Game-theoretic analysis of an aerial jamming attack on a UAV communication network

Sourabh Bhattacharya Tamer Başar

***Abstract*— In this paper, we consider a differential game theoretic approach to compute optimal strategies by a team of UAVs to evade the attack of an aerial jammer on the communication channel. We formulate the problem as a zero-sum pursuit-evasion game. The cost function is the termination time of the game. We use *Isaacs'* approach to derive the necessary conditions to arrive at the equations governing the saddle-point strategies of the players. We illustrate the results through simulations.**

I. INTRODUCTION

In the past few years, a lot of research has been done to deploy multiple UAVs in a decentralized manner to carry out tasks in military as well as civilian scenarios. UAVs have shown promise in a wide range of applications. The recent availability of low-cost UAVs suggests the use of teams of vehicles to perform various tasks such as mapping, surveillance, search and tracking operations [7][25]. For these applications, there has been a lot of focus to deploy teams of multiple UAVs in cooperative or competitive manner [18]. An extensive summary of important milestones and future challenges in network control of multiple UAVs is presented in [21].

In general, the mode of communication among UAVs deployed in a team mission is wireless. This renders the communication channel vulnerable to malicious attacks from aerial intruders flying in the vicinity. An example of such an intruder is an aerial jammer. Jamming is a malicious attack whose objective is to disrupt the communication of the victim network intentionally causing interference or collision at the receiver side. Jamming attack is a well-studied and an active area of research in wireless networks. Many defense strategies have been proposed by researchers against jamming in wireless networks. In [29], Wu *et.al.* propose two strategies to evade jamming. The first strategy, channel surfing, is a form of spectral evasion that involves legitimate wireless devices changing the channel that they are operating on. The second strategy, spatial retreats, is a form of special evasion whereby legitimate devices move away from the jammer. In [27], Wood *et.al.* present a distributed protocol to map jammed region so that the network can avoid routing traffic through it. The solution proposed by Cagalj *et.al.* [5] uses different worm holes (wired worm holes, frequency-hopping pairs, and uncoordinated channel hopping) that lead out of the jammed region to report the alarm to the network operator. In [28], Wood *et.al.* investigate how to deliberately avoid jamming in IEEE 802.15.4 based wireless networks. In [6], Lin Chen *et.al.* propose a strategy to introduce a special node in the network called the anti-jammer to drain the jammer's energy. To achieve its goal, the anti-jammer configures the probability of transmitting bait packets to attract the jammer to transmit.

For a static jammer and mobile nodes, the optimal strategy for the nodes is to retreat away from the jammer after detecting jamming. In case of an aerial jamming attack, optimal strategies for retreat are harder to compute due to the mobility of the jammer and constraints in the kinematics of the UAVs. This attack can be modeled as a zero-sum game [1] between the jammer and the UAVs. Such dynamic games governed by differential equations can be analyzed using tools from differential game theory [12][10]. In the past, differential game theory has been used as a framework to analyze problems in multi-player pursuit-evasion games. Solutions for particular multi-player games were presented by Pashkov and Terekhov [17], Levchenkov and Pashkov [11], Hagedorn and Breakwell [9], Breakwell and Hagedorn [4] and Sriram *et.al.*[22]. More general treatment of multi-player differential games was presented by Starr and Ho [3], Vaisbord and Zhukovskiy [26] and Zhukovskiy and Salukvadze [30] and Stipanović, Hovakimyan and Melikyan [24]. The inherent hardness in obtaining an analytical solution to Hamilton-Jacobi-Bellman-Isaacs equation has led to the development of numerical techniques for the computation of the value function. Recent efforts in this direction to compute an approximation of the reachable sets have been provided by Mitchell and Tomlin [14], Stipanović, Hwang and Tomlin [23] and Stipanović, Sriram and Tomlin [8].

Contrary to the existing literature, our work analyzes the behavior of multiple UAVs in cooperative as well as non-cooperative scenarios in the presence of a malicious intruder in the communication network. In this paper, we envision a scenario in which an aerial jammer intrudes the communication channel in a multiple UAV formation. We model the intrusion as a continuous time pursuit-evasion game between the UAV's and the aerial jammer. In contrast to the previous work in pursuit-evasion games that formulate a payoff based on a geometric quantity in the configuration space of the system, we formulate a payoff based on the capability of the players in a team to communicate among themselves in the presence of a jammer in the vicinity. In particular, we are interested in computing strategies for spatial reconfiguration of a formation of UAVs in the presence of an aerial jammer to reduce the jamming on the communication channel.

Sourabh Bhattacharya and Tamer Başar are with the Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801 {sbhattac,basar1}@illinois.edu

This work was supported in part by a grant from AFOSR.

Section 2 presents the problem formulation. The jamming, communication and mobility models for the nodes are presented. Based on the aforementioned models, a multi-player pursuit-evasion game is analyzed in Section 3. Section 4 extends the solutions to a variant of the problem discussed in Section 3. Section 5 presents the results and the conclusion.

II. Problem formulation

In this section, we first introduce a communication model between two mobile nodes in the presence of a jammer. Then we present the mobility models for the nodes. We conclude the section by formally formulating the problems we study in the paper.

A. Jammer and Communication Model

Consider a mobile node (*receiver*) receiving messages from another mobile node (*transmitter*) at some frequency. Both communicating nodes are assumed to be lying on a plane. Consider a third node that is attempting to jam the communication channel in between the transmitter and the receiver by sending a high power noise at the same frequency. This kind of jamming is referred to as the *trivial jamming*. A variety of metrics can be used to compare the effectiveness of various jamming attacks. Some of these metrics are energy efficiency, low probability of detection, and strong *denial of service* [16] [15]. In this paper, we use the ratio of the jamming-power to the signal-power (JSR) as the metric. [19] provides various models for the JSR (ξ) at the receiver's antenna. In all the models the jammer to signal ratio is dependent on the ratio $\frac{D_{TR}}{D_{JR}}$ where D_{TR} is the Euclidean distance between transmitter and receiver, and D_{JR} is the Euclidean distance between jammer and transmitter.

For digital signals, the jammer's goal is to raise the ratio to a level such that the *bit error rate* [20] is above a certain threshold. For analog voice communication, the goal is to reduce the articulation performance so that the signals are difficult to understand. Hence we assume that the communication channel between a receiver and a transmitter is considered to be jammed in the presence of a jammer if $\xi \geq \xi_{tr}$ where ξ_{tr} is a threshold determined by many factors including application scenario and communication hardware. If all the parameters except the mutual distances between the jammer, transmitter and receiver are kept constant, we can conclude the following from all the above models: If the ratio $\frac{D_{TR}}{D_{JR}} \geq \eta$ then the communication channel between a transmitter and a receiver is considered to be jammed. Here η is a function of ξ, P_{J_T}, P_T, G_{TR}, G_{RT}, G_{JR}, G_{RJ} and D_{TR}. Hence if the transmitter is not within a disc of radius ηD_{JR} centered around the receiver then the communication channel is considered to be jammed. We call this disc as the *perception range*. The *perception range* for any node depends on the distance between the jammer and the node. For effective communication between two nodes, each node should be able to transmit as well as receive messages from the other node. Hence two nodes can communicate if they lie in each other's *perception range*.

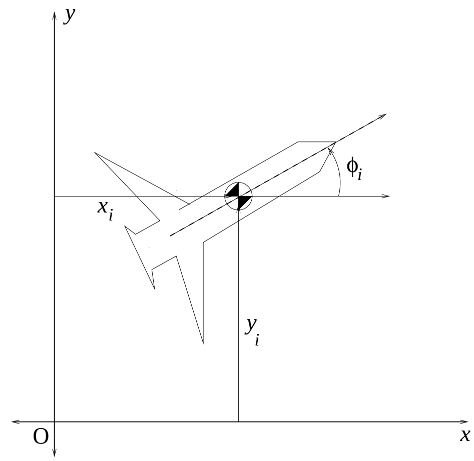

Fig. 1. Configuration of a UAV

In the rest of the paper, we will use the above jamming and communication model.

B. System Model

We now describe the kinematic model of the nodes. In our analysis, each node is a UAV. We consider two UAV's (UAV_1 and UAV_2) in the presence of a third UAV (UAV_j) that is trying to jam the communication link in between them. We assume that the UAV's are having a constant altitude flight. This assumption helps to simplify our analysis to a planar case. Referring to Figure 2, the configuration of each UAV in the global coordinate frame can be expressed in terms of the variables (x_i^g, y_i^g, ϕ_i^g). The subscript *i* is either 1, 2 or j depending on the UAV being referred. The pair (x_i^g, y_i^g) represents the position of a reference point on UAV_i with respect to the origin of the global reference frame and ϕ_i^g denotes the instantaneous heading of the UAV_i in the global reference frame. Hence the state space for UAV_i is $\mathbf{X}_i \cong \mathbb{R}^2 \times \mathrm{S}^1$. In our analysis, we assume that the UAV's are a kinematic system and hence the dynamics of the UAV's are not taken into account in the differential equation governing the evolution of the system. The kinematics of the UAV's are assumed to be the following:

$$\frac{dx_i^g}{dt} = W_i \cos \phi_i^g ; \frac{dy_i^g}{dt} = W_i \sin \phi_i^g ; \frac{d\phi_i^g}{dt} = \sigma_i \qquad (1)$$

where, W_i and σ_i are the speed and angular velocity of UAV_i, respectively. In this paper, we assume that $\sigma_i \in [-1, +1] \quad \forall i$. Moreover, we assume that $W_i = 1 \quad \forall i$.

The state space of the entire system is $\mathbf{X}_1 \times \mathbf{X}_2 \times \mathbf{X}_j \cong \mathbb{R}^6 \times (\mathrm{S}^1)^3$. In order to reduce the dimension of the state space we analyze the system in a coordinate frame fixed to UAV_2 as shown in Figure 2. In the new coordinate frame, the system can be modeled using six independent variables and the equations of motion of the UAV_1 and UAV_j with respect to the new coordinate frame are given by the following [22]:

$$\underbrace{\begin{pmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{\phi}_1 \\ \dot{x}_2 \\ \dot{y}_2 \\ \dot{\phi}_2 \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} -1 + \sigma_2 y_1 + \cos\phi_1 \\ -\sigma_2 x_1 + \sin\phi_1 \\ -\sigma_2 + \sigma_1 \\ -1 + \sigma_2 y_j + \cos\phi_j \\ -\sigma_2 x_j + \sin\phi_j \\ -\sigma_2 + \sigma_j \end{pmatrix}}_{f(\mathbf{x},\sigma_1 . \sigma_2, \sigma_j)} \tag{2}$$

In the above expressions (x_j, y_j, ϕ_j) and (x_1, y_1, ϕ_1) represent the relative position and orientation of the UAV_j and UAV_1 in the reference frame attached to UAV_2 which are the state variables of the system. Hence the state space of the reduced system is isomorphic to $\mathbb{R}^4 \times (\mathbb{S}^1)^2$. Comparing

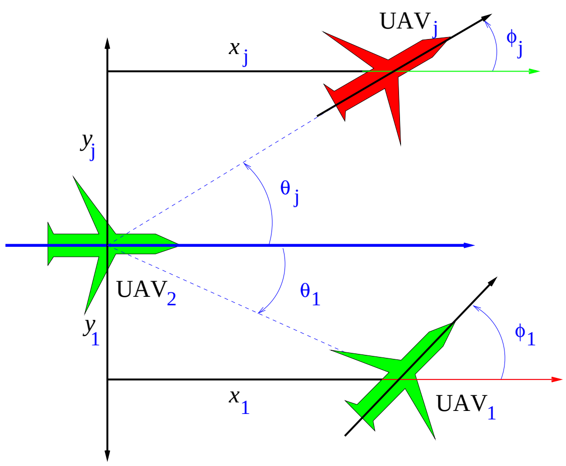

Fig. 2. Relative configuration of UAVs

C. Problem Statement

From the communication and the mobility models proposed in the previous subsections, we formulate the following problems.

- *Problem 1*: Consider a situation in which UAV_1 and UAV_2 are not communicating initially in the presence of a jammer (UAV_j). The objective of the jammer is to maximize the time for which it can jam the communication between UAV_1 and UAV_2. The objective of UAV_1 and UAV_2 is to minimize the time for which communication remains jammed. The game terminates at the first instant at which UAV_1 and UAV_2 are in a position to communicate. We need to compute the optimal strategies for each UAV.

- *Problem 2*: Now consider a situation in which UAV_1 and UAV_2 are communicating initially in the presence of a jammer (UAV_j). The objective of the jammer is to minimize the time in which it can jam the communication channel between UAV_1 and UAV_2. The objective of UAV_1 and UAV_2 is to maximize the time for which communication link between them remains operable. The game terminates at the first instant at which UAV_1 and UAV_2 are about to lose their link. We need to compute the optimal strategies for each UAV.

In both problems, it is assumed that each UAV has a complete knowledge about the state of the system.

In the next section, we analyze the first problem.

III. Analysis of Problem 1

We consider a situation in which UAV_1 and UAV_2 are not communicating initially in the presence of a jammer (UAV_j). The termination condition is defined as the first instant at which UAV_1 and UAV_2 are in a position to communicate. The cost function of the game is the time of termination of the game. The objective of the jammer is to maximize the time for which it can jam the communication between UAV_1 and UAV_2. The objective of UAV_1 and UAV_2 collectively is to minimize the time for which communication remains jammed.

In order to obtain the optimal strategies of the players we need to compute the *saddle-point strategies* since this is a zero-sum game. A set of strategies for the players are said to be in *saddle-point equilibrium* if no unilateral deviation in strategy by a player can lead to a better outcome for that player. Hence there is no motivation for the players to deviate from their equilibrium strategies. In scenarios where the players have no knowledge about each other's strategies, equilibrium strategies are important since they lead to a guaranteed minimum outcome for the players in spite of the other player's strategies.

For a point $\mathbf{x}$ in the state space, let $J(\mathbf{x})$ represent the outcome if the players implement their optimal strategies starting at the point $\mathbf{x}$. In this game, it is the time of termination of the game when the players implement their optimal strategies. It is also called the *value* of the game at $\mathbf{x}$. Let $\nabla J = [J_{x_1} \quad J_{y_1} \quad J_{\phi_1} \quad J_{x_J} \quad J_{y_J} \quad J_{\phi_J}]^T$ denote the gradient of the value function. The Hamiltonian of the system is given by $H = 1 + \nabla J \cdot f(\mathbf{x}, \sigma_1^*, \sigma_j^*, \sigma_2^*, t)$. From the equations of motion of the system, the Hamiltonian is given by the following expression:

$$H = 1 + J_{x_1}\dot{x}_1 + J_{y_1}\dot{y}_1 + J_{\phi_1}\dot{\phi}_1 + J_{x_j}\dot{x}_j + J_{y_j}\dot{y}_j + J_{\phi_j}\dot{\phi}_j$$

Since the jammer wants to maximize the time of termination and the UAV's want to minimize the time of termination, we get the following expressions for the controls from Isaacs' first condition.

$$(\sigma_1^*, \sigma_2^*, \sigma_j^*) = arg \max_{\sigma_j} \min_{\sigma_2 \sigma_1} H$$

Since the Hamiltonian is separable in its controls, the order of taking the extrema becomes inconsequential. Hence the optimal control of the players are given as follows.

$$\begin{aligned} \sigma_2^* = -\text{sign}[&J_{x_1} y_1 - J_{y_1} x_1 - J_{\phi_1} - J_{\phi_j} \\ &- J_{y_j} x_j + J_{x_j} y_j] \end{aligned} \tag{3}$$

$$\sigma_j^* = \text{sign}(J_{\phi_j}) \tag{4}$$

$$\sigma_1^* = -\text{sign}(J_{\phi_1}) \tag{5}$$

The *retrogressive path equations* (RPE) for the system lead to the following equations.

$$\mathring{J}_{x_1} = -\sigma_2^* J_{y_1}, \quad \mathring{J}_{y_1} = \sigma_2^* J_{x_1} \tag{6}$$

$$\mathring{J}_{x_j} = -\sigma_2^* J_{y_j}, \quad \mathring{J}_{y_j} = \sigma_2^* J_{x_j} \tag{7}$$

$$\mathring{J}_{\phi_1} = -J_{x_1} \sin\phi_1 + J_{y_1} \cos\phi_1 \tag{8}$$

$$\mathring{J}_{\phi_j} = -J_{x_j} \sin\phi_j + J_{y_j} \cos\phi_j \tag{9}$$

$^\circ$ denotes derivative with respect to retrograde time.

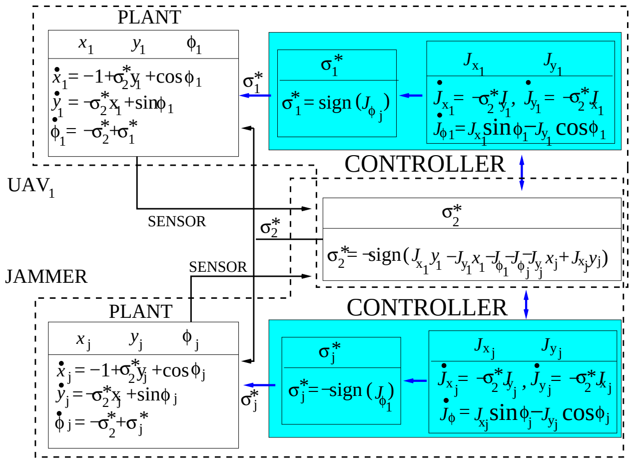

Fig. 3. The Control Loop for the System

Figure 3 summarizes the entire control algorithm. The controller of each UAV takes as input the state variables and runs the RPE to compute the control. This control is then fed into the plant of the respective UAV. The plant updates the state variables based on the kinematic equations governing the UAV. Finally the sensors feedback the state variables into the controllers. In this case the sensors measure the position and the orientation of each UAV.

A. Termination situations

In order to compute the optimal strategies, we need to compute the boundary conditions for the dependent variables of the differential equation. In order to do so, we characterize the terminal conditions of the game in the state space and compute the value of ∇J at the terminal conditions. This section presents the computation of the terminal value of the dependent variables of the differential equations governing the game.

From the communication model, we can conclude that UAV$_1$ can communicate with UAV$_2$ when the following condition holds:

$$\eta \min[d(\text{UAV}_J,\text{UAV}_1), d(\text{UAV}_J,\text{UAV}_2)] \geq d(\text{UAV}_1,\text{UAV}_2)$$

where $d(\text{UAV}_i,\text{UAV}_j)$ is the Euclidean distance between UAV$_i$ and UAV$_j$. Hence the boundary of the game set is the set of positions of the UAV's that satisfies the following condition:

$$\eta \min[d(\text{UAV}_J,\text{UAV}_1), d(\text{UAV}_J,\text{UAV}_2)] = d(\text{UAV}_1,\text{UAV}_2)$$

This leads to two termination manifolds in the state space.

1) The first terminal manifold is characterized by the positions of the UAV's such that UAV$_1$ is at the boundary of the *perception range* of UAV$_2$ and UAV$_2$ is inside the *perception range* of UAV$_1$. In the coordinate system of UAV$_2$ the terminal manifold is represented by the hypersurface $F_1(x_1, y_1, \phi_1, x_j, y_j, \phi_j)$ which is given by the following expression:

$$(\sqrt{x_1^2 + y_1^2} - \eta\sqrt{x_j^2 + y_j^2} = 0) \cap$$
$$((x_1 - x_j)^2 + (y_1 - y_j)^2 - (x_j^2 + y_j^2) \leq 0)$$

2) The second terminal manifold is characterized by the positions of the UAV's such that UAV$_2$ is at the boundary of the *perception range* of UAV$_1$ and UAV$_1$ is inside the *perception range* of UAV$_2$. In the coordinate system attached to UAV$_2$ this terminal manifold is represented by the hypersurface $F_2(x_1, y_1, \phi_1, x_j, y_j, \phi_j)$ is given by the following expression:

$$(\sqrt{x_1^2 + y_1^2} - \eta\sqrt{x_j^2 + y_j^2} = 0) \cap$$
$$((x_1 - x_j)^2 + (y_1 - y_j)^2 - x_j^2 + y_j^2 \geq 0)$$

Both the terminal surfaces are five dimensional manifolds with boundary. Hence they can be parameterized using five independent variables x_1, y_1, x_j, ϕ_1 and ϕ_j. Since $J \equiv 0$ on the terminal manifold, ∇J satisfies the following equations at an interior point in the manifold:

$$J^0_{x_1} + J^0_{y_j}\frac{\partial y_j}{\partial x_1} = 0, \quad J^0_{y_1} + J^0_{y_j}\frac{\partial y_j}{\partial y_1} = 0$$
$$J^0_{x_j} + J^0_{y_j}\frac{\partial y_j}{\partial x_j} = 0, \quad J^0_{\phi_1} = 0, \quad J^0_{\phi_j} = 0 \tag{10}$$

In addition to the above equations Isaacs' second condition leads to the following equation.

$$H(\mathbf{x}, \nabla J, f(\mathbf{x}, \sigma_1^*, \sigma_2^*, \sigma_j^*)) = 0 \tag{11}$$

The value of ∇J at the terminal manifold can be obtained from Equations (11) and (12). Since there are two different terminal manifolds, we have to analyze both of them separately. At first, we compute the value of ∇J on terminal manifold 1. Substituting the expression for $F_1(x_1, y_1, \phi_1, x_j, y_j, \phi_j)$ in Equation (11) and (12), we obtain the following value of J_{y_j} .

$$J^0_{y_j} = y^0_j[\sqrt{(x^0_j)^2 + (y^0_j)^2}(\frac{1}{\eta} - 1) + (x^0_j - \frac{x^0_1}{\eta^2})]^{-1} \tag{12}$$

From the values of ∇J at the terminal manifold, the optimal controls of the UAVs at termination can be computed. An elaborate computation of the optimal control of the UAVs is shown in the [2].

IV. Analysis of Problem 2

For Problem 2 as described in Section 2, Isaacs' first condition leads to the following optimal strategies for the players:

$$(\sigma_1^*, \sigma_2^*, \sigma_j^*) = arg \max_{\sigma_1, \sigma_2} \min_{\sigma_j} H$$

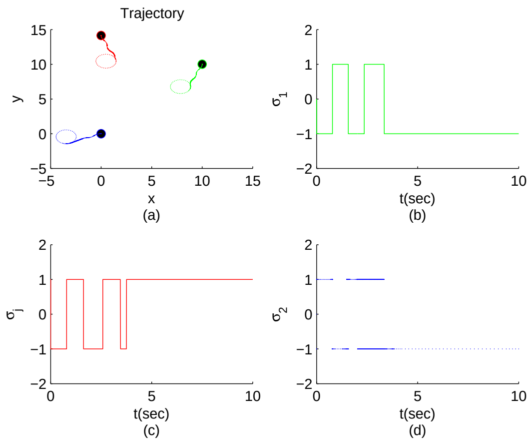

Fig. 4. Figure shows the players leading to Termination condition 1 for Problem 1. The value $\eta = 1$. The player in red is the jammer. The players in green and blue are UAV_1 and UAV_2 respectively. Figure (b) shows the control of the UAV_1. Figure (c) shows the control of the UAV_J. Figure (c) shows the control of the UAV_2.

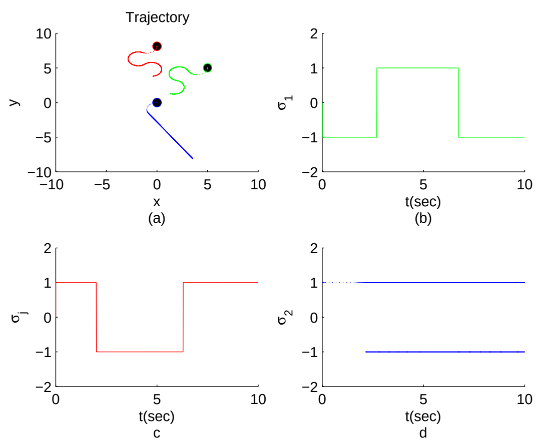

Fig. 5. Figure shows the players leading to Termination condition 2 for Problem 1. The value $\eta = 2$. The player in red is the jammer. The players in green and blue are UAV_1 and UAV_2 respectively. Figure (b) shows the control of the UAV_1. Figure (c) shows the control of the UAV_J. Figure (c) shows the control of the UAV_2.

Hence the optimal control of the players are given as follows:

$$\sigma_2^* = \text{sign}[J_{x_1}y_1 - J_{y_1}x_1 - J_{\phi_1} - J_{\phi_j} - J_{y_j}x_j + J_{x_j}y_j]$$
$$\sigma_j^* = -\text{sign}(J_{\phi_j}), \quad \sigma_1^* = \text{sign}(J_{\phi_1})$$

The retrogressive path equations remain the same as in the previous problem. The terminal conditions also remain the same. Analysis done in the previous section can be extended to this problem. The results obtained by simulating the differential equations governing the optimal control laws and the trajectories are presented in the next section.

V. Results

Figures 4, 5, 6 and 7 show trajectories of the players for both problems along with their optimal controls for various terminal conditions and different values of η. The position of the players corresponding to the termination situation is

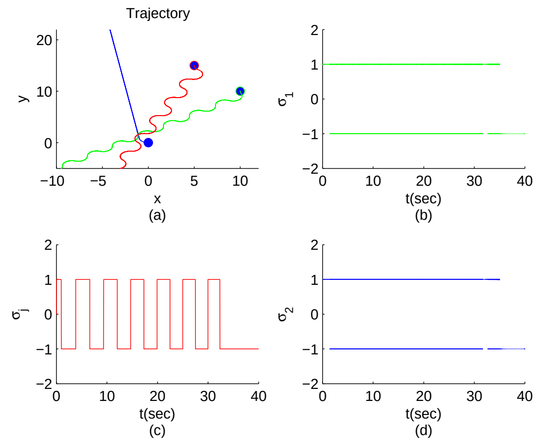

Fig. 6. Figure shows the players leading to Termination condition 1 for Problem 2. The value $\eta = 2$. The player in red is the jammer. The players in green and blue are UAV_1 and UAV_2 respectively. Figure (b) shows the control of the UAV_1. Figure (c) shows the control of the UAV_J. Figure (c) shows the control of the UAV_2.

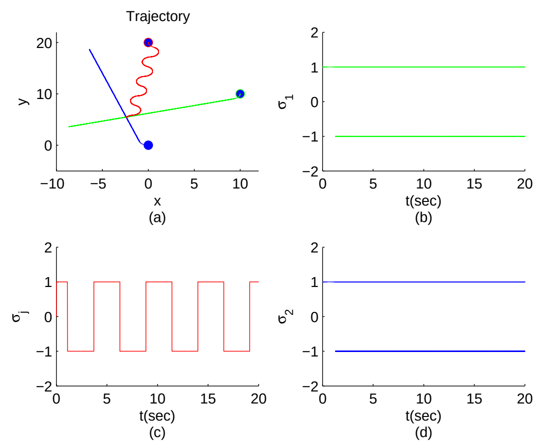

Fig. 7. Figure shows the players leading to Termination condition 2 for Problem 2. The value $\eta = 1$. The player in red is the jammer. The players in green and blue are UAV_1 and UAV_2 respectively. Figure (b) shows the control of the UAV_1. Figure (c) shows the control of the UAV_J. Figure (c) shows the control of the UAV_2.

shown by a small circle in the plots showing the trajectories of the players. Each figure shows the trajectory of the players just before termination for a small time interval. From the expression of the optimal controls in equations (4), (5) and (6), we can infer that the controls of the players are bang-bang. This is also verified from the simulation results. From the nature of the controls and kinematics of the system, we can infer that the optimal paths comprise of arcs of circles and straight line trajectories as motion primitives. Arcs of circles are generated when the UAV keeps its angular velocity saturated at one extrema for a non-zero interval of time. Straight line segments are obtained due to rapid switching between the extremum value of the controls (chattering). An instance of such a behavior is exhibited by UAV_2 in Figure 4. Among the future works are to prevent such an undesired behavior by adding the derivative of the controls in the cost function of the game by considering a dynamic extension of the original system.

VI. Conclusion

In this paper, we considered a differential game theoretic approach to compute optimal strategies by a team of UAVs to evade the attack of an aerial jammer on the communication channel. We considered two variants of the problem in this paper. We formulated the problem as a zero-sum pursuit-evasion game and used *Isaacs'* approach to derive the necessary conditions to arrive at the equations governing the saddle-point strategies of the players. The cost function was picked as the termination time of the game. We illustrated the results through simulations.

Among the future works are to extend the problem to analyze multiple jammers and multiple UAVs in the formation. Another direction of future research is to extend the locally optimal trajectories presented in this paper into the entire phase space. In order to do so construction of various types of singular surfaces [13] is needed. We are also analyzing the case whe th

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