

Appendix to the paper on  
**Significance of the microfluidic concepts for the improvement  
of macroscopic models of transport phenomena**

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Subject of this appendix to the paper submitted to the 3<sup>rd</sup> Micro and Nano Flows Conference at Thessaloniki, Greece, 22-24 August, 2011 (MNF 2011) is additional information about: **a simple mathematical model applicable to the broadest possible range of micro- and macroscopic transport phenomena with help of a minimum of basic concepts of corpuscular physics [2, 3].** The model is the basis of: **Dynamics of disperse systems [9, 7-13]** as since a longer time developed **common background of apparently unrelated phenomena.**

**Main features of the proposed method** in distinction to the most popular methods of fluid mechanics [e.g. 1, 4, 5] are:

- **Explicit use of Newton's law as equation of motion instead of derived equations** like Navier-Stokes [4], Bernoulli equation etc. Thereby Newton's law and conservation laws are applied to non-relativistic motion of particle systems in the range from individual particles characterized by mass and velocity, such as individual atoms (molecules) or even electrons [15-17], over to macroscopic sets of such particles building either solid bodies or flowing systems of traditional mechanics [1, 5], up to celestial bodies of classical astro-physics [6]. Main application field are single- and multi-phase flows in engineering.

- **Departure from differential notation where possible**, thus aiming explicitly at tangible, total effects characterizing conditions in a defined volume or on a surface instead of imaginary conditions at a point. Integral notation allows determining of physical quantities at required scale by choice of the considered reference volumes/surfaces and use of the mean value theorem (MVT) of integral calculus [18].

- **The observed changes of the state of motion define (name) forces**, the exact nature of which is regarded as unknown. Thereby, Newton's second law [7 – 13] and

conservation laws [1] in differential-integral notation build always the same starting point for physically coherent explanation of phenomena rooted in motion of matter, understood as disperse system [2, 3, 5 - 12]. Basic form of Newton's law reads [11, 12]:

$$\underline{F} = \frac{d}{dt} \int_V \rho \underline{v} dV \quad (1)$$

Equation (1) is used to define gravity, similarly as electrostatic forces, or macroscopic results of multiple electrostatic molecular interactions such as pressure or friction. The results of the basic experiments respectively observations are written down in form of "spherical laws"[3, 15 - 17]:

$$F_{AG}(r) = \frac{G m_1 m_2}{r^2} \quad \text{for gravitational}$$

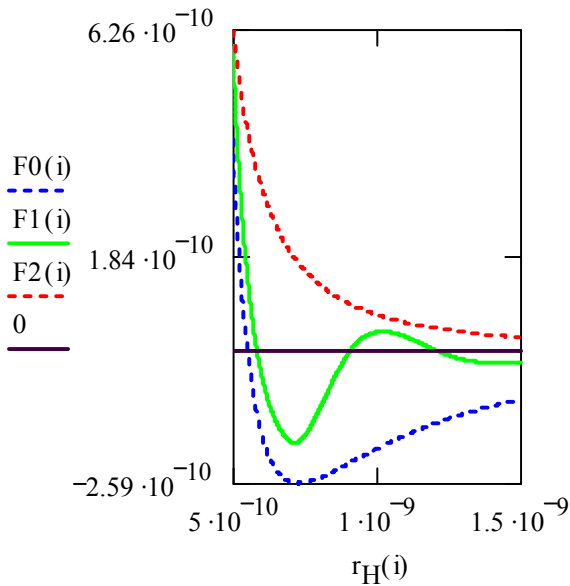
attraction of masses  $m_1$  and  $m_2$  separated by distance of  $r$  with  $G$  universal gravitation constant, or as in case of electrostatic interactions [15-18]:

$$F_{AE}(r) = \frac{1}{4 \pi \epsilon_0} \frac{q_1 q_2}{r^2} \quad \text{for electrostatic}$$

attraction of charges with opposite polarity  $q_1$  and  $q_2$  separated by  $r$  with  $\epsilon_0$  denoting the dielectric constant. Correspondingly  $F_{RE}(r)$  may denote electrostatic repulsion for charges of the same polarity. When dealing with single molecules equation (1) may be written as:

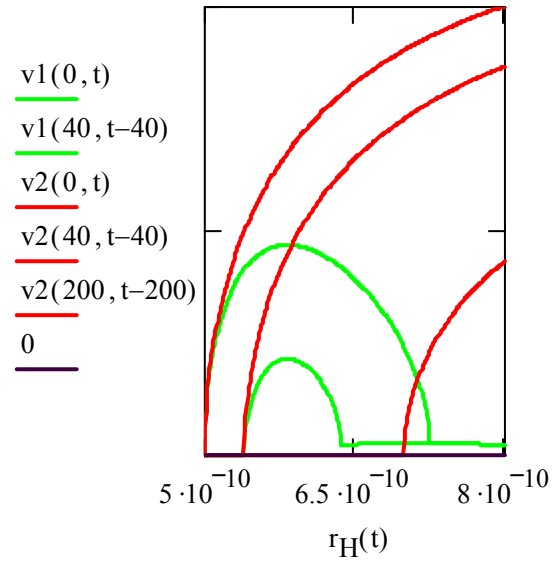
$$\frac{d}{dt} \int_V \rho \underline{v} dV \approx -F_{AG}(r) - F_{AE}(r) + F_{RE}(r) \quad (2)$$

In case of a limited number of molecules it is possible to use approximations of forces (2) e.g. to calculate the velocities of individual molecules and other particle parameters relevant for macroscopic flows. **Fig. 1** below shows a plot of different molecular interaction forces designed for use with model particles of unit charge.



**Fig. 1** Interaction forces for model particles with unit charge and hydrogen mass.

**Fig. 2** below shows the corresponding velocities of the model molecules calculated for interaction forces  $F_0(r)$  and  $F_1(r)$  shown above using form (2) of eq. (1). Thereby forces  $F_0(r)$  and  $F_2(r)$  mark the here adopted approximation ranges.  $F_0(r)$  was constructed for electrostatic repulsion and gravitational attraction.  $F_2(r)$  stresses the influence of electrostatic attraction in presence of gravitation and electrostatic repulsion. Influence of gravitation in the plot of  $F_1(r)$  was stressed to indicate its existence in a single diagram together with much stronger electrostatic forces. **Fig. 1** shows onset of gravitational attraction already at distance of the force centres of just ca.  $1.2 \times 10^{-9}$  m, which was obtained by amplifying gravitational attraction to show all interactions in a single diagram. Calculations were conducted using eq. 2 and data according to [15 -17].



**Fig. 2** Velocities of model particles obtained using interaction forces plotted in **Fig. 1**.

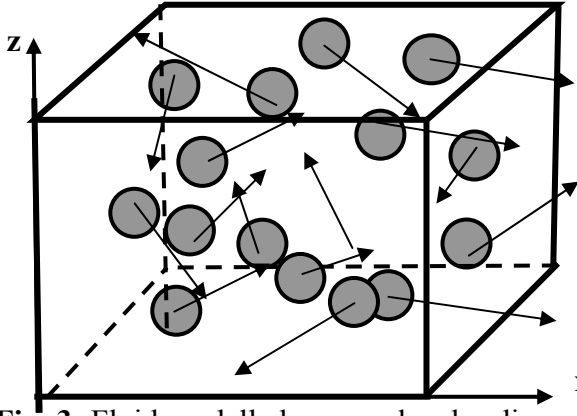
For applications in macroscopic flow situations it is convenient to use the following representation of forces  $\underline{F}$ :

$$\frac{d}{dt} \int_V \rho \underline{v} dV = \int_V \rho \underline{g} dV - \int_A p \underline{dA} + \int_A \underline{\underline{t}}(\eta) \bullet \underline{dA} \quad (3)$$

The forces  $\underline{F}$  on the right side of (3) are identified as [11, 12]: gravity  $\underline{g}$ , pressure  $p$ , friction  $\underline{\underline{t}}$  etc. acting in  $V$  and, respectively, on its limiting surfaces  $A$ . The double underlining denotes the considered term (here  $\underline{\underline{t}}$ ) as tensor of second order. Thereby the so called “**fluid quantities**” are understood as result of averaged application of Newton’s law eq. (1) and momentum conservation law [1] to micro scale particle systems (fluid). Thus the statistical “fluid quantities” like density  $\rho$ , pressure  $p$  and friction  $\underline{\underline{t}}$  (represented as function of the dynamical viscosity  $\eta$ ), are all defined at the micro scale by molecular numbers, masses, volumes, average collision free paths etc. [2, 3, 7 - 13].

Irrespectively of its applicability to macroscopic disperse systems equation (3) does not mix up the particle and the fluid quantities. It uses them consciously together, within the framework of a physically coherent universal method capable of dealing with arbitrary particle or fluid systems characterized by mass and velocity. In

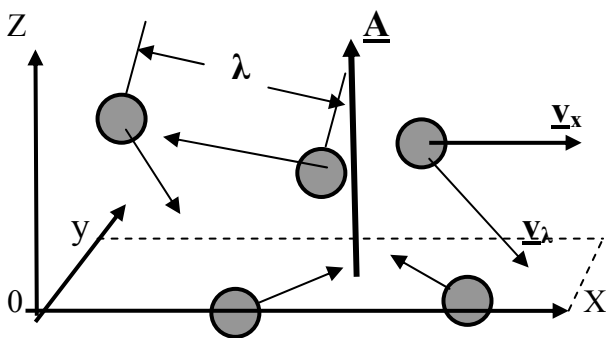
compliance with this “fluid quantities” like pressures  $p$  (or viscosity  $\eta$ ) and such macroscopic equivalents as “body forces” are specialized representations of the averaged molecule impact forces. The term “pressure” will be outlined on example of gas molecules shown schematically below (Fig. 3):



**Fig. 3:** Fluid modelled as a molecular disperse system [7 - 13]. Schematically shown are gas molecules in stochastic thermal motion. Arrows symbolize the thermal molecule velocities.

**Derivations of the statistical physical quantities** follow from the analysis of molecular motions in respect to chosen control surfaces  $A$ . Basic examples are:

**Thermal gas pressure:** We consider the magnified microscopic molecule motion through a macroscopic flat control surface e.g.  $A$  ( $x$ - $0$ - $y$ ), perpendicular to  $z$ -axis and containing the  $x$  and  $y$  -axis Fig. 4. below.



**Fig. 4** Molecular exchange through the control surface  $x$ - $0$ - $y$ .  $\lambda$  is symbolic representation of (averaged) collision free path.  $\underline{v}_\lambda$  is vector of thermal velocity of a molecule.  $\underline{v}_x$  symbolizes the vector of superimposed average bulk flow velocity (if present).  $\underline{A}$  is the surface vector of the considered part of the  $x - 0 - y$  surface.

The surface vector of the control surface  $\underline{A}$  is parallel to the  $z$ -axis. Gas molecules move stochastically in all directions, but in such a way that on the average there is no resultant long term transport across  $A$ . The average macroscopic flow velocity is zero, thus macroscopic friction is not present. Gravity influence is negligible. At these conditions the resulting average pressure force is defined by:

$$\frac{d}{dt} \int_V \rho \underline{v} dV = - \int_A p d\underline{A} = - \underline{F}_p \quad (4)$$

Since the average flow is steady, the local time derivative vanishes and the left hand side of the above equation, which defines the force of total momentum exchange (pressure force), reduces to convective component alone:

$$\begin{aligned} - \underline{F}_p &= \frac{d}{dt} \int_V \rho \underline{v} dV = \frac{\partial}{\partial t} \int_V \rho \underline{v} dV + \int_A (\rho \underline{v}) \underline{v} \cdot d\underline{A} = \\ &= \int_A (\rho \underline{v}) \underline{v} \cdot d\underline{A} \end{aligned} \quad (5)$$

With  $\lambda$  representing the average collision free travelling distance we assume, that the average velocities of the gas molecules are in  $V$  equal. In particular it is:

$$v_\lambda(z \pm \lambda) = v_\lambda(z) \text{ and } v_{x,y,z} = 0 \quad (6)$$

Conditions (6) for averages of molecular velocities are substituted into the convective component, under assumption that half of the surface area  $\underline{A}$  is crossed by molecules travelling e.g. downward and half in reverse direction. Force of momentum exchange by molecule collisions follows as:

$$\begin{aligned} F_{P,Z} &= - \underline{e}_Z \cdot \int_A (\rho_G \underline{v}) \underline{v} \cdot d\underline{A} = \\ &= - \underline{e}_Z \cdot \int_{A/2} \left[ \rho_G \underline{v}_\lambda(z + \lambda) \right] \left( - v_{\lambda Z} dA_Z \right) + \\ &+ \underline{e}_Z \cdot \int_{A/2} \left[ \rho_G \underline{v}_\lambda(z - \lambda) \right] \left( + v_{\lambda Z} dA_Z \right) \end{aligned} \quad (7)$$

Terms  $v_{\lambda Z} dA_Z$  represent results of scalar vector product  $\underline{v}_\lambda \cdot d\underline{A}$  approximating the average diffusive molecule transport through the control surface  $A$ . The „ + “ sign results for transport along the  $Z$ -axis (in Fig. 2 upward), correspondingly „ - “ results for transport reverse to the  $z$ -axis (in Fig.2 downwards).

The  $Z$  index means that the obtained components are directed along the  $z$ -axis, perpendicular to  $A$ . Multiplication with the unit vector  $\underline{e}_z$  results in:

$$\begin{aligned} F_{P,Z} &= -\underline{e}_z \cdot \int_{A/2} \left[ \rho_G \underline{v}_\lambda(z+\lambda) \right] \left( -v_{\lambda Z} dA_Z \right) + \\ &+ \underline{e}_z \cdot \int_{A/2} \left[ \rho_G \underline{v}_\lambda(z-\lambda) \right] \left( +v_{\lambda Z} dA_Z \right) = \\ &= + \int_{A/2} \left[ \rho_G v_{\lambda Z}(z+\lambda) \right] \left( v_{\lambda Z} dA_Z \right) + \\ &- \int_{A/2} \left[ -\rho_G v_{\lambda Z}(z-\lambda) \right] \left( v_{\lambda Z} dA_Z \right) = \\ &= \int_{A/2} \rho_G \left[ +v_{\lambda Z}(z) \right] \left( v_{\lambda Z} dA_Z \right) + \\ &+ \int_{A/2} \rho_G \left[ v_{\lambda Z}(z) \right] \left( v_{\lambda Z} dA_Z \right) \end{aligned}$$

The total interaction force follows after simplifying the last expression and applying the mean value theorem of integral calculus. It reads:  $F_p = \rho_G v_\lambda^2 A_Z$

Pressure  $p$  follows as force component divided by the  $A_Z$  area:  $p = \rho_G v_\lambda^2$  (8)

Similar calculation is performed to find expressions relating e.g.:

**the dynamic gas viscosity  $\eta$**  (fluid parameter) to molecular (particle) transport parameters such as the collision free path  $\lambda$ , average stochastic molecule velocity  $v_\lambda$  and gas density  $\rho_G$ . To this purpose the boundary conditions are changed to model the behaviour of the gas cloud in **Fig. 3** under linear, steady shear load. Conditions concerning the molecular velocities remain unchanged; the main assumption this time concerns the average velocity of the bulk flow i.e. the  $x$ -component of the average gas velocity  $v_x$  does not equal zero as before but changes linearly with  $z$ :

$$v_x(z \pm \lambda) = v_x(z) \pm \lambda \frac{\partial v_x}{\partial z} \quad (9)$$

For otherwise steady average flow velocity field  $\underline{v}_x$  superposed on the stochastic molecule velocity  $v_\lambda$  and, with other assumptions

unchanged it follows, that: molecules crossing the control surface  $A$  and colliding with molecules with different local mean flow velocity  $v_x$  will on the average obtain the local average flow velocity thus changing their original momentum. To find the resulting force balance we substitute again the average velocity field approximation in the convective component as above:

$$\begin{aligned} F_{T,x} &= \underline{e}_x \cdot \int_A \left( \rho_G \underline{v} \right) \underline{v} \cdot d\underline{A} = \\ &\underline{e}_x \cdot \int_{A/2} \left[ \rho_G \underline{v}(z+\lambda) \right] \left( -v_{\lambda Z} dA_Z \right) + \\ &+ \underline{e}_x \cdot \int_{A/2} \left[ \rho_G \underline{v}(z-\lambda) \right] \left( +v_{\lambda Z} dA_Z \right) \end{aligned}$$

The  $x$ -component of the shear force follows from multiplication with a unit vector  $\underline{e}_x$ :

$$\begin{aligned} F_{T,x} &= \underline{e}_x \cdot \int_{A/2} \left[ \rho_G \underline{v}(z+\lambda) \right] \left( -v_{\lambda Z} dA_Z \right) + \\ &+ \underline{e}_x \cdot \int_{A/2} \left[ \rho_G \underline{v}(z-\lambda) \right] \left( +v_{\lambda Z} dA_Z \right) = \\ &= - \int_{A/2} \left[ \rho_G v_x(z+\lambda) \right] \left( v_{\lambda Z} dA_Z \right) + \\ &\int_{A/2} \left[ \rho_G v_x(z-\lambda) \right] \left( v_{\lambda Z} dA_Z \right) = \\ &= \int_{A/2} \rho_G \left[ v_x(z) - \lambda \frac{\partial v_x}{\partial z} \right] \left( v_{\lambda Z} dA_Z \right) + \\ &- \int_{A/2} \rho_G \left[ v_x(z) + \lambda \frac{\partial v_x}{\partial z} \right] \left( v_{\lambda Z} dA_Z \right) \end{aligned}$$

Applying the mean value theorem of integral calculus and simplifying the result, we obtain:

$$F_{T,x} = \rho_G v_{\lambda Z} \lambda A_Z \frac{\partial v_x}{\partial z} \quad (11)$$

The product of the gas density  $\rho_G$ , of the average stochastic molecule velocity  $v_\lambda$  and collision free path  $\lambda$  defines the macroscopic dynamic viscosity of the considered system [2]:  $\eta = \rho_G v_{\lambda Z} \lambda$  (12)

Expressions (11, 12) may be generalized to express the viscous drag force in one dimensional flows as:

$$F_T = k(\text{Re}) \eta A_W \frac{\partial v_x}{\partial z} \quad \text{or, for practical}$$

$$\text{Purposes: } F_T = k(\text{Re}) \eta A_W \frac{\langle v_{\text{rel}} \rangle}{R_H} \quad (13)$$

Thereby  $k(\text{Re})$  is a geometry and flow type (laminar, turbulent) dependent coefficient.  $A_W$  represents the wetted surface of the considered flow geometry. The velocity gradient in the boundary layer may be represented for practical purposes by a quotient of the adequate relative velocity and hydraulic radius  $R_H = V/A_W$ . In particular it was shown by the author, that for one dimensional flows the coefficient  $k(\text{Re})$  allows a very convenient representation of the drag force. Thus for:

**Couette flow (above):**  $k_C(\text{Re}) = 1$

**Pipe with circular cross section:**

$$k_P(\text{Re}) \approx 2 + 0.01 \text{Re}^{0.75} \quad \text{or:}$$

$$k_{P2}(\text{Re}) \approx 2 + 0.01 \text{Re} f_T(\text{Re})$$

**Flow around a sphere:**

$$k_S(\text{Re}) \approx 3 + 0.5 \text{Re}^{0.5} + 0.05 \text{Re}$$

**Flow through fixed particle bed:**

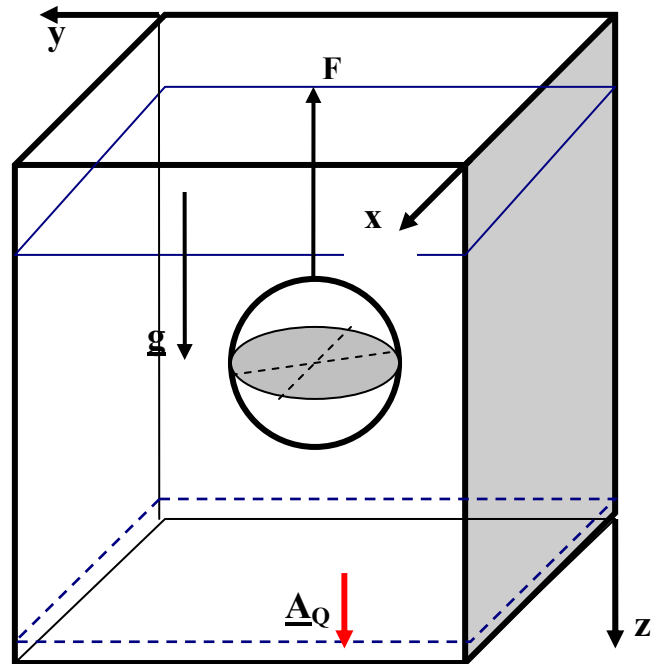
$$k_{FB}(\text{Re}) \approx 4.16 + 0.0486 \text{Re}$$

$f_T(\text{Re})/f_{T,\text{max}}(\text{Re})$  may serve as turbulence intensity indicator [11, 12].

Similar calculations to find expressions relating macroscopic “fluid quantities” to “particle quantities” are given in the submitted paper, e.g. **Heat conductivity etc.[20]**.

Flows through systems of macroscopic particles have similarities with microscopic flows of individual molecules. However, there are also many differences e.g. **in a system of macroscopic particles pressure builds a body force and this body force is experienced not only during particle collisions, but also during the free flights, that particles make between collisions.** To show this and also exactly how to apply the presented method to examples exhibiting similarities of micro- and macroscopic systems we consider:

#### I. Archimedes principle: Fig. 5 (below).



**Fig. 5** Macroscopic particle of density  $\rho_S > \rho_F$  suspended in a container filled with on the average motionless fluid ( $\rho_F$  fluid density). To determine is the force  $\underline{F}$  required to hold the sphere motionless.

**First step** of analysis builds choice of proper theoretical tools. Since in this example we deal with forces in a macroscopic system, the adequate analysis tool will be form (3) of eq. (1). Written for a fluid it reads:

$$\frac{d}{dt} \int_V \rho_F \underline{v} dV = \int_V \rho_F \underline{g} dV - \int_A p \underline{dA} + \int_A \underline{t}(\eta) \bullet \underline{dA} \quad (14)$$

For suspended solid sphere eq. of motion must be extended with external force  $-F \underline{e}_z$ :

$$\begin{aligned} & \frac{d}{dt} \int_V \rho_S \underline{v} dV = \\ & = \int_V \rho_S \underline{g} dV - \int_A p \underline{dA} + \int_A \underline{t} \bullet \underline{dA} + (-F \underline{e}_z) \end{aligned} \quad (15)$$

**Second step** is implementation of adequate simplifying assumptions:

- Time dependence and transport of matter beyond the boundaries of any macroscopic

partial volume (e.g. volume containing more than  $10^{10}$  molecules) vanish in the considered system.

- Friction vanishes because the fluid is in macroscopic sense motionless. From equation of motion (3) for the fluid follows:

$$0 = \int_V \rho_F \underline{g} dV - \int_A p d\underline{A} \quad (16)$$

For submerged sphere, with  $\underline{F} = F \underline{e}_z$ :

$$\underline{0} = \int_V \rho_S \underline{g} dV_S - \int_A p d\underline{A} - \underline{F} \quad (17)$$

For the hydrostatic pressure we have:

$$\int_A p d\underline{A} = \int_V \rho_F \underline{g} dV \quad (18)$$

**The third step** of analysis in all problems allowing it, is use of the mean value theorem (MVT) of integral calculus. Applying it to the above equation, for any horizontal flat surface  $A_Q$  follows:  $p_{A_Q} = \rho_F \underline{g} V$  (19)

**The fourth step** is frequently obtaining the scalar components of the resulting vector equation. To this purpose we multiply the above result with unit vector  $\underline{e}_z$ , it follows:

$$p_{A_Q} = \rho_F g V ; \text{ with: } V = A_Q z$$

and:  $z = h$  for the **hydrostatic pressure** we obtain:  $p = \rho_F g z = \rho_F g h$  (20)

Direct determination of the force suspending the macroscopic sphere from mean value theorem applied to eq. (17) is not possible, because pressure under the integral over sphere surface varies with the depth  $z$  respectively with  $h$ . In such cases it is most convenient to use the **Gauss-Ostrogradski theorem**. It follows:

$$\underline{F} = \int_V \rho_S \underline{g} dV_S - \int_A p d\underline{A} = \int_V \rho_S \underline{g} dV_S - \int_V \nabla p dV_S \quad (21)$$

$$\nabla p = \left( \frac{\partial}{\partial x} \underline{e}_x + \frac{\partial}{\partial y} \underline{e}_y + \frac{\partial}{\partial z} \underline{e}_z \right) \rho_F g z = \rho_F \underline{g} \quad (22)$$

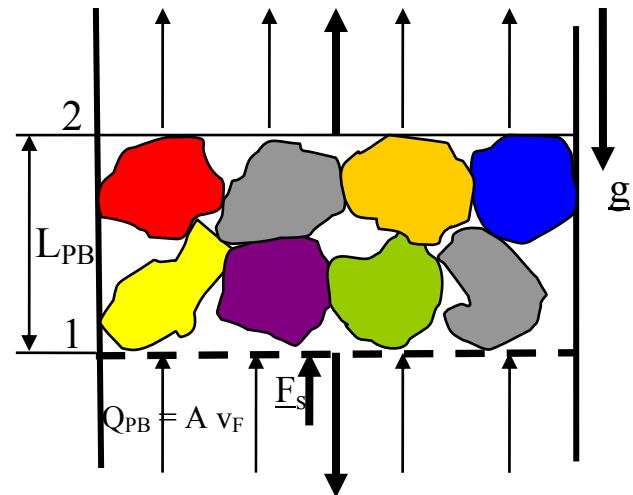
The resulting pressure gradient  $\nabla p = \rho_F \underline{g}$  is constant in the whole fluid volume  $V$  and thus also on all surface parts of the submerged sphere. Applying the MVT, now we obtain:

$$\begin{aligned} \underline{F} &= \int_V \rho_S \underline{g} dV_S - \int_V \nabla p dV_S = \\ &= \rho_S \underline{g} V_S - \rho_F \underline{g} V_S \end{aligned} \quad (23)$$

This result may easily be generalized for an arbitrary shape of a submerged body. We obtain: weight of a body submerged in a fluid (force  $\underline{F}$ ) is apparently reduced by the weight of the displaced fluid, **q.e.d.**

Extending the above example to a system of multiple macroscopic particles we continue with analysis of:

**II. Fixed and fluidized beds, Figs. 6 and 7** below.



**Fig. 6** Schematic representation of particle bed at rest. Particles on the sieve are supported with total force  $\underline{F}_S$  equal to total particle weight less weight of the displaced fluid.

The convenient first step of analysis of an idealized fluidized bed starts with determination of forces acting on particles. These result from equation (3) applied to particles. It reads:

$$\begin{aligned} \frac{d}{dt} \int_{V_p} \rho_p \underline{v} dV &= \\ &= \int_{V_p} \rho_p \underline{g} dV - \int_{A_p} p d\underline{A} + \int_{A_p} \underline{t}(\eta) \bullet d\underline{A} + \underline{F}_S \end{aligned} \quad (25)$$

Thereby  $V_p$  is the volume filled by the particles;  $\rho_p$  is the density of the particles;  $A_p$  is respectively the total surface of all particles regarded here as total wetted surface. Thus:  $A_w = A_p$ ;  $\underline{F}_S$  denotes the support forces acting on particles resting on the supporting sieve. For the upward direction follows [7, 11, 12]:

$$F_S = (\rho_p - \rho_f) g V_p - k_{FB}(Re) \eta A W \frac{v_{rel}}{R_{HP}} \quad (26)$$

Thereby  $\rho_f$  is the fluid density.  $R_{HP}$  is the hydraulic radius estimating the boundary layer thickness in the pores between the particles [8, 11, 12]:  $R_{HP} = V\varepsilon / A_w$ . The above simplified form of eq. (25) is valid for particle phase for situations represented in Fig. 5 during all periods of time in which the average absolute velocity of the particles  $w_g$  along the gravity vector vanishes i.e. :  $w_g = 0$  and  $v_{rel} = v$ .

The first term on the right side of the above eq. (26) represents weight of the particles reduced by weight of the displaced fluid (Archimedes law, above). The second term describes the viscous drag force. The interstitial flow velocity  $v$  in spaces between the particles is controlled by the flow intensity  $Q$  and by the geometry of the interstitial flow:  $Q = v A \varepsilon$  (27)

Where  $A$  is the total cross-section of the column and  $\varepsilon$  represents the porosity of the considered particle ensemble.

Flow intensity  $Q$  defines several operation modes of the system:

**a) Particle bed at rest** is characterized by the condition of total particle weight less weight of the displaced fluid being larger than the drag exerted by the flow i.e., in respect to eq. (26) when:  $F_S \geq 0$  (28)

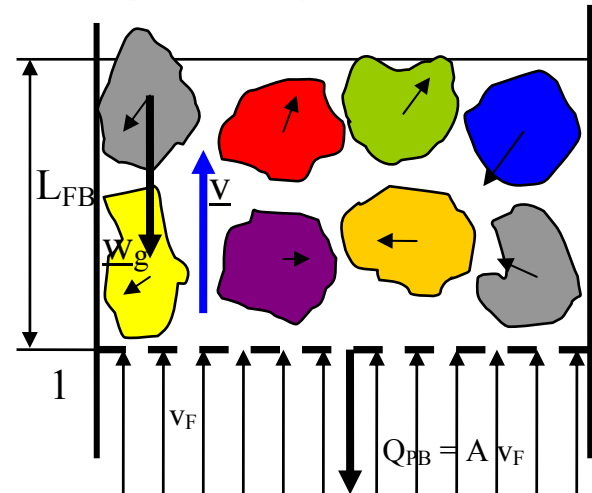
**b) Unstable state** of the bed beginning when the flow drag approaches the total weight of the particle bed diminished by the weight of the displaced fluid, so that the supporting force  $F_S$  (eq. (26)) vanishes (s. Fig. 8 further below). The interstitial fluid velocity in spaces between the particles  $v$  is still smaller than the average absolute sedimentation velocity of the particles  $w_g$ . The porosity of the particles  $\varepsilon$  varies locally between porosity of fixed bed  $\varepsilon_{FB}$  and 1. Then following conditions prevail:

$F_S = 0$ ;  $v < w_g$ ;  $\varepsilon_{FB} \leq \varepsilon \leq 1$

$$F_S = 0 ; \quad v < w_g ; \quad \varepsilon_{FB} \leq \varepsilon \leq 1 \quad (29)$$

**c) Steady state** begins when the interstitial fluid velocity  $v$  reaches the sedimentation

velocity of single particles  $w_g$  throughout the whole particle bed at porosities  $\varepsilon$  just starting to exceed the porosity of a fixed bed  $\varepsilon_{FB}$ . This operation mode is approximated by equilibrium of drag and weight of single particles ( $F_S=0$ ). Individual particles start moving freely in the flow as shown in Fig. 7 below building together a macroscopic model of a liquid (compare Fig. 3 and 7).



**Fig. 7** Fluidized particle bed. Particles are separated from each other by the flow and move freely above the supporting sieve. The average sedimentation velocity of the particles (black arrow  $w_g$ ) approximately equals the average upward interstitial fluid velocity (blue arrow). Flow intensity through the fluidized bed  $Q_{FB}$  is larger than flow intensity through the particle bed at rest  $Q_{PB} > Q_{PB}$ . Length  $L_{FB}$  exceeds length of the particle bed at rest  $L_{FB} > L_{PB}$ .

Porosity of the particles  $\varepsilon$  in steady state of a fluidized bed is estimated by an approximately continuous function of the flow intensity  $Q$  and of characteristic dimensions and velocities in the column as:

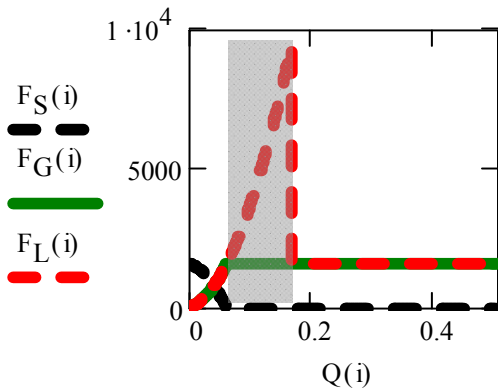
$$\varepsilon = \frac{Q}{W_g A} = \frac{v_F}{W_g} \quad (30)$$

Porosity of the fluidized particle layer increases linearly with increasing flow intensity  $Q$  from porosity of the fixed bed  $\varepsilon_{FB}$  till the value of 1.

**In respect to the calculation of pressures in the column it is to be remarked:**

- **Forces** relevant for the behaviour of the particles remain constant under steady

operation conditions allowing simple analysis and representation of results (Fig. 8 below)

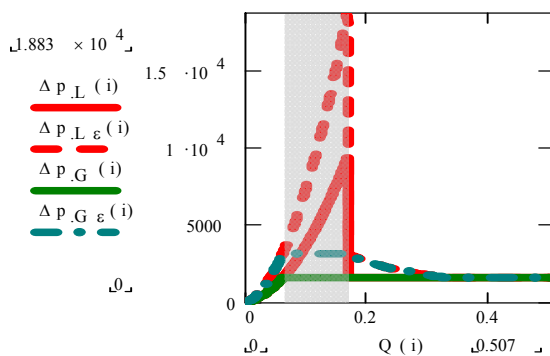


**Fig. 8:**  $F_S(i)$ : force exerted by the particles on the supporting sieve (Archimedes principle).

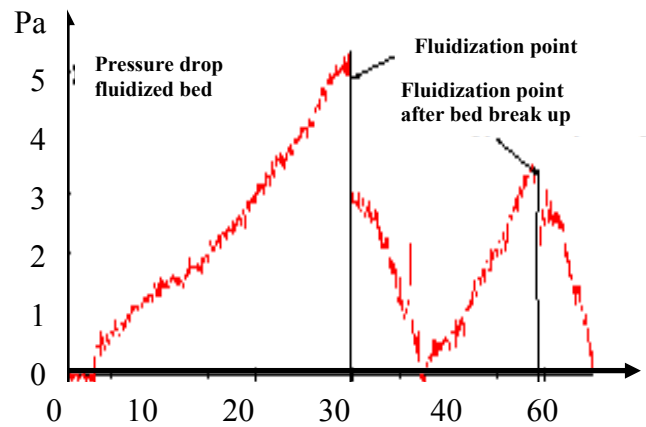
$F_G(i)$ : Drag force on the particles in the fixed bed and in steady state of the fluidized bed (green line in and after the grey block of unstable operation mode),

$F_L(i)$ : viscous drag force exerted by the flow on the particles.  $F_L(i)$  rises till unstable mode (grey block region) is reached. In the unstable region “anything” may occur. Red interrupted line shows viscous drag in the unstable bed increasing till bed breakup occurs (abrupt fall of the interrupted  $F_L(i)$  line to  $F_G(i)$  level).

- **Calculation of pressures** relates forces to the magnitudes of the cross-sections available to the flow. Free cross sections depend on the porosity. Thus pressure is a function of porosity and flow intensity:  $p = f(\epsilon(Q))$ . It may also be calculated by relating forces to the column cross section  $A$ . This **introduces the possibility of multiple interpretations of principally identical results** (s. Fig. 9 below).



**Fig. 9** Calculated pressures: two pressure values ( $\Delta p_L(i)$  and  $\Delta p_{L\epsilon}(i)$ ) result from a single force value  $F_L(i)$  or  $F_G(i)$   $\rho_F \rho_F$  of Fig. 8 above.



**Fig. 10** Cold test of a fluidized bed designed for processing of wood splitters for regenerative energy generation. (By courtesy of Thomas Ziegenhein, c.o. Frank Behrendt)

A serious problem with measurement results in fluidized beds is that they frequently look very much different than theoretical predictions based on idealized assumptions. Causes of deviations from theoretical predictions may be manifold.

Besides inhomogeneity of flow and particle parameters many problems are associated with electrostatic loads acquired by macroscopic particles during processing. Further, other processes like e.g. coalescence, air cleaning etc. require taking under account electrostatic forces, which besides determining the motion of individual molecules (Fig. 1) and parameters of macroscopic flows at micro scale are in similar way highly relevant at macro scale for the behaviour of macroscopic particles, or for design of engineering processes. **Understanding of relevant phenomena oft cannot be limited to their single scale: micro or macro.**

**Conclusions:** Required is a physically coherent, valid in a broad range of scales, yet **adequate for interdisciplinary education** possibly simple description of all phenomena related to the motion of matter.

Experimental evidence gathered in corpuscular physics [3, 15 – 18], in kinetic gas theory [2, 3], as well as in classical astrophysics [6],



confirms Newton's law as the simplest tool suited to describe the non-relativistic motion of matter generally.

On the basis of author's earlier experience [19] and application examples [7 – 13] the submitted paper [20] proposes use of Newton's law in integral notation together with conservation laws as explicit reference standard and starting point of analysis of all phenomena, which may be discussed in terms of dynamics of disperse systems.

## References

- [1] Hughes, W.F.; Brighton, J.A.; Fluid Dynamics; Schaum's Outline Series in Engineering; McGraw-Hill Book Co.; 1967
- [2] Reif, F.; Statistical Physics; McGraw-Hill Book Company; 1967
- [3] Resnick, R.; Halliday, D.; Physics; John Wiley and Sons, Inc.; 1966
- [4] Fefferman; Ch.; L.; Existence and smoothness of the Navier-Stokes equation; Princeton; NJ 08544-1000; 2000
- [5] Hirsh, C.; Numerical Computation of Internal and External Flows; Elsevier; 2007
- [6] Hoyle; F.; Astronomy and Cosmology; Freeman and Co.; 1975
- [7] Krol, M.; Prediction of relative particle fluid displacements in flowing suspensions at medium particle volume fractions by means of a simple stochastic model; 4<sup>th</sup> Miami Int. Conf. on: Multiphase Transport and Particulate Phenomena; Miami, USA; 1986; Multi-Phase Transport and Particulate Phenomena; Vol. 4; Hemisphere Publishing Corporation and Springer Publishing House (1988).
- [8] Krol, M.: Pressure drop in porous media at arbitrary Reynolds numbers; European Aerosol Conference; Helsinki; Sept. 1995;
- [9] Krol, M.: Fluidodynamik – Dynamik disperser Systeme; 1997/98
- [10] Król, M. :Equation of Motion of a compressible Fluid derived from the Kinetic Theory of Gases and Newton's Second Law IMP PAN Rep.No. 3195/2003 and IMP PAN Rep.No. 3197/2003
- [11] Król, M.; Physics of solids, liquids and gases; Lectures (GSW); Poland; 2005 - 2010
- [12] Krol, M.; Physikalische Grundlagen der Ein- und Mehrphasen-Strömungen; Lectures at Technical University of Berlin; Germany 2006 - 2011
- [13] Krol, M.: Momentum Exchange as common physical background of a transparent and physically coherent description of Transport Phenomena; Begell House Inc. Proceedings of the International Symposium "Turbulence, Heat and Mass Transfer 6"; 2009.
- [14] Ziegenhein, Th.; Vergasung von Holz in einer Wirbelschicht mit anschließender katalytischer Reinigung; Term Studie; Fachgebiet Energieverfahrenstechnik und Umwandlungstechniken regenerativer Energien; c/o Frank Behrendt; Technische Universität Berlin; 2010.
- [15] Wichmann; H.,Eyvind; Quantum Physics; Berkeley Physics Course – Volume 4; McGraw – Hill Book Company; 1971
- [16] Purcell; M. Edward; Electricity and Magnetism; Berkeley Physics Course – Volume 2; McGraw – Hill Book Co.; 1965
- [17] Pauling; Linus; Pauling; Peter; Chemistry; W. H. Freeman and Co.: 1975
- [18] Marsden, J.E.; Tromba, A.J.; Vector Calculus; Freeman and Co.; 1976
- [19] Krol, M.; Experimentelle Untersuchung der Partikelbewegung bei hohen Feststoffkonzentrationen in turbulenten Mehrphasenströmungen; Phd. Thesis; University of Kaiserslautern; 1984
- [20] Krol, M.; Significance of the microfluidic concepts for the improvement of macroscopic models of transport phenomena; paper submitted to the 3<sup>rd</sup> Micro and Nano Flows Conference, Thessaloniki, Greece; 2011.