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Stability region analysis in Smith predictor configurations using a PI controller

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Abstract

This paper deals with the stabilization problem of Smith predictor structures using a PI controller. Stability regions that include all stabilizing parameters of a PI controller for the case of perfect matching between the plant and model and for mismatched case are obtained. The models of the plant are assumed to be FOPDT (first-order plus dead time) and SOPDT (second-order plus dead time) transfer functions. Thus, the aim of this study is to determine all stabilizing PI controllers for the Smith predictor scheme and to compare the stability regions obtained for perfectly matched and mismatched models. It is observed that the stability regions obtained for both cases are quite different and the stability regions for FOPDT and SOPDT models are broader than the stability region of the actual model. Furthermore, an approach is presented to find different models of an actual system using the stability region and it is shown that the stability region of these models can fit the stability region of actual system. A simulation example is provided to illustrate the results.

Keywords

Model identification, Smith predictor structure, stability boundary locus, stability region

Introduction

PI/PID controllers have an important role in industrial control. They are preferred in control practice due to their ease of use and robust performance (Astrom and Hagglund, 1995). PI/PID controllers are used to deal with variety of control problems in different fields such as process control, automotive systems, instrumentation, etc. Although these controllers have widespread use, the conventional tuning rules are insufficient to achieve good results for processes with time delay (Astrom and Hagglund, 2001; Kaya, 2003). It is known that dynamic systems encountered in real control applications exhibit noteworthy dead time. The dead time is mainly caused by processing time or time lags of systems elements connected in series. Closed-loop systems could become unstable due to an increase in phase lag caused by dead time. In addition, dead times in the closed-loop control systems complicate the design and analysis of time-delayed systems. Therefore, dead time compensators can be used in order to improve the closed-loop performance of classical controllers such as PI/PID controllers for time-delay systems. The Smith predictor structure presented by Smith (1959) was the first and is a well known dead time compensator. The classical Smith predictor, shown in Figure 1, is composed of a plant with time delay, a model of the plant with time delay and a controller. The controller used in the structure is typically defined as a PI or PID controller whose parameters are determined with respect to the plant model.

Performance of the Smith predictor control dramatically depends on matching between the model and the plant transfer functions (Zhang and Xu, 2001). There are various methods for the estimation of the model or tuning the appropriate controller parameters. In this sense, Kaya and Atherton presented a method using a single relay feedback test with asymmetric limit cycle data (Kaya and Atherton, 2001). Benouarets and Atherton (1994) discussed obtaining appropriate parameters for a Smith predictor controller. Hang et al. (1995) presented self-tuning Smith predictors for processes with long dead time. Palmor and Blan (1994) proposed an automatic tuning algorithm for a Smith dead time compensator. An effective model reduction technique for finding reduced-order models that have similar closed-loop characteristics to those of the original system was given in Thompson (1985, 1989) for closed-loop performance. The reduction method was based on frequency data, which are fitted using

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Levy's complex curve-fitting technique and a time delay can also be introduced into the reduction.

This paper deals with the stability region computation for a Smith predictor scheme. A method is presented for computation of all PI controllers that stabilize classical Smith predictor structures with perfect matching and mismatching between plant and model transfer functions. As mentioned above, the processes with large dead time are difficult to control, as the dead time will reduce gain and phase margins, which may lead to instability (Vodencarevic, 2010). The Smith predictor control structure can be used to overcome this problem and allow larger gain. However, the error between the assumed model and the actual system is very important for the stability. Therefore, the stability analysis in the Smith predictor scheme is an important subject. Many results have been developed on computation of all stabilizing controllers, especially P, PI and PID controllers after the publication of work by Ho et al. (1996, 1997), which are based on the generalized version of the Hermite–Biehler theorem. There are many papers contributing to the field of computing all stabilizing controllers and therefore it is not possible to cite all of these studies here. However, the following works and references therein can provide further details on the subject. One of these studies based on the use of a Nyquist plot was given by Soylemez et al. (2003) for calculating all stabilizing PID controllers. A parameter space approach using the singular frequency approach has been given in Ackermann and Kaesbauer (2003). Computation of the stability region using the stability boundary locus has been given in Tan (2005) and Tan et al. (2006). It is shown in the present paper that if the identified model in the Smith predictor structure is not equal to the actual system, then the characteristic equation includes two time delays terms and the stability region analysis of such systems turns out to be similar to the systems that are called the neutral and retarded systems (Hamamci, 2012). A new identification procedure based on the stability region is proposed using the identification formulas given in Kaya (2004). Using this new identification procedure, it is possible to obtain different first-order plus dead time (FOPDT) models of the exact system. The stability regions of the identified FOPDT models obtained from the presented method are computed and it is shown that there is a good match between the stability region of actual system and the identified FOPDT models. The results presented in this paper will be helpful for the stability analysis of Smith predictor systems. Preliminary versions of some of the results given in this study were presented in Deniz et al. (2013). An illustrative example is given to demonstrate the benefits of the method presented.

The paper is organized as follows: the classical Smith predictor structure is summarized in the next section. Then, the stability boundary locus method and its application to Smith predictor scheme is given. Also, a model identification procedure using the stability region is proposed. Finally, a simulation example and concluding remarks are provided.

Smith predictor structure

The conventional Smith predictor structure is shown in Figure 1, where $C(s)$, $G(s)$ and $G_m(s)$ are the transfer function of controller, plant and model, respectively. The parameters L and L_m are the time delay of the plant and model.

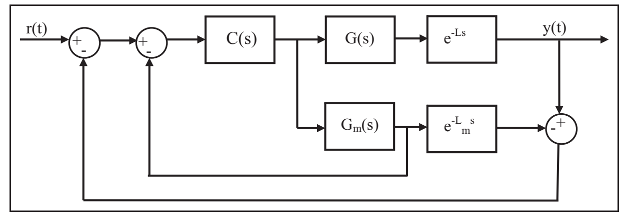

Figure 1. Smith predictor structure.

The closed-loop transfer function can be found for the Smith predictor shown in Figure 1 as

$$T(s) = \frac{G(s)C(s)e^{-Ls}}{1 + [G(s)e^{-Ls} - G_m(s)e^{-L_m s} + G_m(s)]C(s)} \quad (1)$$

Assuming that the model transfer function perfectly matches the plant transfer function, i.e. $G(s)e^{-Ls} = G_m(s)e^{-L_m s}$, the closed-loop transfer function reduces to

$$T(s) = \frac{C(s)G_m(s)e^{-L_m s}}{1 + C(s)G_m(s)} \quad (2)$$

Different approaches can be used to identify tuning parameters of the controller, $C(s)$. Here, tuning parameters of the controller are found by representing the Smith predictor as its equivalent Internal Model Control (IMC) (Abe and Yamanaka, 2003; Morari and Zafiriou, 1989; Rivera et al., 1986), which provides the parameters of the PI or PID controller to be defined in terms of the desired closed-loop time constant, which can be adjusted by the operator, and the parameters of the process model. One of the properties of the IMC system is the perfect controller where the IMC controller is designed to be given by the model inverse. However, this property cannot be realized. Therefore, a low-pass filter with steady-state gain of one must be introduced for physical realizability of the IMC controller (Kaya, 2004). The low-pass filter is usually chosen to have the form $1/(\lambda s + 1)$.

When the Smith predictor is designed using the IMC principles based the assumption that the model transfer function perfectly matches the plant transfer function, the parameters of the controller in the Smith predictor scheme, $C(s)$, which is frequently chosen as a PI/PID, can be adjusted by filter dynamics and the parameters of the model. The most frequently used model is an FOPDT transfer function. For this case, the controller in the Smith predictor scheme is found to be a PI controller such as

$$C(s) = \frac{T_m s + 1}{K_m \lambda s} = k_p + \frac{k_i}{s} = k_p(1 + \frac{1}{T_i s}) \quad (3)$$

This equation can be rearranged as an ideal PI controller, which has the following controller parameters

$$k_p = \frac{T_m}{\alpha K_m \lambda} \quad (4)$$

and

$$T_i = T_m \tag{5}$$

In Equation (4), α, which is a constant and selected to be in the range of $0.2 < \alpha < 1$, is introduced to adjust the speed of closed-loop response (Kaya, 2004). The integral squared error (ISE) criteria can be used to find the filter parameter $\lambda = L_m$.

Similarly, for the second-order plus dead time (SOPDT) model transfer function, the controller in the Smith predictor structure can be found for a PID controller, which has the following tuning parameters

$$k_p = \frac{T_{1m} + T_{2m}}{\alpha K_m L_m} \tag{6}$$

$$T_i = T_{1m} + T_{2m} \tag{7}$$

$$T_d = \frac{T_{1m} T_{2m}}{T_{1m} + T_{2m}} \tag{8}$$

Again α is introduced to adjust the speed of closed-loop response. The details of this approach can be found in Kaya (2004).

Computation of all stabilizing PI controllers

In this section, the stability boundary locus method is used to obtain the stability region of the closed-loop systems with time delay for computing all stabilizing PI controller (Tan et al., 2006). Then, all stabilizing values of the parameters k_p and k_i are determined from the stability region.

Consider the closed-loop control system shown in Figure 2 where $G_p(s)$ is the plant with time delay such as

$$G_p(s) = G(s)e^{-Ls} = \frac{N(s)}{D(s)} e^{-Ls} \tag{9}$$

and $C(s)$ is a PI controller of the form

$$C(s) = k_p + \frac{k_i}{s} = \frac{k_p s + k_i}{s} \tag{10}$$

The characteristic equation of closed-loop system in Figure 2 can be written as

$$\Delta(s) = sD(s) + (k_p s + k_i)N(s)e^{-Ls} \tag{11}$$

Substituting $s = j\omega$ and decomposing the numerator and denominator polynomials of $G(s)$ in Equation (9) into their even and odd parts, then Equation (12) is obtained as

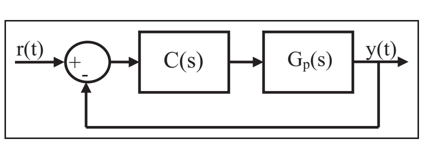

Figure 2. A control scheme for stability analysis.

$$G(j\omega) = \frac{N_e(-\omega^2) + j\omega N_o(-\omega^2)}{D_e(-\omega^2) + j\omega D_o(-\omega^2)} \tag{12}$$

Using Equation (12), the characteristic equation can be rewritten as

$$\begin{aligned} \Delta(j\omega) = {} & [(k_i N_e - k_p \omega^2 N_o)\cos(\omega L) + \omega(k_i N_o + k_p N_e) \\ & \sin(\omega L) - \omega^2 D_o] + j[\omega(k_i N_o + k_p N_e)\cos(\omega L) \\ & -(k_i N_e - \omega^2 k_p N_o)\sin(\omega L) + \omega D_e] \end{aligned} \tag{13}$$

If $\Delta(j\omega)$ is resolved by equating real and imaginary parts of Equation (13) to zero, the following equations are obtained

$$\begin{aligned} k_p Q(\omega) + k_i R(\omega) &= X(\omega) \\ k_p S(\omega) + k_i U(\omega) &= Y(\omega) \end{aligned} \tag{14}$$

From this equation, the parameters k_p and k_i are determined as

$$k_p(\omega) = \frac{X(\omega)U(\omega) - Y(\omega)R(\omega)}{Q(\omega)U(\omega) - R(\omega)S(\omega)} \tag{15}$$

$$k_i(\omega) = \frac{Y(\omega)Q(\omega) - X(\omega)S(\omega)}{Q(\omega)U(\omega) - R(\omega)S(\omega)} \tag{16}$$

Solving Equation (14), the stability boundary locus, $l(k_p, k_i, \omega)$, in the (k_p, k_i) plane can be obtained. The following expressions are used in Equations (15) and (16) to calculate $k_p(\omega)$ and $k_i(\omega)$.

$$Q(\omega) = \omega N_e \sin(\omega L) - \omega^2 N_o \cos(\omega L) \tag{17}$$

$$R(\omega) = N_e \cos(\omega L) + \omega N_o \sin(\omega L) \tag{18}$$

$$X(\omega) = \omega^2 D_o \tag{19}$$

$$S(\omega) = \omega N_e \cos(\omega L) + \omega^2 N_o \sin(\omega L) \tag{20}$$

$$U(\omega) = \omega N_o \cos(\omega L) - N_e \sin(\omega L) \tag{21}$$

$$Y(\omega) = -\omega D_e \tag{22}$$

The (k_p, k_i) plane is divided into stable and unstable regions by the stability boundary locus, $l(k_p, k_i, \omega)$, and the line $k_i = 0$. Thus, the region that includes all stabilizing PI controllers can be estimated using the stability boundary locus (Tan, 2005).

Example 1. Consider the transfer function of the closed-loop system with time delay shown in Figure 2 as

$$G(s) = \frac{s+1}{s^2 + 2s + 5} e^{-s} \tag{23}$$

The stability boundary locus, $l(k_p, k_i, \omega)$, is shown in Figure 3 for $\omega \in [0, 6.8]$. It can be computed that the stability region for all stabilizing PI controllers is obtained using the stability boundary locus for the interval of $\omega \in [0, 2.51]$ and the line $k_i = 0$. All the stabilizing PI controllers are shown in Figure 3 in the coloured region.

Computation of all stabilizing PI controllers for Smith predictor structures

Stability boundary locus analysis is used to obtain the stability region of the Smith predictor configuration. All stabilizing PI controllers are computed for both exactly matched and mismatched models. Let assume that the model of the actual system is $G_m(s) = \frac{N_m(s)}{D_m(s)} e^{-L_m s}$, which can be written as

$$G_m(j\omega) = \frac{N_{me}(-\omega^2) + j\omega N_{mo}(-\omega^2)}{D_{me}(-\omega^2) + j\omega D_{mo}(-\omega^2)} e^{-j\omega L_m} \quad (24)$$

The characteristic equation of mismatched Smith predictor with PI controller is obtained as

$$\Delta(s) = sD(s)D_m(s) + (D_m(s)N(s)e^{-Ls} - D(s)N_m(s)e^{-L_m s} + D(s)N_m(s))(k_p s + k_i) \quad (25)$$

The characteristic equation is rewritten by using Equation (24)

$$\begin{aligned}\Delta(j\omega)l = {} & j\omega(D_e + j\omega D_o)(D_{me} + j\omega D_{mo}) + ((D_{me} + j\omega D_{mo}) \\ & (N_e + j\omega N_o)(\cos(\omega L) - j\sin(\omega L)) \\ & -(D_e + j\omega D_o)(N_{me} + j\omega N_{mo})(\cos(\omega L_m) - j\sin(\omega L_m)) \\ & + (D_e + j\omega D_o)(N_{me} + j\omega N_{mo}))(j\omega k_p + k_i)\end{aligned} \quad (26)$$

Equating the real and imaginary parts of $\Delta(j\omega)$ to zero, the parameters k_p and k_i can be found from Equations (15) and (16). The following expressions are used in Equations (15) and (16) to calculate the stability boundary locus, $l(k_p, k_i, \omega)$, for the Smith predictor scheme

$$\begin{aligned}Q(\omega) = {} & -\omega^2 D_{me}N_o\cos(L\omega) - \omega^2 D_{mo}N_e\cos(L\omega) \\ & + \omega^2 D_e N_{mo}\cos(L_m\omega) + \omega^2 D_o N_{me}\cos(L_m\omega) \\ & + \omega D_{me}N_e\sin(L\omega) - \omega^3 D_{mo}N_o\sin(L\omega) \\ & - \omega D_e N_{me}\sin(L_m\omega) + \omega^3 D_o N_{mo}\sin(L_m\omega) \\ & - \omega^2 D_e N_{mo} - \omega^2 D_o N_{me}\end{aligned} \quad (27)$$

$$\begin{aligned}R(\omega) = {} & D_{me}N_e\cos(L\omega) - \omega^2 D_{mo}N_o\cos(L\omega) \\ & - D_e N_{me}\cos(L_m\omega) + \omega^2 D_o N_{mo}\cos(L_m\omega) \\ & + \omega D_{me}N_o\sin(L\omega) + \omega D_{mo}N_e\sin(L\omega) \\ & - \omega D_e N_{mo}\sin(L_m\omega) - \omega D_o N_{me}\sin(L_m\omega) \\ & + D_e N_{me} - \omega^2 D_o N_{mo}\end{aligned} \quad (28)$$

$$X(\omega) = \omega^2 D_e D_{mo} + \omega^2 D_o D_{me} \quad (29)$$

$$\begin{aligned}S(\omega) = {} & \omega D_{me}N_e\cos(L\omega) - \omega^3 D_{mo}N_o\cos(L\omega) \\ & - \omega D_e N_{me}\cos(L_m\omega) + \omega^3 D_o N_{mo}\cos(L_m\omega) \\ & + \omega^2 D_{me}N_o\sin(L\omega) + \omega^2 D_{mo}N_e\sin(L\omega) \\ & - \omega^2 D_e N_{mo}\sin(L_m\omega) - \omega^2 D_o N_{me}\sin(L_m\omega) \\ & + \omega D_e N_{me} - \omega^3 D_o N_{mo}\end{aligned} \quad (30)$$

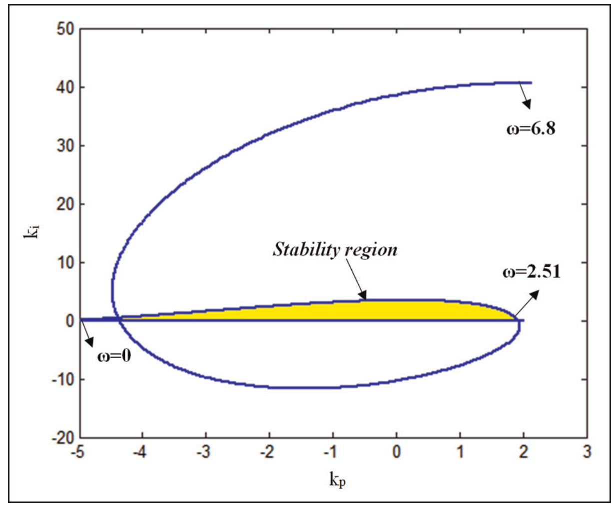

Figure 3. Stability boundary locus.

$$\begin{aligned}U(\omega) = {} & \omega D_{me}N_o\cos(L\omega) + \omega D_{mo}N_e\cos(L\omega) \\ & - \omega D_e N_{mo}\cos(L_m\omega) - \omega D_o N_{me}\cos(L_m\omega) \\ & - D_{me}N_e\sin(L\omega) + \omega^2 D_{mo}N_o\sin(L\omega) \\ & + D_e N_{me}\sin(L_m\omega) - \omega^2 D_o N_{mo}\sin(L_m\omega) \\ & + \omega D_e N_{mo} + \omega D_o N_{me}\end{aligned} \quad (31)$$

$$Y(\omega) = -\omega D_e D_{me} + \omega^3 D_o D_{mo} \quad (32)$$

Then, all stabilizing PI controllers are computed by using the expressions given in Equations (15) and (16).

The characteristic equation of this structure with PI controller under the assumptions of perfect model matching is given by

$$\Delta(s) = sD(s) + N(s)(k_p s + k_i) \quad (33)$$

As seen from Equation (33), the characteristic equation is independent of time-delay parameters. The characteristic equation is written by making the substitution $s = j\omega$ as

$$\Delta(j\omega) = [(k_i N_e - k_p\omega^2 N_o) - \omega^2 D_o] + j[\omega(k_i N_o + k_p N_e) + \omega D_e] \quad (34)$$

The parameters k_p and k_i can be calculated by using Equations (15) and (16) for the following expressions

$$Q(\omega) = -\omega^2 N_o \quad (35)$$

$$R(\omega) = N_e \quad (36)$$

$$X(\omega) = \omega^2 D_o \quad (37)$$

$$S(\omega) = \omega N_e \quad (38)$$

$$U(\omega) = \omega N_o \quad (39)$$

$$Y(\omega) = -\omega D_e \quad (40)$$

In this way, all stabilizing PI controllers can be computed for the Smith predictor with a perfect matching model.

Model identification using stability region

In this section, a model identification method is proposed based on the stability region. The model identification procedure to find model parameters can be performed as follows: first, a stability region is obtained for the perfect matching case. Then, assuming that the equivalent process model is FOPDT, $G_m(s) = \frac{K_m e^{-L_m s}}{(T_m s + 1)}$, the parameters of the model can be estimated by choosing k_p and k_i values from the stability region. Considering the Smith predictor based on IMC, the following equations can be used to determine the parameters:

$$T_m = \frac{k_p}{k_i} \tag{41}$$

$$L_m = \lambda \tag{42}$$

$$K_m = \frac{1}{\alpha k_i \lambda} \tag{43}$$

The filter parameter, λ, is calculated for $\lambda \in [L - \%10, L + \%10]$. Let the gain of the actual plant be K then the constant, α, is selected to make $K_m = K$, from $0.2 < \alpha < 1$ (Kaya, 2004). Since there is a set of stabilizing controllers for actual systems, one can obtain a set of FOPDT approximate models of the system. This provides flexibility to choose an approximate model of the system. Moreover, an FOPDT model from the identified set can be found, which has a stability region that matches the stability region of the actual system. This finding is important, as robust stability results can be achieved in this way. An illustration of the method is given in the following example.

Illustrative example

The high-order plant transfer function given in Kaya (2004) has performed in the Smith predictor control scheme to find a stability region that includes all stabilizing PI controller parameters. The identification method given in Kaya and Atherton (1999, 2001) has been used for FOPDT and SOPDT models in a mismatched Smith predictor structure.

The high-order plant transfer function is given by

$$G(s) = \frac{e^{-20s}}{(3s + 1)(2s + 1)(s + 1)(0.5s + 1)} \tag{44}$$

Assuming that Smith predictor structure perfectly matches the model, i.e. $G(s)e^{-Ls} = G_m(s)e^{-L_m s}$, the stability region shown in Figure 4 is plotted.

To find all stabilizing PI controllers for the mismatched Smith predictor, the FOPDT model given in Kaya (2004) is used, which is

$$G_m(s) = \frac{e^{-23.28s}}{(3.67s + 1)} \tag{45}$$

If Equations (15) and (16) are solved considering Equations (27)–(32), Figure 5 shows the stability regions obtained for a mismatched Smith predictor structure with an FOPDT model using a Pade approximation and the exact value of time delay is plotted.

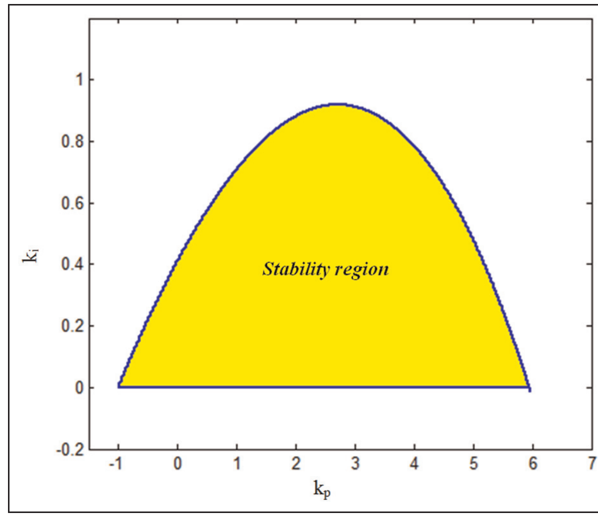

Figure 4. Stability region for $G(s)e^{-Ls} = G_m(s)e^{-L_m s}$ (perfect model).

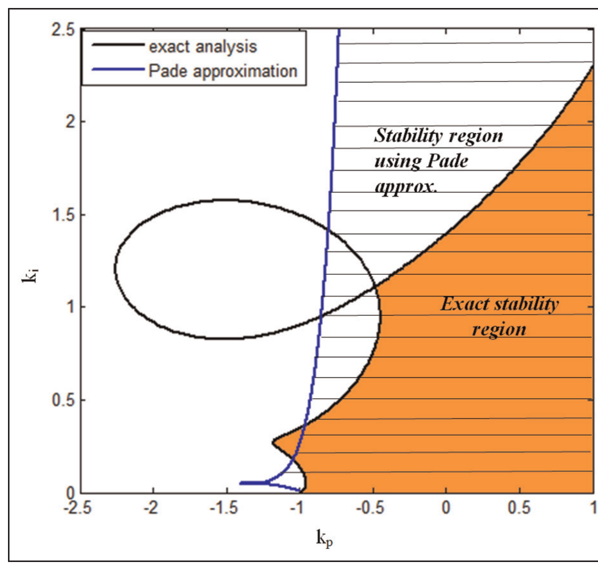

Figure 5. Stability region for the mismatched Smith predictor structure with a first-order plus dead time (FOPDT) model.

If the model of actual system is an SOPDT transfer function (Kaya, 2004) such as

$$G_m(s) = \frac{e^{-21.01s}}{(2.77s + 1)^2} \tag{46}$$

then the stability region shown in Figure 6 is obtained.

In this example, a first-order Pade approximation has been used to obtain a stability region for a mismatched Smith predictor structure in addition to exact analysis for closed-loop systems with time delay. It can be seen that the stability region

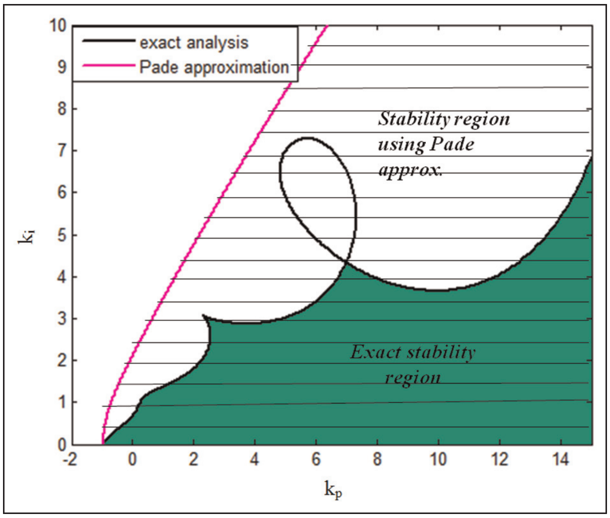

Figure 6. Stability region for the mismatched Smith predictor structure with a second-order plus dead time (SOPDT) model.

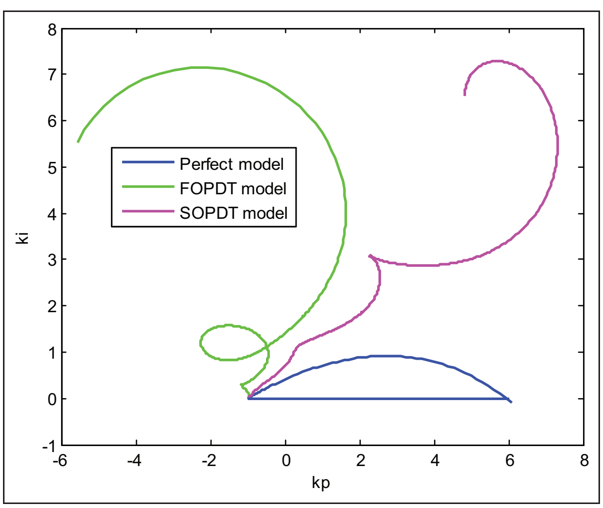

Figure 7. Stability region for the Smith predictor structure with all models.

obtained by using the first-order Pade approximation are inaccurate considering a calculation based on exact analysis. When the system response for different values of k_p and k_i is evaluated, the area on the right side of the curve, the hatched area, obtained by using a Pade approximation is the stability region in Figures 5 and 6. Similarly, the area on the right side of the helix, the coloured area, becomes the stability region of the mismatched Smith predictor structure except for the internal helix.

Figure 7 shows the stability regions for the actual system when there is perfect matching, FOPDT and SOPDT models. This figure clearly shows that the stability regions of the FOPDT and SOPDT models are bigger than the stability region of the actual systems. In terms of the stability region, one can conclude that the FOPDT and SOPDT models given

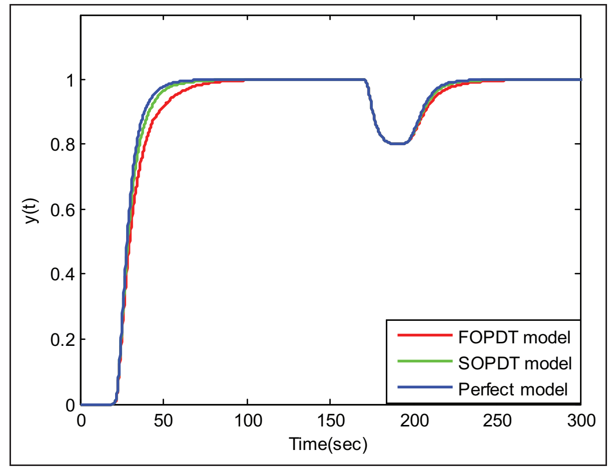

Figure 8. Step responses of all models for $k_p = 0.4$ and $k_i = 0.1$.

in Equations (45) and (46) are not good models for the system. Moreover, if one selects k_p and k_i values out of the stability region of the exact model, which means the system is unstable, a wrong conclusion can be made about the stability of the Smith predictor system, as these selected values for the FOPDT and SOPDT models will be stable.

Beyond stabilization, it is important to design controllers that give the desired performance measures and a good disturbance rejection capability. Stability region includes many controllers that stabilize the system and it will be possible to search over these to find the desired controller. For this example, step responses with a step disturbance at $t = 150$ s obtained by using the values of $k_p = 0.4$ and $k_i = 0.1$, which are selected from the stability regions for all models, are illustrated in Figure 8. From Figure 8, it can be seen that the step response of the SOPDT system for selected controller parameters is close to the step response of the perfect model. However, it is possible that a controller can destabilize the actual system while stabilizing the FOPDT and SOPDT models, as shown in Figure 7. This problem can be eliminated with the identification method based on the stability region explained above. Using this method, the stability region of the actual system and the FOPDT model can be similar by arranging the model parameters. Considering the values of the stabilizing PI controller $k_p = 0.4$ and $k_i = 0.1$, and using the identification formulas given Equations (41)–(43), different FOPDT models can be obtained for $\lambda \in [18, 22]$. The parameters of FOPDT models are presented in Table 1. The FOPDT model described in Equation (47) is selected from Table 1 to obtain the stability region of the Smith predictor configuration.

$$G_m(s) = \frac{e^{-20s}}{(4s+1)} \tag{47}$$

Figure 9 illustrates that the stability region of the Smith predictor configuration including the estimated model for $\lambda = 20$, $K_m = 1$ and $T_m = 4$. As seen in Figure 9, the stability region obtained by using proposed model is quite close to the

Table 1. Models parameters estimated by proposed methods for $\lambda \in [18, 22]$.

$k_p = 0.4$ and $k_i = 0.1$			
λ	K_m	T_m	α
18	1	4	0.55
19	1	4	0.52
20	**1**	**4**	**0.50**
21	1	4	0.47
22	1	4	0.45

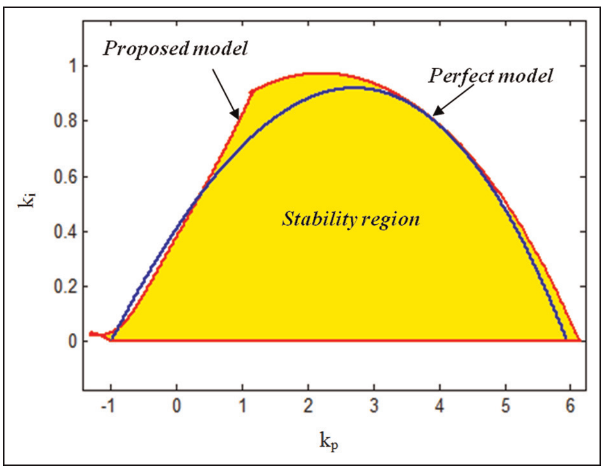

Figure 9. Stability region obtained with the proposed method for a first-order plus dead time (FOPDT) model.

stability region of the Smith predictor including the perfect model. Step responses with a step disturbance at $t = 150$ s of the proposed model, Kaya's model and the perfect model for $k_p = 0.5$ and $k_i = 0.07$ selected from the stability region are illustrated in Figure 10, where it can be seen that the step responses of Equation (47) fit the step response of the actual system with small deviations. Therefore, in terms of the time response, there are no disadvantages to the identified model. A Nyquist plot of the actual system, Kaya's model and the proposed model are shown in Figure 11. Although the Nyquist curve of Kaya's model closely fits the actual system, the stability regions are quite different from the perfect model.

However, it is necessary to point out that not all PI controllers in the stabilizing region can give a good FOPDT approximate model. To clarify this point, the step responses for different PI controllers selected from the stability region are given in Figure 12, which shows that for some PI controllers the step responses are not good.

In order to investigate the robustness of the proposed method in the face of model uncertainties, the model parameters in $G_m(s)$ of Equation (47) are deviated as much as 10% of their nominal values. Thus $G_m(s)$ can be represented by an interval transfer function as

$$G_m(s) = \frac{[0.9, 1.1]e^{-[19,21]s}}{[3.9, 4.1]s + 1} \tag{48}$$

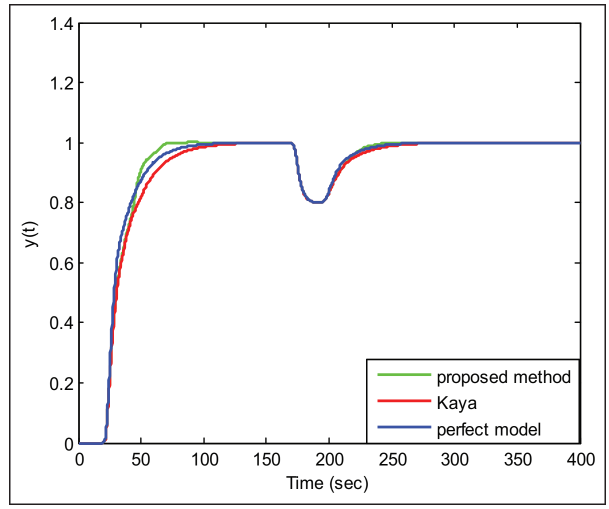

Figure 10. Step responses for $k_p = 0.5$ and $k_i = 0.07$.

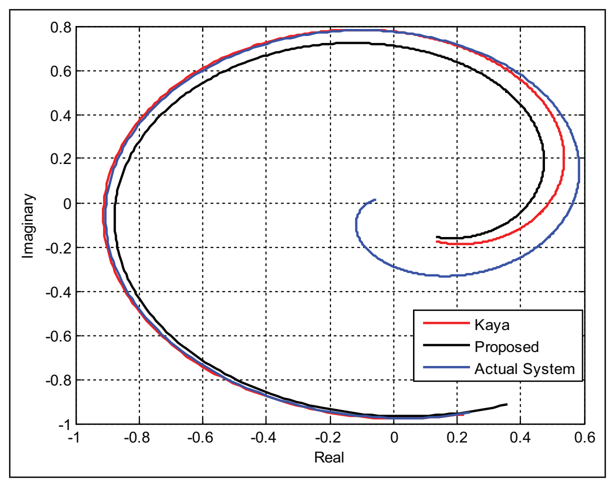

Figure 11. Nyquist plot for the actual model, Kaya's model and the proposed model.

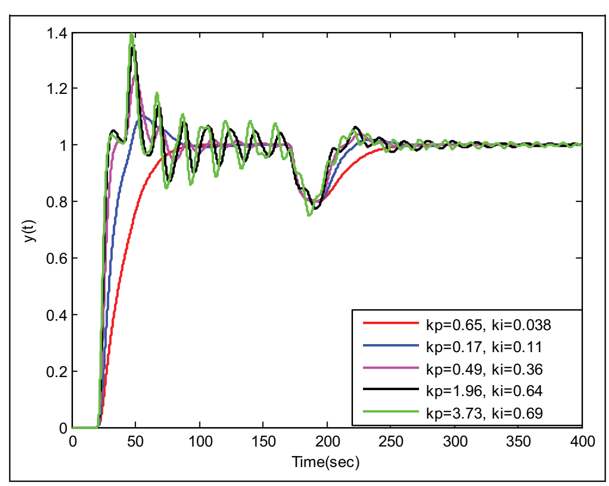

Figure 12. Step responses for different values of k_p and k_i.

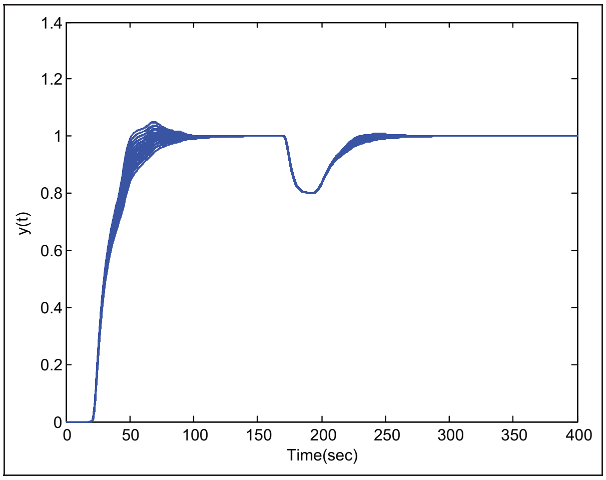

Figure 13. Step responses for different models for $k_p = 0.5$ and $k_i = 0.07$.

Choosing five points within each interval of K_m, T_m and L_m, a total of 5x5x5 = 125 transfer functions are obtained and the step responses of 125 transfer functions are shown in Figure 13. From Figure 13, it can be seen that the results are acceptable and the proposed method is robust when there are parameter uncertainties.

Conclusions

In this study, stabilization of the Smith predictor structure with perfect matched and mismatched models has been investigated. The controller used in the Smith predictor structure has been selected as a PI controller. The stability boundary locus method is used to find stability regions that include all stabilizing PI controllers.

The Smith predictor with a mismatched model has two different time delays belonging to the model and the plant. For this reason, the stability region is similar to a helix, namely the area has no upper limits. Although several studies have been performed that are related to the stabilization of the closed-loop system with two delays known as the neutral and retarded systems, more studies are needed on this topic.

It can be seen in the illustrative example that the stability region of the Smith predictor structure with the SOPDT model is closer to the stability region with the perfect matched model than the FOPDT model. For the purposes of robust controller design, it is important that the stability region obtained using the identified model fits the stability region of the exact system. Therefore, an identification approach based on the stability region has been considered to find the FOPDT models whose stability regions are similar to the stability region of the system. The proposed method was carried out in a simulation example. As seen in the example, the similarity of the stability regions of the model and the plant was increased by using the proposed method.

Conflict of interest

The authors declare that there is no conflict of interest.

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