On the Higher-Order Statistics of the Channel Capacity in Dispersed Spectrum Cognitive Radio Systems over Generalized Fading Channels

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***Abstract*—This work is devoted to the study of dispersed spectrum cognitive radio (CR) systems over independent and non-identically distributed generalized fading channels. More specifically, this is performed in terms of the high-order statistics of the channel capacity over $\eta-\mu$ fading channels. A generic analytic expression is derived for the corresponding n-th statistical moment, which is subsequently employed for deducing exact closed-form expressions for the first four moments. Using these expressions, important statistical metrics such as the amount of dispersion, amount of fading, skewness and kurtosis are derived in closed form and can be efficiently used in providing insights on the performance of dispersed CR systems. The obtained numerical results reveal interesting outcomes that could be useful for the channel selection, either for sharing or aggregation in heterogeneous networks which is the core structure of future wireless communication systems.**

***Index Terms*—Cognitive radio, dispersed spectrum, channel capacity, high-order statistics, amount of dispersion.**

I. Introduction

COGNITIVE radio (CR) is becoming an emerging technology for the next generation of wireless communication systems, i.e. 5th Generation (5G). CR can be efficiently implemented in heterogeneous networks (HetNets), wherein channels can be allocated from heterogeneous bands (i.e. non-adjacent bands) and their sharing or aggregation feature among several basestations and multiple users. These new cognitive-wise technologies deal with the dispersed nature of the heterogeneous spectrum bands. Thus, a *dispersed spectrum CR* over generalized fading channels can be assumed in order to study the performance of the next generation wireless communication systems with CR capabilities (see [1] and the references therein).

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Ergodic capacity, one of the most important performance metrics of wireless communication systems, has been extensively studied in the technical literature (see [2] and references therein). Furthermore, higher-order statistics (HOS) is a useful tool for calibrating the maximum dispersion in the channel capacity [3]. Considering that this dispersion is heavily affected by the heterogeneity that inherently exists in contemporary wireless communication networks, HOS can effectively lead to reliable communication designs. It is notable that HOS of the channel capacity can provide critical information on the dispersion of capacity around a signal-to-noise ratio (SNR) value and thus insightful gains can be extracted for the transmission throughput reliability [3], [4]. The latter is very important in a new feature of 3GPP LTE-Advanced communication systems, called *carrier aggregation*, where multiple component carriers aggregated at the receiver side of a user equipment to get a wider bandwidth and, hence, increased data rate [5].

Prior works on the HOS of the capacity over generalized fading channels have been focused on the diversity receivers. In particular, Yilmaz and Alouini studied in [3] the HOS of the channel capacity for a receiver equipped with a maximal ratio combiner (MRC) over correlated generalized fading channels, where the HOS expressions were given in integral form. In [6], Sagias *et al.* studied the HOS of the capacity for several diversity receivers over independent and non-identically distributed (i.n.i.d.) Nakagami-m fading channels and also proposed a new performance metric, called *fading figure* ($\mathcal{FF}$), which -similarly with the *amount of fading* ($\mathcal{A}o\mathcal{F}$)- is based on the variance of the capacity.

The first published work with performance analysis of ad hoc dispersed spectrum over Nakagami-m fading channels was presented in [1]. In this important paper, Qaraqe *et al.* studied the average symbol error probability of dispersed spectrum CR systems, for both ucorrelated/correlated fading environment. Moreover, in [1] the effective transport capacity of ad hoc dispersed spectrum CR networks in 3D node distribution was investigated. Former works in this topic focused mostly on time delay estimation issues for localization and positioning applications [7]. To the best of our knowledge, the performance of dispersed spectrum CR systems have not been studied considering the HOS of the channel capacity over generalized fading envioronment.

The main of contribution of this paper is based on the

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analysis of HOS of the channel capacity. The study of HOS has attracted a lot of attention in wireless communications; however, most contributions are devoted to the analysis of the level crossing rate and average fade durations. On the contrary, the HOS of the capacity has been addressed in very few published works. Thus, the proposed contribution is meaningful, since it addresses the HOS of the channel capacity of dispersed spectrum CR systems over generalized fading channels. The derived results are novel and also are given in closed form for general fading channels as opposed to previously published works. Specifically, in this paper, a dispersed spectrum CR system with generalized fading channels is considered and the expressions for the HOS of the channel capacity are presented in closed form. The $\eta-\mu$ fading channel model is assumed for the dispersed spectrum CR, as a generalized fading model, which incorporates many special cases like the Nakagami$-q$, Nakagami$-m$, Rayleigh and one-sided Gaussian distribution. Useful statistical metrics such as $\mathcal{FF}$, *amount of dispersion* ($\mathcal{AoD}$), *skewness* ($\mathcal{S}$) and *kurtosis* ($\mathcal{K}$) are analytically presented. Having defined the channel capacity of our model, we obtain the HOS expressions in closed form, which are useful in analyzing statistical metrics such as the $\mathcal{AoD}$ [3], [4]. In addition to the $\mathcal{AoD}$, that measures the maximum dispersion of the channel capacity in CR systems, we derive in closed form the $\mathcal{FF}$, $\mathcal{S}$ and $\mathcal{K}$, highlighting further the behavior of the dispersed spectrum CR. All the aforementioned performance metrics can be efficiently used for the best channel selection in heterogeneous wireless networks with spectrum sharing and/or aggregation capabilities.

The remainder of this paper is organized as follows: Section II describes the considered system and channel model. Section III is devoted to the derivation of the HOS of the channel capacity over $\eta-\mu$ fading channels for the dispersed spectrum cognitive radio. The respective numerical results and analysis are provided in Section IV while concluding remarks are finally given in Section V.

II. System and Channel Model

We consider a dispersed spectrum CR which is similar to the ones studied in [1], [7], [8]. Secondary users (SUs) perform spectrum sensing to identify which bands are available and thus to exploit the benefits of frequency diversity by combining the instantaneous SNRs, γ_is, from each diversity band. By assuming L available frequency diversity bands, the end-to-end SNR at the output of each SU's receiver is given by

$$\gamma_{end} = \sum_{i=1}^{L} \gamma_i. \tag{1}$$

Each frequency diversity channel is assumed to be slow and frequency non-selective and subject to i.n.i.d. $\eta-\mu$ fading. The $\eta-\mu$ distribution is a generalized fading model that has been shown to provide accurate characterization of *small-scale* fading in non-line-of-sight (NLOS) communications. It was presented for first time by the pioneering work of Yacoub in [9] and shown that $\eta-\mu$ is a flexible fading model including as special cases the Nakagami$-q$ (Hoyt), Nakagami$-m$, Rayleigh and one-sided Gaussian distributions. The $\eta-\mu$ fading model is expressed by two parameters, η and μ and is valid for two different formats, namely, *Format*-1 and *Format*-2. In the former, the η parameter represents the ratio of the powers between the multipath waves in the in-phase and quadrature components, whereas in the latter it represents the correlation coefficient between the scattered wave in-phase and quadrature components of each cluster of multipath. Furthermore, the μ parameter denotes, in both formats, the inverse of the normalized variance and is related to the respective number of multipath clusters [9]–[11].

The probability density function (PDF) of the instantaneous SNR in $\eta-\mu$ fading channels is given by [12, Eq. (3)]

$$f_{\gamma_{\eta-\mu}}(\gamma) = \frac{2\sqrt{\pi}\mu^{\mu+\frac{1}{2}}h^{\mu}}{\Gamma(\mu)H^{\mu-\frac{1}{2}}}\frac{\gamma^{\mu-\frac{1}{2}}}{\overline{\gamma}^{\mu+\frac{1}{2}}}e^{-2\mu h\frac{\gamma}{\overline{\gamma}}}I_{\mu-\frac{1}{2}}\left(\frac{2\mu H\gamma}{\overline{\gamma}}\right), \tag{2}$$

where γ and $\overline{\gamma}$ denotes the instantaneous and average SNR, respectively, $\Gamma(x)$ is the Gamma function and $I_n(x)$ is the modified Bessel function of the first kind with argument x and order n. The parameters h and H are defined as follows

$$h_{\mathrm{F}-1} = \frac{(2+\eta^{-1}+\eta)}{4}, \tag{3}$$

$$H_{\mathrm{F}-1} = \frac{(\eta^{-1}-\eta)}{4} \tag{4}$$

with $0 < \eta < \infty$ for *Format*-1 and

$$h_{\mathrm{F}-2} = \frac{1}{(1-\eta^2)}, \tag{5}$$

$$H_{\mathrm{F}-2} = \frac{\eta}{(1-\eta^2)} \tag{6}$$

with $-1 < \eta < 1$ for *Format*-2.

Furthermore,

$$\mu = \frac{\left[\mathbb{E}(R^2)\right]^2}{2\mathrm{Var}(R^2)}\left[1+\frac{H}{h}\right], \tag{7}$$

for both formats with $\mathbb{E}(.)$ and $\mathrm{Var}(.)$ denoting statistical expectation and variance of the envelope R, respectively [9].

With the aid of the finite series representation for integer values of μ [13, eq. (8.467)], $I_n(x)$ in (2) can be equivalently re-written as [11], [14]–[17]

$$\begin{aligned} I_{\mu-\frac{1}{2}}\left(\frac{2\mu H\gamma}{\overline{\gamma}}\right) =& \sum_{l=0}^{\mu-1}\frac{(-1)^l\Gamma(\mu+l)\overline{\gamma}^{l+\frac{1}{2}}e^{\frac{2\mu H\gamma}{\overline{\gamma}}}}{l!\sqrt{\pi}\Gamma(\mu-l)(4\mu H\gamma)^{l+\frac{1}{2}}} \\ &+\sum_{l=0}^{\mu-1}\frac{(-1)^{\mu}\Gamma(\mu+l)\overline{\gamma}^{l+\frac{1}{2}}e^{-\frac{2\mu H\gamma}{\overline{\gamma}}}}{l!\sqrt{\pi}\Gamma(\mu-l)(4\mu H\gamma)^{l+\frac{1}{2}}}. \end{aligned} \tag{8}$$

By substituting (8) into (2) and carrying out basic algebraic manipulations, it follows that

$$\begin{aligned} f_{\gamma_{\eta-\mu}}(\gamma) =& \sum_{k=0}^{\mu-1}\frac{(\mu)_k\mu^{\mu-k}h^{\mu}\gamma^{\mu-k-1}}{k!H^{\mu+k}\Gamma(\mu-k)2^{2k}\,\overline{\gamma}^{\mu-k}} \\ &\times\left[(-1)^k e^{-2\mu(h-H)\frac{\gamma}{\overline{\gamma}}}+(-1)^{\mu}e^{-2\mu(h+H)\frac{\gamma}{\overline{\gamma}}}\right], \end{aligned} \tag{9}$$

which is valid for $\mu \in \mathbb{N}$.

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III. Higher-Order Statistics of the Channel Capacity

The average spectral efficiency (i.e., the average ergodic capacity normalized to the available bandwidth B) can be expressed as [3], [6]

$$\mathbb{E}\left(S^n\right) = \mathbb{E}[\log_2^n(1+\gamma_{end})], \tag{10}$$

where $n \in \mathbb{N}$ denotes the order of the statistics, which is particularly useful in quantifying the maximum dispersion of the channel capacity [2]. By substituting (1) into (10), the following $L-$fold integral is readily deduced,

$$\begin{aligned}\mathbb{E}(S^n) = \int_0^\infty \int_0^\infty \cdots \int_0^\infty \log_2^n \left(1+\sum\nolimits_{i=1}^{L} \gamma_i\right) \\ \times f\left(\gamma_1,\gamma_2,\cdots,\gamma_L\right) d\gamma_1 d\gamma_2 \cdots d\gamma_L,\end{aligned} \tag{11}$$

where $f(\gamma_1,\gamma_2,\cdots,\gamma_L)$ denotes the joint PDF of the instantaneous SNRs. Evaluating (11) in closed form is, unfortunately, intractable even for the case that γ_is are statistically independent. In addition, the involved complexity increases significantly as the number of the aggregated channel bandwidths increase. Motivated by this, we attempt to derive an analytic expression for the higher-order capacity statistics over i.n.i.d $\eta-\mu$ fading channels.

A. Ergodic Capacity for i.n.i.d. fading

For i.n.i.d $\eta-\mu$ fading channels, the PDF of the γ_{end} can be expressed as [18, Eq. (3)]

$$\begin{aligned}f_{\gamma_{end}}(\gamma) = &\sum_{k=1}^{L}\sum_{j=1}^{\mu_k} \frac{C_{kj}}{\Gamma(j)} \gamma^{j-1} e^{-A_k\gamma} \\ &+ \sum_{k=1}^{L}\sum_{j=1}^{\mu_k} \frac{D_{kj}}{\Gamma(j)} \gamma^{j-1} e^{-B_k\gamma},\end{aligned} \tag{12}$$

where C_{kj}, D_{kj}, are the residues of the moment generating function, $M_{\gamma_{end}}(s)$, to the poles $-A_k$ and $-B_k$, respectively, with multiplicity j. Based on this, the residues C_{kj}, D_{kj} in (12) are given by [18, Eqs. (4) and (5)] as

$$\begin{aligned}C_{kj} = &\frac{1}{(\mu_k-j)!}\prod_{i=1}^{L}\left(\frac{2\mu_i}{\bar{\gamma}_i}\right)^{2\mu_i} h^{\mu_i} \\ \times &\left\{\left[\prod_{\substack{i=1\\i\neq k}}^{L}\frac{1}{(s+A_i)^{\mu_i}}\right]\left[\prod_{i=1}^{L}\frac{1}{(s+B_i)^{\mu_i}}\right]\right\}^{\mu_k-j}\Bigg|_{s=-A_k}\end{aligned} \tag{13}$$

and

$$\begin{aligned}D_{kj} = &\frac{1}{(\mu_k-j)!}\prod_{i=1}^{L}\left(\frac{2\mu_i}{\bar{\gamma}_i}\right)^{2\mu_i} h^{\mu_i} \\ \times &\left\{\left[\prod_{i=1}^{L}\frac{1}{(s+A_i)^{\mu_i}}\right]\left[\prod_{\substack{i=1\\i\neq k}}^{L}\frac{1}{(s+B_i)^{\mu_i}}\right]\right\}^{\mu_k-j}\Bigg|_{s=-B_k},\end{aligned} \tag{14}$$

where $A_i = 2\mu_i(h_i - H_i)/\bar{\gamma}_i$ and $B_i = 2\mu_i(h_i + H_i)/\bar{\gamma}_i$.

To this effect, the average spectral efficiency, for the specific case that μ is an arbitrary integer, can be given by [18, eq. (11)]

$$\mathbb{E}(S) = \frac{1}{\ln 2}\sum_{k=1}^{L}\sum_{j=1}^{\mu_k}\frac{C_{kj}}{\Gamma(j)}\mathcal{I}_j(A_k) + \frac{1}{\ln 2}\sum_{k=1}^{L}\sum_{j=1}^{\mu_k}\frac{D_{kj}}{\Gamma(j)}\mathcal{I}_j(B_k), \tag{15}$$

where

$$\mathcal{I}_n(x) = \Gamma(n)e^x \sum_{j=1}^{n}\frac{\Gamma(-n+j,x)}{x^j} \tag{16}$$

and $\Gamma(a,x)$ is the upper incomplete gamma function [13].

B. Higher-Order Statistics

With the aid of (11) and by employing the PDF for MRC in (12), one obtains:

$$\begin{aligned}\mathbb{E}(S^n) = &\frac{1}{\ln^n 2}\int_0^\infty \ln^n(1+\gamma) \\ \times &\left(\sum_{k=1}^{L}\sum_{j=1}^{\mu_k}\frac{C_{kj}}{\Gamma(j)}\gamma^{j-1}e^{-A_k\gamma} + \sum_{k=1}^{L}\sum_{j=1}^{\mu_k}\frac{D_{kj}}{\Gamma(j)}\gamma^{j-1}e^{-B_k\gamma}\right) d\gamma,\end{aligned} \tag{20}$$

which upon expanding (20), it can be equivalently re-written as

$$\begin{aligned}\mathbb{E}(S^n) = &\frac{1}{\ln^n 2}\sum_{k=1}^{L}\sum_{j=1}^{\mu_k}\frac{C_{kj}}{\Gamma(j)}\int_0^\infty \frac{\ln^n(1+\gamma)}{\gamma^{1-j}e^{A_k\gamma}}d\gamma \\ &+ \frac{1}{\ln^n 2}\sum_{k=1}^{L}\sum_{j=1}^{\mu_k}\frac{D_{kj}}{\Gamma(j)}\int_0^\infty \frac{\ln^n(1+\gamma)}{\gamma^{1-j}e^{B_k\gamma}}d\gamma.\end{aligned} \tag{21}$$

Importantly, the above integrals can be expressed in closed-form with the aid of [19, eqs. (33) and (37)]. To this end, by performing the necessary change of variables, substituting in (22) and carrying out some algebraic manipulations yields

$$\begin{aligned}\mathbb{E}(S^n) = &\frac{1}{\ln^n 2}\sum_{k=1}^{L}\sum_{j=1}^{\mu_k}\frac{C_{kj}}{\Gamma(j)}\mathcal{J}_{j,n}\left(1,\frac{1}{A_k},1\right) \\ &+ \frac{1}{\ln^n 2}\sum_{k=1}^{L}\sum_{j=1}^{\mu_k}\frac{D_{kj}}{\Gamma(j)}\mathcal{J}_{j,n}\left(1,\frac{1}{B_k},1\right),\end{aligned} \tag{22}$$

where

$$\begin{aligned}\mathcal{J}_{j,n}\left(1,\frac{1}{A_k},1\right) = &n! e^{A_k}\sum_{l=0}^{j-1}\frac{(-1)^{j-l-1}}{A_k^{1+l}}\binom{j-1}{l} \\ &\times G_{n+1,n+2}^{n+2,0}\left(A_k\left|\begin{matrix}\overbrace{1,1,\ldots,1}^{n+1\ 1's} \\ \underbrace{0,0,\ldots,0}_{n+1\ 0's},1+l\end{matrix}\right.\right)\end{aligned} \tag{23}$$

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and

$$\mathcal{J}_{j,n}\left(1,\frac{1}{B_k},1\right)=n!e^{B_k}\sum_{l=0}^{j-1}\frac{(-1)^{j-l-1}}{B_k^{1+l}}\binom{j-1}{l}\times G_{n+1,n+2}^{n+2,0}\left(B_k\middle|\begin{matrix}\overbrace{1,1,\ldots,1}^{n+1\,1's}\\ \underbrace{0,0,\ldots,0}_{n+1\,0's},1+l\end{matrix}\right) \tag{24}$$

with $G(.)$ denoting the Meijer $G-$function [13, eq. (9.30)]. Therefore, by substituting (23) and (24) into (22), the following closed-form expression is deduced

$$\begin{aligned}\mathbb{E}(S^n)=&\sum_{k=1}^{L}\sum_{j=1}^{\mu_k}\sum_{l=0}^{j-1}\frac{(-1)^{j-l-1}C_{kj}n!e^{A_k}}{\ln^n(2)\Gamma(j)A_k^{1+l}}\binom{j-1}{l}\\&\times G_{n+1,n+2}^{n+2,0}\left(A_k\middle|\begin{matrix}\overbrace{1,1,\ldots,1}^{n+1\,1's}\\ \underbrace{0,0,\ldots,0}_{n+1\,0's},1+l\end{matrix}\right)\\&+\sum_{k=1}^{L}\sum_{j=1}^{\mu_k}\sum_{l=0}^{j-1}\frac{(-1)^{j-l-1}D_{kj}n!e^{B_k}}{\ln^n(2)\Gamma(j)B_k^{1+l}}\binom{j-1}{l}\\&\times G_{n+1,n+2}^{n+2,0}\left(B_k\middle|\begin{matrix}\overbrace{1,1,\ldots,1}^{n+1\,1's}\\ \underbrace{0,0,\ldots,0}_{n+1\,0's},1+l\end{matrix}\right).\end{aligned} \tag{25}$$

For the special case of $n=1$, it readily follows that:

$$\begin{aligned}\mathbb{E}(S)=&\sum_{k=1}^{L}\sum_{j=1}^{\mu_k}\sum_{l=0}^{j-1}\frac{(-1)^{j-l-1}}{\ln(2)\Gamma(j)}\binom{j-1}{l}\\&\times\left[\frac{C_{kj}e^{A_k}}{A_k^{1+l}}G_{2,3}^{3,0}\left(A_k\middle|\begin{matrix}1,1\\0,0,1+l\end{matrix}\right)\right.\\&\left.+\frac{D_{kj}e^{B_k}}{B_k^{1+l}}G_{2,3}^{3,0}\left(B_k\middle|\begin{matrix}1,1\\0,0,1+l\end{matrix}\right)\right].\end{aligned} \tag{26}$$

It is noted here that according to [13, eq. (9.31.1)], the Meijer $G-$function in (26) can be alternatively expressed as

$$G_{2,3}^{3,0}\left(A_k\middle|\begin{matrix}1,1\\0,0,1\end{matrix}\right)=G_{1,2}^{2,0}\left(A_k\middle|\begin{matrix}1\\0,0\end{matrix}\right)=\Gamma\left(0,A_k\right). \tag{27}$$

As a result, (26) can be also expressed as

$$\begin{aligned}\mathbb{E}(S)=&\sum_{k=1}^{L}\sum_{j=1}^{\mu_k}\sum_{l=0}^{j-1}\frac{(-1)^{j-k-1}}{\ln(2)\Gamma(j)}\binom{j-1}{l}\\&\times\left[\frac{C_{kj}e^{A_k}}{A_k^{1+l}}\Gamma\left(0,A_k\right)+\frac{D_{kj}e^{B_k}}{B_k^{1+l}}\Gamma\left(0,B_k\right)\right],\end{aligned} \tag{28}$$

which is a simplified algebraic representation. To this effect, by substituting the above expression into (26), one obtains the analytic expression in (15). Likewise, the second, third and fourth moment are readily deduced yielding:

$$\begin{aligned}\mathbb{E}(S^2)=&\sum_{k=1}^{L}\sum_{j=1}^{\mu_k}\sum_{l=0}^{j-1}\frac{2(-1)^{j-l-1}C_{kj}e^{A_k}}{\ln^2(2)\Gamma(j)A_k^{1+l}}\binom{j-1}{l}\\&\times G_{3,4}^{4,0}\left(A_k\middle|\begin{matrix}1,1,1\\0,0,0,1+l\end{matrix}\right)\\&+\sum_{k=1}^{L}\sum_{j=1}^{\mu_k}\sum_{l=0}^{j-1}\frac{2(-1)^{j-l-1}D_{kj}e^{B_k}}{\ln^2(2)\Gamma(j)B_k^{1+l}}\binom{j-1}{l}\\&\times G_{3,4}^{4,0}\left(B_k\middle|\begin{matrix}1,1,1\\0,0,0,1+l\end{matrix}\right),\end{aligned} \tag{29}$$

$$\begin{aligned}\mathbb{E}(S^3)=&\sum_{k=1}^{L}\sum_{j=1}^{\mu_k}\sum_{l=0}^{j-1}\frac{6(-1)^{j-l-1}C_{kj}e^{A_k}}{\ln^3(2)\Gamma(j)A_k^{1+l}}\binom{j-1}{l}\\&\times G_{4,5}^{5,0}\left(A_k\middle|\begin{matrix}1,1,1,1\\0,0,0,0,1+l\end{matrix}\right)\\&+\sum_{k=1}^{L}\sum_{j=1}^{\mu_k}\sum_{l=0}^{j-1}\frac{6(-1)^{j-l-1}D_{kj}e^{B_k}}{\ln^3(2)\Gamma(j)B_k^{1+l}}\binom{j-1}{l}\\&\times G_{4,5}^{5,0}\left(B_k\middle|\begin{matrix}1,1,1,1\\0,0,0,0,1+l\end{matrix}\right),\end{aligned} \tag{30}$$

and

$$\begin{aligned}\mathbb{E}(S^4)=&\sum_{k=1}^{L}\sum_{j=1}^{\mu_k}\sum_{l=0}^{j-1}\frac{24(-1)^{j-l-1}C_{kj}e^{A_k}}{\ln^4(2)\Gamma(j)A_k^{1+l}}\binom{j-1}{l}\\&\times G_{5,6}^{6,0}\left(A_k\middle|\begin{matrix}1,1,1,1,1\\0,0,0,0,0,1+l\end{matrix}\right)\\&+\sum_{k=1}^{L}\sum_{j=1}^{\mu_k}\sum_{l=0}^{j-1}\frac{24(-1)^{j-l-1}D_{kj}e^{B_k}}{\ln^4(2)\Gamma(j)B_k^{1+l}}\binom{j-1}{l}\\&\times G_{5,6}^{6,0}\left(B_k\middle|\begin{matrix}1,1,1,1,1\\0,0,0,0,0,1+l\end{matrix}\right),\end{aligned} \tag{31}$$

respectively. In what follows, the above analytic expressions are analyzed in determining useful statistical measures.

C. Performance Metrics

Using (25), new analytic expressions can be straightforwardly deduced for important system's performance metrics. To this end, the first four statistical moments are derived, which are respectively presented at the top of this page. Based on these expressions, the following measures, can be introduced as

- *Variance of the channel capacity*, ($\mathcal{V}ar$):

$$\mathcal{V}ar=\mathbb{E}(S^2)-(\mathbb{E}(S))^2, \tag{32}$$

- *Fading Figure*, ($\mathcal{FF}$):

$$\mathcal{FF}=\frac{\mathbb{E}(S^2)}{(\mathbb{E}(S))^2}-1, \tag{33}$$

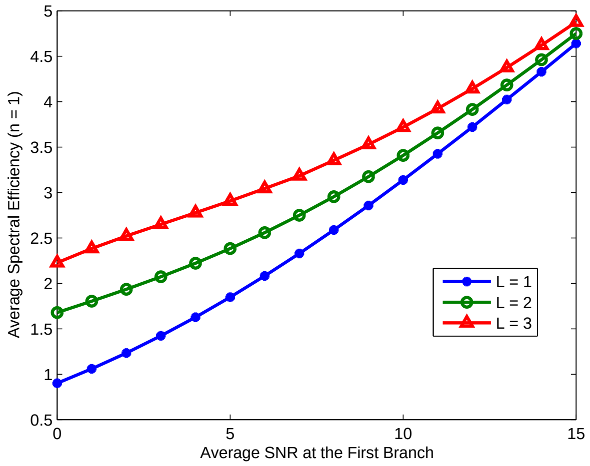

Fig. 1. Average spectral efficiency versus average SNR in the first branch for different values of L and unequal SNRs and fading parameters ($\overline{\gamma}_2 = 2\text{dB}$, $\overline{\gamma}_3 = 4\text{dB}$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\mu_3 = 3.0$ and $\eta_1 = 2.3$, $\eta_2 = 1.2$, $\eta_3 = 0.4$)

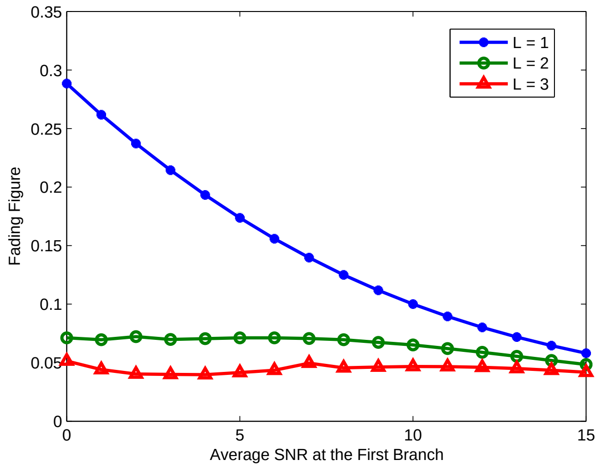

Fig. 2. Fading Figure versus average SNR in the first branch for different values of L and unequal SNR and fading parameters ($\overline{\gamma}_2 = \overline{\gamma}_3 = 1\text{dB}$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\mu_3 = 3.0$ and $\eta_1 = 2.3$, $\eta_2 = 1.2$, $\eta_3 = 0.4$).

- *Amount of Dispersion*, ($\mathcal{AoD}$):

$$\mathcal{AoD} = \frac{\mathbb{E}(S^2)}{\mathbb{E}(S)} - \mathbb{E}(S), \tag{34}$$

- *Skewness*, ($\mathcal{S}$):

$$\mathcal{S} = \frac{\mathbb{E}(S^3) - (\mathbb{E}(S))^3}{\sqrt{\{\mathbb{E}(S^2)] - (\mathbb{E}(S))^2\}^3}}, \tag{35}$$

- *Kurtosis*, ($\mathcal{K}$):

$$\mathcal{K} = \frac{\mathbb{E}(S^4) - (\mathbb{E}(S))^4}{\{\mathbb{E}(S^2) - (\mathbb{E}(S))^2\}^2}. \tag{36}$$

Notably, by substituting (26), (29), (30) and (31) into the above metrics accordingly, exact closed-form expressions are readily deduced for the above performance measures.

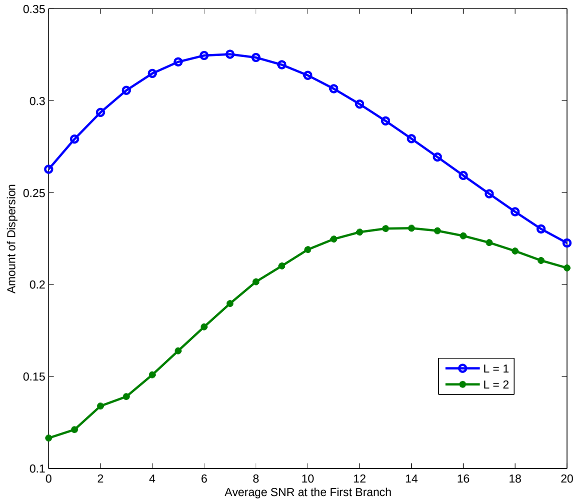

Fig. 3. Amount of dispersion versus average SNR in the first branch for different values of L and unequal SNR and fading parameters($\overline{\gamma}_2 = \overline{\gamma}_3 = 1\text{dB}$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\mu_3 = 3.0$ and $\eta_1 = 2.3$, $\eta_2 = 1.2$, $\eta_3 = 0.4$).

IV. Numerical Results an Discussion

The proposed closed-form expressions are validated via Monte Carlo simulations, where an excellent match between analytical and simulation results is shown. This is performed by means of the average spectral efficiency, the fading figure, the $\mathcal{AoD}$, the $\mathcal{S}$ and the $\mathcal{K}$ for different number of receiving paths. Specifically, Fig. 1 demonstrates the average spectral efficiency as a function of the average SNR at the first branch for the case of one, two and three branches. The average SNR in the other two branches are arbitrarily selected as $\overline{\gamma}_2 = 2$ dB and $\overline{\gamma}_3 = 4$ dB whereas the corresponding fading parameters are $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\mu_3 = 3.0$ and $\eta_1 = 2.3$, $\eta_2 = 1.2$, $\eta_3 = 0.4$, respectively[1]. One can notice that the average spectral efficiency increases significantly as $\overline{\gamma}_1$ increases while it is shown that the effect and usefulness of frequency diversity of dispersed spectrum CR is particularly critical at lower SNR values. For example, for $\overline{\gamma}_1 = 5$ dB, there is 15% difference between one and two branches and about 45% difference between one and three branches.

Figs. 2 and 3 illustrate the fading figure and the amount of dispersion as a function of the average SNR in the first branch. Regarding the former, one can observe a rapid reduction as the value of SNR and/or L increase. Indicatively, the difference between $\overline{\gamma}_1 = 2\text{dB}$ and $\overline{\gamma}_1 = 10\text{dB}$ is about 60% for $L = 1$. Likewise, for $\overline{\gamma}_1 = 1$ dB, the fading figure for the case of $L = 1$ is over 75% larger compared to the case of $L = 2$. The same also holds inversely for the amount of dispersion which appears to increase for low and moderate SNR values, contrary to the high SNR regime where it begins to decrease both for one or two antenna receivers at the UE. Also, for $L = 1$ the $\mathcal{AoD}$ reaches its highest value around 5 dB whereas for $L = 2$ the highest value of $\mathcal{AoD}$ is around 13 dB. Moreover,

[1]Also, an exponential power decay profile can also be applied in our examples where similar plots to the ones presented in [6], [18] will be derived.

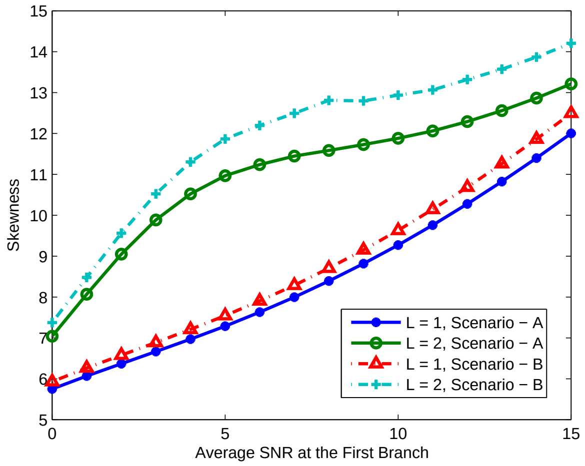

Fig. 4. Skewness versus average SNR in the first branch for two different scenarios($\text{Scenario A}: \overline{\gamma}_2 = 2\text{dB};\ \mu_1 = 1$ and $\mu_2 = 2;\ \eta_1 = 2.3$ and $\eta_2 = 1.2$. $\text{Scenario B}: \overline{\gamma}_2 = 3\text{dB};\ \mu_1 = 1$ and $\mu_2 = 2;\ \eta_1 = 1.2$ and $\eta_2 = 0.4$.)

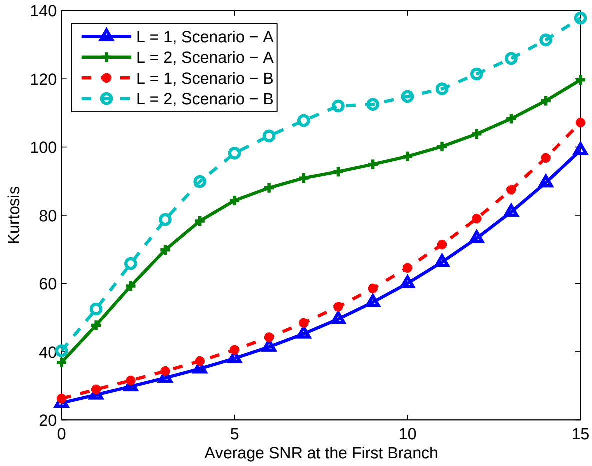

Fig. 5. Kurtosis versus average SNR in the first branch for two different scenarios ($\text{Scenario A}: \overline{\gamma}_2 = 2\text{dB};\ \mu_1 = 1$ and $\mu_2 = 2;\ \eta_1 = 2.3$ and $\eta_2 = 1.2$. $\text{Scenario B}: \overline{\gamma}_2 = 3\text{dB};\ \mu_1 = 1$ and $\mu_2 = 2;\ \eta_1 = 1.2$ and $\eta_2 = 0.4$.)

considering the reliability percentage i.e. 100 $\times(1 - \mathcal{A}o\mathcal{D})$ [3], the data throughput is reliable when the channel capacity values do not variate significantly. Therefore, for reliable transmission of the information signals at the cognitive UE, the SNR for transmission should be selected greater than the SNR for which the $\mathcal{A}o\mathcal{D}$ takes its maximum value. As expected, in both cases the deviation between different values of L reduces at the high SNR regime since the effect of fading becomes relatively less critical in comparison to moderate or low SNR values.

The corresponding $\mathcal{S}$ and $\mathcal{K}$ statistical measures are depicted in Figs. 4 and 5, respectively, as a function of the average SNR in the first branch for unequal SNR and fading values. Specifically, two scenarios are considered for one and two branches: $\text{Scenario A}: \overline{\gamma}_2 = 2$ dB; $\mu_1 = 1$ and $\mu_2 = 2$; $\eta_1 = 2.3$ and $\eta_2 = 1.2$. $\text{Scenario B}: \overline{\gamma}_2 = 3$ dB; $\mu_1 = 1$ and $\mu_2 = 2$; $\eta_1 = 1.2$ and $\eta_2 = 0.4$. One can notice the corresponding increase as the number of paths increases by one. In fact, for the considered scenario it is shown that the increased severity of multipath fading can be compensated by increasing the number of paths. Yet, this appears to be particularly critical for low and moderate SNR values as in the high SNR regime the effect of fading and multipath branches is, as expected, relatively reduced.

V. Conclusion

The present work was devoted to the analysis of the HOS of the channel capacity of dispersed spectrum CR systems over generalized fading channels. Due to the inherent structure of the considered system, it was effectively represented as a respective maximal ratio combining system. Based on this, new analytic expression was derived for the n-th channel capacity moment which was subsequently employed in deriving closed-form expressions for critical HOS metrics. The derived expressions were extensively validated with respective results from computer simulations and they were utilized in analyzing the performance of the system in terms of the average channel capacity, the fading figure, the amount of dispersion, the skewness and the kurtosis for unequal power and fading parameters. It was shown that the for reliable transmission in heterogeneous bands, the SNR at the transmitter should be controlled and selected greater than the SNR for which the $\mathcal{A}o\mathcal{D}$ of the channel capacity is in its higher level.

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