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Reservoir Monitoring and Continuous Model Updating Using Ensemble Kalman Filter

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Abstract

The use of ensemble Kalman filter techniques for continuous updating of reservoir model is demonstrated. The ensemble Kalman filter technique is introduced, and thereafter applied on two 2-D reservoir models. One is a synthetic model with two producers and one injector. The other model is a simplified 2-D field model, which is generated by using a horizontal layer from a North Sea field model.

By assimilating measured production data, the reservoir model is continuously updated. The updated models give improved forecasts. Both dynamic variables, as pressure and saturations, and static variables as the permeability are updated in the reservoir model.

Introduction

In the management of reservoirs it is important to utilize all available data in order to make accurate forecasts. For short time forecasts, in particular, it is important that the initial values are consistent with recent measurements. The ensemble Kalman filter[1] is a Monte Carlo approach, which is promising with respect to achieving this goal through continuous model updating and reservoir monitoring.

In this paper, the ensemble Kalman filter is utilized to update both static parameters, such as the permeability, and dynamic variables, such as the pressure and saturation of the reservoir model. The filter computations are based on an ensemble of realizations of the reservoir model, and when new measurements are available new updates are obtained by combining the model predictions with the new measurements. Statistics about the model uncertainty is built from the ensemble. While new measurements become available, the filter is used to update all the realizations of the reservoir model. This means that an ensemble of updated realizations of the reservoir model is always available.

The ensemble Kalman filter has previously been successfully applied for large-scale nonlinear models in oceanography[2] and hydrology[3]. In those applications only dynamic variables were tuned. Tuning of model parameters and dynamic variables was done simultaneously in a well flow model used for underbalanced drilling[4]. In two previous papers[5,6], the filter has been used to update static parameters in near-well reservoir models, by tuning the permeability field. In this paper, the filter has been further developed to tune the permeability for simplified real field reservoir simulation models.

We present results from a synthetic model as well as a simplified real field model. The measurements are well bottomhole pressures, water cuts and gas/oil ratios. A synthetic model gives the possibility of comparing the solution obtained by the filter to the true solution, and the performance of the filter can be evaluated. It is shown how the reservoir model is updated as new measurements becomes available, and that good forecasts are obtained. The convergence of the reservoir properties to the true solution as more measurements becomes available is investigated.

Since the members of the ensemble are updated independently of each other, the method is very suitable for parallel processing. It is also conceptually straightforward to extend the methodology to update other reservoir properties than the permeability.

Based on the updated ensemble of models, production forecasts and reservoir management studies may be performed on a single "average" model, which is always consistent with the latest measurements. Alternatively, the entire ensemble may be applied to estimate the uncertainties in the forecasts.

Updating reservoir models with ensemble Kalman filter

The Kalman filter was originally developed to update the states of linear systems to take into account available measurements[7]. In our case, the system is a reservoir model, using black oil, and three phases (water, oil and gas). For this model, the solution variables of the system are the pressure, the water saturation, in addition to a third solution variable that depends on the oil and gas saturation. If the gas saturation is zero, the third solution variable becomes the solution gas/oil ratio, if the oil saturation

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is zero it becomes the vapor oil/gas ratio. Otherwise the third solution variable is the gas saturation. The states of this system are the values of the solution variables for each grid block of the simulation model. This model is non-linear.

An early attempt to extend the ideas of the Kalman filter to non-linear models is the extended Kalman filter which is based on linearization of the non-linear model. The extended Kalman filter is however not suitable for very large models and also fails if the non-linearities are too severe. Therefore an alternative is needed[1]. The ensemble Kalman filter has shown to be a promising approach for large scale non-linear atmospheric and oceanographic models.

For the reservoir model, however, there are poorly determined model parameters that have a governing influence. In this study we will focus on using the ensemble Kalman filter to update the permeability, in addition to the above mentioned state variables. This means that we extend the state vector with the permeability of each grid block (assuming that the permeability is isotropic). Note that in principle any input parameter in the model can be updated.

The literature on estimating dynamic variables and model parameters simultaneously for non-linear models using some variants of a Kalman filter is modest. The topic is discussed by Wan and Nelson[8]. They give further references, but do not discuss the use of ensemble Kalman filter to solve such a task.

The ensemble Kalman filter has previously been used to update both dynamic variables and model parameters for a two-phase well flow model used in underbalanced drilling[4,9]. The approach has also been used for updating the permeability and dynamic variables in a two-phase near-well reservoir setting[5,6].

The ensemble Kalman filter is based on a Monte-Carlo approach, using an ensemble of model representations to evaluate the necessary statistics. We have used 100 members in the ensemble, based on experience from atmospheric data assimilation, where this have been sufficient[10].

The filter consists of sequentially running a forecast step followed by an analysis step. The input to the forecast step is the result obtained from the analysis step, which is an updated description of the model after assimilating a new set of measurements. The forecast step consists of running the reservoir simulator for each of the model realizations. The forward simulations are ending at the next point in time where new measurements are to be assimilated. The state vector after running the forecast step is denoted by $\mathbf{s}_k^f$ and the state vector after the analysis step is denoted by $\mathbf{s}_k^a$.

The filter is initialized by generating an initial ensemble. The generation of the initial ensemble can be described by the equation

$$\mathbf{s}_{0,i}^a = \overline{\mathbf{s}_0^a} + \mathbf{e}_{0,i}^m, \tag{1}$$

where i runs from 1 to the number of members in the ensemble. Here $\overline{\mathbf{s}_0^a}$ is the mean of the initial ensemble. The terms $\mathbf{e}_{0,i}^m$ are drawn from a mixture distribution with zero mean (we use the term *mixture distribution*[11] if a random variable have a distribution that depends on a quantity which also have a distribution). Further details on the distribution that we use in generating the terms $\mathbf{e}_{0,i}^m$ is presented below.

The forecast step consists of running the model (i.e. the simulator solving the reservoir model) and giving an expression for the uncertainty in the model output. In these studies we have used a commercial reservoir simulator. We denote running the reservoir simulator forward to the next point in time where measured data is going to be assimilated by $\mathbf{f}$. The reservoir simulator is run once for each member of the ensemble. For the i'th member of the ensemble at time level k, we denote the forecasted state vector by $\mathbf{s}_{k,i}^f$ and the analyzed state by $\mathbf{s}_{k,i}^a$. In the forecast step the simulator is run from the current time (say time level $k-1$) to the point in time where the next measurement becomes available (time level k). To take into account the model uncertainty, we add model noise to the ensemble members prior to each simulation. This gives the equation

$$\mathbf{s}_{k,i}^f = \mathbf{f}(\mathbf{s}_{k-1,i}^a + \mathbf{e}_{k,i}^m). \tag{2}$$

The terms $\mathbf{e}_{k,i}^m$ is drawn from a mixture distribution.

Generation of the initial ensemble and the generation of the model noise at each step is done along the same lines, represented by the terms $\mathbf{e}_{0,i}^m$ and $\mathbf{e}_{k,i}^m$ in Equations 1 and 2, respectively. The model noise is basically placed in the permeability. The noise added to the permeability is independent of the noise added to the time dependent variables of the model (pressure, saturations, etc.).

To generate stochastic realizations of the permeability field we assume that the permeability is spatially correlated with a Gaussian correlation model, i.e. that the permeability in grid block (i_1, j_1) is correlated with the permeability in grid block (i_2, j_2) with correlation coefficient

$$e^{-\left(\frac{i_1-i_2}{l}\right)^2-\left(\frac{j_1-j_2}{l}\right)^2}. \tag{3}$$

Using the correlations specified in Equation 3 a correlation matrix $\mathbf{C}$ is computed. From this correlation matrix, the covariance matrices used while generating the initial permeability field and the model noise is computed as $\sigma^2\mathbf{C}$, where σ is the standard deviation in the permeability of each grid block. The standard deviation used while generating the initial permeability field (Eq. 1) is usually much higher than the used while adding the model noise in Eq. 2. The correlation length, l, is also treated as a normally distributed stochastic variable. This means that (the noise added to) each ensemble member is generated by first drawing l from a normal distribution, then generating $\mathbf{C}$ and finally draw a permeability field from a multinormal distribution with zero mean and covariance matrix $\sigma^2\mathbf{C}$.

To generate the noise added to time dependent variables of the model, we draw from a normal distribution with zero mean and covariance matrix $\mathbf{Q}_d$. In the examples we have used $\mathbf{Q}_d = \epsilon\mathbf{I}$, where $\epsilon \approx 2 \cdot 10^{-16}$. In generation of the initial ensemble there is no noise in the state variables, as all the ensemble members are started using the same equilibrium condition of the reservoir.

The analyzed state at time level k is computed by taking into account the measurement vector at time level k. The theory assumes that there is a linear relationship between the measurements, $\mathbf{d}_k$, and the states, $\mathbf{s}_k$, expressed by the equation

$$\mathbf{d}_k = \mathbf{H}\mathbf{s}_k. \tag{4}$$

To take into account the fact that there is a non-linear relationship between the observed quantities (measurements in the well), and the state variables, we extend the state vector by one state for each non-linear measurement. This brings the correspondence between all measurements and the state variables on the form (4). We assume that the measurement noise has a multinormal distribution with zero mean and covariance matrix $\mathbf{R}_k$. The index k is included since the covariance matrix of the measurement noise may be time dependent.

It is shown that if the measurement is not treated as a random variable, the updated ensemble will have correct mean, but to low variance[12]. This means that to get consistent error propagation in the ensemble Kalman filter one has to treat the observations as random variables. This is done by using the actual measurement as reference, and adding random noise, reflecting our assumptions on the measurement noise. This means that the actual measurement $\mathbf{d}_k$ serves as the reference observation. For each member of the ensemble an observation vector $\mathbf{d}_{k,i}$ is generated randomly as

$$\mathbf{d}_{k,i} = \mathbf{d}_k + \mathbf{e}^o_{k,i} \tag{5}$$

where $\mathbf{e}^o_{k,i}$ is drawn from a multinormal distribution with zero mean and covariance $\mathbf{R}_k$.

The error covariance matrix for the state variables of the model is defined in terms of the true state as the expectation

$$E\left((\mathbf{s}^f_k - \mathbf{s}^t_k)(\mathbf{s}^f_k - \mathbf{s}^t_k)^T\right). \tag{6}$$

Since the true state is not known, we approximate the true state by the mean of the ensemble

$$\mathbf{s}^t_k \approx \widehat{\mathbf{s}^f_k} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{s}^f_{k,i} \tag{7}$$

where N is the sample size of the ensemble. With this approximation of the true state, an approximation of a left factor of the error covariance matrix of the model is

$$\mathbf{L}^f_k = \frac{1}{\sqrt{N-1}}\left[\begin{array}{ccc}(\mathbf{s}^f_{k,1} - \widehat{\mathbf{s}^f_k}) & \ldots & (\mathbf{s}^f_{k,N} - \widehat{\mathbf{s}^f_k})\end{array}\right], \tag{8}$$

a matrix with N columns. The approximation of the model error covariance matrix then becomes

$$\mathbf{P}^f_k = \mathbf{L}^f_k(\mathbf{L}^f_k)^T. \tag{9}$$

The expression of the Kalman gain matrix is[7]

$$\mathbf{K}_k = \mathbf{P}^f_k\mathbf{H}^T(\mathbf{H}\mathbf{P}^f_k\mathbf{H}^T + \mathbf{R}_k)^{-1}. \tag{10}$$

The analyzed state of each member of the ensemble is computed as

$$\mathbf{s}^a_{k,i} = \mathbf{s}^f_{k,i} + \mathbf{K}_k(\mathbf{d}_{k,i} - \mathbf{H}\mathbf{s}^f_{k,i}). \tag{11}$$

The analyzed error covariance matrix, $\mathbf{P}^a_k$, of the model can be computed along the same lines as $\mathbf{P}^f_k$. Since the updating of the ensemble is linear, the new estimate of the true state, based on the ensemble after the analysis step is

$$\mathbf{s}^t_k \approx \widehat{\mathbf{s}^a_k} = \widehat{\mathbf{s}^f_k} + \mathbf{K}_k(\mathbf{d}_k - \mathbf{H}\widehat{\mathbf{s}^f_k}), \tag{12}$$

and the model error covariance matrix after the analysis step is

$$\mathbf{P}^a_k = (\mathbf{I} - \mathbf{K}_k\mathbf{H})\mathbf{P}^f_k. \tag{13}$$

The underlying assumptions behind the filter are that there is zero covariance between the model error and the measurement error and that both the model error and measurement error are uncorrelated in time.

In evaluating the quality of the solution we use the root mean square (RMS) error defined as

$$\sqrt{\frac{1}{M}\sum_{j=1}^{M}\left(X_j - X^t_j\right)^2}, \tag{14}$$

where M is the number of state variables, X_j is the estimate of state variable j, and X^t_j is the true value of state variable j.

Example 1: Synthetic case

A two-dimensional $50 \times 50 \times 1$ grid is defined as shown in Figure 1, modeling a reservoir with a constant dip. The model contains two production wells, P2 and P4, and a single water injector, WI. The water injector is located below the initial water-oil contact, as shown in Figure 1.

A ”true” permeability distribution is created, where the permeability varies linearly as a function of the x-coordinate in the interval $[200, 2000]\,\mathrm{mD}$. A constant value of 0.15 is used for the porosity in all grid cells. As relative permeabilities and PVT properties we have used values similar to those used in a North Sea field.

The production data is generated by running the reservoir simulator using group control for the two production wells, with a constant total liquid rate of 15.000 Sm3/d. This total rate is distributed between the two production wells according to their production potential. Voidage replacement is used for the injection well, leading to a water injection rate which may vary with time if pressure fells below the bubble point and free gas is produced in either of the wells. No pressure constraints are used for the wells. The measurements are generated by running the simulator with the “true” permeability, and adding noise to the resulting well pressures, oil rates, GORs and water cuts. Measurement are assimilated after 4 days, then after 22 days, then once every month for 50 months.

The measurement uncertainties are presented in Table 1. We apply the filter to take into account the information gained from 8 measurements at each assimilation, 4 for each of the production wells. The measured quantities are the pressure, the oil production rate, the water cut and gas/oil ratio. There are no measurements from the injector.

While running the ensemble Kalman filter we keep all quantities constant except for the permeability (which is isotropic) and the state variables, which are the pressure, the saturation of gas and water, and the solution gas/oil ratio. In this case there are no vaporized oil. The reservoir simulator is run in “history-matching mode”, that is, the wells are controlled by reservoir volume rates equal to the once used when generating the (synthetic) history data. Thus the reservoir volume offtake is the same for all ensemble members. The same applies for the injection rates. In the computation with the Kalman filter we use the natural logarithm of the permeability instead of the permeability.

The ensemble consists of 100 members. The initial ensemble is generated using a mean correlation length of 16 grid blocks with a standard deviation of 6. The initial ln-permeability fields for the Kalman filter is generated using a standard deviation of 0.5. For the model noise (Eq. 2) the standard deviation is set to $5 \cdot 10^{-4}$.

For comparison we have run a reference solution using the mean permeability of the initial ensemble (i.e., approximately 1000mD). The initial ensemble is generated using the mean of the ln-permeability of the three grid blocks perforated by a well.

In Figure 2 we present the outcome of the filter for each of the 8 measured variables, together with the reference solution. As we can see, the filter essentially tracks all the measurements, with a small exception for the gas/oil ratio for the well P4.

In Figure 3 we show forecasts based on the ensemble mean after 4 days, 604 days and 908 days. As expected, the quality of the forecasts are generally improved as more data is assimilated. It should be noted, however, that because of the pressure measurements, the permeability trend is recovered by the filter very soon. Already after 4 days, the permeability in the region from P2 to P4 is almost correct, as can be seen in Figure 4 showing the development in the estimated permeability. After 300 days, long before any water breakthrough, the trend is correct in the entire reservoir. At later times, however, the permeability estimates move away from the correct values, and the effect of the later measurements, including the water cut development, is not quite clear. Still, the uncertainty estimates shown in Figure 5 seems reasonable, being lowest close to the producers, where the measurements are obtained, and decreasing with time. The highest uncertainty is close to the boundary of the reservoir.

In Figure 6 we show how the root mean square (RMS) error defined in Eq. 14 evolves. Again, we see that the error in estimated permeability decreases very rapidly, before it starts increasing. At late times the RMS error is the same as for the reference solution. Here, the correct permeability trend is probably more important for the predictions than the global RMS error, and since the trend is recovered, the forecasts will still improve with time. However, an increase in the RMS error, despite of more available data is not satisfactory, and the reason for this behavior should be further investigated. For the dynamic variables, we should bear in mind that all the members of the ensemble are initiated from a model with a known equilibrium condition, giving zero RMS error. The RMS therefore has to increase initially. Theoretically, the error should stabilize at a level given by the data error and model error. It is seen that RMS error of the dynamic variables stabilize after some time, but because of the complicated relation between data, model and state variables, it is difficult to verify that the stabilized values of the RMS are correct.

Example 2: Simplified field model

A simplified 2D field model has been constructed by taking out a horizontal layer from a model of a North Sea field. The dimensions of the simplified model are 39×55, but with some inactive blocks. There are 1931 active grid blocks. The simulation spans a time range over 3955 days. The reservoir model is shown in Figure 7.

Measurements are assimilated at least once a month, but also when new wells are starting production, or when wells are shut in. Altogether there are 171 points in time when measurements are assimilated. The measurements are assimilated from 14 producing wells. Two of them are starting from the first day. The reservoir is produced by gas injection, and the producers are located along the oil-water contact. There are 4 gas injectors. There are no measurements from the injectors.

In addition to the 14 producers and 4 injectors there are some dummy wells included in the simulation to adjust for flow through the boundary. The rates of the dummy wells are kept constant while generating the data and running the ensemble Kalman filter.

The quantities that are tuned with the Kalman filter are the horizontal permeability, which is assumed to be the same in the x- and y-direction, together with the dynamic variables, which are the pressure, saturation of water and gas, the solution gas/oil ratio and the vapor oil/gas ratio. This means that we both have vaporized oil and dissolved gas.

The "true" model is obtained from a stochastic realization of the field. Relative permeabilities and PVT properties were taken from the original field model. The measurements are generated by running the simulator with the "true" permeability, and adding noise to the values obtained.

While running the ensemble Kalman filter we keep all quantities except the state variables described above constant. Like in Example 1, the reservoir simulator is run in "history-matching mode", and the natural logarithm of the permeability is used in the computations with the Kalman filter.

The ensemble consists of 100 members. The initial ensemble is generated using a mean correlation length of 16 grid blocks with a standard deviation of 6 grid blocks. The initial ln-permeability fields for the Kalman filter is generated using a standard deviation of 0.5. For the model noise (Eq. 2) the standard deviation is set to $5 \cdot 10^{-4}$. The initial mean of the permeability of the ensemble is set to 1000 for all the grid blocks.

The measurement uncertainties are as in Example 1 (Table 1). We apply the filter to take into account the information gained from 4 measurements for each of the production wells. The measured quantities are pressure, oil production rate, water cut and gas/oil ratio.

In Figure 8 the development in the estimated permeability is shown together with the "true" permeability and the reference solution. The reference solution is the mean of the initial ensemble, and since the initial ensemble is based on a model with constant mean permeability of $1000\,\mathrm{mD}$, the reference model will have a permeability of approximately $1000\,\mathrm{mD}$ in all cells. One can observe that after 1611 days a good match of the permeability field is obtained, especially close to the producing wells. The location of the producers are shown with circles. For the permeabilities along the boundaries, the permeability starts drifting towards the end of the simulation. Further development of the filter is needed to avoid this effect.

The uncertainty in the ln-permeability is presented in Figure 9. The uncertainty is lowest close to the producers, where the measurements are obtained. The highest uncertainty is close to the boundary of the reservoir. The uncertainty is decreasing as more measurements are assimilated.

In Figure 10 we present the outcome of the filter for some of the measured variables, together with the reference solution. We have selected to present results from measurements that have among the largest relative deviations between the truth and the solution obtained from the Kalman filter.

In Figure 11 we show forecasts based on the ensemble mean after 4 days, 819 days and 3050 days for the same measurements as presented in Figure 10. As expected, the quality of the forecasts is improved as more data is assimilated.

In Figure 12 we show how the root mean square (RMS) error defined in Eq. 14 evolves. Like in Example 1, the RMS error for the permeability decreases very rapidly and then increases. Still the predictions clearly improve with time as additional measurements become available. Note also that with the exception for RS and RV, the RMS values for the state variables stabilize relatively fast at quite low levels.

Discussion

The ensemble Kalman filter technology seems promising for tuning permeability and dynamic variables, such as the pressure and saturations, in reservoir simulator models. In both the examples presented, the filter were able to recover the main trends of the permeability field rapidly due to the continuous downhole pressure measurements. However, although new measurements are added continuously, the error in the estimated permeability increases late in time. The reason for this instability is not fully understood.

At this stage, all the variables that are not tuned, are kept fixed while running the reservoir simulator. This includes the initial state of the reservoir. Therefore the difference between the estimate obtained from the filter and the true value of the dynamic variables have to increase in the beginning. After a while the differences stabilize.

It is crucial for a successful application of the methodology that the ensemble of models correctly reflects the uncertainty at all times. So the generation of the initial ensemble and how to add model noise are important questions which also have to be further addressed.

Technically, there is no limit with respect to how many model parameters which can be updated, but the effect of adding a large number of parameters on filter stability, etc., has not yet been considered.

Conclusions

An ensemble Kalman filter technology is developed for continuous updating of reservoir simulation models. Examples are shown in 2D, but the methodology can easily be extended to 3D and applied to any existing reservoir simulator.

The filter is used to tune the permeability and dynamic variables like pressure and saturations. The continuous updating ensures that the predictions always start from a solution that matches the observed production data.

The methodology is applied on two 2D reservoir models: One synthetic model with one injector and two producers, and one simplified field model with 14 producers and 3 injectors. In both examples the ensemble Kalman filter is able to track the measurements and tune the permeability field, and as more measurements are assimilated, the forecasts are improved.

Although the presented results are promising, this technology is novel to reservoir simulation, and further development of the filter is needed for this application.

Acknowledgments

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Nomenclature

$\mathbf{C}$ = correlation matrix
$\mathbf{d}$ = measurement vector
E = expectation
$\mathbf{e}$ = stochastic noise
$\mathbf{f}$ = output from reservoir simulator
$\mathbf{H}$ = measurement matrix
i = index (members of ensemble / grid block coordinates)
$\mathbf{I}$ = identity matrix
j = index (state variables / grid block coordinates)
k = index (assimilation time)
$\mathbf{K}$ = Kalman gain matrix
l = correlation length
$\mathbf{L}$ = left factor of covariance matrix of model unceratinty
M = number of states
N = number ensemble members
$\mathbf{P}$ = covariance matrix of model unceratinty
$\mathbf{Q}$ = covariance matrix of modelling error
$\mathbf{R}$ = covariance matrix of measurement error
$\mathbf{s}$ = state vector
σ = standard deviation
σ^2 = variance

Subscripts

d = dynamic variables
i = index (ensemble members)
j = index (state variables)
k = index (assimilating time)

Superscripts

a = analyzed (aposteriori)
f = forecasted (apriori)
m = model noise
T = matrix transpose
t = true

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Type	Std. dev.
Pressure	1 bar
Oil production	5 %
GOR	5 %
Water cut	5 %

Table 1– Measurement uncertainties used in both the examples.

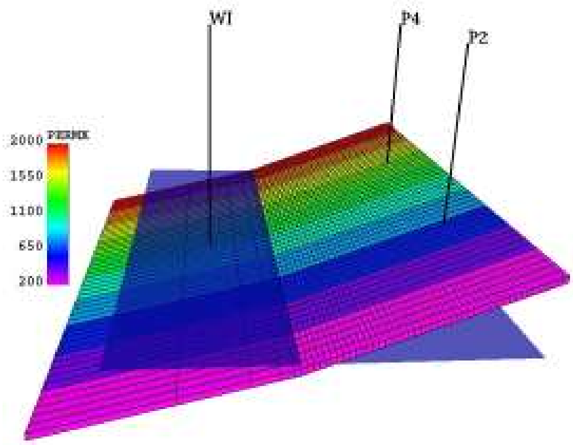

Figure 1– Example 1: Test model with true permeability field and initial oil-water contact.

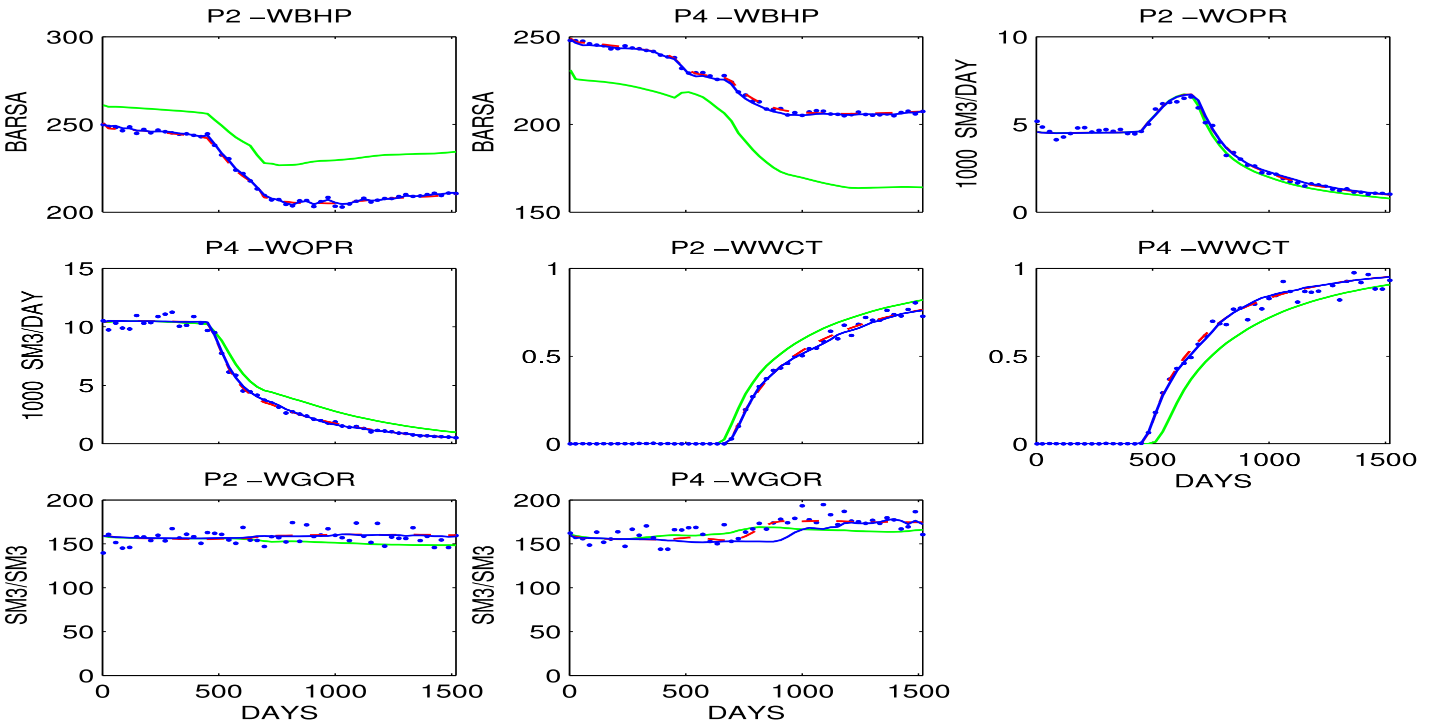

Figure 2– Example 1: Measured values (blue dots), filter solution (blue line), true solution (red stapled) and the reference value (green line) for the measured quantities. The reference value is the solution obtained from the simulation with the mean of the initial ensemble.

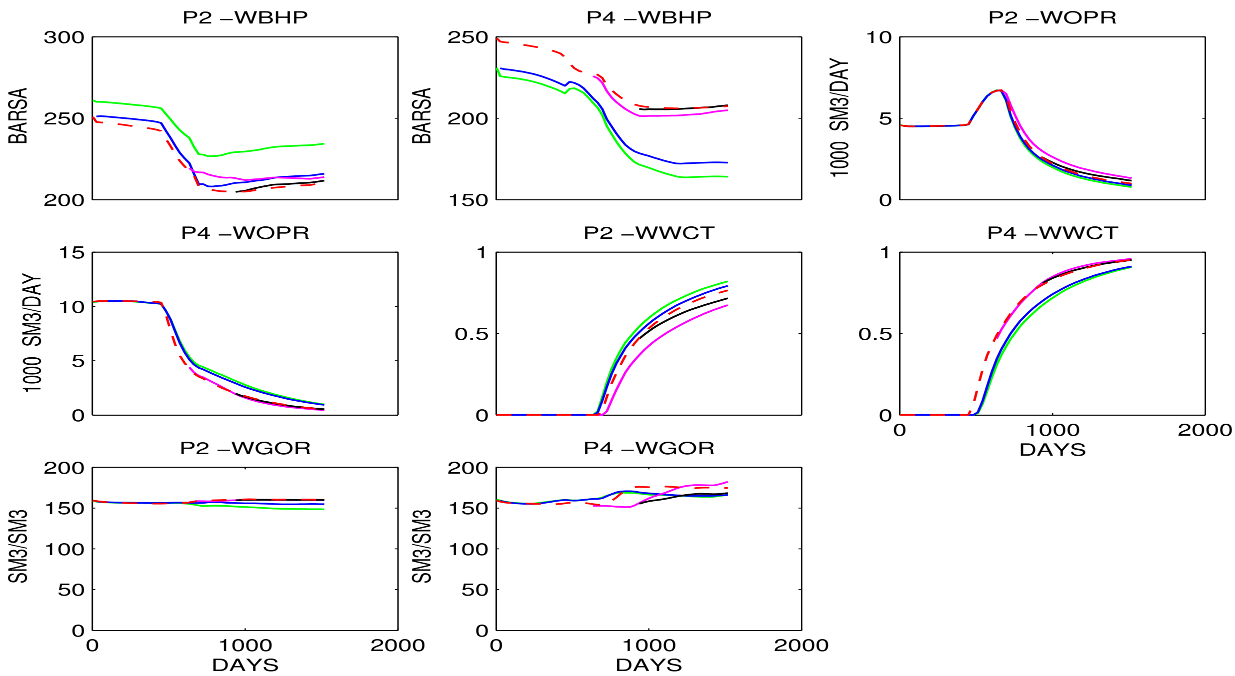

Figure 3– Example 1: Forecasted values. The forecast based on the mean of the initial ensemble (green), the forecast after 4 days (blue), the forecast after 604 days (magneta) and the forecast after 908 days (black) and the true solution (red stapled).

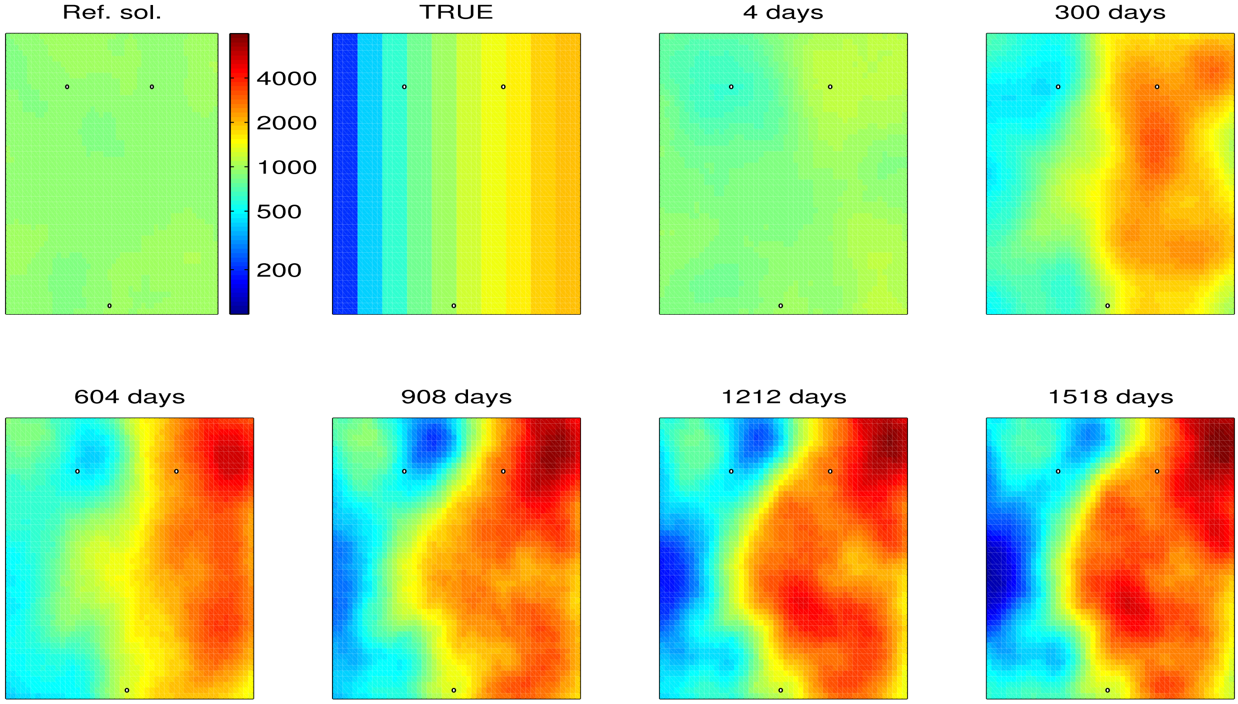

Figure 4– Example 1: True and estimated permeabilities. The wells are located by small circles. The injector, WI, in the bottom, the producer P2 up to the left and the producer P4 up to the right.The upper left plot shows the reference permeability which is the mean of the initial ensemble.

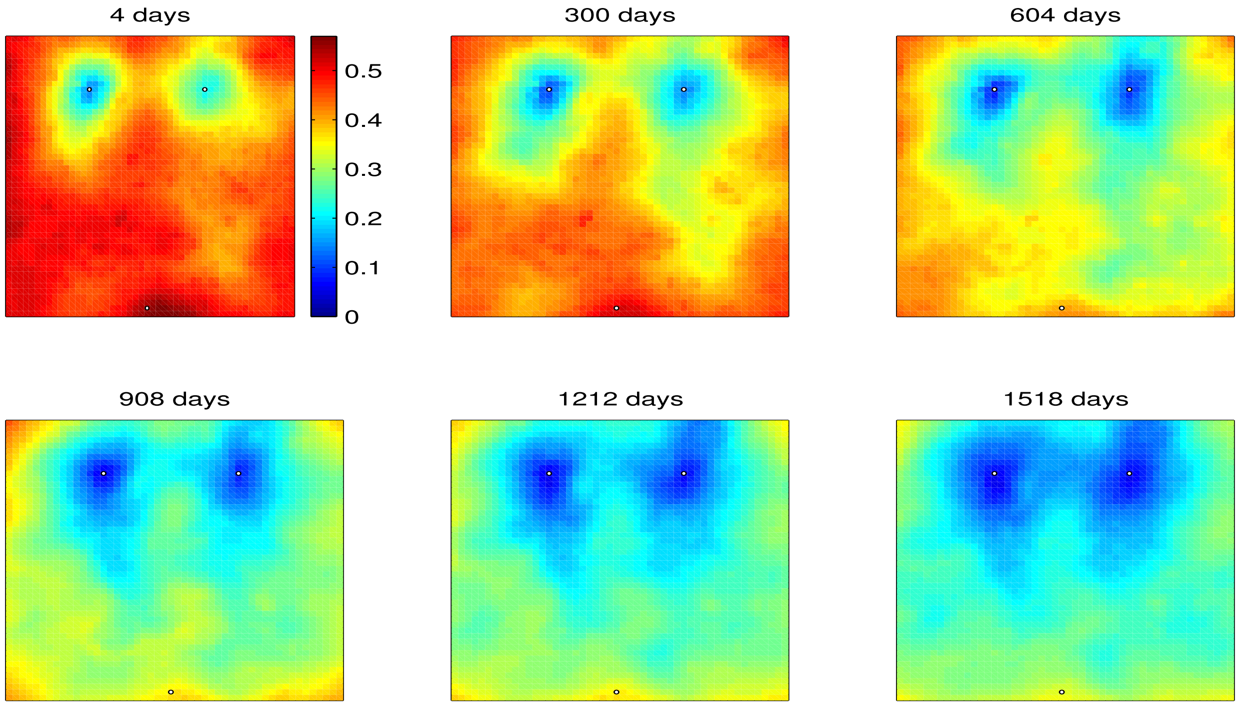

Figure 5– Example 1: Uncertainty in estimated ln-permeabilities

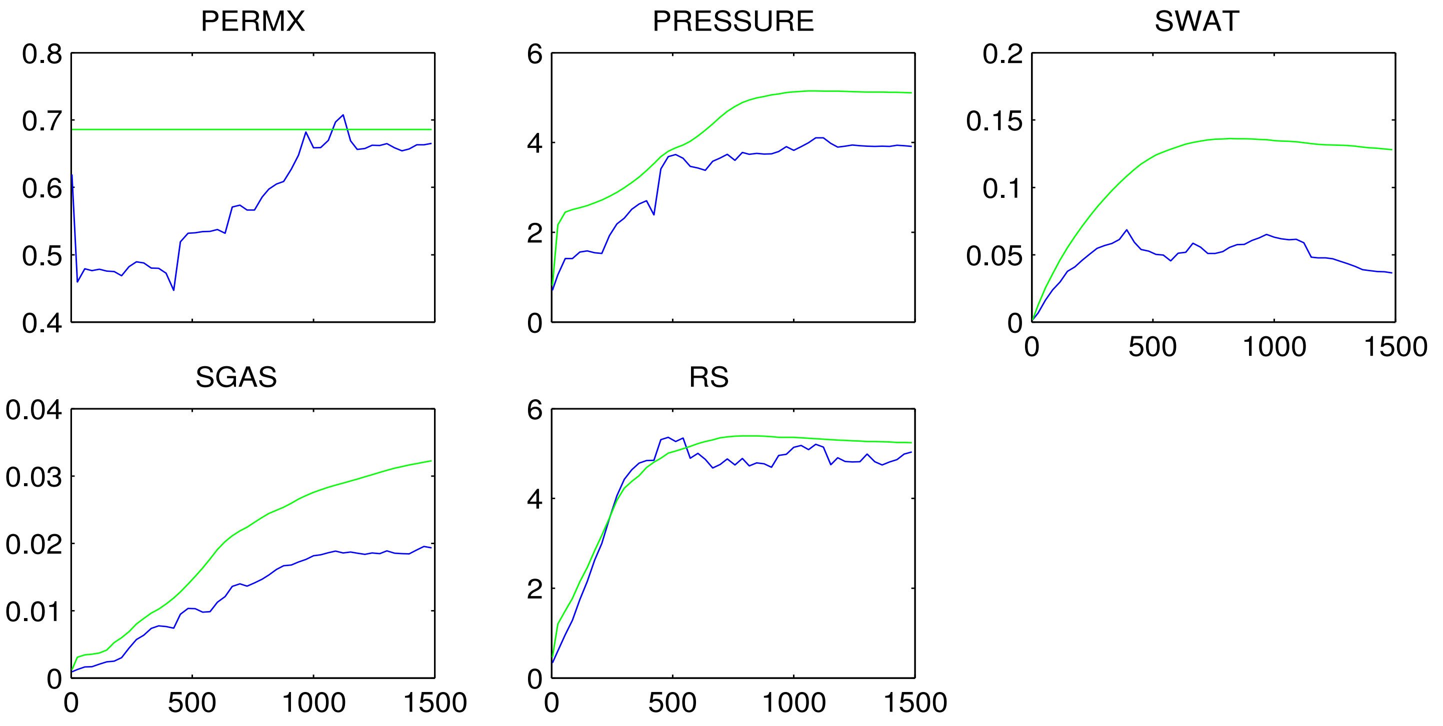

Figure 6– Example 1: RMS error between estimated and true variables (blue) and between the reference solution and the true variables (green). PERMX is the ln-permeability.

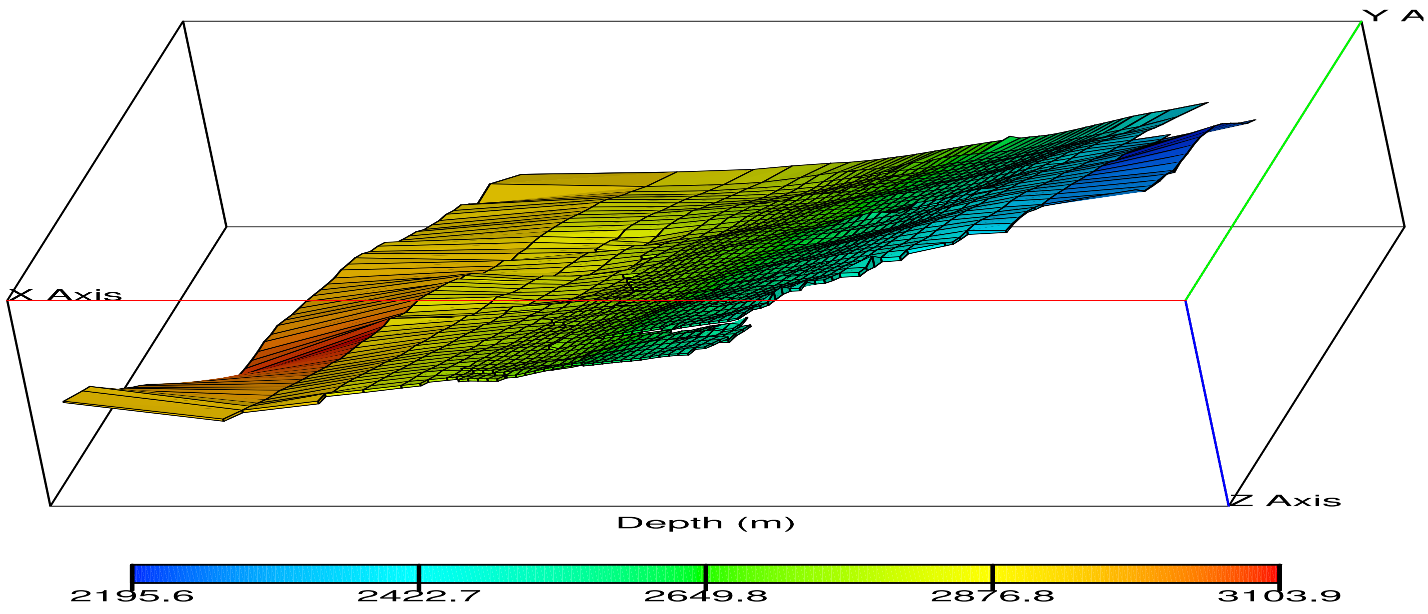

Figure 7– Example 2: Description of reservoir.

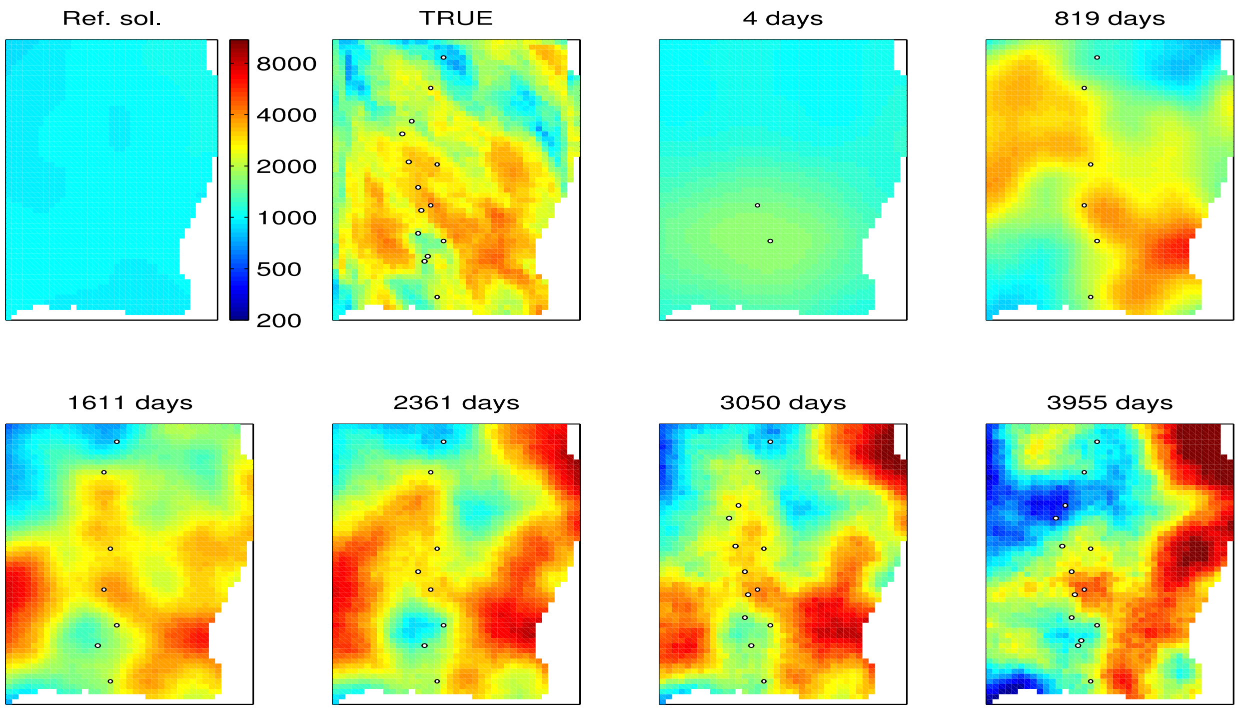

Figure 8– Example 2: True and estimated permeabilities. The reference solution is the mean of the initial ensemble (before assimilating any measurements). The location of the producers where the measurements are obtained are shown by circles. In the plot of the estimated permeabilities, only the wells that have been producing prior to the time of obtaining the estimate are included.

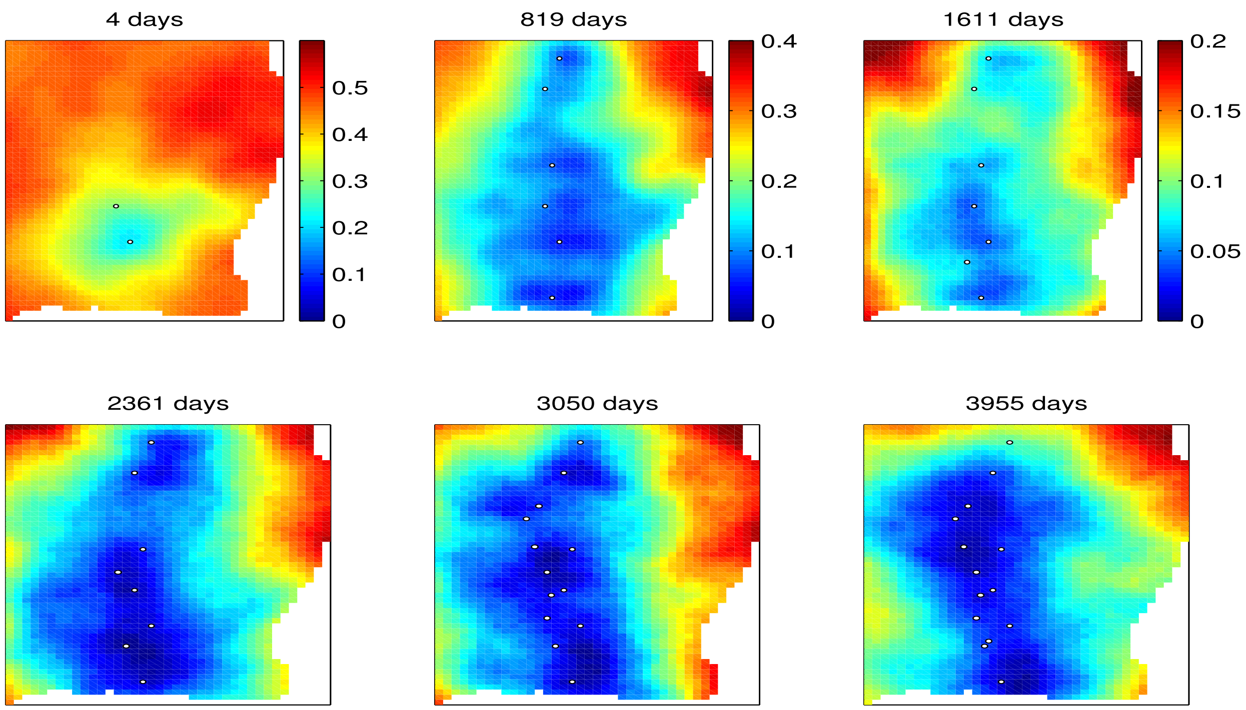

Figure 9– Example 2: Uncertainty in estimated ln-permeabilities. The plots in the bottom row use the same color as the upper right plot.

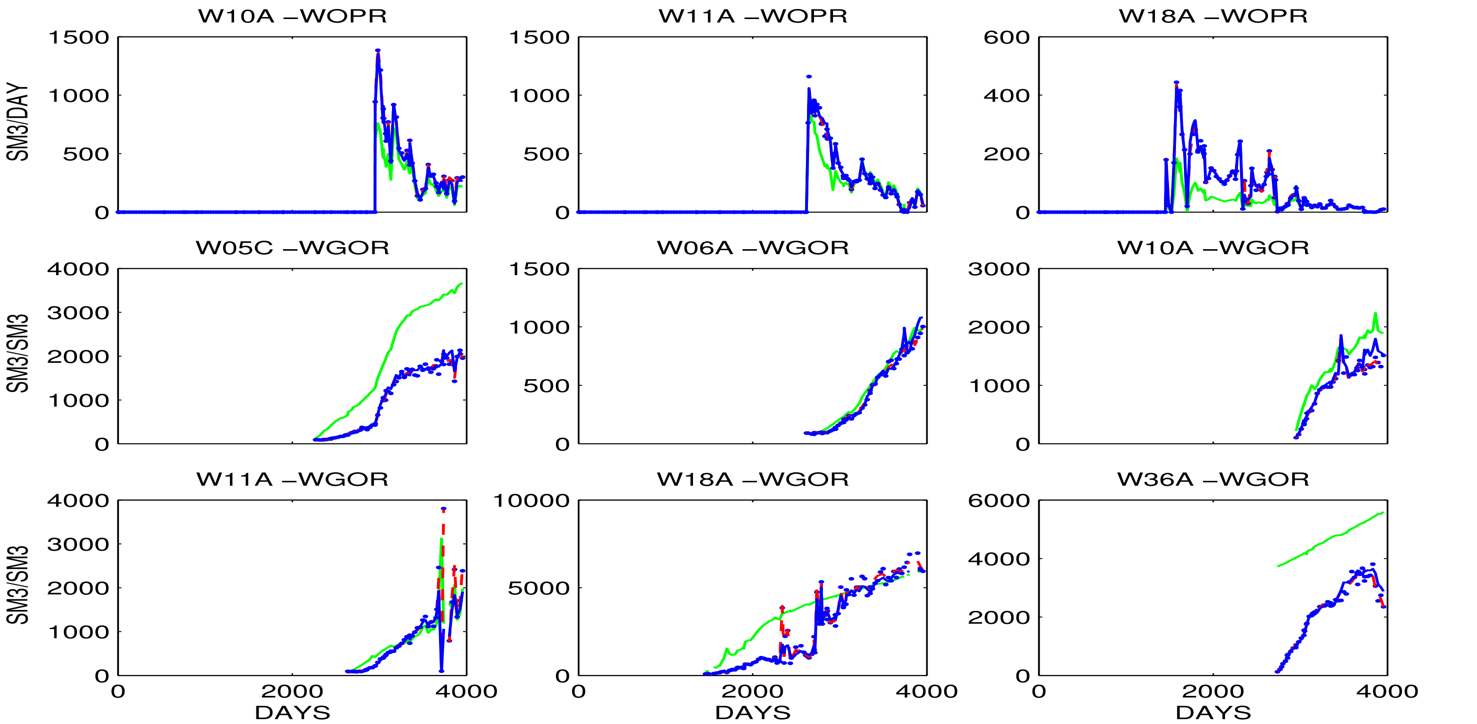

Figure 10– Example 2: Measured values (blue dots), filter solution (blue line), true solution (red stapled) and the reference solution (green) for some measured quantities. The true solution is mostly hidden behind the filter solution.

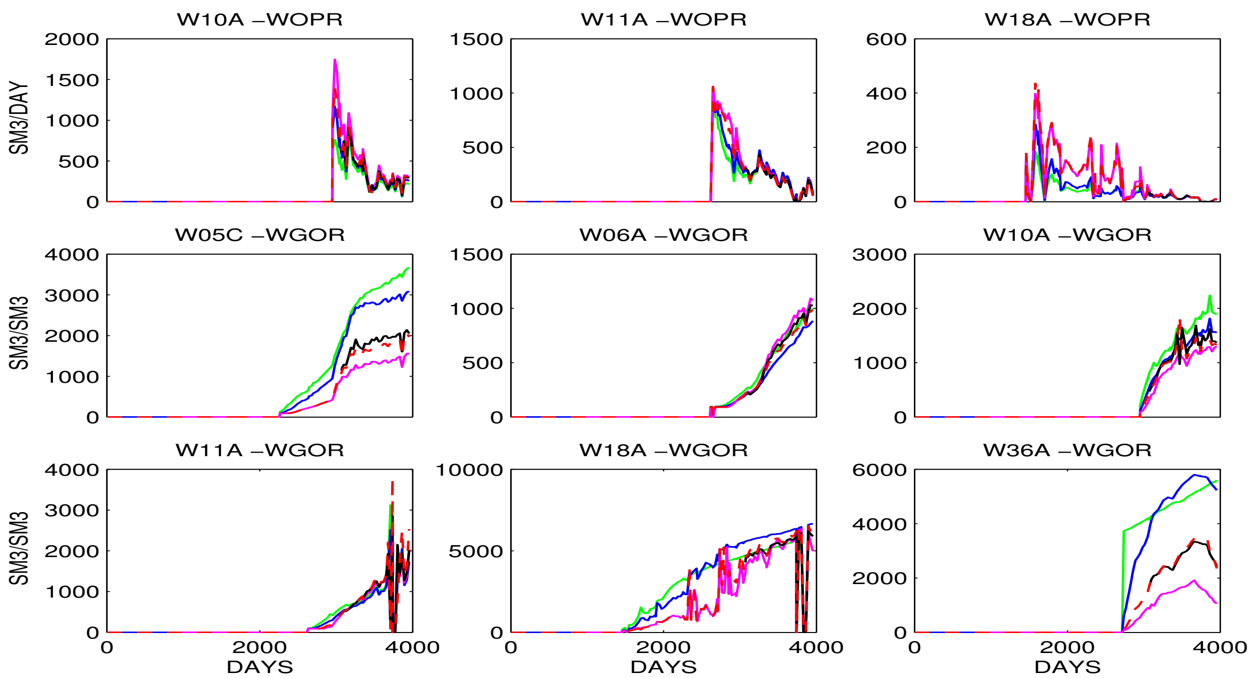

Figure 11– Example 2: Forecasted values for some of the measured quantities. The forecast based on the mean of the initial ensemble (green), the forecast after 4 days (blue), after 819 days (magneta), after 3050 days (black) and the true solution (red stapled).

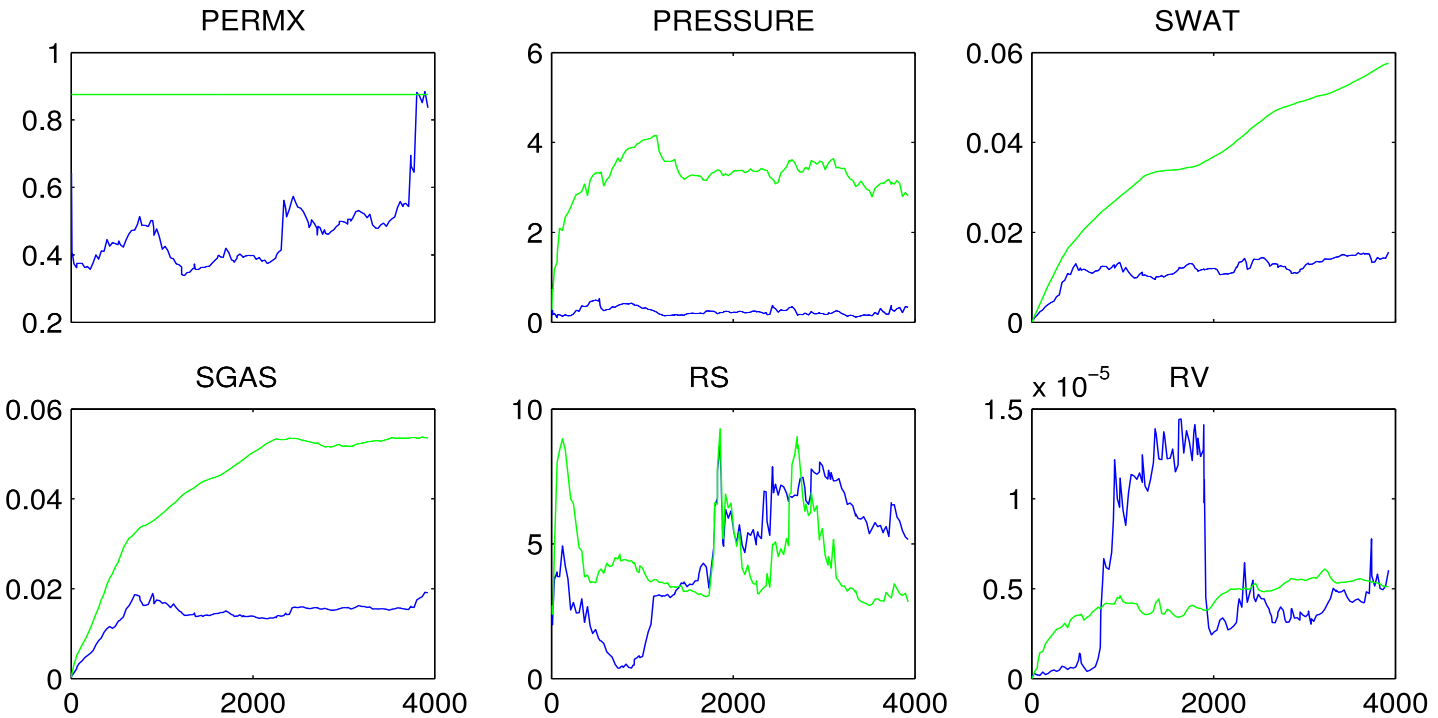

Figure 12– Example 2: RMS error between estimated and true variables (blue) and between the reference solution and the true variables (green). PERMX is the ln-permeability.