

# Multicast Lifetime Maximization for Energy-Constrained Wireless Ad-hoc Networks with Directional Antennas

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*Abstract* —We consider the problem of maximizing the lifetime of a given multicast connection in wireless networks that use directional antennas and have limited energy resources. We first provide a globally optimal solution to this problem for the special case of using omni-directional antennas. This graph theoretic approach provides insights into more general case of using directional antennas later on, and inspires us to produce two heuristic algorithms. Experiment results show that minimum total power consumption does not guarantee maximum lifetime for either broadcasting or multicasting, and our algorithms outperform the group of minimum-energy multicast algorithms significantly.

## I. INTRODUCTION

Broadcasting/multicasting is an important mechanism in wireless ad hoc networks, since many routing protocols for wireless ad-hoc networks need such mechanism to communicate the updates of their states and maintain the routes between nodes. When power efficiency is considered, an energy-efficiency broadcast/multicast routing protocol has become imperative. Typically, there are two main power-aware metrics to gauge and to optimize the energy-efficiency of a broadcast or multicast routing algorithm: (1) the total transmission power assigned to all nodes to be minimized [1-5], and (2) the multicast/broadcast lifetime to be maximized [7-10]. Maximum lifetime broadcast/multicast routing algorithms can distribute packet-relaying loads for each node in a manner that prevents nodes from being overused or abused. Most recent work for the broadcast/multicast using the minimum total transmission power as optimization metric is based on the obvious intuition that conserving power will increase the network lifetime. However, this assumption may not always be valid, and will be demonstrated later in this paper.

Since minimum energy multicast/broadcast routing cannot guarantee the maximum multicast/broadcast lifetime, we have chosen the second optimization metric. Previous work closest to the present paper consists of articles [7-10], however these papers contain neither a complete global optimal solution of the problem in the case of using omni-directional antennas, nor theoretical analysis and heuristic algorithms in the case of using directional antennas.

In this paper, we have systematically explored and solved the Multicast/Broadcast Lifetime Maximization (MLM/BLM) problem. We study this problem in the omni-directional antenna scenarios first because it provides insights into more general case of using directional antennas. Our work has extended the BLM results in [7] to the more general MLM problem. From the theoretical analysis, we have derived a group of efficient algorithms with a lower complexity than those in [7]. We then studied the same problem with directional antennas. Two polynomial-time algorithms, called

S-DPMT and D-DPMT, have been implemented to handle significantly large networks with directional antennas. Experiment results show that minimum total power consumption does not guarantee maximum lifetime for either broadcasting or multicasting, and our D-DPMT algorithm provides much better performance (over double lifetime) than D-MIP [5, 6], which is one of the best minimum-energy multicast algorithms in directional antenna applications.

The remaining of this paper is organized as follows. Section 2 presents our analytical models. Section 3 provides globally optimal solutions for the MLM/BLM problem in the special case of using omni-directional antennas. Section 4 presents two heuristic approaches that can easily handle larger networks with directional antennas. Simulation results assessing the performance using several algorithms for many network examples are discussed in Section 5. Section 6 summarizes our findings and points out several future research problems.

## II. PRELIMINARIES

### A. The Directional Antenna Model

We use an idealized adaptive antenna propagation model, where the antenna directionality at node  $v$  is specified as the angle of beamwidth  $\theta_v$ , such that  $\theta^{\min} \leq \theta_v \leq 360^\circ$ . By ignoring the possibility of sidelobe interference, the transmitted energy is assumed to be uniformly distributed across the beamwidth. Based on this model, the minimal transmitted power required by node  $v$  to support a link between two nodes  $v$  and  $u$  separated by a distance  $r_{vu}$  ( $r_{vu} > 1$ ) is proportional to  $r_{vu}^\alpha$  and its beamwidth  $\theta_v$ , where the propagation loss exponent  $\alpha$  typically takes on a value between 2 and 4. Without loss of generality, all receivers are assumed to have the same signal detection threshold, which is typically normalized to one. Then the transmission power  $p_{vu}$  needed by node  $v$  to reach node  $u$  can be expressed as

$$p_{vu} = \frac{\theta_v}{360} r_{vu}^\alpha. \quad (1)$$

### B. Network Model and Multicast Tree

The network is modelled by a simple directed graph  $G(N, A)$ , where  $N$  is a finite node set, and  $A$  is an arc set corresponding to the unidirectional wireless communication links. As commonly done in others works [1-10], we assume the network is static. Let the energy supply set  $E = \{e_1, e_2, \dots, e_n\}$ ,  $n = |N|$ , be the initial energy level associated with each node. We assume that any node  $v \in N$  can choose its power level  $p_{vu}$ , up to some maximum value  $p_{max}$ . Any directed arc  $(v, u) \in A$  if and only if  $p_{vu} \leq p_{max}$ . The maximal lifetime  $\tau_{vu}$  of an arc  $(v, u) \in A$  is therefore

$$\tau_{vu} = \frac{e_v}{P_{vu}} \quad (2)$$

We consider source-initiated multicast operation where any node is permitted to initiate a multicast session. Each multicast session is supported by a set of multicast members  $M$  (consisting of the source node and all destination nodes), and other relay nodes. Formally, a multicast tree  $T_s$  with a source node  $s$  is modeled by a rooted tree of  $G$ , with a node set  $N(T_s) \subseteq N$ , and an arc set  $A(T_s) \subseteq A$ . A property of a rooted tree is that, for any node  $u$  in the tree, there exists a single directed path from  $s$  to  $u$  in the tree. The multicast lifetime  $\tau_M(T_s)$ , with respect to the multicast tree  $T_s$ , is defined as the duration of the network operation time until the battery depletion of the first node in  $N(T_s)$ . Given an initial energy supply  $\{e_1, e_2, \dots, e_n\}$ , the multicast lifetime  $\tau_M(T_s)$  can be easily obtained and is given below. A detailed derivation can be found in [7].

$$\tau_M(T_s) = \min_{(v,u) \in A(T_s)} \{\tau_{vu}\} \quad (3)$$

### C. Min-Max Multicast Tree

In a directed graph  $G(N, A)$ , each arc  $(v, u) \in A$  associates a positive weight  $w_{vu}$ . Given a source node  $s$  and a subset  $M$  of nodes ( $s \in M \subseteq N$ ), the min-max Steiner or multicast tree (MMMT) problem is to determine a directed tree rooted at node  $s$  that spans all nodes in  $M$  such that the maximum of the tree arc weight is minimized. The arc with the maximum weight of a multicast tree  $T_s$  is called the bottleneck arc, denoted as  $w_M(T_s)$ .

$$w_M(T_s) = \max_{(v,u) \in A(T_s)} \{w_{vu}\}, M \subseteq N \quad (4)$$

Let  $\Omega_M$  be the family of the trees  $T_s$  of  $G$  spanning all the nodes in  $M$ . Objective of our multicast lifetime maximization (MLM) problem is to find a multicast tree with the maximum multicast lifetime  $\tau_M^*$  defined as:

$$\begin{aligned} \tau_M^* &\equiv \max_{T_s \in \Omega_M} \{\tau_M(T_s)\} = \max_{T_s \in \Omega_M} \left\{ \min_{(v,u) \in A(T_s)} \{\tau_{vu}\} \right\} \quad (5) \\ &= \frac{1}{\min_{T_s \in \Omega_M} \left\{ \max_{(v,u) \in A(T_s)} \left\{ \frac{1}{\tau_{vu}} \right\} \right\}}, M \subseteq N \end{aligned}$$

We observe from Equation 5 that if we define  $w_{vu} = 1/\tau_{vu}$  as the weight for each arc  $(v, u)$ , then the MLM problem is equivalent to the MMMT problem. The min-max multicast tree  $T_s^*$  corresponding to the optimal solution of the MMMT problem, is therefore a multicast tree with the maximum multicast lifetime. As a special case of MLM problem when  $M = N$ , the BLM problem is finding a min-max spanning or broadcast tree (MMBT) of  $G$ .

### III. USING OMNI-DIRECTIONAL ANTENNAS

In this section, we investigate the MLM/BLM problem in energy-constrained wireless ad-hoc networks with omnidirectional antennas. We assume that once a broadcast/multicast tree is established at the beginning of a broadcast/multicast session, the same tree is used for the whole session duration.

#### A. The BLM Problem

The solution of the BLM problem can be directly derived from the MMBT problem as explained in the last section. We like to point out that the minimum spanning rooted tree of a directed graph  $G$  is not necessarily a min-max broadcast tree of  $G$  (and vice versa), because the *minimum* and *min-max* problems have been shown to be not the same as in the undirected case [7]. However, we shall observe later that a min-max broadcast tree of a directed graph  $G$  can be simply obtained using the well-known Prim's algorithm, which was originally designed to find an MST in an undirected graph.

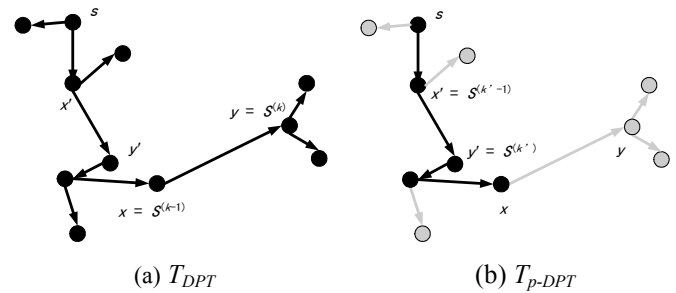


Figure 1. Illustration of the proof for Theorems 1 and 2, (a) a directed Prim tree, (b) the corresponding pruned directed Prim tree.

We define a directed Prim tree (DPT) as the spanning rooted tree constructed by Prim's algorithm. In fact, if  $T_{DPT}$  is a directed Prim tree,  $T_{DPT}$  is also a min-max broadcast tree. In order to prove this, first recall the standard Prim algorithm for the formation of a minimum spanning tree. It maintains throughout its execution a single tree rooted at the source node. Initially, the rooted tree includes the source node only. Subsequently, new nodes are added to the tree iteratively one at a time on a minimum weight basis until all nodes are included in the tree. Let  $S^{(i)}$  be the new node chosen at the  $i$ -th iteration of the tree incremental formation, where  $i = 1, 2, \dots, |N|-1$ , and  $S^{(0)} = s$ . At the same time, a new arc  $(S^{(i-1)}, S^{(i)})$  is added in the tree with the minimum weight, i.e.

$$w_{S^{(i-1)}S^{(i)}} \leq w_{vu}, \forall v \in N_1^i \cup \bigcup_{j=0}^i S^{(j)}, u \in N_2^i \equiv N - N_1^i, (v, u) \in A. \quad (6)$$

**Theorem 1:** In a directed graph  $G$ , if  $T_{DPT}$  is a directed prim tree of  $G$ ,  $T_{DPT}$  is also a min-max broadcast tree of  $G$ .

*Proof:* We assume that  $(x, y)$  is the arc of maximum weight of  $T_{DPT}$  as shown in Fig. 1 (a), and it is added into the tree at the  $k$ -th iteration, i.e.  $w_{xy} = w_{S^{(k-1)}S^{(k)}} = w_N(T_{DPT})$ . For any broadcast tree  $T_s \in \Omega_N$ , there must exist an arc  $(v', u')$  to connect two node-disjoint sets  $N_1^k$  and  $N_2^k$  ( $v' \in N_1^k$  and  $u' \in N_2^k$ ) in order to maintain the connectivity of the tree, and therefore  $w_{xy} \leq w_{v'u'}$  from Constraint (6). Finally, we have  $w_N(T_{DPT}) = w_{xy} \leq w_{v'u'} \leq \max_{(v,u) \in A(T_s)} \{w_{vu}\} = w_N(T_s), \forall T_s \in \Omega_N$ . That is, the maximum of the tree arc weight is minimized in  $T_{DPT}$  over  $\Omega_M$ . Hence  $T_{DPT}$  is a min-max broadcast tree of  $G$ . ■

When  $G$  is a complete undirected graph, the best known Prime algorithm has an  $O(|N|^2)$  complexity. We extend it to deal with a directed graph, and denote the modified version as

the DPBT (Directed Prim Broadcast Tree) algorithm. The advantage of our approach using the DPBT algorithm is that it has only an  $O(|N|^2)$  complexity. This can be compared to the complexity of  $\Theta(|N|^2 \log |N|)$  in the min-max broadcast tree algorithm [7].

### B. The MLM Problem

Similarly, the MLM problem can be formulated as an MMT problem. We provide in the following a theorem that would give us a global optimal solution of MLM problem using a simple procedure PRUN, which returns the corresponding multicast tree spanning all the nodes in the multicast group  $M$  by pruning from a broadcast tree all transmissions that are not needed to reach the nodes in  $M$ .

*Theorem 2:* In a directed graph  $G$ , if  $T_{p-DPBT}$  is a pruned directed prim tree of  $G$ , then  $T_{p-DPBT}$  is also a min-max multicast tree of  $G$ .

*Proof:* We assume that  $(x, y)$  and  $(x', y')$  are the arcs of maximum weight of  $T_{DPBT}$  and  $T_{p-DPBT}$  respectively, i.e.  $w_{xy} = w_N(T_{DPBT})$  and  $w_{x'y'} = w_M(T_{p-DPBT})$ , as shown in Fig. 1 (b).

We further assume  $w_{xy} = w_{S^{(k-1)}S^{(k)}}$  and  $w_{x'y'} = w_{S^{(k'-1)}S^{(k')}}$ . Note that there is at least one multicast member  $z$  belonging to  $N_2^{k'}$ , i.e.  $z \in N_2^{k'} \cap M$ , since otherwise any node in  $N_2^{k'}$  should be pruned, which contradicts the fact  $y' \in N_2^{k'}$ . For any multicast tree  $T_s \in \Omega_M$ , there must exist an arc  $(v', u') \in A(T_s)$  connecting  $N_1^{k'}$  and  $N_2^{k'}$  ( $v' \in N_1^{k'}$  and  $u' \in N_2^{k'}$ ) in order to satisfy the requirement that there exists a directed path from  $s$  to the multicast member  $z$ . Therefore, from Constraint (6) and Equation (4) we have  $w_M(T_{p-DPBT}) = w_{x'y'} \leq w_{v'u'} \leq \max_{(v,u) \in A(T_s)} \{w_{vu}\} = w_M(T_s)$ ,  $\forall T_s \in \Omega_M$ . Hence  $T_{p-DPBT}$  is a min-max multicast tree of  $G$ . ■

Theorem 2 immediately suggests our algorithm for the MLM problem, called DPMT (Directed Prim Multicast Tree) algorithm. To obtain the optimal solution of the MLM problem based on a min-max broadcast tree, our approach only yields an additional  $O(|M|)$  complexity on the procedure PRUN.

## IV. USING DIRECTIONAL ANTENNAS

We now turn our attention to the more difficult task of multicast time maximization using direction antennas. We must first incorporate the directionality of the antenna in the weight on each arc. By substituting Equation (1) into Equation (2) and taking reciprocal, we easily obtain:

$$w_{vu} = \frac{r_{vu}^\alpha \cdot \theta_v(T_s)}{360 \cdot e_v} \quad (7)$$

where  $\theta_v(T_s) \in [\theta^{\min}, 360^\circ]$  is the antenna beamwidth applied by node  $v$  in the tree  $T_s$ . If  $v$  is a leaf node, then  $\theta_v(T_s) = \theta^{\min}$ . Otherwise ( $v$  is an internal node)  $\theta_v(T_s)$  is set to be the minimum possible width that can cover all its children in  $T_s$ .

In fact, in an energy-limited wireless ad-hoc network, it is desirable to be able to precompute the min-max broadcast/multicast tree for any given  $M$ . However our

approaches in Section 3 are only for fixed weight  $r_{vu}^\alpha / e_v$  in the case of omni-direction antennas, but not necessarily valid for direction antennas (see Equation 7) since the value of  $\theta_v(T_s)$  normally remains unknown until the tree is completely constructed. Unfortunately, we do not have any scalable solutions for the general MLM/BLM problem using adaptive antennas. In fact, we suspect and conjecture that this problem is NP-complete. Therefore we shall design heuristic algorithms for the MLM/BLM problem to explore the properties we have studied in Section 3. Similar to [5, 6], we have considered two approaches for broadcasting and multicasting with adaptive antennas. They are called the static weight approach and the dynamic weight approach, and their details are given in Section IV-A and IV-B respectively.

### A. Static Weight Approach

This approach disregards the item  $\theta_v(T_s)/360$  in Equation (7) and directly applies DPBT or DPMT, under the assumption that the transmitting antennas are omni-directional. Then, after the tree is constructed in this manner, each internal node reduces its antennas beamwidth to the smallest possible value that provides coverage of the node's downstream neighbors in the tree, subject to the constraint  $\theta^{\min} \leq \theta_v(T_s) \leq 360^\circ$ . Thus in the process of the tree formation, the tree structure is independent of  $\theta^{\min}$ . When applied to the DPBT algorithm, we shall call this scheme Static-weight DPBT (S-DPBT). Likewise, when applied to the DPMT algorithm, the resulting scheme is called Static-weight DPMT (S-DPMT).

### B. Dynamic Weight Approach

In this section, we introduce and describe the D-DPBT (Dynamic-weight DPBT) algorithm, which is another major contribution of this paper. D-DPBT is similar in principle to DPBT algorithm for the formation of directed Prim tree, in the sense that new nodes are added to the tree one at a time on a minimum weight basis until all nodes are included in the tree. Unlike the input arc weights  $w_{vu} = r_{vu}^\alpha / e_v$  in the DPBT, which are precomputed and remain unchanged throughout the execution of the algorithm, D-DPBT must dynamically update the weights  $w_{vu} = r_{vu}^\alpha \cdot \theta_v(T_s) / (360 \cdot e_v)$  at each step to reflect the value  $\theta_v(T_s)$ . A pseudo code of the D-DPBT algorithm is given below where  $T_s^{(i)}$  is the intermediate tree constructed at the  $i$ -th iteration using our D-DPBT algorithm.

#### The D-DPBT( $G, s$ ) Algorithm

- 1) Initialize  $T_s^{(0)}$  by setting  $N(T_s^{(0)}) = \{s\}$  and  $A(T_s^{(0)}) = \emptyset$ , and initialize arc weight as  $w_{vu} = r_{vu}^\alpha \cdot \theta^{\min} / (360 \cdot e_v)$ .
- 2) For ( $i = 1$  to  $|N|-1$ )
  - i) Find the arc  $(x, y)$  connecting tree node outside node such that the value  $w_{xy}$  is minimized, i.e.  $w_{xy} = \min\{w_{vu} | v \in N(T_s^{(i-1)}), u \in N - N(T_s^{(i-1)}), \text{ and } (v, u) \in A\}$ .
  - ii) Construct  $T_s^{(i)}$  by setting  $N(T_s^{(i)}) = N(T_s^{(i-1)}) \cup \{y\}$  and  $A(T_s^{(i)}) = A(T_s^{(i-1)}) \cup \{(x, y)\}$ .
  - iii) Update  $w_{xu} = r_{xu}^\alpha \cdot \theta_x(T_s^{(i)}) / (360 \cdot e_x)$  for any node  $u \in N - N(T_s^{(i)})$  and  $(x, u) \in A$ , where  $N(T_s^{(i)}) = N(T_s^{(i-1)}) \cup \{u\}$  and  $A(T_s^{(i)}) = A(T_s^{(i-1)}) \cup \{(x, u)\}$ .
- 3) Return the final broadcast tree  $T_s^{(|N|-1)}$ .

### C. Example

Table 1. Network configuration

$i$	0	1	2	3	4	5	6	7	8	9
$x_i$	3.8	8.0	6.1	8.2	8.4	5.5	1.6	6.8	4.0	7.1
$y_i$	8.3	1.0	0.8	4.6	7.9	5.6	4.5	8.1	7.4	7.8
$e_i$	854	593	415	695	560	274	662	1001	698	244

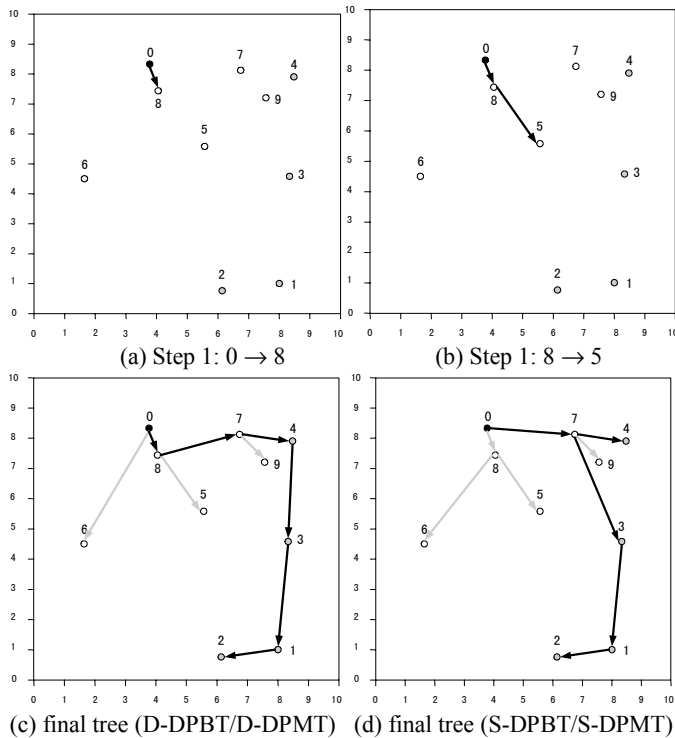


Figure 2. Examples of broadcast/multicast tree construction using D-DPBT/D-DPMT and S-DPBT/S-DPMT algorithms (Solid arcs indicate multicast tree arcs; light arcs indicate the pruned branches from the broadcast tree.)

This simple example uses a ten-node network to illustrate the basic tree construction steps in D-DPBT. Nodes  $\{0, 1, 2, 3, 4\}$  are multicast members and Node 0 is the source. Each node is equipped with a directional antenna that has a minimal beamwidth of  $30^\circ$ . The node placement  $(x_i, y_i)$  and initial energy supply  $e_i$  are listed in Table 1. No restrictions are placed on the maximum transmission power, and a propagation-loss exponent of  $\alpha = 2$  is assumed.

**Step 1:** Initially, the tree consists of only the source node (i.e.  $N(T_0^{(0)}) = \{0\}$  and  $A(T_0^{(0)}) = \emptyset$ ), and the initial weight value  $w_{vu}$  on each arc  $(v, u)$  is set as  $[(x_v - x_u)^2 + (y_v - y_u)^2] \cdot 30 / (e_v \cdot 360)$ . We then determine the node  $u$  that Node 0 can reach with the minimum arc weight  $w_{0u}$  ( $u = 1, \dots, 9$ ). Based on the information from table 1, Node 8 is chosen and arc  $(0, 8)$  is added to the tree. Thus  $N(T_0^{(1)}) = \{0, 8\}$  and  $A(T_0^{(1)}) = \{(0, 8)\}$  as shown in Fig. 2 (a), where  $0 \rightarrow 8$  denotes the arc chosen to be included in the tree at this step. We then update weigh for each arc  $(0, u)$  ( $u \in \{1, 2, 3, 4, 5, 6, 7, 9\}$ ). For an example, after the new arc  $(0, 8)$  is included, the value of  $w_{07}$  is changed accordingly from  $[(x_0 - x_7)^2 + (y_0 - y_7)^2] \cdot 30 / (e_0 \cdot 360)$  to  $[(x_0 - x_7)^2 + (y_0 - y_7)^2] \cdot \angle 807 / (e_0 \cdot 360)$ , where  $\angle 807$  indicates the degree of angle between arc  $(0, 8)$  and arc  $(0, 7)$ .

**Step 2:** We then again determine the next new node to be added into the tree. We see that there are two alternatives, Node 0 or Node 8. Either node can adjust its transmission power level and antenna beamwidth to reach a new node outside  $\{0, 8\}$ . Since  $w_{85} = \min_{u \neq 0, 8} \{w_{0u}, w_{8u}\}$ , Node 5 is chosen

at this time, and a new arc  $(8, 5)$  is added into the tree. At this point, we have obtained  $N(T_0^{(2)}) = \{0, 5, 8\}$  and  $A(T_0^{(2)}) = \{(0, 8), (8, 5)\}$  as shown in Fig. 2 (b). Consequently, we update the weight for each arc  $(8, u)$ ,  $u \in \{1, 2, 3, 4, 6, 7, 9\}$ . For example, the value of  $w_{87}$  is changed accordingly from  $\frac{[(x_8 - x_7)^2 + (y_8 - y_7)^2] \cdot 30}{e_8 \cdot 360}$  to  $\frac{[(x_8 - x_7)^2 + (y_8 - y_7)^2] \cdot \angle 587}{e_8 \cdot 360}$ .

**The following iterative steps:** This procedure is continued until all nodes are included in the tree, as shown in Fig. 2 (c). The order in which the nodes were added in the subsequent steps is:  $8 \rightarrow 7, 7 \rightarrow 9, 7 \rightarrow 4, 4 \rightarrow 3, 3 \rightarrow 1, 1 \rightarrow 2, 0 \rightarrow 6$ .

Figures 2 (c) and (d) also show the multicast trees pruned from the corresponding broadcast trees produced by D-DPMT and S-DPMT respectively.

### D. Time Complexity

The complexity of constructing the initial broadcast tree in the S-DPBT algorithm is  $O(|N|^2)$  as explained in Section III. The overall complexity of the beam reduction operation for all transmitting nodes using the sector-choosing algorithm [6] is  $O(|N|^2 \log |N|)$ . A similar complexity analysis can be made for the S-DPMT algorithm. Therefore, the time complexity of both algorithms is  $O(|N|^2 \log |N|)$ .

The time complexity of D-DPBT is  $O(|N|^3 \log |N|)$ . This can be explained as follows. In Step 2-i), a minimum-weight arc  $(x, y)$  must be chosen, and the complexity of this search is at most  $O(|N|^2)$ . Then in Step 2-ii), the new arc  $(x, y)$  is added into the tree with a complexity of  $O(1)$ . Finally, in Step 2-iii) the weights that relate to the new child's parent  $x$  must be updated. For each outside node  $u$ , the complexity of the weight update operation is  $O(|N| \log |N|)$  using the sector-choosing algorithm [6], which involves to calculate the minimum beamwidth of node  $x$  after the new child  $u$  is added. Since the number of children outside the tree the node  $x$  may have is at most  $|N| - 1$ , the overall complexity of Step 2-iii) is  $O(|N|^2 \log |N|)$ . Therefore, the complexity at each iteration of Step 2) is at most  $O(|N|^2 \log |N|)$ . Since there are  $|N|$  steps, the overall complexity is at most  $O(|N|^3 \log |N|)$ . Similar to the approach we have applied in Section III-B, the D-DPMT (Dynamic-weight DPMT) algorithm simply prunes the broadcast tree obtained using D-DPBT. It has the same complexity of  $O(|N|^3 \log |N|)$  as D-DPBT.

## V. PERFORMANCE EVALUATION

We have evaluated the performance of a set of heuristic algorithms  $I = \{S-DPMT, D-DPMT, RB-MIP, D-MIP\}$ , where RB-MIP and D-MIP are two well-known minimum-energy multicast algorithms using directional antennas [5, 6]. We specify a normal distributed initial energy supply with a mean of 5000-unit energy and a variance of 2000 in a 100-node network. Other experimental setup is the same as in Section IV-C. In all cases, our results are based on the performance of 100 randomly generated network examples.

Table 2. Mean and variance of normalized multicast lifetime with normal distributed energy supply

$ M $	S-DPMT	D-DPMT	RB-MIP	D-MIP	S-DPMT	D-DPMT	RB-MIP	D-MIP
	$\theta_{\min} = 30^\circ$				$\theta_{\min} = 60^\circ$			
25	(0.53, 0.044)	(0.98, 0.005)	(0.26, 0.023)	(0.54, 0.076)	(0.74, 0.040)	(0.98, 0.005)	(0.32, 0.030)	(0.47, 0.062)
50	(0.49, 0.031)	(0.99, 0.003)	(0.24, 0.021)	(0.51, 0.073)	(0.71, 0.040)	(0.98, 0.004)	(0.31, 0.028)	(0.49, 0.077)
100	(0.47, 0.026)	(0.98, 0.005)	(0.20, 0.013)	(0.52, 0.081)	(0.68, 0.032)	(0.99, 0.003)	(0.29, 0.027)	(0.47, 0.073)
	$\theta_{\min} = 90^\circ$				$\theta_{\min} = 360^\circ$			
25	(0.88, 0.024)	(0.98, 0.004)	(0.38, 0.045)	(0.45, 0.071)	(1.00, 0.000)	(1.00, 0.000)	(0.45, 0.061)	(0.45, 0.061)
50	(0.87, 0.026)	(0.98, 0.004)	(0.37, 0.041)	(0.48, 0.080)	(1.00, 0.000)	(1.00, 0.000)	(0.46, 0.066)	(0.46, 0.066)
100	(0.85, 0.027)	(0.99, 0.002)	(0.35, 0.038)	(0.45, 0.071)	(1.00, 0.000)	(1.00, 0.000)	(0.45, 0.058)	(0.45, 0.058)

To gauge the performance of our algorithm, we compare the *normalized multicast lifetime*  $\tau_M^i / \tau_M^{\text{BEST}}$  defined as the ratio of actual multicast lifetime  $\tau_M^i$  obtained using heuristic algorithm- $i$  to the best solution  $\tau_M^{\text{BEST}} = \max\{\tau_M^i \mid i \in I\}$ . This metric allows us to facilitate the comparison of different algorithms over a wide range of network examples.

Performance results for 100-node networks with normal distributed energy supply are shown in Table 2. For all the cases, D-DPMT performs much better than the other algorithms. On the average, D-DPMT has about double the lifetime for all possible minimum beamwidth when compared to D-MIP, which is one of the best minimum-energy multicast algorithms in directional antenna applications. This improvement ratio, when compared to RB-MIP, can be up to 2 ~ 5. We attribute the improved performance of the D-DPMT algorithm to the DPMT algorithm's property of finding min-max multicast tree. In fact, as an extension of DPMT, the D-DPMT algorithm partially possesses this property.

## VI. CONCLUSION

In this paper, we have studied some of the fundamental issues associated with maximum lifetime broadcasting and multicasting in energy-constrained wireless ad-hoc networks, and we have presented preliminary algorithms for cases of both omni-directional antennas and adaptive antennas. Our analysis and experiment show that: (1) our DPMT/DPBT algorithm always maximizes the multicast/broadcast lifetime in the case of using omni-directional antennas, (2) the D-DPMT/D-DPBT algorithm, which exploits the property of minimizing the bottleneck arc weight, provides much better performance than the other algorithms over a wide range of network examples. A major challenge, and a topic of continued research, is the development of distributed algorithms that provide the benefits that have been demonstrated in this paper.

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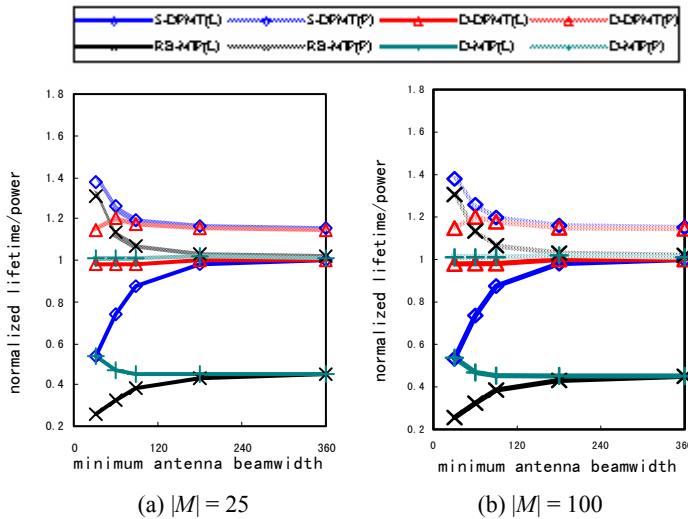


Fig 3. Performance comparison based on *normalized multicast lifetime* (denoted by L) and *normalized tree power* (denoted by P) for 100-node networks with normal distributed energy supply.

In order to investigate the relationship of two types of energy-efficiency related performance metrics, *normalized multicast lifetime* and *normalized tree power*, we depict them together as shown in Fig. 3. We observe that the minimal total power consumption does not guarantee maximum lifetime for a network either for broadcast or multicast. For example in Fig. 3, D-DPMT uses about 20% more energy than D-MIP, but improves multicast lifetime over 100% than D-MIP.