theta}(m)}{\partial\left\|{}^i\nabla\varphi_i\right\|} = \frac{4\sqrt{2}\left\|{}^i\nabla\varphi_i\right\|\pi^{9/2}}{\left(4\left\|{}^i\nabla\varphi_i\right\|+\sqrt{2}\cdot\pi^{3/2}\right)^3} \geq 0$. Activating the constraint $\left\|{}^i\nabla\varphi_i\right\| \geq \varepsilon_1$ we get: $\varepsilon_2 = \min(m) = \frac{2\varepsilon_1^2\pi^3}{\left(4\varepsilon_1+\sqrt{2}\cdot\pi^{3/2}\right)^2}$.

Substituting in the time derivative of the Lyapunov function, we have that: $\dot{V}_i \leq \frac{\partial\varphi_i}{\partial t} - \left|\partial\varphi_i/\partial t\right| c\frac{\varepsilon_2}{\varepsilon_2+1}$, so choosing $c > \frac{\varepsilon_2+1}{\varepsilon_2}$ we get that

$$\dot{V}_i \leq \left|\partial\varphi_i/\partial t\right| \left(sign\left(\partial\varphi_i/\partial t\right) - k\right) \leq 0$$

since

$$k = c\frac{\varepsilon_2}{\varepsilon_2+1} > 1$$

The equality holds when

$$\left(q_i = q_{d_i}\right) \wedge \left(\partial\varphi_i/\partial t = 0\right)$$

We assume that the system's initial conditions are in the set $\mathcal{W}_i \backslash \mathcal{S}_i$ where the set $\mathcal{S}_i = \left\{p_i : \left\|{}^i\nabla\varphi_i\right\| < \varepsilon_1\right\}$. ε_1 can be chosen to be arbitrarily small such that the set $\mathcal{S}_i$ includes arbitrarily small regions only around the saddle points and the target. Since we are considering convergence to the set B_i, we have that

$$\dot{V}_i < 0,\ \forall q_i \in W_{free}\backslash\left\{\bar{B}_i \cup \left\{q_i : \left\|{}^i\nabla\varphi_i\left(q_i\right)\right\| < \varepsilon_1\right\}\right\}$$

, where the bar denotes the set internal.

Remark 1: In practice we can choose for each agent i an ε_{1_i} such that $\varepsilon_{1_i} < \min\left\{\left\|{}^i\nabla\varphi_i\left(q_i(0)\right)\right\|,\ \varepsilon_1\right\}$. Since saddle points are excluded, as a set of measure zero, we can always calculate a finite c_i for the proposed controller, based on the initial conditions.

■

Appendix B
Proof of proposition 2

Taking $V_i = \varphi_{\mathrm{int}_i}$ as a Lyapunov function candidate, we have for the time derivative:

$$\dot{V}_i = \dot{\mathbf{x}}\cdot\nabla\varphi_{\mathrm{int}_i} = u_i\left({}^i\nabla\varphi_{\mathrm{int}_i}\cdot\eta_i\right) + w_i\,\partial\varphi_{\mathrm{int}_i}/\partial y_i$$

. We can now discriminate two cases, depending on the level of Δ_i:

1) $\Delta_i < 0$. Then $\dot{V}_i = -K_{u_i}K_{z_i}\left|{}^i\nabla\varphi_{\mathrm{int}_i}\cdot\eta_i\right| + K_{\theta_i}\left(\theta_{inh_i} - \theta_i\right)\partial\varphi_{\mathrm{int}_i}/\partial y_i = \Delta_i < 0$
2) $\Delta_i \geq 0$. Then $\dot{V}_i = -K_{u_i}K_{z_i}\left|{}^i\nabla\varphi_{\mathrm{int}_i}\cdot\eta_i\right| - K_{\theta_i}\left(\partial\varphi_{\mathrm{int}_i}/\partial y_i\right)^2 \leq 0$, with the equality holding only at the origin. ■