Capacity Of a Wireless Ad Hoc Network With Infrastructure

Benyuan Liu
Dept. of Computer Science
University of Massachusetts
Lowell, MA
bliu@cs.uml.edu

Patrick Thiran
EPFL-I&C-LCA
CH-1015 Lausanne
Switzerland
patrick.thiran@epfl.ch

Don Towsley
Dept. of Computer Science
University of Massachusetts
Amherst, MA
towsley@cs.umass.edu

ABSTRACT

In this paper we study the capacity of wireless ad hoc networks with infrastructure support of an overlay of wired base stations. Such a network architecture is often referred to as hybrid wireless network or multihop cellular network. Previous studies on this topic are all focused on the two-dimensional disk model proposed by Gupta and Kumar in their original work on the capacity of wireless ad hoc networks. We further consider a one-dimensional network model and a two-dimensional strip model to investigate the impact of network dimensionality and geometry on the capacity of such networks. Our results show that different network dimensions lead to significantly different capacity scaling laws. Specifically, for a one-dimensional network of n nodes and b base stations, even with a small number of base stations, the gain in capacity is substantial, increasing linearly with the number of base stations as long as $b \log b \leq n$. However, a two-dimensional square (or disk) network requires a large number of base stations $b = \Omega(\sqrt{n})$ before we see such a capacity increase. For a 2-dimensional strip network, if the width of the strip is at least on the order of the logarithmic of its length, the capacity follows the same scaling law as in the 2-dimensional square case. Otherwise the capacity exhibits the same scaling behavior as in the 1-dimensional network. We find that the different capacity scaling behaviors are attributed to the percolation properties of the respective network models.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and DesignWireless Communications

General Terms

Performance, Design, Theory.

Keywords

Capacity, wireless ad hoc networks, infrastructure support.

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1. INTRODUCTION

A wireless ad hoc network allows nodes to communicate with each other over a common wireless channel without support from fixed infrastructure. Recently there has been a growing interest in understanding how the throughput capacity scales with the number of nodes in such a network. In particular, the seminal work of Gupta and Kumar [7] has shown that the per-node capacity decreases at a rate of $1/\sqrt{n}$ in a network of n optimally located nodes, as n goes to infinity. Also, if the nodes are randomly located, the per-node capacity decreases by a factor of $\sqrt{\log n}$ to $1/\sqrt{n \log n}$. In a recent work by Franceschetti, Dousse, Tse, and Thiran [4], the authors show that a per-node capacity of $1/\sqrt{n}$ is indeed achievable in a random wireless network, thus providing a tight lower bound and closing the gap in the capacity of random wireless networks.

While some wireless ad hoc networks have no fixed infrastructure, other networks have some amount of infrastructure (e.g., base stations connected by high bandwidth links) available. These base stations neither produce data nor consume data. They support the underlying ad hoc networks by relaying data packets through the infrastructure. In such a network, data packets can be transported in a multi-hop fashion as in an ad hoc network or through the infrastructure as in a cellular network. The resulting mix of ad hoc and cellular network architecture is often referred to as hybrid wireless network or multihop cellular network [10, 3, 12, 2, 8]. The hybrid network architecture combines the advantages of both types of networks. It offers the local flexibility of ad hoc networks and efficient long-distance routing of infrastructure. An interesting question then is: how much does the additional infrastructure improve the capacity of pure wireless ad hoc networks.

In [11], the authors show the benefit of the infrastructure depends on the number of base stations relative to the number of ad hoc nodes. For a two-dimensional hybrid network of n ad hoc nodes and b base stations, the capacity only starts to increase effectively when the number of base stations is sufficiently large, namely, $b = \Omega(\sqrt{n})$. Otherwise, there is no significant capacity increase. Similar results are also reported in [13, 14]. A more specific scenario where the number base stations is on the same order as the number of nodes (i.e., $b = \Theta(n)$) is considered in [9]. If the nodes are asymptotically connected even without infrastructure, the per-node capacity is shown to scale as $\Theta\left(1/\log n\right)$. Moreover, when a less stringent requirement is imposed on the network connectivity, even though the capacity can be further improved, a per-node capacity arbitrarily close to $\Theta(1)$

cannot be achieved. This was further studied in [1], where the authors show that by power control and properly choosing the number of nodes assigned to a base station, it is possible to provide a capacity of $\Theta(1)$ to any fraction f, $0 < f < 1$, of nodes.

In this paper we revisit the capacity of a hybrid wireless network. Our work differs from and extends the previous work in the following critical ways.

First, in addition to the 2-dimensional network model considered in most of the previous network capacity studies, we further study the capacity of 1-dimensional and 2-dimensional strip hybrid wireless networks, which have not been studied before. The 1-dimensional network can be used to model scenarios such as highway, while the 2-dimensional strip can be used to model regions with a long extension in one particular direction such as valleys. In this work we investigate the impact of network dimensionality on the capacity. Our results show interesting different capacity scaling laws for these models.

Secondly, in most of the previous studies (e.g., [7, 11, 9, 1]), a node is assumed to transmit at a fixed rate when the interference condition is satisfied. This fixed transmission rate model does not capture the dependency of the achievable transmission rate of a point-to-point link on the SINR at the receiver. More realistically, the transmission rate decreases as SINR decreases in order to guarantee certain successful packet reception rate (e.g., rate control in IEEE 802.11). In this work we adopt a more realistic communication model that takes all interference into account and uses Gaussian channel capacity for achievable data rate between two nodes. Based on this model, recently established tight bounds on the capacity of random wireless networks will be used in our study [4].

The main results of this paper are summarized as follows. For a 1-dimensional network of n nodes and b base stations, even with a small number of base stations, the gain in capacity is significant, increasing linearly with the number of base stations as long as $b \log b \leq n$. However, this is not the case for a 2-dimensional square network, which requires a large number of base stations $b = \Omega(\sqrt{n})$ before we see such a capacity increase. An intuitive explanation is that in a 2-dimensional network, due to percolation, nodes form a wireless backbone (also referred to as percolation highway) to relay the traffic [4]. Therefore, the number of base stations has to be sufficiently large to have significant effects. However, it is impossible for a 1-dimensional network to percolate at finite node densities or emitting powers. As a result, wired base stations do compensate for the lack of wireless highway backbone and the benefit of infrastructure is more pronounced than in the 2-dimensional case. For a 2-dimensional strip network, we find that if the width of the strip is on the order of the logarithm of its length, the capacity behaves the same as in the 2-dimensional square case. Otherwise, if the width of the strip is asymptotically smaller than the logarithm of its length, the capacity is the same as for the 1-dimensional network case. This is because only at this length to width ratio, the *percolation highway*, in this case horizontal disjoint paths, start to emerge in the strip, resulting in the similarity between the 2-dimensional strip and square cases.

The rest of the paper is organized as follows. Section 2 presents the network model and the main results. The capacity of 1-dimensional interval, 2-dimensional square, and 2-dimensional strip networks are obtained in Section 3, 4, and 5, respectively. Finally, Section 6 concludes the paper.

2. NETWORK MODEL AND MAIN RESULTS

In this section, we describe the network model, summarize the main results, and provide explanations on the results.

2.1 Network model

We make the following assumptions for the random wireless network model.

- We consider three different network scenarios, namely, 1-dimensional interval $A_{\text{1-dim}} = [0, n]$, 2-dimensional square $A_{\text{2-dim}} = [0, \sqrt{n}] \times [0, \sqrt{n}]$ with side length $\sqrt{n}$, and 2-dimensional strip $A_{\text{2-dim strip}} = [0, n/w(n)] \times [0, w(n)]$ with length $n/w(n)$ and width $w(n)$. We assume that nodes are randomly distributed according to a Poisson point process of unit density. The length of the 1-dimensional interval model is n. The areas of the 2-dimensional square and 2-dimensional strip are both n. Therefore, the expected number of nodes is n for each of the network models.

- A total number of $b = b(n)$ base stations are regularly placed in the network. These base stations neither produce data nor consume data. They act solely as relay nodes and only engage in routing and forwarding data for normal wireless nodes. This scenario is often referred to as hybrid or multi-cellular network. The base stations are connected by high bandwidth long range links and there is no capacity constraint within the infrastructure. Also, the base stations are line powered and can transmit at any power level. We assume that the number b of base stations tends to infinity as $n \to \infty$, possibly at a much slower rate.

- Each node i can transmit with the same power P, and node j receives the transmitted signal with power $Pl(i, j)$, where $l(\cdot)$ indicates the path loss between i and j. Denote the Euclidean distance between the two nodes as d_{ij}. In this paper we assume $l(i, j) = \min\{1, d_{ij}^{-\alpha}\}$ with $\alpha > 1$ for 1-dimensional case and $\alpha > 2$ for 2-dimensional case. The results should readily extend to the case $l(i, j) = \min\{1, d_{ij}^{-\alpha} e^{-\gamma d_{ij}}\}$ with $\alpha > 0, \gamma > 0$ for both 1-dimensional and 2-dimensional networks.

- Source-destination pairs are chosen uniformly at random so that each node is the destination of exactly one source.

- The wireless channel follows a Gaussian white noise model with noise $N \sim \mathcal{N}(0, N_0)$. The achievable data rate from node i to node j is given by the channel capacity: $R_{ij} = \frac{1}{2}\log(1+\text{SINR})$ bps/Hz, where $\text{SINR} = Pl(i, j)/(N + \sum_{k \neq i} Pl(k, j))$ represents the signal to interference and noise ratio. Note that this channel model is more realistic than the fixed transmission model used in many previous studies on the capacity of hybrid wireless networks.

We say that an event occurs with high probability (w.h.p.) if its probability tends to 1 as $n \to \infty$. The per-node throughput capacity $T(n)$ of the network is defined as the number of bits per second that every node can transmit w.h.p. to its destination. The delay of a packet is the time it takes for the packet to reach the destination from the source. Thus, the per packet delay $D(n,b)$ is the sum of the times a packet spends at each relay node. As in previous capacity and delay studies [5, 4], we scale the packet size by the per-node capacity $T(n,b)$ so the transmission delay at each node is constant. Hence, the per-packet delay corresponds to the number of hops needed to reach its destination. Due to scaling, the per-packet delay $D(n,b)$ better captures the dynamics of the network. Since the base stations are well connected by high-bandwidth and long-range links, we do not account for the delay within the infrastructure.

2.2 Main results

The main results of this paper are summarized as follows.

THEOREM 1. *In a 1-dimensional network of interval* $[0,n]$*, assuming a power attenuation function of* $l(i,j)=\min\{1,d_{ij}^{-\alpha}\}$ *with* $\alpha>1$*, and a number of base stations* b *that goes to infinity as* $n\to\infty$*, a per-node capacity of*

$$T_{\textit{1-dim}}(n,b)=\Omega\left(\min\{b/n,1/\log b\}\right)$$

bits/sec is achievable w.h.p., with a corresponding average packet delay of at most

$$E_{\textit{1-dim}}[D(n,b)]=O(n/b\log n).$$

THEOREM 2. *In a 2-dimensional square network of size* $[0,\sqrt{n}]\times[0,\sqrt{n}]$*, assuming a power attenuation function of the type* $l(i,j)=\min\{1,d_{ij}^{-\alpha}\}$ *with* $\alpha>2$*, and a number of base stations* b *that goes to infinity as* $n\to\infty$*, a per-node throughput capacity of*

$$T_{\textit{2-dim}}(n,b)=\begin{cases}\Omega\left(1/\sqrt{n}\right) & \textit{if } b=O(\sqrt{n})\\ \Omega\left(\min\{b/n,1/\log b\}\right) & \textit{otherwise.}\end{cases}$$

bits/sec is achievable w.h.p., with a corresponding average packet delay of at most

$$E_{\textit{2-dim}}[D(n,b)]=\begin{cases}O(\sqrt{n}) & \textit{if } b=O(\sqrt{n})\\ O(\sqrt{n/b\log n}) & \textit{otherwise.}\end{cases}$$

THEOREM 3. *In a 2-dimensional strip network of size* $[0,n/w(n)]\times[0,w(n)]$*, assuming a power attenuation function of the type* $l(i,j)=\min\{1,d_{ij}^{-\alpha}\}$ *with* $\alpha>2$*, and a number of base stations* b *that goes to infinity as* $n\to\infty$*.*

- *If* $w(n)=\Omega(\log n)$*, the per-node capacity and average packet delay are the same as those of a 2-dimensional square network presented above.*
- *If* $w(n)=o(\log n))$*, the per-node capacity and average packet delay are the same as those of a 1-dimensional network.*

2.3 Discussions of results

For a 1-dimensional hybrid wireless network, our results show that even with a small number of base stations, the gain in capacity is significant, increasing linearly with the number of base stations b as long as $b\log b\le n$. However, this is not the case for a 2-dimensional network, which requires $b=\Omega(\sqrt{n})$ before we see such an capacity increase. If the number of base stations is not sufficiently large, i.e., $b=O(\sqrt{n})$, there is no capacity benefit from deploying infrastructure. The likely explanation is the impossibility for a 1-dimensional network to percolate at finite node densities or emitting powers, but not in the 2-dimensional network. As a result, in the 2-dimensional network, nodes can form a backbone highway to relay the traffic [4], without any additional wired infrastructure. This is impossible in a 1-dimensional network, where wired base stations do compensate for the absence of wireless highway backbone. We will also see that contrary to the 2-dimensional setting, pure ad hoc transmission is never used for any value of b in a 1-dimensional network. A similar observation on the influence of network dimensions on the usefulness of fixed infrastructure was already shown for network connectivity in the simple Boolean model [3]. We see that it carries on to capacity.

In fact, taking $b=\Theta(\sqrt{n})$, we find that $T_{\text{1-dim}}(n,b)=\Omega(1/\sqrt{n})$, which is the same result as the one that would be obtained in 2-dimensional networks. In the absence of infrastructure, different network dimensions can lead to significant differences in capacity. The addition of fixed infrastructure compensates for this difference between 1-dimensional and 2-dimensional networks. We know that for pure ad hoc networks, in the 1-dimensional and 2-dimensional cases,

$$\begin{aligned}T_{\text{1-dim}}(n) &= \Theta(1/n)\text{ bit/sec}\\ T_{\text{2-dim}}(n) &= \Theta(1/\sqrt{n})\text{ bit/sec,}\end{aligned}$$

whereas with $b=\Omega(\sqrt{n})$, these values become identical

$$T_{\text{1-dim}}(n,b)=T_{\text{2-dim}}(n,b)=\Omega(1/\sqrt{n})\text{ bit/sec.}$$

For a 2-dimensional strip network, if the width of the strip is on the order of the logarithmic of its length, the capacity exhibits the same scaling behavior as in the 2-dimensional square case. Otherwise the capacity exhibits the same scaling behavior as in the 1-dimensional network. This is because at this width to length ratio, the *percolation highway*, in this case, the horizontal disjoint paths, emerges and provides wireless backbone for data transport, as in a 2-dimensional square network. There is no percolation highway below this width to length ratio.

3. ONE-DIMENSIONAL NETWORK

For one-dimensional networks, we construct a data delivery scheme which consists of the following three phases.

1. *Phase 1: Multi-hop access to the base station.* Each source node sends packets in a multiple-hop fashion to the nearest base station.

2. *Phase 2: Infrastructure transport.* Packets are transported via the infrastructure from the ingress base station to the egress base station closest to the destination.

3. *Phase 3: Cellular broadcast to the destination.* The egress base station broadcasts the data packets to the destinations in its cell.

Figure 1 illustrates the three phases of the delivery scheme. In the following we describe each of the delivery phases in detail and derive the corresponding scaling laws for capacity and delay. Finally, we summarize the results and prove Theorem 1.

Phase 1: Multi-hop access to the base station.

We adopt TDMA scheme and detail the number of time slots that are needed to move one bit from every source

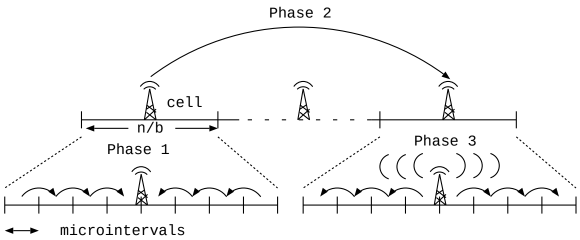

Figure 1: The 1-dimensional network is partitioned into cells of length n/b. Each cell is divided into micro intervals of length $c \log n$ so that all micro intervals are occupied with high probability. In Phase 1, packets are relayed along adjacent micro intervals to the nearest base station. In Phase 2, packets are transported from the ingress base station to the egress base station that is the closest to the destination. In Phase 3, packets are broadcast to destinations by the egress base station or routed using the reverse scheme from that of Phase 1 in a multi-hop fashion.

towards the nearest base station in a cell, in a multiple hop fashion. We do not use any result on the relay channel, but simply use the physical model defined by Gupta and Kumar [7], which treats all interferences as noise.

We first divide the interval $A = [0, n]$ into $n/c \log n$ *micro intervals*, of length $c \log n$ each, with $c > 1$. By adjusting c, we can adjust the probability that a micro interval contains at least one node (we will declare such a micro interval *occupied*). Indeed, this probability is

$$1 - \exp(-c \log n) = 1 - \frac{1}{n^c},$$

from which we find that the probability that all micro intervals are occupied is $\left(1 - \frac{1}{n^c}\right)^{n/c \log n}$, which tends to 1 as $n \to \infty$. We have thus the following lemma:

LEMMA 1. *Divide the network interval in micro intervals of length $c \log n$, with $c > 1$. Every micro interval is occupied w.h.p.*

We choose one node in each micro interval to relay traffic towards the nearest base station from the source. For these nodes, we find a TDMA schedule that is able to serve one node in each micro interval so that when a node transmits, other nodes that are sufficiently far away can simultaneously transmit, without causing excessive interference. Lemma 2 makes this precise, ensuring that a constant rate R, independent of n, can be achieved on all the simultaneous transmissions as $n \to \infty$. The proof is included in the appendix.

LEMMA 2. *For any given integer $d > 0$ there exists a TDMA scheme with $k = 2(d + 1)$ time slots, such that one node per micro interval can transmit to any destination located within a distance of d micro intervals with fixed rate $R(d)$ independent of n.*

The b base stations divide the network interval $[0, n]$ in b *cells*, each of length n/b. The following lemma gives an upper bound on the number of nodes in each cell.

LEMMA 3. *Suppose that there are $b = b(n)$ base stations, with $\lim_{n\to\infty} b(n) = \infty$. Then each cell contains at most $3 \max\{n/b, \log b\}$ nodes w.h.p.*

PROOF. Let us denote by X_i the number of nodes in the ith cell, $1 \le i \le n/b$. As it is a Poisson random variable of parameter n/b, Chernoff's inequality implies that for all $s > 0$

$$P(X_i > 3 \max\{n/b, \log b\}) \le \exp\left(-3s \max\{n/b, \log b\}\right) E\left[e^{sX}\right], \tag{1}$$

with $E[e^{sX}] = \exp((e^s - 1)n/b)$. We can always write

$$(e - 1)n/b - 3 \max\{n/b, \log b\} \le (e - 4) \log b.$$

Plugging this inequality in (1) and taking $s = 1$, we find therefore that

$$P(X_i > 3 \max\{n/b, \log b\}) \le b^{e-4},$$

from which we get that

$$P(X_i \le 3 \max\{n/b, \log b\} \text{ for all } i) \ge \left(1 - b^{e-4}\right)^b.$$

The right-hand-side of the above equation tends to 1 as n (and thus b) go to infinity. Hence, the probability at the left-hand-side tends to 1 also. □

All the nodes in a cell relay traffic to the base station located in it, in a multiple hop fashion. Lemma 1 ensures that there is at least one relay node in every micro interval and Lemma 2 ensures that it is possible to have direct transmissions between adjacent micro intervals at a rate R independent of n, using a given number, k, of time-slots, also independent of n. A node may have to relay traffic for at most all the nodes that are in a cell, which is at most $3 \max\{n/b, \log b\}$ according to Lemma 3. Consequently, in at most $3k \max\{n/b, \log b\}$ time slots, every node will have transmitted 1 bit towards a base station.

The length of each cell is n/b and the length of each micro interval is $c \log n$. There are a total number of $n/b \log n$ micro intervals in each cell. Since each packet is relayed along adjacent cells to the base station in the same cell, the average number of transmission hops is $O(n/b \log n)$.

Phase 2: Infrastructure transport.

Each packet is transported from the ingress base station to the egress base station closest to the destination. As we assume an infinite capacity between base stations, this can be considered as incurring zero delay.

In reality, the link capacity between base stations is finite. From the above arguments, at any given time, only one node communicates with each base station. Therefore, there is no additional constraint on the capacity if the capacity of each wired link is at least the rate R at which each node sends data to the destination when given the token by the TDMA scheme.

Phase 3: Cellular broadcast to the destination.

If we assume a cellular broadcast from the base station to every node in a cell (since the base station emitting power is not limited), we can get from the base station to the destination in a single hop. We adjust the power of the base station so that we can schedule transmissions in non-adjacent cells. Replacing "micro interval" by "cell" in Lemma 2, we need only $k = 2$ time slots to serve each node once per cell. As there are no more than $3 \max\{n/b, \log b\}$ nodes in a cell according to Lemma 3, we need at most $3k \max\{n/b, \log b\}$ time slots for this phase.

Alternatively, we can use a scheme that is the reverse of that of Phase 1, which requires also $3k \max\{n/b, \log b\}$ time slots, with k given by Lemma 5.

The proof of Theorem 1 follows directly from the above arguments.

Proof of Theorem 1. In Phase 1 and 3, every node can transmit one bit towards a base station once in every $3k \max\{n/b, \log b\}$ time slots. There is no capacity constraint in Phase 2. Hence, the capacity is

$$\begin{aligned} T_{\text{1-dim}}(n,b) &= 1/(3k \max\{n/b, \log b\}) \\ &= \Omega\left(\min\{b/n, 1/\log b\}\right) \text{ bit/sec.} \end{aligned}$$

Combining the number of transmission hops in Phase 1, 2, and 3, the average number of transmission hops per packet from source to destination is $D(n,b) = O(n/b \log n)$.

□

4. TWO-DIMENSIONAL NETWORK

In a two-dimensional square network with infrastructure, data packets can be delivered in the following two ways.

1. Data can be delivered through a *percolation highway* as described in [4].
2. Data can be delivered through *infrastructure* via the base stations.

We first briefly review on the capacity of random wireless networks without infrastructure. In such a network, when the density is above a certain threshold, nodes form an infinite connected cluster that percolates the network. The authors of [4] show that due to percolation, the wireless nodes form a rich wireless backbone (also referred to as percolation highway) that can carry packets across the network at constant rate. For data delivery, a node first sends its packets to the nearby entry point of the percolation highway, which carries the packets to the exit point near the destination, and finally makes the delivery to the destination. The capacity of this delivery scheme is proved to scale as $\Omega(1/\sqrt{n})$ and average packet delay is $\sqrt{n}$. Note that this delivery scheme achieves the capacity upper bound of $1/\sqrt{n}$ for an arbitrary wireless network [7], thus providing a tight lower bound for the capacity of random wireless networks. In the following we will use this tight lower bound in our derivation of the capacity of two-dimensional wireless networks with infrastructure support.

We divide the wireless channel frequency into two parts so that the two data delivery schemes (via percolation highway and via infrastructure) use different sub-channels. Thus there is no interference between these two delivery schemes. Assume the fraction of wireless channel bandwidth allocated to the *percolation highway* delivery scheme is f. The capacity of the *percolation highway* delivery scheme is known to be $\Omega(1/\sqrt{n})$ [4]. We only need to derive the capacity of the *infrastructure* delivery scheme, which is similar to that in the 1-dimensional case (illustrated in Figure 2).

Phase 1: Multi-hop access to the base station.

Lemma 4. *Divide the network into micro cells of size* $\sqrt{c \log n} \times \sqrt{c \log n}$*, with* $c > 1$*. Every micro cell is occupied w.h.p.*

The proof is the same as in the 1-dimensional case. Data traffic from a node is relayed along the micro cells that lie on

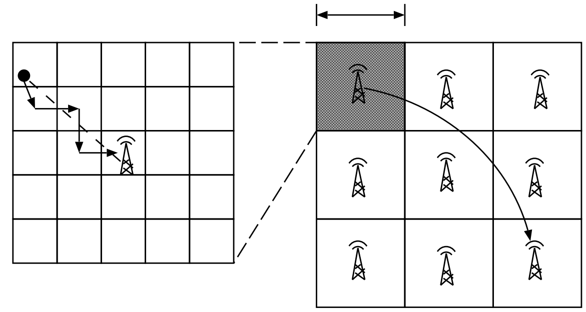

Figure 2: The 2-dimensional square network is partitioned into cells of size $\sqrt{n/b} \times \sqrt{n/b}$. Each cell is further divided into micro cells of size $\sqrt{c \log n} \times \sqrt{c \log n}$ so that all micro cells are occupied w.h.p. The delivery scheme is similar to that for 1-dimensional networks. Within a cell, packets are relayed between adjacent micro cells alone the line from the source to the base station.

the straight line from the node to the nearest base station (see Figure 2). We choose one node in each micro cell to relay the traffic going through the micro cell. For these nodes, we use a TDMA schedule that is able to serve one node in each micro cell so that when a node transmits, other nodes that are sufficiently far away can simultaneously transmit, without causing excessive interference. The following lemma makes this precise, ensuring that a constant rate R, independent of n, can be achieved on all the simultaneous transmissions as $n \to \infty$. The proof of this lemma is similar to that of Theorem 4 in [4], and therefore omitted here.

Lemma 5. *For any given integer* $d > 0$ *there exists a TDMA scheme with* $k = 2(d+1)$ *time slots, such that one node per micro interval can transmit to any destination located within a distance of* d *micro intervals with fixed rate* $R(d)$ *independent of* n*.*

The b base stations divide the square $[0, \sqrt{n}] \times [0, \sqrt{n}]$ in b *cells*, each of size $\sqrt{n/b} \times \sqrt{n/b}$. Since the number of base stations, b, tends to infinity as $n \to \infty$, the following lemma yields an upper bound on the number of nodes in each cell.

Lemma 6. *Suppose that there are* $b = b(n)$ *base stations, with* $\lim_{n\to\infty} b(n) = \infty$*. Then each cell contains at most* $3 \max\{n/b, \log b\}$ *nodes w.h.p.*

The proof is the same as in the 1-dimensional case. All the nodes in a cell relay traffic to the base station located in it, in a multiple hop fashion. Lemma 4 ensures that there is at least one relay node in every micro cell and Lemma 5 ensures that it is possible to adjust the power to have direct transmissions between adjacent micro cell at a rate R independent of n, using a given number k of time-slots, also independent of n. A node will relay traffic for at most all nodes in a cell, which is at most $3 \max\{n/b, \log b\}$ according to Lemma 6. Consequently, in at most $3k \max\{n/b, \log b\}$ time slots, every node will have transmitted one bit towards a base station. The per-node capacity is thus $\min\{b/n, 1/\log b\}$.

Each cell of size $\sqrt{n/b} \times \sqrt{n/b}$ is divided into micro cells of size $\sqrt{c \log n} \times \sqrt{c \log n}$. Each packet is relayed between adjacent micro cells along the line from source to the base station, the average number of hops is $O(\sqrt{n/b}/\sqrt{c\log n}) = O(\sqrt{n/b\log n})$.

The arguments for **Phase 2 (Infrastructure transport)** and **Phase 3 (Cellular broadcast to the destination)** remain the same as in the 1-dimensional network case. We are now ready to complete the proof of Theorem 2.

Proof of Theorem 2. Per above, the capacity of the *infrastructure* delivery scheme is the same as in the 1-dimensional case. From [4], the capacity of the delivery scheme through the *percolation highway* is $1/\sqrt{m}$. Therefore, the overall per-node capacity is

$$T_{\text{2-dim}}(n,b) = f/\sqrt{n} + (1-f)\min\{b/n, 1/\log b\} \qquad (2)$$

The scaling behavior of the per-node capacity depends on the number of base stations relative to the number of nodes.

- If $b = O(\sqrt{n})$, the capacity is maximized if the whole channel bandwidth is allocated to the delivery scheme through the *percolation highway* as in [4], i.e., $f = 1$. The corresponding capacity is $\Omega(1/\sqrt{n})$ bits/sec, and the delay is $\sqrt{n}$.
- If $b = \omega(\sqrt{n})$, the capacity is maximized if the whole channel bandwidth is allocated to the delivery scheme through the *infrastructure*, i.e., $f = 0$. The corresponding capacity is $\Omega(\min\{b/n, 1/\log b\})$ bit/sec. The average per-packet delay is $D(n,b) = O(\sqrt{n/b\log n})$.

□

5. TWO-DIMENSIONAL STRIP NETWORK

The two previous sections show that the increase in capacity brought by the infrastructure network strongly depends on the geometry of the area A where the network is deployed, and more specifically, by the emergence of giant connected component at a finite node density (percolation). This phase transition occurs in the plane, but not on the line. It is therefore interesting to further investigate the impact of the geometry of the area of deployment of network on the lower bounds we can obtain for transport capacity, by considering a random network where nodes are distributed on a 2-dimensional strip $[0, n/w(n)] \times [0, w(n)]$, where nodes are randomly distributed according to a Point process of unit density. The strip has width $0 < w(n) \le \sqrt{n}$, and the expected number of nodes in the interval is therefore n, as in the two other cases.

If $w(n) \ge C\log n$ (in other words, if $w(n) = \Omega(\log n)$), with $C > 1$ large enough, then according to [4], there exist a number of $\Omega(\sqrt{n})$ disjoint connected horizontal paths (percolation highway) in the strip, and the per-node capacity by routing packets through the percolation highway is the same as that in a 2-dimensional square network, i.e., $1/\sqrt{n}$. The capacity of the delivery scheme through infrastructure can be obtained using the same arguments as for the 2-dimensional square case. Therefore, the per-node capacity and delay remain the same as in a 2-dimensional square case.

In contrast, if $\lim_{n\to\infty} w(n)/\log n = 0$ (in other words, if $w(n) = o(\log n)$), then a simple adaptation of Theorem 11.55 in [6], p. 304, establishes that the probability that there is a path connecting the left and right sides of the strip is zero as $n \to \infty$, and therefore excludes the existence of the highway backbone. The proof is provided in the Appendix. In this case, the only lower bound that we can use is the 1-dimensional bound.

6. CONCLUSIONS

In this paper we investigate the impact of network dimensionality and geometry on the capacity of hybrid wireless networks. Our results show that the benefit of the infrastructure to the network capacity mainly depends on the percolation properties of the network models. Different network dimensions can lead to significantly different scaling laws in capacity.

7. ACKNOWLEDGMENTS

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APPENDIX

A. PROOF OF LEMMA 2

Let us label each micro interval with an integer coordinate, ranging from 1 to $(n/c \log n)$. Consider micro intervals whose coordinate is a multiple of k where $k = 2(d+1)$ for integer $d > 0$. By translation, we can construct k disjoint equivalent subsets, as illustrated in Figure 3. This allows us to build the following TDMA scheme: we define k time slots, during which only nodes from a particular subset are allowed to emit. We assume also that at most one node per micro interval emits at the same time, and that they all emit with the same power P.

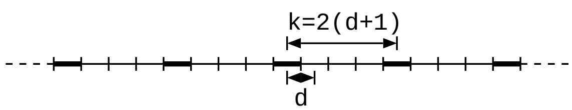

Figure 3: Construction of equivalent subsets in the TDMA scheme.

To find an upper bound on the interferences, we observe that with this choice, the emitters in the 2 first closest micro intervals are located at a distance at least $2(d+1) - d = d+2$ (in the integer coordinate of micro intervals) from the receiver. This means that the Euclidean distance between the receiver and the 2 closest interferers is at least $c(d+1)$. The 2 next closest squares are at distance at least $2 \times 2(d+1) - d = 3d+4$ (in micro intervals), and the Euclidean distance between the receiver and the 2 next interferers is therefore at least $c(3d+3)$, and so on. The sum of the interferences $I(d)$ can be bounded as follows:

$$
\begin{aligned}
I(d) &\leq \sum_{i=1}^{\infty} 2\, Pl(c(2i-1)(d+1)) \\
&= 2P \sum_{i=1}^{\infty} P \min\{1, [c(2i-1)(d+1)]^{-\alpha}\} \\
&\leq 2P \sum_{i=1}^{\infty} [c(2i-1)(d+1)]^{-\alpha} \\
&= 2P[c(d+1)]^{-\alpha} \sum_{i=1}^{\infty} (2i-1)^{-\alpha}.
\end{aligned}
$$

The sum in $I(d)$ clearly converges if $\alpha > 1$.

Now we want to bound from below the signal received from the emitter. We observe first that the distance between the emitter and the receiver is at most $c(d+1)$. The strength $S(d)$ of the signal at the receiver is therefore lower bounded by

$$S(d) \geq Pl(c(d+1)) = P \min\{1, [c(d+1)]^{-\alpha}\}.$$

Finally, we obtain a bound on the signal-to-interference ratio

$$
\begin{aligned}
SINR(d) &= \frac{S(d)}{N_0 + I(d)} \\
&\geq \frac{P \min\{1, [c(d+1)]^{-\alpha}\}}{N_0 + 2P[c(d+1)]^{-\alpha} \sum_{i=1}^{\infty} (2i-1)^{-\alpha}},
\end{aligned}
$$

which does not depend on n. The proof is concluded by noting that the capacity between two nodes is $\frac{1}{2} \log(1 + SINR)$ bps/Hz.

B. PROOF OF THE ABSENCE OF HIGHWAY BACKBONE WHEN $W(N) = O(\log N)$

We prove that the highway backbone network disappears when the width of the strip $\lim_{n \to \infty} w(n)/\log n = 0$. Remember that to use this approach [4], the strip A is divided in squares of size $c \times c$ for some $c > \log 2$ so that the corresponding bond percolation model in the plane is supercritical, because the open edge probability $p = 1 - \exp(-c^2) > 1/2$ [4]. Using a direct adaptation of the proof of Theorem 11.55 in [6], p. 304, we can indeed prove that if the area of the strip is too narrow, then the network is sub-critical, even if $p > 1/2$. More precisely, for any $c > 0$ that does not depend on n, and thus any $p = 1 - \exp(-c^2) < 1$, the probability that there is an open path crossing the strip horizontally from its left side to its right side is zero as $n \to \infty$.

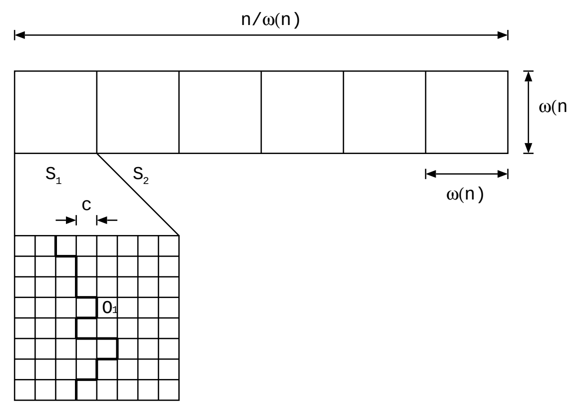

Figure 4: The 2-dimensional strip is divided into a number of $n/w^2(n)$ squares of size $w(n) \times w(n)$. The center of square S_k is O_k. The bond percolation model is constructed by dividing the strip into grids of side length c. Each edge is open with probability p. A path is open or closed if it contains only open or closed edges, respectively. A close path in S_1 is depicted using bold lines.

Assume for simplicity and without loss of generality that n is a multiple of $w^2(n)$ and that $w(n)$ is even, divide the strip A in $n/w^2(n)$ adjacent and disjoint square boxes $S_k = [(k-1)w(n), kw(n)] \times [0, w(n)]$ for $1 \leq k \leq n/w^2(n)$, whose centers are respectively denoted by $O_k = ((2k-1)w(n)/2, w(n)/2)$, as depicted in Figure 4.

Denote by $O_k \to \overline{S_k}$ the existence of a closed (or open) path from O_k to the top side of S_k, by $O_k \to \underline{S_k}$ the existence of a closed (or open) path from O_k to the bottom side of S_k, by $O_k \leftrightarrow \Box S_k$ the existence of a closed (or open) path from O_k to both the top and bottom sides of S_k, and by

$O_k \leftrightarrow \partial S_k$ the existence of a closed (or open) path from O_k to the perimeter of S_k. The path property (open or closed) will be indicated on top of the arrow. Let A_k be the event that there is a closed path connecting the top and bottom sides of S_k. With the usual notation $\mathbb{P}_p(\cdot)$ for the product measure on the grid when the open edge probability is p, we find

$$\begin{aligned}
\mathbb{P}_p(A_k) &\geq \mathbb{P}_p(O_k \overset{\text{closed}}{\longleftrightarrow} \square S_k) \\
&= \mathbb{P}_{1-p}(O_k \overset{\text{open}}{\longleftrightarrow} \square S_k) \\
&\geq \mathbb{P}_{1-p}(O_k \overset{\text{open}}{\longrightarrow} \overline{S_k}) \cdot \mathbb{P}_{1-p}(O_k \overset{\text{open}}{\longrightarrow} \underline{S_k}) \quad (3) \\
&= \left(\frac{1}{4}\mathbb{P}_{1-p}(O_k \overset{\text{open}}{\longleftrightarrow} \partial S_k)\right)^2 \quad (4) \\
&\geq \left(\frac{\rho}{4w(n)} \exp\left(-\frac{w(n)}{2\xi(1-p)}\right)\right)^2 \quad (5)
\end{aligned}$$

where (3) follows from FKG's inequality (see [6], p. 34), (4) follows from symmetry arguments, and (5) follows from Theorem 6.10 in [6], p. 120 (Remember that $p > 1/2$). In this latter expression, $\rho > 0$ is independent of p and n, and $\xi(1-p)$ is the correlation length when the open edge probability is $(1-p)$ [6], p. 127. It only depends on $1 - p = \exp(-c^2)$. The analytical expression of $\xi(q)$ as a function of $0 < q < 1/2$ is not known, but it enjoys a few properties, namely to be continuous and strictly increasing in its argument $0 < q < 1/2$, with $\xi(q) \to 0$ when $q \downarrow 0$ and $\xi(q) \to \infty$ when $q \uparrow 1/2$.

Summing over all boxes $1 \leq k \leq n/w^2(n)$, we find that

$$\sum_{k=1}^{n/w^2(n)} \mathbb{P}_p(A_k) \geq \frac{\rho^2}{16}\frac{n}{w^4(n)} \exp\left(-\frac{w(n)}{\xi(1-p)}\right). \quad (6)$$

Now, as $\lim_{n\to\infty} w(n)/\log n = 0$ and as $\xi(1-p) = \xi \exp(-c^2) > 0$ for all possible $c > 0$, there is some $N \in \mathbb{N}$ and some $0 < \varepsilon < 1$ such that $w(n) \leq (1-\varepsilon)\xi(1-p)\log n$ for all $n \geq N$. Therefore for $n \geq N$, (6) yields that

$$\sum_{k=1}^{n/w^2(n)} \mathbb{P}_p(A_k) \geq \frac{\rho^2}{16(1-\varepsilon)^4\xi^4(1-p)}\frac{n^\varepsilon}{\log^4 n}$$

and thus that as $n \to \infty$,

$$\sum_{k=1}^{n/w^2(n)} \mathbb{P}_p(A_k) \to \infty,$$

which implies that a.s. infinitely many of the events A_k occur. As the events A_k are independent because the boxes S_k are disjoint, it implies therefore that (almost surely) no open path connects the left and right sides of the strip, and thus that no highway exists in this network.