INTUITIONISTIC FUZZY IDEALS OF NEAR-RINGS

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Abstract. In this paper, we introduce the concept of intuitionistic fuzzy ideals of near-rings, and obtain some related properties.

1. Introduction and Preliminaries W.Liu ([1]) has studied fuzzy ideals of a ring, and many researchers are engaged in extending the concepts. S.Abou-Zaid([2]) introduced the notion of a fuzzy subnear-ring and studied fuzzy left(resp.right) ideals of a near-ring, and many followers ([3,4,5,6]) discussed further properties of fuzzy ideals in near-rings. The idea of " intuitionistic fuzzy set" was first published by Atanassov ([7,8]), as a generalization of the notion of fuzzy sets. In this paper, we introduce the concept of intuitionistic fuzzy ideals of near-rings and investigate some related properties.

By a near-ring we mean a non-empty set R with two binary operations "+ " and " $\cdot$ " satisfying the following axioms:

(i)$(R, +)$ is a group

(ii)$(R, \cdot)$ is a semigroup

(iii) $x \cdot (y+z) = x \cdot y + x \cdot z$ for all $x, y, z \in R$.

Precisely speaking, it is a left near-ring because it satisfies the left distributive law. We will use the word " near-ring " instead of " left near-ring ". We denote xy instead of $x \cdot y$. An ideal of a near-ring R is a subset I of R such that

(i) $(I, +)$ is a normal subgroup of $(R, +)$

(ii)$RI \subseteq I$

(iii) $(x+i)y - xy \in I$ for all $i \in I$ and $x, y \in R$.

By a fuzzy set μ in a non-empty set X, we mean a function $\mu : X \to [0,1]$, and the complement of μ, denoted by $\overline{\mu}$, is the fuzzy set in X given by $\overline{\mu}(x) = 1 - \mu(x)$ for all $x \in X$. For any $t \in [0,1]$, and a fuzzy set μ in a non-empty set X, the set $U(\mu; t) = \{x \in X | \mu(x) \geq t\}$ is called an upper t-level cut of μ, and the set $L(\mu; t) = \{x \in X | \mu(x) \leq t\}$ is called a lower t-level cut of μ.

An intuitionistic fuzzy set (briefly, IFS) A in a nonempty set X is an object having the form $IFSA = \{(x, \alpha_A(x), \beta_A(x)) | x \in X\}$, where the functions $\alpha_A : X \to [0,1]$ and $\beta_A : X \to [0,1]$ denote the degree of membership and the degree of nonmembership, respectively, and $0 \leq \alpha_A(x) + \beta_A(x) \leq 1, x \in X$.

An intuitionistic fuzzy set $IFSA = \{(x, \alpha_A(x), \beta_A(x)) | x \in X\}$ in X can be identified to an order pair (α_A, β_A) in $I^X \times I^X$. For the sake of simplicity, we shall use the symbol $IFSA = (\alpha_A, \beta_A)$ for the $IFSA = \{(x, \alpha_A(x), \beta_A(x)) | x \in X\}$.

Definition 1.1. **([5])** A fuzzy set μ in a near-ring R is called a fuzzy ideal of R if it satisfies:

(F1) $\mu(x-y) \geq min\{\mu(x), \mu(y)\}$

(F2) $\mu(y+x-y) \geq \mu(x)$

(F3) $\mu(xy) \geq \mu(y)$

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(F4) $\mu((x+z)y - xy) \geq \mu(z)$
for all $x, y, z \in R$.

Definition 1.2. **([6])** A fuzzy set μ in a near-ring R is an anti fuzzy ideal of R if it satisfies:
(AF1) $\mu(x-y) \leq max\{\mu(x), \mu(y)\}$
(AF2) $\mu(y+x-y) \leq \mu(x)$
(AF3) $\mu(xy) \leq \mu(y)$
(AF4) $\mu((x+z)y - xy) \leq \mu(z)$
for all $x, y, z \in R$.

2. Main Results

Definition 2.1. An $IFSA = (\alpha_A, \beta_A)$ in a near-ring R is called an intuitionistic fuzzy ideal of R if it satisfies:
(IF1) $\alpha_A(x-y) \geq min\{\alpha_A(x), \alpha_A(y)\}$
(IF2) $\alpha_A(y+x-y) \geq \alpha_A(x)$
(IF3) $\alpha_A(xy) \geq \alpha_A(y)$
(IF4) $\alpha_A((x+z)y - xy) \geq \alpha_A(z)$
(IF5) $\beta_A(x-y) \leq max\{\beta_A(x), \beta_A(y)\}$
(IF6) $\beta_A(y+x-y) \leq \beta_A(x)$
(IF7) $\beta_A(xy) \leq \beta_A(y)$
(IF8) $\beta_A((x+z)y - xy) \leq \beta_A(z)$
for all $x, y, z \in R$.

Example 2.2. Let $R = \{a, b, c, d\}$ be a set with two binary operation as follows:

$+$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

$\cdot$	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	b	b

Then $(R, +, \cdot)$ is a near-ring. Let $IFSA = (\alpha_A, \beta_A)$ in R defined by $\alpha_A(a) = 0.8, \alpha_A(b) = 0.6, \alpha_A(c) = \alpha_A(d) = 0.3$ and $\beta_A(a) = 0.2, \beta_A(b) = 0.3, \beta_A(c) = \beta_A(d) = 0.7$. It's easy to show that $IFSA = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of R.

Proposition 2.3. Every intuitionistic fuzzy ideal $IFSA = (\alpha_A, \beta_A)$ of a near-ring R, then $\alpha_A(0) \geq \alpha_A(x)$ and $\beta_A(0) \leq \beta_A(x)$ for all $x \in R$.

Proof. Straightforward.

Lemma 2.4. An $IFSA = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of a near-right R if and only if α_A and $\overline{\beta}_A$ are fuzzy ideals of R.

Proof. Let $IFSA = (\alpha_A, \beta_A)$ be an intuitionistic fuzzy ideal of R. Clearly α_A is a fuzzy ideal. For every $x, y, z \in R$, we have (IF5) $\overline{\beta}_A(x-y) = 1 - \beta_A(x-y) \geq 1 - max\{\beta_A(x), \beta_A(y)\} = min\{1-\beta_A(x), 1-\beta_A(y)\} = min\{\overline{\beta}_A(x), \overline{\beta}_A(y)\}$, (IF6) $\overline{\beta}_A(y+x-y) = 1-\beta_A(y+x-y) \geq 1-\beta_A(x) = \overline{\beta}_A(x)$, (IF7) $\overline{\beta}_A(xy) = 1-\beta_A(xy) \geq 1-\beta_A(y) = \overline{\beta}_A(y)$, (IF8) $\overline{\beta}_A((x+z)y-xy) = 1-\beta_A((x+z)y-xy) \geq 1-\beta_A(z) = \overline{\beta}_A(z)$. Hence $\overline{\beta}_A$ is a fuzzy ideal of R.

Conversely, assume that α_A and $\overline{\beta}_A$ are fuzzy ideals of R. For every $x, y, z \in R$, we get (IF5) $\overline{\beta}_A(x-y) \geq min\{\overline{\beta}_A(x), \overline{\beta}_A(y)\}$, and that, $1-\beta_A(x-y) \geq min\{1-\beta_A(x), 1-\beta_A(y)\} = 1 - max\{\beta_A(x), \beta_A(y)\}$, that is, $\beta_A(x-y) \leq max\{\beta_A(x), \beta_A(y)\}$, (IF6) $\overline{\beta}_A(y+x-y) \geq \overline{\beta}_A(x)$, and that , $1-\beta_A(y+x-y) \geq 1-\beta_A(x)$, that is $\beta_A(y+x-y) \leq \beta_A(x)$, (IF7)$\overline{\beta}_A(xy) \geq \overline{\beta}_A(y)$, and that, $1-\beta_A(xy) \geq 1-\beta_A(y)$, that is, $\beta_A(xy) \leq \beta_A(y)$, (IF8) $\overline{\beta}_A((x+z)y-xy) \geq \overline{\beta}_A(z)$, and that, $1-\beta_A((x+z)y-xy) \geq 1-\beta_A(z)$, that is, $\beta_A((x+z)y-xy) \leq \beta_A(z)$. Hence $IFSA = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of R.

Theorem 2.5. Let $IFSA = (\alpha_A, \beta_A)$ in R. Then $IFSA = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of a near-ring R if and only if $\Box A = (\alpha_A, \overline{\alpha}_A)$ and $\Diamond A = (\overline{\beta}_A, \beta_A)$ are intuitionistic fuzzy ideals of R.

Proof. If $IFSA = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of R, then $\alpha_A = \overline{\overline{\alpha}}_A$ and $\overline{\beta}_A$ are fuzzy ideals of R from Lemma 2.4, hence $\Box A = (\alpha_A, \overline{\alpha}_A)$ and $\Diamond A = (\overline{\beta}_A, \beta_A)$ are intuitionistic fuzzy ideals of R. Conversely, if $\Box A = (\alpha_A, \overline{\alpha}_A)$ and $\Diamond A = (\overline{\beta}_A, \beta_A)$ are intuitionistic fuzzy ideals of R, then the fuzzy sets α_A and $\overline{\beta}_A$ are fuzzy ideals of R. Hence $IFSA = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of R.

Theorem 2.6. An $IFSA = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of a near-ring R if and only if for all $s, t \in [0, 1]$, the non-empty sets $U(\alpha_A; t)$ and $L(\beta_A; s)$ are ideals of R.

Proof. Let $IFSA = (\alpha_A, \beta_A)$ be an intuitionistic fuzzy ideal of R. First, for any $s, t \in [0, 1]$, let $x, y \in U(\alpha_A; t)$, then $\alpha_A(x) \geq t$ and $\alpha_A(y) \geq t$. Hence $\alpha_A(x-y) \geq min\{\alpha_A(x), \alpha_A(y)\} \geq t$ and so $x - y \in U(\alpha_A; t)$. Second, for any $x \in U(\alpha_A; t)$ and $y \in R$, we get $\alpha_A(y + x - y) \geq \alpha_A(x) \geq t$, and that $y + x - y \in U(\alpha_A; t)$. Third, for any $r \in R$ and $x \in U(\alpha_A; t)$, we have $\alpha_A(xr) \geq \alpha_A(x) \geq t$ and so $xr \in U(\alpha_A; t)$. At last , for any $i \in U(\alpha_A; t)$ and $x, y \in R$, then $\alpha_A((x + i)y - xy) \geq \alpha_A(i) \geq t$, and that $(x + i)y - xy \in U(\alpha_A; t)$. Therefore $U(\alpha_A; t)$ is an ideal of R. Now, let $x, y \in L(\beta_A; s)$, then $\beta_A(x) \leq s$ and $\beta_A(y) \leq s$. Hence $\beta_A(x - y) \leq max\{\beta_A(x), \beta_A(y)\} \leq s$, and so $x-y \in L(\beta_A; s)$. Secondly, for any $x \in L(\beta_A; s)$ and $y \in R$, we get $\beta_A(y+x-y) \leq \beta_A(x) \leq s$, and that $y + x - y \in L(\beta_A; s)$. Moreover, for any $r \in R$ and $x \in L(\beta_A; s)$, we have $\beta_A(xr) \leq \beta_A(x) \leq s$, and so $xr \in L(\beta_A; s)$. At last, for any $i \in L(\beta_A; s)$ and $x, y \in R$, we have $\beta_A((x + i)y - xy) \leq \beta_A(i) \leq s$, and that $(x + i)y - xy \in L(\beta_A; s)$, and therefore $L(\beta_A; s)$ is an ideal of R.

Conversely, assume that for each $s, t \in [0, 1]$, the non-empty $U(\alpha_A; t)$ and $L(\beta_A; s)$ are ideals of R. If there exist $x_0, y_0 \in R$ such that $\alpha_A(x_0 - y_0) < min\{\alpha_A(x_0), \alpha_A(y_0)\}$, putting $t_0 = ((\alpha_A(x_0 - y_0) + min\{\alpha_A(x_0), \alpha_A(y_0)\})/2$, then $0 \leq \alpha_A(x_0 - y_0) < t_0 < min\{\alpha_A(x_0), \alpha_A(y_0)\} \leq 1$. It follows that $x_0 \in U(\alpha_A; t_0)$ and $y_0 \in U(\alpha_A; t_0)$, but $x_0 - y_0 \notin U(\alpha_A; t_0)$, that is, $U(\alpha_A; t_0)$ is not an ideal of R. This is a contradiction. Secondly, suppose that there exist $x_0, y_0 \in R$ such that $\alpha_A(y_0 + x_0 - y_0) < \alpha_A(x_0)$, setting $t_0 = (\alpha_A(y_0 + x_0 - y_0) + \alpha_A(x_0))/2$, we have $0 \leq \alpha_A(y_0 + x_0 - y_0) < t_0 < \alpha_A(x_0) \leq 1$. It follows that $x_0 \in U(\alpha_A; t_0)$, but $y_0 + x_0 - y_0 \notin U(\alpha_A; t_0)$. This is a contradiction. Thirdly, if there exist $y_0 \in R$ and $x_0 \in U(\alpha_A; t_0)$ such that $\alpha_A(x_0y_0) < \alpha_A(x_0)$. Taking $t_0 = (\alpha_A(x_0y_0) + \alpha_A(x_0))/2$, we have $0 \leq \alpha_A(x_0) < t_0 < \alpha_A(x_0y_0) \leq 1$. It follows that $x_0y_0 \notin U(\alpha_A; t_0)$, but $x_0 \in U(\alpha_A; t_0)$. This is a contradiction. At last, suppose that there exist $x_0, y_0, z_0 \in R$ such that $\alpha_A((x_0+z_0)y_0 - x_0y_0) < \alpha_A(z_0)$. Putting $t_0 = (\alpha_A((x_0+z_0)y_0 - x_0y_0) + \alpha_A(z_0))/2$, we get $0 \le \alpha_A((x_0+z_0)y_0 - x_0y_0) < t_0 < \alpha_A(z_0) \le 1$. It follows that $z_0 \in U(\alpha_A; t_0)$, but $(x_0+z_0)y_0 - x_0y_0 \notin U(\alpha_A; t_0)$. This is a contradiction. Hence, α_A satisfies (IF1)-(IF4). Similarly, we can prove that β_A satisfies (IF5)-(IF8).This completes the proof.

Let Λ be a non-empty subset of $[0,1]$.

Theorem 2.7. Let $\{I_t | t \in \Lambda\}$ be a collection of ideals of a near-ring R such that (i) $R = \bigcup_{t\in\Lambda} I_t$; (ii) $s > t$ if and only if $I_s \subset I_t$ for all $s, t \in \Lambda$.

Then an IFSA=(α_A, β_A) in X defined by $\alpha_A(x) = \sup\{t \in \Lambda \mid x \in I_t\}, \beta_A(x) = inf\{t \in \Lambda \mid x \in I_t\}$ for all $x \in X$ is an intuitionistic fuzzy ideal of R.

Proof. According to Theorem 2.6, it is sufficient to show that $U(\alpha_A; t)$ and $L(\beta_A; s)$ are ideals of R for every $t \in [0, \alpha_A(0)]$ and $s \in [\beta_A(0), 1]$. In order to prove that $U(\alpha_A; t)$ is an ideal of R, we divide the proof into the following two cases:

(i) $t = \sup\{q \in \Lambda \mid q < t\}$

(ii) $t \neq \sup\{q \in \Lambda \mid q < t\}$

The case (i) implies that $x \in U(\alpha_A; t) \Leftrightarrow x \in I_q, \forall q < t \Leftrightarrow x \in \bigcap_{q<t} I_q$ so that $U(\alpha_A; t) = \bigcap_{q<t} I_q$, which is an ideal of R. For the case (ii), we claim that $U(\alpha_A; t) = \bigcup_{q\ge t} I_q$, If $x \in \bigcup_{q\ge t} I_q$, then $x \in I_q$ for some $q \ge t$. It follows that $\alpha_A(x) \ge q \ge t$, so that $x \in U(\alpha_A; t)$. This shows that $\bigcup_{q\ge t} I_q \subseteq U(\alpha_A; t)$. Now assume $x \notin \bigcup_{q\ge t} I_q$. Then $x \notin I_q$ for all $q \ge t$. Since $t \neq \sup\{q \in \Lambda \mid q < t\}$, there exists $\varepsilon > 0$ such that $(t-\epsilon, t) \bigcap \Lambda = \phi$. Hence $x \notin I_q$ for all $q > t-\varepsilon$, which means that $x \in I_q$, then $q \le t-\varepsilon < t$. Thus $\alpha_A(x) \le t-\varepsilon$, and so $x \notin U(\alpha_A; t)$. Therefore $U(\alpha_A; t) \subseteq \bigcup_{q\ge t} I_q$, and that $U(\alpha_A; t) = \bigcup_{q\ge t} I_q$, which is an ideal of R. Next, we prove that $L(\beta_A; s)$ is an ideal of R. We consider the following two cases:

(iii) $s = inf\{r \in \Lambda \mid s < r\}$

(iv) $s \neq inf\{r \in \Lambda \mid s < r\}$

For the case (iii), we have

$$x \in L(\beta_A; s) \Leftrightarrow x \in I_r, \forall s < r \Leftrightarrow x \in \bigcap_{s<r} I_r$$

and hence $L(\beta_A, s) = \bigcap_{s<r} I_r$, which is an ideal of R. For the case(iv), there exists $\varepsilon > 0$ such that $(s, s+\varepsilon) \bigcap \Lambda = \phi$, we will show that $L(\beta_A; s) = \bigcup_{s\ge r} I_r$. If $x \in \bigcup_{s\ge r} I_r$, then $x \in I_r$ for some $r \le s$. It follows that $\beta_A(x) \le r \le s$. So that $x \in L(\beta_A; s)$. Hence $\bigcup_{s\ge r} I_r \subseteq L(\beta_A; s)$. Conversely, if $x \notin \bigcup_{s\ge r} I_r$, then $x \notin I_r$ for all $r \le s$, which implies that $x \notin I_r$ for all $r < s+\varepsilon$, that is, if $x \in I_r$, then $r \ge s+\varepsilon$. Thus $\beta_A(x) \ge s+\varepsilon > s$, that is, $x \notin L(\beta_A; s)$. Therefore $L(\beta_A; s) \subseteq U_{s\ge r} I_r$ and consequently $L(\beta_A; s) = \bigcup_{s\ge r} I_r$, which is an ideal of R. This completes the proof.

A map f from a near-ring R into a near-ring S is called a homomorphism if $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$ for all $x, y \in R$. Let $f : R \to S$ be a homomorphism of near-rings. For any $IFSA = (\alpha_A, \beta_A)$ in S, we define a new $IFSA^f = (\alpha_A^f, \beta_A^f)$ in R by $\alpha_A^f(x) = \alpha_A(f(x))$, $\beta_A^f(x) = \beta_A(f(x))$, for all $x \in R$.

Theorem 2.8. Let $f : R \to S$ be a homomorphism of near-rings. If an $IFSA = (\alpha_A, \beta_A)$ in S is an intuitionistic fuzzy ideal of S, then an $IFSA^f = (\alpha_A^f, \beta_A^f)$ in R is an intuitionistic fuzzy ideal of R.

Proof. For any $x, y, z \in R$, (IF1) $\alpha_A^f(x-y) = \alpha_A(f(x-y)) = \alpha_A(f(x)-f(y)) \geq min\{\alpha_A(f(x)), \alpha_A(f(y))\} = min\{\alpha_A^f(x), \alpha_A^f(y)\}$. (IF2) $\alpha_A^f(y+x-y) = \alpha_A(f(y+x-y)) = \alpha_A(f(y)+f(x)-f(y)) \geq \alpha_A(f(x)) = \alpha_A^f(x)$. (IF3) $\alpha_A^f(f(xy)) = \alpha_A(f(xy)) = \alpha_A(f(x)f(y)) \geq \alpha_A(f(x)) = \alpha_A^f(x)$ and (IF4) $\alpha_A^f((x+z)y-xy) = \alpha_A(f((x+z)y-xy)) = \alpha_A((f(x)+f(z))f(y)-f(x)f(y)) \geq \alpha_A(f(z)) = \alpha_A^f(z)$. Moreover, (IF5) $\beta_A^f(x-y) = \beta_A(f(x-y)) = \beta_A(f(x)-f(y)) \leq max\{\beta_A(f(x)), \beta_A(f(y))\} = max\{\beta_A^f(x), \beta_A^f(y)\}$. (IF6) $\beta_A^f(y+x-y) = \beta_A(f(y+x-y)) = \beta_A(f(y)+f(x)-f(y)) \leq \beta_A(f(x)) = \beta_A^f(x)$. (IF7) $\beta_A^f(xy) = \beta_A(f(xy)) = \beta_A(f(x)f(y)) \leq \beta_A(f(x)) = \beta_A^f(x)$ and (IF8) $\beta_A^f((x+z)y-xy) = \beta_A(f((x+z)y-xy)) = \beta_A((f(x)+f(z))f(y)-f(x)f(y)) \leq \beta_A(f(z)) = \beta_A^f(z)$. Hence $IFSA^f = (\alpha_A^f, \beta_A^f)$ is an intuitionistic fuzzy ideal of R.

If we strengthen the condition of f, then we can construct the converse of Theorem 2.8 as follows.

Theorem 2.9. Let $f : R \to S$ be an epimorphism of near-rings and let $IFSA = (\alpha_A, \beta_A)$ in S. If $IFSA^f = (\alpha_A^f, \beta_A^f)$ is an intuitionistic fuzzy ideal of R, then $IFSA = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of S.

Proof. Let $x, y, z \in S$, then there exist $a, b, c \in R$ such that $f(a) = x, f(b) = y$ and $f(c) = z$. (IF1) $\alpha_A(x-y) = \alpha_A(f(a)-f(b)) = \alpha_A(f(a-b)) = \alpha_A^f(a-b) \geq min\{\alpha_A^f(a), \alpha_A^f(b)\} = min\{\alpha_A(f(a)), \alpha_A(f(b))\} = min\{\alpha_A(x), \alpha_A(y)\}$. (IF2) $\alpha_A(y+x-y) = \alpha_A(f(b)+f(a)-f(b)) = \alpha_A(f(b+a-b)) = \alpha_A^f(b+a-b) \geq \alpha_A^f(a) = \alpha_A(f(a)) = \alpha_A(x)$. (IF3) $\alpha_A(xy) = \alpha_A(f(a)f(b)) = \alpha_A(f(ab)) = \alpha_A^f(ab) \geq \alpha_A^f(a) = \alpha_A(f(a)) = \alpha_A(x)$. (IF4) $\alpha_A((x+z)y-xy) = \alpha_A((f(a)+f(c))f(b)-f(a)f(b)) = \alpha_A(f((a+c)b-ab)) = \alpha_A^f((a+c)b-ab) \geq \alpha_A^f(c) = \alpha_A(f(c)) = \alpha_A(z)$. Moreover, (IF5) $\beta_A(x-y) = \beta_A(f(a)-f(b)) = \beta_A(f(a-b)) = \beta_A^f(a-b) \leq max\{\beta_A^f(a), \beta_A^f(b)\} = max\{\beta_A(f(a)), \beta_A(f(b))\} = max\{\beta_A(x), \beta_A(y)\}$. (IF6) $\beta_A(y+x-y) = \beta_A(f(b)+f(a)-f(b)) = \beta_A(f(b+a-b)) = \beta_A^f(b+a-b) \leq \beta_A^f(a) = \beta_A(f(a)) = \beta_A(x)$. (IF7) $\beta_A(xy) = \beta_A(f(a)f(b)) = \beta_A(f(ab)) = \beta_A^f(ab) \leq \beta_A^f(a) = \beta_A(f(a)) = \beta_A(x)$. (IF8) $\beta_A((x+z)y-xy) = \beta_A((f(a)+f(c))f(b)-f(a)f(b)) = \beta_A(f((a+c)b-ab)) = \beta_A^f((a+c)b-ab) \leq \beta_A^f(c) = \beta_A(f(c)) = \beta_A(z)$. Hence $IFSA = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of S.

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