# Outage Analysis for Underlay relay-assisted Cognitive Networks

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***Abstract*—Cooperative relay technology was recently introduced into cognitive radio networks in order to enhance network capacity, scalability, and reliability of end-to-end communication. In this paper, we investigate an underlay cognitive network where the quality of service of the secondary link is maintained by triggering an opportunistic regenerative relaying once it falls under an unacceptable level. We first provide the exact cumulative density function (CDF) of received signal-to-noise (SNR) over each hop with co-located relays. Then, the CDFs are used to determine very accurate closed-form expression for the outage probability for a transmission rate $R$. We validate our analysis by showing that simulation results coincide with our analytical results in Rayleigh fading channels.**

## I. Introduction

Cognitive radio (CR) is an emerging technique which is proposed to improve the wireless spectrum resources utilization efficiency. This improvement is achieved by allowing the unlicensed users, referred to as secondary users (SU), to get dynamic access to the spectrum of the licensed users, referred to as primary users (PU), using interweave, overlay, or underlay paradigms [1]. In particular, in spectrum underlay cognitive radio, the SU coexists with PU in the licensed band by keeping their transmission power under a certain threshold to avoid any problem for the detection at the primary receiver. Thus, traditional underlay cognitive networks have predominantly used direct point-to-point secondary link where the transmission power is constrained [2]. Accordingly, the quality of service (QoS) of the secondary user may be under expectation. At the same time, cooperative diversity schemes have been very attractive for small-size and antenna-limited wireless devices, and opportunistic relaying (OR) techniques have been proposed where only the best relay from a set of $K$ available candidate relays is selected to cooperate [3], [4].
Recently, several works investigated cognitive wireless relay networks consisting of a source node that intends to communicate with a destination node aided by a total number of $K$ relays nodes based on the underlay approach [5]–[9]. More specifically, in [5], the authors considered a relay selection based on max-min criterion where the direct secondary link was omitted, and they derived the outage probability. However, the direct secondary link was considered and only a tight lower bound of the outage probability was provided in [6]. Besides, the authors opted for relay selection based on the second hope, and they did not consider a maximum transmission power which means that transmission power will increase as more as the interference channel is in deep fading. In [7], the authors investigated a cognitive relay network under PU's outage constraint, where the SU's transmission should not affect the PU's transmission and maintain the outage probability at the primary receiver under a predetermined value. However, and considering that direct secondary link may or may not be used, they derived the outage probability of SU based on an upper bound expression of the PU's outage probability, and the number of involved relays is determined by exhaustive comparisons. By investigating several relay selection strategies, the authors in [8] derived asymptotic outage analysis of the secondary system, whereas the authors in [9] derived exact outage analysis by relaxing the maximum transmission power constraint and by reducing the number of active relays to one. Both works [8] and [9] did not consider direct link in the cognitive network. Obviously, involving relays within underlay cognitive networks may lead to better performances by addressing the aforementioned drawbacks.
Our goal in this paper is to evaluate the end-to-end outage analysis of cognitive relay networks where an incremental opportunistic DF relaying is considered. Our contribution is three folds. First, we provide exact statistics of the received signal-to-noise (SNR) over each hop with co-located relays, in terms of cumulative density function (CDF). Since the received SNRs by the secondary receiver and the selected relay are dependent, we provide their joint CDF. Finally, we derive the end-to-end outage probability for each scheme.
To the best of our knowledge, such performance analysis for underlay cognitive network using incremental opportunistic regenerative relaying has not been considered in the literature, and is crucial to offer accurate outage of such systems.

## II. System model

In this section, we describe our proposed underlay cognitive network using an incremental regenerative relaying which is triggered based on the received SNR at the secondary receiver, and we note that only a selected relay from a cluster is targeted to cooperate. The secondary networks consists of a source (S-Tx), a receiver (S-Rx), and a cluster of $K$ potential relays, whereas the primary network consists of a source (P-Tx) and a receiver (P-Rx), where each node is equipped with a single antenna and each relay works in DF mode. For the secondary network, we denote $h_{sr_k}$, $h_{ss}$ and $h_{r_ks}$ as the coefficients of the channels between S-Tx and the $k^{th}$

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relay, S-Tx and S-Rx, and the $k^{th}$ relay and S-Rx, modeled as flat fading and Rayleigh distributed with variances $\lambda_{sr_k}$, $\lambda_{ss}$ and $\lambda_{r_k s}$, respectively. Similarly, we denote $h_{sp}$ and $h_{r_k p}$ as the coefficients of the interference channels between S-Tx and P-Rx, and the $k^{th}$ relay and P-Rx, and modeled as flat fading and Rayleigh distributed with variances $\lambda_{sp}$ and $\lambda_{r_k p}$, respectively. We assume that the relays are close to each other and forming a cluster[1] and accordingly, we assume that $\lambda_{sr} = \lambda_{sr_k}$, $\lambda_{rs} = \lambda_{r_k s}$ and $\lambda_{rp} = \lambda_{r_k p}$ for all $k$.

While a conventional terminal transmits with a constant power, under the underlay paradigm, S-Tx is able to optimize its broadcasting power $P_s$ according to the radio environments by satisfying the following conditions

$$\begin{cases} Q_s \leq \mathrm{I} \\ P_s \leq \bar{P}, \end{cases} \tag{1}$$

where $\bar{P}$ is the maximum S-Tx transmit power constraint, $Q_s$ is the interference induced at P-Rx by the simultaneous transmission of S-Tx over the same band with P-Tx, and given by

$$Q_s \triangleq |h_{sp}|^2 \frac{P_s}{N_0}, \tag{2}$$

where $N_0$ is the noise variance, and I is the maximum acceptable level of interference tolerated at P-Rx. As a consequence, the received SNR at S-Rx, labeled as $\gamma_{ss}$, can be given by

$$\gamma_{ss} = \min\left(\frac{\mathrm{I}N_0}{|h_{sp}|^2}, \bar{P}\right) \frac{|h_{ss}|^2}{N_0} = \min\left(\frac{\mathrm{I}}{|h_{sp}|^2}, \bar{\gamma}\right) |h_{ss}|^2, \tag{3}$$

where $|h_{sp}|^2$ is considered to be known at S-Tx and the first inequality in (1) is well satisfied. It follows that CDF of $\gamma_{ss}$ is given by

$$F_{\gamma_{ss}}(\mathrm{x}) = 1 - e^{-\frac{\mathrm{x}}{\lambda_{ss}\bar{\gamma}}} \left(1 - e^{-\frac{\mathrm{I}}{\lambda_{sp}\bar{\gamma}}}\right) - \frac{1}{1 + \frac{\lambda_{sp}\mathrm{x}}{\lambda_{ss}\mathrm{I}}} e^{-\left(\frac{\mathrm{I}}{\lambda_{sp}\bar{\gamma}} + \frac{\mathrm{x}}{\lambda_{ss}\bar{\gamma}}\right)} \tag{4}$$

When $\gamma_{ss}$ exceeds a certain threshold, we consider the direct transmission is enough and no need to cooperate. Otherwise, the cooperation is needed to maintain the QoS at S-Rx, which sends a binary feedback to S-Tx and relays requesting them to retransmit. Therefore a relayed copy is needed and the received signal is decoded and forwarded by a relay $r_*$ which is selected following the rule

$$r_* = \arg\max_k \min\left(|h_{sr_k}|^2, |h_{r_k s}|^2\right). \tag{5}$$

We can deduce that received SNRs at $r_*$ and S-Tx can be given by

$$\gamma_{sr_*} = \min\left(\frac{\mathrm{I}}{|h_{sp}|^2}, \bar{\gamma}\right) |h_{sr_*}|^2, \tag{6}$$

and

$$\gamma_{r_* s} = \min\left(\frac{\mathrm{I}}{|h_{r_* p}|^2}, \bar{\gamma}\right) |h_{r_* s}|^2, \tag{7}$$

respectively.

[1]We assume short distances between the relays compared to the distances (S)-cluster, cluster-(D) and cluster-(P-RX), respectively.

*Lemma 1:* For a (S-Tx)-(S-Rx) pair with $K$ relays sharing the same spectrum witn a (P-Tx)-(P-Rx) pair using opportunistic DF-relaying scheme in Rayleigh fading channels, the CDF and the probability density function (PDF) of $\gamma_{r_* s}$ are given by (8) and (9), respectively, where $\lambda = \frac{\lambda_{sr}\lambda_{rs}}{\lambda_{sr}+\lambda_{rs}}$, $\xi = \lambda_{sr}/\lambda_{rs}$, $A(i)$ and $B(i)$ are given by

$$A(i) = \frac{i\xi}{\lambda_{sr}(i(\xi+1)-\xi)}, \tag{10}$$

$$B(i) = \left(\frac{i}{\lambda_{rs}} - \frac{i}{\lambda_{sr}(i(1+(1/\xi))-1)}\right), \tag{11}$$

and $\mathcal{I}_b(.)$ is given by

$$\mathcal{I}_b(a) = \frac{1}{1+ab} e^{-\frac{\mathrm{I}}{\bar{\gamma}}\left(a+\frac{1}{b}\right)}. \tag{12}$$

*Proof:* See Appendix A. ■

## III. Outage Analysis

The end-to-end outage probability of the underlay cognitive radio using the opportunistic DF relaying at the operating transmission rate $R$ is given by

$$\begin{aligned} P_{out} =& P_r\left[\gamma_{ss} < \Phi', \gamma_{sr_*} < \Phi\right] \\ &+ P_r\left[\gamma_{ss} < \Phi', \gamma_{sr_*} \geq \Phi, \gamma_{ss} + \gamma_{r_* s} < \Phi\right], \end{aligned} \tag{13}$$

where $\Phi' = 2^R - 1$ and $\Phi = 2^{2R} - 1$, and the first probability in (13) can be given by the joint CDF[2] of $\gamma_{ss}$ and $\gamma_{sr_*}$, $F_{\gamma_{ss},\gamma_{sr_*}}(.,.)$, which is given by (See Appendix B)

$$F_{\gamma_{ss},\gamma_{sr_*}}(\Phi', \Phi) = P_1(\Phi', \Phi) + P_2(\Phi', \Phi), \tag{14}$$

where $P_1(.,.)$ and $P_2(.,.)$ are detailed in (15), and $\mathfrak{g}_{\lambda_{sp}}(.,.)$ is given by

$$\mathfrak{g}_{\lambda_{sp}}(\alpha, \beta) = \int_{\frac{\mathrm{I}}{\bar{\gamma}}}^{\infty} (1 - e^{-\alpha \mathrm{z}})(1 - e^{-\beta \mathrm{z}}) \frac{e^{-\mathrm{z}/a}}{a}\, d\mathrm{z} = \tag{16}$$
$$e^{-\frac{\mathrm{I}}{\lambda_{sp}\bar{\gamma}}} \left[1 - \frac{e^{-\frac{\mathrm{I}\beta}{\bar{\gamma}}}}{1+\beta\lambda_{sp}} - \frac{e^{-\frac{\mathrm{I}\alpha}{\bar{\gamma}}}}{1+\alpha\lambda_{sp}} + \frac{e^{-\frac{\mathrm{I}(\alpha+\beta)}{\bar{\gamma}}}}{1+(\alpha+\beta)\lambda_{sp}}\right]$$

In (13), the second probability can be defined as

$$P_r\left[\gamma_{ss} < \Phi', \gamma_{sr_*} \geq \Phi, \gamma_{ss} + \gamma_{r_* s} < \Phi\right] = \tag{17}$$
$$\int_0^{\Phi} P_r\left[\gamma_{ss} < \min(\Phi', \Phi - \mathrm{z}), \gamma_{sr_*} \geq \Phi | \mathrm{z}\right] p_{\gamma_{r_* s}}(\mathrm{z}) d\mathrm{z},$$

which can be derived as function of $F_{\gamma_{ss}}(.)$, $F_{\gamma_{r_* s}}(.)$, and $F_{\gamma_{ss},\gamma_{sr_*}}(.,.)$, as shown in (18). Moreover, by using (4) and (9), $I_1$ can be given by (19) where $S_i$ are detailed in (20) with the help of the following identities.

In (20), $\mathfrak{I}(.,.)$ is given by

$$\mathfrak{I}(a, b) = \frac{e^{(a-b)\Phi}}{a-b} \left(1 - e^{-(a-b)\Phi'}\right), \tag{21}$$

$\mathfrak{I}_1(.,.,.)$ is defined in [10, Eq. (3.352.3)], and given by

$$\mathfrak{I}_1(a, b, c) = \left[\frac{e^{-\frac{a-b}{c}}}{c} E_i\left((a-b)\left(\mathrm{x} + \frac{1}{c}\right)\right)\right]_{\Phi-\Phi'}^{\Phi} \tag{22}$$

[2]Looking at (3) and (6), it can be noted that $\gamma_{ss}$ and $\gamma_{sr_*}$ are not independent.

$$F_{\gamma_{r_*s}}(\mathrm{x}) = 1 - \sum_{i=1}^{K} \binom{K}{i} \frac{i(-1)^{i-1}\lambda}{\lambda_{sr}(i\lambda_{rs}-\lambda)} \left(\lambda_{rs}\mathcal{I}_{\lambda_{rp}}\left(\frac{\mathrm{x}}{\lambda_{rs}\mathrm{I}}\right) - \frac{\lambda}{i}\mathcal{I}_{\lambda_{rp}}\left(\frac{i\mathrm{x}}{\lambda\mathrm{I}}\right)\right) - \sum_{i=1}^{K}\binom{K}{i}(-1)^{i-1}\frac{\lambda}{\lambda_{rs}}\mathcal{I}_{\lambda_{rp}}\left(\frac{i\mathrm{x}}{\lambda\mathrm{I}}\right)$$
$$-\left(1-e^{-\frac{\mathrm{I}}{\lambda_{rp}\bar{\gamma}}}\right)\left[\sum_{i=1}^{K}\binom{K}{i}\frac{(-1)^{i-1}}{\lambda_{sr}}\frac{i\lambda}{i\lambda_{rs}-\lambda}\left(\lambda_{rs}e^{-\frac{\mathrm{x}}{\lambda_{rs}\bar{\gamma}}} - \frac{\lambda}{i}e^{-\frac{i\mathrm{x}}{\lambda\bar{\gamma}}}\right) + \sum_{i=1}^{K}\binom{K}{i}(-1)^{i-1}\frac{\lambda}{\lambda_{rs}}e^{-\frac{i\mathrm{x}}{\lambda\bar{\gamma}}}\right], \quad (8)$$

---

$$p_{\gamma_{r_*s}}(\mathrm{x}) = \frac{1}{\bar{\gamma}}\left(1-e^{-\frac{\mathrm{I}}{\lambda_{rp}\bar{\gamma}}}\right)\left[\sum_{i=1}^{K}\binom{K}{i}(-1)^{i-1}A(i)e^{-\frac{\mathrm{x}}{\lambda_{rs}\bar{\gamma}}} + \sum_{i=1}^{K}\binom{K}{i}(-1)^{i-1}B(i)e^{-\frac{i\mathrm{x}}{\lambda\bar{\gamma}}}\right] + e^{-\frac{\mathrm{I}}{\lambda_{rp}\bar{\gamma}}}\times$$
$$\left[\sum_{i=1}^{K}\binom{K}{i}(-1)^{i-1}A(i)\left(\frac{\frac{1}{\bar{\gamma}}e^{-\frac{\mathrm{x}}{\lambda_{rs}\bar{\gamma}}}}{1+\frac{\lambda_{rp}\mathrm{x}}{\lambda_{rs}\mathrm{I}}} + \frac{\frac{\lambda_{rp}}{\mathrm{I}}e^{-\frac{\mathrm{x}}{\lambda_{rs}\bar{\gamma}}}}{\left(1+\frac{\lambda_{rp}\mathrm{x}}{\lambda_{rs}\mathrm{I}}\right)^2}\right) + \sum_{i=1}^{K}\binom{K}{i}(-1)^{i-1}B(i)\left(\frac{\frac{1}{\bar{\gamma}}e^{-\frac{i\mathrm{x}}{\lambda\bar{\gamma}}}}{1+\frac{i\lambda_{rp}\mathrm{x}}{\lambda\mathrm{I}}} + \frac{\frac{\lambda_{rp}}{\mathrm{I}}e^{-\frac{i\mathrm{x}}{\lambda\bar{\gamma}}}}{\left(1+\frac{i\lambda_{rp}\mathrm{x}}{\lambda\mathrm{I}}\right)^2}\right)\right], \quad (9)$$

---

$$P_1(\mathrm{x},\mathrm{y}) = \left(1-e^{-\frac{\mathrm{x}}{\lambda_{ss}\bar{\gamma}}}\right)\left(1-e^{-\frac{\mathrm{I}}{\lambda_{sp}\bar{\gamma}}}\right)\left[\sum_{i=1}^{K}\frac{\binom{K}{i}i\xi(-1)^{i-1}}{i(1+\xi)-1}\left(1-e^{-\frac{\mathrm{y}}{\lambda_{sr}\bar{\gamma}}}\right) + \sum_{i=1}^{K}\frac{\binom{K}{i}(i-1)(-1)^{i-1}}{i(1+\xi)-1}\left(1-e^{-\frac{i\mathrm{y}}{\lambda\bar{\gamma}}}\right)\right],$$
$$P_2(\mathrm{x},\mathrm{y}) = \sum_{i=1}^{K}\frac{\binom{K}{i}i\xi(-1)^{i-1}}{i(1+\xi)-1}\mathfrak{g}_{\lambda_{sp}}\left(\frac{\mathrm{x}}{\lambda_{ss}\mathrm{I}},\frac{\mathrm{y}}{\lambda_{sr}\mathrm{I}}\right) + \sum_{i=1}^{K}\frac{\binom{K}{i}(i-1)(-1)^{i-1}}{i(1+\xi)-1}\mathfrak{g}_{\lambda_{sp}}\left(\frac{\mathrm{x}}{\lambda_{ss}\mathrm{I}},\frac{i\mathrm{y}}{\lambda\mathrm{I}}\right), \quad (15)$$

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$$P_r\left[\gamma_{ss}<\Phi',\gamma_{sr_*}\geq\Phi,\gamma_{ss}+\gamma_{r_*s}<\Phi\right] = \left(F_{\gamma_{ss}}(\Phi') - P_r[\gamma_{s_*s}<\Phi',\gamma_{sr_*}<\Phi]\right)F_{\gamma_{r_*s}}(\Phi-\Phi')$$
$$+\underbrace{\int_{\Phi-\Phi'}^{\Phi}P_r\left[\gamma_{ss}<\Phi-\mathrm{z}|\mathrm{z}\right]p_{\gamma_{r_*s}}(\mathrm{z})d\mathrm{z}}_{I_1} - \underbrace{\int_{\Phi-\Phi'}^{\Phi}P_r\left[\gamma_{ss}<\Phi-\mathrm{z},\gamma_{sr_*}<\Phi|\mathrm{z}\right]p_{\gamma_{r_*s}}(\mathrm{z})d\mathrm{z}}_{I_2}, \quad (18)$$

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$$I_1 = \left[F_{\gamma_{r_*s}}(\Phi) - F_{\gamma_{r_*s}}(\Phi-\Phi')\right] - \left(1-e^{-\frac{\mathrm{I}}{\lambda_{sp}\bar{\gamma}}}\right)e^{-\frac{\Phi}{\lambda_{ss}\bar{\gamma}}}\left[(1-e^{-\frac{\mathrm{I}}{\lambda_{rp}\bar{\gamma}}})(S_1+S_2) + e^{-\frac{\mathrm{I}}{\lambda_{rp}\bar{\gamma}}}(S_3+S_4)\right]$$
$$-\ e^{-\frac{\mathrm{I}}{\lambda_{sp}\bar{\gamma}}}\ e^{-\frac{\Phi}{\lambda_{ss}\bar{\gamma}}}\left[(1-e^{-\frac{\mathrm{I}}{\lambda_{rp}\bar{\gamma}}})(S_5+S_6) + e^{-\frac{\mathrm{I}}{\lambda_{rp}\bar{\gamma}}}(S_7+S_8)\right], \quad (19)$$

---

where $E_i(\mathrm{x}) = \int_{-\infty}^{\mathrm{x}}\frac{e^t}{t}\,dt$ is the exponential integral function, and $\mathfrak{I}_2(.,.,.)$ is given by (23). $\mathfrak{g}(.,.,.,.)$ can be deduced from (19) as

$$\mathfrak{g}(a,b_1,c,b_2) = \frac{1}{a}\ \mathfrak{I}_1\left(b_1,b_2,-\frac{c}{a}\right), \quad (24)$$

and $\mathfrak{g}_1(.,.,.,.,.)$ and $\mathfrak{g}_2(.,.,.,.,.)$ are given by (25) and (26), respectively.

Similarly, by using (15) and (9), $I_2$ can be given by (23) where $\Lambda_{\Phi,\Phi'}(.)$ is given by (28), where $S_1$, $S_2$, $S_3$ and $S_4$ are already given by (20), and $\Omega_{\lambda_{sp}}(.,.)$ is defined by

$$\Omega_{\lambda_{sp}}\left(\frac{1}{\lambda_{ss}\mathrm{I}},\beta\right) \triangleq \int_{\Phi-\Phi'}^{\Phi}\mathfrak{g}_{\lambda_{sp}}\left(\frac{\Phi-\mathrm{z}}{\lambda_{ss}\mathrm{I}},\beta\right)p_{\gamma_{r_*s}}(\mathrm{z})d\mathrm{z}, \quad (29)$$

which is given by (30), where $s'_5$, $s'_6$, $s'_7$ and $s'_8$ are given by (31).

## IV. PERFORMANCE RESULTS

In this section, we confirm our performance analysis in terms of closed-form expression of the outage probability derived in Section III through comparisons with simulation results. We use $\lambda_{sp}=1$ and $\lambda_{ij}=d_{ij}^{-\nu}$, where $\nu=4$, $i\in\{s,r\}$ and $j\in\{s,r,p\}$[3], and all distances are normalized by $d_{sp}$ ($0<d_{ij}<1$). Therefore $d_{rp}=1-d_{sr}$ and $d_{rs}=d_{ss}-d_{sr}$. We evaluate the outage probability expression of our scheme as function of several parameters, for a transmission rate $R$ ($=2\ bit/s/Hz$) and a number ($K=4$) of active relays in the secondary system, and for several interference threshold I.

We first show the performance results as function of $\bar{\gamma}$ where S-Rx is fixed at the mid-distance to P-Rx. At $\mathrm{I}=0\ dB$, the use of four relays provides an excellent percentage of outage when the relay cluster is also placed at the mid-distance of S-Rx. The outage probability is about to 7% when $\mathrm{I}=-5\ dB$. Nevertheless, the QoS of the secondary system can be improved, even if the relay cluster is close to S-Rx, when the latter leaves the vicinity of P-Rx ($d_{sr}=0.4$).

Based on results shown in Fig. 1, we opted in Fig. 2 for positioning S-Rx at $d_{ss}=0.5$ and fixing $\bar{\gamma}$ at 30 $dB$, and varying the relay cluster position ($d_{sr}$) to optimize the overall QoS of the system. It is shown that secondary system performance achieves significant gains when $d_{sr}=0.25$.

[3] $s$ refers to S-Rx

$$
\begin{aligned}
S_1 &= \sum_{i=1}^{K} \binom{K}{i} \frac{(-1)^{i-1}}{\bar{\gamma}} \; A(i)\mathfrak{I}\left(\frac{1}{\lambda_{ss}\bar{\gamma}}, \frac{1}{\lambda_{rs}\bar{\gamma}}\right), \quad S_2 = \sum_{i=1}^{K} \binom{K}{i} \frac{(-1)^{i-1}}{\bar{\gamma}} \; B(i)\mathfrak{I}\left(\frac{1}{\lambda_{ss}\bar{\gamma}}, \frac{i}{\lambda\bar{\gamma}}\right), \\
S_3 &= \sum_{i=1}^{K} \binom{K}{i} (-1)^{i-1} A(i) \left[\left(\frac{1}{\bar{\gamma}}\right) \mathfrak{I}_1\left(\frac{1}{\lambda_{ss}\bar{\gamma}}, \frac{1}{\lambda_{rs}\bar{\gamma}}, \frac{\lambda_{rp}}{\lambda_{rs}\mathrm{I}}\right) + \left(\frac{\lambda_{rp}}{\mathrm{I}}\right) \mathfrak{I}_2\left(\frac{1}{\lambda_{ss}\bar{\gamma}}, \frac{1}{\lambda_{rs}\bar{\gamma}}, \frac{\lambda_{rp}}{\lambda_{rs}\mathrm{I}}\right)\right], \\
S_4 &= \sum_{i=1}^{K} \binom{K}{i} (-1)^{i-1} B(i) \left[\left(\frac{1}{\bar{\gamma}}\right) \mathfrak{I}_1\left(\frac{1}{\lambda_{ss}\bar{\gamma}}, \frac{i}{\lambda\bar{\gamma}}, \frac{i\lambda_{rp}}{\lambda\mathrm{I}}\right) + \left(\frac{\lambda_{rp}}{\mathrm{I}}\right) \mathfrak{I}_2\left(\frac{1}{\lambda_{ss}\bar{\gamma}}, \frac{i}{\lambda\bar{\gamma}}, \frac{i\lambda_{rp}}{\lambda\mathrm{I}}\right)\right], \\
S_5 &= \sum_{i=1}^{K} \binom{K}{i} (-1)^{i-1} \frac{A(i)}{\bar{\gamma}} \mathfrak{g}\left(1 + \frac{\lambda_{sp}\Phi}{\lambda_{ss}\mathrm{I}}, \frac{1}{\lambda_{ss}\bar{\gamma}}, \frac{\lambda_{sp}}{\lambda_{ss}\mathrm{I}}, \frac{1}{\lambda_{rs}\bar{\gamma}}\right), S_6 = \sum_{i=1}^{K} \binom{K}{i} (-1)^{i-1} \frac{B(i)}{\bar{\gamma}} \; \mathfrak{g}\left(1 + \frac{\lambda_{sp}\Phi}{\lambda_{ss}\mathrm{I}}, \frac{1}{\lambda_{ss}\bar{\gamma}}, \frac{\lambda_{sp}}{\lambda_{ss}\mathrm{I}}, \frac{i}{\lambda\bar{\gamma}}\right), \\
S_7 &= \sum_{i=1}^{K} \binom{K}{i} (-1)^{i-1} A(i) \left[\left(\frac{1}{\bar{\gamma}}\right) \mathfrak{g}_1\left(1 + \frac{\lambda_{sp}\Phi}{\lambda_{ss}\mathrm{I}}, \frac{1}{\lambda_{ss}\bar{\gamma}}, \frac{\lambda_{sp}}{\lambda_{ss}\mathrm{I}}, \frac{1}{\lambda_{rs}\bar{\gamma}}, \frac{\lambda_{rp}}{\lambda_{rs}\mathrm{I}}\right)\right. \\
&\qquad\qquad\qquad \left. + \left(\frac{\lambda_{rp}}{\mathrm{I}}\right) \mathfrak{g}_2\left(1 + \frac{\lambda_{sp}\Phi}{\lambda_{ss}\mathrm{I}}, \frac{1}{\lambda_{ss}\bar{\gamma}}, \frac{\lambda_{sp}}{\lambda_{ss}\mathrm{I}}, \frac{1}{\lambda_{rs}\bar{\gamma}}, \frac{\lambda_{rp}}{\lambda_{rs}\mathrm{I}}\right)\right] \\
S_8 &= \sum_{i=1}^{K} \binom{K}{i} (-1)^{i-1} B(i) \left[\left(\frac{1}{\bar{\gamma}}\right) \mathfrak{g}_1\left(1 + \frac{\lambda_{sp}\Phi}{\lambda_{ss}\mathrm{I}}, \frac{1}{\lambda_{ss}\bar{\gamma}}, \frac{\lambda_{sp}}{\lambda_{ss}\mathrm{I}}, \frac{i}{\lambda\bar{\gamma}}, \frac{i\lambda_{rp}}{\lambda\mathrm{I}}\right)\right. \\
&\qquad\qquad\qquad \left. + \left(\frac{\lambda_{rp}}{\mathrm{I}}\right) \mathfrak{g}_2\left(1 + \frac{\lambda_{sp}\Phi}{\lambda_{ss}\mathrm{I}}, \frac{1}{\lambda_{ss}\bar{\gamma}}, \frac{\lambda_{sp}}{\lambda_{ss}\mathrm{I}}, \frac{i}{\lambda\bar{\gamma}}, \frac{i\lambda_{rp}}{\lambda\mathrm{I}}\right)\right]. 
\end{aligned} \tag{20}
$$

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$$
\mathfrak{I}_2(a,b,c) \triangleq \int_{\Phi-\Phi'}^{\Phi} \frac{e^{(a-b)\mathrm{x}}}{(1+c\,\mathrm{x})^2}\, d\mathrm{x} = \left[\frac{e^{-\frac{a-b}{c}}}{c^2(1+c\,\mathrm{x})} \; \left((a-b)(1+c\,\mathrm{x})E_i\left((a-b)\left(\mathrm{x}+\frac{1}{c}\right)\right) - c\, e^{(a-b)\left(\mathrm{x}+\frac{1}{c}\right)}\right)\right]_{\Phi-\Phi'}^{\Phi}, \tag{23}
$$

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$$
\mathfrak{g}_1(a,b_1,c_1,b_2,c_2) = \left[\frac{e^{-\frac{b_1-b_2}{c_2}}}{c_1+ac_2} E_i\left((b_1-b_2)\left(\mathrm{x}+\frac{1}{c_2}\right)\right) - \frac{e^{(b_1-b_2)\frac{a}{c_1}}}{c_1+ac_2} E_i\left((b_1-b_2)\left(\mathrm{x}-\frac{a}{c_1}\right)\right)\right]_{\Phi-\Phi'}^{\Phi} \tag{25}
$$

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$$
\begin{aligned}
\mathfrak{g}_2(a,b_1,c_1,b_2,c_2) = &\left[-\frac{c\, e^{(b_1-b_2)\frac{a}{c_1}}}{(ac_2+c_1)} \; E_i\left((b_1-b_2)\left(\mathrm{x}-\frac{a}{c_1}\right)\right)\right. \\
&\left.+\frac{((b_1-b_2)(ac_2+c_1)+(c_1c_2))}{c_2(ac_2+c_1)^2} \; E_i\left((b_1-b_2)\left(\mathrm{x}+\frac{1}{c_2}\right)\right) - \frac{e^{(b_1-b_2)\left(\mathrm{x}+\frac{1}{c_2}\right)}}{(1+c_2\;\mathrm{x})(ac_2+c_1)}\right]_{\Phi-\Phi'}^{\Phi}.
\end{aligned} \tag{26}
$$

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$$
\begin{aligned}
I_2 = &\Lambda_{\Phi,\Phi'}\left(\frac{1}{\lambda_{ss}\bar{\gamma}}\right)\left(1-e^{-\frac{\mathrm{I}}{\lambda_{sp}\bar{\gamma}}}\right)\left[\sum_{i=1}^{K}\binom{K}{i}\frac{i\xi(-1)^{i-1}}{i(\xi+1)-1}\;\left(1-e^{-\frac{\Phi}{\lambda_{sr}\bar{\gamma}}}\right)+\sum_{i=1}^{K}\binom{K}{i}\frac{(i-1)(-1)^{i-1}}{i(1+\xi)-1}\;\left(1-e^{-\frac{i\Phi}{\lambda\bar{\gamma}}}\right)\right] \\
&+\sum_{i=1}^{K}\binom{K}{i}\frac{i\xi(-1)^{i-1}}{i(\xi+1)-1}\;\Omega_{\lambda_{sp}}\left(\frac{1}{\lambda_{ss}\bar{\gamma}},\frac{\Phi}{\lambda_{sr}\mathrm{I}}\right)+\sum_{i=1}^{K}\binom{K}{i}\frac{(i-1)(-1)^{i-1}}{i(1+\xi)-1}\;\Omega_{\lambda_{sp}}\left(\frac{1}{\lambda_{ss}\bar{\gamma}},\frac{i\Phi}{\lambda\mathrm{I}}\right),
\end{aligned} \tag{27}
$$

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$$
\Lambda_{\Phi,\Phi'}\left(\frac{1}{\lambda_{ss}\bar{\gamma}}\right) = \left[F_{\gamma_{r_*s}}(\Phi) - F_{\gamma_{r_*s}}(\Phi-\Phi')\right] - e^{-\frac{\Phi}{\lambda_{ss}\bar{\gamma}}}\;\left[(1-e^{-\frac{\mathrm{I}}{\lambda_{rp}\bar{\gamma}}})(S_1+S_2)+e^{-\frac{\mathrm{I}}{\lambda_{rp}\bar{\gamma}}}(S_3+S_4)\right], \tag{28}
$$

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Fig. 3 shows performance results function of $\lambda_{ss}$ ($\lambda = 1$) with varying interference power constraints I. Using $d_{sr} = d_{ss}/2$, $\lambda_{rp}$ can be given by

$$
\lambda_{rp} = \left[1 - \frac{1}{2}\left(\frac{\lambda_{ss}}{\lambda_{sp}}\right)^{-\frac{1}{\nu}}\right]^{-\nu}.
$$

$$\Omega_{\lambda_{sp}}\left(\frac{1}{\lambda_{ss}\mathrm{I}},\beta\right)=e^{-\frac{\mathrm{I}}{\lambda_{sp}\bar{\gamma}}}\left[\left(1-\frac{e^{-\frac{\mathrm{I}\beta}{\bar{\gamma}}}}{1+\beta\lambda_{sp}}\right)\left[F_{\gamma_{r_*s}}(\Phi)-F_{\gamma_{r_*s}}(\Phi-\Phi')\right]-e^{-\frac{\Phi}{\lambda_{ss}\bar{\gamma}}}\left[(1-e^{-\frac{\mathrm{I}}{\lambda_{rp}\bar{\gamma}}})(S_5+S_6)\right.\right.$$
$$\left.\left.+\,e^{-\frac{\mathrm{I}}{\lambda_{rp}\bar{\gamma}}}(S_7+S_8)\right]\right]+e^{-\frac{\mathrm{I}}{\bar{\gamma}}\left(\frac{\Phi}{\lambda_{ss}\mathrm{I}}+\beta\right)}\left[(1-e^{-\frac{\mathrm{I}}{\lambda_{rp}\bar{\gamma}}})(s'_5+s'_6)+e^{-\frac{\mathrm{I}}{\lambda_{rp}\bar{\gamma}}}(s'_7+s'_8)\right], \tag{30}$$

$$\begin{aligned}
s'_5 &= \sum_{i=1}^{K}\binom{K}{i}(-1)^{i-1}\frac{A(i)}{\bar{\gamma}}\mathfrak{g}\left(1+\frac{\lambda_{sp}\Phi}{\lambda_{ss}\mathrm{I}}+\lambda_{sp}\beta,\frac{1}{\lambda_{ss}\bar{\gamma}},\frac{\lambda_{sp}}{\lambda_{ss}\mathrm{I}},\frac{1}{\lambda_{rs}\bar{\gamma}}\right),\\
s'_6 &= \sum_{i=1}^{K}\binom{K}{i}(-1)^{i-1}\frac{B(i)}{\bar{\gamma}}\,\mathfrak{g}\left(1+\frac{\lambda_{sp}\Phi}{\lambda_{ss}\mathrm{I}}+\lambda_{sp}\beta,\frac{1}{\lambda_{ss}\bar{\gamma}},\frac{\lambda_{sp}}{\lambda_{ss}\mathrm{I}},\frac{i}{\lambda\bar{\gamma}}\right),\\
s'_7 &= \sum_{i=1}^{K}\binom{K}{i}(-1)^{i-1}A(i)\left[\left(\frac{1}{\bar{\gamma}}\right)\mathfrak{g}_1\left(1+\frac{\lambda_{sp}\Phi}{\lambda_{ss}\mathrm{I}}+\lambda_{sp}\beta,\frac{1}{\lambda_{ss}\bar{\gamma}},\frac{\lambda_{sp}}{\lambda_{ss}\mathrm{I}},\frac{1}{\lambda_{rs}\bar{\gamma}},\frac{\lambda_{rp}}{\lambda_{rs}\mathrm{I}}\right)\right.\\
&\qquad\left.+\left(\frac{\lambda_{rp}}{\mathrm{I}}\right)\mathfrak{g}_2\left(1+\frac{\lambda_{sp}\Phi}{\lambda_{ss}\mathrm{I}}+\lambda_{sp}\beta,\frac{1}{\lambda_{ss}\bar{\gamma}},\frac{\lambda_{sp}}{\lambda_{ss}\mathrm{I}},\frac{1}{\lambda_{rs}\bar{\gamma}},\frac{\lambda_{rp}}{\lambda_{rs}\mathrm{I}}\right)\right]\\
s'_8 &= \sum_{i=1}^{K}\binom{K}{i}(-1)^{i-1}B(i)\left[\left(\frac{1}{\bar{\gamma}}\right)\mathfrak{g}_1\left(1+\frac{\lambda_{sp}\Phi}{\lambda_{ss}\mathrm{I}}+\lambda_{sp}\beta,\frac{1}{\lambda_{ss}\bar{\gamma}},\frac{\lambda_{sp}}{\lambda_{ss}\mathrm{I}},\frac{i}{\lambda\bar{\gamma}},\frac{i\lambda_{rp}}{\lambda\mathrm{I}}\right)\right.\\
&\qquad\left.+\left(\frac{\lambda_{rp}}{\mathrm{I}}\right)\mathfrak{g}_2\left(1+\frac{\lambda_{sp}\Phi}{\lambda_{ss}\mathrm{I}}+\lambda_{sp}\beta,\frac{1}{\lambda_{ss}\bar{\gamma}},\frac{\lambda_{sp}}{\lambda_{ss}\mathrm{I}},\frac{i}{\lambda\bar{\gamma}},\frac{i\lambda_{rp}}{\lambda\mathrm{I}}\right)\right].
\end{aligned} \tag{31}$$

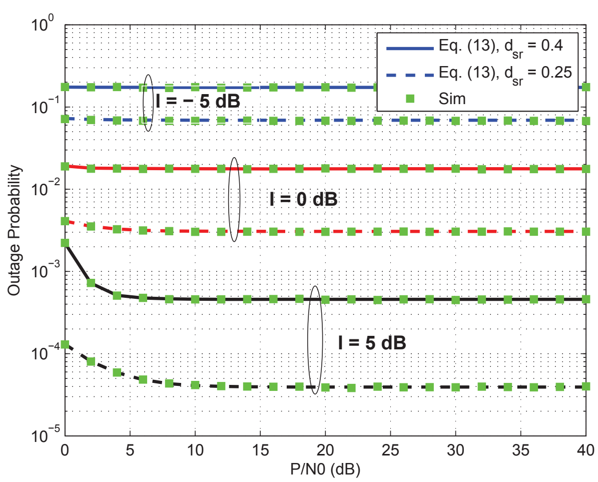


Fig. 1. Outage probability versus $\bar{\gamma}$ using different interference constraint's level I, when $d_{ss} = 0.5$.

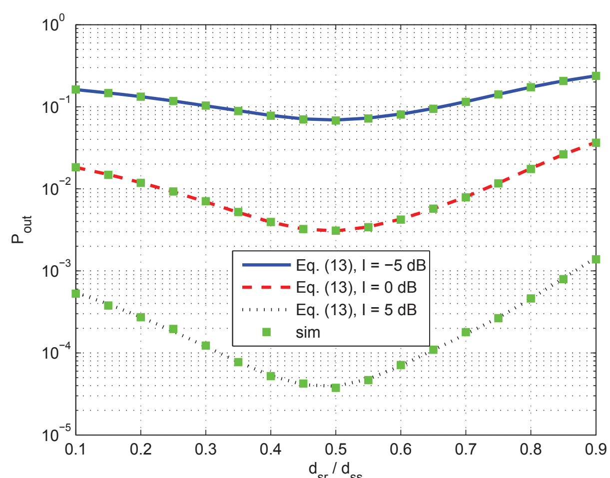


Fig. 2. Outage probability versus $d_{sr}/d_{ss}$ using different interference constraint's level I, when $\bar{\gamma} = 30dB$ and $d_{ss}$ fixed to 0.5 (middle between S-Tx and P-Rx).

When $\lambda_{ss}/\lambda_{sp}$ $4\ dB$, which corresponds to $d_{ss} = 0.9$, the outage probability is about 7% for $\mathrm{I} = 5\ dB$, whereas the same performance can be achieved at $d_{ss} = 0.63$ and $d_{ss} = 0.5$ for $\mathrm{I} = 0\ dB$ and $\mathrm{I} = -5\ dB$, respectively. For an outage probability at 2%, $d_{ss} = 0.7$ ($6\ dB$) could be enough even when $\mathrm{I} = -5\ dB$.

## V. Conclusion

In this work, we evaluated the outage probability of a relay-assisted secondary system coexisting with a primary system under the underlay paradigm. Our performance results are in perfect match with simulations, and show that by using relays the interference constraint level can be lowered while maintaining significantly the system performance.

## Appendix

### A. Derivation of Eqs. (8) and (9)

Based on $\gamma_{r_*s}$ expression in (7), its CDF can be given by

$$F_{\gamma_{r_*s}}(\mathrm{x}) = \int_0^{\frac{\mathrm{I}}{\bar{\gamma}}} \Pr\left[|h_{r_*s}|^2 \le \frac{\mathrm{x}}{\bar{\gamma}}\right] p_{|h_{r_*p}|^2}(\mathrm{y})d\mathrm{y}$$

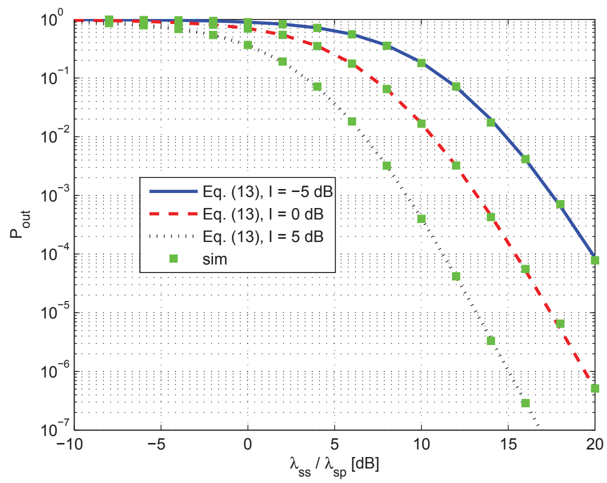


Fig. 3. Outage probability versus $\lambda_{ss}/\lambda_{sp}$ using different interference constraint's level I, when $\bar{\gamma} = 20dB$ and $d_{ss}$ is fixed to 0.5 (middle between S-Tx and P-Rx).

$$+\int_{\frac{\mathrm{I}}{\bar{\gamma}}}^{\infty} \Pr\left[|h_{r_*s}|^2 \leq \frac{\mathrm{xy}}{\mathrm{I}}|\mathrm{y}\right] p_{|h_{r_*p}|^2}(\mathrm{y})d\mathrm{y}, \qquad (32)$$

where $p_{|h_{r_*p}|^2}(.)$ is the PDF of $|h_{r_*p}|^2$, which is given by

$$p_{|h_{r_*p}|^2}(\mathrm{y}) = \frac{1}{\lambda_{rp}} e^{-\frac{\mathrm{y}}{\lambda_{rp}}}, \qquad \mathrm{y} > 0, \qquad (33)$$

and probability expressions in (32) can be computed with the help of $p_{|h_{r_*s}|^2}(.)$, the PDF of $|h_{r_*s}|^2$, which can be deduced from [4] and be given by

$$p_{|h_{r_*s}|^2}(\mathrm{z}) = \sum_{i=1}^{K} \binom{K}{i} (-1)^{i-1} A(i) e^{-\mathrm{z}/\lambda_{rs}} + \sum_{i=1}^{K} \binom{K}{i} (-1)^{i-1} B(i) e^{-i\mathrm{z}/\lambda}, \qquad (34)$$

where $\lambda = \frac{\lambda_{sr}\lambda_{rs}}{\lambda_{sr}+\lambda_{rs}}$, $\xi = \lambda_{sr}/\lambda_{rs}$, and

$$A(i) = \frac{i\xi}{\lambda_{sr}(i(\xi+1)-\xi)},$$
$$B(i) = \left(\frac{i}{\lambda_{rs}} - \frac{i}{\lambda_{sr}(i(1+(1/\xi))-1)}\right).$$

Eq. (8) can be found using (33) and (34), and Eq. (9) is the derivative of (8).

### B. Derivation of Eq. (14)

Looking at (3) and (6), it can be noted that $\gamma_{ss}$ and $\gamma_{sr_*}$ are not independent. As a consequence, the joint CDF of $\gamma_{ss}$ and $\gamma_{sr_*}$, $F_{\gamma_{ss},\gamma_{sr_*}}(.,.)$, can be derived by solving the following integration

$$F_{\gamma_{ss},\gamma_{sr_*}}(\mathrm{x},\mathrm{y}) = \int_0^{\infty} \Pr\left[\gamma_{ss} \leq \mathrm{x}, \gamma_{sr_*} \leq \mathrm{y}|\mathrm{z}\right] p_{|h_{r_*p}|^2}(\mathrm{z})d\mathrm{z}, \qquad (35)$$

where $p_{|h_{r_*p}|^2}(.)$ was defined above, and the probability expression can be given by

$$\Pr\left[\gamma_{ss} \leq \mathrm{x}, \gamma_{sr_*} \leq \mathrm{y}|\mathrm{z}\right] = F_{|h_{ss}|^2}\left(\frac{\mathrm{x}}{\min\left(\frac{\mathrm{I}}{\mathrm{z}},\bar{\gamma}\right)}\right) \times F_{|h_{sr_*}|^2}\left(\frac{\mathrm{y}}{\min\left(\frac{\mathrm{I}}{\mathrm{z}},\bar{\gamma}\right)}\right), \qquad (36)$$

where $F_{|h_{ss}|^2}(.)$ and $F_{|h_{sr_*}|^2}(.)$ are the CDFs expressions given by

$$F_{|h_{ss}|^2}(\mathrm{x}) = 1 - e^{-\frac{\mathrm{x}}{\lambda_{ss}}}, \qquad (37)$$

and deduced from [4],

$$F_{|h_{sr_*}|^2}(\mathrm{y}) = \sum_{i=1}^{K} \frac{\binom{K}{i} i\xi(-1)^{i-1}}{i(1+\xi)-1}\left(1 - e^{-\frac{\mathrm{y}}{\lambda_{sr}}}\right) + \sum_{i=1}^{K} \frac{\binom{K}{i}(i-1)(-1)^{i-1}}{i(1+\xi)-1}\left(1 - e^{-\frac{i\mathrm{y}}{\lambda}}\right), \qquad (38)$$

respectively. After some manipulations, and using (16), the result can be found.

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