

# k-t SPARSE: High frame rate dynamic MRI exploiting spatio-temporal sparsity

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## Introduction

Recently rapid imaging methods that exploit the spatial sparsity of images using under-sampled randomly perturbed spirals and non-linear reconstruction have been proposed [1,2]. These methods were inspired by theoretical results in sparse signal recovery [1-5] showing that sparse or compressible signals can be recovered from randomly under-sampled frequency data. We propose a method for high frame-rate dynamic imaging based on similar ideas, now exploiting both spatial and temporal sparsity of dynamic MRI image sequences (dynamic scene). We randomly under-sample k-t space by random ordering of the phase encodes in time (Fig. 1). We reconstruct by minimizing the  $L_1$  norm of a transformed dynamic scene subject to data fidelity constraints. Unlike previously suggested linear methods [7, 8], our method does not require a known spatio-temporal structure nor a training set, only that the dynamic scene has a sparse representation. We demonstrate a 7-fold frame-rate acceleration both in simulated data and *in vivo* non-gated Cartesian balanced-SSFP cardiac MRI.

## Theory

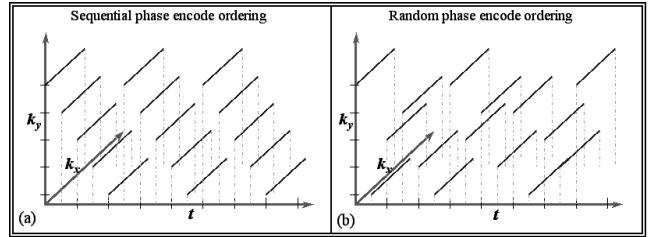
Dynamic MR images are highly redundant in space and time. By using linear transformations (such as wavelets, Fourier etc.), we can represent a dynamic scene using only a few sparse transform coefficients. Inadequate sampling of the spatial-frequency -- temporal space (k-t space) results in aliasing in the spatial -- temporal-frequency space (x-f space). The aliasing artifacts due to random under-sampling are incoherent as opposed to coherent artifacts in equispaced under sampling. More importantly the artifacts are incoherent in the sparse transform domain. By using the non-linear reconstruction scheme in [1-5] we can recover the sparse transform coefficients and as a consequence, recover the dynamic scene. We exploit sparsity by constraining our reconstruction to have a sparse representation and be consistent with the measured data by solving the constrained optimization problem:  $\text{minimize } \|\Psi m\|_1 \text{ subject to: } \|Fm - y\|_2 < \epsilon$ . Here  $m$  is the dynamic scene,  $\Psi$  transforms the scene into a sparse representation,  $F$  is randomized phase encode ordering Fourier matrix,  $y$  is the measured k-space data and  $\epsilon$  controls fidelity of the reconstruction to the measured data.  $\epsilon$  is usually set to the noise level.

## Methods

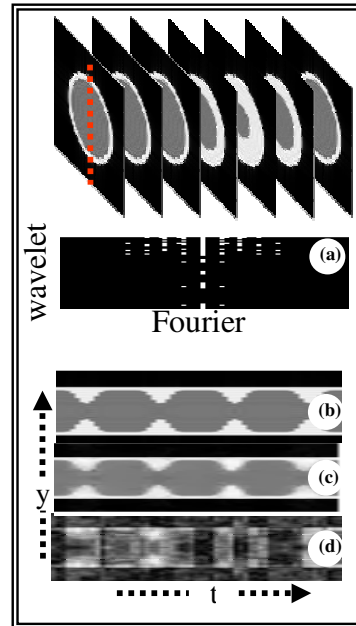
For dynamic heart imaging, we propose using the wavelet transform in the spatial dimension and the Fourier transform in the temporal. Wavelets sparsify medical images [1] whereas the Fourier transform sparsifies smooth or periodic temporal behavior. Moreover, with random k-t sampling, aliasing is extremely incoherent in this particular transform domain. To validate our approach we considered a simulated dynamic scene with periodic heart-like motion. A random phase-encode ordered Cartesian acquisition (See Fig. 2) was simulated with a  $TR=4ms$ ,  $64 \text{ pixels}$ , acquiring a total of 1024 phase encodes ( $4.096 \text{ sec}$ ). The data was reconstructed at a frame rate of 15FPS (a 4-fold acceleration factor) using the  $L_1$  reconstruction scheme implemented with non-linear conjugate gradients. The result was compared to a sliding window reconstruction (64 phase encodes in length). To further validate our method we considered a Cartesian balanced-SSFP dynamic heart scan ( $TR=4.4$ ,  $TE=2.2$ ,  $\alpha=60^\circ$ ,  $res=2.5mm$ ,  $slice=9mm$ ). 1152 randomly ordered phase encodes ( $5sec$ ) were collected and reconstructed using the  $L_1$  scheme at a 7-fold acceleration (25FPS). Result was compared to a sliding window (64 phase encodes) reconstruction. The experiment was performed on a 1.5T GE Signa scanner using a 5inch surface coil.

## Results and discussion

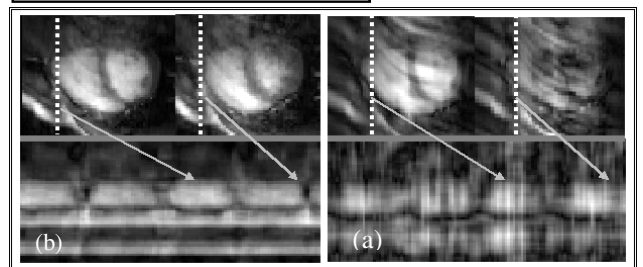
Figs. 2 and 3 illustrate the simulated phantom and actual dynamic heart scan reconstructions. Note, that even at 4 to 7-fold acceleration, the proposed method is able to recover the motion, preserving the spatial frequencies and suppressing aliasing artifacts. This method can be easily extended to arbitrary trajectories and can also be easily integrated with other acceleration methods such as phase constrained partial k-space and SENSE [1]. In the current, Matlab<sup>TM</sup> implementation we are able to reconstruct a  $64 \times 64 \times 64$  scene in an hour. This can be improved by using newly proposed reconstruction techniques [5,6]. Previously proposed linear methods [7,8] exploit known or measured spatio-temporal structure. The advantage of the proposed method is that the signal need not have a known structure, only sparsity, which is a very realistic assumption in dynamic medical images [1,7,8]. Therefore, a training set is not required.



**Figure 1:** (a) Sequential phase encode ordering. (b) Random Phase encode ordering. The k-t space is randomly sampled, which enables recovery of sparse spatio-temporal dynamic scenes using the  $L_1$  reconstruction.



**Figure 2:** Simulated dynamic data. (a) The transform domain of the cross section is truly sparse. (b) Ground truth cross-section. (c)  $L_1$  reconstruction from random phase encode ordering, 4-fold acceleration (64) (d) Sliding window (64) reconstruction from random phase encode ordering.



**Figure 3:** Dynamic SSFP heart imaging with randomized ordering. 7-fold acceleration (25FPS). The images show two frames of the heart phase and a cross section evolution in time (a) Sliding window (64) recon. (b)  $L_1$  recon. The signal is recovered in both time and space using the  $L_1$  method.

## References

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