S. C. Liu

S. J. Hu

Department of Mechanical Engineering and Applied Mechanics, The University of Michigan, Ann Arbor, MI 48109

T. C. Woo

Department of Industrial Engineering, The University of Washington, Seattle, WA 98194

Tolerance Analysis for Sheet Metal Assemblies

Traditional tolerance analyses such as the worst case methods and the statistical methods are applicable to rigid body assemblies. However, for flexible sheet metal assemblies, the traditional methods are not adequate: the components can deform, changing the dimensions during assembly. This paper evaluates the effects of deformation on component tolerances using linear mechanics. Two basic configurations, assembly in series and assembly in parallel, are investigated using analytical methods. Assembly sequences and multiple joints beyond the basic configurations are further examined using numerical methods (with finite element analysis). These findings constitute a new methodology for the tolerancing of deformable parts.

1 Introduction

Sheet metal assembly is practiced widely in such industries as automobile, aerospace, electronics and furniture-making. One metric for the overall quality of the product is the amount of variation from nominal dimensions, as caused by the variation in the parts. A typical method for determining the overall variation is called *stack-up:* by "adding" the variations in the parts to arrive at the variation of the whole. Because the components are put together "end to end," the assembly and the tolerances are in *series.* This paper examines an alternative method: assembling and tolerancing in *parallel.* For two components with variations ν_1 and ν_2, the resulting variation ν depends on the configurations, viz.:

$$\textit{series} \quad \nu = \nu_1 + \nu_2 \tag{1}$$

$$\textit{parallel} \quad \nu = f(\nu_1, \nu_2) \tag{2}$$

where the function f in the parallel assembly is to be determined. For example, if the function f is $f(\nu_1, \nu_2) = (1/3)\nu_1 + (1/3)\nu_2$, then in concept a decrease in the assembly variation would result. The function f depends on the coefficients of stiffness of the components in the parallel assembly.

In his statistical study of production data, Takezawa (1980) observed that, for flexible sheet metal assemblies, "the conventional addition theorem of variance is no longer valid." Furthermore, he noted that "the assembly variance has decreased and is closer to the variance of the stiffer components." This paper seeks an explanation of this phenomenon by using analytical mechanics. The effect of assembly sequences will also be investigated when involving flexible parts. To gain an insight in the importance of sequencing, an automotive body assembly is illustrated. There are n components as shown in Fig. 1. The basic inquiry is: In what sequence should the components be assembled such that the resulting variation is minimized? For simplicity, suppose the components are classified as either "rigid" or "flexible." An underbody and a door ring could be considered as rigid while a quarter panel or a shelf is flexible. Common sense would suggest that one builds a "cage" of rigid components—in series—and then lays the flexible panels on the rigid cage—in parallel. Alternatively, a flexible "shell" of panels could be built in series and then laid on the rigid cage in parallel. Indeed, different sequencing of the components would result in different variation in the final assembly.

Tolerance is defined as the permissible variations of a dimension in engineering drawings or designs (ANSI 1994). It may be treated deterministically. A dimension d is said to have a nominal value D with an upper bound τ_u and a lower bound τ_l, i.e., $d \in [D - \tau_l, D + \tau_u]$. Since a permissible dimension can occur anywhere in this range, the notion of a random variable arises with the probabilistic treatment. Assuming an underlying probability distribution, the nominal dimension then corresponds to the mean, and the tolerance corresponds to the standard deviation (or variance). Typically, the range $[D - \tau_l, D + \tau_u]$ corresponds to 6 standard deviations (6σ, the natural tolerance). Therefore, there are correspondingly two methods for the analysis of assembly tolerance: worst case (deterministic) and statistical analysis (probabilistic).

The *worst case* method evaluates an assembly by assuming that the dimensions of all components take on their extreme values (Chase and Greenwood, 1987; Dong and Soom, 1990; Dong et al., 1994; Fortini, 1967; Greenwood and Chase, 1987; Spotts 1978). While simple and efficient, this method overestimates the assembly tolerance, because the probability of all the components in the assembly having the "worst case" dimensions simultaneously is very small. The effect of applying this technique is that very tight tolerances for the components are required in order for the final assembly tolerance to meet the design specifications. It is known that tolerances are inversely proportional to manufacturing cost (Wu et al., 1988). Therefore, an unnecessarily high cost may result.

In *statistical analysis,* the component tolerances are assumed to adopt known distributions. Calculating the joint distribution of some linear or nonlinear function of component distributions as a result of the assembly is the main task (Chase and Greenwood, 1987; Fortini, 1967; Greenwood and Chase, 1987, 1990; Lee and Woo, 1990; Spotts, 1978; and Treacy et al., 1991). Detailed reviews of tolerancing methods are available (Chase and Parkinson, 1991; Juster, 1992; Roy et al., 1991; and Wu et al., 1988). Statistical analysis is more effective in supporting interchangibility for mass production and efficient for cost reduction.

Whether using worst case or statistical analysis, there is an underlying assumption in these state-of-the-art tolerance analyses: that the components are rigid (e.g., independent of the applied forces). In other words, during assembly, the components do not deform, hence their dimensions do not change. Sheet metal components, on the other hand, are *flexible:* their dimensions may change during locating, clamping and joining. This paper investigates such changes. Section 2 examines an assembly in series and in parallel. The equations for analyzing the tolerances are derived for the basic configurations. Linear mechanics is employed, hence identifying deflection with tolerance, denoted by the variable ν. Section 3 explores tolerance stack-up in configurations beyond the basic. As the number of simultaneous equations inevitably exceeds the ability to solve

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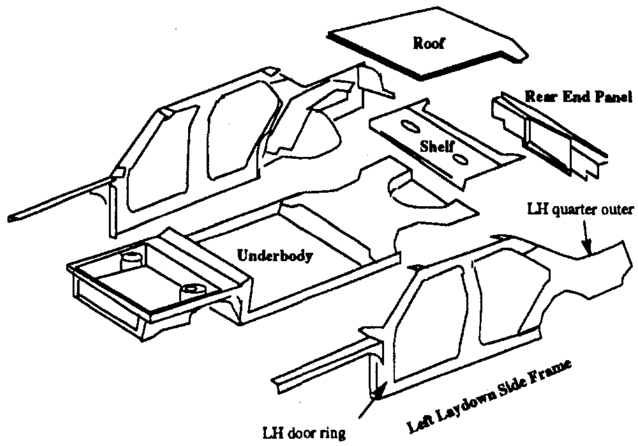

Fig. 1 Automotive body panel structure

them manually, the finite element method is invoked. Statistical simulation is also conducted. In doing so, the variable ν is treated as a random variable in the following sense. Consider a linear system with two independent variables ν_1 and ν_2:

$$\nu = c_1\nu_1 + c_2\nu_2 \tag{3}$$

as a generalization of Eqs. (1) and (2), where ν is the system response, and c_1 and c_2 are the parameters to be determined by the assembly configuration (series or parallel). For a normal distribution with mean μ and variance σ^2, Eq. (3) becomes:

$$\mu = c_1\mu_1 + c_2\mu_2 \tag{3a}$$

$$\sigma^2 = c_1^2\sigma_1^2 + c_2^2\sigma_2^2 \tag{3b}$$

And if the tolerance τ is set at the 6σ level, then the assembly tolerance is related to the component tolerances by:

$$\tau = \sqrt{c_1^2\tau_1^2 + c_2^2\tau_2^2} \tag{3c}$$

2 Problem and Its Analysis

Tolerance analysis in sheet metal assemblies can be stated simply. Components, each with its own tolerance, are fixtured together and joined; the overall tolerance of the assembly is sought. While the problem is easily understood, the considerations are manifold. External forces are applied in order to bring the flexible components together. The components are joined—mechanically (such as by riveting) or by thermal means (such as by resistance welding, arc welding, or laser welding). The external loading (from clamping and joining) as well as internal loading (from residual stresses, for example) may cause buckling, for which there is no closed form solution. Thus the problem of tolerance analysis of sheet metal assemblies does not seem to yield easily. And, indeed, this explains the scarcity of literature on flexible components (Lee and Haynes, 1987; and Menassa and DeVries, 1991) despite the immense utility of sheet metal assemblies in industry. In explaining the phenomenon of reduced variation after flexible components are fixtured and assembled, simplicity is adopted for clarity. Later, generalizations are made.

2.1 Linear Mechanics. Let a piece of sheet metal be idealized as a cantilevered beam of length L and consider the variation of a single point p, as shown in Fig. 2. The fixed end is due to fixturing or due to a prior weldment. In the context of tolerancing, such a fixed point is the datum. The free end p reflects the variation ν. In general ν has three components (ν_x, ν_y, and ν_z). In Fig. 2 only ν_y is shown and in the subsequent discussion ν_y is identified with ν, without loss of generality.

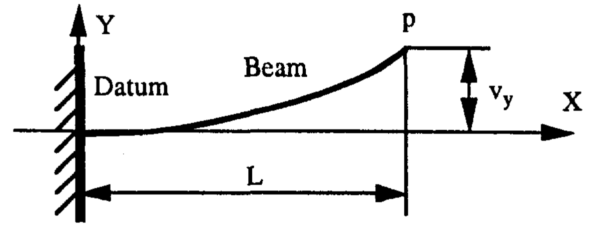

Fig. 2 Datum and variation

Next, consider two components with variations ν_1 and ν_2 at their respective free ends joined mechanically. (The assumption of mechanical joining is made so that later calculations are unencumbered by thermal considerations. In other words, a weld nugget is indistinguishable from a rivet.) The problem is to determine the parameters c_i in Eq. (3). Two basic configurations are examined.

Configuration S: Two components are assembled in *series;* see Fig. 3(a). A clamp (Clamp 1) exerting a force F_1 in the Y-direction on Beam 1 brings ν_1 to its nominal; likewise, a force F_2 from Clamp 2 brings ν_2 to nominal; see Fig. 3(b). (Forces and moments at the datum are not shown.) A joining agent, shown as a welding or riveting gun, is brought in to join the two free ends. Then, as shown in Fig. 3(c), the clamps are released, upon which a reactive force F results, which must necessarily be equal to the sum of F_1 and F_2 in magnitude though opposite in direction (indicated by a reverse direction of the arrow):

$$F = F_1 + F_2 \tag{4}$$

When the fixture on Beam 2 is released, the reactive force F_2 and a reactive moment M balancing F_2 from Clamp 2 must necessarily be released. (The force and the moment at the Datum for balancing F_1 from Clamp 1 do not affect the final dimension of the assembly.) Finally, the variation of the assembly ν_a can be calculated at two points of interest: ν_{a1} where

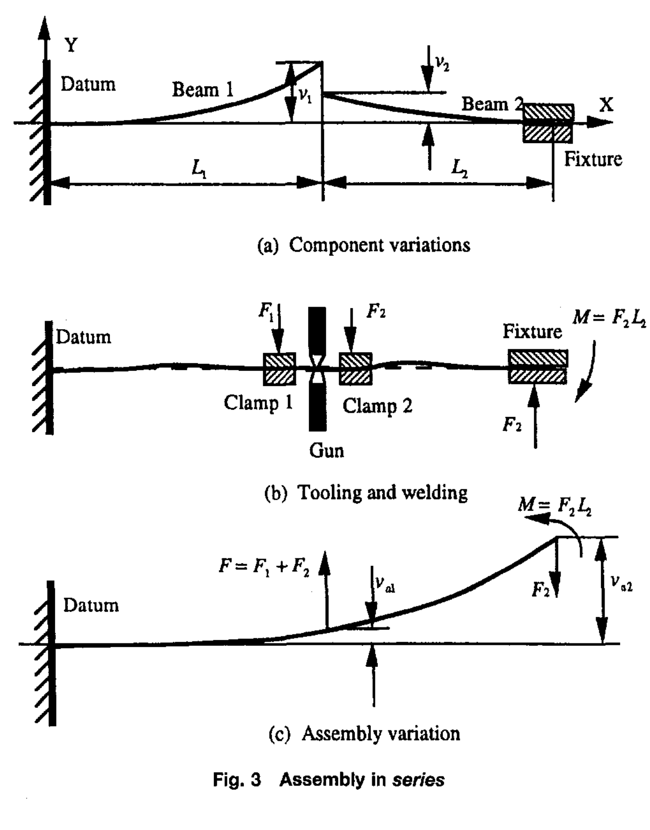

Fig. 3 Assembly in *series*

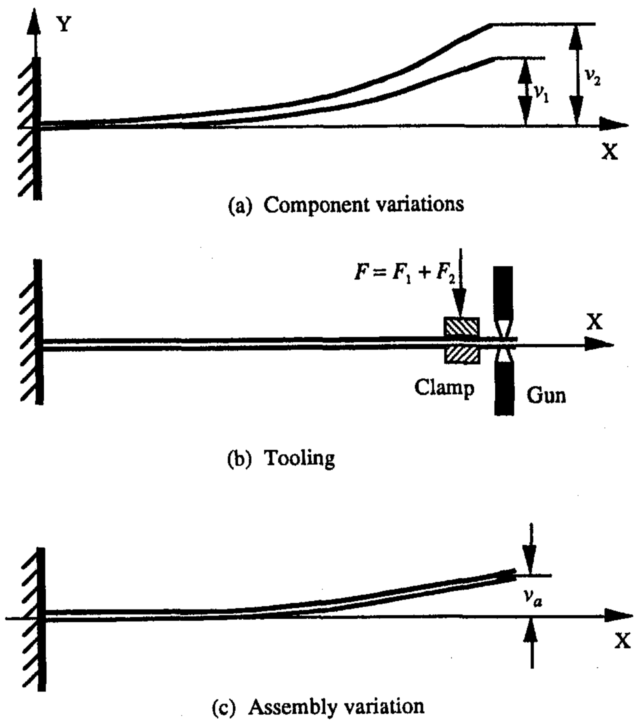

Fig. 4 Assembly in *parallel*

the free ends were joined and ν_{a2} which was a fixed end for Beam 2.

Configuration P: The assembly of two components in parallel is illustrated in Fig. 4. While the same forces as in Eq. (4) are involved, there is a difference in the structure—the resulting assembly is "thicker," hence less subject to deflection. The relation between ν and F is established first.

A linear model of a cantilevered beam acts as a spring. A force F causes a deflection ν according to Hooke's law:

$$F = K\nu \tag{5}$$

where K is the spring constant (coefficient of stiffness). Specifically, ν is related to F by (Skalmierski, 1979):

$$\nu = \frac{FL^3}{3EI} \tag{6}$$

where L is the length of the beam, E is the Young's modulus of the material, and I is the moment of inertia of the cross-section of the beam. Then the coefficient of stiffness K can be calculated by substituting Eq. (6) to Eq. (5):

$$K = \frac{3EI}{L^3} \tag{7}$$

2.2 Tolerance Analysis. For assemblies in series, the variations (spring-back) ν_{a1} and ν_{a2} at the two points shown in Fig. 3(c) can be calculated using the following equations (the derivations of which are omitted):

$$\textit{series} \quad \nu_{a1} = \frac{F_1L_1^3}{3EI_1} = \nu_1 \tag{8}$$

$$\nu_{a2} = \frac{F_1L_1^3}{3EI_1}\left(1 + \frac{3L_2}{2L_1}\right) + \frac{F_2L_2^3}{6EI_2}$$

$$= \left(1 + \frac{3L_2}{2L_1}\right)\nu_1 + 0.5\nu_2 \tag{9}$$

Note that the assembly variation is independent of the material properties (Young's modulus and the coefficient of stiffness); only the geometry (the length) contributes to the parameters c_i in Eq. (3). Equations (8) and (9) indicate that the assembly tolerances will be affected by part tolerances and the geometry of the parts in serial assembly.

For assembly in parallel, as shown in Fig. 4(b), the reactive force F is also the sum of the clamping forces F_1 and F_2, each is related to the component variations ν_1 and ν_2 by Eq. (5):

$$F = F_1 + F_2 = K_1\nu_1 + K_2\nu_2 \tag{10}$$

Force F causes an assembly variation ν_a, with a stiffness K_p, for assembly in parallel. The spring model results in:

$$F = K_p\nu_a \tag{11}$$

By comparing Eqs. (10) and (11), the variation takes on the form of:

$$\textit{parallel} \quad \nu_a = \frac{K_1}{K_p}\nu_1 + \frac{K_2}{K_p}\nu_2 \tag{12}$$

Interestingly, if one of the two components has a large coefficient of stiffness and a negligible variation, the assembly variation becomes very small. (This is often the case in industrial practice. The automotive chassis, for example, is much stiffer than the body panels. And the chassis serves as a reference onto which the panels assemble.) Assume that $K_1 \gg K_2$ and $\nu_1 \sim 0$. Then Eq. (12) becomes:

$$\nu_a \sim \frac{K_2}{K_p}\nu_2 \sim \frac{K_2}{K_1}\nu_2 \ll \nu_2 \tag{13}$$

In other words, the assembly variation can be much smaller than the component variation, provided that the assembly is done in parallel and that one of the two components is stiff and almost error-free.

3 Examples and Validation

The phenomenon that the assembly variation can be less than the component variation (Takezawa, 1980) has been explained by Eq. (13), using simple mechanics, for two components. When the number of components exceeds two, or when multiple joints are used, the composite stiffness coefficient involves the solution of simultaneous equations. In the following, two examples are offered beyond the basic configurations. The first example involves three components and examines the effects of sequencing on the assembly tolerance. The second example involves two components resulting from multiple joints.

3.1 Assembly Sequences. Consider the various sequences of assembling the components shown in Fig. 5. There are three components: panel P_1, panel P_2, and chassis C; with

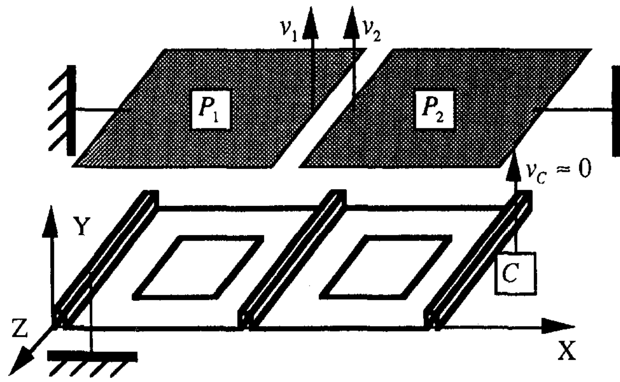

Fig. 5 Two panels and a chassis

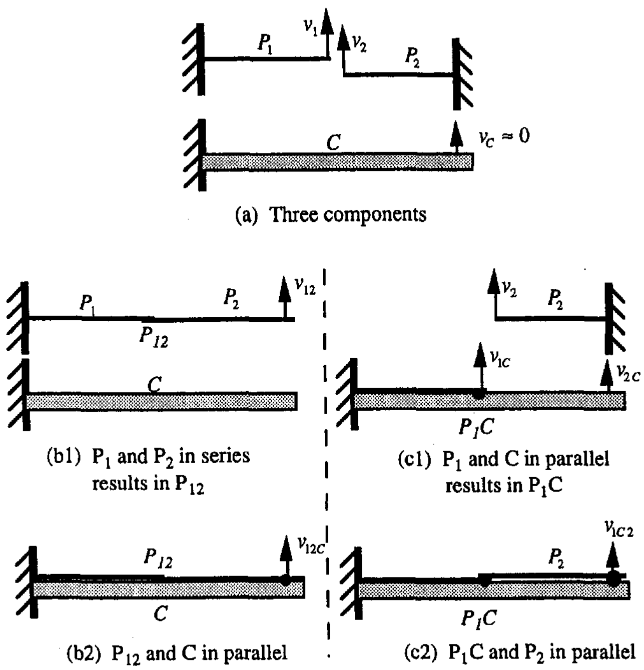

Fig. 6 Two assembly sequences

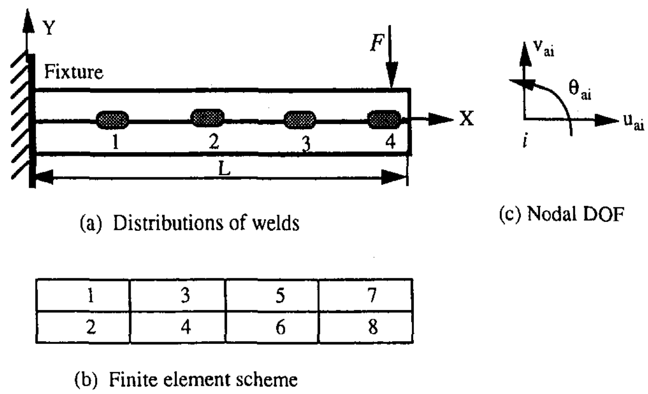

Fig. 7 Assembly with multiple welds

variations ν_1, ν_2 and ν_C; and coefficients of stiffness K_1, K_2 and K_C, respectively. As C is the chassis, $K_C \gg K_1$, $K_C \gg K_2$, and $\nu_C \sim 0$ are assumed.

To simplify the computations, the components are treated as cantilevered beams, so that the previously derived equations can be used directly; see Fig. 6(*a*). Two sequences are examined; they are:

Sequence SP: Panels P_1 and P_2 are first joined in *series* and then the resulting panel P_{12} is joined to the chassis C in parallel. As illustrated in Fig. 6(*b*1), the "left" side of P_1 serves as the datum and the "right" side of the resulting panel P_{12} exhibits a variation ν_{12}. In the subsequent step, illustrated in Fig. 6(*b*2), the left side of the chassis again serves as the datum and the variation on the right side ν_{12C} is the assembly variation.

Sequence PP: Panel P_1 is first joined to the chassis C in parallel resulting in P_1C. Then panel P_2 is joined to P_1C, also in parallel. In joining P_1 to C, the left side of C is the datum and the right side of P_1 shows a variation ν_{1C}, while the right side of C yields a variation ν_{2C}; refer to Fig. 6(*c*1). Similarly, in joining P_2 to P_1C, the left side of P_2 is the datum and its right side gives a variation ν_{1C2} as shown in Fig. 6(*c*2).

The dimensions of both panels and the chassis are shown in Table 1. The material is mild steel, with Young's modulus $E = 207{,}000$ N/mm^2

Variations in the X- and Z-directions are set to zero so as to simplify the computation. For simplicity, assume the variations ν_1 and ν_2 are identical. The variation of the chassis C is set to zero, and finite element analysis (CBAR beam element, MSC/NASTRAN 1988) is used to obtain the coefficients of stiffness of the two sequences. The results are:

Table 1 Dimensions of panels and chassis (mm)

Components	Panels P_1 and P_2	Chassis C
Length L	500	1000
Width W	500	500
Thickness T	1	2

$$\textit{Sequence SP:}\quad \nu_{12C} = 0.0983\nu_1 + 0.0197\nu_2 \tag{14}$$

$$\textit{Sequence PP:}\quad \nu_{1C2} = 0.0658\nu_1 + 0.0094\nu_2 \tag{15}$$

If the component standard deviations are identical and are set at 1 mm,

$$\sigma_1 = \sigma_2 = 1 \text{ mm} \tag{16}$$

then from Eq. (3*b*), the assembly standard deviation for *Sequence SP* is:

$$\sigma_{sp} = \sqrt{0.0983^2\sigma_1^2 + 0.0197^2\sigma_2^2} = 0.1 \text{ mm} \tag{17}$$

and the assembly standard deviation for *Sequence PP* is:

$$\sigma_{pp} = \sqrt{0.0658^2\sigma_1^2 + 0.0094^2\sigma_2^2} = 0.066 \text{ mm} \tag{18}$$

It is clear that *PP* results in less variation than *SP*.

3.2 Multiple Joints. The effect that additional joints have on a large panel is of interest since the joints serve the same purpose as multiple parallel assemblies of "smaller" panels. The effect is most vivid if the undercarriage from the previous example is "softened"; that is, two sheet metal components of equally low stiffness are joined, in parallel, at multiple spots.

Consider the assembly of two cantilevered beams with four joints. The thickness of both components is 1 mm. The dimensions are length L = 100 mm, width W = 10 mm (in Z-direction). The material used is steel. The four joints are distributed *uniformly* along the beams, see Fig. 7(*a*). Force F is the same as that shown in Eq. (10).

Finite element analysis is again used to find the coefficients of stiffness. There are eight beam elements (CBAR, MSC/NASTRAN 1988) in the assembly; see Fig. 7(*b*). The beam element is a three degree-of-freedom node element. The nodes coincide to the spot welds shown in Fig. 7(*a*). At each node i (i = 1 to 4) there are three degrees of freedom (DOF): two translations and one rotation; see Fig. 7(*c*). Since each weld constrains 3 DOF between the two cantilevered beams, the overall stiffness of the welded structure will be affected by the welds. Thus the number of welds will influence the assembly dimension. In total, there are 12 degrees of freedom (4 nodes, with 3 DOF at each node). The equations are:

$$
\begin{bmatrix}
331200 & 0 & 0 & -165600 & & & & & & & & \\
 & 530 & 0 & 0 & -265 & 3312 & & & & & & \\
 & & 193200 & 0 & -3312 & -13800 & & & & & & \\
 & & & 331200 & 0 & 0 & -165600 & & & & & \\
 & & & & 530 & 0 & 0 & -265 & 3312 & & & \\
 & & & & & 193200 & 0 & -3312 & -13800 & & & \\
 & & & & & & 331200 & 0 & 0 & -165600 & & \\
 & & & & & & & 530 & 0 & 0 & -265 & 3312 \\
 & & & & & & & & 193200 & 0 & -3312 & -13800 \\
 & & & & & & & & & 165600 & 0 & 0 \\
s & y & m & m & e & t & r & i & c & & 256 & -3312 \\
 & & & & & & & & & & & 96600
\end{bmatrix}
\begin{Bmatrix} u_{a1} \\ \nu_{a1} \\ \theta_{a1} \\ u_{a2} \\ \nu_{a2} \\ \theta_{a2} \\ u_{a3} \\ \nu_{a3} \\ \theta_{a3} \\ u_{a4} \\ \nu_{a4} \\ \theta_{a4} \end{Bmatrix}
=
\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ F \\ 0 \end{Bmatrix}
\tag{19}
$$

where the force released (F) is calculated from Eq. (10). If the geometry and Young's modulus are substituted into Eqs. (7) and (10), then an explicit form for F is obtained:

$$F = K_1\nu_1 + K_2\nu_2 = 0.5175\nu_1 + 0.5175\nu_2 \tag{20}$$

Variations in the Y-direction at the four nodes can be solved from Eq. (19) with the solution:

$$
\begin{Bmatrix} \nu_{a1} \\ \nu_{a2} \\ \nu_{a3} \\ \nu_{a4} \end{Bmatrix}
=
\begin{Bmatrix} 0.0122 \\ 0.0420 \\ 0.0835 \\ 0.1309 \end{Bmatrix} \nu_1 +
\begin{Bmatrix} 0.0122 \\ 0.0420 \\ 0.0835 \\ 0.1309 \end{Bmatrix} \nu_2
\tag{21}
$$

A statistical simulation of ν_1 and ν_2 reveals the effect of multiple joints in Eq. (21) graphically (MATLAB 1994). Suppose both variables ν_1 and ν_2 are normally distributed with zero mean and unit standard deviation as shown in the upper-right box of Fig. 8. The variations at nodes 1 through 4 decreases towards the fixed end (datum) of the assembly.

Tolerance stack-up is now analyzed based on the tolerance model, Eq. (21). For example, given unit standard deviations of ν_1 and ν_2, as in Eq. (16), the standard deviations of the selected points after assembly can be obtained from Eq. (3b):

$$
\begin{Bmatrix} \sigma_{a1} \\ \sigma_{a2} \\ \sigma_{a3} \\ \sigma_{a4} \end{Bmatrix}
=
\begin{Bmatrix} 0.0173 \\ 0.0594 \\ 0.1181 \\ 0.1851 \end{Bmatrix}
\tag{22}
$$

where σ_{ai} represents the standard deviation of the assembly at joint i ($i = 1$ to 4). Equation (22) shows that the largest variation appears at the free end ($i = 4$) of the assembly. Variation at $i = 1$ is very small. These calculations agree with the results of the Monte Carlo simulation of Fig. 8.

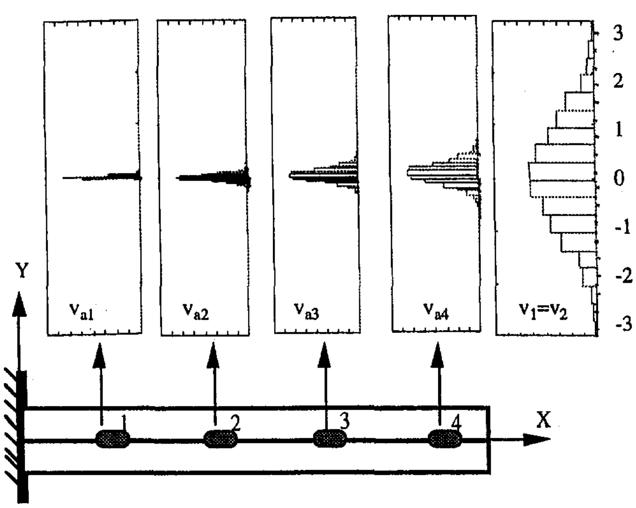

Fig. 8 Monte Carlo simulation

The effect of having multiple joints, as given by Eq. (22) can be compared to that of a single joint, as illustrated in Fig. 4. The finite element modeling for a single joint assembly gives:

$$
\begin{bmatrix} 41400 & 0 & 0 \\ 0 & 4.1 & -207 \\ 0 & -207 & 24150 \end{bmatrix}
\begin{Bmatrix} u_a \\ \nu_a \\ \theta_a \end{Bmatrix}
=
\begin{Bmatrix} 0 \\ F \\ 0 \end{Bmatrix}
\tag{23}
$$

The assembly stiffness coefficient (K_p) relates to the spring-back (ν_a) at the free end and the force released (F) by inverting Eq. (23):

$$\nu_a = \frac{F}{K_p} = \frac{F}{2.3657} \tag{24}$$

As expected, the assembly stiffness K_p is larger than the sum of the component stiffness K_1 and K_2; refer to Eq. (20):

$$2.3657 > 0.5175 + 0.5175$$

The difference is contributed from the tensile stiffness of the joint. (After joining, the components can no longer slide along each other). Now, Eq. (12) becomes:

$$\nu_a = \frac{F}{K_p} = \frac{K_1}{K_p}\nu_1 + \frac{K_2}{K_p}\nu_2 = \frac{0.5175}{2.3657}\nu_1 + \frac{0.5175}{2.3657}\nu_2 \tag{25}$$

$$= 0.2188\nu_1 + 0.2188\nu_2$$

$$\sigma_a = \sqrt{0.2188^2\sigma_1^2 + 0.2188^2\sigma_2^2} = 0.31 \text{ mm} \tag{26}$$

by assuming Eq. (16). By comparing Eq. (26) which gives the standard deviation of an assembly at the free end with one joint and Eq. (22) which shows the results with four joints, the variation (tolerance) reduction at the free end of the assemblies is about:

$$\frac{0.31 - 0.1851}{0.31} \times 100\% \approx 40\% \tag{27}$$

A similar analysis can be performed to show the "diminishing return" of additional joints in an assembly.

4 Summary

Two basic configurations, i.e., assembly in series and in parallel have been presented. The importance of assembly sequencing has been illustrated and the effect of multiple joints assessed.

Assembly tolerances are affected not only by the geometry of the components, but also by the stiffness of the components in parallel assembly. This study provides an explanation as to why the assembly tolerance can be less than the "stacked up" tolerances of the components in flexible bodies. In serial assembly, only the geometry of the components affects the tolerance of the assembly.

In this work, linear mechanics is employed which by no means explains the possibly nonlinear behaviors in assemblies, series or parallel. No thermal consideration was taken either. It is the hope that this work will generate further interest in investigating these phenomena.

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