

Exit selection strategy in pedestrian evacuation simulation with multi-exits*

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(Received 23 June 2013; revised manuscript received 14 October 2013; published online 25 March 2014)

A mixed strategy of the exit selection in a pedestrian evacuation simulation with multi-exits is constructed by fusing the distance-based and time-based strategies through a cognitive coefficient, in order to reduce the evacuation imbalance caused by the asymmetry of exits or pedestrian layout, to find a critical density to distinguish whether the strategy of exit selection takes effect or not, and to analyze the exit selection results with different cognitive coefficients. The strategy of exit selection is embedded in the computation of the shortest estimated distance in a dynamic parameter model, in which the concept of a jam area layer and the procedure of step-by-step expanding are introduced. Simulation results indicate the characteristics of evacuation time gradually varying against cognitive coefficient and the effectiveness of reducing evacuation imbalance caused by the asymmetry of pedestrian or exit layout. It is found that there is a critical density to distinguish whether a pedestrian jam occurs in the evacuation and whether an exit selection strategy is in effect. It is also shown that the strategy of exit selection has no effect on the evacuation process in the no-effect phase with a low density, and that evacuation time and exit selection are dependent on the cognitive coefficient and pedestrian initial density in the in-effect phase with a high density.

Keywords: pedestrian evacuation, exit selection strategy, critical density, dynamic parameters

PACS: 05.50.+q, 05.20.Jj, 64.70.-p

DOI: 10.1088/1674-1056/23/5/050512

1. Introduction

Pedestrian dynamics is one of the basic theories for the management, control, and guidance of pedestrian evacuation and the design of pedestrian facilities, which contribute to the humanism design of pedestrian facilities and pedestrian evacuation rules. Pedestrian evacuation simulation is a topic of pedestrian flow studies in the field of traffic and granular flow. Recently, considerable researches have been conducted on the issue of pedestrian dynamics based on mathematics,^[1–4] experiments,^[5–7] and simulations. Many microscopic simulation models of pedestrian dynamics have been developed, such as the social force model,^[8] centrifugal force model,^[9] lattice gas model,^[10] two process model,^[11] floors field model,^[12] dynamic parameters model,^[13,14] discrete choice model,^[15,16] etc. Based on the characteristics of individual pedestrian and system surroundings, these simulation models are extended, combined or modified to approximate pedestrian dynamics.

From the view of pedestrian occupying site, pedestrians can occupy more than one cell in the multi-site lattice gas model^[17] or a cell can be occupied by more than one pedestrian in the alternative floor field model.^[18] In order to describe pedestrian preference to move along a certain side of the road, a new drift parameter D_2 was introduced in the extended lattice gas model.^[19] A new right-hand parameter was

introduced in the dynamic parameters model.^[20] The behaviors of pedestrian overtaking and swirling were considered in the behavior-based lattice gas model.^[21]

The study of interactive forces between pedestrians is one of the essential issues in the emergency evacuation with a crowd. The agitated behaviors and elastic characteristics in pedestrian movement are incorporated in the alternative floor field model.^[18] A vector-based particles field was introduced in the floor field model.^[22] The multi-grid model^[23] introduced the force concept of the social force model into the lattice gas model. A friction parameter was introduced in the floor field model^[24,25] to describe the friction influences occurring between pedestrians. The game-theoretic approach was introduced to research crowd dynamic conflicts during the evacuation processes, in which the effect of rationality and herding and the cost of conflicts were taken into the evacuation strategy.^[26]

From the view of pedestrian evacuation from a room with visible adverse effects or life-threatening conditions, the behavior termed “flow with the stream” was simulated in emergency evacuation from a large smoke-filled compartment with the visibility range affected by smoke concentration.^[27] A fire floor field was introduced in the extended floor field model to investigate the dynamics of pedestrian evacuation with the influence of the fire spreading.^[28] A communication field was

*Project supported by the National Basic Research Program of China (Grant No. 2012CB725400), the National Natural Science Foundation of China (Grant No. 11172035), the Fundamental Research Funds for the Central Universities of China (Grant No. 2013JBM046), the China Postdoctoral Science Foundation (Grant Nos. 20090460184 and 201003036), and the Talent Foundation of Beijing Jiaotong University, China (Grant No. 2012RC026).

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introduced in the extended evacuation field model to simulate pedestrian evacuation flow in an emergency under the control of evacuation assistants.^[29] The pedestrian sight radius was introduced in the dynamic parameters model to simulate the evacuation flow with an affected visual field.^[30] The view radius was introduced in the social force model to simulate pedestrian evacuate from a hall full of smoke.^[31]

From the point of a special layout of obstacles or exits in the evacuation room, a logit-based discrete choice principle was introduced in the floor field model^[32] to simulate pedestrian evacuation flow with internal obstacles and multi-exits. The field model and social force model were involved to simulate pedestrian evacuation with obstacles and multi-exits.^[33] The effect of placing an obstacle near the exit on improving the evacuation time under panic was studied by the social force model and the effect of “clever is not always better” was found.^[34] A force-driving cellular automata model and social force model were jointed to investigate the evacuation behaviors of pedestrians at a T-shaped intersection.^[35] The social force model was adopted to investigate the effect of complex building architecture on the uncoordinated crowd motion during urgent evacuation.^[36] A direction visual field was introduced in the floor field model to simulate pedestrian evacuation from a room with multi-exits, which predicted the propagation of pedestrian flow along some directions in the evacuation.^[37] The pedestrian evacuations with asymmetrical exits and pedestrian layout were simulated using a dynamic parameter model,^[38,39] in which the mechanism of max–min evacuation path selection, jam cognition coefficient, and imbalance coefficient were presented.

From the effect of surroundings on pedestrian dynamics, an evolutionary algorithm of particle swarm optimization was introduced in the pedestrian evacuation model, in which both the maximum velocity and the size of the pedestrian depended on the local pedestrian density.^[40] A proxemic floor field was introduced in the floor field model to describe the characteristics of the proxemic behavior of pedestrians, in which the “motivation to move” was dependent on the crowdedness of the neighborhood of each pedestrian.^[41] An anticipation floor field was introduced in the floor field model to focus on the non-local interaction between pedestrians.^[42] A heterogeneous lattice gas model was developed to simulate emergency evacuation, in which the concepts of local population density and exit crowded degree were introduced.^[43]

In these original or improved models mentioned above, researches were focused on the pedestrian movement mechanism,^[8–16] pedestrian route choice,^[2,7,44,45] pedestrian crowd,^[3,4] pedestrian occupying size,^[17,18] pedestrian walking preferences,^[19–21] force effects in a crowd,^[18,22–26] evacuation with adverse condition^[7,27–31] and with obstacles,^[32–34,45,46] evacuation with multi-

exits^[14,32,33,37–39,47] and the effect of the surroundings on the dynamic.^[40,43] In most pedestrian evacuation studies and simulations, the pedestrian facilities were often supposed to be rectangle rooms with one exit. As for the pedestrian evacuation with multi-exits, the logit-based discrete choice^[32] and the direction visual field^[37] were used to formulate the exit choice behavior, and the pedestrian density around exits was considered in the pedestrian movement rules.^[33,47] There has been relatively little research on evacuation with multi-exits, especially on the strategy of exit selection with multi-exits.

However, facilities with more than one exit particularly with asymmetrical pedestrian or exit layout can be frequently observed in real life. The strategy of exit selection will affect the exit selection results, evacuation process, and evacuation time. The evacuation imbalance often is caused by the asymmetry of exits and pedestrian layout under the condition that pedestrian leave the room within the shortest possible movement distance. Pedestrians, who are familiar with the evacuation circumstances without panicking and intend to leave room within the shortest possible evacuation time, will try to reduce the evacuation imbalance to enhance evacuation efficiency. An effective strategy of exit selection should cover the motivation to evacuation with the shortest possible movement distance or evacuation time. Moreover, evacuation time or process depends on the initial pedestrian density in the room. Pedestrian crowding occurring before exits will increase the evacuation time and affect the results of exit selection. Consequently, the critical density is a key factor in the pedestrian arrangement to distinguish whether pedestrian congestion occurs in an evacuation and the strategy of exit selection is in effect.

In the paper, a mixed strategy of exit selection in a pedestrian evacuation simulation with multi-exits is constructed by fusing the distance-based and time-based strategies through a cognitive coefficient, which is embedded in the computation of the shortest estimated distance in the dynamic parameter model. The concept of a jam area layer and the procedure of step-by-step expending are introduced in the computation of the shortest estimated distance, in which the jam count-area of imaginary distance can be dynamically updated and the evacuation room can be completely divided. The balance of the evacuation process, the critical density of the occurring crowd, and the results of exit selection are studied by simulation. The simulation model is also compared with those of the published results. In the following sections, Section 2 describes the pedestrian evacuation model, in which the exit selection strategy is embedded. Section 3 presents the evacuation balance, critical density, and exit selection results in the simulation, and a further comparison and discussions are presented in Section 4. The final section is the conclusion.

2. Model

2.1. Basic rules

The simulation model is defined on a discrete $(W + 2) \times (W + 2)$ cell grid in the two-dimensional system Ω^2 , where the movement area in the room is marked out $W \times W$ cells and W is the system size. The cells next to the system boundary are occupied as the wall, while the blank cells are reserved as room exits. Each cell can either be empty or occupied by no more than one pedestrian. Pedestrians must leave the evacuation system through room exits. After leaving the evacuation system, pedestrians will not re-enter the room

$$\begin{cases} \text{Wall} = \{(x, y) = 2, x = 1, W + 2; y = 1, W + 2\}, \\ \text{Exit} = \{(x, y) = 0, x = 1, W + 2; y = 1, W + 2\}, \\ (x, y) \in \Omega^2, \\ (x, y) = \begin{cases} 1, & \text{for occupied cells by pedestrian,} \\ 0, & \text{for empty cells.} \end{cases} \end{cases} \quad (1)$$

The simulation procedure is divided into discrete time steps. In the time step, the Moore field is adopted as the movement field $F_{(x,y)}$, in which the pedestrian occupies the core cell (x, y) (see Fig. 1(a)). Pedestrians choose to wait or move to any cell in the $F_{(x,y)}$ according to the corresponding transition payoff, and a 3×3 matrix of transition payoff $P = (P_{ij})$ (see Fig. 1(b)) is constructed to describe the transition payoffs for the pedestrian to make choices

$$F_{(x,y)} = \{(m, n) \mid |m - x| \leq 1, |n - y| \leq 1, (m, n) \in \Omega^2\}. \quad (2)$$

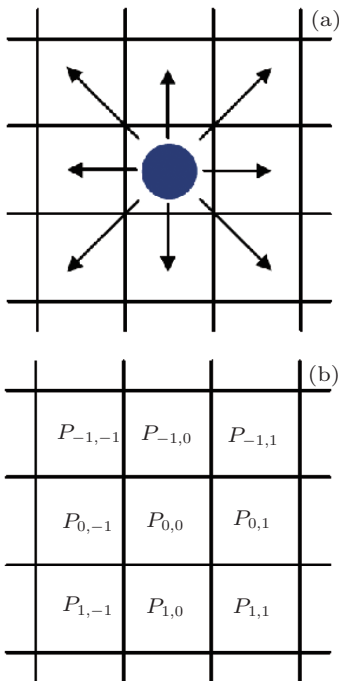


Fig. 1. (color online) The allowed action choices and the associated matrix of transition payoffs. (a) The movement field with a pedestrian occupying the core cell. (b) The associated matrix of transition payoffs.

In the $F_{(x,y)}$, every cell possesses two dynamic parameters: direction-parameter D_{ij} and empty-parameter E_{ij} , and accordingly two associated 3×3 matrices are constructed. The P_{ij} corresponding to the transition payoff for the pedestrian's choice of cell (i, j) as the possible target position in the $F_{(x,y)}$, is determined by two dynamic parameters D_{ij} and E_{ij} of the cell (i, j)

$$\begin{aligned} P_{ij} &= D_{ij} + E_{ij}, \\ D_{ij} &= \frac{S_{xy} - S_{mn}}{\sqrt{(m-x)^2 + (n-y)^2}}, \\ E_{ij} &= \begin{cases} \max(D_{ij}), & (m, n) = 0, \\ 0, & m = x, n = y, \\ -\max(D_{ij}), & (m, n) = 1, (m, n) \neq (x, y), \end{cases} \\ S_{xy} &= \min_k (S_{xy}^k), \\ i &= m - x, \quad j = n - y, \\ (m, n) &\in \Omega^2, \quad (x, y) \in \Omega^2, \end{aligned} \quad (3)$$

where S_{xy} is the shortest estimated distance from the cell (x, y) to the evacuation exits; S_{xy}^k is defined to be the shortest estimated distance from cell (x, y) to the k -th exit.

In the multi-exit simulation model, the strategy of exit selection is embedded in the computation of the shortest estimated distance. These factors affecting the pedestrian to select an exit are fused into the shortest estimated distance, which constructs the strategy of exit selection in an evacuation.

2.2. Exit selection strategy

When there is more than one exit in an evacuation, pedestrians will adopt different strategies of exit selection to evacuate from the room based on different evacuation environments such as panic or normal evacuations. In different evacuations, the evacuation time and movement distance are two key factors that influence a pedestrian's exit choice. Therefore, the strategies of exit selection are classified into two basic types: distance-based strategy and time-based strategy. Pedestrians pursue the shortest evacuation movement distance in an evacuation process with a distance-based strategy; however, the short evacuation time is pursued in a time-based strategy.

In a distance-based strategy, the distance between the pedestrian site and the room exit is a major factor affecting exit choice. Pedestrians, who are unfamiliar with evacuation circumstances with panic, will select the nearest exit from them as their final exit, which can be observed in a panic and an irrational evacuation environment. Because of exit width and pedestrian jam not being considered, the imbalances of the evacuation process and exit utilization often occur in a multi-exit evacuation with a distance-based strategy, in which the jam often occurs before the nearest exit to the pedestrian and the farthest exit is not fully utilized, which induces a longer evacuation time and lower evacuation efficiency. Under the

condition with a certain exit layout, the total amount and initial site of the pedestrian affects the evacuation time, which is decided by the time of the last pedestrian leaving the room. Therefore, the asymmetrical layout of the exit and the pedestrians will increase the evacuation time.

In a time-based strategy, the evacuation aim is to leave the room within the shortest possible time. Pedestrians, who are familiar with evacuation circumstances without panics, will intend to select a slight or no jam exit as their final exit, which can be observed in a normal and rational evacuation environment. Because rational pedestrians are intelligent and adaptive to the dynamic conditions around them by constantly seeking and choosing an optimum exit, the width and layout of exits and the pedestrian jam before each exit will be considered in an evacuation. The evacuation imbalance caused by the asymmetry of exits and pedestrian layout will be lessened in the multi-exit evacuation with a time-based strategy. In a balanced evacuation with a time-based strategy, the crowd is evenly distributed before exits based on exit width and every exit is fully utilized, which induces a shorter evacuation time and a higher evacuation efficiency. Under the condition with a certain exit layout, the evacuation time is dependent on the total amount of pedestrians and the total width of exits. Therefore, the effect of the asymmetrical layout of exits and pedestrians on the evacuation time will be lessened.

These distance-based and time-based evacuations are two basic exit selection strategies, which respectively describe the concerns of pedestrian selection from distance and time in evacuation. However, the evacuation distance and time are often simultaneously taken into account in a realistic evacuation, because of the difference of pedestrians in jam cognition, familiarity with the facilities, selection rationality, etc. Therefore, a mixed strategy is constructed to fuse the distance-based and time-based strategies and describe the consideration of both distance and time in an evacuation. In a realistic evacuation, pedestrian jam is a dominant factor that affects the exit selection, so the pedestrian jam around an exit is adopted to describe the exit selection strategy.

In order to describe the distance-based, time-based, and mixed strategy of exit selection, the actual distance, waiting distance, imaginary distance are introduced in the computation of the shortest estimated distance, in which these strategies are embedded. It is well accepted that the time of passing through an exit with a jam is longer than one without a jam. Therefore, it is assumed that the evacuation time consists of two parts: actual movement time and queue waiting time when a jam occurs around exits. The actual movement time indicates the time that a pedestrian moves from his or her site to the exit at normal evacuation speed without considering a jam. The queue waiting time indicates the time that the pedestrian waits to pass through exits and leave the room. The M_{xy}^i is defined

to indicate the actual distance from cell (x, y) to the i -th exit, which corresponds to the actual movement time, H_{xy}^i is defined to indicate the waiting distance that a pedestrian moves in the queue and the waiting time from cell (x, y) to the i -th exit, Q_{xy}^i is defined to indicate an imaginary distance that the pedestrian moves in evacuation time T_{xy}^i from cell (x, y) to pass through the i -th exit.

In the distance-based strategy, the distance between pedestrian sites and room exits is taken into account in an evacuation. Pedestrians select the nearest exit from them as their final exit without considering the jam before the exit. Therefore, the M_{xy}^i is adopted as the S_{xy}^i , S_{xy}^i is given by

$$S_{xy}^i = M_{xy}^i = \min_j (\sqrt{(x-x_j^i)^2 + (y-y_j^i)^2} + 1), \quad (4)$$

where S_{xy}^i is defined to be the shortest estimated distance from cell (x, y) to the i -th exit; (x, y) are the coordinates of the cell (x, y) in the evacuation system; (x_j^i, y_j^i) are the coordinates of the j -th cell in the i -th exit door. When cell (x, y) is in the exit, $M_{xy}^i = 1$, which indicates that pedestrians need one time step to leave the system from an exit.

In a time-based strategy, the jam before an exit will affect the pedestrian to select exits. The degree of the jam before the exit is described by waiting distance H_{xy}^i . When a large jam occurs before an exit, the number of pedestrians in jam areas and the width of exits are two dominant factors for evacuation time, and the Q_{xy}^i consists of two parts: H_{xy}^i and M_{xy}^i . Because $H_{xy}^i \geq 0$, $Q_{xy}^i \geq M_{xy}^i$. The H_{xy}^i and Q_{xy}^i are respectively given by

$$H_{xy}^i = \frac{2 \times N_{xy}^i}{l_i} - M_{xy}^i, \quad (5)$$

$$Q_{xy}^i = M_{xy}^i + H_{xy}^i = \frac{2 \times N_{xy}^i}{l_i}, \quad (6)$$

where l_i is the width of the i -th exit; N_{xy}^i is the number of pedestrians in the jam count-area for the i -th and cell (x, y) . Moreover, it is found that $H_{xy}^i < 0$ and $Q_{xy}^i < M_{xy}^i$ from Eqs. (5) and (6), when no jam exists around an exit. Therefore, the evacuation time is decided by M_{xy}^i or Q_{xy}^i with a larger value, S_{xy}^i is given by

$$S_{xy}^i = \max(M_{xy}^i, Q_{xy}^i). \quad (7)$$

In Eq. (7), when no jam exists around an exit, $M_{xy}^i \geq Q_{xy}^i$, and M_{xy}^i is the dominant factor in S_{xy}^i , and when a large jam exists around an exit, $Q_{xy}^i \geq M_{xy}^i$ and Q_{xy}^i is the dominant factor in S_{xy}^i .

In a mixed strategy, a cognition coefficient α is introduced in the computation of Q_{xy}^i to describe the cognitive ability of the jam cognition, facilities familiarity and selection rationality of the pedestrian. The Q_{xy}^i of the mixed strategy is given by

$$Q_{xy}^i = M_{xy}^i + \alpha H_{xy}^i, \quad (8a)$$

$$Q_{xy}^i = \alpha \frac{2 \times N_{xy}^i}{l_i} + (1 - \alpha) M_{xy}^i. \quad (8b)$$

From the view of the pedestrian, α indicates the cognitive capability to jam. From the view of the jam, α represents the degree of effect on pedestrian selection. From the view of exit selection, α is a coefficient to weight the evacuation time and evacuation distance. From the view of a mixed strategy, α is a coefficient to weight the time-based and distance-based strategies.

When $1 > \alpha > 0$, the mixed strategy is constructed by fusing the time-based and distance-based strategies, in which movement distance and evacuation time are both considered. α is the weight coefficient of two types of strategy. When $\alpha = 1$, the mixed strategy is reduced to the time-based strategy, in which the effect of a jam around an exit is fully considered, and the pedestrian is entirely sensible to the jam effect and leaves the room within the shortest possible time. When $\alpha = 0$, the mixed strategy is reduced to the distance-based strategy, in which the effect of the jam is not considered and the actual distance determines the selection of the pedestrian's evacuation exit, and the pedestrian leaves the room within the shortest possible movement distance.

2.3. The shortest estimated distance computation

In the computation of Q_{xy}^i , the concept of a jam area layer is introduced to describe the jam count-area. It is supposed that the pedestrian at cell (x, y) can judge the movement direction and destination of every pedestrian and the crowd in the jam count-area around an exit is considered to affect the evacuation time. In the count-area of Q_{xy}^i , these pedestrians are counted, whose movement direction and destination is the i -th exit, and whose actual distance of reaching the i -th exit is shorter than the pedestrian at cell (x, y) . The jam count-area is depicted with different layers. These cells with a j -th level are extended from one with the $(j - 1)$ -th level. The cells in an exit are initialized with the first layer. The layer of the j -th level includes these cells, whose actual distance $M_{xy}^i \in [j, j + 1)$ (see Fig. 2(a)). In order to realize that the jam count-area varies in real time according to the evacuation condition in the room, the procedure of step-by-step expanding is introduced in the computation of S_{xy} .

In the S_{xy} computation, every cell is assigned three tags: F-tag, T-tag, and N-tag. The F-tag indicates the cell has been handled with a final S_{xy} value. The T-tag indicates the cell is with a temporary M_{xy} value. The N-tag indicates the number of cells belonging to an exit and the jam count-areas of Q_{xy} (see Fig. 2(b)). Every exit is assigned a temporary shortest estimated distance ES, and ES^j indicates the ES possessed by the j -th exit.

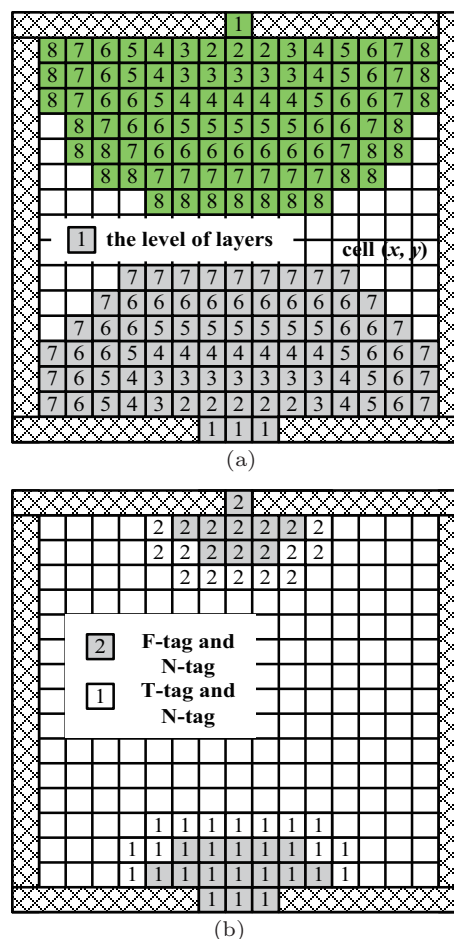


Fig. 2. (color online) The illustration of the jam count-area for an imaginary distance and cells with an F-tag, T-tag and N-tag in the computation of the shortest estimated distance. (a) The room has two exits respectively with two different colors. The tag of cells is the level of layers. The jam count-area for cell (x, y) consists of these cells with the same color as the attached exit and their layer levels are less than that of cell (x, y) . (b) The room has two exits labeled respectively No. 1 and No. 2. These cells with an F-tag are grouped with different exits and possess the same number as the attached exit.

The S_{xy} is assigned as follows.

(I) **Initialization** These cells in an exit are respectively assigned two tags: a T-tag, N-tag and the level of the jam area layer. A T-tag with $M_{xy} = 1$ indicates that the pedestrian needs one time step to leave the system from this site. An N-tag with the number of the exit indicates that these cells belong to the exit. The ES^j for the j -th exit is initialized with a 1. The level of the jam area layer is labeled with a 1.

(II) **Selecting the expended exit and cell** The exit with the N-tag of " i " is selected as the expended exit, which possesses the smallest value $\min(ES^j)$ and T-tag cells exist in these cells with an N-tag of " i ". The cell with the smallest value $\min(M_{xy}^i)$ in these T-tag cells with an N-tag of " i " is selected as the expended cell (with the coordinates (x, y)).

(III) **Expanding cell** Then all cells (m, n) adjacent to the expended cell (x, y) are assigned a T-tag, N-tag and the level of the jam area layer according to the following rules:

1) If the cell (m, n) is without an F-tag or T-tag, then the

cell is assigned an N-tag with “ i ”, a T-tag with M_{mn}^i , and the jam area layer level cell (m, n) is the nearest integer less than or equal to M_{mn}^i

$$M_{mn}^i = (\sqrt{(m-x)^2 + (n-y)^2} + 1). \quad (9)$$

2) If the cell (m, n) is with a T-tag, there are conflicts between the new and old M_{mn}^i value. The new M_{mn}^i is computed by Eq. (9). The minimum possible is assigned to the cell (m, n) and the relevant N-tag and the level of the jam area layer also are changed in the conflict.

(IV) Computation The jam count-area of Q_{xy}^i consists of these cells, whose N-tag is “ i ” and the level of the jam area layer is less than cell (x, y) . The S_{xy}^i is computed by Eq. (7). The cell (x, y) is assigned an F-tag with S_{xy} and $S_{xy} = S_{xy}^i$. The S_{xy} of the room wall is a very large positive number, which indicates that the wall cells do not have any attractiveness to pedestrians.

(V) Updating ESⁱ The S_{xy}^i is selected as the ESⁱ of the i -th exit. Then the computation step skips into step II.

(VI) End condition The process is repeated until all cells are labeled with an F-tag.

2.4. Update rules

Finally, an overall outline of the update rules is as follows.

(i) The pedestrian at cell (x, y) chooses the target cell in the $F_{(x,y)}$ based on the transition payoffs P_{ij} obtained at the current time step.

(ii) The pedestrian would choose the cell with the largest value $P_{\max} = \max(P_{ij})$ as his or her target position at the next time step. If more than one possible target cells ranks P_{\max} , only one of them will be chosen as the target position randomly with equal probability.

(iii) A conflict occurs when any two or more pedestrians attempt to move to the same target position. Only one of them will be chosen randomly with equal probability. The selected pedestrian moves to the corresponding cell at the next time step and the unselected pedestrians stay at the original position and will not move to any other cell.

(iv) If and only if two pedestrians simultaneously choose each other’s presently occupied cell as their target position, will the mutual position exchange occur between the two pedestrians.

(v) When a pedestrian is at cell (x, y) in exit, i.e., $(x, y) \in \text{Exit}$, the pedestrian will leave the room system at the next time step.

(vi) When all the pedestrians leave the room, i.e., $(x, y) \neq 1$, $(x, y) \in \Omega^2$, the simulation procedure is terminated.

All the rules must be applied to all the pedestrians at each time step and a parallel update of rules is adopted.

3. Simulation and results

In the simulation system, the initial density K is defined as the number of total pedestrians existing in the system at the initial time, divided by the total number of cells $W \times W$. The evacuation time T is the sum of time steps the system takes until the last pedestrian leaves the room. The unit evacuation time t is defined as the T divided by the number of total pedestrians at the initial time. The values of T and t are computed according to the statistics of ten simulation runs. Initially, the pedestrians are distributed randomly in the system and there is no pedestrian in the exit cell.

The strategy of a pedestrian selecting an exit in an evacuation with multi-exits is examined through two types of simulation systems: a four-exit room and a two-exit room. In the four-exit room, three types of layout are adopted respectively from ID = 1 to ID = 3, and the exit widths l are equal to each other (Figs. 3(a)–3(c)). In the two-exit room, exits A and B are face-by-face fitted in the middle of the room wall (Fig. 3(d)).

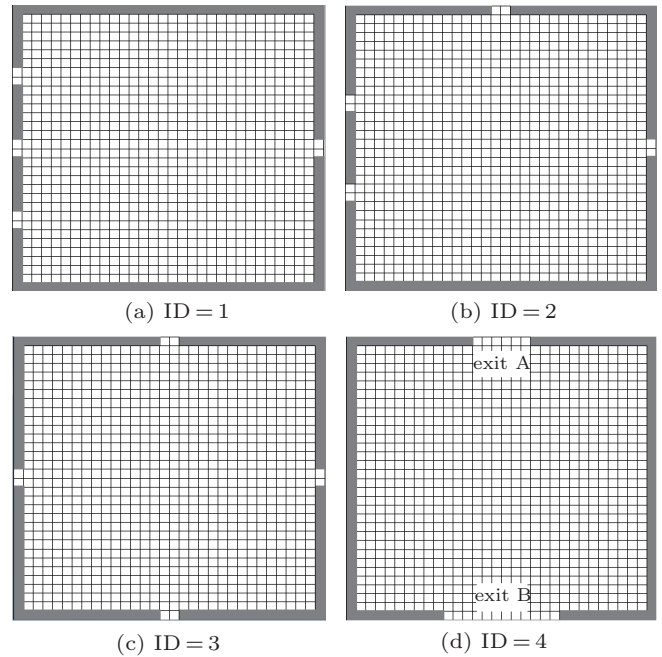


Fig. 3. Schematic illustrations of the four-exit room and the two-exit room. Panels (a)–(c) describe the four-exit room and panel (d) describes the two-exit room. (a) ID = 1 with asymmetry layout. (b) ID = 2 with asymmetry layout. (c) ID = 3 with symmetry layout. (d) ID = 4 with exits A and B in the two-exit room.

The strategy of exit selection is embedded in the computation of the shortest estimated distance. Distance-based and time-based strategies respectively with $\alpha = 0$ and $\alpha = 1$ are two special cases in a mixed strategy with different α . Since distance-based and time-based strategies will result in different evacuation processes, evacuation balance is analyzed in Subsection 3.1. From Eqs. (7) and (8), it is found that pedestrian density and weight coefficient are two key factors to affect exit selection results and evacuation time. Moreover, the degree of pedestrian jam determines whether the strategy is in

effect in exit selection. Therefore, these simulation curves of evacuation time against K or α and the critical density distinguish whether pedestrian congestion occurs in an evacuation are analyzed in Subsection 3.2. These simulation curves of exit selection against K or α are analyzed in Subsection 3.3.

3.1. Evacuation balance

Figure 4 shows the simulation curves of the mean evacuation time $\langle T \rangle$ against the initial density K with $\alpha = 1$, $W = 30$, $l = 2$, and different exit layouts under a time-based strategy, in which the contrast of simulation curves of the distance-based strategy with $\alpha = 0$ and the time-based strategy with $\alpha = 1$ is also shown.

It is found that the time-based selection strategy can realize the evacuation balance and reduce the effect of the multi-exit asymmetry layout on the evacuation. When pedestrians are entirely sensible to the effect of a jam around an exit, i.e., under time-based strategy with $\alpha = 1$, the evacuation time will linearly increase with K rising and do not depend on the exit layout, which indicates that under normal evacuation conditions with rational pedestrians, the evacuation time depends on the total width of all exits and the amount of pedestrians in the room.

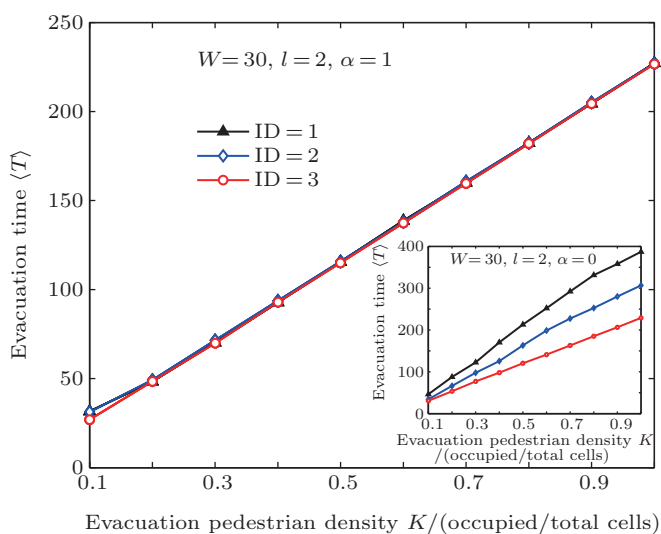


Fig. 4. (color online) The simulation curves of the mean evacuation time $\langle T \rangle$ against the initial densities K with $W = 30$, $l = 2$, $\alpha = 1$, and different exit layouts under the time-based strategy. The inset are simulation curves with $\alpha = 0$ under the distance-based strategy.

3.2. Critical density

Figures 5 and 6 respectively show the simulation curves of the mean evacuation time $\langle T \rangle$ and mean unit evacuation time $\langle t \rangle$ against K with $W = 20$, $ID = 1$, $l = 1$ and different α under the mixed strategy. It is found that the $\langle T \rangle$ will increase with K rising and the $\langle t \rangle$ will decrease with K rising under the mixed strategy with $0 \leq \alpha \leq 1$. The $\langle T \rangle$ depends on the time of the last pedestrian leaving the evacuation room. Since the total width of exits is fixed, pedestrian crowd occurs and gets

worse around exits with K rising. The pedestrian jam increases the $\langle T \rangle$. Because the $\langle t \rangle$ depends on the $\langle T \rangle$ and the amount of pedestrians in the evacuation room, the amount of pedestrian will increase with K rising, therefore the $\langle t \rangle$ will decrease with K rising.

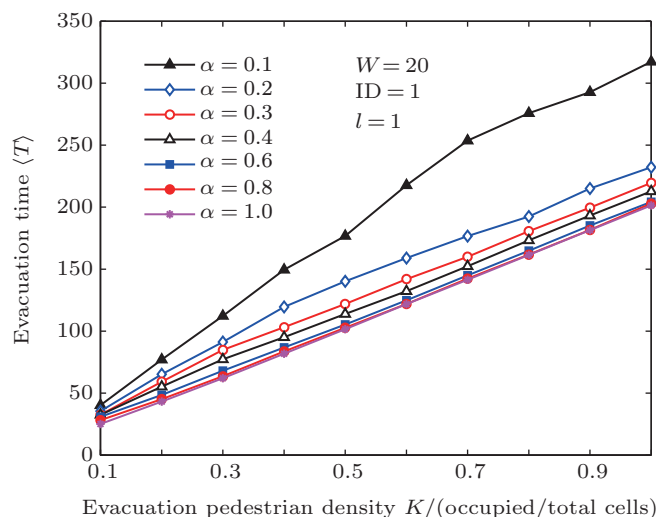


Fig. 5. (color online) The simulation curves of the mean evacuation time $\langle T \rangle$ against initial density K with $W = 20$, $ID = 1$, $l = 1$, and different α under the mixed strategy.

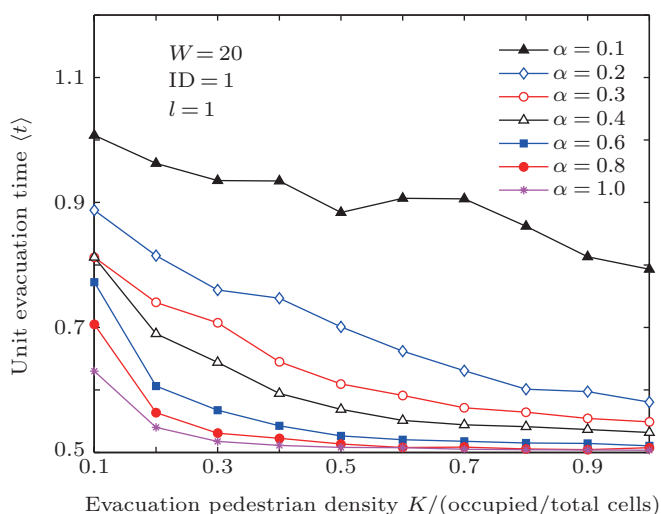


Fig. 6. (color online) The simulation curves of the mean unit evacuation time $\langle t \rangle$ against initial densities K with $W = 20$, $ID = 1$, $l = 1$, and different α under the mixed strategy.

Figures 7 and 8 respectively show the simulation curves of the mean evacuation time $\langle T \rangle$ and mean unit evacuation time $\langle t \rangle$ against α with $W = 20$, $ID = 1$, $l = 1$ and different K . From Figs. 7 and 8, the simulation curves show different characteristics with the low density (e.g., $K = 0.03$) and high density (e.g., $K = 1$). There is a critical density K_c and a corresponding critical amount N_c to distinguish whether pedestrian evacuation is with a low or high density, and whether an exit selection strategy takes effect or not, owing to the effect of the pedestrian jam before an exit on the evacuation process. In the low density with initial density $K \leq K_c$ or pedestrian

amount $N \leq N_c$, there is no pedestrian jam around every exit and so the exit selection strategy is in vain; however, when in the high density with $K > K_c$ or $N > N_c$, pedestrian crowding occurs around exits and so the exit selection strategy takes effect.

dependent on α . Because no jam takes place around exits and the exit width cannot be completely utilized in the evacuation, the evacuation time mainly depends on the pedestrian initial site.

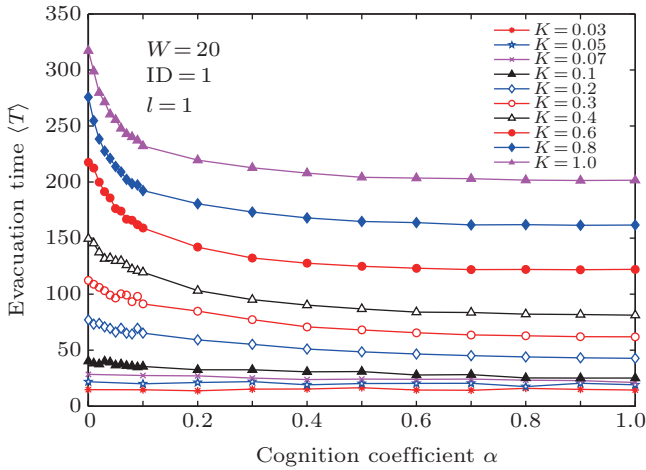


Fig. 7. (color online) Simulation curves of the mean evacuation time $\langle T \rangle$ against the cognition coefficient with $W = 20$, $ID = 1$, $l = 1$, and different initial densities.

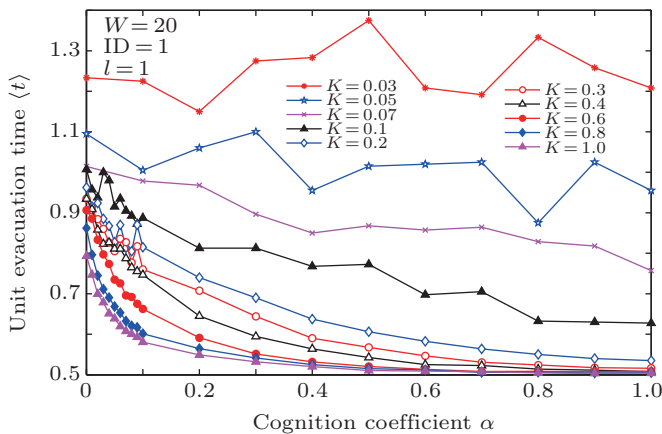


Fig. 8. (color online) Simulation curves of the mean unit evacuation time $\langle t \rangle$ against the cognition coefficient with $W = 20$, $ID = 1$, $l = 1$, and different initial densities.

Under the condition with a high density (e.g., $K = 1$), in which an exit selection strategy is in effect, the $\langle T \rangle$ and $\langle t \rangle$ will reduce with α rising. When pedestrian jam occurs around exits with a high density, the capability of perceiving the jam effect on evacuation is enhanced with α rising. Pedestrians will select a no or relatively slight jam exit to leave the evacuation room. The larger the α , the stronger the intention to deviate from the exit with a serious jam. The difference of jam degree between exits will gradually decrease with α increasing, so the evacuation time will decline accordingly. Under the condition that there are no pedestrian crowds around an exit with a low density (e.g., $K = 0.03$), the simulation curves show a flat tendency with a slight oscillation, which indicates that an exit selection strategy has no effect and the $\langle T \rangle$ and $\langle t \rangle$ are not

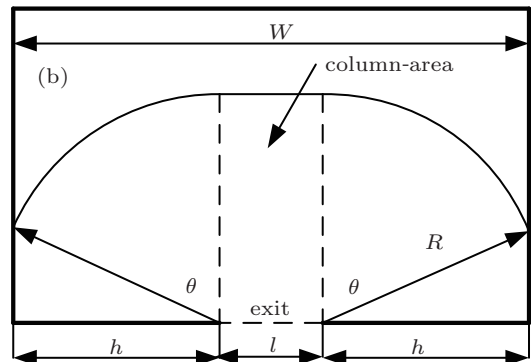
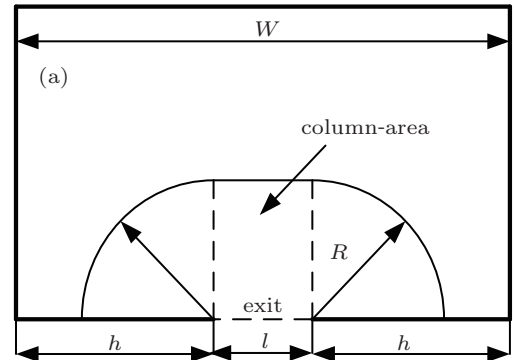


Fig. 9. Schematic illustrations of the count-area and column-area with only one exit. The count-area is encompassed by solid line. The column-area is encompassed by dashed lines. (a) $R \leq h$, (b) $R > h$.

When pedestrians are randomly evenly-distributed before an exit, the condition of whether a crowd occurs around the exit is that $M_{xy} = Q_{xy}$. Since the exit is the bottleneck of pedestrian evacuation, the values of K_c and N_c depend on the width of the exit. The K_c and N_c can be computed by the condition of $M_{xy} = Q_{xy}$. When there is only an exit in a rectangle simulation room, the sizes of the jam count-area vary according to the range of the count-area and the width of the exit (Fig. 9). The size of jam count-area A_c can be computed by

$$A_c = \begin{cases} Rl + \frac{\pi R^2}{2}, & R \leq h, \\ Rl + \theta R^2 + h\sqrt{R^2 - h^2}, & R > h, \end{cases} \quad (10a)$$

$$\theta = \arcsin\left(\frac{h}{R}\right), \quad h = \frac{W-l}{2}, \quad 1 \leq l \leq W, \quad (10b)$$

where l is the width of the exit and R is the range of the jam count-area, W is the length of the wall in which the exit is fixed. The K_c and N_c can be gained by

$$\frac{2A_c K_c}{l} = \frac{2N_c}{l} = R, \quad (11a)$$

$$K_c = \begin{cases} \frac{l}{\pi R + 2l}, & R \leq h, \\ \frac{l}{\theta R + 2l + \sqrt{1 - (h/R)^2}h}, & R > h, \end{cases} \quad (11b)$$

$$\lim_{l \rightarrow W} K_c = 0.5, \tag{11c}$$

$$N_c = \frac{Rl}{2}. \tag{11d}$$

From Eq. (11c), it can be accepted that the evacuation pedestrian flow is transformed into the unidirectional flow with $K_c = 0.5$, when the width of the exit increases to its limit. The unidirectional flow is turned from the free phase to the jam phase around K_c in the ideal condition, which is studied in Ref. [20]. From Eq. (11d), it is found that the value of N_c depends on the rectangle area before the exit, which is named the column-area, the lengths of whose sides are respectively l and R (Fig. 9).

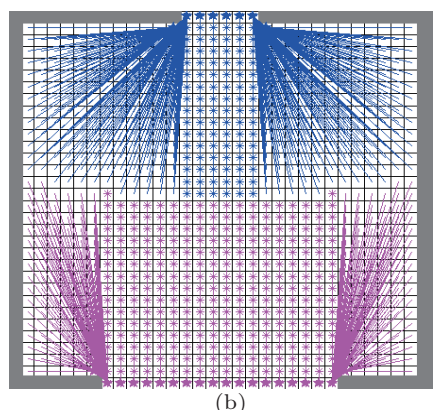
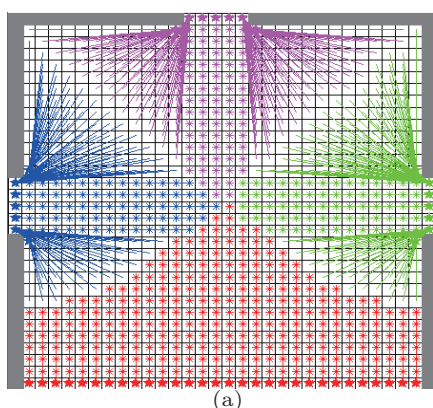


Fig. 10. (color online) The schematic illustrations of the room being divided into different sub-areas according to the shortest estimated distance with $\alpha = 0$, and pedestrian movement room is divided into different sub-areas with different colors. The column-area before the exit is labeled by the start mark * and with the same color as the sub-areas. (a) The room with four exits. (b) The room with two exits.

In the evacuation room with multi-exits and randomly evenly-distributed pedestrians, when $K \leq K_c$, since $M_{xy}^i \geq Q_{xy}^i$ and there is no pedestrian jam around exits, pedestrians select the nearest exit to them as their objective exit and the M_{xy} determines the exit selection with $S_{xy} = M_{xy}$. The S_{xy} can be computed through $\alpha = 0$. The cells in the evacuation room are designated to different exits and the evacuation room is divided into different sub-areas, in which the column area before exits is selected as the exit column-area (Fig. 10), and every exit possesses its sub-area, column-area, and critical density, K_c^i , the critical density of the i -th exit, is given by

$$K_c^i = \frac{N_c^i}{a_i} = \frac{a_i'}{2a_i}, \tag{12}$$

where N_c^i is the critical amount of the i -th exit, a_i is the size of the sub-area of the i -th exit with $\alpha = 0$, a_i' is the size of the column-area of the i -th exit with $\alpha = 0$. In an evacuation with multi-exits, the K_c and N_c of the evacuation system are given by

$$K_c = \min_i(K_c^i), \tag{13a}$$

$$N_c = AK_c, \tag{13b}$$

$$A = \sum_i a_i. \tag{13c}$$

When $K \leq K_c$, the evacuation time depends on the initial sites of the distributed pedestrians. The pedestrian with the largest M_{xy} in the sub-area is the last one to leave the room, who determines the evacuation time. When $K > K_c$, pedestrian jam occurs before the exit and the size of every sub-area varies with K changing (see Fig. 11). The evacuation time depends on the degree of the pedestrian jam and cognition coefficient. The pedestrian with the largest Q_{xy} in the sub-area is the last one leaving the room, who determines the evacuation time. Therefore, under the condition of the mixed selection strategy, actual distance M_{xy} is the dominant factor in the low density with $K \leq K_c$; and imaginary distance Q_{xy} is the dominant factor in the high density with $K > K_c$.

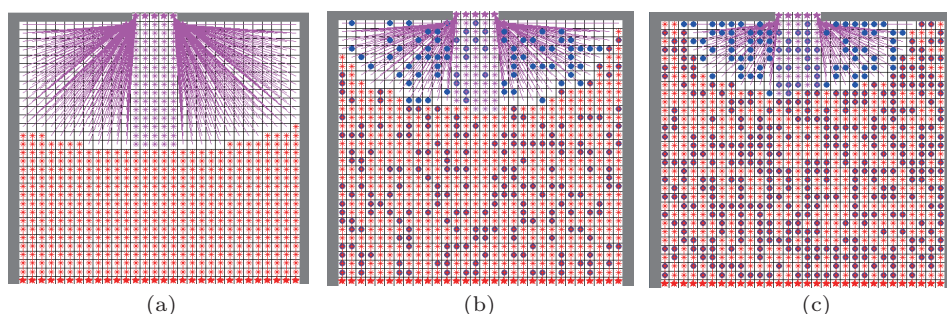


Fig. 11. (color online) The schematic illustrations of the size of sub-area varying with the pedestrian density changing in the two-exit simulation system. The room has two exits and pedestrian movement room is divided into two sub-areas with different colors. (a) $K = 0$, (b) $K = 0.3$, and (c) $K = 0.5$.

3.3. Exit selection result

The percentage of exit sub-area is adopted to describe the results of the pedestrian selecting an exit, which is computed by

$$P_{\text{sub}}^A = \frac{S_{\text{sub}}^A}{\sum_i S_{\text{sub}}^i}, \quad (14a)$$

$$\sum_i P_{\text{sub}}^i = 1, \quad (14b)$$

where P_{sub}^A is the percentage of sub-area for exit A, S_{sub}^A is the size of the sub-area for exit A. The larger the percentage of sub-area for exit A, the more the number of pedestrians selecting exit A.

Figure 12 shows the curves of the percentage of the sub-area for exit A against K in the two-exit simulation system with $W = 30, l_A = 6, l_B = 24$, and different α under the mixed strategy. Figure 13 shows the curves of the percentage of the sub-area for exit A against α in the two-exit simulation system with $l_A = 6, l_B = 24$ and different K .

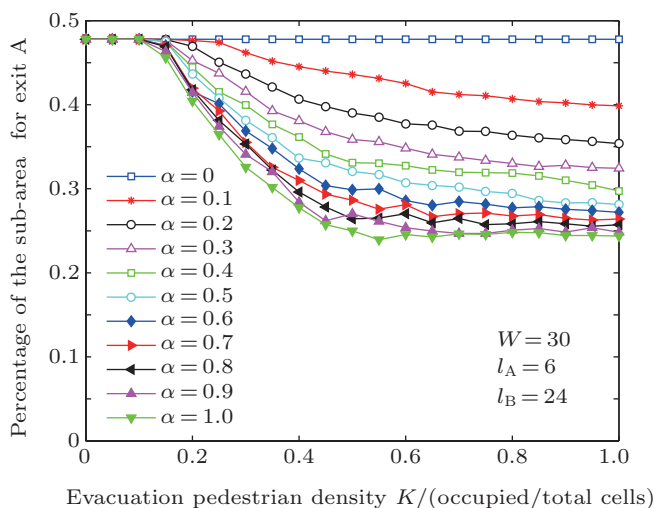


Fig. 12. (color online) The curves of the percentage of the sub-area for exit A against K in the two-exit simulation system with $l_A = 6, l_B = 24$, and different cognition coefficients under the mixed strategy.

In the two-exit system with $\alpha = 0$, since the width of exit B is longer than exit A, the size of the column-area for exit B is larger than for exit A, and $K_c^A \approx 0.1$, the critical density of exit A, is lower than $K_c^B \approx 0.4$, i.e., $K_c^A < K_c^B$. Therefore, the critical density $K_c = K_c^A$.

From Figs. 12 and 13, it can be found there are phase transitions in the curves of sub-area percentage against K . There are two phases: no-effect phase and in-effect phase. There is a critical density K_c^A , around which the evacuation system enters the in-effect phase from the no-effect phase. When $K < K_c^A$, the evacuation system is in the no-effect phase and an exit selection strategy is in vain, in which α and K do not affect the percentage of exit sub-area. The percentage of exit sub-area

does not vary with the rise of K or α . When $K > K_c^A$, the evacuation system is in the in-effect phase and the exit selection strategy is in effect, in which α and K have an effect on the percentage of the exit sub-area. The percentage of sub-area for exit A will decrease with K or α increasing.

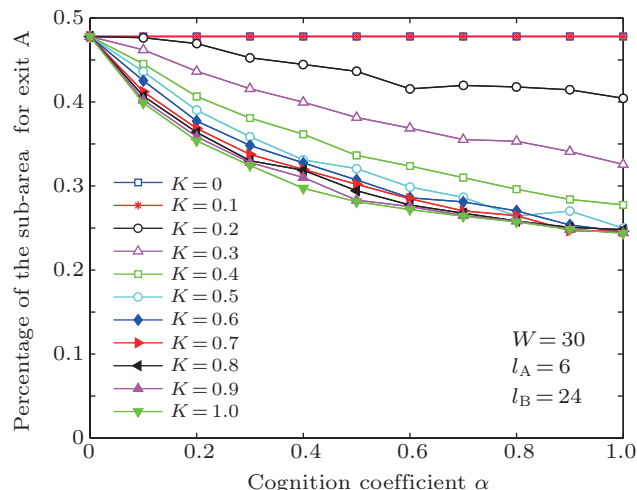


Fig. 13. (color online) The curves of the percentage of the sub-area for exit A against cognition coefficient in the two-exit simulation system with $l_A = 6, l_B = 24$, and different K .

In the two-exit system, since the width of exit B is longer than that of exit A, pedestrian jam occurs more easily before exit A than exit B with a high density. Under the condition with a high density, pedestrian jams around exit A become heavier than around exit B with K rising. The capacity of a pedestrian selecting a slightly jammed exit and giving up a seriously jammed exit will be enhanced with α rising. Therefore, the number of pedestrians selecting exit A and the percentage of sub-area for exit A will decrease with K or α rising.

It is assumed that there is a site point (x, y) in the boundary line between sub-areas. The point (x, y) is named equilibrium point (EP) with $S_{xy} = S_{xy}^A = S_{xy}^B, M_{xy}^A = R, \text{ and } M_{xy}^B = W - R$ (see Fig. 14). Figure 15 shows the relationship between equilibrium point moving and K rising.

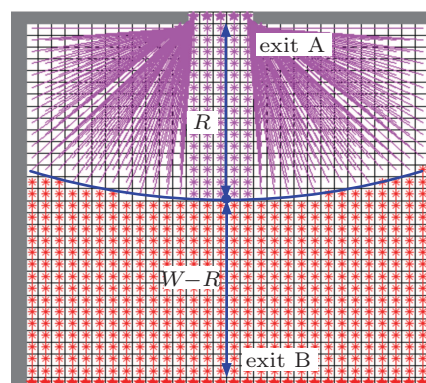


Fig. 14. (color online) The schematic illustration of an equilibrium point in the boundary line between sub-areas.

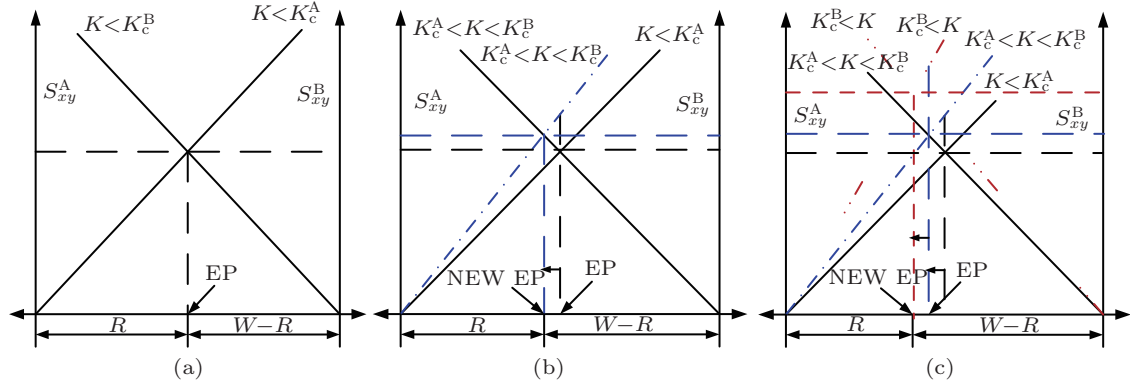


Fig. 15. (color online) The illustration of the equilibrium point movement with K rising. EP is the equilibrium point. (a) $K < K_c^A < K_c^B$, (b) $K_c^A \leq K < K_c^B$, and (c) $K_c^A \leq K_c^B < K$.

When $K < K_c^A < K_c^B$, there is no pedestrian jam before exits A and B, and $S_{xy}^A = M_{xy}^A > Q_{xy}^A$, $S_{xy}^B = M_{xy}^B > Q_{xy}^B$. When $K_c^A \leq K < K_c^B$, pedestrian jam occurs only before exit A and $S_{xy}^A = Q_{xy}^A > M_{xy}^A$, $S_{xy}^B = M_{xy}^B > Q_{xy}^B$. When $K_c^A \leq K_c^B < K$, pedestrian jam occurs before both exits A and B, and $S_{xy}^A = Q_{xy}^A > M_{xy}^A$, $S_{xy}^B = Q_{xy}^B > M_{xy}^B$. The partial derivative of S_{xy} against K are respectively given by

$$\frac{\partial S_{xy}^A}{\partial K} = \frac{\partial M_{xy}^A}{\partial K} = 0, \quad (K \leq K_c^A), \quad (15a)$$

$$\frac{\partial S_{xy}^A}{\partial K} = \frac{\partial Q_{xy}^A}{\partial K} = \frac{2\alpha S_{sub}^A}{l_A}, \quad (K_c^A \leq K), \quad (15b)$$

$$\frac{\partial S_{xy}^B}{\partial K} = \frac{\partial M_{xy}^B}{\partial K} = 0, \quad (K \leq K_c^B), \quad (15c)$$

$$\frac{\partial S_{xy}^B}{\partial K} = \frac{\partial Q_{xy}^B}{\partial K} = \frac{2\alpha S_{sub}^B}{l_B}, \quad (K_c^B \leq K), \quad (15d)$$

where l_A and l_B respectively are the widths of exits A and B, and S_{sub}^A and S_{sub}^B respectively are the sizes of sub-areas for exits A and B.

When $K < K_c^A < K_c^B$, since

$$\frac{\partial S_{xy}^A}{\partial K} = \frac{\partial S_{xy}^B}{\partial K} = 0,$$

the variation of K does not affect the percentage of exit sub-area (see Fig. 15(a)). When $K_c^A \leq K < K_c^B$, since

$$\frac{\partial S_{xy}^A}{\partial K} > \frac{\partial S_{xy}^B}{\partial K},$$

the increase of S_{xy}^A is larger than S_{xy}^B . Therefore, an evacuation system needs to reduce R to find a new equilibrium point to meet $S_{xy}^A = S_{xy}^B$ (see Fig. 15(b)). When $K_c^A \leq K_c^B < K$, since $Q_{xy}^A = Q_{xy}^B$, through Eq. (8b)

$$\frac{2\alpha S_{sub}^A}{l_A} - \frac{2\alpha S_{sub}^B}{l_B} = \frac{(1-\alpha)}{K}(W-2R). \quad (16)$$

The reducing R leads to $(W-2R) > 0$. Through Eqs. (15b), (15d), and (16), it is found that

$$\frac{\partial S_{xy}^A}{\partial K} > \frac{\partial S_{xy}^B}{\partial K},$$

so the percentage of sub-area for exit A will decrease with K increasing when $K > K_c^B$.

In the no-effect phase, since there is no pedestrian jam before exits, α has no effect on the percentage of exit sub-area. Therefore, the simulation curves show the level line with α varying. In the in-effect phase, the percentage of sub-area for exit A will reduce with α rising. Because $\frac{\partial S_{xy}^A}{\partial \alpha} > \frac{\partial S_{xy}^B}{\partial \alpha}$ at the equilibrium point, the simulation system needs to reduce R to find a new equilibrium point with α rising.

In the evacuation with multi-exits, the actual distance M_{xy} and imaginary distance Q_{xy} are two dominant factors to influence the exit selection under the mixed strategy. Pedestrian density is a key factor that causes a jam before an exit. Therefore, there is a critical density to distinguish whether pedestrian evacuation is in a low or high density, which determines whether the exit selection strategy had an effect. In the no-effect phase with a low density, an exit selection strategy is in vain. In other words, the distance-based, time-based, and mixed strategies have the same selection result and exit selection strategy has no effect on evacuation process. Because the pedestrian selects the nearest exit from him or her with the smallest actual distance, the selection of an exit does not depend on the density of pedestrians and the cognition coefficient, but relies on the site of the pedestrian layout in the no-effect phase. In the in-effect phase with a high density, the exit selection strategy is in effect, in which the distance-based, time-based, and mixed strategies have different selection results and an exit selection strategy has an effect on the evacuation process. Because the pedestrian selects the least slightly congested exits from him or her with the smallest imaginary distance, the selection of exit depends on the density of pedestrians and the jam cognition coefficient in the in-effect phase.

4. Comparison with the published model

The pedestrian evacuation with multi-exits is simulated in Ref. [14], in which the evacuation imbalance is caused by

the asymmetry of exit layout and without the jam effect considered. An improvement in pedestrian evacuation with multi-exits is presented in Ref. [38], in which the “max–min” selection of actual and imaginary distance is proposed to reduce evacuation imbalance. Pedestrian evacuation with asymmetrical pedestrian layout is studied in Ref. [39], in which the waiting distance is introduced and the imbalance coefficient is discussed from the viewpoint of the utilization of exits.

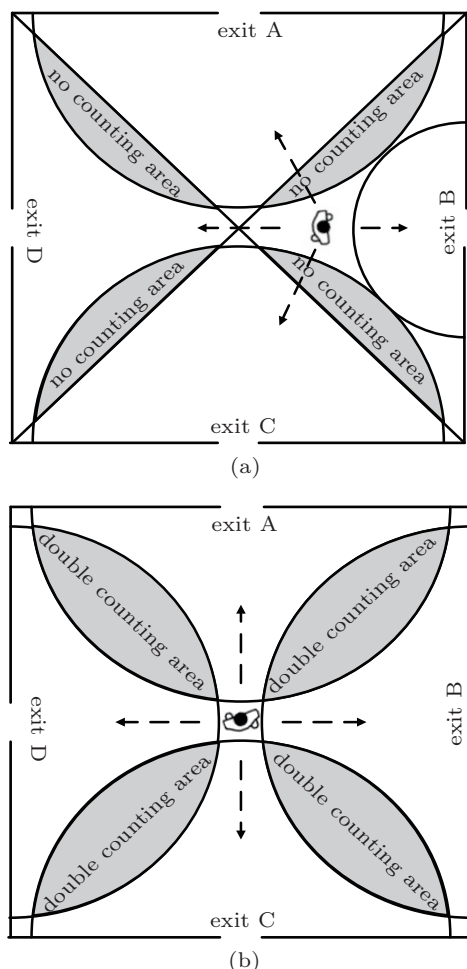


Fig. 16. A schematic illustration of the no-counting area and double-counting area in the jam count-area of imaginary distance with four exits. (a) The no-counting area. The count-area only includes the local area before an exit. There is a no-counting area in the pedestrian evacuation room. (b) The double-counting area. The count-area includes the range between the pedestrian and the exit. There is a double-counting area in the pedestrian evacuation room.

The jam count-area only includes a triangle or semi-circle before an exit in Ref. [38] (see Fig. 16(a)). The jam count-area includes the area between the pedestrian and the exit in Ref. [39] (see Fig. 16(b)). The jam count-area of an imaginary distance is fixed ahead and the evacuation room is divided with a no-counting area^[38] or double-counting area^[39] in the computation of the shortest estimated distance. In order to realize the dynamic variation of the jam count-area with the density changing, the procedure of step-by-step expanding and the concept of a jam area layer are introduced in the computation of the shortest estimated distance, in which the pedestrian

movement room is completely partitioned into different sub-areas without a double-counting area and a no-counting area.

The adjustment of α to a pedestrian jam is carried out through αQ_{xy}^i in Ref. [38], H_{xy}^i is introduced in Ref. [39] in order to delete the adjustment to M_{xy}^i , because reasonable pedestrians only perceive the degree of the jam around an exit with a fixed M_{xy}^i . Therefore, it is adopted that α only adjusts H_{xy}^i through $Q_{xy}^i = M_{xy}^i + \alpha H_{xy}^i$. The models in Refs. [38] and [39] are respectively named models A and B, and the improved model in this paper is named as model C. Figure 17 shows the comparison between simulation curves of the mean evacuation time $\langle T \rangle$ against α with $ID = 1$, $W = 20$, and $l = 1$ and a high K .

From Fig. 17, it can be found that $\langle T \rangle$ will reduce with α rising in the only adjustment to H_{xy}^i in models B and C. However, the simulation curves show the phase transition and critical point in model A. It is also found that the simulation curves show the same variation tendency and are overlapped by each other in models A and C with a high α . The $\langle T \rangle$ of models A and C is lower than that of model B, which indicates models A or C can more effectively reduce the evacuation imbalance caused by the asymmetry layout of exits. Therefore, the simulation model can not only show the characteristics of the $\langle T \rangle$ gradually changing against α , but also the effectiveness of reducing evacuation imbalance.

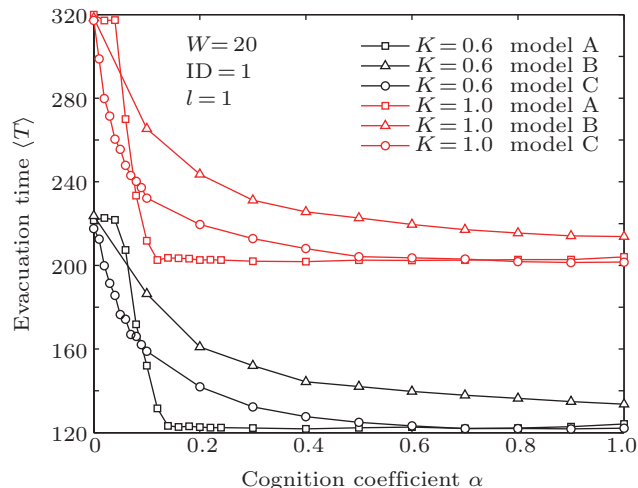


Fig. 17. (color online) The simulation curves of the mean evacuation time $\langle T \rangle$ against the cognition coefficient with $W = 20$, $ID = 1$, $l = 1$, and a high K in the different models.

5. Conclusion

In this paper, the simulation of pedestrian evacuation from a room with multi-exits is presented based on an improved dynamic parameter model. Distance-based and time-based strategies of exit selection in a simulation are analyzed respectively from the movement distance and the evacuation time. A mixed strategy of exit selection is constructed by fusing the distance-based and time-based strategies through a cognitive coefficient, which is embedded in the computation

of the shortest estimated distance. The concept of a jam area layer and the procedure of step-by-step expanding are introduced in the computation of the shortest estimated distance, in which the jam count-area of imaginary distance is dynamically updated and the evacuation room is completely divided into different sub-areas without a no-counting area and a double-counting area. The mixed strategy can effectively reduce the evacuation imbalance caused by the asymmetry of exits or the pedestrian layout. It is found that there is a critical density to distinguish whether pedestrian congestion occurs in a pedestrian evacuation where an exit selection strategy is in effect, that there are two phases: a no-effect phase with a low density and an in-effect phase with a high density. In the no-effect phase, there is no pedestrian jam in the evacuation, and the strategy has no effect on the evacuation process, and the results of exit selection and evacuation time depend on the initial site of the pedestrian layout. In an in-effect phase, pedestrian jams occur in an evacuation, and the strategy is in effect; the results of exit selection and evacuation time depend on the cognitive coefficient and the initial density. Through the comparison between the improved and the published models, it is found that the simulation model shows the characteristics of the evacuation time gradually changing against the jam cognition coefficient and the effectiveness of reducing the evacuation imbalance.

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