

ASYMPTOTIC ANALYSIS OF DOWNLINK MISO SYSTEMS OVER RICIAN FADING CHANNELS

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ABSTRACT

In this work, we focus on the ergodic sum rate in the downlink of a single-cell large-scale multi-user MIMO system in which the base station employs N antennas to communicate with K single-antenna user equipments. A regularized zero-forcing (RZF) scheme is used for precoding under the assumption that each link forms a homogeneous spatially correlated MIMO Rician fading channel. The analysis is conducted assuming N and K grow large with a non trivial ratio and perfect channel state information is available at the base station. Recent results from random matrix theory and large system analysis are used to compute an asymptotic expression of the signal-to-interference-plus-noise ratio as a function of the system parameters, the spatial correlation matrix and the Rician factor. Numerical results are used to evaluate the performance gap in the finite system regime under different operating conditions.

1. INTRODUCTION

Large-scale multiple-input multiple-output (MIMO) systems (also known as massive MIMO systems) are considered as one of the most promising technology for next generation wireless communication systems [1–3] because of their considerable spatial multiplexing gains. The use of large-scale MIMO systems is beneficial not only in terms of communication performances (such as better coverage and efficient radio resource utilization) but also in terms of energy-saving. In this complex system model, a number of practical factors such as correlation effects and line-of-sight (LOS) components need to be included, which occur due to the space limitation of user equipments (UEs) and the densification of the antenna arrays resulting in a visible propagation path from the UEs, respectively. For typical systems of hundreds of antennas and tens of UEs, even computer simulations become challenging, which makes performance analysis of large-scale MIMO systems an important and a new subject of research.

In this work, we consider the downlink of a single-cell large-scale MIMO system in which the base station (BS), equipped with N antennas, makes use of regularized zero-forcing (RZF) precoding to communicate with K single-antenna UEs. In particular, we are interested in evaluating the ergodic sum rate of the system when a power constraint is imposed at the BS. The analysis is conducted assuming that N and K grow large with a non trivial ratio under the assumption that perfect channel state information is available at the BS. Differently from most of the existing literature [4–8], we consider

a spatially correlated MIMO Rician fading model, which is more general and accurate to capture the fading variations when there is a LOS component. Compared to the Rayleigh fading channel, a Rician model makes the asymptotic analysis of large-scale MIMO systems much more involved. To overcome this issue, recent results from random matrix theory and large system analysis [8–10] are used to compute an asymptotic expression of the signal-to-interference-plus-noise ratio (SINR), which is eventually used to approximate the ergodic sum rate of the system. As shall be seen, the results are found to depend only on the system parameters, the spatial correlation matrix and the Rician factor. As a notable outcome of this work, the above analysis provides an analytical framework that can be used to evaluate the performance of the network under different settings without resorting to heavy Monte Carlo simulations and to eventually get insights on how the different parameters affect the performance.

The main literature related to this work is represented by [4, 7, 11–13]. Tools from random matrix theory are used in [4] to compute the ergodic sum rate in a single-cell setting with Rayleigh fading and different precoding schemes while the multicell case is analyzed in [7]. In [11], the authors investigate a LOS-based conjugate beamforming transmission scheme and derive some expressions of the statistical SINR under the assumption that N grows large and K maintain fixed. In [12], the authors study the fluctuations of the mutual information in a small cell network of a Rician fading channel under the form of a central limit theorem and provide explicit expression of the asymptotic variance. In [13], a deterministic equivalent of the ergodic sum rate and an algorithm for evaluating the capacity achieving input covariance matrices for the uplink of a large-scale MIMO are proposed for spatially correlated MIMO channel with LOS components. The analysis of the uplink rate with both zero-forcing and maximum ratio combining receivers is performed in [14].

The following notation is used throughout this work. The superscript H stands for the conjugate transpose operation. The operators $\text{Tr}(\mathbf{X})$ and $\|\mathbf{X}\|$ denote trace and spectral norm of matrix \mathbf{X} , respectively. The $N \times N$ identity matrix is denoted by \mathbf{I}_N . A random variable x is a standard complex Gaussian variable if $\sim \mathcal{CN}(0, 1)$.

2. SYSTEM AND CHANNEL MODEL

We consider the downlink of a network in which K UEs are served by a single BS equipped with $N \geq K$ antennas. The signal y_k received by UE k takes the form [4]

$$y_k = \mathbf{h}_k^H \mathbf{x} + n_k \quad \forall k \in \{1, \dots, K\} \quad (1)$$

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where $\mathbf{h}_k \in \mathbb{C}^N$ is the random channel from the BS to user k , $\mathbf{x} \in \mathbb{C}^N$ is the transmit vector, and $n_k \sim \mathcal{CN}(0, \sigma^2)$ accounts for thermal noise. The transmit vector \mathbf{x} is obtained as a linear combination of the independent user symbols $\{s_k\}$ and can be written as

$$\mathbf{x} = \sum_{k=1}^K \sqrt{p_k} \mathbf{g}_k s_k \quad (2)$$

where $p_k \geq 0$ and $\mathbf{g}_k \in \mathbb{C}^N$ are the signal power and precoding vector of UE k , respectively. For analytic tractability, we assume that the BS is able to acquire perfect channel state information from the uplink pilots. The RZF scheme is used as precoding technology [4]. Therefore, the precoding matrix $\mathbf{G} = [\mathbf{g}_1 \dots \mathbf{g}_K] \in \mathbb{C}^{N \times n}$ takes the form

$$\mathbf{G} = \xi \left(\mathbf{H}\mathbf{H}^H + \lambda \mathbf{I}_N \right)^{-1} \mathbf{H} \quad (3)$$

where $\lambda > 0$ is the so-called regularization parameter and ξ is chosen so as to satisfy the average total power constraint given by $\text{Tr}(\mathbf{P}\mathbf{G}^H\mathbf{G}) = P_T$ where $P_T > 0$ denotes the total available transmit power and $\mathbf{P} = \text{diag}\{p_1, \dots, p_K\}$. Therefore, it follows that

$$\xi^2 = \frac{P_T}{\text{Tr}(\mathbf{P}\mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \lambda \mathbf{I}_N)^{-2}\mathbf{H})}. \quad (4)$$

Under the assumption of Gaussian signaling, i.e., $s_k \sim \mathcal{CN}(0, 1)$ and single user detection with perfect channel state information at the receiver, the SINR γ_k of user k takes the form

$$\gamma_k = \frac{p_k |\mathbf{h}_k^H \mathbf{g}_k|^2}{\sum_{i=1, i \neq k}^K p_i |\mathbf{h}_k^H \mathbf{g}_i|^2 + \sigma^2}. \quad (5)$$

The rate r_k of UE k is given by

$$r_k = \log_2(1 + \gamma_k) \quad (6)$$

whereas the ergodic same rate is defined as

$$r_E = \sum_{k=1}^K \mathbb{E}[\log_2(1 + \gamma_k)] \quad (7)$$

where the expectation is taken over the random channels \mathbf{h}_k . The channel matrix $\mathbf{H} = [\mathbf{h}_1 \dots \mathbf{h}_K] \in \mathbb{C}^{N \times K}$ represents the Rician channel matrix modeling fast fading with a deterministic line of sight path, which is modelled as

$$\mathbf{H} = \sqrt{\frac{1}{1+\rho}} \mathbf{\Theta}^{1/2} \frac{1}{\sqrt{N}} \mathbf{W} + \sqrt{\frac{\rho}{1+\rho}} \mathbf{A} \quad (8)$$

where $\mathbf{A} = [\mathbf{a}_1, \dots, \mathbf{a}_K] \in \mathbb{C}^{N \times K}$ is a deterministic matrix and $\mathbf{W} \in \mathbb{C}^{N \times K}$ accounts for the random fast fading component, which is composed of independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables with zero-mean and unit variance. The scalar $\rho \geq 0$ is the Rician factor¹ denoting the power ratio between \mathbf{A} and \mathbf{W} whereas the matrix $\mathbf{\Theta}^{1/2}$ is obtained from the Cholesky decomposition of $\mathbf{\Theta} \in \mathbb{C}^{K \times N}$, which accounts for the channel correlation matrix at the BS antennas. As seen, in this work we consider an homogeneous system with common UE channel correlation matrix [4, 15, 16]. This makes the problem analytically more tractable. Although possible, the extension to the case in which UEs have different channel correlation matrices is mathematically more involved and would require many technical details that could not be properly addressed for space limitations. For this reason, all this is left for the extended version, which includes also the multicell setting [17].

¹Note that $\rho = 0$ corresponds to a Rayleigh fading channel model while $\rho \rightarrow \infty$ corresponds to the non-fading channel model.

3. MAIN RESULT

We exploit the statistical distribution for the channel \mathbf{H} and the large dimensions of N and K to compute the deterministic approximation of γ_k , which will be eventually used to find an approximation of the ergodic sum rate. In doing so, we assume the following growth rate of system dimensions:

Assumption 1. *The dimensions N and K grow to infinity at the same pace, that is:*

$$1 \leq \liminf N/K \leq \limsup N/K < \infty. \quad (9)$$

The above assumption will be referred to as $N, K \rightarrow \infty$ in the sequel. For technical reasons, the following reasonable assumptions are also imposed on the system settings [4, 9, 10].

Assumption 2. *As $N, K \rightarrow \infty$, the correlation matrix $\mathbf{\Theta}$ has uniformly bounded spectral norm on N , i.e.,*

$$\sup \|\mathbf{\Theta}^{1/2}\| < \infty \quad \inf \frac{1}{N} \text{Tr}(\mathbf{\Theta}) > 0 \quad (10)$$

Also, as $N, K \rightarrow \infty$

$$\sup \|\mathbf{A}\| < \infty \quad (11)$$

which implies that the Euclidean norm of the columns \mathbf{a}_k are uniformly bounded in N, K .

Assumption 3. *The maximum transmit power $p_{\max} = \max_i p_i$ is of order $O(1/K)$. Moreover, we denote by $p_i = \bar{p}_i/K$ with \bar{p}_i uniformly bounded in K for $\alpha > 0$.*

Let us now introduce the fundamental equations that are needed to express a deterministic equivalent of γ_k . The following set of equations:

$$\delta = \frac{1}{N} \text{Tr} \frac{\mathbf{\Theta}}{1+\rho} \left(\lambda \left(\mathbf{I}_N + \frac{\tilde{\delta}}{1+\rho} \mathbf{\Theta} \right) + \frac{\rho}{1+\rho} \frac{\mathbf{A}\mathbf{A}^H}{1+\delta} \right)^{-1} \quad (12)$$

$$\tilde{\delta} = \frac{1}{N} \text{Tr} \left(\lambda (1+\delta) \mathbf{I}_K + \frac{\rho}{1+\rho} \mathbf{A}^H \left(\mathbf{I}_N + \frac{\tilde{\delta}}{1+\rho} \mathbf{\Theta} \right)^{-1} \mathbf{A} \right)^{-1} \quad (13)$$

admits a unique positive solution [9, 10]. The matrices $\mathbf{R}_{[k]}$ and $\mathbf{R}_{[i,k]}$ are given by:

$$\mathbf{R}_{[k]} = \left(\lambda \left(\mathbf{I}_N + \frac{\tilde{\delta}}{1+\rho} \mathbf{\Theta} \right) + \frac{\rho}{1+\rho} \frac{\mathbf{A}_{[k]}\mathbf{A}_{[k]}^H}{1+\delta} \right)^{-1} \quad (14)$$

$$\mathbf{R}_{[i,k]} = \left(\lambda \left(\mathbf{I}_N + \frac{\tilde{\delta}}{1+\rho} \mathbf{\Theta} \right) + \frac{\rho}{1+\rho} \frac{\mathbf{A}_{[i,k]}\mathbf{A}_{[i,k]}^H}{1+\delta} \right)^{-1} \quad (15)$$

We also define

$$u_k = \delta + \frac{\rho}{1+\rho} \mathbf{a}_k^H \mathbf{R}_{[k]} \mathbf{a}_k \quad (16)$$

$$u_{i,k} = \delta + \frac{\rho}{1+\rho} \mathbf{a}_i^H \mathbf{R}_{[i,k]} \mathbf{a}_i \quad (17)$$

and

$$s_{i,k} = \frac{1}{N} \text{Tr} \left(\frac{\mathbf{\Theta}}{1+\rho} \mathbf{R}_{[i,k]} \left(\frac{1}{N} \frac{\mathbf{\Theta}}{1+\rho} + \frac{\rho}{1+\rho} \mathbf{a}_i \mathbf{a}_i^H \right) \mathbf{R}_{[i,k]} \right). \quad (18)$$

The following theorem summarizes the main result of this work.

Theorem 1. Let Assumptions 1 – 3 hold true. Then, we have that $\gamma_k - \bar{\gamma}_k \rightarrow 0$ almost surely with

$$\bar{\gamma}_k = \frac{p_k \bar{t}_k^2}{\bar{s}_k + \bar{\psi} \frac{\sigma^2}{P_T}} \quad (19)$$

where

$$\bar{t}_k = \frac{u_k}{1 + u_k} \quad (20)$$

$$\bar{s}_k = \sum_{i=1, i \neq k}^K p_i \frac{c_{i,k} + \left| \frac{\rho}{1+\rho} \mathbf{a}_k^H \mathbf{R}_{[i,k]} \mathbf{a}_i \right|^2}{(1 + u_k)^2 (1 + u_{i,k})^2} \quad (21)$$

$$\bar{\psi} = \sum_{i=1}^K p_i \frac{u'_i}{(1 + u_i)^2} \quad (22)$$

with $u'_i = -\frac{du_i}{d\lambda}$.

Proof. The proof is very much involved and relies on results in random matrix theory [9] as well as some recent ones on the deterministic equivalent of bilinear forms [10, Theorem 1]. Due to the space limitations, it is omitted. A complete proof will be provided in the extended version, which is currently under preparation [17]. Details will be provided upon request. \square

Let us now consider the two extreme cases of the channel model in (8), namely, the Rayleigh fading channel and the LOS channel. If a Rayleigh fading channel model is considered (i.e., $\rho = 0$), the following results are obtained, which coincide with those in [4].

Corollary 1. Let Assumptions 1 – 3 hold true. If a Rayleigh fading model is considered, then $\bar{\gamma}_k - \bar{\gamma}_k^{(0)} \rightarrow 0$ almost surely

$$\bar{\gamma}_k^{(0)} = \frac{p_k \bar{t}_k^2}{\bar{s}_k + \bar{\psi} \frac{\sigma^2}{P_T}} \quad (23)$$

where

$$\bar{t}_k = \frac{\delta}{1 + \delta} \quad (24)$$

$$\bar{s}_k = \frac{1}{N^2} \text{Tr}(\Theta \mathbf{R} \Theta \mathbf{R}) \frac{\sum_{i=1, i \neq k}^K p_i}{\left(1 + \frac{1}{N} \text{Tr}(\Theta \mathbf{R})\right)^4} \quad (25)$$

$$\bar{\psi} = \frac{\frac{1}{N} \text{Tr}(\Theta \mathbf{R}^2)}{1 - \frac{K}{N} \frac{\frac{1}{N} \text{Tr}(\Theta \mathbf{R} \Theta \mathbf{R})}{\left(1 + \frac{1}{N} \text{Tr}(\Theta \mathbf{R})\right)^2}} \frac{\sum_{i=1}^K p_i}{\left(1 + \frac{1}{N} \text{Tr}(\Theta \mathbf{R})\right)^2} \quad (26)$$

where δ is the unique positive solution of $\delta = 1/N \text{Tr}(\Theta \mathbf{R})$ with $\mathbf{R} = (\lambda \mathbf{I}_N + \frac{K}{N} \frac{\Theta}{1+\delta})^{-1}$.

Proof. From Theorem 1, it follows that if $\rho = 0$ then

$$u_k = u_{i,k} = \delta = \frac{1}{N} \text{Tr}(\Theta \mathbf{R}) \quad (27)$$

since $\bar{\delta} = K/(N\lambda(1+\delta))$. Moreover, we have that $u'_i = -\delta' = -d\delta/d\lambda = -1/N \text{Tr}(\Theta \mathbf{R}')$ with

$$\mathbf{R}' = \frac{d\mathbf{R}}{d\lambda} = -\mathbf{R}^2 + \frac{K}{N} \frac{\mathbf{R} \Theta \mathbf{R}}{(1+\delta)^2} \delta'. \quad (28)$$

Solving $\delta' = 1/N \text{Tr}(\Theta \mathbf{R}')$ with respect to δ' yields the desired result. \square

If a LOS environment is considered (i.e., $\rho \rightarrow \infty$), then we have that:

Corollary 2. Let Assumptions 1 – 3 hold true. If a LOS environment is considered, then $\bar{\gamma}_k - \bar{\gamma}_k^{(\infty)} \rightarrow 0$ almost surely

$$\bar{\gamma}_k^{(\infty)} = \frac{p_k \bar{t}_k^2}{\bar{s}_k + \bar{\psi} \frac{\sigma^2}{P_T}} \quad (29)$$

where

$$\bar{t}_k = \frac{\mathbf{a}_k^H \mathbf{R}_{[k]} \mathbf{a}_k}{1 + \mathbf{a}_k^H \mathbf{R}_{[k]} \mathbf{a}_k} \quad (30)$$

$$\bar{s}_k = \sum_{i=1, i \neq k}^K p_i \frac{|\mathbf{a}_k^H \mathbf{R}_{[i,k]} \mathbf{a}_i|^2}{(1 + \mathbf{a}_k^H \mathbf{R}_{[k]} \mathbf{a}_k)^2 (1 + \mathbf{a}_i^H \mathbf{R}_{[i,k]} \mathbf{a}_i)^2} \quad (31)$$

$$\bar{\psi} = \sum_{i=1}^K p_i \frac{\mathbf{a}_i^H \mathbf{R}_{[i]}^2 \mathbf{a}_i}{(1 + \mathbf{a}_i^H \mathbf{R}_{[i]} \mathbf{a}_i)^2} \quad (32)$$

where δ is the unique positive solution of $\delta = 1/N \text{Tr}(\Theta \mathbf{R})$ with $\mathbf{R} = (\lambda \mathbf{I}_N + \frac{\mathbf{A} \mathbf{A}^H}{1+\delta})^{-1}$

$$\mathbf{R}_{[k]} = \left(\lambda \mathbf{I}_N + \mathbf{A}_{[k]} \mathbf{A}_{[k]}^H \right)^{-1} \quad (33)$$

$$\mathbf{R}_{[i,k]} = \left(\lambda \mathbf{I}_N + \mathbf{A}_{[i,k]} \mathbf{A}_{[i,k]}^H \right)^{-1}. \quad (34)$$

Proof. From Theorem 1, it follows that if $\rho \rightarrow \infty$ then $\delta \rightarrow 0$ since $\bar{\delta}$ is upper bounded by $1/N \text{Tr}(\lambda \mathbf{I}_K + \mathbf{A}^H \mathbf{A})^{-1}$. Moreover, we have that $c_{i,k} \rightarrow 0$ whereas $\mathbf{R}_{[k]}$ and $\mathbf{R}_{[i,k]}$ reduce to (33) and (34). Also, $u'_i = \mathbf{a}_i^H \mathbf{R}_{[i]}^2 \mathbf{a}_i$ since $u'_i = -du_i/d\lambda = -\mathbf{a}_i^H \mathbf{R}'_{[i]} \mathbf{a}_i$ with $\mathbf{R}'_{[i]} = d\mathbf{R}_{[i]}/d\lambda = -\mathbf{R}_{[i]}^2$. \square

We are ultimately interested in the individual rates $\{r_k\}$ and the ergodic sum rate r_E . Since the logarithm is a continuous function, by applying the continuous mapping theorem, from the almost sure convergence results of Theorem 1 it follows that $r_k - \bar{r}_k \rightarrow 0$ almost surely with [4]

$$\bar{r}_k = \log_2(1 + \bar{\gamma}_k). \quad (35)$$

An approximation of r_E is obtained as follows [4]

$$\bar{r}_E = \sum_{k=1}^K \log_2(1 + \bar{\gamma}_k) \quad (36)$$

such that $\frac{1}{K}(r_E - \bar{r}_E) \rightarrow 0$ holds true almost surely.

4. NUMERICAL RESULTS

Monte-Carlo simulations are now used to validate the above asymptotic analysis for a network with finite size. We consider a cell of radius $R = 500$ m. The transmission bandwidth is $W = 10$ MHz and the total noise power $W\sigma^2$ is -104 dBm. To allow for reproducibility of our results², we assume that the UEs are positioned at a distance of $x = 2/3R$ meters and use the standard correlation model $[\Theta]_{i,j} = \sqrt{\beta} \nu^{|i-j|}$ where β is the path loss function obtained as $\beta = 2L_{\bar{x}}(1 + x^\kappa/\bar{x}^\kappa)^{-1}$. The parameter $\kappa > 2$ is the path loss

²To enable simple testing of other parameter values, the code is also available for download at the following address <https://github.com/lucasanguinetti/downlink-Rician-MISO-systems>.

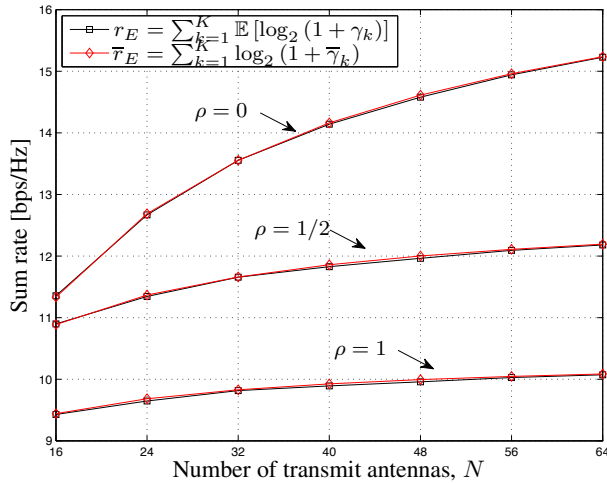


Fig. 1. Sum rate vs. N when $N/K = 2$ and the Rician factor ρ is 0, $1/2$ and 1.

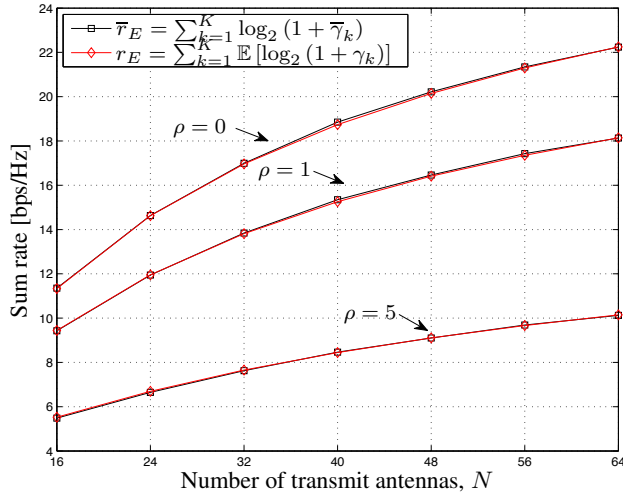


Fig. 2. Sum rate vs. N when $K = 8$ and the Rician factor ρ is 0, 1 and 5.

exponent, $\bar{x} > 0$ is some cut-off parameter and $L_{\bar{x}}$ is a constant that regulates the attenuation at distance \bar{x} . Unless otherwise specified, we assume $\nu = 0.9$, $\kappa = 3.5$, $L_{\bar{x}} = -86.5$ dB and $\bar{x} = 25$ m. The results are obtained for 1000 different channel realizations. We assume that a uniform linear array (ULA) is adopted at the BS, the (n, k) th entry of \mathbf{A} is given by

$$[\mathbf{A}]_{n,k} = e^{-i(n-1)(2\pi \frac{\Delta}{\lambda}) \sin \theta_k} \quad (37)$$

where Δ is the transmit antenna spacing, λ is the wavelength, and θ_k is the arrival angle of the k th UE. Moreover, we set $\Delta = \lambda/2$, which means that there is no correlation between receive antennas. The transmit power P_T is fixed to 10 Watt with $p_k = P_T/K$ for $k = 1, \dots, K$ and the regularization parameter λ is computed as $\lambda = \sigma^2 / (\beta P_T)$.

Fig. 1 illustrates r_E and \bar{r}_E when N grows large and $N/K = 2$ for different values of the Rician factor K . As seen, the approximation matches very well with Monte Carlo simulations in all the

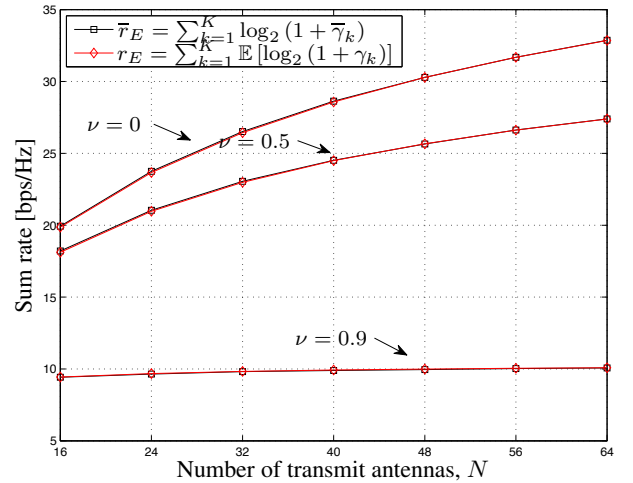


Fig. 3. Sum rate vs. N when $N/K = 2$ and $\rho = 1$ for different values of the correlation factor among BS antennas.

investigated scenarios. As expected, increasing the Rician factor reduces the system performance because of the reduced spatial diversity of the channel. When the LOS component has the same power of the Rayleigh counterpart, i.e. $\rho = 1$, the network performance improves marginally as N, K grow. In Fig. 2, a classical massive MIMO setting is considered in which N grows large and K is kept fixed to 8. As before, the approximation is very accurate and substantial improvements are observed as N increases also for a Rician factor of 5. The impact of the correlation matrix is analyzed in Fig. 3 under the same operating conditions of Fig. 1 for $\rho = 1$. As expected, reducing the correlation factor largely improves the system performance as it increases the spatial multiplexing capabilities of the channel model.

5. CONCLUSION

In this work, we analyzed the ergodic sum rate in the downlink of a single-cell large-scale MIMO system operating over a Rician fading channel. A regularized zero-forcing precoding scheme under the assumption of perfect channel state information. Recent results from large-scale random matrix theory allowed us to give concise approximations of the SINRs. Such approximations turned out to depend only on the long-term channel statistics, the Rician factor and the deterministic component. Numerical results indicated that these approximations are very accurate. Applied to practical networks, such results may lead to important insights on how the different parameters affect the performance and allow to simulate the network behavior without the need of extensive Monte Carlo simulations. More details and insights on these aspects will be given in the extended version where a more general multi cell setting will be also investigated under the assumption of different spatial correlation matrices and imperfect channel state information due to pilot contamination [17].

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