SUBJECTIVISM, OBJECTIVISM AND OBJECTIVITY IN BRUNO DE FINETTI'S BAYESIANISM

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The paper will focus on Bruno de Finetti's position, which combines Bayesianism with a strictly subjective interpretation of probability. For de Finetti, probability is always subjective and expresses the degree of belief of the evaluating subject. His perspective does not accommodate a notion of "objective chance" in the way other subjectivists, including Frank Ramsey, do. To de Finetti's eyes, objectivism, namely the idea that probability depends entirely on some aspects of reality, is a distortion, and the same holds for the idea that there exists an absolute notion of objectivity, to be grounded on objective facts. For him there is no problem of objectivity beyond that of the evaluation of probabilities in a Bayesian framework. This is a complex procedure, which includes subjective elements as well as the consideration of objective elements like observed frequencies.

1. De Finetti's subjectivism.

Bruno de Finetti used to call his perspective *"subjective Bayesianism"* (de Finetti 1969, p. 3), to stress that in his conception Bayes' scheme is assigned a central role, and that it goes hand in hand with a subjective view of probability. Inspired by what we would today call a radically "anti-realist" philosophy, de Finetti finds in the Bayesian approach a way of combining empiricism and pragmatism. The resulting position is not only incompatible with any perspective based on an objective notion of probability, neither can it be assimilated to other subjective views of probability. While being opposed both to frequentism and logicism, taken as "objective" views of probability, de Finetti's perspective strays from Ramsey's subjectivism in important respects.

De Finetti entrusted his philosophy of probability, called "probabilism", to the paper *Probabilismo* [1], which he regarded as his philosophical manifesto. Its starting point is a refusal of the

[1]The paper was written in 1929 and published in 1931: see de Finetti 1931b.

notion of truth, and the related notions of determinism and *"immutable and necessary"* laws. In their place, de Finetti reaffirms a conception of science seen as a human activity, a product of thought, having as its main tool probability. *"...no science* - says de Finetti - *will permit us say: this fact will come about, it will be thus and so because it follows from a certain law, and that law is an absolute truth. Still less will it lead us to conclude skeptically: the absolute truth does not exist, and so this fact might or might not come about, it may go like this or in a totally different way, I know nothing about it. What we can say is this: I foresee that such a fact will come about, and that it will happen in such and such a way, because past experience and its scientific elaboration by human thought make this forecast seem reasonable to me"* (de Finetti 1931b, p. 170 of the English edition). Probability is precisely what makes a forecast possible. And since a forecast is always referred to a subject, being the product of his experience and convictions, *"the logical instrument that we need is the subjective theory of probability"*. In other words, probabilism represents for de Finetti an escape from the antithesis between absolutism and skepticism, and at its core one finds the subjective notion of probability.

Following the subjectivist approach, probability *"means degree of belief (as actually held by someone, on the ground of his whole knowledge, experience, information) regarding the truth of a sentence, or event E (a fully specified 'single' event or sentence, whose truth or falsity is, for whatever reason, unknown to the person)" (*de Finetti 1968, p.45). According to de Finetti, one can show not only that this notion of probability is the only non contradictory one, but also that it covers all uses of probability in science and everyday life. This program is realized in two steps: firstly, an operational definition of probability is worked out, secondly, it is argued that the notion of objective probability is reducible to that of subjective probability.

The operational definition moves along well known lines: probability is defined in terms of betting quotients, namely the degree of probability assigned by an individual to a certain event is identified with the betting quotient at which he *would be* ready to bet a certain sum on its occurrence. The fundamental and unique criterion one must obey to avoid sure losses is that of coherence. The individual in question should be thought of as one in a condition to bet whatever sum against any gambler whatsoever, free to choose the betting conditions, like someone holding the bank at a

gambling-casino. Probability can be defined as the fair betting quotient he would attach to his bets. Coherence is a sufficient condition for the fairness of a betting system, and a behaviour conforming to coherence satisfies the principles of probability calculus, which can be derived from the notion of coherence defined in the specified way. This result was certainly grasped by Ramsey, but is fully worked out only by de Finetti in *Sul significato soggettivo della probabilità* (de Finetti 1931a).

Here de Finetti, in addition to the quantitative introduces a qualitative definition of subjective probability, based on the relation of "at least as probable as". He then argues that it is not essential to adopt a quantitative notion of probability expressed by a real number; the latter is the most common way of talking about probability, and also the simplest one, but is in no way the only one. This illustrates the role that de Finetti assigns to betting quotients within his theory: they offer an apt device for measuring probability and defining it operationally, but they do not represent an essential component of the notion of probability, which is a primitive notion, expressing *"the psychological sensation of an individual"* (de Finetti 1931a, p. 302).

This point has been overlooked by the literature. The idea that probability can be defined in various ways is a central feature of de Finetti's perspective, where the scheme of bets represents only a convenient device for talking about probability in a way that makes it understandable to the *"man in the street".* Also, in his *Theory of Probability* de Finetti points out that the scheme of bets is just a useful tool, leading to *"simple and useful insights"* (de Finetti 1970a, English edition , p. 180). In addition to the scheme of bets, he adopts another way of measuring probability by means of scoring rules based on penalties, which is shown to be equivalent to the first one. Something more will be said on this method in the following pages. It is worth noting that the autonomous value assigned by the author to the notion of probability marks a difference between his position and that of the other major supporters of subjectivism, namely F.P. Ramsey and L.J. Savage[2]. Unlike these authors, de Finetti does not see probability as strictly connected with utility and claims that probability and utility have *"different 'cogent values': an indisputable value in the case of probability, a rather uncertain value in the case of ...utility"* (de Finetti 1955, p. 7).

[2]See Ramsey 1926 and Savage 1954.

The second part of de Finetti's program amounts to the reduction of objective to subjective probability. This is done by means of the so-called "representation theorem", which was obtained by de Finetti already in 1928, though its best known formulation is contained in *La prévision: ses lois logiques, ses sources subjectives* (de Finetti 1937). This result is crucial, because it gives applicability to subjective probability by bridging degrees of belief and observed frequencies. The fundamental notion here is that of "exchangeability", which can be defined as follows: events belonging to a sequence are *exchangeable* if the probability of h successes in n events is the same, for whatever permutation of the n events, and for every n and $h \leq n$. The representation theorem says that the probability of exchangeable events can be represented as follows: imagine the events were probabilistically independent, with a common probability of occurrence p. Then the probability of a sequence with h occurrences in n would be $p^h (1 - p)^{n-h}$. But if the events are only exchangeable, the sequence has a probability $\omega^{(n)}_h$, representable according to de Finetti's representation theorem as a *mixture* over the $p^h (1 - p)^{n-h}$ with varying values of p:

$$\omega^{(n)}_h = \int p^h (1 - p)^{n-h} \, f(p) \, dp.$$

Here $f(p)$ is a uniquely defined density for the variable p, or in other words, it gives the weights $f(p)$ for the various values $p^h (1 - p)^{n-h}$ in the above mixture.

In order to understand de Finetti's position, it is useful to start by considering how an objectivist would proceed when assessing the probability of an unknown event. An objectivist would assume an objective success probability p. But its value would in general remain unknown. One could give weights to the possible values of p, and determine the weighted average. The same applies to the probability of a sequence with h successes in n independent repetitions. Note that because of independence it does not matter where the successes appear. De Finetti focuses on the latter, calling *exchangeable* those sequences where the places of successes don't make a difference in probability. These need not be independent sequences. An objectivist who wanted to explain subjective probability, would say that the weighted averages are precisely the subjective probabilities. But de Finetti proceeds in the opposite direction, with his representation theorem. It says in his interpretation: *starting* from the subjective judgment of exchangeability, one can show that there is only one way of giving weights to the possible

values of the unknown objective probabilities. According to this interpretation, objective probabilities become useless and subjective probability can do the whole job.

In the course of a comment on the notion of exchangeability, de Finetti reaffirms that the latter represents the correct way of expressing the idea that is usually conveyed by the phrase *"independent events with constant but unknown probability"*. If we take an urn of unknown composition, says de Finetti, the above phrase means that, relative to each of all possible compositions of the urn, the events can be seen as independent with constant probability. Then he points out that *"...what is unknown is the composition of the urn, not the probability: the latter is always known and depends on the subjective opinion about the composition, which opinion is modified as new drawings are made, and observed frequencies are taken into account" (*de Finetti 1995, p. 214). It should not pass unnoticed that for de Finetti subjective probability, being the expression of the feelings of the subjects evaluating it, is always definite and known: *"Probability as degree of belief is surely known by anyone"* (de Finetti1973, p. 356).

An example, taken from the article *Logical Foundations and Measurement of Subjective Probability* illustrates in what sense *"the concept of unknown probability... must be seen as fictitious and misleading"* (de Finetti 1970b, p. 144). The example compares the processes of Bayes-Laplace and Pólya:

> *"It is well-known that the processes of Bayes-Laplace and Pólya are identical as probabilistic models although very different in the way they are produced. Bayes-Laplace model is a Bernoulli process: in the drawing of balls (with replacement) from an urn containing white and black balls in an unknown proportion, the probability distribution of this proportion is uniform over the interval (0, 1). A Pólya process (contagious probabilities) consists in drawing balls from an urn containing, in the beginning, two balls, one white and one black, and where after each draw, not only is the ball drawn replaced, but also another one of the same color is added. After N = W + B drawings (W = number of white, B = number of black) there are N + 2 balls (W +1) white and (B +1) black; the probability of the next trial is (W + 1)/ (N + 2). But, surprisingly enough, this is the same that happens in the Bayes-Laplace model: that is the*

famous Laplace succession rule. What is the lesson? In the Bayes-Laplace version it is correct to call 'unknown probability' the 'unknown proportion' (which has a real existence). The wording would be: 'the probability of each trial conditional to the knowledge of the unknown proportion and given the fact that my subjective opinion agrees with the standard assumption that the drawings are stochastically independent and that all the balls have equal probability'. In the Pólya version it is formally possible to think of a fictitious urn of Bayes-Laplace type existing in some supposed world of Platonic ideas ... But that, outside Platonism, is obviously a pointless fiction. In conclusion, the recourse to concepts like 'objective unknown probability' in a problem is neither justified nor useful for intrinsic reasons. It may correspond to something realistic under particular factual features, not of a probabilistic model, but of a specific device" (ibid.).

From a philosophical point of view, the reduction of objective to subjective probability is to be seen in a pragmatic perspective. It is performed in the same pragmatic spirit that inspires the operational definition of subjective probability in terms of coherent betting quotients, and complements the latter. If such a reduction is based on consideration of the role played by objective probability in statistical reasoning, it is again the role played by subjective probability in life and science that gives an operational basis for its definition. *"Probability* - says de Finetti - *is actually already defined implicitly by the role played, with respect to the decisional criterion of an individual, by the fact that he evaluates it in a certain way"* (de Finetti 1963, p. 66).

The representation theorem does not serve only the purpose of reducing objective to subjective probability; it also shows how subjective probability can be applied to statistical inference. In this connection the representation theorem plays a vital role within subjectivism, a role whose importance can hardly be overrated. According to de Finetti, statistical inference can be entirely performed by exchangeability in combination with Bayes' rule. If the notion of probability as degree of belief is grounded in an operational definition, probabilistic inference - taken in a subjective sense - is grounded in Bayes' theorem. Therefore, de Finetti's probabilism is intrinsically Bayesian; one could say that for him Bayesianism represents the crossroads where pragmatism and empiricism meet

subjectivism. He thinks that one needs to be Bayesian in order to be a subjectivist, but on the other hand subjectivism is a choice to be made if one embraces a pragmatist and empiricist philosophy.

As reflected by the article *Initial Probabilities: A Prerequisite for any Valid Induction* the shift from prior (or initial) to posterior (or final) probabilities, is considered by de Finetti the cornerstone of statistical inference[3]. In this connection he takes a *"radical approach"* by which *"all the assumptions of an inference ought to be interpreted as an overall assignment of initial probabilities"* (de Finetti 1969, p. 9). Though this shift is given a subjective interpretation, in the sense that going from prior to posterior assessments involves a shift from one subjective probability to another, it also involves consideration of objective factors.

Before we face this issue, it is worth noting that for de Finetti updating one's mind in view of new evidence does not mean changing opinion: *"If we reason according to Bayes' theorem we do not change opinion. We keep the same opinion and we update it to the new situation. If yesterday I said 'Today is Wednesday', today I say 'It is Thursday'. Yet I have not changed my mind, for the day following Wednesday is indeed Thursday"* (de Finetti 1995, p. 100). If the idea of *correcting* previous opinions is completely alien to this perspective, so is the notion of self-correcting procedure, which occupies a central place within the perspective of other authors, such as Hans Reichenbach[4]. De Finetti's attitude is grounded in the conviction that there are no "correct" and "rational" probability assignments: *"The subjective theory...* - he says - *does not contend that the opinions about probability are uniquely determined and justifiable. Probability does not correspond to a self- proclaimed 'rational' belief, but to the effective personal belief of anyone"* (de Finetti 1951, p. 218). Incidentally, we might notice that his attitude in this connection marks a sharp difference from the logicism of Rudolf Carnap and Harold Jeffreys[5], who believe that there are "correct" probability evaluations. In this sense, logicism attributes to probability theory a normative aspect which is absent from subjectivism.

[3]On the problem of the choice of initial probabilities de Finetti wrote a joint paper with Savage: see de Finetti and Savage 1962.

[4]See Reichenbach 1949.

[5]See Carnap 1951 and Jeffreys 1931and 1939.

2. Objectivism and objectivity

De Finetti's subjective Bayesianism is intransigent, even dogmatic. Not only is subjective Bayesianism the sole acceptable way of addressing probabilistic inference and the whole of statistical methodology, but it makes any form of "objectivism" look silly. In de Finetti's words:

> *"The whole of subjective statistics is based on this simple theorem of probability calculus [Bayes' theorem]. Consequently, subjective statistics has a very simple and general foundation. Moreover, being grounded only on the basic axioms of probability, subjective statistics does not depend on those definitions of probability that would narrow its range of application (like, for instance, the definitions based on the idea of equally probable events). Nor - once one endorses this view - is there any need to resort to empirical formulae, in order to characterize inductive reasoning. Objectivist statisticians, on the contrary, make extensive use of empirical formulae. The need to do so stems only from their refusal to admit the use of initial probability P(E). They reject the use of initial probability because they reject the idea of a probability that depends on the state of information. However, by doing so they distort everything: not only do they make probability an objective entity...they even make it a theological entity: they claim that 'true' probability exists, outside us, independently of a person's judgment"* (de Finetti 1995, p. 99).

This passage highlights a main feature of de Finetti's position, namely his refusal of objective probability, which is deemed not only useless, but even meaningless, like all metaphysical notions. Throughout his life, de Finetti held that *"probability does not exist"*. This claim, which appears in capital letters in the Preface to the English edition of his *Theory of Probability*, is the *leit-motiv* of his production. *"Objective probability never exists*" he says in *Il significato soggettivo della probabilità* (de Finetti 1931a), and almost fifty years later he opens the article *Probabilità* in the *Einaudi Encyclopedia* with the words: *"is it true that probability 'exists'? What could it be? I would say no, it does not exist"* (de Finetti 1980, p. 1146) . Such aversion to "objective" probability is inspired by the desire to keep

probability free from metaphysical "contaminations"[6].

De Finetti's refusal to attach an "objective" meaning to probability ends with a denial of the notions of "chance" and "physical probability". No doubt, the lack of consideration for the notions of "chance" and "physical probability" represents a limitation of de Finetti's perspective[7]. Spurred by his anti-realism, de Finetti never paid much attention to the use made of probability in science, in the conviction that science is just a continuation of everyday life and subjective probability is all that is needed. Only the volume *Filosofia della probabilità*, containing the text of a course given by de Finetti in 1979, includes a few remarks to the effect that probability distributions belonging to statistical mechanics can be taken as more solid grounds for subjective opinions (de Finetti 1995, p. 117). These remarks suggest that late in his life de Finetti might have entertained the idea that when probability assignments are strictly related to scientific theories, they acquire a special meaning.

The road to a more flexible form of subjectivism, which can accommodate these concepts, has been paved by the other "father" of modern subjectivism, Frank Ramsey. He defines "chance" and "probability in physics" in terms of "systems of beliefs" making reference to theories accepted by the scientific community. Ramsey thought that the probabilities we encounter in physics are derived from physical theories. Their objective character descends from the objectivity ascribed to theories that are commonly accepted as true. Within Ramsey's perspective, this idea is combined with a pragmatic approach to theories and truth that would have been quite congenial to de Finetti, had he been acquainted with it[8]. In fact, his remarks contained in *Filosofia della probabilità* lean in the same direction. But de Finetti did not grasp the insights of Ramsey's philosophy, though he knew about his subjective definition of probability, to which the French probabilist Maurice Fréchet called his attention around 1937. In the Cambridge of the Twenties another Bayesian often praised by de Finetti, Harold Jeffreys, put forward the idea that one can make sense of physical probability in an epistemic

[6]See Galavotti 1989 for an exposition of the anti-metaphysical and anti-realist basis of de Finetti's subjectivism.

[7]This is argued in Galavotti 1995-96 and 1997.

[8]For a comparison between the philosophy of probability of Ramsey and de Finetti see Galavotti 1991. For Ramsey's notion of chance see Galavotti 1995 and 1999.

framework, holding a position akin to that of Ramsey[9]. To be sure, Jeffreys was a logicist more than a subjectivist. More recently, however, the idea that subjectivism should be flexible enough to accommodate for a notion of physical probability has been heralded by statisticians as well as philosophers, as testified, for instance, by the work of I.J. Good and R.C. Jeffrey[10].

Having refused the notion of "objective" probability and denied that there are "correct" probability assignments, the radical subjectivist de Finetti still faces the problem of objectivity of probability evaluations. Let us examine his position on this issue. His point of departure is the conviction that the process through which probability judgments are obtained is more complex than is supposed by the other interpretations of probability, which define probability on the basis of a unique criterion. While subjectivists distinguish between the definition and the evaluation of probability, and do not mix them up, upholders of the other interpretations confuse them: they look for a unique criterion - be it frequency, or symmetry - and use it as grounds for both the definition and the evaluation of probability. In so doing, they embrace a "rigid" attitude towards probability, an attitude which consists *"in defining (in whatever way, according to whatever conception) the probability of an event, and in univocally determining a function"* (de Finetti 1933, p. 740). On the contrary, subjectivists adopt an "elastic" approach, which *"consists in demonstrating that all functions f have all the necessary and sufficient properties to represent probability evaluations (also in this case, defined according to whatever conception, in whatever way) which are not intrinsically contradictory, leaving to a second (extra-mathematical) stage the discussion and analysis of reasons and criteria for the choice of a particular among all possible ones"* (ibid., p. 741). In other words, for the subjectivist all coherent functions are admissible; far from being committed to a single rule or method, the choice of one particular function is seen as the result of a complex and largely context-dependent procedure, which necessarily involves subjective elements.

The explicit recognition of the role played by subjective elements within the complex process of the formation of probability judgments is for de Finetti a prerequisite for the appraisal of objective

[9] See Jeffreys 1955.

[10] See Good 1965 and Jeffrey 1997.

elements: *"Subjective elements* - he says - *will noways destroy the objective elements nor put them aside, but bring forth the implications that originate only after the conjunction of both objective and subjective elements at our disposal"* (de Finetti 1973, p. 366). To be sure, Bayesian subjectivism requires that objective elements also be taken into account, but such objective elements are not seen as the only basis for judgment. *"Subjectivism* - de Finetti says - *is one's degree of belief in an outcome, based on an evaluation making the best use of all the information available to him and his own skill...Subjectivists...believe that every evaluation of probability is based on available information, including objective data"* (de Finetti 1974b, p. 16). In conclusion, *"Every probability evaluation essentially depends on two components: (1) the objective component, consisting of the evidence of known data and facts; and (2) the subjective component, consisting of the opinion concerning unknown facts based on known evidence"* (de Finetti 1974a, p. 7).

De Finetti warns that the objective component of probability judgments, namely factual evidence, is in many ways context-dependent: evidence must be collected carefully and skillfully, its exploitation depends on the judgment on what elements are relevant to the problem under consideration, and can be useful to the evaluation of related probabilities. In addition, the collection and exploitation of evidence depends on economic considerations varying in practical cases. So, one can say that the collection and exploitation of factual evidence involves subjective elements of various sorts. Equally subjective is the decision on how to let objective elements influence belief. Typically, one relies on information regarding frequencies. For de Finetti frequencies, like symmetry considerations, are useful and important ingredients of probability evaluations, *provided that* they are not used uncritically as automatic rules and simply equated with probability. Those who do so, namely frequentists, are simply committed to "superstition":

> *"There is no worse conceptual distortion than that owing to which, starting from the premise that any sequence can occur, one defines probability in terms of a property (that of exhibiting a certain frequency) pertaining only to a portion of all sequences... when we define probability in terms of frequency, we define it thoughtlessly. The only objective thing is the set of all possible sequences, but it does not say anything concerning their probability. The probability of*

> *sequences can only be the feeling we had before and which characterized our expectation. Here we have a perversion of language, logic and common sense. Such a logical mistake is unacceptable, because the set of all possible sequences (which is logically determined) cannot be confused with probability (which, on the contrary, is subjective)"* (de Finetti 1995, pp. 140-141).

Keeping in mind the distinction between the definition and the evaluation of probability, one can make good use of frequencies within probability evaluations. It is precisely in this connection that exchangeability enters the stage, giving the reason *"why expected future frequencies should be guessed according to past observed frequencies",* and thereby creating a strong connection between *"subjectivistic and objectivistic interpretations"* (De Finetti 1970b, p. 143). As already stressed, de Finetti assigns to exchangeability a subjective interpretation, according to which exchangeability represents a:

> *"directly observable property of the probability evaluation. It means that for every set of n of the events concerned, the probability that all events occur is the same; it depends only on n... Under such a clear subjective condition (and a few side restrictions to avoid special cases, such as that of repeated trials with a known probability that remains unchanged), one is perfectly free to improve the evaluation of probabilities for any future events according to the frequency of the observed ones. This improvement generally entails modifying the initial evaluation... so as to approach gradually the obtained frequency of the events observed up to that time"* (de Finetti 1974a, p. 12).

Therefore exchangeability allows probability judgments to be improved in view of observed frequencies in an empiricist fashion fully in tune with the subjectivist approach.

Since the present contribution is meant to be historically oriented, it is not out of place to make a brief digression on the origin of exchangeability. It is de Finetti's merit to have combined the subjective notion of probability in terms of coherent beliefs with that of exchangeability. In so doing, he was able to guarantee the applicability of subjective probability to practical situations, including those encountered within experimental science. Exchangeability is the missing aspect in Ramsey's

perspective, which could have made him see the link between degrees of belief and observed frequencies. It can be conjectured that by the time of his death Ramsey came very close to recognizing such a link. Evidence that he was intrigued by the relationship between frequency and degree of belief is offered by his note *Miscellaneous Notes on Probability* [11], where he ponders over the idea that *"degree γ of belief means acting appropriately to a frequency γ"*, of which he says that *"it is [the]...one which makes calculus of frequencies applicable to degrees of belief"* (Ramsey 1991, p. 275). The justification of this claim lies precisely with exchangeability, a property that Ramsey knew through his teacher and colleague in Cambridge William Ernest Johnson, a logicist who is seen as a forerunner of Carnap's inductive logic, which assigns a privileged role to the same probabilistic property he calls *"symmetry"* [12]. Ramsey himself made use of this property, named by him *"equiprobability of all permutations"*, in a short note called "Rule of Succession" (in Ramsey 1991), which contains a derivation of Laplace's Rule of Succession from the property in question. However, Ramsey was unable to connect it with degree of belief in the way de Finetti did.

Going back to the evaluation of probabilities, when a considerable amount of information about frequencies is available, it influences probability assignments through the assumption of exchangeability. Often, however, this kind of information is scant, and in this case the problem of how to obtain good probability evaluations is open. The problem is addressed by de Finetti in a number of works, especially starting from the Sixties. The approach adopted is based on penalty methods, like the so called "Brier's rule", named after the meteorologist Brier who applied it to weather forecasts. De Finetti did extensive work on scoring rules, partly in cooperation with Savage. He even tested the goodness of this method through an experiment among his students, who were asked to make forecasts on the results of soccer matches in the Italian championship. A simple description of such methods, referred to the case of three possible results (as with soccer matches) is the following:

> *"everybody participating in...[an experiment on probabilistic forecasts] is asked to indicate the probabilities of (for instance) the three possible results - victory, or draw, or defeat - of the*

[11]The note was written in 1928 and appears in Ramsey 1991.

[12]On this point see Zabell 1988.

home-team in a specified football match; say, e.g. 50% - 30% - 20%. A scoring rule indicates how much the participant is to be penalised depending on the 'distance' between the assessed probabilities and the actual result. The simplest and most practical scoring rule is Brier's rule; if someone indicates as his own opinion P(E) = p, the score (i.e. the penalisation) is the square of the distance between forecast and result: $p^2 (= (p - 0)^2)$ if the result is 0 (E does not happen), and $(1 - p)^2$ if the result is 1 (E does happen). The fact that Brier's rule is a proper one is proved since for a person indicating as his probability assessment a $\bar{p}$ different from his own effective opinion p, expected penalisation is increased by $(p - p)^2$. Analogously, when the possible results are three (as in football) indicating them with the vertices of an equilateral triangle, and the forecast with the centre of gravity of weights indicating the probabilities of each vertex, a proper scoring rule is $(P - P_0)^2$ (square of the distance between probability assessment and effectual result" (de Finetti 1981, p. 55).

Scoring rules of this kind are based on the idea that the device in question should oblige those who make probability evaluations to be as accurate as they can and, if they have to compete with others, to be honest. In fact any deviation from the true *p*, on the part of the person who evaluates probability, enables the opponent to engage him in a disadvantageous bet. Such rules play a twofold role within de Finetti's approach. In the first place, they offer a suitable tool for an operational definition of probability. As recollected, in his late works de Finetti adopted such a device to define subjective probability. In addition, these rules offer a method for improving probability evaluations made both by a single person and by several people, because they can be employed as methods for exercising *"self-control"*, as well as a *"comparative control"* over probability evaluations (de Finetti 1980, p. 1151).

De Finetti assigns these methods, which are quite widespread among Bayesian statisticians, a straightforward interpretation in tune with subjectivism: *"The objectivists, who reject the notion of personal probability because of the lack of verifiable consequences of any evaluation of it, are faced with the question of admitting the value of such a 'measure of success' as an element sufficient to soften their fore-judgments. The subjectivists, who maintain that a probability evaluation, being a measure of someone's beliefs, is not susceptible of being proved or disproved by the facts, are faced*

with the problem of accepting some significance of the same 'measure of success' as a measure of the 'goodness of the evaluation'" (de Finetti 1962a, p. 360). The following remark further clarifies de Finetti's position: *"though maintaining the subjectivist idea that no fact can prove or disprove belief, I find no difficulty in admitting that any form of comparison between probability evaluations (of myself, of other people) and actual events may be an element influencing my further judgment, of the same status as any other kind of information"* (ibid.). De Finetti's work on scoring rules is in tune with a widespread attitude among Bayesians, an attitude that has given rise to a vast literature on *"well-calibrated"* estimation methods.

If these methods provide us with useful devices for improving probability evaluations, a whole series of elements seem to be relevant in this connection. De Finetti mentions an array of such elements, including :

> *"(1) degree of competence or care in forecasts concerning different subject matters, epochs or regions;*
>
> *(2) optimistic or pessimistic attitudes...*
>
> *(3) degree of influence of the most recent facts...*
>
> *(4) degree of deviation from statistical standards, according to specific knowledge of each item...*
>
> *(5) stability or flexibility (evolutionary or oscillating) of opinions without a change in the available information, by thinking about or by the influence of another's opinions...*
>
> *(6) conscious or unconscious adaptation of the opinion to standard patterns of statistical theory and practice...."* (De Finetti 1970b, pp. 141-142).

To sum up, the evaluation of probability is seen as a most complex procedure, resulting from the concurrence of all sorts of factors. Starting from the recognition of the fact that probability is subjective, and that there is no unique, "rational" way of assessing probability, one can make room for a whole array of elements that can influence probability evaluations, suggesting various ways of ameliorating them. De Finetti's remarks in this connection may sound very general, but his warnings against pre-confectioned recipes for the evaluation of probabilities should be taken seriously.

In a paper dealing with issues related to economic theory, de Finetti discusses the *"dangerous mirage"* of *"identifying objectivity and objectivism"* (de Finetti 1962b, p. 344) and exhorts *"to fight against the ambush of pseudo-objectivity which is concealed under the false shield of 'objectivism', boasting of it as if it were a chrism of 'scientificity'"* (ibid., p. 360). Since objectivism is nothing but a conceptual distortion, an absolute idea of objectivity grounded on it can only be a chimera. A more viable notion of objectivity lies with a "deep analysis of problems", aimed at avoiding hasty judgments, superficial intuitions and careless conclusions, to form evaluations which are the best one can attain in the light of the available information. Such a deep analysis of problems will include consideration of objective elements, in the awareness that, taken by themselves, these are neither necessary not sufficient to guarantee objectivity in an absolute sense. This is because absolute objectivity does not exist: "only a honest reflection, careful of facts and other people's ideas can lead to the maximum possible objectivity" (ibid., p. 367).

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