Modulation Doping for Iterative Demapping of Bit-Interleaved Coded Modulation

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***Abstract*— In this letter, the problem of the transmitter design for bit-interleaved coded modulation (BICM) with the receiver employing iterative demapping, is addressed. Conventionally, the design is focused on the appropriate choice of the constellation mapping so that the iterative process converges to small values of bit- or block- error rates. Here, instead of a difficult design of a new mapping, we propose to use symbols modulated using two different mappings. Through numerical simulation, our simple design is shown to outperform the conventional approach without increase in the receiver's complexity.**

***Index Terms*— Bit Interleaved Coded Modulation (BICM), iterative demapping, iterative decoding.**

I. Introduction

BIT INTERLEAVED coded modulation (BICM) is a coded modulation transmission scheme suited for fading and non-fading channels [1]. To improve the performance of the BICM, iterative exchange of information between the de-mapper and the decoder was proposed in [2] [3]. Such BICM with iterative demapping (BICM-ID), provides gains throughout the iterations if the bits-to-symbol *mapping* is appropriately chosen. To design the mapping, heuristics were used in [4] and computer search was employed in [5]. The main challenge of the design lies in the lack of a well defined criterion reflecting the performance [expressed e.g., in terms of bit error rate (BER) or block error rate (BLER)] for a limited number of iterations and for a given signal to noise ratio (SNR). To avoid this issue, the design proposed in [5] optimizes the performance when a perfect a priori information about the bits is provided by the decoder (the so-called perfect feedback [4]). The mappings designed in such way will be called herein *anti-Gray* (name used, e.g., in [6],[7]).

The assumption of the perfect feedback indeed materializes after sufficiently large number of iterations and for the SNR high enough to trigger the convergence [4] [5]. On the other hand, for the low SNR and/or for a small number of iterations the feedback is not perfect and the performance of the BICM-ID is adversely affected by the anti-Gray mapping. To take the "imperfect feedback" into account, a method to predict the performance of BICM-ID for arbitrary mapping would be needed. Unfortunately such method does not exist, so the only way to design the mapping would be via extensive computer simulations and the results would not generalize to different codes, block lengths or channel characteristics.

In this letter we present a new approach allowing us to take advantage of the BICM-ID in all ranges of SNR, for an arbitrary number of iterations, and for arbitrary block-length. To circumvent the difficulty of the aforementioned extensive simulations search we propose to employ, within the same data block, two different mappings: the Gray and the anti-Gray. Mixture of symbols obtained with different mappings ensures desired behavior of the iterative process. Transmission with such *modulation doping*[1] implies that only one parameter (a doping ratio - a scalar defining the proportion of Gray and anti-Gray mapped symbols) has to be chosen, which greatly reduces the search complexity. The very idea of modulation doping was presented first in [9][10] and was called therein *irregular modulation*. The doping ratio was selected in [9] using the so-called EXIT charts. Our approach is slightly different being based on the actual outcome of the simulations; in this way the finite block-length and/or limited number of iteration are taken care of.

This letter is organized as follows. In Section II the system model and principle of iterative demapping are introduced. In Section III we show how to select the system's parameter and we show that the proposed approach yields gain over the conventional BICM-ID (with anti-Gray mapping only); the so-called EXIT chart is also used to give an explanation of the obtained improvement. The conclusions are drawn in Section IV.

II. System Model

We consider the BICM system shown in Fig. 1 [4][5], where a block of information bits $b(\tau),\ \tau = 1, \ldots, N_{\mathrm{b}}$ is encoded by a convolutional encoder of rate R. The output of the encoder is randomly interleaved through $\Pi\{\cdot\}$ and transformed into binary codewords of length m, $\mathbf{c}(t) = [c_1(t), \ldots, c_m(t)] \in \{0,1\}^m$, where $t = 1, \ldots, N_{\mathrm{s}}$ denotes discrete time and $N_{\mathrm{s}} = \frac{N_{\mathrm{b}}}{R\cdot m}$. The codewords $\mathbf{c}(t)$ are mapped into symbols $s(t)$ taken from the complex constellation $\mathcal{X}$ using the one-to-one mapping $\mu_t\{\cdot\}$, i.e., $s_m(t) = \mu_t\{\mathbf{c}_m(t)\} \in \mathcal{X}$. The constellation is unbiased $\sum_{a\in\mathcal{X}} a = 0$ and power-normalized $\frac{1}{2^m}\sum_{a\in\mathcal{X}} |a|^2 = 1$. The mapping may vary in time t; here we consider only the case where $\mu_t\{\cdot\} \equiv \mu_1, t = 1, \ldots, N_1$ and $\mu_t\{\cdot\} \equiv \mu_2, t = N_1 + 1, \ldots, N_{\mathrm{s}}$, i.e., only two different mappings $\mu_1\{\cdot\}$ and $\mu_2\{\cdot\}$, being respectively anti-Gray and Gray, are used. As we learned, the full credit for this idea goes to [9][10]; this modulation doping being an important

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[1]Name coined by analogy to the method employed in [8] in the context of repeat-accumulate codes.

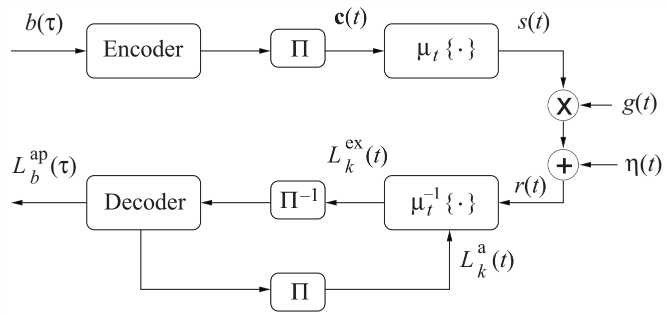

Fig. 1. System model.

distinctive feature comparing to the conventional schemes proposed for BICM-ID, e.g. in [4] [5], practically does not increase the complexity of the de-mapper, which only needs to change the operation mode depending on the time t.

The modulated signals are transmitted over a memoryless channel yielding the signal $r(t) = g(t)s(t) + \eta(t)$, where $g(t)$ is the complex channel's gain and $\eta(t)$ is a complex, zero-mean white Gaussian noise whose real and imaginary parts are independent, each with variance $N_0/2$. Since the channel identification is out of scope of this letter, the channel state [i.e., $g(t)$ and N_0] is assumed known, cf. [4] [5].

The extrinsic logarithmic likelihood ratios (LLRs) $L_k^{\rm ex}(t)$ for the coded bits $c_k(t)$ are produced by the de-mapper $\mu_t^{-1}\{\cdot\}$ using max-log simplification

$$L_k^{\rm ex}(t) = \min_{a \in \mathcal{X}_0^k} \left\{ \frac{|r(t) - g(t)a|^2}{N_0} - \sum_{j \neq k} L_j^{\rm a}(t)\lambda_j\{a\} \right\} - \min_{a \in \mathcal{X}_1^k} \left\{ \frac{|r(t) - g(t)a|^2}{N_0} - \sum_{j \neq k} L_j^{\rm a}(t)\lambda_j\{a\} \right\}, \quad (1)$$

where $L_k^{\rm a}(t)$ is the a priori LLR for the coded bit $c_k(t)$ obtained from the decoder in the previous iteration, $\lambda_j\{a\}$ is an operator extracting the j-th bit labelling the symbol $a \in \mathcal{X}$ and $\mathcal{X}_\beta^k$ is the set of symbols having the k-th bit set to β, i.e. $\mathcal{X}_\beta^k = \{a : \lambda_k\{a\} = \beta\}$. Since the sets $\mathcal{X}_\beta^k$ depend on the mappings $\mu_t\{\cdot\}$, the de-mapper must adjust its operation according to the time t. The decisions about the information bits $b(\tau)$ are made from the sign of a-posteriori LLRs $L_b^{\rm ap}(\tau)$ produced by the decoder.

The LLRs calculated by the demapper are deinterleaved (inverting the operation of the bit-level interleaver $\Pi\{\cdot\}$ applied prior to the modulation) and fed to the soft-input soft-output (SISO) a-posteriori decoder [11], implemented here using the max-log simplification. Thanks to the deinterleaver the LLRs obtained from symbols with different mappings are spread throughout the data block. The extrinsic LLRs related to the coded bits and produced by the decoder are used in the subsequent iterations as a-priori LLRs in (1).

For the numerical simulations presented in this letter, $\mathcal{X}$ is a 16-ary quadrature amplitude modulation (16-QAM) and Gray and anti-Gray mappings are shown in Fig. 3; the latter taken from [5]. A systematic recursive convolutional code of rate $R = \frac{1}{2}$ with generating polynomials $\{1\ 23/35\}_8$ is used [5], bits are not flushed at the block's end, i.e., the trellis is

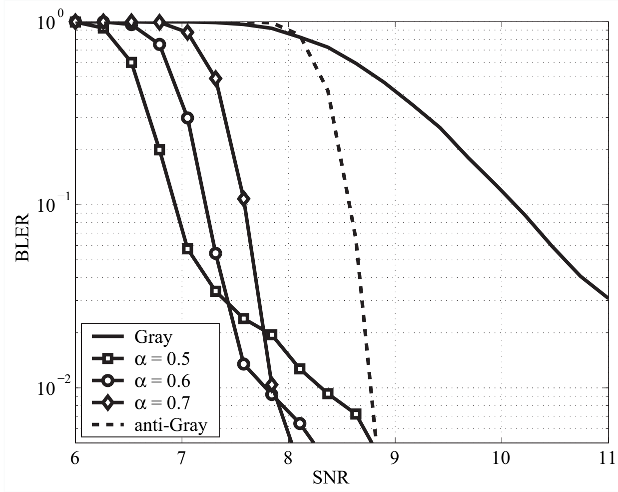

Fig. 2. Performance of the BICM-ID after 8 turbo-iterations in AWGN channel for different values of the doping ratio .

not terminated. The interleaver is generated randomly and the number of information bits in the block is equal to $N_{\rm b} = 2000$, thus $N_{\rm s} = 1000$.

We consider here additive white Gaussian noise (AWGN) channel, i.e. $g(t) \equiv 1$ so the SNR is equal to $SNR = \frac{1}{N_0}$, however, the same approach can be used in fading channels.

III. Design Procedure

Using the model defined in Section II, the system's design consists in choosing the doping ratio $\alpha = \frac{N_1}{N_{\rm s}} \in \{0, 1\}$ to obtain a desirable performance.

The design itself is very simple indeed: the value of α producing the best results (e.g., in terms of BER or BLER) is chosen on the basis of the results of simulations. Since only the scalar parameter α needs to be adjusted, the search over whole range of its values does not introduce a significant computational overhead and may be easily repeated if the system's parameters (code or block length) change. We show in Fig. 2 the BLER obtained in AWGN channel after eight turbo-iterations at the receiver.

We observe that, to optimize the performance (BLER), α should vary with SNR, which may be accomplished through the SNR measurement at the receiver sent over a feedback channel to the transmitter. If we target the BLER $BLER_{\rm t} = 10^{-1}$ we should use $\alpha = 0.5$, but for $BLER_{\rm t} = 10^{-2}$, $\alpha = 0.6$ is the most appropriate. Only for sufficiently high SNR, using pure anti-Gray mapping ($\alpha = 1$) provides the best results (not shown in the figure), but the improvement is rather marginal since the BLER value is already very low (less than 10^{-2}), so the effective throughput is very close to the nominal spectral efficiency (here, 2 bits per channel use). A reasonable tradeoff is obtained with constant $\alpha = 0.6$; then the proposed BICM-ID scheme outperforms the conventional pure anti-Gray mapping design by 1.4dB (at $BLER_{\rm t} = 10^{-1}$) or 1dB at $BLER_{\rm t} = 10^{-2}$. Such improvement is obtained without increasing the complexity of the receiver.

The value of α may be adjusted depending on the number of iterations, the latter being dictated by available computational resources at the receiver (results not shown for lack of space).

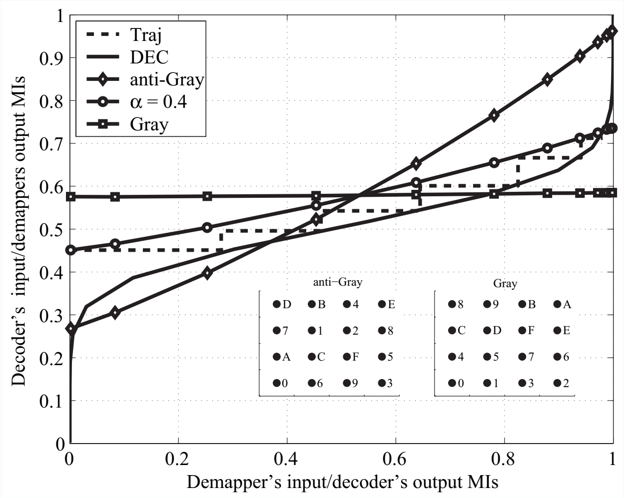

Fig. 3. Decoder's EXIT function (DEC) and the demapper's functions obtained for =6.5dB when using mappings Gray, anti-Gray and for the proposed modulation doping with = 0 4. For the latter, the EXIT trajectory (TRAJ) is shown to illustrate the evolution of the MI throughout the iterations. The labellings Gray and anti-Gray are shown in hexadecimal format.

Finally, to give the reader an intuitive understanding of the working principle of the proposed scheme we present in Fig. 3 the so-called extrinsic information transfer (EXIT) chart [12], obtained in AWGN channel. The axes show mutual information (MI) between extrinsic LLRs and the corresponding coded bits measured at the input/output of the de-mapper/decoder. For more details about EXIT charts we refer the reader to [6] [12].

As known, the Gray mapping's EXIT function ($\alpha = 0$) is quasi-flat [6], thus only slight improvement over iterations may be obtained. On the other hand, the anti-Gray mapping's EXIT function ($\alpha = 1$) increases with MI, thus leads to improvement over iterations, provided it does not intersect the decoder' EXIT function too early (as it is the case for SNR=6.5dB in Fig. 3). To make the EXIT functions cross for high value of MI, the demapper's function should be an average of the Gray and the anti-Gray's functions. Employing both mappings within the same data block corresponds indeed to such MI averaging. In the shown example, using $\alpha = 0.4$, the de-mapper's function is flattened enough (with respect to the anti-Gray mapping) to open the so-called "tunnel" [12], which allows for iterative improvement, as indicated by the so-called trajectory, shown as well in the figure.

Using the EXIT chart interpretation, the dependance of optimal (i.e, minimizing the BLER) α on the SNR, is intuitively clear: the anti-Gray mapping produces the best results for high SNR; on the other hand, for moderate SNR, the convergence is not triggered and proposed modulation doping improves the performance.

IV. Conclusions

In this letter we have presented a new design paradigm for the bit-interleaved coded modulation when the iterative demapping is used at the receiver (BICM-ID). Instead of a difficult approach of designing the mapping for a limited SNR and/or for a small number of iterations, we propose to use two different but known mappings to modulate the symbols within the same transmitted data block. The symbols from each mapping must be inserted with adequate doping ratio, which is easy to find through simulations. The proposed approach does not increase the processing complexity of the receiver, leads to a simple and flexible design and is shown to outperform the conventional approach of using only one prescribed mapping.

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References

[1] G. Caire, G. Tarrico, and E. Biglieri, "Bit-interleaved coded modulation," *IEEE Trans. Inform. Theory*, vol. 44, pp. 927–946, May 1998.

[2] X. Li and J. A. Ritcey, "Bit-interleaved coded modulation with iterative decoding," *IEEE Commun. Lett.*, vol. 1, pp. 169–171, Nov. 2002.

[3] S. ten Brink, "Iterative demapping for QPSK modulation," *IEE Electronics Lett.*, vol. 34, pp. 1459–1460, July 1998.

[4] A. Chindapol and J. A. Ritcey, "Design, analysis, and performance evaluation for BICM-ID with square QAM constellations in rayleigh fading channels," *IEEE J. Select. Areas Commun.*, vol. 19, pp. 944–957, May 2001.

[5] F. Schreckenbach, N. Görtz, J. Hagenauer, and G. Bauch, "Optimization of symbol mappings for bit-interleaved coded modulation with iterative decoding," *IEEE Commun. Lett.*, vol. 07, pp. 593–595, Dec. 2003.

[6] S. ten Brink, "Convergence of iterative decoding," *IEE Electronics Lett.*, vol. 35, pp. 806–808, May 1999.

[7] M. Tüchler, "Design of serially concatenated systems depending on the block length," *IEEE Trans. Commun.*, vol. 52, pp. 209–218, Feb. 2004.

[8] S. ten Brink and G. Kramer, "Design of repeat-accumulate codes for iterative detection and decoding," *IEEE Trans. Signal Processing*, vol. 51, pp. 2764–2772, Nov. 2003.

[9] F. Schreckenbach and G. Bauch, "Irregular signal constellations, mappings and precoder," in *Proc. International Symposium on Information Theory and its Applications (ISITA)*, Oct. 2004.

[10] F. Schreckenbach and G. Bauch, "Adaptive bit-interleaved coded irregular modulation," in *Proc. 14th IST Mobile & Wireless Communications Summit*, pp. 1332–1336, June 2005.

[11] S. Benedetto, D. Divsalar, G. Montorsi, and F. Pollara, "A soft-input soft-output APP module for iterative decoding of concatenated codes," *IEEE Commun. Lett.*, vol. 1, pp. 22–24, Jan. 1997.

[12] S. ten Brink, "Convergence behaviour of iteratively decoded parallel concatenated codes," *IEEE Trans. Commun.*, vol. 49, pp. 1727 –1737, Oct. 2001.