Shamos's Catalog of the Real Numbers

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Preface

Have you ever looked at the first few digits the decimal expansion of a number and tried to recognize which number it was? You probably do it all the time without realizing it. When you see 3.14159... or 2.7182818... you have no trouble spotting π and *e*, respectively. But are you on familiar terms with 0.572467033... or 0.405465108...? These are $\frac{\pi^2-3}{12}$ and $\log 3-\log 2$, both of which have interesting stories to tell. You will find them, along with those of more than 10,000 other numbers, listed in this book. But why would someone bother to compile a listing of such facts, which may seem on the surface to be a compendium of numerical trivia?

Unless you had spent some time with $\frac{\pi^2-3}{12}$, you might not realize that it is the sum of two very different series, $\sum_{k=1}^{\infty}(-1)^k\frac{\cos k}{k^2}$ and $\sum_{k=1}^{\infty}k\big(\zeta(2k)-\zeta(2k+1)\big)$. The proof that each sum equals $\frac{\pi^2-3}{12}$ is elementary; what connection there may be between the two series remains elusive. The number $\log 3-\log 2$ turns up as the sum of several series and the value of certain definite integrals: $\sum_{k=1}^{\infty}\frac{1}{3^k k}$, $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^k k}$, $\int_1^{\infty}\frac{dx}{(x+1)(x+2)}$ and $\int_0^{\infty}\frac{dx}{2e^x+1}$. If your research led to either number, this information might well lead you to additional findings or help you explain what you had already discovered.

Mathematics is an adventure, and a very personal one. Its practitioners are all exploring the same territory, but without a roadmap they all follow different paths. Unfortunately, the mathematical literature dwells primarily on final results, the destinations, if you will, rather than the paths the explorers took in arriving at them. We are presented with an orderly sequence of definitions, lemmas, theorems and corollaries, but are often left mystified as to how they were discovered in the first place.

The real empirical process of mathematics, that is, the laborious trek through false conjectures and blind alleys and taking wrong turns, is a messy business, ill-suited to the crisp format of referred journals. Possibly the unattractiveness of the topic accounts for the scarcity of writing on the subject. How many unsuccessful models of computation did Turing invent before he hit on the Turing machine? How did he even generate the models? How did he realize that the Halting Problem was interesting to consider, let alone imagine that it was unsolvable? What did his scrap paper look like?

This book is a field guide to the real numbers, similar in many ways to a naturalist's handbook. When a birdwatcher spots what he thinks may be a rare species, he notes some of its characteristics, such as color or shape of beak, and looks it up in a field guide to verify his identification. Likewise, when you see part of a number (its initial decimal digits), you should be able to look it up here and find out more about it. The book is just a lexicographic list of 10,000 numbers arranged by initial digits of their decimal expansions, along with expressions whose values share the same initial digits.

This book was inspired by Neil Sloane's *A Handbook of Integer Sequences*. That work, first published in 1971, is an elaborately-compiled list of the initial terms of over 2000 integer sequences arranged in lexicographic order. When one is confronted with the first few term of an unfamiliar sequence, such as {1, 2, 5, 16, 65, 326, 1957,...}, a glance at Sloane reveals that this sequence, number 589, is the total number of permutations of all subsets of n objects, that is, $\sum_{k=1}^{n} k!\binom{n}{k} = \sum_{k=1}^{n} \frac{n!}{(n-k)!} = n!\sum_{k=1}^{n} \frac{1}{j!}$, which is the closest integer to $n!e$. Sloane's book is extremely useful for finding relationships among combinatorial problems. It leads to discoveries, which help in turn generate conjectures that hopefully lead to theorems.

What impelled me to compile this catalog was an investigation into Riemann's ζ-function that I began in 1990. It struck me as incredible that we have landed men on the moon but still have not found a closed-form expression for $\zeta(3) = \sum_{k=1}^{\infty} \frac{1}{k^3}$. I say incredible because in the 18th century Euler proved that $\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s}$ is a rational multiple of π^s for all even s, and gave a formula for the coefficient in terms of Bernoulli numbers. No such formula is known for any odd s. It is not even known whether a closed form exists.

I began exploring such sums as $\sum_{k=2}^{\infty} \frac{1}{k^s - k^{s-t}}$, $\sum_{k=1}^{\infty} \frac{1}{k^s + k^{s-t}}$ and $\sum_{k=2}^{\infty} \frac{(-1)^k}{k^s - k^{s-t}}$, whose values can often be written as simple expressions involving various values of the ζ-function. For example, $\sum_{k=2}^{\infty} \frac{1}{k^5 - k^3} = \frac{5}{4} - \zeta(3)$, $\sum_{k=1}^{\infty} \frac{1}{k^4 - k^2} = \frac{\pi^2}{6} - \frac{1+\pi\coth\pi}{2}$ and the remarkable $\sum_{k=2}^{\infty} \frac{1}{k^4 - k^{-2}} = \frac{1}{12}$. To keep track of these many results, I kept them in a list by numerical value, conveniently obtained using Mathematica®. Very quickly, after experimenting with sums of the form $\sum_{k=1}^{\infty} \zeta(ak+b) - 1$, I began to notice connections that led to the following theorem.

Theorem 1. For $a > 1$ an integer, b an integer such that $a+b \geq 2$ and c a positive real number, then $\sum_{k=1}^{\infty} \frac{1}{ck^{a+b} - k^b} = \sum_{j=1}^{\infty} \frac{\zeta(aj+b)}{c^j}$.

A proof is given at the end of the chapter. The proof is elementary, but I never would have been motivated to write down the theorem in the first place without studying the emerging list of expressions sorted by numerical value.

After a time, I became somewhat obsessed with expanding the list of values and expressions and found that as it grew it became more and more useful. I began to add material not directly related to my research and found that it raised more questions than it answered. It became for me both a handbook and a research manifesto.

For example, when I observed that the sums $\sum_{k=1}^{\infty}\frac{1}{2^{2^k}}$ and $\sum_{k=1}^{\infty}\frac{\mu(2k)}{4^k-1}$ shared the same 20-digit prefix, this evoked considerable interest, since sums of the form $\sum_{k>0}\frac{1}{a^{b^k}}$ are very difficult to treat analytically. After verifying that the sums were equal to over 500 digits, I began to look for generalizations, exploring such sums as $\sum_{k>0}\frac{1}{2^{3^k}}$, $\sum_{k>0}\frac{1}{4^{4^k}}$, $\sum_{k=1}^{\infty}\frac{\mu(2k)}{2^k-1}$ and $\sum_{k=1}^{\infty}\frac{\mu(3k)}{4^k-1}$, to give a few examples. The results suggested a conjecture, which soon yielded the following theorem:

Theorem 2. For $c > 1$ real and p prime, $\sum_{i=1}^{\infty}\frac{1}{c^{p^i}} = -\sum_{k=1}^{\infty}\frac{\mu(kp)}{c^{kp}-1}$, where $\mu(n)$ is the Moebius function.

For a proof, see the end of the chapter. Alternatively, the theorem may be expressed in the form $\sum_{k=1}^{\infty}\frac{\mu(kp)}{a^k-1} = -\sum_{k=1}^{\infty}\frac{1}{\left(a^{1/p}\right)^{\left(p^k\right)}}$. This allows us to relate such strange expressions as $\sum_{k=1}^{\infty}\frac{1}{\left(\sqrt{\pi}\right)^{2^k}}$ and $\sum_{k=1}^{\infty}\frac{1}{\left(\sqrt[3]{e}\right)^{3^k}}$ to Moebius function sums. Unfortunately, the theorem seems to shed no light on trickier sums like $\sum_{k=1}^{\infty}\frac{1}{k^{2^k}}$ and $\sum_{k=1}^{\infty}\frac{1}{k^{k^k}}$.

This book is what is known in computer science as a hash table, a structure for indexing in a small space a potentially huge number of objects. It is similar to the drawers in the card catalog of a library. Imagine that a library only has 26 card drawers and that it places catalog cards in each draw without sorting them. To find a book by Taylor, you must go to drawer “T” and look through all of the cards. If the library has only a hundred books and the authors’ names are reasonably distributed over the alphabet, you can expect to have to look at only two or three cards to find the book you want (if the library has it). If the library has a million books, this method would be impractical because you would have to leaf through more than 20,000 cards on the average. That’s because about 40,000 cards will “hash” to the same drawer. However, if the library had more card drawers, let’s say 17,576 of them labeled “AAA” through “ZZZ,” you would have a much easier time since each drawer would only have about 60 cards and you would expect to leaf through 30 of them to find a book the library holds (all 60 to verify that the book is not there).

Hashing was originally used to create small lookup tables for lengthy data elements. The term “hashing” in this context means cutting up into small pieces. Suppose that a company has 300 employees and used social security numbers (which have nine digits) to identify them. A table large enough to accommodate all possible social security numbers would need 10^9 entries, too large even to store on most computer hard disks. Such a table seems wasteful anyway, since only a few hundred locations are going to be occupied. One might try using just the first three digits of the number, which reduces the space required to just a thousand locations. A drawback is that the records for several employees would have to be stored at that location (if their social security numbers began with the same three digits). An attempt to store a record in a location where one already exists is called a “collision” and must be resolved using other methods. There will not be many collisions in a sparse table unless the numbers tend to cluster strongly (as the initial digits of social security numbers do). Using this scheme in a small community where workers have lived all their lives might result in everyone hashing to the same location because of the geographical scheme used to assign social security numbers. A better method would be to use the three least-significant digits or to construct a “hash function,” one that is designed to spread the records evenly over the storage locations. A suitable hash function for social security numbers might be to square them and use the middle three digits. (I haven’t tried this.)

Getting back to the library, notice that any two cards in a drawer may be identical (for multiple copies of the same book) or different. The only thing they are guaranteed to have in common is the first three letters of the author’s last name. Now imagine that the library sets up 26^{20} drawers. Aside from the fact that you might have to travel for some time to get to a drawer, once you arrive there you will probably find very few cards in it, and the chances are excellent that if there is more than one card, you are dealing with the same author or two authors who have identical names. Of course, it’s still possible for two authors to have different names that share the same 20 first letters.

This book is a hash table for the real numbers. It has 10^{20} drawers, but to save paper only the nonempty drawers are shown. But it differs very significantly from the card catalog of a library, in which the indexing terms (the “keys”) have finite length. When you examine two catalog cards, you can tell immediately whether the authors have the same name, even if their names are very long. When you look up an entry in this book, you can’t tell immediately whether the expressions given are identically equal or not. That’s because their decimal expansions are of infinite length and all you know is that they share the same first 20 digits. With that warning, I can tell you there is no case in this book known to the author in which two listed quantities have the same initial 20 digits but are known not to be equal. There are many cases, however, in which the author has no idea whether two expressions for the same entry are equal, but therein lies the opportunity for much research by author and reader alike.

As a final example, seeing with some surprise that $\sum_{k=1}^{\infty}\frac{\sin k}{k}=\sum_{k=1}^{\infty}\frac{\sin^2 k}{k^2}=\frac{\pi-1}{2}$ led me to an investigation of the conditions under which the sum of an infinite series equals the sum of the squares of its individual terms. (We may call such series “element-squaring.”) Element-squaring series rarely occur naturally but are in fact highly abundant:

Theorem 3. Every convergent series of reals $B=\sum_{k=1}^{\infty}b_k$ such that $B^{(2)}=\sum_{k=1}^{\infty}{b_k}^2<\infty$ can be transformed into an element-squaring real series $B'=\sum_{k=0}^{\infty}b_k$ by prepending to B a single real term b_0 iff $B^{(2)}-B\le\frac{1}{4}$. Proof: see the end of the chapter.

I hope that these examples show that the list offers both information and challenges to the reader. It is a reference work, but also a storehouse of questions. Anyone who is fascinated by mathematics will be able to scan through the pages here and wonder if, how and why the entries for a given number are related.

Eventually it became clear that perusing the catalog in pursuit of gems by eyeball would not be effective. I then wrote software to search for relationships among the entries, seeking, for example, numbers *p*, *q*, *r* and *s* such that *p* = *f* (*q*, *r*, *s*) for interesting functions *f*. With 10,000 entries, such searches are fruitful. With a million entries more computing power would be needed. Some of the results of these investigations can be found in the author's papers, e.g., *Overcounting Functions*[1] and *Property Enumerators and a Partial Sum Theorem*[2]. I contend that this method of search yields a rich stream of conjectures, which in turn lead to interesting mathematical investigations.

The selection of entries in this book necessarily reflects the interests and prejudices of the author. This will explain the relatively large number of expressions involving the ζ-function. For this I make no apology; there are more interesting real numbers than would fit in any book, even listing only their first 20 digits. The reader is encouraged to develop his own supplements to this catalog by using Mathematica to develop a list of numbers useful in his or her own field of study.

Professor George Hardy once related the now-famous story of his visit to an ailing Ramanujan in which Ramanujan asked him the number of the taxicab that brought him. Hardy thought the number, 1729, was a dull one, but Ramanujan instantly countered that it was the smallest integer that can be written as the sum of two cubes in two different ways, as $1^3 + 12^3$ and $9^3 + 10^3$. Hardy marveled that Ramanujan seemed to be on intimate terms with the integers and regarded them as his personal friends[3]. It is hoped that this book will help make the real numbers the reader's personal friends.

Proofs of Theorems 1-3

Theorem 1. For $a > 1$ an integer, b an integer such that $a + b \geq 2$ and c a positive real number, then $\sum_{j=1}^{\infty} \frac{\zeta(aj+b)}{c^j} = \sum_{k=1}^{\infty} \frac{1}{ck^{a+b} - k^b}$. A proof is given at the end of the chapter.

Proof: The left-hand sum may be written as the double sum $\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} \frac{1}{k^{aj+b} c^j}$. Reversing the order of summation, $\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} \frac{1}{k^{aj+b} c^j} = \sum_{k=1}^{\infty} \frac{1}{k^b} \sum_{j=1}^{\infty} \frac{1}{k^{aj} c^j} = \sum_{k=1}^{\infty} \frac{1}{k^b} \sum_{j=1}^{\infty} \frac{1}{\left(ck^a\right)^j}$, which equals $\sum_{k=1}^{\infty} \frac{1}{k^b} \frac{1}{ck^a - 1} = \sum_{k=1}^{\infty} \frac{1}{ck^{a+b} - k^b}$. QED.

Theorem 2. For *c* > 1 real and *p* prime, $\sum_{i=1}^{\infty} \frac{1}{c^{p^i}} = -\sum_{k=1}^{\infty} \frac{\mu(kp)}{c^{kp} - 1}$, where $\mu(n)$ is the Moebius function.

Proof: $\sum_{k=1}^{\infty} \frac{\mu(kp)}{c^{kp} - 1} = \sum_{k=1}^{\infty}\sum_{j=1}^{\infty} \frac{\mu(kp)}{c^{jkp}}$. Note that every term of $\sum_{i=1}^{\infty} \frac{1}{c^{p^i}}$ appears at least once in the double sum, certainly in the case when $k = 1$ and $j = p^i$. We will show that the sum of all

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[1] Available at http://euro.ecom.cmu.edu/people/faculty/mshamos/Overcounting.pdf

[2] Available at http://euro.ecom.cmu.edu/people/faculty/mshamos/PST.pdf

[3] This incident is partially related in Kanigel, R. *The Man Who Knew Infinity*. New York: Washington Square Press (1991). ISBN 0-671-75061-5. The book is a fascinating biography of Srinivasa Ramanujan.

coefficients of $\frac{1}{c^{p^i}}$ in the double sum is zero except when $k = 1$ and *j* is a power of *p*, in which case $\mu(kp) = -1$, and the result follows.

Suppose that jk is a power of *p*. Then if $k > 1$, $\mu(kp) = 0$ since *kp* has a repeated factor. Therefore, only the terms with $k = 1$ contribute to the sum and $\mu(kp) = -1$ each *j* a power of *p*.

Now consider all pairs *j*, *k* with $jk = i$ not a power of *p*. Suppose that *i* has the factorization $p_1^{a_1} \dots p_q^{a_q}$, with each of the $a_i > 0$. If $p|k$ or if *k* has any repeated prime factors, then $\mu(kp) = 0$. Since $k|i$, it suffices to consider only those *k* whose factors are products of subsets P_k of distinct primes taken from the set $P = \{p_1 \dots p_q\} - \{p\}$, a set having cardinality $q-1$ or *q*, depending on whether P contains *p*. If P_k has an odd number of elements, then $\mu(kp) = 1$; otherwise, if P_k has an even number of elements, then $\mu(kp) = -1$. $\mu(kp) = 0$ cannot be zero since *p* does not divide *k*. But the number of subsets of P_k having an odd number of elements is the same as the number of subsets having an even number of elements. Therefore, the terms cancel each other and terms with *jk* not a power of *p* do not contribute to the double sum. QED.

Theorem 3. Every convergent series of reals $B = \sum_{k=1}^{\infty} b_k$ such that $B^{(2)} = \sum_{k=1}^{\infty} b_k^2 < \infty$ can be transformed into an element-squaring series $B' = \sum_{k=0}^{\infty} b_k$ by appending to B a single real term b_0 iff $B^{(2)} - B \le \frac{1}{4}$.

Proof: For B' to be element-squaring, it is necessary that $\sum_{k=0}^{\infty} b_k^2 = b_0^2 + \sum_{k=1}^{\infty} b_k^2 = b_0^2 + B^{(2)} = b_0 + B$. Solving this quadratic equation for b_0 yields $b_0 = \frac{1 \pm \sqrt{1 - 4(B^{(2)} - B)}}{2}$, which is real iff $B^{(2)} - B \le \frac{1}{4}$. Since b_0 is finite, $B' = b_0 + B$ converges. It is also elementary to show that if $B^{(2)} - B > \frac{1}{4}$, then there is no real element that can be prepended to B to make it element-squaring. QED.

Note that $B < \infty$ does not necessarily imply that $B^{(2)} < \infty$. For example, let $b_k = \frac{(-1)^k}{\sqrt{k}}$. Then $B = \zeta\left(\frac{1}{2}, \frac{1}{2}\right)$, which is finite, but $B^{(2)} = \sum_{k=1}^{\infty} \frac{1}{k}$, which diverges. Likewise, $B^{(2)} < \infty$ does not imply that $B < \infty$. If $b_k = \frac{1}{k}$, then $B^{(2)} = \frac{\pi^2}{6}$ but B diverges.

Using This Book

This book is a catalog of almost 10,000 real numbers. A typical entry looks like:

$$\begin{array}{rll} .20273255405408219099\ldots \approx & \dfrac{\log 3-\log 2}{2} = \operatorname{arctanh}\dfrac{1}{5} & \text{AS 4.5.64, J941, K148} \\ = & \displaystyle\sum_{k=0}^{\infty}\frac{1}{5^{2k+1}(2k+1)} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^{k+1}k} & \\ = & \displaystyle\int_{1}^{\infty}\frac{\log x dx}{(2x+1)^2} & \\ = & \displaystyle\int_{0}^{1}\frac{\psi(x)\sin\pi x\sin 5\pi x}{x}dx & \text{GR 1.513.7} \end{array}$$

Each entry begins with a real number at the left, with its integer and fractional parts separated. Entries are aligned on the decimal point. Three dots are used to indicate either (1) a non-terminating expansion, as above, or (2) a rational number whose period is greater than 18. Three dots are always followed by the symbol ≈, which denotes "approximately equal to."

The center portion of an entry contains a list of expressions whose numerical values match the portion of the decimal expansion shown. The entries in the list are separated by the equal sign =; however, it is not to be inferred that the entries are identically equal, merely that their decimal expansions share the same prefix to the precision shown.

There is no precise ordering to the expressions in an entry. In general, closed forms are given first, followed by sums, products and then integrals. At the right margin next to expression may appear one or more citations to references relating the expression to the numerical value given or another expression in the entry. When more than one expression appears on a line containing a citation, it means that the citation refers to at least one expression on that line. See also, "Citations," below.

Format of Numbers

Numbers are given to 19 decimal places in most cases. Where fewer than 15 digits are given, no more are known to the author. Rational numbers are specified by repeating the periodic portion two or more times, with the digits of the last repetition underlined. For example, 1/22 would be written as $.045045\underline{45}$, which is an abbreviation for the non-terminating decimal .0454545454545045... . Since rational numbers can be specified exactly, the = symbol rather than ≈ is used to the right of the decimal. Where space does not permit a repetition of the periodic part, it is given once only. If the period is longer than 18 decimals, the ≈ sign is used to denote that the decimal has not been written exactly. No negative entries appear.

Ordering of Entries

Entries are in lexicographic order by decimal part. Entries having the same fractional part but different integer parts are listed in increasing magnitude by integer part. Rational numbers are treated as if their decimal expansions were fully elaborated. For example, these entries would appear in the following order:

.505002034457717739... ≈
$.50501\underline{50501}$ =
.505015060150701508... ≈
$.50\underline{50}$ =
.505069928817260012... ≈

Arrangement of Expressions

In general, terms in a multiplicative expression are listed in decreasing order of magnitude except that constants, including such factors as $(-1)^k$ appear at the left. For example, $4k!2^k k^3$ is used in preference to $4k^3 2^k k!$. Additive expressions are ordered to avoid initial minus signs, where possible. For example, $3-2k$ is used instead of $-2k+3$. These principles are applied separately in the numerators and denominators of fractions.

Citations

Abbreviations flushed against the right margin next to an expression denote a reference to an equation, page or section of a reference work. Page numbers are prefixed by the letter p, as in "p. 434." A list of cited works appears at the end of the book.

Other Catalogs

Simon Plouffe maintains a server at the University of Quebec at Montreal that allows searching among 215 million real numbers for equivalent symbolic expressions. See http://pi.lacim.uqam.ca/. This book, by contrast, contains only about 10,000 entries. One would expect, therefore, that almost all of the numbers in the book would also be found in Plouffe's index. To test this hypothesis, I had a student write a script to look up each of the 10,000 entries on Plouffe's server to see how many it would find. The result was approximately five percent. The reason for such a low degree of overlap, it appears, is that the expressions in Plouffe's index were primarily generated by computer from templates, which the real numbers in this book were assembled for the most part from the mathematical literature.

WARNING

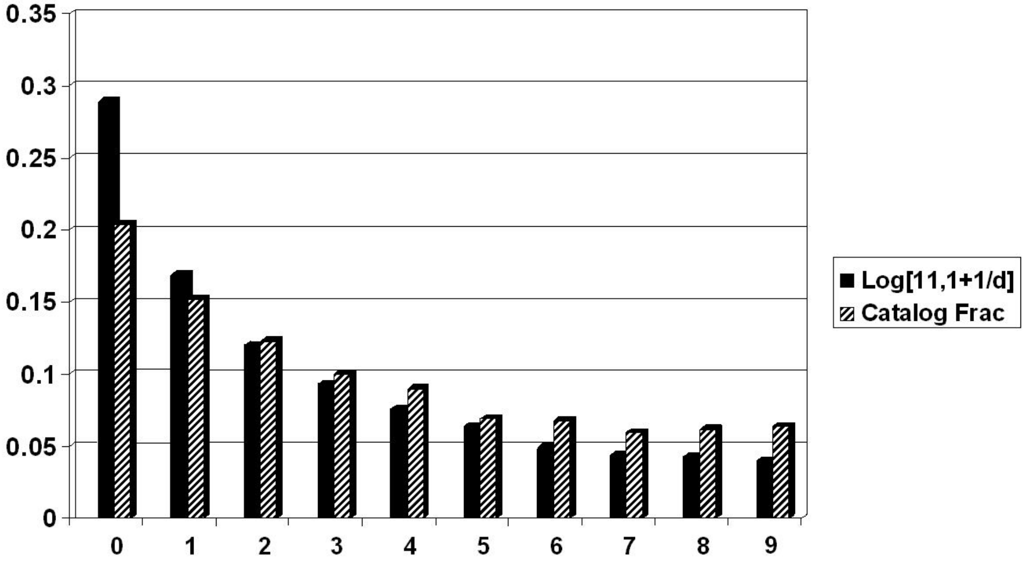
While great effort has been expended to ensure accuracy, the numerical values and expressions in this book have not been checked thoroughly and should not be relied upon unless verified independently.

The First-Digit Phenomenon

In 1881, Simon Newcomb observed that in books of logarithmic tables pages near the front tended to receive more wear than the later pages, suggesting that the earlier pages were used more frequently. That implies that the numbers whose logarithms were being looked up were not uniformly distributed. In particular, numbers whose leading digit was a 1 occurred more frequently than those beginning with 2 (or any other digit). Newcomb proposed that the frequency of first digit n in a random list of numbers (assuming meaning can be ascribed to such a concept), should be $\log_{10}(n+1) - \log_{10} n$. That is, the frequency of numbers beginning with 1 ought to be about 30%, while only 4.6% of numbers should begin with 9.

This phenomenon was rediscovered by Frank Benford in 1938 and it is now referred to as Benford's law. Its generalization to numbers in base b is that the frequency of digit d is $\log_b (d+1) - \log_b d$. This catalog (and Plouffe's collection) provide experimental data concerning the first digits of the fractional parts of lists of numbers. In fact, both lists exhibit the first digit phenomenon very strongly. Because the fractional first digits range from 0 to 9 (rather than 1 to 9), the appropriate form of Benson's law is to take b to be 11, not 10.

The bar chart below shows a plot of $\log_{10}(d+1) - \log_{10} d$ (solid black bars) versus the actual percentage of entries in the catalog whose fractional part begins with the digit d (diagonal shading).



No explanation is offered to explain the deviation from the numerical predictions of Benford's law in this instance, but the shape of the decay as d increases is similar. Various mathematical derivations of Benford's law have been offered, but none are rigorous because of the futility of attempting to define a random sample of numbers form an unbounded set. It is somewhat easier to understand the law informally. When I explained Benford's Law to the late John Gaschnig, an artificial intelligence graduate student at Carnegie Mellon in the late 1970's, he pondered it for a few minutes and then announced, "Yes, it's because 1 is first."

Notation

$a(n)$	greatest odd divisor of integer *n*.	
$Ai(x)$	Airy function, solution to $y'' - xy = 0$.	AS 10.4.1
$b(x)$	$\sum_{k=0}^{\infty} \frac{(-1)^k}{x+k}$.	
B_k	kth Bernoulli number, $B_k = B_k(0)$.	AS 23.1.2
$B_k(x)$	kth Bernoulli polynomial, defined by $\frac{te^{xt}}{e^t - 1} = \sum_{k=0}^{\infty} B_k(x) \frac{t^k}{k!}$.	AS 23.1.1
$bp(n)$	number of divisors of *n* that are primes or powers of primes.	
c_k	coefficient of x^k in the expansion of $\Gamma(x)$.	
$C(x)$	$C(x) = \frac{1}{2}\int_0^x \cos \pi t^2 \, dt = \sum_{k=0}^{\infty} \frac{(-1)^k \pi^{2k} x^{4k+1}}{(2k)! 2^{2k} (4k+1)}$.	AS 7.3.1, 7.3.11
$Ci(x)$	cosine integral, $Ci(x) = \gamma + \log x + \int_0^x \frac{\cos t - 1}{t} dt = \gamma + \log x + \sum_{k=1}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!(2k)}$.	AS 5.2.2, AS 5.2.16
$d(n)$	number of divisors of *n*, $d(n) = \sigma_0(n)$	
d_n	nth derangement number, $d_1 = 0, d_2 = 1, d_3 = 2, d_4 = 9$, nearest integer to $n!/e$.	Riordan 3.5
$e(n)$	number of exponents in prime factorization of *n*.	
$E(x)$	complete elliptic integral $E(x) = \int_0^{\pi/2} \sqrt{1 - x \sin^2 \theta} \, d\theta$.	
E_n	nth Euler number, $E_k = 2^k E_k\left(\frac{1}{2}\right)$.	AS 23.1.2
$E_n(x)$	nth Euler number, defined by $\frac{2e^{xt}}{e^t + 1} = \sum_{k=0}^{\infty} E_k(x) \frac{t^k}{k!}$.	AS 23.1.1
$Ei(x)$	exponential integral $Ei(x) = -\int_{-x}^{\infty} \frac{e^{-t} dt}{t} = \gamma + \log x + \sum_{k=1}^{\infty} \frac{x^k}{k!k}$.	AS 5.5.1
$Erf(x)$	error function $\frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{k!(2k+1)} = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} e^{-x^2} \frac{(-1)^k 2^k x^{2k+1}}{(2k+1)!!}$.	AS 7.1.5
$Erfc(x)$	complimentary error function = $1 - Erf(x)$.	AS 7.1.2
$f(n)$	number of representations of *n* as an ordered product of factors	
F_n	nth Fibonacci number, $F_1 = F_2 = 1;\ F_n = F_{n-1} + F_{n-2}$	

$_0\mathrm{F}_1(;a;x)$	hypergeometric function $\sum_{k=0}^{\infty}\frac{x^k}{k!a_{(k)}}$, where $a_{(k)} = a(a-1)\ldots(a-k)$.	Wolfram A.3.9
$_1\mathrm{F}_1(a;b;x)$	Kummer confluent hypergeometric function $\sum_{k=0}^{\infty}\frac{x^k a_{(k)}}{k!b_{(k)}}$, where $a_{(k)} = a(a-1)\ldots(a-k)$.	Wolfram A.3.9
$_2\mathrm{F}_1(a;b;c;x)$	hypergeometric function $\sum_{k=0}^{\infty}\frac{x^k a_{(k)}b_{(k)}}{k!c_{(k)}}$, where $a_{(k)} = a(a-1)\ldots(a-k)$.	Wolfram A.3.9
G	Catalan's constant $\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)^2} \approx 0.91596559417721901505\ldots$	
G_n	$1+\sum_{k=1}^{\infty}\left(\frac{1}{(3k+1)^n}+\frac{1}{(3k-1)^n}\right)$	
g_n	$1+\sum_{k=1}^{\infty}\left(\frac{1}{(3k+1)^n}-\frac{1}{(3k-1)^n}\right)$	
$gd(z)$	Gudermannian, $gd(z) = 2\arctan e^z - \frac{\pi}{2}$	
H_n	harmonic number: $H_n = \sum_{k=1}^{n}\frac{1}{k}$	
$H^e{}_n$	even harmonic sum: $H^e{}_n = \sum_{k=1}^{n}\frac{1}{2k}$	
$H^o{}_n$	odd harmonic sum: $H^o{}_n = \sum_{k=1}^{n}\frac{1}{2k-1}$	
$H^{(j)}{}_n$	generalized harmonic number: $H^{(j)}{}_n = \sum_{k=1}^{n}\frac{1}{k^j} = \zeta(j) + \frac{(-1)^{j+1}\psi^{(j-1)}(n+1)}{(j-1)!}$, $j>1$	
h_n	$\sum_{k=0}^{\infty}\left(\frac{(-1)^k}{(6k+1)^n}-\frac{(-1)^k}{(6k+5)^n}\right)$	J314
$hg(x)$	Ramanujan's generalization of the harmonic numbers, $hg(x) = \sum_{k=1}^{\infty}\frac{x}{k(k+x)}$	
$I_n(x)$	modified Bessel function of the first kind, which satisfies the differential equation $z^2y'' + zy' - (z^2+n^2)y = 0$.	Wolfram A.3.9
j_n	$1+\sum_{k=1}^{\infty}\left(\frac{1}{(3k-1)^n}-\frac{1}{(3k+1)^n}\right)$	J311
$J_n(x)$	Bessel function of the first kind $\sum_{k=0}^{\infty}\frac{(-1)^k x^{n+2k}}{k!(n+k)!2^{n+2k}}$	LY 6.579
k'	$\sqrt{1-k^2}$	
$K(x)$	complete elliptic integral of the first kind $\int_0^{\pi/2}\frac{1}{\sqrt{1-x\sin^2\theta}}d\theta$	
$K_n(x)$	modified Bessel function of the second kind, which satisfies the differential equation	

	$z^2y''+zy'-(z^2+n^2)y=0$.	Wolfram A.3.9
$l(x)$	$\sum_{k=1}^{\infty}\left(\frac{\log k}{k}-\frac{\log(k+x)}{k+x}\right)=\sum_{k=1}^{n}\frac{\log k}{k}$, when *x* is an integer *n*.	
L_n	"little" Fibonacci numbers: $L_n=2L_{n-1}+L_{n-2};\ L_0=L_1=1$ OR	
L_n	Lucas numbers: $L_n=L_{n-1}+L_{n-2};\ L_0=2;\ L_1=1$	
$li(x)$	logarithmic integral $li(x)=-\int_0^x\frac{dt}{\log t}$	
$Li_n(x)$	polylog function $Li_n(x)=\sum_{k=1}^{\infty}\frac{x^k}{k^n}$	
$\log x$	natural logarithm $\log_e x$	
p	a prime number. $\sum_p$ denotes a sum over all primes.	
$pd(n)$	product of divisors $pd(n)=\prod_{d\mid n}d$	
$pf(n)$	partial factorial $pf(n)=\sum_{k=0}^{n}\frac{1}{k!}$	
$pfac(n)$	product of primes not exceeding *n*, $pfac(n)=\prod_{p\le n}p$	
$pq(n)$	product of quadratfrei divisors of *n*, $pq(n)=2^{pr(n)}$	
$pr(n)$	number of prime factors of *n*	
$prime(n)$	*n*th prime	
$r(n)$	$4\sum_{d\mid n}\chi(d)$	HW 17.9
$s(n)$	smallest prime factor of *n*	
$sd(n)$	sum of the divisors of *n*, $sd(n)=\sigma_1(n)$	
$S(x)$	$S(x)=\int_0^x\sin\left(\frac{\pi t^2}{2}\right)dt=\sum_{k=1}^{\infty}\frac{(-1)^k(\pi/2)^{2k}}{(2k+1)!(4k+3)}x^{4k+3}$.	AS7.3.2, AS 7.3.13
$S_1(n,m)$	Stirling number of the first kind, defined by the recurrence relation:	
	$x(x-1)\ldots(x-n+1)=\sum_{m=0}^{n}S_1(n,m)x^m$.	AS 24.1.3
$S_2(n,m)$	Stirling number of the first kind, $S_2(n,m)=\frac{1}{m!}\sum_{k=0}^{m}(-1)^{m-k}\binom{m}{k}k^n$.	AS 24.1.4
$Si(x)$	sine integral, $Si(x)=\int_0^x\frac{\sin t}{t}dt=\sum_{k=0}^{\infty}\frac{(-1)^k x^{2k+1}}{(2k+1)!(2k+1)}$.	AS 5.2.1, AS 5.2.14

$t_k(n)$	$\prod_i \binom{k-1+e_i}{k-1}$, $t_k(1) = 1$, where $n = \prod_i p_i^{e_i}$ This is the number of ways of writing *n* as a product of *k* factors.	
$T(s)$	$\sum_k \frac{1}{k^s}$, where *k* has an odd number of prime factors	
$u_k(n)$	number of unordered *k*-tuples $\{a_1 ..., a_k\}$ whose product is *n*.	
$w(x)$	$w(x) = e^{-x^2} erfc(-ix) = \sum_{k=0}^{\infty} \frac{(ix)^k}{\Gamma(k/2+1)}$	
W_n	$\sum_{k=0}^{\infty} \left(\frac{(-1)^k}{(6k+1)^n} + \frac{(-1)^k}{(6k+5)^n} \right)$	J313
$z(n)$	number of factorizations of *n*.	
$z_2(x)$	$z_2(x) = \sum_{k=2}^{\infty} (\zeta(k)-1)x^k = \frac{2}{1-x} - x(\gamma + \psi(1-x))$	
$\beta(s)$	$\beta(s) = \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{(2k+1)^s}$	
γ	Euler's constant $\gamma = \lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{k} - \log n \approx 0.57721566490153286$	AS 6.1.3
γ_f	$\lim_{n \to \infty} \left(\sum_{k=1}^{n} f(k) - \int_1^{\infty} f(x) dx \right)$	
$\zeta(s)$	Riemann zeta function $\sum_{k=1}^{\infty} \frac{1}{k^s}$	
$\zeta^{(m)}(s)$	derivative of the Riemann zeta function $\sum_{k=1}^{\infty} \frac{(-\log k)^m}{k^s}$	
$\zeta_p(s)$	Prime zeta function $\sum_{k=1}^{\infty} \frac{1}{prime(k)^s}$	
$\eta(s)$	$\eta(s) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^s} = (1-2^{1-s})\zeta(s)$	
$\lambda(s)$	$\lambda(s) = \sum_{k=0}^{\infty} \frac{1}{(2k+1)^s} = (1-2^{-s})\zeta(s)$	
$\Lambda(k)$	Mangoldt lambda, $\Lambda(k) = \log k$ if *k* is a prime power; 0 otherwise	
$\mu(n)$	Moebius function, $\mu(1) = 1$, $\mu(n) = (-1)^k$ if *n* has exactly k distinct prime factors, 0 otherwise.	
$\nu(n)$	number of different prime factors of *n*.	

v_1	$v_1=(1-i\sqrt{3})\left(\frac{3-i\sqrt{2}}{2}\right)+(1+i\sqrt{3})\left(\frac{3+i\sqrt{2}}{2}\right)$	
$\xi(s)$	$\Gamma\left(\frac{s}{2}\right)(s-1)\pi^{-s/2}\zeta(s)$	
$\sigma_k(n)$	sum of k^{th} powers of divisors of *n*, $\sum_{d\|n} d^k$	
ϕ	golden ratio, $\frac{1+\sqrt{5}}{2} \approx 1.61803397498948482...$	
$\phi(n)$	Euler totient function, the number of positive integers less than and relatively prime to *n*.	
$\chi_n(z)$	$\frac{1}{2}\left(Li_n(z) - Li_n(-z)\right) = \sum_{k=0}^{\infty}\frac{z^{2k+1}}{(2k+1)^n}$	[Ramanujan] Berndt Ch. 9
$\psi(n)$	polygamma function $\psi(n) = -\gamma+\sum_{k=1}^{n-1}\frac{1}{k}$	
$\psi(x)$	k^{th} derivative of the polygamma function $\psi^{(k)}(x) = \frac{d^k\,\psi(x)}{dx^k}$	
$\Phi(n)$	$\Phi(n) = \sum_{k=1}^{n}\phi(k)$	
$\xi(s)$	$\xi(s)=\Gamma\left(1+\frac{s}{2}\right)(s-1)\pi^{-s/2}\zeta(s)$	
$\Omega(n)$	overcounting function $\Omega(n) =$ -1 + the number of divisors of the g.c.d. of the exponents of the prime factorization of *n*.	

References

Citations given in the right margin of the number list refer to works in which more information about a number or equation may be found. The form of citation is generally a word or abbreviation followed by a reference giving a page, section or equation number. In the case of journals, the citation is by volume and page. Following is an alphabetical listing of reference abbreviations used in the text.

Andrews	Andrews, L. *Special Functions for Engineers and Applied Mathematicians*. New York: Macmillan.
AMM	American Mathematical Monthly
AS	Abramowitz, M. and Stegun, I. A. *Handbook of Mathematical Functions*. New York: Dover (1964).
Berndt	Berndt, B. C. *Ramanujan's Notebooks*. New York: Springer-Verlag (1985). ISBN 0-387-96110-0
CFG	Croft, H. T., Falconer, K. J. and Guy, R. K. *Unsolved Problems in Geometry*. New York: Springer-Verlag (1991). ISBN 0-387-97506-3.
CRC	*Handbook of Mathematical Tables*. Cleveland: Chemical Rubber Co. (2nd ed., 1964)
Dingle	Dingle, R. B. Asymptotic Expansions: Their Derivation and Interpretation. London: Academic Press (1973).
ex	exercise
GKP	Graham, R. E., Knuth, D. E. and Patashnik, O. *Concrete Mathematics*. Reading, MA: Addison-Wesley (1989). ISBN 0-201-14236-8.
GR	Gradshteyn, I. S. and Ryzhik, I. M. *Table of Integrals, Series and Products*. San Diego: Academic Press (1979). ISBN 1-12-294760-6.
Edwards, H. M.	Riemann's Zeta Function. New York: Academic Press (1974). ISBN 0-12-232750-0.
Henrici	Henrici, P. *Applied and Computational Complex Analysis*.
Hirschman	Hirschman Jr., I. I. *Infinite Series*. New York: Holt (1962). ISBN 0-03-11040-8.
HW	Hardy, G. H. and Wright, E. M. *Theory of Numbers*. Glasgow: Oxford (4th ed., 1960)
J	Jolley, L. B. W. *Summation of Series*. New York: Dover (1961).
K	Knopp, K. *Theory and Application of Infinite Series*. New York: Dover (1990).
LY	Lyusternik, L. A. and Yanpol'skii, A. R. *Mathematical Analysis: Functions, Limits, Series and Continued Fractions*. Oxford: Pergamon (1965).
Marsden	Marsden, J. E. *Basic Complex Analysis*. San Francisco: Freeman (1973). ISBN 0-7167-0451-X.
Melzak	Melzak, Z. A. *Companion to Concrete Mathematics*. New York: Wiley (1973). ISBN 0-471-59338-9.
MI	*Mathematical Intelligencer*

Riordan	Riordan, J. *An Introduction to Combinatorial Analysis*. New York: Wiley (1958).
Seaborn	Seaborn, J. B. *Hypergeometric Functions and Their Applications*. New York: Springer-Verlag (1991). ISBN 0-387-97558-6.
Sloane	Sloane, N. J. A. *A Handbook of Integer Sequences*. New York: Academic Press (1973). ISBN 0-12-648550-X.
Titchmarsh	The Theory of the Riemann Zeta Function. London: Oxford (1951).
Vardi	Vardi, I. *Computational Recreations in Mathematica*. Redwood City: Addison-Wesley (1991). ISBN 0-201-52989-0
Wilf	Wilf, H. S. *generatingfunctionology*. Boston: Academic Press (1989). ISBN 0-12-751955-6.
Wolfram	Wolfram, S. *Mathematica: A System for Doing Mathematics by Computer*. Reading, MA: Addison-Wesley (2nd ed., 1991). ISBN 0-201-51502-4.

Acknowledgments

No book like this one has previously appeared, probably because of the monumental difficulty of performing the required calculations even with computer assistance. However, with the development of the Mathematica® program, a product of Wolfram Research, Inc., it has been possible to obtain 20-digit expansions of most of the entries without undue difficulty. The symbolic summation capability of that program has been invaluable in developing closed-form expressions for many of the sums and products listed. Even though my involvement with computers extends back more than 40 years, I have no hesitation in declaring that Mathematica is the most important piece of mathematical software that has ever been written and my debt to its authors is, paradoxically, incalculable.

The tendency to be fascinated with numbers, as opposed to other, abstract structures of mathematics, is not uncommon among mathematicians, though to be afflicted to the same degree as this author may be unusual. If so, it because of the role played by a collection of people and books in my early life. My grandfather, Max Shamos, was a surveyor who, to the end of his 93-year life, performed elaborate calculations manually using tables of seven-place logarithms. The work never bored him — he took an infectious pride in obtaining precise results. When calculators were developed (and, later, computers), he never used them, not out of stubbornness or a refusal to embrace new technology, but because, like a cabinetmaker, he needed to use his hands.

I read later in Courant and Robbins' *What is Mathematics*, of the accomplishments of the calculating prodigies like Johann Dase and of William Shanks, who devoted decades to a manual determination of π to 707 decimal places (526 of them correct). At the time, I couldn't imagine why someone would do that, but I later understood that he did it because he just had to know what lay out there. I expect that people will ask the same question about my compiling this book, and the answer is the same. The challenge set by Shanks is still alive and well — every year brings a story of a new calculation — π is now known to several billion digits.

My father, Morris Shamos, was an experimental physicist with a healthy knowledge of mathematics. When I was in high school, I was puzzled by the formula for the moment of inertia of a sphere, $M = \frac{2}{5} m r^2$. That is was proportional to the square of the radius made sense enough, but I couldn't understand the factor of 2/5. Dad set me straight on the road to advanced calculus by showing me how to calculate the moment by evaluating a triple integral.

As a teenager during the 1960s, I worked for Jacob T. ("Jake," later "Jack") Schwartz at the New York University Computer Center, programming the IBM 7094. (That machine, which cost millions, had a main memory equivalent to about 125,000 bytes, about one-tenth of the capacity of a single floppy disk that today can be purchased for a dollar.) NYU had a large filing cabinet with punched-cards containing FORTRAN subroutines for calculating the values of numerous functions, such as the double-precision (in the parlance of the day) arctangent. By poring over these cards, I learned something of the trouble even a computer must go through in its calculations of mathematical functions. It was this exposure that drew me to field that later came to be called computer science.

For the past 25 years I have been an enthusiastic user of Neil Sloane's *A Handbook of Integer Sequences* (now co-authored with Simon Plouffe) in combinatorial research, so it is particularly pleasing to note that this book itself, in a sense, consists entirely of such sequences.

My greatest debt, however, is to Stephen Wolfram for his creation of *Mathematica*, the single greatest piece of software ever written. In developing this catalog, I used *Mathematica* on and off over a 15-year period to explore symbolic sums and integrals and to compute most of the constants that appear in this book to 20 decimal places.

About the Author

Michael Ian Shamos was born in New York in 1947. He attended the Horace Mann School and the Columbia University Science Honors Program during high school. He learned computer programming at Columbia and the NYU Computer Center on the IBM 1401, 1620 and 7094 computers during the 1960s. He graduated from Princeton University in 1968 with an A.B. degree in Physics and a minor in Mathematics. His senior thesis, written under the guidance of Prof. John Wheeler, was devoted to the theory of gravitational radiation.

He joined IBM as an engineer in 1968, developing programming systems to operate automated circuit testing equipment and obtained an M.A. from Vassar College in 1970, working under Prof. Morton Tavel in the field of acoustics. In 1970, he joined the National Cancer Institute to write software in support of the Institute's research in cancer chemotherapy.

In 1970, he began graduate study at the Yale University Department of Computer Science where he laid the foundations for the field of computational geometry and was awarded the M.S., M.Phil. and Ph.D. degrees, the latter in 1978. In 1975, he joined the faculty of the Computer Science Department at Carnegie Mellon University, doing research in combinatorics and the analysis of algorithms.

In 1979, Dr. Shamos founded Unilogic, Ltd., a computer software company in Pittsburgh. He obtained the J.D. degree from Duquesne University School of Law in 1981 and has engaged in the practice of intellectual property law since then, dividing his time with Unilogic until 1987. He practiced intellectual property law with the Webb Law Firm in Pittsburgh, concentrating in computer-related matters, from 1990-1998. From 1980-2000, he has acted as an examiner of computerized voting systems for the Secretary of the Commonwealth of Pennsylvania and the Attorney General of Texas.

Dr. Shamos is now Distinguished Career Professor in the School of Computer Science at Carnegie Mellon, where he directs a graduate program in eBusiness Technology.

Dr. Shamos is the author of four published books, *Computational Geometry* (1985), with F. P. Preparata, *Pool* (1990), *The Illustrated Encyclopedia of Pool, Billiards and Snooker* (1993), and *Shooting Pool* (1998). He serves as Curator of the Billiard Archive, a non-profit organization devoted the preservation of the history of cue games, as Contributing Editor of *Billiards Digest* magazine, and Chairman of the Statistics and Records Committee of the Billiard Congress of America.

Dr. Shamos lives in the Shadyside neighborhood of Pittsburgh with his wife Julie. They were married in 1973. Their son Alex is a student at the University of Arizona in Tucson. Their daughter Josselyn Crane lives nearby with her husband Dan and their infant daughter Harlow.

For more information, consult the Author's web page:
http://euro.ecom.cmu.edu/shamos.html

.0000000000000000000$\underline{0}$ $= \sum_{k=0}^{\infty} \frac{(-1)^k (k+1)}{k!} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^2}{k!}$ GR 1.212

$= \sum_{k=0}^{\infty} \frac{(-1)^k 2^k (k+2)}{k!} = \sum_{k=1}^{\infty} \frac{(-1)^k k}{k!!} = \sum_{k=1}^{\infty} (-1)^k \binom{2k}{k} \frac{k^2}{2^k}$

$= \sum_{k=1}^{\infty} \frac{\mu(k)}{k}$ Berndt Ch. 24 (14.3)

$= \sum_{k=1}^{\infty} \frac{\mu(k)}{2^k + 1}$

$= \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\sin^3 k}{k}$

$= \int_0^{\infty} \frac{\log^2 x}{x^2 - 1}$

1 .00000000000000000000$\underline{0}$ $= \sum_{k=1}^{\infty} \frac{1}{k(k+1)}$ J396

$= \sum_{k=1}^{\infty} \frac{1}{2k^2 - 1}$

$= \sum_{k=1}^{\infty} \frac{k}{(k+1)!}$ GR 0.245.4

$= \sum_{k=0}^{\infty} \frac{2^k k}{(k+2)!}$

$= \sum_{k=0}^{\infty} \frac{1}{(k+1)! + k!}$

$= \sum_{k=0}^{\infty} \left(\frac{k!!}{(k+3)!} + \frac{1}{((k+2)!! + k!!} \right)$

$= \sum_{k=2}^{\infty} \frac{2k^4 + k^3 + 1}{k^7 - k} = \sum_{k=1}^{\infty} \frac{3k^2 + 3k + 1}{k^6 + 3k^5 + 3k^4 + k^3}$

$= \sum_{k=1}^{\infty} \binom{2k}{k} \frac{1}{4^k (k+1)} = \sum_{k=1}^{\infty} \frac{(2k-1)!!}{(2k)!\,(k+1)}$

$= \sum_{k=0}^{\infty} \binom{2k}{k}^2 \frac{4k+3}{2^{4k+2} (k+1)^2}$

$= -\sum_{k=1}^{\infty} \frac{\mu(k) \log k}{k}$

$= \sum_{k=2}^{\infty} \frac{F_{k-1}}{F_k F_{k+1}}$

$= \sum_{k=1}^{\infty} \frac{F_{4k-2}}{8^k}$

$$= \sum_{k=1}^{\infty} \frac{H_k}{(k+1)(k+2)}$$

$$= \sum_{k=2}^{\infty} \frac{H_k}{2k^2 - 2k}$$

$$= \sum_{k=1}^{\infty} \frac{L_k}{3^k}$$

$$= \sum_{k=2}^{\infty} \left(\zeta(k) - 1\right)$$

$$= \sum_{k=2}^{\infty} \frac{\Omega(k)}{k}$$

$$= -\sum_{k=1}^{\infty} \frac{\mu(k)\log k}{k}$$

Berndt Ch. 24, Eq. 14.3

$$= \sum_{k=2}^{\infty} \left(\frac{\mu(k)}{k(k-1)} - \frac{\mu(k)}{k-1} \right)$$

$$= \sum_{\omega \in S}^{\infty} \frac{1}{\omega - 1}$$

, where S is the set of all non-trivial integer powers

(Goldbach, 1729) JAMA 134, 129-140(1988)

$$= \sum_{n=1}^{\infty} \frac{1}{n!} \sum_{k=2}^{\infty} \frac{\log^n k}{k!}$$

$$= \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\sin 2k}{k}$$

$$= \prod_{k=1}^{\infty} 1 + \frac{(-1)^{k+1}}{k}$$

J1065

$$= \int_1^{\infty} \frac{dx}{x^2} = \int_1^{\infty} \frac{\log x \, dx}{x^2}$$

$$= \int_0^{\infty} \frac{dx}{(x^2+1)^{3/2}}$$

$$= \int_0^{1} \frac{dx}{(1+x)\sqrt{1-x^2}}$$

$$= \int_0^{\infty} \frac{x \log(1+x)}{e^x} dx$$

$$= -\int_0^{\infty} \left(\cos x - \frac{\sin x}{x} \right) \frac{dx}{x}$$

Berndt I, p. 318

$$= \int_0^{\pi/2} \frac{dx}{(\sin x + \cos x)^2}$$

$$= \int_1^\infty \frac{\arctan\sqrt{x}}{x^2}\,dx$$

$$2 \quad .00000000000000000000\underline{0} \quad = \quad \Phi(½,-1,0) \;=\; \sum_{k=1}^\infty \frac{k}{2^k} \;=\; \sum_{k=1}^\infty \frac{k!2^k}{(2k)!} \;=\; \sum_{k=1}^\infty \frac{k}{(k+1)!!}$$

$$= \frac{1}{512}\left(\psi^{(2)}\left(\frac{9}{8}\right)-\psi^{(2)}\left(\frac{1}{8}\right)\right) \;=\; \frac{1}{216}\left(\psi^{(2)}\left(\frac{7}{6}\right)-\psi^{(2)}\left(\frac{1}{6}\right)\right)$$

$$= \sum_{k=1}^\infty \frac{k}{(k^2-1/4)^2}$$ Prud. 5.1.25.31

$$= \sum_{k=1}^\infty \frac{(k-1)H_k}{2^k} \;=\; \sum_{k=1}^\infty \frac{F_k}{2^k} \;=\; \sum_{k=0}^\infty \frac{1}{4^k(k+1)}\binom{2k}{k}$$

$$= \exp\sum_{k=1}^\infty \frac{\zeta(2k)-1}{k}$$

$$= \sum_{k=1}^\infty \frac{z(k)}{k^2}$$

$$= \sum_{k=1}^\infty \left(\frac{\phi(2k)}{2^{2k}-1}+\frac{\phi(2k-1)}{2^{2k-1}-1}\right)$$

$$= \prod_{k=1}^\infty \left(1+\frac{1}{k(k+2)}\right) \;=\; \prod_{k=2}^\infty \frac{k^2}{k^2-1} \;=\; \prod_{k=1}^\infty \frac{k^2+2k+1}{k^2+2k}$$

$$= \prod_{k=0}^\infty \left(1+\frac{1}{2^{2^k}}\right)$$ Prud. 6.2.3.1

$$= \prod_{k=1}^\infty e^{(-1)^{k+1}/k} \;=\; \prod_{k=1}^\infty e^{1/2^k k}$$

$$= \int_1^\infty \frac{\log^2 x}{x^2}\,dx \;=\; \int_o^\infty \frac{x^2}{e^x}\,dx \;=\; \int_0^{2\pi} \frac{d\theta}{(\cos\theta)+\sqrt{\pi^2+1}}$$

$$= \int_0^{\pi/2} \frac{x-\sin x}{1-\cos x}\,dx$$ GR 3.791.1

$$= \int_0^{\pi/2} \left(\log\sin x\right)^2 \cos x\,dx$$

$$= \int_0^1 e^{\sqrt{x}}\,dx$$

$$= \int_0^\infty \frac{x^{11}\,dx}{e^{x^3}}$$

$$3 \quad .00000000000000000000\underline{0} \quad = \quad \sum_{k=0}^\infty \frac{k}{6^k}\binom{2k+1}{k}$$

$$= \sum_{k=1}^{\infty}\frac{F_k L_k}{3^k}$$

$$= \prod_{k=1}^{\infty}\left(1+\frac{2}{k(k+3)}\right)$$

$$4 \quad .00000000000000000000\underline{0} \quad = \quad \sum_{k=1}^{\infty}\frac{k}{2^{k-1}} = \sum_{k=2}^{\infty}\frac{k(k-1)}{2^k} = \sum_{k=0}^{\infty}\frac{1}{4^k(k+1)}\binom{2k+2}{k}$$

$$= \sum_{k=2}^{\infty}\frac{(2k)!!}{(2k-1)!!\,(k^2-k)} = \sum_{k=1}^{\infty}\frac{(2k)!!}{(2k-1)!!\,k(k+\tfrac{1}{2})}$$

$$= \prod_{k=1}^{\infty}\left(1+\frac{3}{k^2+4k}\right)$$

$$6 \quad .00000000000000000000\underline{0} \quad = \quad \Phi(\tfrac{1}{2},-2,0) = \sum_{k=1}^{\infty}\frac{k^2}{2^k} = \sum_{k=1}^{\infty}\frac{F_k F_{k+3}}{3^k}$$

$$= \prod_{k=3}^{\infty}\frac{k^2}{(k-2)(k+2)}$$

$$= \int_0^{\infty}\frac{x^3 dx}{e^x} = \int_1^{\infty}\frac{\log^3 x dx}{x^2}$$

$$10 \quad .00000000000000000000\underline{0} \quad = \quad \sum_{k=0}^{\infty}\frac{(k+4)}{2^k} = \sum_{k=1}^{\infty}\frac{F_k k}{2^k}$$

$$12 \quad .00000000000000000000\underline{0} \quad = \quad \sum_{k=0}^{\infty}\frac{(k+1)^2}{2^k} = \int_2^{\infty}\frac{dx}{x\log x} = \int_0^{\infty} x e^{-\sqrt{x}} dx$$

$$15 \quad .00000000000000000000\underline{0} \quad = \quad \Phi(1/3,-4,0) = \sum_{k=1}^{\infty}\frac{k^4}{3^k}$$

$$20 \quad .00000000000000000000\underline{0} \quad = \quad \prod_{k=4}^{\infty}\frac{k^2}{(k-3)(k+3)}$$

$$24 \quad .00000000000000000000\underline{0} \quad = \quad \int_0^{\infty}\frac{\log^4 x dx}{x^2}$$

$$26 \quad .00000000000000000000\underline{0} \quad = \quad \Phi(\tfrac{1}{2},-3,0) = \sum_{k=1}^{\infty}\frac{k^3}{2^k}$$

$$52 \quad .00000000000000000000\underline{0} \quad = \quad \sum_{k=0}^{\infty}\frac{(k+1)^3}{2^k}$$

$$70 \quad .00000000000000000000\underline{0} \quad = \quad \prod_{k=5}^{\infty}\frac{k^2}{(k-4)(k+4)}$$

$$94 \quad .00000000000000000000\underline{0} \quad = \quad \sum_{k=1}^{\infty}\frac{F_k k^2}{2^k}$$

$$150 \quad .00000000000000000000\underline{0} \quad = \quad \Phi(\tfrac{1}{2},-4,0) = \sum_{k=1}^{\infty}\frac{k^4}{2^k}$$

1082 .00000000000000000000$\underline{0}$ = $\Phi(\frac{1}{2},-5,0)=\sum_{k=1}^{\infty}\frac{k^5}{2^k}$

1330 .00000000000000000000$\underline{0}$ = $\sum_{k=1}^{\infty}\frac{F_k k^3}{2^k}$

25102. 00000000000000000000$\underline{0}$ = $\sum_{k=1}^{\infty}\frac{F_k k^4}{2^k}$

1 .00000027557319224027... ≈ $\sum_{k=1}^{\infty}\frac{1}{(10k)!}$

1 .000002739583286472... ≈ $\frac{697\pi^8}{6613488}=W_8$ J313

.00000484813681109536... ≈ $\frac{\pi}{648000}$, the number of radians in one second

1 .000006974709035616233... ≈ $\coth 2\pi$

.00001067827922686153... ≈ π^{-10}

1 .0000115515612717522... ≈ $\frac{547\cdot 61\pi^7}{2^{10}3^9 5}=\sum_{k=1}^{\infty}\left(\frac{(-1)^{k+1}}{(6k-5)^7}-\frac{(-1)^{k+1}}{(6k-1)^7}\right)$ J328

1 .000014085447167... ≈ W_7 J313

1 .0000248015873015873... ≈ $\sum_{k=1}^{\infty}\frac{1}{(k^3)!}$

.00003354680357208869... ≈ π^{-9}

1 .00003380783857369614... ≈ $\sec 1°$

.00004439438838973293... ≈ $\frac{1}{72}\left(12\arctan 2+6\arctan\left(4+\sqrt{3}\right)+6\arctan\left(4-\sqrt{3}\right)-4\left(3+\sqrt{3}\right)\right)$

$+\frac{1}{72}\left(3\log 3+\log 343+3\sqrt{3}\left(2\arctan\frac{5}{\sqrt{3}}+\log\frac{37+20\sqrt{3}}{13}\right)\right)$

= $\frac{1}{22528}\,{}_2F_1\left(\frac{11}{12},1,\frac{23}{12},\frac{1}{4096}\right)=\int_2^{\infty}\frac{dx}{x^{12}-1}$

.00004539992976248485... ≈ e^{-10}

1 .0000513451838437726... ≈ $\lambda(9)=\frac{511\zeta(9)}{512}=\sum_{k=0}^{\infty}\frac{1}{(2k+1)^9}$ AS 23.2.20

1 .00005516079486946347... ≈ $\frac{1}{358318080}\left(\psi^{(5)}\left(\frac{1}{12}\right)+\psi^{(5)}\left(\frac{5}{12}\right)-\psi^{(5)}\left(\frac{7}{12}\right)-\psi^{(5)}\left(\frac{11}{12}\right)\right)$

= $\sum_{k=1}^{\infty}\left(\frac{(-1)^{k+1}}{(6k-5)^6}-\frac{(-1)^{k+1}}{(6k-1)^6}\right)$

1	.0000733495124908981...	≈	$W_6 = \frac{91\pi^6}{87480}$	J313
	.00010539039165349367...	≈	π^{-8}	
	.00012340980408667955...	≈	e^{-9}	
6	.00014580284304486564...	≈	$-\zeta^{(3)}(2) = \sum_{k=1}^{\infty} \frac{\log^3 k}{k^2}$	
1	.000155179025296119303...	≈	$\lambda(8) = \frac{17\pi^8}{161280} = \frac{255\zeta(8)}{256} = \sum_{k=0}^{\infty} \frac{1}{(2k+1)^8}$	AS 23.2.20
1	.0002024735580587153...	≈	$\frac{\pi^3}{31}$	
	.0002480158734938207...	≈	$\sum_{k=1}^{\infty} \frac{1}{(8k)!}$	
1	.000249304876753261748...	≈	$-\frac{1}{8192}\psi^{(2)}\left(\frac{1}{16}\right) = \sum_{k=0}^{\infty} \frac{1}{(16k+1)^3}$	
	.0002499644008724365...	≈	$\sum_{k=2}^{\infty} \frac{1}{k^{12} - k^6} = \sum_{k=1}^{\infty} \left(\zeta(6k+6) - 1\right)$	
1	.0002572430074464511...	≈	$\frac{5 \cdot 61\pi^7}{2^7 3^6} = \sum_{k=1}^{\infty} \left(\frac{(-1)^{k+1}}{(6k-5)^5} - \frac{(-1)^{k+1}}{(6k-1)^5} \right)$	J328
	.0002604166666666666$\underline{6}$	=	$\frac{1}{3840} = \frac{1}{5!2^5}$	
	.0002908882086657216...	≈	$\frac{\pi}{10800}$, the number of radians in one minute of arc.	
	.000291375291375$\underline{291375}$	=	$\frac{1}{3432} = \prod_{k=8}^{\infty} \left(1 - \frac{49}{k^2}\right)$	
	.00029347092362294782...	≈	$\frac{\zeta(3)}{4096} = -\frac{1}{8192}\psi^{(2)}(1) = \sum_{k=1}^{\infty} \frac{1}{(16k)^3}$	
	.00033109368017756676...	≈	π^{-7}	
	.00033546262790251184...	≈	e^{-8}	
1	. 00038992014989212800...	≈	$W_5 = \frac{3751\zeta(5)}{3888} = \sum_{k=1}^{\infty} \left(\frac{1}{(6k+1)^5} + \frac{1}{(6k+5)^5} \right)$	J313
1	.000443474605655004...	≈	$\frac{307\pi^7}{655360\sqrt{2}}$	
		=	$\sum_{k=1}^{\infty} \left(\frac{(-1)^{k+1}}{(4k-3)^7} + \frac{(-1)^{k+1}}{(4k-1)^7} \right)$	J326
		=	$\sum_{k=0}^{\infty} \frac{(-1)^{\lfloor k/2 \rfloor}}{(2k+1)^7}$	Prud. 5.1.4.3

1 .0004642141285359797... ≈ $\frac{1}{7776}\psi^3\left(\frac{1}{6}\right) = \sum_{k=1}^{\infty}\frac{1}{(6k-1)^4}$

1 .00047154865237655476... ≈ $\lambda(7) = \frac{127\zeta(7)}{128} = \sum_{k=0}^{\infty}\frac{1}{(2k+1)^7}$ AS 23.2.20

1 .00049418860411946456... ≈ $\zeta(11) = \sum_{k=1}^{\infty}\frac{1}{k^{11}}$

1 .00056102267483432351... ≈ $-\frac{1}{3456}\psi^{(2)}\left(\frac{1}{12}\right) = \sum_{k=1}^{\infty}\frac{1}{(12k-11)^3}$

1 .000588927172046... ≈ $r(16)$ Berndt 8.14.1

$$= \frac{5}{8}\log 2 + \frac{\log(1+\sqrt{2})}{4\sqrt{2}} + \frac{\sqrt{2+\sqrt{2}}}{16}\log\frac{2+\sqrt{2+\sqrt{2}}}{2-\sqrt{2+\sqrt{2}}} + \frac{\sqrt{2-\sqrt{2}}}{16}\log\frac{2+\sqrt{2-\sqrt{2}}}{2-\sqrt{2-\sqrt{2}}}$$

.00064675959761549737... ≈ $1 + \frac{1}{2}\zeta(2) + \frac{21}{2}\zeta(4) - \frac{9}{2}\zeta(3) - \frac{15}{2}\zeta(5) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^3}{(k+2)^5}$

.00069563478192106151... ≈ $\frac{\zeta(3)}{1728} = \sum_{k=1}^{\infty}\frac{1}{(12k)^3}$

1 .00076240287712890293... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k\zeta^2(2k)}{2^k}$

.000807441557192450666... ≈ $\frac{1}{64}\left(\frac{1}{2} - \frac{3}{\pi} + \frac{5}{\pi^2}\right) = \int_0^{\infty}\frac{x\cos(2\pi x)}{e^{2\pi\sqrt{x}}-1}dx$

.00088110828277842903... ≈ $-\frac{1}{3456}\psi^{(2)}\left(\frac{11}{12}\right) = \sum_{k=1}^{\infty}\frac{1}{(12k-1)^3}$

.00091188196555451621... ≈ e^{-7}

1 .00097663219262887431... ≈ $\sum_{k=1}^{\infty}\frac{1}{k^{5k}}$

1 .00099457512781808534... ≈ $\zeta(10) = \sum_{k=1}^{\infty}\frac{1}{k^{10}}$

.00099553002208977438... ≈ $\sum_{k=1}^{\infty}\left(\zeta(10k)-1\right)$

.000998407381073899767... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^{10}-k^2}$

$$= 1 - \frac{\pi^2}{6} + \frac{1}{16}\left(-5 + \pi\left(2\coth\pi - (1-i)\sqrt{2}\left(\cot\frac{(1+i)}{\sqrt{2}}\right) - \coth\frac{1+i}{\sqrt{2}}\right)\right)$$

.00101009962219514182... ≈ $\frac{13}{12} - \zeta(4) = \sum_{k=1}^{\infty}\left(\zeta(6k+4)-1\right) = \sum_{k=2}^{\infty}\frac{1}{k^{10}-k^4}$

.00102614809103260947... ≈ $\sum_{k=1}^{\infty}\left(\zeta(5k+5)-1\right) = \sum_{k=2}^{\infty}\frac{1}{k^{10}-k^5} = \sum_{k=2}^{\infty}\frac{\Omega(k)}{k^5}$

.001040161473295852296... $\approx$ π^{-6}

.0010598949016150109... $\approx$ $\frac{15}{8}+\frac{\pi}{4}\coth\pi-\zeta(2)-\zeta(6)=\sum_{k=1}^{\infty}\big(\zeta(4k+6)-1\big)=\sum_{k=2}^{\infty}\frac{1}{k^{10}-k^{6}}$

.00108225108225$\underline{108225}$ $=$ $\frac{1}{924}=\prod_{k=7}^{\infty}\left(1-\frac{36}{k^{2}}\right)$

.00114484089113910728... $\approx$ $-\frac{1}{3456}\psi^{(2)}\left(\frac{5}{6}\right)=\sum_{k=1}^{\infty}\frac{1}{(12k-2)^{3}}$

1 .00115374373790910569... $\approx$ $\frac{1}{124416}\left(\psi^{(3)}\left(\frac{1}{12}\right)+\psi^{(3)}\left(\frac{5}{12}\right)-\psi^{(3)}\left(\frac{7}{12}\right)-\psi^{(3)}\left(\frac{11}{12}\right)\right)$

$=$ $\sum_{k=1}^{\infty}\left(\frac{(-1)^{k+1}}{(6k-5)^{4}}-\frac{(-1)^{k+1}}{(6k-1)^{4}}\right)$

1 .00124297812019553 8... $\approx$ $-\frac{1}{1458}\psi^{(2)}\left(\frac{1}{9}\right)=\sum_{k=1}^{\infty}\frac{1}{(9k-8)^{3}}$

3 .00126030124568644264... $\approx$ $\sum_{k=1}^{\infty}\frac{2^{k}k^{2}}{k+1}\big(\zeta(k+1)-1\big)=\frac{\pi^{2}}{3}-\frac{\gamma}{2}$

.001264489267349618680... $\approx$ $\frac{22}{7}-\pi=\int_{0}^{1}\frac{x^{4}(1-x)^{4}}{1+x^{2}}dx$

1 .00130144245443407196... $\approx$ $\frac{1}{31457280}\left(\psi^{(5)}\left(\frac{1}{8}\right)+\psi^{(5)}\left(\frac{3}{8}\right)-\psi^{(5)}\left(\frac{5}{8}\right)-\psi^{(5)}\left(\frac{7}{8}\right)\right)$

$=$ $\sum_{k=1}^{\infty}\left(\frac{(-1)^{k+1}}{(4k-3)^{6}}-\frac{(-1)^{k+1}}{(4k-1)^{6}}\right)$

.001311794958417665... $\approx$ $\frac{197}{27\cdot 64}-\frac{3\zeta(3)}{32}=\sum_{k=0}^{\infty}\frac{(-1)^{k}}{(2k+8)^{3}}$

.00138916447355203054... $\approx$ $\frac{e}{4}-\frac{1}{4e}-1-\frac{\cos 1}{2}=\sum_{k=1}^{\infty}\frac{1}{(4k+2)!}$

3 .00139933814196238175... $\approx$ γ^{-2}

1 .0013996605974732513... $\approx$ $\frac{3+\sqrt{3}}{6}\log 2+\frac{\log 3}{4}-\frac{\log(\sqrt{3}-1)}{\sqrt{3}}=r(12)$ Bendt 8.14.1

.00142506011172369955... $\approx$ $\sum_{k=2}^{\infty}\frac{1}{k^{10}\log k}$

.00144498563598179575... $\approx$ $1-\frac{960}{\pi^{6}}=1-\frac{1}{\lambda(6)}=\sum_{k=2}^{\infty}\frac{\mu(2k)}{k^{6}}$

.00144687406599163371... $\approx$ $\sum_{k=2}^{\infty}\frac{\mu(2k)}{k^{6}-1}$

1 .0014470766409421219... $\approx$ $\frac{\pi^{6}}{960}=\lambda(6)=\frac{63\zeta(6)}{64}=\sum_{k=1}^{\infty}\frac{1}{(2k+1)^{6}}$ AS 23.2.20

= $\frac{1}{1-\sum_{k=2}^{\infty}\frac{\mu(2k)}{k^{6}}}$

.00146293937923861579... ≈ $-\frac{1}{8192}\psi^{(2)}\left(\frac{9}{16}\right) = \sum_{k=0}^{\infty}\frac{1}{(16k+9)^{3}}$

24 .00148639373646157098... ≈ $\zeta^{(4)}(2) = \sum_{k=1}^{\infty}\frac{\log^{4}k}{k^{2}}$

.00153432674083843739... ≈ $-\frac{1}{3456}\psi^{(2)}\left(\frac{3}{4}\right) = \sum_{k=1}^{\infty}\frac{1}{(12k-3)^{3}}$

.0015383430770987471... ≈ $\frac{930\zeta(5)-\pi^{6}}{1920} = \sum_{k=1}^{\infty}\frac{k}{(2k+1)^{6}}$

.0015383956660985045... ≈ $\frac{4-\zeta(3)}{256} = \sum_{k=0}^{\infty}\frac{(-1)^{k}}{(4k+8)^{3}}$

1 .001712244255991878... ≈ $-\frac{1}{1024}\psi^{(2)}\left(\frac{1}{8}\right) = \sum_{k=1}^{\infty}\frac{1}{(8k-7)^{3}}$

1 .00171408596632857117... ≈ $\frac{5\zeta(3)}{6}$

1 .00181129167264869533... ≈ $\frac{1}{1536}\psi^{(3)}\left(\frac{1}{4}\right) = \sum_{k=0}^{\infty}\frac{1}{(4k+1)^{4}}$

= $\frac{1}{6}\int_{0}^{1}\frac{\log^{3}x}{x^{4}-1}dx$

.001815222323088761... ≈ $\frac{5\pi^{2}}{54}-\frac{197}{216} = \sum_{k=1}^{\infty}\frac{1}{k^{2}(k+1)^{2}(k+2)^{2}(k+3)^{2}}$ LY 6.21

.00187137275936602737 9... ≈ $\frac{7\pi^{3}}{360}-\frac{\zeta(3)}{2} = \sum_{k=1}^{\infty}\frac{1}{k^{3}\left(e^{2\pi k}-1\right)}$ Ramanujan

= $\sum_{k=1}^{\infty}\frac{\sigma_{-3}(k)}{e^{2\pi k}-1}$

.00187180916165245621 0... ≈ $\sum_{k=1}^{\infty}\frac{1}{k^{2}\left(e^{2\pi k}-1\right)} = \sum_{k=1}^{\infty}\frac{\sigma_{-2}(k)}{e^{2\pi k}}$

.00187268244976854611 6... ≈ $\sum_{k=1}^{\infty}\frac{1}{k\left(e^{2\pi k}-1\right)} = \sum_{k=1}^{\infty}\frac{\sigma_{-1}(k)}{e^{2\pi k}}$

.00187443047777494092... ≈ $\sum_{k=1}^{\infty}\frac{1}{e^{2\pi k}-1} = \sum_{k=1}^{\infty}\frac{\sigma_{0}(k)}{e^{2\pi k}}$

.00187795166148433638 4... ≈ $\int_{0}^{\infty}\frac{x^{6}dx}{e^{2\pi x}-1} = \frac{45\zeta(7)}{8\pi^{7}}$

.001912334879124743... ≈ $\frac{15e^{2}-109}{960} = \sum_{k=0}^{\infty}\frac{2^{k}}{(k+6)!}$

1 .001949804343... ≈ j_9 J311

1 .00195748553... ≈ G_9 J309

$.001984126984126\underline{984126}$ = $\frac{1}{504} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^5}{e^{\pi k}+(-1)^k} = \int_0^{\infty}\frac{x^5}{e^{2\pi x}-1}dx$

2 .001989185473323053259... ≈ $\frac{\sinh\pi}{\pi^5}\left(\sin\left((-1)^{1/10}\pi\right)\sin\left((-1)^{3/10}\pi\right)\sin\left((-1)^{7/10}\pi\right)\sin\left((-1)^{9/10}\pi\right)\right)$

= $\prod_{k=1}^{\infty}\left(1+\frac{1}{k^{10}}\right)$

1 .00200839282608221442... ≈ $\zeta(9)$

.00201221758463940331... ≈ $\sum_{k=1}^{\infty}\left(\zeta(9k)-1\right) = \sum_{k=2}^{\infty}\frac{1}{k^9-1}$

.00201403420628152655... ≈ $\frac{3}{4}\zeta(6)+\frac{3}{2}\zeta(5)-\frac{1}{2}\zeta^2(3)+\frac{5}{4}\zeta(4)+\frac{9}{8}\zeta(3)+\frac{17}{16}\zeta(2)-\frac{387}{64}$

= $\sum_{k=1}^{\infty}\frac{H_{k-1}}{(k+2)^5}$

.00201605994409566126... ≈ $\sum_{k=1}^{\infty}\left(\zeta(8k+1)-1\right) = \sum_{k=2}^{\infty}\frac{1}{k^9-k}$

.00201681670359358... ≈ $\frac{3277}{3375}-\frac{\pi^3}{32} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+7)^3}$

.00202379524407751486... ≈ $\sum_{k=1}^{\infty}\left(\zeta(7k+2)-1\right) = \sum_{k=2}^{\infty}\frac{1}{k^9-k^2}$

.002037712107418497211... ≈ $MHS(9,1) = \frac{7}{4}\zeta(10)-\zeta^2(5)-\zeta(3)\zeta(7)$

.00203946556435117678... ≈ $\sum_{k=1}^{\infty}\left(\zeta(6k+3)-1\right) = \sum_{k=2}^{\infty}\frac{1}{k^9-k^3}$

.00205429646536063477... ≈ $\frac{7\zeta(3)}{4096} = -\frac{1}{8192}\psi^{(2)}\left(\frac{1}{2}\right) = \sum_{k=0}^{\infty}\frac{1}{(16k+8)^3}$

$.00208\underline{3}$ = $\frac{1}{480} = \int_0^{\infty}\frac{x^7dx}{e^{2\pi x}-1}$

.00210716107047916356... ≈ $\sum_{k=1}^{\infty}\left(\zeta(5k+4)-1\right) = \sum_{k=2}^{\infty}\frac{1}{k^9-k^4}$

.002124728805039749548... ≈ $1-\frac{\pi^4}{90}+\log\frac{4\pi}{\sinh\pi} = \sum_{k=1}^{\infty}\frac{\zeta(4k)-1}{k} = \sum_{k=2}^{\infty}\left(\log\left(1-\frac{1}{k^4}\right)-\frac{1}{k^4}\right)$

.0021368161356763007... ≈ $-\frac{1}{3456}\psi^{(2)}\left(\frac{2}{3}\right) = \sum_{k=1}^{\infty}\frac{1}{(12k-4)^3}$

.00213925209462515473... $\approx$ $\frac{13}{8}-\frac{\gamma}{2}-\zeta(5)+\frac{1}{4}\big(\psi(2+i)-\psi(2-i)\big)$

$= \sum_{k=1}^{\infty}\big(\zeta(4k+5)-1\big)=\sum_{k=2}^{\infty}\frac{1}{k^9-k^5}$

1 .0021511423251279551... $\approx$ $\frac{5\pi^4}{486}=W_4$ J313

1 .00220358587275955997... $\approx$ $\frac{19\pi^5}{11520}+\frac{\pi^3\log^2 2}{96}+\pi\frac{\log^4 2}{48}+\frac{\zeta(3)\pi\log 2}{8}$

$= \sum_{k=0}^{\infty}\frac{1}{4^k(2k+1)^5}\binom{2k}{k}$

.00227204396400239669... $\approx$ $-\frac{1}{1458}\psi^{(2)}\left(\frac{8}{9}\right)=\sum_{k=1}^{\infty}\frac{1}{(9k-1)^3}$

.00228942996522361328... $\approx$

$2-\zeta(3)-\zeta(6)+\left(\frac{1}{6}+\frac{i}{2\sqrt{3}}\right)\Big(\psi\big(2+(-1)^{1/3}\big)-\psi\big(2-(-1)^{1/3}\big)\Big)$

$= \sum_{k=1}^{\infty}\frac{1}{k^9-k^6}=\sum_{k=1}^{\infty}(\zeta(3k+6)-1)$

.00234776738898358259... $\approx$ $\frac{\zeta(3)}{512}=\sum_{k=1}^{\infty}\frac{1}{(8k)^3}$

1 .00237931966451004015... $\approx$ $\frac{\pi\log 2}{4}+\frac{G}{2} = -\int_0^{\pi/4}\log\sin x\,dx$ GR 4.224.2

$= -\int_0^{1/\sqrt{2}}\frac{\log x}{\sqrt{1-x^2}}dx$ GR 2.241.6

1 .0024250560555062466... $\approx$ $\frac{2\log 2}{5}+\frac{\log 5}{4}+\frac{3}{2\sqrt{5}}\log\frac{1+\sqrt{5}}{2} = r(10)$ Berndt 8.14.1

.002450822732290039104... $\approx$ $\frac{3}{4\pi^5} = \int_{-\infty}^{\infty}\frac{x^4e^{-x}}{1+e^{-2\pi x}}dx$

.00245775047043566779... $\approx$ $17e-\frac{1109}{24}=\sum_{k=1}^{\infty}\frac{k^2}{(k+5)!}$

.002478752176666358... $\approx$ e^{-6}

2 .0025071947052832538... $\approx$ $6\log 2+8\log^2 2-6=\sum_{k=0}^{\infty}\frac{k^2H_k}{2^k(k+2)}$

.00254132611404785053... $\approx$ $\frac{3\zeta(5)}{4\pi^5} = \int_0^{\infty}\frac{x^4dx}{e^{2\pi x}-1}$

.002597505554170387853... $\approx$ $-\frac{1}{432}\psi^{(2)}\left(\frac{4}{3}\right)$

$.002604161616161616\underline{16}$ = $\frac{1}{284}=\frac{1}{4!2^4}$

1 .00260426354837675067... ≈ $\frac{1}{2}\left(\cos\frac{1}{2}+\cosh\frac{1}{2}\right) = \frac{1}{4\sqrt{e}} + \frac{\sqrt{e}}{4} + \frac{1}{2}\cos\frac{1}{2} = \sum_{k=0}^{\infty}\frac{1}{(4k)!16^k}$

.00260853718903279685 8... ≈ $\frac{1}{8}\log\frac{5}{3} - \frac{1}{4}\arctan\frac{1}{4} = \int_2^{\infty}\frac{dx}{x^7 - x^{-1}}$

.00261360465209153398 2... ≈ $\int_2^{\infty}\frac{dx}{x^7-1}$

.00262472616135652810... ≈ $\log 2 - \frac{\log 63}{6} = \int_2^{\infty}\frac{dx}{x^7 - x}$

.00264711657512301799... ≈ $\frac{315\zeta(9)}{4\pi^9} = \int_0^{\infty}\frac{x^8dx}{e^{2\pi x}-1}$

.00264953284061841586... ≈

$$\frac{1}{20}\left(\left(2\sqrt{5+2\sqrt{5}}\right)\left(\pi-\arctan\sqrt{13-\frac{22}{\sqrt{5}}}\right)+\log 31-2\sqrt{10-2\sqrt{5}}\arctan\sqrt{13+\frac{22}{\sqrt{5}}}-10+2\sqrt{5}\,\mathrm{arccoth}\frac{6}{\sqrt{5}}\right)$$

= $\int_2^{\infty}\frac{dx}{x^7 - x^2}$

1 .002651309852400201... ≈ $\frac{2\sqrt{2\pi}}{5} = \int_0^{\infty}\frac{\sin x - x\cos x}{x^{7/2}}dx$

.00270640594149767080... ≈ $\frac{1}{4}\log\frac{5}{3} - \frac{1}{8} = \int_2^{\infty}\frac{dx}{x^7 - x^3}$

1 .00277832894710406108... ≈ $\cosh 1 - \cos 1$

.0028437975415075410... ≈ $\log 2 - \frac{\log 7}{3} - \frac{1}{24} = \int_2^{\infty}\frac{dx}{x^7 - x^4}$

.00286715218149708931... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^9\log k}$

.00286935966734132299... ≈ $\frac{195353}{216000} - \frac{3\zeta(3)}{4} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k+5)^3}$

= $\int_0^1\int_0^1\int_0^1\frac{x^5y^5z^5}{1+xyz}dx\,dy\,dz$

5 .0029142333248880669... ≈ $\sum_{k=0}^{\infty}\frac{2^k}{k!+1}$

.002928727553540092... ≈ $\frac{\zeta(3)}{36} - \frac{\pi^2}{324} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}k}{(k+3)^3}$

.00302826097746345169... ≈ $-\frac{1}{8192}\psi^{(2)}\left(\frac{7}{16}\right) = \sum_{k=0}^{\infty} \frac{1}{(16k+7)^3}$

.0031244926731183736... ≈ $-\frac{1}{3450}\psi^{(2)}\left(\frac{7}{12}\right) = \sum_{k=1}^{\infty} \frac{1}{(12k-5)^3}$

.00318531972162645303... ≈ $\frac{\pi}{2}e^{-3\sqrt{2}}\sin 3 = \int_{-\infty}^{\infty} \frac{\sin 3x}{x^2+2x+3}$

.00321603622589046372... ≈ $\log 2 - \frac{\log 3}{2} - \frac{9}{24} = \int_{2}^{\infty} \frac{dx}{x^7 - x^5}$

.00326776364305338547... ≈ π^{-5}

.003286019155251709... ≈ $-\frac{1}{1458}\psi^{(2)}\left(\frac{7}{9}\right) = \sum_{k=1}^{\infty} \frac{1}{(9k-2)^3}$

.0033002236853241029... ≈ $\mathrm{Re}\left\{\zeta(i)\right\}$

.0033104481564221302... ≈ $\frac{1}{16} - \frac{7\pi^4}{45\cdot 256}$

.0033178346712119643... ≈ $\frac{3\zeta(3)}{32} - \frac{7}{64} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+6)^3}$

= $\int_0^1\int_0^1\int_0^1 \frac{x^5y^5z^5}{1+x^2y^2z^2}\,dx\,dy\,dz$

24 .00333297476905227... ≈ $\frac{e(e^3+11e^2+11e+1)}{(e-1)^5} = \Phi\left(\frac{1}{e},-4,0\right) = \sum_{k=1}^{\infty} \frac{k^4}{e^k}$

.0033697704469093... ≈ $\frac{15\zeta(5)}{16} - \frac{31}{32} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+3)^5}$

.00337787815787790335... ≈ $-\frac{1}{1024}\psi^{(2)}\left(\frac{7}{8}\right) = \sum_{k=1}^{\infty} \frac{1}{(8k-1)^3}$

.00347222222222222222$\underline{2}$ = $\frac{1}{288} = \sum_{k=1}^{\infty} \frac{k}{(k+1)(k+2)(k+3)(k+4)(k+5)}$

.00364656750482608467... ≈ $\frac{1}{27} - \frac{\zeta(3)}{36} = \sum_{K=1}^{\infty} \frac{(-1)^{k+1}}{(3k+3)^3}$

= $\int_0^1\int_0^1\int_0^1 \frac{x^5y^5z^5}{1+x^3y^3z^3}\,dx\,dy\,dz$

.00368755153479527... ≈ $\frac{\pi^3}{36\sqrt{3}} + \frac{91\zeta(3)}{216} = -\frac{1}{432}\psi^{(2)}\left(\frac{1}{6}\right)$ Berndt 7.12.3

= $\sum_{k=1}^{\infty} \frac{1}{(6k-5)^3}$

.0037145966378051377... ≈ $\log(-\log(-\log(\log 2)))$

1 .0037215043023101292 7... ≈ $-\log(-\log(\log 2))$

.0037418731973212882 0... ≈ $\coth\pi - 1 = \dfrac{2}{e^{2\pi}-1}$

1 .0037418731973212882 0... ≈ $\coth\pi = \dfrac{e^{\pi}+e^{-\pi}}{e^{\pi}-e^{-\pi}}$

$.003750000000000000\underline{0}$ = $\dfrac{3}{800} = \displaystyle\sum_{k=1}^{\infty}\frac{1}{(k+2)(k+3)(k+4)(k+5)}$

1 .0037556863956550109 9... ≈ $\dfrac{19\pi^5}{2^{12}\sqrt{2}} = \displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(4k-3)^5} + \frac{(-1)^{k+1}}{(4k-1)^5}$ J 326

= $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{\lfloor k/2\rfloor}}{(2k+1)^5}$ Prud. 5.1.4.3

$.0037878787878787\underline{87}\underline{8}$ = $\dfrac{1}{264} = \displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^9}{e^{\pi k}+(-1)^k}$

1 .003795666917135695... ≈ $\dfrac{\pi + 4\arctan 3 + 2\log 5}{8\sqrt{2}} = \displaystyle\int_0^{\sqrt{2}}\frac{dx}{1+x^4}$

.00388173171751883... ≈ $\dfrac{32-\pi^3}{256} = \displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(4k+2)^3}$

.0038845985984860509... ≈ $\dfrac{22}{7} + 2\log\dfrac{2}{3} = \displaystyle\sum_{k=0}^{\infty}\frac{(-1)^k k^3}{2^k(k+1)}$

1 .0038848218538872141... ≈ $\arctan\dfrac{\pi}{2}$

1 .003893431348... ≈ j_8 J 311

1 .0039081319092642885... ≈ $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k^{4k}}$

1 .0039243189542013209... ≈ $\dfrac{656\pi^8}{6200145} = G_8$ J 309

$.0039682539682539682\underline{5}$ = $\dfrac{1}{252} = -\zeta(-5) = \displaystyle\prod_{k=6}^{\infty}\left(1-\frac{25}{k^2}\right)$

2 .0040170020015391198 7... ≈ $\displaystyle\prod_{k=1}^{\infty}\left(\frac{1}{1-k^{-9}}\right)$

= $1/\left(\Gamma\left(-(-1)^{1/9}\right)\Gamma\left((-1)^{2/9}\right)\Gamma\left(-(-1)^{1/3}\right)\Gamma\left((-1)^{4/9}\right)\right)$

$\bullet\, 1/\left(\Gamma\left(-(-1)^{5/9}\right)\Gamma\left((-1)^{2/3}\right)\Gamma\left(-(-1)^{7/9}\right)\Gamma\left((-1)^{8/9}\right)\right)$

.0040255657517126268 6... ≈ $\dfrac{7\zeta(4)}{8} - \dfrac{7}{8} - \dfrac{\pi}{4\sinh\pi} = \displaystyle\sum_{k=2}^{\infty}\frac{(-1)^k}{k^8-k^4}$

$= \quad \sum_{k=2}^{\infty} \frac{(-1)^k \Omega(k)}{k^4} = \sum_{\substack{\omega \text{ a non-trivial} \\ \text{integer power}}} \frac{(-1)^{\omega}}{\omega^4 - 1}$

.00403144180414993614 8... ≈ $\frac{1}{8\pi^3} = \int_0^{\infty} \frac{x^2 e^{-x}}{1 + e^{-2\pi x}} dx$

.0040607988744848531 7... ≈ $1 - \frac{9450}{\pi^8} = \frac{\zeta(8) - 1}{\zeta(8)}$

.00406140536651783056 1... ≈ $\sum_{p \text{ prime}} p^{-8} = \sum_{k=1}^{\infty} \frac{\mu(k)}{k} \log\left(\zeta(8k)\right)$

1 .0040773561979443394... ≈ $\frac{\pi^8}{9450} = \zeta(8)$

1 .0040780274963726575 7... ≈ $\frac{i}{2\pi^4} \left(\sin\left((-1)^{1/8}\pi\right) \sin\left((-1)^{3/8}\pi\right) \sin\left((-1)^{7/8}\pi\right) \sinh\left((-1)^{1/8}\pi\right) \right)$

$= \quad \prod_{k=2}^{\infty} \left(1 + k^{-8}\right)$

.0040926982992862873 1... ≈ $\frac{15}{16} - \frac{\pi}{8} \coth \pi + \frac{\pi\sqrt{2}}{8} \frac{\sin \pi\sqrt{2} + \sinh \pi\sqrt{2}}{\cos \pi\sqrt{2} - \cosh \pi\sqrt{2}}$

$= \quad \sum_{k=1}^{\infty} \left(\zeta(8k) - 1\right) = \sum_{k=2}^{\infty} \frac{1}{k^8 - 1}$

1 .0040933723113992278 3... ≈ $\frac{16\pi^3 \operatorname{csch} \pi}{\cosh \pi\sqrt{2} - \cos \pi\sqrt{2}} = \prod_{k=2}^{\infty} \frac{1}{1 - k^{-8}}$

.0041056420979153835 5... ≈ γ^{10}

.0041081847620803150 2... ≈ $\sum_{k=2}^{\infty} \frac{1}{k^8 - k} = \sum_{k=1}^{\infty} \left(\zeta(7k + 1) - 1\right)$

.0041395734331186539 8... ≈ $\frac{3}{4} - \zeta(2) + \frac{\pi}{2\sqrt{3}} \tanh \frac{\pi\sqrt{3}}{2}$

$= \quad \sum_{k=2}^{\infty} \frac{1}{k^8 - k^2} = \sum_{k=1}^{\infty} \left(\zeta(6k + 2) - 1\right)$

.00414721082882274135 3... ≈ $\frac{319}{512} - \frac{35\pi^2}{768} - \frac{\pi^4}{576}$

$= \quad \sum_{k=2}^{\infty} \frac{1}{(k^2 - 1)^5}$

1 .0041514158449728636 1... ≈ $16 - 16\sqrt{2} + 4\pi - \frac{\pi^2}{2} = \int_0^1 \frac{\arccos^2 x}{\sqrt{1 + x}} dx$

$.0041666666666666666\underline{6}$ = $\frac{1}{240} = -\zeta(-7) = \int_0^{\infty} \frac{x^3 dx}{e^{2\pi x} - 1}$

.004170242045482641209... ≈ $MHS(8,1) = 4\zeta(9) - \zeta(2)\zeta(7) - \zeta(4)\zeta(5) - \zeta(3)\zeta(6)$

.0042040099042187... ≈ $\sum_{k=2}^{\infty} \frac{1}{k^8 - k^3} = \sum_{k=1}^{\infty} \left(\zeta(5k+3) - 1\right)$

.0043397425545712... ≈ $\frac{15}{8} - \zeta(4) - \frac{\pi}{4} \coth \pi$

= $\sum_{k=2}^{\infty} \frac{1}{k^8 - k^4} = \sum_{k=1}^{\infty} \left(\zeta(4k+4) - 1\right) = \sum_{k=1}^{\infty} \frac{\Omega(k)}{k^4}$

.00442764193431403234... ≈ $\frac{\zeta(2)}{2} + \frac{7\pi^4}{180} - 3\zeta(3) - 1 = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^2}{(k+2)^4}$

1 .00449344967863012707... ≈ $\frac{1}{486} \psi^{(3)}\left(\frac{1}{3}\right) = \sum_{k=0}^{\infty} \frac{1}{(3k+1)^4} = \frac{1}{6} \int_0^1 \frac{\log^3 x}{x^3 - 1} dx$

.00450339052463230128... ≈ $\sum_{k=2}^{\infty} \frac{\mu(2k)}{k^5}$

.00452050160996510208... ≈ $\sum_{k=2}^{\infty} \frac{\mu(2k)}{k^5 - 1}$

3 .00452229793774208627... ≈ $\frac{\pi^2}{2} \csc^2 \pi\sqrt{2} - \frac{\pi\sqrt{2}}{4} \cot \pi\sqrt{2} - 2$

= $\sum_{k=1}^{\infty} k 2^k \left(\zeta(2k) - 1\right) = \sum_{k=2}^{\infty} \frac{2k^2}{(k^2 - 2)^2}$

1 .00452376279513961613... ≈ $\frac{31\zeta(5)}{32} = \lambda(5) = \sum_{k=0}^{\infty} \frac{1}{(2k+1)^5}$ AS 23.2.20

.004548315464154810203... ≈ $\frac{3\pi^2}{256} - \frac{1}{9} = \sum_{k=1}^{\infty} \frac{1}{(2k-1)^2 (2k+1)^2 (2k+3)^2}$ LY 6.21

.0045635776995554368... ≈ $1 + \frac{G}{2} - \frac{3\pi^3}{64} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k}{(2k+3)^3}$

.00460551389327864275... ≈ $\log 2 - \frac{661}{960} = \int_2^{\infty} \frac{dx}{x^7 - x^6}$

.004641520347027523161... ≈

$2 - \zeta(5) - \zeta(2) + \frac{\gamma}{3} + \frac{1}{6}\left(\left(1 - i\sqrt{3}\right)\psi\left(2 + (-1)^{1/3}\right) + \left(1 + i\sqrt{3}\right)\psi\left(2 - (-1)^{1/3}\right)\right)$

= $\sum_{k=2}^{\infty} \frac{1}{k^8 - k^5} = \sum_{k=1}^{\infty} \left(\zeta(3k+5) - 1\right)$

.00475611798059498829... ≈ $-\frac{1}{8192} \psi^{(2)}\left(\frac{3}{8}\right) = \sum_{k=0}^{\infty} \frac{1}{(16k+6)^3}$

2 .004758639329020080 3... ≈ $G + \frac{\pi \log 2}{2} = \int_0^1 \frac{\log(1+x^2)}{1+x^2} dx$ GR 4.295.6

= $-\int_0^{\pi/2} \log\sin\left(\frac{x}{2}\right) dx$

1 .00475980063006056 6114... ≈ $\log 2 + \frac{\sqrt{2}}{4}\log\left(1+\sqrt{2}\right) = r(8)$

.004764821927567051 0... ≈ $\frac{67}{64} - \frac{\pi^2}{96} - \frac{\pi^4}{360} - \frac{\zeta(3)}{8} - \frac{\zeta(5)}{2} = \sum_{k=2}^{\infty} \frac{1}{k(k+2)^5}$

1 .0048330405652719501 3... ≈ $\frac{7\pi^3}{216} = \sum_{k=1}^{\infty}\left(\frac{(-1)^{k+1}}{(6k-5)^3} + \frac{(-1)^{k+1}}{(6k-1)^3}\right)$

.00486944347344743055... ≈ $\frac{7\zeta(3)}{1728} = \sum_{k=1}^{\infty} \frac{1}{(12k-6)^3}$

.00491550994358136550... ≈ $\frac{\sin\pi^2}{\pi^2 - \pi^4} = \prod_{k=2}^{\infty}\left(1 - \frac{\pi^2}{k^2}\right)$

.00498320$\underline{0}$... ≈ mil/sec

.00506504565493641652... ≈ $-\frac{1}{1458}\psi^{(2)}\left(\frac{2}{3}\right) = \sum_{k=1}^{\infty}\frac{1}{(9k-3)^3}$

.00507713440452621921... ≈ $\frac{\pi^4-93\zeta(5)}{192} = \sum_{k=1}^{\infty}\frac{k}{(2k+1)^5}$

2 .005110575642302907627... ≈ $\frac{4}{3}+\frac{10\pi\sqrt{3}}{81}$

$= \sum_{k=1}^{\infty}\frac{k^2}{\binom{2k}{k}} = \frac{1}{2}\,{}_2F_1\left(2,2,\frac{3}{2},\frac{1}{4}\right) + \frac{1}{3}\,{}_2F_1\left(3,3,\frac{5}{2},\frac{1}{4}\right)$

.00513064033265867701... ≈ $\frac{3\zeta(3)}{4}-\frac{1549}{1728} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k+4)^3}$

$= \int_0^1\int_0^1\int_0^1\frac{x^4y^4z^4}{1+xyz}\,dx\,dy\,dz$

.005161189593421394431... ≈ $\frac{13}{30}-\frac{\pi}{8}\coth\frac{\pi}{2} = \sum_{k=1}^{\infty}\frac{\zeta(k)-1}{16^k}$

.0051783527503297262... ≈ $\frac{7\zeta(3)}{128}-\frac{\pi^3}{512} = -\frac{1}{1024}\psi^{(2)}\left(\frac{3}{4}\right) = \sum_{k=1}^{\infty}\frac{1}{(8k-2)^3}$

.0053996374561862323... ≈ $\frac{15}{4}-\zeta(2)-\zeta(4)-\zeta(6) = \sum_{k=1}^{\infty}\left(\zeta(2k+6)-1\right)$

$= \sum_{k=2}^{\infty}\frac{1}{k^8-k^6}$

.005491980326903542827... ≈ $\frac{13-4\pi}{8\pi^2} = \int_0^{\infty}\frac{x\cos(\pi x/2)}{e^{2\pi\sqrt{x}}-1}\,dx$ Prud. 2.5.38.10

.0055555555555555555$\underline{5}$ = $\frac{1}{180} = \sum_{k=1}^{\infty}\frac{k}{(k+2)(k+3)(k+4)(k+5)}$

6 .005561414313810886999... ≈ $\frac{2}{\log^3 2} = \int_0^{\infty}\frac{x^2}{2^x}\,dx$

.00567775514336992633... ≈ $\zeta(5,3)$

1 .00570213235367585932... ≈ $\frac{1}{2}+\frac{\log 2}{3}+\frac{\log 3}{4} = \sum_{k=1}^{\infty}\frac{1}{216k^3-6k^2}$ Ramanujan] Berndt Ch. 2

.005787328838611000493... ≈ $\sum_{k=1}^{\infty}\frac{1}{k^8\log k}$

.005899759143515937... ≈ $\frac{45\zeta(7)}{8\pi^6}$ Berndt 9.27.12

1 .005912144457743732... ≈ $\frac{1}{250}\psi^{(2)}\left(\frac{1}{5}\right) = \sum_{k=1}^{\infty}\frac{1}{(5k-4)^3} = \frac{1}{2}\int_0^1 \frac{\log^2 x}{1-x^5}\,dx$

1 .005913809064703791 3... ≈ $\frac{\pi^3\sqrt{3}}{243} + \frac{91\zeta(3)}{486} + \frac{1}{324}\psi^{(1)}\left(\frac{1}{6}\right) + \frac{1}{17496}\psi^{(3)}\left(\frac{1}{6}\right)$

$= \sum_{k=0}^{\infty}\frac{(2k+1)^2}{(6k+1)^4}$

.005983183296406418... ≈ $\frac{\pi^3}{32} - \frac{26}{27} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+5)^3}$

.00603351696087563779... ≈ $-\zeta'(7) = \sum_{k=2}^{\infty}\frac{\log k}{k^7}$

.00605652195492690338... ≈ $-\frac{1}{4096}\psi^{(2)}\left(\frac{7}{16}\right) = \int_1^{\infty}\frac{\log^2 x}{x^8 - x^{-8}}\,dx$

.00609098301750747937... ≈ $-\frac{1}{3375}\psi^{(2)}\left(\frac{7}{15}\right) = \int_1^{\infty}\frac{\log^2 x}{x^8 - x^{-7}}\,dx$

.00613294338346731778... ≈ $\frac{\zeta(3)}{196} = -\frac{1}{2744}\psi^{(2)}\left(\frac{1}{2}\right) = \int_1^{\infty}\frac{\log^2 x}{x^8 - x^{-6}}\,dx$

1 .00615220919858899772... ≈ $\sum_{k=2}^{\infty}\left(\frac{\zeta(k)}{\zeta(k+1)} - \frac{\zeta(k+1)}{\zeta(k)}\right)$

2 .00615265522741426944... ≈ $\sum_{k=1}^{\infty}\frac{\sqrt{k}}{k^2+1}$

.00618459927831671541... ≈ $-\frac{1}{2197}\psi^{(2)}\left(\frac{7}{13}\right) = \int_1^{\infty}\frac{\log^2 x}{x^8 - x^{-5}}\,dx$

.00624898534623674710... ≈ $-\frac{1}{1728}\psi^{(2)}\left(\frac{7}{12}\right) = \int_1^{\infty}\frac{\log^2 x}{x^8 - x^{-4}}$

.00625941279499622882... ≈ $-\frac{1}{4}\arctan 2 + \frac{\sqrt{2}}{16}\log\left(\frac{5-2\sqrt{2}}{5+2\sqrt{2}}\right) +$

$+ \frac{\pi(1-\sqrt{2})}{8} + \frac{\sqrt{2}}{8}\left(\arctan\left(1+\sqrt{2}\right) - \arctan\left(1-\sqrt{2}\right)\right) + \frac{\log 3}{8} + \frac{\sqrt{2}}{16} = \int_2^{\infty}\frac{dx}{x^6 - x^{-2}}$

31 .006276680299820175 8... ≈ $\pi^3 = \int_0^{\infty}\log^2 x\,\frac{x}{(1+x)\sqrt{x}}\,dx$ GR 4.261.10

$= \int_{-\infty}^{\infty}\frac{x^2 e^{-x/2}}{1+e^{-x}}\,dx$ GR 3.4.11.16

.00628052611482317663... ≈ $\frac{\pi^2}{48} - \frac{\log 2}{8} - \frac{3\zeta(3)}{32} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{(2k+2)^3}$

.00629484324054357244... ≈ $\frac{1}{2\sqrt{3}}\arctan\frac{5}{\sqrt{3}}+\frac{\log 21}{12}-\frac{\pi}{3\sqrt{3}}$

= $\frac{1}{160}\,{}_2F_1\left(\frac{5}{6},1,\frac{11}{6},\frac{1}{64}\right) = \int_2^\infty \frac{dx}{x^6-1}$

.00633038459106079398... ≈ $-\frac{1}{1331}\psi^{(2)}\left(\frac{7}{11}\right) = \int_1^\infty \frac{\log^2 x}{x^8-x^{-3}}dx$

.00634973966291606023... ≈ $\log 2-\frac{1}{5}\log 31 = \int_2^\infty \frac{dx}{x^6-x}$

.0064000000000000000$\underline{0}$ = $\frac{4}{625} = \sum_{k=1}^\infty (-1)^{k+1}\frac{k^3}{4^k}$

.00643499287419092255... ≈ $-\frac{1}{1000}\psi^{(2)}\left(\frac{7}{10}\right) = \int_1^\infty \frac{\log^2 x}{x^8-x^{-2}}$

.006476876667430481... ≈ $\frac{\log 3}{4}-\frac{\arctan 2}{2}-\frac{1}{2}+\frac{\pi}{4} = \int_2^\infty \frac{dx}{x^6-x^2}$

6 .006512796636760148... ≈ $\frac{e\left(e^2+4e+1\right)}{(e-1)^4}=\Phi\left(\frac{1}{e},-3,0\right)=\sum_{k=0}^\infty \frac{k^3}{e^k}$

102 .00656159509381775425... ≈ $\frac{\pi^5}{3}$

.006572038310503418... ≈ $-\frac{1}{729}\psi^{(2)}\left(\frac{7}{9}\right) = \int_1^\infty \frac{\log^2 x}{x^8-x^{-1}}dx$

.0066004473706482057... ≈ $\zeta(i)+\zeta(-i)$

.0066201284733196607... ≈ $\frac{\log 2}{4}-\frac{1}{6}=\int_0^1 \frac{\log x}{(x+1)^5}dx$

.00664014983663361581... ≈ $\sum_{k=1}^\infty \frac{\zeta(2k+1)-1}{\pi^{2k+1}}$

.0067379469990854671... ≈ e^{-5}

.00675575631575580671... ≈ $-\frac{1}{512}\psi^{(2)}\left(\frac{7}{8}\right) = \int_1^\infty \frac{\log^2 x}{x^8-1}dx$

.0067771148086586796... ≈ $\left(\zeta(4)-1\right)^2 = \sum_{i=2}^\infty\sum_{j=2}^\infty \frac{1}{(jk)^4} = \sum_{k=1}^\infty \frac{f_2(k)}{k^4}$

.00677885613384857755... ≈ $\frac{3}{4}\zeta(6)+\zeta(5)-\frac{1}{2}\zeta^2(3)+\zeta(4)+\zeta(3)+\zeta(2)-5$

= $\sum_{k=1}^\infty \frac{H_{k-1}}{(k+1)^5}$

.0067875324087718381... ≈ $\frac{\log 7}{6}+\frac{1}{\sqrt{3}}\arctan\frac{5}{\sqrt{3}}-\frac{\pi\sqrt{3}}{6}-\frac{1}{8} = \int_2^\infty \frac{dx}{x^6-x^3}$

.00695061706903604783... ≈ $\frac{\sqrt{2}}{8}\arctan\frac{1}{\sqrt{2}} - \frac{11}{108}$

1 .00698048496251558702... ≈ $\frac{\pi}{96}\left(2\pi^2\log 2 + 8\log^3 2 + 12\zeta(3)\right) = \sum_{k=0}^{\infty}\frac{1}{4^k(2k+1)^4}\binom{2k}{k}$

.00700907815253407747... ≈ $\frac{2\zeta(3)}{343} = \int_1^{\infty}\frac{\log^2 x}{x^8 - x}dx$

.00702160271746929417... ≈ $\frac{7\pi^2}{48} + \frac{\pi^4}{180} + \frac{3\zeta(3)}{4} - \frac{23}{8} = \sum_{k=1}^{\infty}\frac{1}{k(k+1)(k+2)^4}$

.00709355636667009771... ≈ $\frac{\pi}{16} + \frac{3\pi^3}{128} - G = \int_1^{\infty}\frac{\log^2 x}{(x^2+1)^3}dx$

.00711283900899634188... ≈ γ^9

.00712055882855767784... ≈ $\frac{3}{8} - \frac{1}{e} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k+4)!}$

6 .00714826080960966608... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta^4(2k) - 1\right)$

1 .00716066957145795517 5...≈ $\sum_{k=0}^{\infty}\frac{1}{k^3+2}$

.0071609986849739239... ≈ $\frac{1}{24}\left(54\zeta(3) - 2\pi^2 - 45\right)$

.0071918833558263656... ≈ $e^{-\pi^2/2} = i^{i\pi} = \cos(\pi\log i) + i\sin(\pi\log i)$

.00737103069590539413... ≈ $-\frac{1}{216}\psi^{(2)}\left(\frac{7}{6}\right) = \int_1^{\infty}\frac{dx}{x^8 - x^2}$

2 .00737103069590539413... ≈ $-\frac{1}{216}\psi^{(2)}\left(\frac{1}{6}\right) = 2 + \frac{1}{216}\psi^{(2)}\left(\frac{7}{6}\right) = -\int_0^1\frac{\log^2 x\,dx}{x^6 - 1}$

.00745592551331455 0565...≈ $\sum_{k=1}^{\infty}\operatorname{sech}^2(k\pi)$

.00747701837520388... ≈ $1 - \frac{7\pi^4}{360} + \frac{3\zeta(3)}{4} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(k+2)^4}$

.00748486855363624861... ≈ $G - \frac{10016}{11025} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k+7)^2}$

.00751172357477133 0898...≈ $\frac{1}{6} - \frac{1}{2\pi} = \sum_{k=1}^{\infty}\operatorname{csch}^2(k\pi)$ [Ramanujan] Berndt Ch. 26

$.00757575757575757575\underline{75}$ = $\frac{1}{132} = -\zeta(-9)$

.00763947766738817903... ≈ $\frac{\log 3}{2} - \frac{13}{24} = \int_2^{\infty}\frac{dx}{x^6 - x^4}$

1 .00776347048... ≈ j_7 J 311

1 .0078160724631199185... ≈ $\sum_{k=1}^{\infty} \frac{1}{(k!)^7}$

.0078161009852685688... ≈ $1716 - 154\pi^2 - \frac{28\pi^4}{15} - \frac{2\pi^6}{135} = \sum_{k=1}^{\infty} \frac{1}{k^7 (k+1)^7}$

.0078437109295513413... ≈ $\frac{119}{72} - \frac{\pi^2}{6} = \sum_{k=1}^{\infty} \frac{1}{k(k+1)^2 (k+2)^2 (k+3)}$

1 .00788821... ≈ G_7 J 309

.0079130289886268556... ≈ $-\frac{1}{125}\psi^{(2)}\left(\frac{7}{5}\right) = \int_1^{\infty} \frac{\log^2 x}{x^8 - x^3} dx$

.0079838114502686242 8... ≈ $\frac{3\zeta(5)}{4\pi^4} = \zeta'(-4) = -c_4$ Berndt 9.27.12

.0080628836082998722 96... ≈ $\frac{1}{4\pi^3} = \int_{-\infty}^{\infty} \frac{x^2 e^{-x}}{1 + e^{-2\pi x}} dx$

.0080845993622212534 6... ≈ $\frac{\sqrt{\pi}}{4}\left(1 - \log 2 - \frac{\gamma}{2}\right) = \int_0^{\infty} x^2 e^{-x^2} \log x \, dx$

.0081426980566204283... ≈ $-\frac{1}{8192}\psi^{(2)}\left(\frac{5}{16}\right) = \sum_{k=0}^{\infty} \frac{1}{(16k+5)^3}$

2 .0081560544927453151 5... ≈ $\frac{1}{\pi^4}\left(\sin(\pi(-1)^{1/8} \sin(\pi(-1)^{3/8} \sin(\pi(-1)^{5/8} \sin(\pi(-1)^{7/8}\right)$

= $\prod_{k=1}^{\infty}\left(1 + \frac{1}{k^8}\right)$

.0082455366501036024 5... ≈ $\frac{1}{6144}\left(96\pi^3 - 1536G - \psi^{(3)}\left(\frac{1}{4}\right) + \psi^{(3)}\left(\frac{3}{4}\right)\right)$

.0082493755919660668 8... ≈ $\left(\zeta(3) - 1\right)^3 = \sum_{k=2}^{\infty} \frac{f_3(k)}{k^3}$ Titchmarsh 1.2.14

.0082776188463344292... ≈ $-\frac{1}{3456}\psi^{(2)}\left(\frac{5}{12}\right) = \sum_{k=0}^{\infty} \frac{1}{(12k+5)^3}$

.0082801441615556895 7... ≈ $1 - \frac{1}{\zeta(7)}$

.0082838328561335925 35... ≈ $\sum_{p\ prime} \frac{1}{p^7} = \sum_{k=1}^{\infty} \frac{\mu(k)}{k} \log \zeta(7k)$

.0083085567866721969 2... ≈ $\frac{1}{2} - \frac{1}{3e} + \frac{2\sqrt{e}}{3}\cos\left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(3k+2)!}$

1 .0083360892258489817 7... ≈ $\frac{\sin 1 + \sinh 1}{2} = \frac{\sin 1}{2} + \frac{e}{4} + \frac{1}{4e} = \sum_{k=0}^{\infty} \frac{1}{(4k+1)!}$

.0083388452789606915 4... ≈ $\frac{1}{8}(\cos 1 - \sin 1) + \frac{1}{8e} = \sum_{k=1}^{\infty} \frac{k}{(4k+1)!}$

1 .00834927738192282684... ≈ $\zeta(7)$

$$= -\frac{8\pi^4\zeta(3)}{1143} + \frac{64\pi^2\zeta(5)}{381} - \frac{64\pi^6}{381}\sum_{k=0}^{\infty}\frac{\zeta(2k)}{4^k(2k+1)(2k+2)(2k+3)(2k+4)}$$

$$= -\frac{\pi^6}{7560}\sum_{k=0}^{\infty}\frac{(248k^3+2604k^2+9394k+11757)\zeta(2k)}{4^k(2k+1)(2k+3)(2k+5)(2k+6)(2k+7)}$$

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$$= \zeta(4)\prod_{p\ prime}\frac{1+p^{-1}+p^{-2}+p^{-3}}{1+p^{-1}+p^{-2}+p^{-3}+p^{-4}+p^{-5}+p^{-6}}$$

$$= \zeta(6)\prod_{p\ prime}\frac{1+p^{-1}+p^{-2}+p^{-3}+p^{-4}+p^{-5}}{1+p^{-1}+p^{-2}+p^{-3}+p^{-4}+p^{-5}+p^{-6}}$$

.0083799853056448421... ≈ $\sum_{k=1}^{\infty}\left(\zeta(8k-1)-1\right) = \sum_{k=2}^{\infty}\frac{1}{k^7-k^{-1}}$

.00838389135409569500... ≈ $MHS(7,4) = 4\zeta(5)\zeta(6) + 21\zeta(4)\zeta(7) + 84\zeta(2)\zeta(9) - \frac{331}{2}\zeta(11)$

9 .00839798699900984124... ≈ γ^{-4}

.00841127767185517548... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(6k+1)-1}{k} = -\sum_{k=2}^{\infty}\frac{\log(1-k^{-6})}{k}$

.00847392921877574231... ≈ $\frac{1}{4} - \frac{2\gamma}{3} - \frac{1}{3}\left(\psi\left(\frac{1+i\sqrt{3}}{2}\right) + \psi\left(\frac{1-i\sqrt{3}}{2}\right)\right)$

$$= \sum_{k=2}^{\infty}\frac{1}{k^7-k} = \sum_{k=1}^{\infty}\left(\zeta(6k+1)-1\right)$$

.0085182264132904860... ≈ $-\frac{1}{1458}\psi^{(2)}\left(\frac{5}{9}\right) = \sum_{k=1}^{\infty}\frac{1}{(9k-4)^3}$

.00860324727442514399... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^7-k^2} = \sum_{k=1}^{\infty}\left(\zeta(5k+2)-1\right)$

.008650529099561105501...≈ $MHS(7,1) = \frac{\pi^8}{7560} - \zeta(3)\zeta(5)$

.00866072400601531257... ≈ $-\frac{1}{1024}\psi^{(2)}\left(\frac{5}{8}\right) = \sum_{k=1}^{\infty}\frac{1}{(8k-3)^3}$

.00877956993120154517... ≈ $-\frac{1}{64}\psi^{(2)}\left(\frac{7}{4}\right) = -\frac{1}{64}\psi^{(2)}\left(\frac{3}{4}\right) - \frac{2}{27} = \int_1^{\infty}\frac{\log^2 x}{x^8-x^4}dx$

.00887608960241063354... ≈ $\frac{7}{8} - \zeta(3) + \frac{\gamma}{2} + \frac{1}{4}\left(\psi(i)+\psi(-i)\right)$

$$= \sum_{k=2}^{\infty}\frac{1}{k^7-k^4} = \sum_{k=1}^{\infty}\left(\zeta(3k+4)-1\right)$$

1 .00891931476945941517... ≈ $\frac{\zeta(6)}{\zeta(7)} = \sum_{k=1}^{\infty}\frac{\varphi(k)}{k^7}$ Titchmarsh 1.2.13

5 .00898008076228346631... ≈ $\sum_{k=0}^{\infty}\frac{1}{({}^{k}\!/_{2})!}$

.00898329102112942789... ≈ $e^{-3\pi/2} = i^{3i}$

.00915872712911285823... ≈ $\frac{91\zeta(3)}{216} - \frac{\pi^3}{36\sqrt{3}} = -\frac{1}{432}\psi^{(2)}\left(\frac{5}{6}\right) = \sum_{k=1}^{\infty}\frac{1}{(6k-1)^3}$

.00919770361155757433 8... ≈ $\frac{3}{8} - \frac{\pi}{4\sqrt{3}}\left(\cot\frac{\pi}{\sqrt{3}} + \coth\frac{\pi}{\sqrt{3}}\right) = \sum_{k=1}^{\infty}\frac{\zeta(4k)-1}{9^k}$

.00935868207596499462 2... ≈ $\frac{52}{4e^2} - \frac{7}{4} = \sum_{k=0}^{\infty}(-1)^k\frac{2^k}{k!(k+4)(k+7)}$

.0094497563422752927... ≈ $3\zeta(3) + \log 2 - \frac{\pi^2}{3} - 1 = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2}{(k+2)^3}$

.00948402884097088413... ≈ $\frac{5}{3} - \frac{2\gamma}{3} - \frac{\pi^4}{90} - \frac{1}{3}\left(\psi\left(\frac{3+i\sqrt{3}}{2}\right) + \psi\left(\frac{3-i\sqrt{3}}{2}\right)\right)$

= $\sum_{k=2}^{\infty}\frac{1}{k^7-k^4} = \sum_{k=1}^{\infty}\left(\zeta(3k+4)-1\right)$

.00953282949724591758... ≈ $\frac{7\pi^4}{45\cdot 16} - \frac{15}{16} = \frac{7\zeta(4)}{8} - \frac{15}{16} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+3)^4}$

2 .00956571464674566328... ≈ $\frac{\sqrt{\pi}\left(16G+\pi^2\right)}{64}\Gamma\left(\frac{1}{4}\right)\Gamma^{-1}\left(\frac{3}{4}\right) = \int_0^1\frac{\log^2 x\,dx}{\sqrt{1-x^4}}$

.00958490817198552203... ≈ $\frac{\pi}{96\left(2+\sqrt{2}\right)} = \sum_{k=1}^{\infty}\frac{1}{(8k-7)(8k-5)(8k-3)(8k-1)}$ J243

$.0096000000000000000\underline{0}$ = $\frac{6}{625} = \int_1^{\infty}\frac{\log^3 x}{x^6}dx$

.00961645522527675428... ≈ $\frac{\zeta(3)}{125} = \sum_{k=1}^{\infty}\frac{1}{(5k)^3}$

.00963211289494928567 5... ≈ $\frac{\zeta(3)}{8} - \frac{9}{64}$

= $\int_0^1\int_0^1\int_0^1\frac{x^5y^5z^5}{1-x^2y^2z^2}dx\,dy\,dz$

2 .00965179272267959834... ≈ $\sqrt{3}\cosh\pi\left(\operatorname{csc}h\pi\sqrt{3}\right)\sinh\pi = \prod_{k=1}^{\infty}\left(1+\frac{1}{k^2+3}\right)$

2 .00966608113054390026... ≈ $\frac{7\pi^3}{108} = \int_0^{\infty}\frac{\log^2 x}{x^6+1}dx$

.009692044900729199735... ≈ $\frac{\zeta(3)}{4\pi^3} = \int\limits_0^\infty \frac{x^2 dx}{e^{2\pi x}-1}$

153 .00984239264072663137... ≈ $\frac{\pi^5}{2}$

.00988445549319781198... ≈ $\frac{9\zeta(3)-\pi^2}{96} = \sum_{k=1}^\infty (-1)^{k+1}\frac{k}{(2k+2)^3}$

.009999202740867977 8... ≈ $\frac{1}{3072}\left(\psi^{(3)}\left(\frac{1}{4}\right)-\psi^{(3)}\left(\frac{3}{4}\right)\right) - \frac{\pi^3}{64}$

.010007793923494035492... ≈ $\frac{2}{\sqrt{3}}\operatorname{csch}\pi\sqrt{3} \;=\; -\frac{1}{6} + \sum_{k=0}^{\infty}\frac{(-1)^k}{k^2+3}$

2 .0100283440867821521... ≈ $\frac{i\sqrt{2}}{4}\left(\psi^{(1)}\left(\frac{i}{\sqrt{2}}\right) - \psi^{(1)}\left(-\frac{i}{\sqrt{2}}\right) + \psi^{2}\left(-\frac{i}{\sqrt{2}}\right) - \psi^{2}\left(\frac{i}{\sqrt{2}}\right)\right)$

$+\,\gamma \;+\; \frac{\sqrt{2}}{2}\,\gamma\,\pi\coth\frac{\pi}{\sqrt{2}}$

= $\sum_{k=1}^{\infty}\frac{H_k}{k^2+{}^{1}\!/_{2}}$

.010097259642724159142... ≈ $\frac{61}{126} - \frac{\pi}{16}\left(1+\sqrt{2}\,\right) = \sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{64^k}$

.01026128066531735403... ≈ $\frac{3\zeta(3)}{2} - \frac{1549}{864} \;=\; \int_1^{\infty}\frac{\log^2 x}{x^6+x^5}\,dx$

.01026598225468433519... ≈ π^{-4}

.01031008886672620565... ≈ $-\frac{1}{27}\psi^{(2)}\left(\frac{7}{3}\right) \;=\; \int_1^{\infty}\frac{\log^2 x}{x^8-x^5}\,dx$

.01036989801169337524... ≈ $\frac{69}{16} - \frac{3\pi^2}{8} - \frac{\zeta(3)}{2} \;=\; \sum_{k=1}^{\infty}\frac{1}{k(k+1)^2(k+2)^3}$

1 .01037296826200719010... ≈ $\frac{7\zeta(3)}{16} + \frac{\pi^3}{64} \;=\; -\frac{1}{128}\psi^{(2)}\left(\frac{1}{4}\right)$ Berndt 7.12.3

= $-\frac{1}{1024}\left(\psi^{(2)}\left(\frac{1}{8}\right) + \psi^{(2)}\left(\frac{5}{8}\right)\right) \;=\; \sum_{k=1}^{\infty}\frac{1}{(4k-3)^3}$

.01037718999605679651... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k^6} \;=\; \sum_{k=2}^{\infty}\left(Li_6\left(\frac{1}{k}\right) - \frac{1}{k}\right)$

.01041511660008006356... ≈ $1-\cos\frac{1}{2}\cosh\frac{1}{2} \;=\; \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(4k)!4^k}$ GR 1.413.2

.0104166666666666666$\underline{6}$ = $\frac{1}{96} \;=\; \sum_{k=1}^{\infty}\frac{k^2}{(k+1)(k+2)(k+3)(k+4)(k+5)}$

1 .01044926723267323166... ≈ $4\sinh\frac{1}{4} \;=\; \prod_{k=1}^{\infty}\left(1+\frac{1}{16\pi^2k^2}\right)$

.010466588676332594722... ≈ $\frac{1}{8\pi^3}\big((2\gamma-3+2\log 2\pi)\zeta(3) - 2\zeta'(3)\big) = \int_0^{\infty}\frac{x^2\log x\,dx}{e^{2\pi x}-1}$

.01049303297362745807... ≈ $\frac{\pi}{16}\cot\frac{7\pi}{8} \;-\; \frac{1}{56} \;+\; \frac{\log 2}{2} \;+\; \frac{\sqrt{2}}{8}\log\left(1+\sqrt{2}\right)$

= $\sum_{k=2}^{\infty}\frac{1}{64k^2-8k} \;=\; \sum_{k=2}^{\infty}\frac{\zeta(k)-1}{8^k}$

.01049435966734132299... ≈ $\frac{197}{216} - \frac{3\zeta(3)}{4} \;=\; \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+4)^3}$

= $\displaystyle\int_0^1\int_0^1\int_0^1 \frac{x^3y^3z^3}{1+xyz}\,dx\,dy\,dz$

1 .01050894057394275299... ≈ $\displaystyle\frac{1}{24576}\left(\psi^{(3)}\left(\frac{1}{8}\right)+\psi^{(3)}\left(\frac{3}{8}\right)-\psi^{(3)}\left(\frac{5}{8}\right)-\psi^{(3)}\left(\frac{7}{8}\right)\right)$

= $\displaystyle\sum_{k=1}^{\infty}\left(\frac{(-1)^{k+1}}{(4k-3)^4}+\frac{(-1)^{k+1}}{(4k-1)^4}\right)$

= $\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^{\lfloor k/2\rfloor}}{(2k+1)^4}$

.01065796470989861952... ≈ $\displaystyle 2\zeta(5)-\zeta(2)\zeta(3)+\frac{3}{2}\zeta(4)+\frac{5}{4}\zeta(3)+\frac{9}{8}\zeta(2)-\frac{81}{16}$

= $\displaystyle\sum_{k=1}^{\infty}\frac{H_{k-1}}{(k+2)^4}$

.01077586137283670648... ≈ $\displaystyle 1-\frac{\log 3}{8}-\frac{\log 2}{4}+\frac{\sqrt{3}}{12}\log\frac{\sqrt{3}-1}{\sqrt{3}+1}-\frac{\pi}{8}\frac{\sqrt{3}+1}{\sqrt{3}-1}$

= $\displaystyle\frac{1}{12}\sum_{k=1}^{\infty}\frac{1}{12k^2+k} = \frac{1}{12}hg\left(\frac{1}{12}\right) = \sum_{k=1}^{\infty}(-1)^k\frac{\zeta(k)}{12^k}$

.01085551389327864275... ≈ $\displaystyle\log 2-\frac{131}{192} = \int_2^{\infty}\frac{dx}{x^6-x^5}$

.010864219555727274797... ≈ $\displaystyle\frac{1}{12}\left(6\csc 1-7\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{\pi^4k^4-\pi^2k^2}$

6 .01086775003589217196... ≈ $\displaystyle\frac{1}{256}\psi^{(3)}\left(\frac{1}{4}\right) = 6\sum_{k=0}^{\infty}\frac{1}{(k+1)^4} = \int_0^1\frac{\log^3 x\,dx}{x^4-1}$

.01097098867094099842... ≈ $\displaystyle\frac{\pi}{60}-\frac{49}{3600} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)(2k+2)(2k+6)(2k+7)}$

.01101534169703578827... ≈ $\displaystyle\frac{9}{4}-\zeta(3)-\zeta(5) = \sum_{k=2}^{\infty}\frac{1}{k^7-k^5}$

= $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k(k+1)^5(k+2)} = \sum_{k=1}^{\infty}\left(\zeta(2k+5)-1\right)$

.01105544825889466389... ≈ $\displaystyle 1-\frac{1}{1536}\left(\psi^{(3)}\left(\frac{1}{4}\right)-\psi^{(3)}\left(\frac{3}{4}\right)\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+3)^4}$

1 .0110826597832979188... ≈ $\displaystyle\sum_{k=1}^{\infty}\frac{1}{(2^k-1)k^5} = \sum_{k=1}^{\infty}\frac{\sigma_{-5}(k)}{2^k}$

1 .011119930921598195267... ≈ $\displaystyle\frac{1}{2}\left(\cosh\sqrt{2}-\cos\sqrt{2}\right)$

= $\displaystyle -\sin(-1)^{1/4}\sin(-1)^{3/4} = -\sin\left(\frac{(-1+i)\sqrt{2}}{2}\right)\sin\left(\frac{(1+i)\sqrt{2}}{2}\right)$

$= \prod_{k=1}^{\infty}\left(1+\frac{1}{\pi^4 k^4}\right)$

7 .01114252399629706133... ≈ $\pi\sqrt{3} - 12\log 2 + 9\log 3 = \sum_{k=0}^{\infty}\frac{1}{(k+1/2)(k+1/3)}$

.01121802464158449834... ≈ $\frac{1}{4}\left(2 - \cot 1 - \coth 1\right)$

$= \frac{\csc 1}{4(e^2-1)}\left(e^2\cos 1 - \cos 1 + 3\sin 1 - e^2 \sin 1\right)$

$= \sum_{k=1}^{\infty}\frac{1}{k^4\pi^4 - 1}$

1 .01130477870987731055... ≈ $\sum_{j=5}^{\infty}\sum_{k=1}^{\infty}\left(\zeta(jk-3)-1\right)$

.01134023029066286157... ≈ $\frac{1}{36\sqrt{6}} = \sum_{k=0}^{\infty}\frac{(-1)^k k^3}{8^k}\binom{2k}{k}$

1 .01140426470735171864... ≈ $\frac{\log 3}{2} + \frac{2\log 2}{3} = r(6)$

1 .01143540792841495861... ≈ $32\log 2 - 8\zeta(2) - 4\zeta(3) - 2\zeta(4) - \zeta(5) = \sum_{k=1}^{\infty}\frac{1}{2k^6 - k^5}$

$= \sum_{k=1}^{\infty}\frac{\zeta(k+5)}{2^k}$

.01147971498443532465... ≈ $720\left(\zeta(7)-1\right) = \int_0^{\infty}\frac{x^6}{e^x(e^x-1)}dx$

726 .01147971498443532465... ≈ $720\zeta(7) = -\psi^{(7)}(1)$

1 .01151515992746256836... ≈ $\frac{\pi}{3\sqrt{2}\left(\sqrt{3}-1\right)} = \frac{\pi}{3}\cos\frac{\pi}{12} = \int_0^{\infty}\frac{dx}{1+x^{12}} = \int_0^{\infty}\frac{x^{10}dx}{1+x^{12}}$

2 .01171825870300665613... ≈ $\sum_{k=1}^{\infty}\frac{2^{1/k}}{k^7}$

.0117335296527018915... ≈ $\frac{\pi}{\sinh 2\pi} = \frac{1}{2}\prod_{k=1}^{\infty}\frac{k^2}{k^2+4}$

$= \int_0^1 \frac{\cos(2\log x)}{(1+x)^2}dx$ GR 3.883.1

.01174227447013291929... ≈ $\sum_{k=2}^{\infty}\frac{1}{k\log^7 k} = -\int\left(\zeta(s)-1\right)ds\Big|_{s=7}$

.01175544134736910981... ≈ $\pi\coth\pi - \pi = \frac{2\pi}{e^{2\pi}-1}$

2 .01182428891548746447... ≈ $-\frac{1}{125}\psi^{(2)}\left(\frac{1}{5}\right) = \sum_{k=1}^{\infty}\frac{1}{(5k-4)^3} = \int_0^1\frac{\log^2 x\,dx}{1-x^5}$

.0118259564890464719... $\approx \frac{\pi^2}{144}-\frac{49}{864} = \sum_{k=1}^{\infty}\frac{H_k}{k(k+1)(k+2)(k+3)(k+4)}$

.01184674928339526517... $\approx \frac{3\zeta(3)}{2}+\frac{\pi^2}{2}+2\log 2-4 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(k+1)(k+2)^3}$

.01196636659281283504... $\approx \frac{\pi^3}{16}-\frac{52}{27} = \int_1^{\infty}\frac{\log^2 x}{x^6+x^4}dx$

.0121188016321911204... $\approx \frac{7\pi^4}{5760}+\frac{5\pi^2}{192}-\frac{93}{256} = \sum_{k=2}^{\infty}\frac{(-1)^k}{(k^2-1)^4}$

.01215800309158281960... $\approx \frac{e^2-7}{32} = \sum_{k=0}^{\infty}\frac{2^k}{(k+5)!}$

.01227184630308512984... $\approx \frac{\pi}{256} = \int_0^{\infty}\frac{dx}{(x^2+16)^2}$

.01228702536616798183... $\approx \frac{\csc 1}{6}\left(2\sin 1-3\cos 1\right) = \sum_{k=1}^{\infty}\frac{1}{\pi^4k^4-\pi^2k^2}$

.012307165328788035744...$\approx \frac{1}{8}-\frac{3\zeta(3)}{32} = \int_0^1\int_0^1\int_0^1\frac{x^3y^3z^3}{1+x^2y^2z^2}dx\,dy\,dz$

.01232267147533101011... $\approx \gamma^8$

1 .01232891528356646996... $\approx 5\zeta(2)-6\zeta(3)$

.01236617586807707787... $\approx \frac{\pi^4+30\pi^2}{768}-\frac{1}{2} = \sum_{k=1}^{\infty}\frac{1}{(4k^2-1)^4}$ J373

.01243125524701590587... $\approx \frac{18G+13}{32\pi}-\frac{9\log 2}{32}-\frac{13}{128}$ J385

.0124515659374656125... $\approx \frac{80\pi^2}{3}+\frac{8\pi^4}{45}+320\log 2+48\zeta(3)-560 = \sum_{k=1}^{\infty}\frac{1}{k^4(2k+1)^4}$

.01250606673761113456... $\approx 1-4\zeta(4)-8\zeta(6)+\zeta(3)+12\zeta(5) = \sum_{k=1}^{\infty}\frac{k^3}{(k+2)^6}$

1 .01251960216027655609... $\approx \frac{3\pi\sqrt{3}}{8}-\frac{\pi}{2}+\frac{9\log 3}{8}-\log 2 = \sum_{k=0}^{\infty}\frac{1}{(2k+1)(4k+1)(6k+1)}$

.01256429785698989747... $\approx 160-44\zeta(2)-\frac{15}{2}\zeta(4)-80\log 2-20\zeta(3) = \sum_{k=1}^{\infty}\frac{1}{k^3(2k+1)^4}$

.01261355010396303533... $\approx \frac{\pi^4}{720}+\frac{5\pi^2}{96}-\frac{163}{256}$

$= \sum_{k=2}^{\infty}\frac{1}{(k^2-1)^4}$

.012625017203357165027...$\approx \frac{4\pi^2}{9}-\frac{1}{6}-\frac{7\zeta(3)}{2}$ Borwein-Devlin, p. 56

.01266514795529222143 0... ≈ $\frac{1}{8\pi^2} = \int_0^\infty \frac{xe^{-x}}{1+e^{-2\pi x}}dx$

.01270663057023925266 0... ≈ $\frac{15\zeta(5)}{4\pi^5} = \int_0^\infty \frac{dx}{e^{2\pi\, x^{1/5}}-1}$

2 .0127932466328548535 7... ≈ $\frac{1}{\sqrt{5}}\left(I_0\left(\sqrt{5}+1\right)-I_0\left(\sqrt{5}-1\right)\right) = \sum_{k=1}^\infty \frac{F_k}{k!k!}$

1 .0128442424770655552 9... ≈ $\frac{91\zeta(3)}{108} = \frac{1}{432}\left(\psi^{(2)}\left(\frac{5}{6}\right)-\psi^{(2)}\left(\frac{1}{6}\right)\right) = W_3$

.0128521651317957250 8... ≈ $-\zeta'(6) = \sum_{k=1}^\infty \frac{\log k}{k^6}$

.0128667399315433592 2... ≈ $\frac{1}{1536}\psi^{(3)}\left(\frac{3}{4}\right) = \sum_{k=0}^\infty \frac{1}{(4k+3)^4}$

.0128813592289992976 2... ≈ $\frac{31-3\pi^2}{108} = \sum_{k=1}^\infty \frac{1}{k(k+1)(k+2)(k+3)^2}$

.0128978828974203426... ≈ $\frac{3\zeta(3)}{4} + 4\log 2 - \frac{\pi^2}{24} - \frac{13}{4} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{k(k+1)^3(k+2)^2}$

6 .0129068539507677306 4... ≈ $\frac{5\pi^4}{81} = \int_0^\infty \frac{\log^3 x\ dx}{x^6-1}$

.0129232947116698738 3... ≈ $\frac{209}{225} - G = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{(2k+5)^2}$

.01292830656030189083 2... ≈ $-2\ \mathrm{csch}\ \pi\sqrt{3}\sin\pi\sqrt{3} = \prod_{k=1}^\infty \frac{k^2+2k-2}{k^2+2k+4}$

1 .0129537533348609619 1... ≈ $\mathrm{arccot}\ 2 + \frac{\log 3}{2} = \frac{1}{2}\left(\log 3 + i\log\left(1-\frac{i}{2}\right) - i\log\left(1+\frac{i}{2}\right)\right)$

= $\sum_{k=0}^\infty \frac{1}{16^k(4k+1)} = 2\int_2^\infty \frac{dx}{x^2+x^{-2}}$

.0130395598910641381 2... ≈ $1 - \frac{\pi^2}{10}$

.0131224431498849407 5... ≈ $\frac{1}{216}\left(\psi^{(2)}\left(\frac{4}{3}\right)-\psi^{(2)}\left(\frac{5}{6}\right)\right) = \int_1^\infty \frac{\log^2 x\ dx}{x^6+x^3}$

1 .01321183515... ≈ $\sum_{k=1}^\infty (-1)^{k+1}\frac{\mu(k)}{k^2}$

5 .0132565492620010048 3... ≈ $\sqrt{8\pi}$

1 .0133023030122599952 8... ≈ $\frac{6}{5-4\cos 1}\sin\frac{1}{2} = \sum_{k=0}^\infty \frac{1}{2^k}\sin\frac{2k+1}{2}$

.0135255064552159253 8... $\approx \frac{7\zeta(3)}{4} - \frac{7054}{3375} = -\frac{1}{8}\psi^{(2)}\left(\frac{7}{2}\right) = \int_1^\infty \frac{\log^2 x}{x^8 - x^6}\,dx$

.01358311877719198759... $\approx \frac{1}{4}\left(3\zeta(3) + 12\log 2 - \pi^2 - 2\right) = \sum_{k=0}^{\infty} \frac{(-1)^{k+1}k^3}{(k+1)^3}$

.01361111111111111111 $= \frac{49}{3600} = \sum_{k=0}^{\infty} \frac{1}{(2k+1)(2k+2)(2k+6)(2k+7)}$

.01375102324650707998... $\approx \frac{\pi^2}{32} + \frac{3\pi^4}{128} - \frac{21}{16}\zeta(3) - 1 = \sum_{k=1}^{\infty} \frac{k^2}{(2k+3)^4}$

.01387346911505558557... $\approx \zeta(2) + 6\zeta(4) - \frac{65}{8} = -\sum_{k=2}^{\infty} \frac{1-4k+k^2}{k(k+1)^4}$

$= \sum_{k=1}^{\infty} (-1)^k k^3 \left(\zeta(k+2) - 1\right)$

1 .01389384950459390246... $\approx \cos 1 + \sin 1 - \frac{1}{e} = \sum_{k=0}^{\infty} \frac{1}{(4k)!(2k+1)}$

.01389695950004687007... $\approx \frac{1}{512}\left(\psi^{(2)}\left(\frac{9}{8}\right) - \psi^{(2)}\left(\frac{5}{8}\right)\right) = \int_1^\infty \frac{\log^2 x}{x^6 + x^2}\,dx$

1 .01395913236076850429... $\approx \frac{i}{2}\left(Li_2\left(e^{-i}\right) - Li_2\left(e^{i}\right)\right) = \sum_{k=1}^{\infty} \frac{\sin k}{k^2}$

.013983641297271329... $\approx e^{-e\pi/2} = i^{ie}$

.01406321433149292917 8... $\approx \frac{\pi}{2}\coth\pi + \zeta(4) - \zeta(2) - 1$

$= \sum_{k=1}^{\infty} (-1)^{k+1}\left(\zeta(2k+4) - 1\right) = \sum_{k=2}^{\infty} \frac{1}{k^6 + k^4}$

1 .01406980328946130324... $\approx \frac{\log 3}{12}\left(\pi^2 + \log^2 3\right) = \int_0^\infty \frac{\log^2 x}{(x-1)(x+3)}\,dx$

.01408660433390149553... $\approx \frac{3\zeta(3)}{256} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(4k+k)^3}$

.01417763649649285569... $\approx \frac{2\pi\sqrt{3}}{27} - \frac{7}{18} = \int_1^\infty \frac{dx}{(x^2+x+1)^3}$

.0142857142857$\underline{142857}$ $= \frac{1}{70} = \prod_{k=5}^{\infty}\left(1 - \frac{16}{k^2}\right)$

2 .01432273354831573658... $\approx \frac{e^{\sqrt{5}} - 1}{\sqrt{5}}\, e^{(1-\sqrt{5})/2} = \sum_{k=1}^{\infty} \frac{F_k}{k!}$

5021 .01432933734541210554... $\approx \frac{127\pi^8}{240} = -\int_0^1 \frac{\log^7 x\,dx}{1+x}$ GR 4.266.1

3 .014359217465414272 4... $\approx \frac{e^{\sqrt{5}} - 1}{\sqrt{5}e^{2/(1+\sqrt{5})}} = \sum_{k=1}^{\infty} \frac{1}{F_k}$

.01442468283791513142... ≈ $\frac{3\zeta(3)}{250} = \int_1^\infty \frac{\log^2 x\,dx}{x^6+x}$

1 .0145709024296292091... ≈ $\sum_{k=1}^\infty \left(e^{\zeta(k)-1}-1\right)$

1 .01460720903672859326... ≈ $\sqrt{e}\left(1+\frac{\gamma}{2}-\frac{1}{2}Ei\left(-\frac{1}{2}\right)-\frac{\log 2}{2}\right)-1 = \sum_{k=1}^\infty \frac{kH_k}{k!2^k}$

.01462276787138248... ≈ $\sum_{k=2}^\infty \frac{\mu(2k)}{k^4-1}$

1 .01467803160419205455... ≈ $\frac{\pi^4}{96} = \frac{7\zeta(4)}{8} = \lambda(4) = \sum_{k=1}^\infty \frac{1}{(2k-1)^4}$ AS 23.2.20, J342

.01476275322760796269... ≈ $\frac{7\zeta(3)}{8}-\frac{28}{27}$

= $\int_0^1\int_0^1\int_0^1 \frac{x^4y^4z^4}{1-x^2y^2z^2}\,dx\,dy\,dz$

.01479302112711200034... ≈ $\frac{1}{1728}\left(\psi^{(2)}\left(\frac{11}{12}\right)-\psi^{(2)}\left(\frac{5}{12}\right)\right) = \int_1^\infty \frac{\log^2 x\;dx}{x^6-1}$

6 .01484567464494796828... ≈ $\sum_{k=2}^\infty \frac{2^k\zeta(k)}{k!} = \sum_{k=1}^\infty \left(e^{2/k}-\frac{2}{k}-1\right)$

1 .014877322758252568429...≈ $\frac{1}{4}\log\frac{27}{8}-1+\frac{3\sqrt{3}}{4}\log\left(2+\sqrt{3}\right) = \sum_{k=1}^\infty \frac{H_{2k+1}}{3^k}$

1 .01503923463935248777... ≈ $\frac{e\sqrt{\pi}}{4}erf\,(1) = \int_0^\infty e^{-x^2}\sinh x\cosh x\;dx$

1 .01508008428612328783... ≈ $\frac{\sqrt{2}}{\pi}\Gamma^4\left(\frac{3}{4}\right)$ Berndt 8.17.16

.01520739899850402833... ≈ $\frac{17}{16}-\frac{\pi^2}{48}-\frac{\pi^4}{180}-\frac{\zeta(3)}{4} = \sum_{k=1}^\infty \frac{1}{k(k+2)^4}$

.01521987685245300483... ≈ $\zeta(4)+3\zeta(6)-3\zeta(5)-\zeta(7) = \sum_{k=1}^\infty \frac{k^3}{(k+1)^7}$

.01523087098933542997... ≈ $\frac{\pi^2}{648} = \sum_{k=0}^\infty \frac{(2k+1)^2}{(6k+3)^4}$

.01529385672063108627... ≈ $\frac{\log 2}{8}-\frac{\pi}{16}+\frac{1}{8} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{(4k+2)(4k+4)}$

.01534419711274536931... ≈ $\frac{\pi^2-5}{4}-\zeta(3) = \sum_{k=1}^\infty \frac{1}{k(k+1)^3(k+2)^2}$

.015439636746... ≈ j_6 J311

.01555356082097039944... ≈ $\zeta(3)-\frac{\gamma+6}{3}-\frac{1+i\sqrt{3}}{6}\psi\left(\frac{3-i\sqrt{3}}{2}\right)-\frac{1-i\sqrt{3}}{6}\psi\left(\frac{3+i\sqrt{3}}{2}\right)$

$= \sum_{k=1}^{\infty} (-1)^{k+1}\left(\zeta(3k+3)-1\right) = \sum_{k=2}^{\infty} \frac{1}{k^6+k^3}$

.015603887018732408909... $\approx \frac{45\zeta(5)}{4} - 84\zeta(3) + 504\log 2 + 462 = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^6(k+1)^6}$

2 .015641477955609996 54... $\approx \psi(8)$

1 .01564643870362333537... $\approx \sum_{k=1}^{\infty} \frac{1}{(k!)^6}$

.01564678558976431415... $\approx 2\zeta(6) + 42\zeta(4) + 252\zeta(2) - 462 = \sum_{k=1}^{\infty} \frac{1}{k^6(k+1)^6}$

1 .01567586490084791444... $\approx \sum_{k=1}^{\infty} \frac{1}{k^{3k}}$

.0157870776290938623... $\approx -\frac{1}{8192}\psi^{(2)}\left(\frac{1}{4}\right) = \sum_{k=0}^{\infty} \frac{1}{(16k+4)^3}$

1 .0158084056589008209... $\approx \frac{\log 2}{2} - \gamma + \frac{1}{\sqrt{2}}\left(\log\frac{\sqrt{2}+1}{\sqrt{2}} - \log\frac{\sqrt{2}-1}{\sqrt{2}}\right) = \sum_{k=1}^{\infty} \frac{\psi(2k)}{2^k}$

.01584647647919286276... $\approx \int_2^{\infty} \frac{dx}{x^5-1}$

.01593429224574911842... $\approx -\sum_{k=1}^{\infty} \frac{\mu(4k-1)}{4^{4k-1}-1}$

1 .01594752966348281716... $\approx \frac{104\pi^6}{98415} = G_6$ J309

.01594968819427129848... $\approx -\frac{1}{3456}\psi^{(2)}\left(\frac{1}{3}\right) = \frac{243+32\pi^3\sqrt{3}}{124416} - \frac{1}{3456}\psi^{(2)}\left(\frac{5}{3}\right)$

$= \sum_{k=1}^{\infty} \frac{1}{(12k-8)^3} = \frac{13\zeta(3)}{1728} + \frac{\pi^3}{2592\sqrt{3}}$

$.0160000000000000000\underline{0}$ $= \frac{2}{125} = \int_1^{\infty} \frac{\log^2 x}{x^6}dx$

1 .01609803521875068827... $\approx \sum_{k=1}^{\infty} \frac{1}{k^{k+4}}$

.01613463028439279292... $\approx \log 2 - \frac{\log 15}{4} = \int_2^{\infty} \frac{dx}{x^5-x}$

.01614992628790568331... $\approx 9 - \frac{2\pi^2}{3} - 2\zeta(3) = \sum_{k=2}^{\infty} \frac{k-1}{k^2(k+1)^3}$

$= \sum_{k=1}^{\infty} (-1)^{k+1}k^2\left(\zeta(k+3)-1\right)$

.0161516179237856869... $\approx 10e - \frac{163}{6} = \sum_{k=1}^{\infty} \frac{k^2}{(k+4)!}$

.0162406578502357031... ≈ $\frac{31\zeta(5)}{8} - \frac{7\pi^2\zeta(3)}{32} - \frac{\pi^4\log 2}{48} = \sum_{k=1}^{\infty}\frac{H_k}{(2k+1)^4}$

2 .0162791183236353425 1... ≈ $\sum_{k=1}^{\infty}\frac{2^k}{k!k^7}$

1 .0163544190511628273 7... ≈ $\sum_{k=1}^{\infty}\frac{1}{k!k^5}$

.0163897496007479593... ≈ $1 + 2G - \frac{\pi}{2} - \frac{\pi^3}{16} + \log 2 = \sum_{k=2}^{\infty}\frac{(-1)^k}{k(2k-1)^3}$

.0164061421239163510 60... ≈ $\frac{\pi^2}{6} + \frac{\pi\left(\sin\pi\sqrt{2} - \sinh\pi\sqrt{2}\right)}{2\sqrt{2}} - \frac{1}{2}$

= $\sum_{k=2}^{\infty}\frac{1}{k^6+k^2} = \sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(4k+2)-1\right)$

.0164149791532220206... ≈ $6 - \zeta(2) - \zeta(3) - \zeta(4) - \zeta(5) - \zeta(6)$

= $\sum_{k=2}^{\infty}\frac{1}{k^7-k^6} = \sum_{k=1}^{\infty}\frac{1}{k(k+1)^6} = \sum_{k=1}^{\infty}\left(\zeta(k+6)-1\right)$

.0164343717228850781 2... ≈ $\frac{7\zeta(3)}{512} = \sum_{k=1}^{\infty}\frac{1}{(8k-4)^3}$

.0166317317921115726... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k\Omega(k)}{k^3}$

.01666666666666666666$\underline{6}$ = $\frac{1}{60} = \int_0^{\infty}\frac{dx}{e^{2\pi x^{1/4}}-1}$

2 .0167070176498317204 44... ≈ $\prod_{k=1}^{\infty}\left(1+k^{-7}\right)$

$= 1/\left(\Gamma(-(-1)^{1/7})\Gamma((-1)^{2/7})\Gamma(-(-1)^{3/7})\Gamma((-1)^{4/7})\Gamma(-(-1)^{5/7})\Gamma((-1)^{6/7})\right)$

.0167339005856450062 55... ≈ $\sum_{k=1}^{\infty}\frac{\mu(k)}{k!+1}$

.0168491839429992636 1... ≈ $\frac{\log 7}{6} - \frac{1}{2} + \frac{\pi\sqrt{3}}{6} + \frac{\sqrt{3}}{3}\arctan\frac{5}{\sqrt{3}} = \int_2^{\infty}\frac{dx}{x^5-x^2}$

.0168636931567544158 2... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^6+k} = \sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(5k+1)-1\right)$

.0168770970905985686 2... ≈ $\frac{6155-63\pi^4}{1080} = \int_1^{\infty}\frac{\log^3 x\,dx}{x^5+x^4}$

.0169114778685576626 8... ≈ $\frac{\pi^2}{12} - \frac{29}{36} = \sum_{k=1}^{\infty}\frac{1}{k(k+1)(k+2)^2(k+3)}$

1 .0169428198056403842 40... ≈ $\sum_{k=1}^{\infty}\frac{nfac(k)}{2^k}$

.0170474077354195802... ≈ $1-\frac{945}{\pi^6} = \frac{\zeta(6)-1}{\zeta(6)}$

.017070086850636512954... ≈ $\sum_{p\ prime}\frac{1}{p^6} = \sum_{k=1}^{\infty}\frac{\mu(k)}{k}\log\zeta(6k)$

.01709452908541040576... ≈ $\frac{13\zeta(3)}{216}-\frac{\pi^3}{324\sqrt{3}} = -\frac{1}{432}\psi^{(2)}\left(\frac{2}{3}\right) = \sum_{k=1}^{\infty}\frac{1}{(6k-2)^3}$

.017100734033216426153... ≈ $\frac{\pi}{6}\coth\pi - 1 + \frac{\pi}{6}\sqrt{\frac{1+i\sqrt{3}}{2}}\cot\pi\sqrt{\frac{1+i\sqrt{3}}{2}}$

$+\frac{\pi}{12}\sqrt{2-2i\sqrt{3}}\cot\frac{\pi}{2}\sqrt{2-i\sqrt{3}}$

= $\sum_{k=2}^{\infty}\frac{1}{k^6+1} = \sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(6k)-1\right)$

1 .01714084272965151097... ≈ $\frac{3\sqrt{\pi}\left(2-\sqrt{2}\right)}{8}\zeta\left(\frac{3}{2}\right) = \int_0^{\infty}\frac{dx}{e^{x^{2/3}}+1}$

1 .01729042976987861804... ≈ $6\log 2 - \pi = \sum_{k=1}^{\infty}\frac{2}{4k^2-k} = \sum_{k=2}^{\infty}\frac{\zeta(k)}{2^{2k-3}}$

1 .01734306198444913971... ≈ $\frac{\pi^6}{945} = \zeta(6)$

1 .01735897287127529182... ≈ $\frac{1}{120}\sum_{k=2}^{\infty}\frac{\log^5 k}{k(k-1)} = -\frac{1}{5!}\sum_{m=2}^{\infty}\zeta^{(5)}(m)$

1 .01736033215192280239... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^2+k^{-2}-2}$

1 .01736613603473227207... ≈ $\frac{703\pi^{12}}{638512875} = \frac{703}{691}\zeta(12) = \sum_{k=1}^{\infty}\frac{H^{(6)}{}_k}{k^6}$

1 .01737041750471453179... ≈ $\frac{\sinh\pi}{4\pi^3}\left(\cosh\pi-\cos\pi\sqrt{3}\right) = \prod_{k=2}^{\infty}\left(1+\frac{1}{k^6}\right)$

.01740454950499025083... ≈ $\frac{3}{16}+\frac{\pi}{8}\coth\pi+\left(\frac{1-i}{16}\right)\pi\sqrt{2}\left(\cot\left(\frac{(1+i)\pi}{\sqrt{2}}\right)-\coth\left(\frac{(1+i)\pi}{\sqrt{2}}\right)\right)$

= $\sum_{k=2}^{\infty}\frac{1}{k^6-k^{-2}} = \sum_{k=1}^{\infty}\left(\zeta(8k-2)-1\right)$

.01745240643728351282... ≈ $\sin 1^\circ$

.01745329251994329577... ≈ $\frac{\pi}{180}$, the number of radians in one degree

.01745506492821758577... ≈ $\tan 1^\circ$

.01746673680343934044... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^6-k^{-1}} = \sum_{k=1}^{\infty}\left(\zeta(7k-1)-1\right)$

.01746739278296249906... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(6k)-1}{k} = -\sum_{k=2}^{\infty}\log(1-k^{-6})$

.01758720202617971085... ≈ $\zeta(3) - 3\zeta(4) + \frac{33}{16} = \sum_{k=1}^{\infty}\frac{k}{(k+3)^4}$

.01759302638532157621... ≈ $\frac{11}{12}-\frac{\pi}{2\sqrt{3}}\tanh\frac{\pi\sqrt{3}}{2} = \sum_{k=2}^{\infty}\frac{1}{k^6-1} = \sum_{k=1}^{\infty}\left(\zeta(6k)-1\right)$

.01759537266628388376... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(5k+1)-1}{k} = -\sum_{k=2}^{\infty}\frac{\log(1-k^{-5})}{k}$

1 .01762083982618704928... ≈ $6\pi^2\operatorname{sech}^2\left(\frac{\pi\sqrt{3}}{2}\right) = \prod_{k=2}^{\infty}\left(\frac{k^6}{k^6-1}\right)$

.01765194199771948957... ≈ $\frac{\pi^2-8G}{144} = \sum_{k=1}^{\infty}\frac{1}{(12k-3)^2}$

.017716230658780156731...≈ $-\frac{\pi^4}{1440}\log 2 - \frac{15}{16}\zeta'(4) = \sum_{k=0}^{\infty}\frac{\log(2k+1)}{(2k+1)^4}$ Prud. 5.5.1.5

.01772979515817195598... ≈ $\frac{1}{6}-\frac{11\log 2}{18}+\frac{\log 3}{4} = \int_1^{\infty}\frac{dx}{e^x(e^x+1)(e^x+2)(e^x+3)}$

1 .017739549085798905616...≈ $\frac{10G}{9} = \frac{1}{72}\left(\psi^{(1)}\left(\frac{1}{12}\right)+\psi^{(1)}\left(\frac{5}{12}\right)\right)-\frac{\pi^2}{9}$

= $\frac{1}{144}\left(\psi^{(1)}\left(\frac{1}{12}\right)+\psi^{(1)}\left(\frac{5}{12}\right)-\psi^{(1)}\left(\frac{7}{12}\right)-\psi^{(1)}\left(\frac{11}{12}\right)\right)$

= $\sum_{k=1}^{\infty}\left(\frac{(-1)^{k+1}}{(6k-5)^2}+\frac{(-1)^{k+1}}{(6k-1)^2}\right)$

.017796701498469380675...≈ $\frac{1}{12}\left(\pi^2\log^3 2 - 18\log^2 2\,\zeta'(2) + 18\zeta''(2)\log 2 + 6\zeta^{(3)}(2)\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}\log^3 k}{k^2}$

.01785302516320311736... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^6-k}=\sum_{k=1}^{\infty}\left(\zeta(5k+1)-1\right)$

10 .01787492740990189897... ≈ $\sinh 3 = \frac{e^3-e^{-3}}{2} = \sum_{k=0}^{\infty}\frac{3^{2k+1}}{(2k+1)!}$ AS 4.5.62

.01793192101883973082... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2k(2k+1)9^k} = 3\arctan\frac{1}{3} + \log\frac{10}{9} - 1$

2 .01820187236017492412... ≈ $\frac{1}{72}\left(\pi^2 - 6\left(2\log^2 2 + 2\log 2 - 1\right)\right) = \sum_{k=1}^{\infty}\frac{1}{B_{2k}}$

1 .01829649667637074039... ≈ $\sqrt{\zeta(5)}$

.01831563888873418029... ≈ e^{-4}

.01835592831749446598 8... ≈ $3\zeta(7) - \zeta(2)\zeta(5) - \zeta(3)\zeta(4) = MHS(6,1) = \sum_{k=1}^{\infty}\frac{H_k}{(k+1)^6}$

.01839913772968859482 8... ≈ $\sum_{k=2}^{\infty}\mu(2k)\big(\zeta(2k)-1\big)$

.01839913959279820706 5... ≈ $\sum_{k=2}^{\infty}\left|\mu(2k)\right|\big(\zeta(2k)-1\big)$

.01840295688606415059... ≈ $\frac{7}{8} - \zeta(2) + \frac{\pi}{4}\coth\pi = \sum_{k=2}^{\infty}\frac{1}{k^6-k^2} = \sum_{k=1}^{\infty}\big(\zeta(4k+2)-1\big)$

.01840776945462769476... ≈ $\frac{3\pi}{512} = \int_0^{\infty}\frac{dx}{(x^2+4)^3}$

.0185185185185185185$\underline{185}$ = $\frac{1}{54} = \sum_{k=1}^{\infty}\frac{H_k}{(k+1)(k+2)(k+3)(k+4)}$

.01856087933022647259... ≈ $\frac{7\zeta(3)}{16} - \frac{\pi^4}{192} = \sum_{k=1}^{\infty}\frac{k}{(2k+1)^4}$

1 .01868112699866705508... ≈ $\sum_{k=2}^{\infty}\nu(k)\big(\zeta(k)-1\big)$

.01874823366450212957... ≈ $\frac{5}{3} - 3\operatorname{arctanh}\frac{1}{2} = \sum_{k=1}^{\infty}\frac{1}{4^k(2k+1)(2k+3)}$

.01878213911186866071... ≈ $\frac{\zeta(3)}{64} = \sum_{k=1}^{\infty}\frac{1}{(4k)^3}$

.01884103622589046372... ≈ $\log 2 - \frac{\log 3}{2} - \frac{1}{8} = \int_2^{\infty}\frac{dx}{x^5-x^3}$

.01891895796178806334... ≈ $\frac{\log 2}{3} + \frac{\log 3}{4} - \frac{1}{30} - \frac{\pi}{4\sqrt{3}} = \sum_{k=2}^{\infty}\frac{1}{36k^2-6k}$

= $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{6^k}$

1 .01898180765636222260... ≈ $-\frac{1}{4}\left(H\left(\frac{1}{2^{1/4}}\right) + H\left(\frac{i}{2^{1/4}}\right) + H\left(-\frac{1}{2^{1/4}}\right) + H\left(-\frac{i}{2^{1/4}}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{\zeta(4k+1)}{2^k} = \sum_{k=1}^{\infty}\frac{1}{2k^5-k}$

.01923291045055350857... ≈ $\frac{2\zeta(3)}{125} = \int_1^{\infty}\frac{\log^2 x}{x^6-x}dx$

1 .01925082490926758147... ≈ $\frac{\zeta(5)}{\zeta(6)} = \sum_{k=1}^{\infty}\frac{\varphi(k)}{k^6}$ Titchmarsh 1.2.13

3 .01925198919165474986... $\approx \frac{e\pi}{2\sqrt{2}} = -\int_0^\infty \frac{dx}{e^x(x^4+\frac{1}{2})}$

.01930023477927761455 3... $\approx \frac{1}{32}\left(\psi^{(1)}\left(\frac{7}{8}\right)-\psi^{(1)}\left(\frac{9}{8}\right)\right) = \sum_{k=1}^\infty \frac{k\zeta(2k+1)}{64^k}$

.0193153758425229159 6... $\approx \frac{3}{4}\log\frac{2}{3} + \frac{\log^2 2}{2} - \log 2\log 3 + \frac{\log^2 3}{2} + Li_2\left(\frac{1}{3}\right) - \frac{1}{8}$

$= \sum_{k=2}^\infty \frac{(-1)^k}{2^k(k^4-k^2)}$

.019378701873... $\approx \sum_{\substack{k\ a\ non-trivial\\ \text{int}\,eger\ power}} \frac{1}{k^3}$

.01938167868472179014... $\approx 1+\frac{\pi^2}{12}-\frac{3\zeta(3)}{2} = \sum_{k=1}^\infty \frac{(-1)^{k+1}k}{(k+2)^3}$

.01939671967440508500... $\approx \frac{G}{2}-\frac{\pi}{16}-\frac{\pi^3}{128} = \sum_{k=1}^\infty \frac{(-1)^{k+1}k^2}{(2k+1)^3}$

.01942719099991587856... $\approx \frac{2}{\sqrt{5}}-\frac{7}{8} = \sum_{k=2}^\infty \frac{(-1)^k(2k-1)!!}{(2k)!\,4^k}$

.01953464060278682017 5... $\approx \frac{1}{1728}\left(219-10\pi^2-72\zeta(3)\right) = \sum_{k=1}^\infty \frac{H_k H_{k+1}}{k(k+1)(k+2)(k+3(k+4)}$

.01956335398266840592... $\approx J_3(1)$

.01959332075386166857... $\approx \frac{14}{9}+6\log\frac{2}{3}-2Li_2\left(-\frac{1}{2}\right) = \sum_{k=1}^\infty \frac{(-1)^{k+1}k^3}{2^k(k+1)^2}$

.01963249194967275299... $\approx \frac{4+\gamma}{3}-\zeta(3)+\frac{1}{6}\left(\left(1-i\sqrt{3}\right)\psi\left(\frac{3-i\sqrt{3}}{2}\right)+\left(1+i\sqrt{3}\right)\psi\left(\frac{3+i\sqrt{3}}{2}\right)\right)$

$= \sum_{k=2}^\infty \frac{1}{k^6-k^3} = \sum_{k=1}^\infty \left(\zeta(3k+3)-1\right) = \sum_{k=2}^\infty \frac{1}{k^3-1}-\zeta(3)+1$

$= \sum_{\substack{\omega\ \text{a non-trivial}\\ \text{integer power}}} \frac{1}{\omega^3-1} = \sum_{k=1}^\infty \frac{\Omega(k)}{k^3}$

306 .01968478528145326274... $\approx \pi^5$

.01972588517509853131... $\approx \frac{1}{9}-\frac{\pi^2}{108} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{(3k+3)^2}$

.01982323371113819152... $\approx \frac{\pi^4}{90}-\frac{17}{16} = \zeta(4,3) = \sum_{k=1}^\infty \frac{1}{(k+2)^4}$

.0198603854199589820 63... ≈ $\frac{3\log 2}{4} - \frac{1}{2} = \sum_{k=1}^{\infty} \frac{1}{64k^3 - 4k^2}$ [Ramanujan] Berndt Ch. 2

.0198626889385327809... ≈ $\frac{3}{8} - \frac{7\pi^4}{1920} = \int_1^{\infty} \frac{dx}{x^5 + x^3}$

.0199048057076455380 5... ≈ $\frac{3}{64} + \frac{3}{32}\log\frac{3}{4} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^2 H_k}{3^k}$

1 .0199234266990522067... ≈ $\frac{\log 5}{2} + \frac{1}{\sqrt{5}}\log\frac{1+\sqrt{5}}{2} = r(5)$

1 .0199340668482264365... ≈ $\frac{\pi^2}{6} - \frac{5}{8} = \sum_{k=2}^{\infty} \frac{k^2 + 1}{(k^2 - 1)^2} = \sum_{k=1}^{\infty} k^2\big(\zeta(2k) - \zeta(2k+2)\big)$

.0200000000000000000$\underline{0}$ = $\frac{1}{50}$

1 .0200208006525427694 7... ≈ $\sum_{k=0}^{\infty} \frac{1}{\binom{2k}{k}(2k+1)^3}$

.0200542945589048403 1... ≈ $\frac{2}{9} - \frac{7\log 2}{24} = \int_1^{\infty} \log\left(1 + \frac{1}{x}\right)\frac{dx}{(x+1)^4} = \int_2^{\infty} \log\frac{x}{x-1} \cdot \frac{dx}{x^4}$

.0203443179819896350 4... ≈ $\frac{10}{9\pi} - \frac{1}{3} = -\sum_{k=1}^{\infty} \left(\frac{(2k-1)!!}{(2k+1)!!}\right)^2 \frac{1}{2k-3}$ J385

.0204081632653061224 5... ≈ $\frac{1}{49}$

1 .0204685844268084989 3... ≈ $\frac{62\zeta(5)}{63} = \sum_{k=1}^{\infty} \frac{a(k)}{k^6}$ Titchmarsh 1.2.13

.0204749888885212246 6... ≈ $4\zeta(5) - 4\zeta(4) + \zeta(3) - 1 = \sum_{k=1}^{\infty} \frac{k^2}{(k+2)^5}$

.0204919541493373728 30... ≈ $\zeta(3) - \frac{\zeta(3)}{\zeta(6)} = \sum_{k=1}^{\infty} \frac{(1 - |\mu(k)|)}{k^3} = \sum_{n\ not\ squarefree} \frac{1}{n^4}$ Berndt 6.30

1 .0206002693428741088... ≈ $\frac{1}{9}\csc\frac{8\pi}{9} = \int_0^{\infty} \frac{dx}{x^9 + 1}$

.0207459365240143802 1... ≈ $\frac{7\zeta(3)}{8} + \frac{\pi^3}{32} - 2 = -2 - \frac{1}{64}\psi^{(2)}\left(\frac{1}{4}\right) = \int_1^{\infty} \frac{\log^2 x}{x^6 - x^2} dx$

2 .0207459365240143802 1... ≈ $\frac{7\zeta(3)}{8} + \frac{\pi^3}{32} = -\frac{1}{64}\psi^{(2)}\left(\frac{1}{4}\right) = \int_0^1 \frac{\log^2 x}{1 - x^4} dx$

.020747041268399142... ≈ $\frac{\sqrt{e}}{2\left(\sqrt{e} - 1\right)} - \frac{5}{4} = \sum_{k=1}^{\infty} \frac{B_{2k}}{(2k)!4^k}$

1 .02078004443336310282... ≈ $\frac{13\zeta(3)}{27}+\frac{2\pi^3}{81\sqrt{3}} = -\frac{1}{54}\psi^{(2)}\left(\frac{1}{3}\right) = \sum_{k=1}^{\infty}\frac{1}{(3k-2)^3}$ Berndt 7.12.3

= $\frac{1}{2}\int_0^1 \frac{\log^2 x}{1-x^3}dx$

33 .02078982774748560951... ≈ $4+\frac{16\pi}{\sqrt{3}} = \sum_{k=0}^{\infty}\frac{(k!)^2 3^k}{(2k)!}$

.0208333333333333333$\underline{3}$ = $\frac{1}{48}$

1 .020837513725616828... ≈ $-2\gamma-\zeta(3)-\psi\left(1+\frac{1}{\sqrt{2}}\right)-\psi\left(1-\frac{1}{\sqrt{2}}\right) = \sum_{k=1}^{\infty}\frac{1}{2k^5-k^3}$

= $\sum_{k=1}^{\infty}\frac{\zeta(2k+3)}{2^k}$

1 .02083953399112117969... ≈ $\frac{1}{2}\left(\cos\frac{1}{\sqrt[4]{2}}+\cosh\frac{1}{\sqrt[4]{2}}\right) = \sum_{k=0}^{\infty}\frac{1}{(4k)!2^k}$

.020839548935264637084... ≈ $\frac{1}{6}-\frac{\pi}{4\sqrt{2}}\left(\cot\frac{\pi}{\sqrt{2}}+\coth\frac{\pi}{\sqrt{2}}\right) = \sum_{k=1}^{\infty}\frac{\zeta(4k)-1}{4^k}$

3 .02087086732388650526... ≈ $\sum_{k=1}^{\infty}\frac{\Phi(k)}{k!}$

2 .02093595742098418518... ≈ $\frac{\pi^3}{125}\csc\frac{\pi}{5}\left(1+2\cot^2\frac{\pi}{5}\right) = \int_0^{\infty}\frac{\log^2 x}{x^5+1}dx$

= $\frac{4\pi^3\sqrt{2}\left(25-3\sqrt{5}\right)}{125\left(5-\sqrt{5}\right)^{5/2}}$

.02098871933468264598... ≈ $\frac{197}{108}-\frac{3\zeta(3)}{2} = \int_1^{\infty}\frac{\log^2 x\,dx}{x^5+x^4}$

1 .02100284076754382916... ≈ $\frac{1}{8}\Phi\left(\frac{1}{2},3,\frac{1}{2}\right) = \int_0^1\frac{\log^2 x}{2-x^2}dx$

= $\frac{1}{\sqrt{2}}\left(Li_3\left(\frac{1}{\sqrt{2}}\right)-Li_3\left(-\frac{1}{\sqrt{2}}\right)\right)$

.02103789210760432503... ≈ $\frac{\gamma}{9}+\left(\frac{1}{18}-\frac{i}{6\sqrt{3}}\right)\psi\left(\frac{9-i\sqrt{3}}{6}\right)+\left(\frac{1}{18}+\frac{i}{6\sqrt{3}}\right)\psi\left(\frac{9+i\sqrt{3}}{6}\right)$

= $\sum_{k=1}^{\infty}\frac{1}{(3k+1)^3-1}$

.02108081020345922845... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k^5} = \sum_{k=2}^{\infty}\left(Li_5\left(\frac{1}{k}\right)-\frac{1}{k}\right)$

.021092796092796$\underline{092796}$ = $\frac{691}{32760} = \zeta(-11)$

.02127659574468085106... ≈ $\frac{1}{47}$

1 .02129180311957246996... ≈ $\frac{1}{3}\psi^{(1)}\left(\frac{2}{3}\right)=\sum_{k=1}^{\infty}\frac{3}{(3k-1)^2}=\sum_{k=1}^{\infty}\frac{k\zeta(k+1)}{3^k}$

.02129496275413122889... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{4^k k} = -\sum_{k=1}^{\infty}\left(\frac{1}{4k}+\log\left(1-\frac{1}{4k}\right)\right)$

.02134847029391195214... ≈ γ^7

.021491109059244877187... ≈ $-\sqrt{\frac{3}{2}}\operatorname{csch}\pi\sqrt{2}\sin\pi\sqrt{3} = \prod_{k=1}^{\infty}\frac{k^2+2k-2}{k^2+2k+3}$

.021566684583513603674... ≈ $\frac{\operatorname{sech}\pi}{4} = \prod_{k=0}^{\infty}\frac{k^2+1}{k^2+4}$

1 .021610099269294925320... ≈ $\frac{3\pi}{4}\zeta(3)+2\pi\log 2-\frac{\pi^3}{6}\log 2-\frac{\pi^3}{12} = \int_0^{\infty}\frac{\arctan^4 x}{x^4}dx$

1 .02165124753198136641... ≈ $2\log\frac{5}{3} = 4\operatorname{arctanh}\frac{1}{4} = 2Li_1\left(\frac{2}{5}\right) = \sum_{k=0}^{\infty}\frac{1}{16^k(2k+1)}$

.0217391304347826087... ≈ $\frac{1}{46}$

60 .02182669455857804485... ≈ $42 + 26\log 2 = \sum_{k=1}^{\infty}\frac{k^3 H_k}{2^k}$

1 .02210057524924977233... ≈ $2\sqrt{2}\arcsin\frac{1}{2\sqrt{2}}=\sum_{k=0}^{\infty}\frac{1}{32^k(2k+1)}\binom{2k}{k}$

.02219608194134170119... ≈ $\frac{7\pi^4}{30720} = \int_1^{\infty}\frac{\log^3 x\,dx}{x^5+x}$

.0222222222222222222$\underline{2}$ = $\frac{1}{45}$

.02222900171596697005... ≈ $\frac{\pi^2}{8}+\frac{\zeta(3)}{2}-\frac{29}{16} = \sum_{k=1}^{\infty}\frac{1}{k(k+1)(k+3)^2}$

.02224289490044537014... ≈ $\frac{\pi^2}{54}-\frac{2\gamma}{9}-\left(\frac{1}{9}-\frac{i}{3\sqrt{3}}\right)\psi\left(\frac{5-i\sqrt{2}}{2}\right)-\left(\frac{1}{9}+\frac{i}{3\sqrt{3}}\right)\psi\left(\frac{5+i\sqrt{2}}{2}\right)$

$-\left(\frac{1}{18}+\frac{i}{6\sqrt{3}}\right)\psi^{(1)}\left(\frac{5-i\sqrt{2}}{2}\right)-\left(\frac{1}{18}-\frac{i}{6\sqrt{3}}\right)\psi^{(1)}\left(\frac{5+i\sqrt{2}}{2}\right)$

= $\sum_{k=2}^{\infty}\frac{1}{\left(k^3-1\right)^2} = \sum_{k=1}^{\infty}(k-1)\left(\zeta(3k)-1\right)$

.022436419216546791773... ≈ $\frac{1}{12}-\frac{\zeta(3)}{2\pi^2} = \sum_{k=1}^{\infty}\frac{\zeta(2k)}{4^k(2k+2)(2k+3)}$

= $\frac{1}{6}\sum_{k=1}^{\infty}\left(24k^2-1-48k^3\operatorname{arctanh}\frac{1}{2k}-12k^2\log\left(1-\frac{1}{4k^2}\right)\right)$

.02258872223978123767... ≈ $4\log 2 - \frac{11}{4} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k}{(k+1)(k+2)(k+3)}$

.02271572685331510314... ≈ $\frac{\log 2}{3} - \frac{5}{24} = \int_0^1 \frac{\log x}{(x+1)^4} dx$

.022727272727272727$\underline{27}$ = $\frac{1}{44}$

.02274269944063537720... ≈ $\frac{11}{4} - \zeta(2) - \zeta(4) = \sum_{k=2}^{\infty} \frac{1}{k^6 - k^4} = \sum_{k=1}^{\infty} \frac{1}{k(k+1)^4 (k+2)}$

.02277922916008203587... ≈ $\frac{17}{16} - \frac{3}{2}\log 2 = \int_0^{\pi/4} \sin^4 x \tan^4 x \, dx$

.022891267882240749138... ≈ $\zeta^3(3) + \frac{197}{24}\zeta(9) + \frac{\pi^2 \zeta(7)}{2} - \frac{11}{120}\pi^4 \zeta(5) - \frac{37}{7560}\pi^6 \zeta(3)$

= $\sum_{k=1}^{\infty} \frac{H_k^{\,3}}{(k+1)^6}$ Borwein-Devlin, p. 63

.02289908811502851203... ≈ $\frac{9}{8} - \frac{\pi^2}{12} - \frac{3\log 2}{4} + \frac{\log^2 2}{2} = \sum_{k=2}^{\infty} \frac{1}{2^k (k^4 - k^2)}$

.02291843300216453709... ≈ $\frac{3}{2}\log 3 - \frac{13}{8} = \sum_{k=2}^{\infty} \frac{1}{9k^3 - k} = \sum_{k=1}^{\infty} \frac{\zeta(2k+1) - 1}{9^k}$

1 .0229247413409167683... ≈ $\sum_{k=2}^{\infty} \frac{1}{k^4 - 15}$

1 .0229259639348026338... ≈ $\sum_{k=1}^{\infty} \frac{1}{(2^k - 1)k^4} = \sum_{k=1}^{\infty} \frac{\sigma_{-4}(k)}{2^k}$

.02297330144608439494... ≈ $\frac{50G + 43}{128\pi} - \frac{28\log 2}{128} - \frac{71}{1536} = \sum_{k=1, k\neq 3}^{\infty} \left(\frac{(2k-1)!!}{(2k)!!}\right)^2 \frac{1}{2k-6}$

ϑ385

1 .02307878236628305099... ≈ $\frac{\pi}{64}\left(2\pi - \sqrt{2}\sin \pi\sqrt{2}\right)\sec^2 \frac{\pi}{\sqrt{2}} = \sum_{k=1}^{\infty} \frac{1}{((2k-1)^2 - 2)^2}$

.02309570896612103381... ≈ $\frac{\gamma}{2} + \frac{\log \pi}{2} + 1 - \log 2\pi = \sum_{\rho} \frac{1}{\rho}$, ρ a zero of $\xi(s)$

Edwards 3.8.4

1 .02310387968177884861... ≈ $\frac{31\zeta(5)}{8} - \frac{7\pi^2 \zeta(3)}{32} - \frac{7\zeta(3)}{4} - \frac{\pi^4 \log 2}{48} + \frac{\pi^4}{48} + \frac{\pi}{4} - 2\log 2$

= $\sum_{k=1}^{\infty} \frac{H_k}{(2k-1)^4}$

1 .02313872642793929553... ≈ $\sum_{k=2}^{\infty} \frac{\log k}{k^2 - 1} = \frac{\log 2}{2} + \sum_{k=1}^{\infty} \frac{\zeta(2k) - 1}{2k - 1}$

= $\frac{\log 2}{4} + \sum_{k=1}^{\infty} H_{2k-1}\left(\zeta(2k) - 1\right) = \frac{\log 2}{2} + \sum_{k=2}^{\infty} \left(\frac{1}{k} \operatorname{arctanh} \frac{1}{k}\right)$

$$.023148148148148148\underline{148} \quad = \quad \frac{5}{216} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^3}{5^k}$$

1 $$.0231629187630801764 6... \quad \approx \quad \frac{\pi^2 (2+\sqrt{3})}{36} = \sum_{k=1}^{\infty} \left(\frac{1}{(12k-1)^2} + \frac{1}{(12k-11)^2} \right)$$

$$= \quad \int_0^{\infty} \frac{\log x \, dx}{x^{12}-1}$$

$$.02317871150152727739... \quad \approx \quad \frac{233}{1920} - \frac{\pi \coth 2\pi}{32} = \sum_{k=3}^{\infty} \frac{1}{k^4 - 16}$$

$$.02325581395348837209... \quad \approx \quad \frac{1}{43}$$

1 $$.0233110122363703231... \quad \approx \quad \frac{\pi^3}{48} + \frac{\pi \log^2 2}{4} = \sum_{k=1}^{\infty} \frac{1}{4^k (2k+1)^3} \binom{2k}{k}$$

$$= \quad -\int_0^1 \frac{\arcsin x \log x}{x} dx$$

$$.02334259570291055312... \quad \approx \quad 3 - 2\gamma - 3\zeta(3) + 2\gamma\zeta(3) - 2\zeta'(3) = -\int_0^{\infty} \frac{x^2 \log x}{e^x \left(e^x - 1\right)} dx$$

1 $$.02335074992098443049... \quad \approx \quad \frac{27}{2} - 18 \log 2 = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k(k^2 - 1/9)}$$

$$.02336435879556467065... \quad \approx \quad \frac{5}{36} - \frac{\log 2}{6} = \sum_{k=1}^{\infty} \frac{1}{2^k (9k^2 + 18k)}$$

$$= \quad \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)(2k+2)(2k+4)(2k+5)}$$ K ex 107b

$$.0234375000000000000\underline{0} \quad = \quad \frac{3}{128} = \sum_{k=1}^{\infty} \frac{(-1)^k k^3}{3^k} = \int_1^{\infty} \frac{\log^3 x}{x^5} dx$$

$$.02346705930540378299... \quad \approx \quad \frac{2\pi}{\sinh 2\pi} = \prod_{k=1}^{\infty} \left(\frac{k^2}{k^2 + 4} \right)$$

$$.02351246536656649216... \quad \approx \quad \frac{26\zeta(3)}{27} - \frac{4\pi^3}{81\sqrt{3}} - \frac{1}{4} = -\frac{1}{27} \psi^{(2)}\left(\frac{5}{3}\right)$$

$$= \quad \frac{2\zeta(3)}{3} - \frac{8\pi^3}{81\sqrt{3}} + \frac{1}{6(\sqrt{3}+i)} \left(4\left(\sqrt{3} - i\right) Li_3\left(\frac{1+i\sqrt{3}}{2}\right) - \left(\sqrt{3} + 7i\right) Li_3\left(\frac{-1+i\sqrt{3}}{2}\right) \right)$$

$$= \quad \int_1^{\infty} \frac{\log^2 x}{x^6 - x^3} dx$$

$$.02353047298585523747... \quad \approx \quad 2\zeta(3) - \frac{28567}{12000} = -\psi^{(2)}(7) = \int_1^{\infty} \frac{\log^2 x}{x^8 - x^7} dx$$

.02375236632261848595... ≈ $\frac{\pi^4}{90}+\frac{\pi^2\log^2 2}{24}-Li_4\left(\frac{1}{2}\right)-\frac{\log 2}{24}\left(21\zeta(3)+\log^3 2\right)$

= $\sum_{k=3}^{\infty}\frac{S_1(k,3)}{k!k}$

$.0238095238095\underline{238095}$ = $\frac{1}{42} = B_3$

.02385592231300210713... ≈ $\frac{\pi^2}{12}-\frac{115}{144}=\sum_{k=0}^{\infty}\frac{(-1)^k}{(k+5)^2}=\int_1^{\infty}\frac{dx}{x^6+x^5}$

1 .023884750384121991821... ≈ $\frac{\pi^2}{\pi^2-1}\cosh\frac{1}{2} = \prod_{k=1}^{\infty}\left(1+\frac{1}{\pi^2(2k+1)^2}\right)$

1 .02394696928321706214... ≈ $\sum_{j=4}^{\infty}\sum_{k=1}^{\infty}\left(\zeta(jk-2)-1\right)$

.02402666421487355693... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^6\log k} = -\int(\zeta(s)-1)dx\Big|_{s=6}$

.02408451239117029266... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^k(2k+1)}{2^{k(k+1)/2}} = \prod_{k=1}^{\infty}\left(1-\frac{1}{2^k}\right)^3$ HW Thm. 357

1 .02418158153589244247... ≈ $16\log 2 - 4\zeta(2) - 2\zeta(3) - \zeta(4) = \sum_{k=1}^{\infty}\frac{1}{2k^5-k^4}$

= $\sum_{k=1}^{\infty}\frac{\zeta(k+4)}{2^k}$

2 .02423156789529607325... ≈ $\sum_{k=1}^{\infty}\frac{2^{1/k}}{k^6}$

1 .02434757508833937419... ≈ $\frac{G}{8}+\frac{\pi^2}{64}-\frac{1}{256}\psi^{(2)}\left(\frac{1}{4}\right)-\frac{1}{6144}\psi^{(3)}\left(\frac{1}{4}\right) = \sum_{k=0}^{\infty}\frac{(2k+1)^2}{(4k+1)^4}$

= $\frac{G}{8}+\frac{\pi^2}{64}+\frac{\pi^3}{128}+\frac{7\zeta(3)}{32}-\frac{1}{6144}\psi^{(3)}\left(\frac{1}{4}\right)$

$.02439024390243 9\underline{02439}$ = $\frac{1}{41}$

.02439486612255724836... ≈ $\zeta(3,5) = \zeta(3)-\frac{2035}{1728}$

= $\int_0^1\int_0^1\int_0^1\frac{x^4y^4z^4}{1-xyz}\,dx\,dy\,dz$

3 .02440121749914375973... ≈ $\sum_{k=0}^{\infty}\frac{\zeta(k+3)}{k!} = \sum_{k=1}^{\infty}\frac{e^{1/k}}{k^3}$

.02461433065757607149... ≈ $\frac{4-3\zeta(3)}{16} = \int_1^{\infty}\frac{\log^2 x\,dx}{x^5+x^3}$

1 .02464056431481555547... ≈ $2\pi^2(1-\log 2)+\dfrac{\pi^4}{72}-64\log 2\ +12\log^2 2\ +22\zeta(3)\ =\ \sum_{k=1}^{\infty}\dfrac{H_k}{k^3(2k-1)^2}$

.02475518879810900399... ≈ $\dfrac{61-6\pi^2}{72}\ =\ \sum_{k=1}^{\infty}\dfrac{1}{k(k+1)^2(k+2)(k+3)}$

.02481113982432774736... ≈ $\sum_{k=1}^{\infty}\dfrac{H_k}{2^k k(k+1)(k+2)(k+3)}$

7 .02481473104072639316... ≈ $\pi\sqrt{5}$

.0249410205141828796... ≈ $\log\Gamma\left(\dfrac{5}{6}\right)-\dfrac{\gamma}{6}\ =\ \sum_{k=2}^{\infty}\dfrac{\zeta(k)}{6^k k}$

.024991759249294471972... ≈ $\dfrac{\gamma}{4}-2-\dfrac{43\log 2}{12}+3\log 3+\dfrac{\log\pi}{2}-3\zeta'(1)\ =\ \sum_{k=2}^{\infty}\dfrac{(-1)^k\big(\zeta(k)-1\big)}{2^k k(k+1)}$

$.0250000000000000000\underline{0}$ = $\frac{1}{40}$

.0252003973997703426... ≈ $\zeta(2)-\zeta(3)+\zeta(4)-\frac{3}{2} = \sum_{k=2}^{\infty}\frac{1}{k^5+k^4}$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(k+4)-1\right)$

.0253302959105844428 6... ≈ $\frac{1}{4\pi^2}$

$.025400050800101 6\underline{0}$ = $\frac{1000}{3937}$ = meters/inch

.0255320819086914121 0... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(3k)-1}{8^k} = \sum_{k=2}^{\infty}\frac{1}{8k^3-1}$

2 .0256058527663465935 1... ≈ $\frac{4430}{2187} = \sum_{k=1}^{\infty}(-1)^k\frac{k^8}{2^k}$

$.025641025641\underline{025641}$ = $\frac{1}{39}$

1 .0257108717464466500 9... ≈ $2\sqrt{\frac{3}{7}}\operatorname{arcsinh}\frac{\sqrt{3}}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}3^k}{k\binom{2k}{k}}$

1 .0258279838738500474 0... ≈ $\frac{\pi^2}{8}\csc^2\frac{\pi}{\sqrt{2}}+\frac{\pi}{4\sqrt{2}}\cot\frac{\pi}{\sqrt{2}}-\frac{1}{2} = \sum_{k=1}^{\infty}\frac{1}{(2k^2-1)^2}$ J840

= $\sum_{k=1}^{\infty}\frac{k\zeta(2k+2)}{2^{k+1}}$

1 .0258371414015320133 7... ≈ $\zeta(6)\zeta(7) = \sum_{k=1}^{\infty}\frac{\sigma_1(k)}{k^7}$ HW Thm. 290

.0258436393842234815 3... ≈ $\sum_{k=2}^{\infty}\frac{k-1}{k^6\log k} = \int_5^6\left(\zeta(x)-1\right)dx$

.025865363708402647 9... ≈ $2\zeta(5)-\zeta(2)\zeta(3)+\zeta(4)+\zeta(3)+\zeta(2)-4 = \sum_{k=1}^{\infty}\frac{H_k}{(k+2)^4}$

.0259702755157400321 6... ≈ $\frac{1}{2}-\frac{\pi}{16}(\sqrt{2}+1) = \sum_{k=1}^{\infty}\frac{1}{(8k-1)(8k+1)}$ GR 0.239.7

1 .0261721529770308888 7... ≈ $\frac{\pi}{4\sqrt{2-\sqrt{2}}} = \frac{\pi}{8}\csc\frac{\pi}{8} = \int_0^{\infty}\frac{dx}{x^8+1}$

.0262279565260190530 46...≈ $\frac{161}{900}-\frac{G}{6} = -\int_0^1 \mathrm{x}^5\operatorname{arccot}x\log x\,dx$

.0262424596442487372 19...≈ $\frac{\pi}{128}\left(1+\sqrt{2}\right)\left(\pi\sqrt{2}-4\right) = \sum_{k=1}^{\infty}\frac{k\zeta(2k)}{64^k}$

$$.\underline{0263157894736842105} = \frac{1}{38}$$

$$.02632650867185557837\ldots \approx \frac{1}{2} - \frac{7}{2e^2} = \sum_{k=0}^{\infty} (-1)^k \frac{2^k k^2}{(k+1)!}$$

241 $$.02644307825201289114\ldots \approx 12e^3 = \sum_{k=1}^{\infty} \frac{3^k k^2}{k!}$$

$$.02648051389327864275\ldots \approx \log 2 - \frac{2}{3} = \sum_{k=1}^{\infty} \frac{1}{2^k(8k+24)} = \int_1^{\infty} \frac{dx}{x(x+1)^4} = \int_2^{\infty} \frac{dx}{x^5 - x^4}$$

$$.02649027604107518271\ldots \approx \frac{\pi^3}{64} - \frac{G}{2} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k}{(2k+1)^3}$$

$$.02654267736969571405\ldots \approx \frac{3\zeta(3)}{4} - \frac{7}{8} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+3)^3}$$

$$= \int_0^1 \int_0^1 \int_0^1 \frac{x^2 y^2 z^2}{1 + xyz} \, dx \, dy \, dz$$

1 $$.02669788816903316941 5\ldots \approx \log\left(\frac{\pi}{\sqrt{2}} \csc \frac{\pi}{\sqrt{2}} \right) = \sum_{k=1}^{\infty} \frac{\zeta(2k)}{2^k k}$$

$$= -\sum_{k=1}^{\infty} \log\left(1 - \frac{1}{2k^2} \right)$$

1 $$.02670520569941729272\ldots \approx 4\zeta(7) - \zeta(2)\zeta(5) - \zeta(3)\zeta(4) = \sum_{k=1}^{\infty} \frac{H_k}{k^6}$$

$$.02671848900111377452\ldots \approx \beta(\pi, \pi)$$

$$.02681802747491541739\ldots \approx 2 - \frac{5\pi^3}{81\sqrt{3}} - \frac{13\zeta(3)}{18} = \frac{1}{216}\left(\psi^{(2)}\left(\frac{7}{6}\right) - \psi^{(2)}\left(\frac{2}{3}\right) \right)$$

$$= 2 - \frac{10\pi^3}{81\sqrt{3}} - \frac{\zeta(3)}{2} - \frac{1}{3(\sqrt{3}+i)}\left(4i\, Li_3\left(\frac{1+i\sqrt{3}}{2}\right) + 2\left(\sqrt{3} - i\right) Li_3\left(\frac{1-i\sqrt{3}}{2}\right) \right)$$

$$= \int_1^{\infty} \frac{\log^2 x \, dx}{x^5 + x^2}$$

$$.02688374054040548325 3\ldots \approx 1 - \frac{12}{5} \log \frac{3}{2} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\binom{2k}{k} 6^k (2k+1)}$$

6 $$.02696069807178076242\ldots \approx \frac{1}{81} \psi^{(3)}\left(\frac{1}{3}\right) = 6 \sum_{k=0}^{\infty} \frac{1}{(3k+1)^4} = \int_0^1 \frac{\log^3 x \, dx}{x^3 - 1}$$

$$.027027027027207207\underline{027} = \frac{1}{37}$$

.02707670528833012617... ≈ $G - \frac{8}{9} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+5)^2} = \int_1^{\infty} \frac{\log x\; dx}{x^6+x^4}$

.02715493933253428647... ≈ $\sum_{k=2}^{\infty} \frac{\zeta(k)-1}{(2k)!} = \sum_{k=2}^{\infty} \left(-1 - \frac{1}{2k} + \cosh \frac{1}{\sqrt{k}} \right)$

.02737781481649722160... ≈ $\frac{\pi^2}{64} - \frac{\pi^4}{768} = \sum_{k=1}^{\infty} \frac{k(k+1)}{2(2k+1)^4}$ J349

.02737844362978441947... ≈ $\frac{3}{2} - \frac{5\pi}{32} = \int_0^{\pi/4} \sin^4 x \, \tan^2 x \; dx$

.02742569312329810612... ≈ $\pi^{-\pi}$

1 .0275046341122482764... ≈ $\frac{1}{4} - Li_2(1-e) = \sum_{k=0}^{\infty} \frac{B_{2k}}{(2k+1)!}$

.02753234031735682758... ≈ $\frac{3\log^2 2}{2} - \log 2 = -\int_0^{\infty} \frac{x \log x}{\cosh^2 x} dx$

.02755619219153047054... ≈ $-\frac{\zeta'(5)}{\zeta(5)} = \sum_{p\,prime} \frac{\log p}{p^5-1} = \frac{1}{\zeta(5)} \sum_{k=1}^{\infty} \frac{\log k}{k^5}$

= $\sum_{k=1}^{\infty} \frac{\Lambda(k)}{k^6}$

3 .0275963897427775429... ≈ $\cosh \sqrt{\pi} = \frac{e^{\sqrt{\pi}} + e^{-\sqrt{\pi}}}{2} = \sum_{k=0}^{\infty} \frac{\pi^k}{(2k)!}$ GR 1.411.4

.02768166089965397924... ≈ $\frac{8}{289} = \int_0^{\infty} \frac{x \sin x}{e^{4x}} dx$

.02772025198050593623... ≈ $\sum_{k=2}^{\infty} \frac{\zeta(k)}{k^6} = \sum_{k=1}^{\infty} \left(Li_6\left(\frac{1}{k}\right) - \frac{1}{k} \right)$

1 .02772258593685856788... ≈ $\frac{3\pi^3}{64\sqrt{2}} = \sum_{k=1}^{\infty} \left(\frac{(-1)^{k+1}}{(4k-3)^3} + \frac{(-1)^{k+1}}{(4k-1)^3} \right)$ J326, J340

= $\sum_{k=0}^{\infty} \frac{(-1)^{\lfloor k/2 \rfloor}}{(2k+1)^3}$ Prud. 5.1.4.3

.0277777777777777777$\underline{7}$ = $\frac{1}{36} = \sum_{k=0}^{\infty} \frac{1}{(2k+1)(2k+2)(2k+4)(2k+5)}$ K ex. 107c

= $\sum_{k=0}^{\infty} \frac{k}{(k+1)(k+2)(k+3)(k+4)}$

.02785584179904046214 0... ≈ $\sum_{k=2}^{\infty} \frac{(-1)^k}{4^k \zeta(k)} = -\sum_{k=1}^{\infty} \frac{\mu(k)}{4k+1} = \sum_{k=1}^{\infty} \frac{\mu(k)}{4k(4k+1)}$

.02788022955309069406... ≈ $1 - \frac{15\zeta(5)}{16} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+2)^5}$

1 .02797279921331664752... ≈ $7e - 18 = \sum_{k=0}^{\infty} \frac{k^2}{k!(k+3)}$

2 .02797279921331664752... ≈ $7e - 17 = \sum_{k=0}^{\infty} \frac{k^4}{(k+2)!}$

19 .02797279921331664752... ≈ $7e$

1 .0281424933935921577... ≈ $1 + \frac{1}{2}\log\left(-\sin\frac{\pi}{\sqrt{e}}\csc\pi\sqrt{e}\right) = \sum_{k=1}^{\infty} \frac{\sinh k}{k}\left(\zeta(2k)-1\right)$

.02817320866780299106... ≈ $\frac{3\zeta(3)}{128} = \int_1^{\infty} \frac{\log^2 x\, dx}{x^5 + x} = \int_0^{\infty} \frac{x^2\, dx}{e^{4x}+1}$

1 .02819907222448865115... ≈ $\frac{\pi^{\pi}}{\pi^{\pi}-1}$

.02835017583077031521... ≈ $\frac{1}{16}\left(\pi\cot\frac{7\pi}{8} + 8\log 2 + 2\sqrt{2}\log\left(1+\sqrt{2}\right)\right)$

= $\sum_{k=2}^{\infty} \frac{\zeta(k)}{8^k} = \sum_{k=1}^{\infty} \frac{1}{64k^2 - 8k}$

.02839399521893587901... ≈ $\frac{1-2\cos 1}{4\cos 1 - 5} = \frac{1-\frac{1}{2}(\cos 1)}{\frac{5}{4}-\cos 1} = \sum_{k=1}^{\infty} \frac{\cos k}{2^k}$ GR 1.447.1

.02848223531423071362... ≈ $\frac{3}{2} - \frac{4}{e} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}k}{(k+3)!}$

.0285714285714$\underline{285714}$ = $\frac{1}{35}$

.02857378050946295008... ≈ $-\zeta'(5) = \sum_{k=1}^{\infty} \frac{\log k}{k^5}$

.02874500302121581899... ≈ $\frac{\pi}{54} - \frac{19}{108} + \frac{\pi^2}{216} + \frac{\log 2}{27} + \frac{\log\pi}{2} + \frac{\zeta(3)}{16} = \int_0^1 x^2 \log^2 x \arctan x\, dx$

1 .02876865332981955131... ≈ $\sum_{c=2}^{\infty}\sum_{k=1}^{\infty} \frac{\zeta(ck)-1}{k^2} = \sum_{c=1}^{\infty}\sum_{k=2}^{\infty} Li_2\left(\frac{1}{k^c}\right)$

.028775355933941373288...≈ $2 - \frac{13}{4\sqrt{e}} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}k^3}{(k+1)!2^k}$

.02883143502699708281... ≈ $\frac{3\zeta(3)}{2} + \log 2 - \frac{\pi^2}{4} = \int_1^{\infty} \frac{\log^3 x}{(x+1)^4}dx$

18 .02883964878196093788... ≈ $\int_0^1 \log\left(1+\frac{1}{x^3}\right)\log^2 x\, dx$

= $12 + \frac{2\pi}{\sqrt{3}} + 4\log 2 + \frac{1}{6}\left(\psi^{(1)}\left(\frac{7}{6}\right) - \psi^{(1)}\left(\frac{2}{3}\right)\right) + \frac{1}{72}\left(\psi^{(2)}\left(\frac{2}{3}\right) - \psi^{(2)}\left(\frac{7}{6}\right)\right)$

.0290373315797836370... ≈ $\frac{1}{1000}\left(\psi^{(2)}\left(\frac{9}{10}\right) - \psi^{(2)}\left(\frac{2}{5}\right)\right) = \int_1^{\infty} \frac{\log^2 x\, dx}{x^5+1}$

.029076134702187599206... ≈ $\frac{3\zeta(3)}{4\pi^3} = \int_0^\infty \frac{dx}{e^{2\pi x^{1/3}}-1}$

.0290888208665721596... ≈ $\frac{\pi}{108} = \int_0^\infty \frac{dx}{(x^2+9)^2}$

3 .02916182001228824928... ≈ $\frac{\sinh \pi\sqrt{3}}{7\pi\sqrt{3}} = \prod_{k=1}^\infty \left(1+\frac{3}{(k+2)^2}\right)$

.029280631484544157522... ≈ $\frac{1}{12}\left(19\log 2 - 3\gamma + 6\left(\log\pi + 6\zeta'(1) - 2\right)\right) = \sum_{k=2}^\infty \frac{\zeta(k)-1}{2^k k(k+1)}$

= $\sum_{k=2}^\infty \left(1-\frac{1}{4k}+(2k-1)\log\left(1-\frac{1}{2k}\right)\right)$

.02929178853562162658... ≈ $\frac{1}{48}+\frac{1}{16e^2} = \sum_{k=0}^\infty \frac{(-1)^k 2^k}{(k+4)!}$

.029292727281254639... ≈ $\sum_{k=2}^\infty \frac{\log k}{k^5-1}$

9 .02932234585424892342... ≈ $\sum_{k=2}^\infty \left(4^k(\zeta(k)-1)^2-1\right)$

.02941176470588235294... ≈ $\frac{1}{34}$

.029524682084021688... ≈ $c_3 = \frac{1}{24}\left(\gamma^4 - 12\gamma^2\zeta(2) + 8\gamma\zeta(3) + 3\zeta^2(2) - 6\zeta(4)\right)$

Patterson ex. A4.2

.0295255064552159253... ≈ $\frac{7\zeta(3)}{4} - \frac{56}{27} = 2\sum_{k=2}^\infty \frac{1}{(2k+1)^3}$

= $\int_1^\infty \frac{\log^2 x}{x^6-x^4}dx$

= $\int_0^1 \frac{x^4\log^2 x}{1-x^2}dx$ GR 4.261.13

.02956409407168559657... ≈ $\frac{1}{4}+\frac{1}{36}\left(\psi^{(1)}\left(\frac{5}{6}\right)-\psi^{(1)}\left(\frac{1}{6}\right)\right) = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{(3k+2)^2}$

= $\frac{1}{36}\left(\psi^{(1)}\left(\frac{5}{6}\right)-\psi^{(1)}\left(\frac{4}{3}\right)\right) = \int_1^\infty \frac{\log x\, dx}{x^6+x^3}$

1 .02958415460387793411... ≈ $HypPFQ\left[\left\{\frac{1}{2},1,1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},\frac{1}{8}\right] = \sum_{k=0}^\infty \frac{1}{\binom{2k}{k}2^k(2k+1)^2}$

.029648609903896948156... ≈ $\frac{1}{6}-\frac{9\zeta(3)}{8\pi^2} = \sum_{k=1}^\infty \frac{\zeta(2k)}{4^k(2k+1)(2k+3)}$

1 .02967959373171800359... ≈ $\frac{\pi^{3/2}}{16}\Gamma\left(\frac{1}{4}\right)\Gamma^{-1}\left(\frac{3}{4}\right) = -\int_0^1 \frac{\log x dx}{\sqrt{1-x^4}}$

.029700538379251529... ≈ $\frac{2}{\sqrt{3}} - \frac{9}{8} = \sum_{k=2}^{\infty}\frac{(2k-1)!!}{(2k)!\,4^k}$

.02974960554835212735 3... ≈ $\frac{\pi^2+9\pi}{24} - \frac{9\log 2}{4} = \sum_{k=2}^{\infty}\frac{\zeta(k)-\zeta(k+1)}{4^k}$

1 .0297616606527557773... ≈ $\frac{\pi}{3\sqrt{3}}\left(\log 3 + \frac{\pi}{3\sqrt{3}}\right) = \int_0^{\infty}\frac{x\ dx}{\sqrt[3]{e^{3x}-1}}$ GR 3.456.1

= $-\int_0^1 \frac{\log x\ dx}{\sqrt[3]{1-x^3}}$ GR 4.244.2

.0299011002723396547... ≈ $\frac{\pi^2}{4} - \frac{39}{16} = \sum_{k=1}^{\infty}\frac{1}{\left(k(k+1)(k+2)\right)^2}$ J241, K ex. 111

.0299472777990186765... ≈ $\frac{3\log 2}{2} + \frac{\log^2 2}{2} - \frac{5}{4} = \sum_{k=1}^{\infty}\frac{H_k}{2^k(k+1)(k+2)(k+3)}$

.0300344524524566209... ≈ $\frac{2}{27}\log\frac{3}{2} = \sum_{k=1}^{\infty}(-1)^k\frac{H_k k^2}{2^k}$

.0300690429769907224... ≈ $\log\Gamma\left(\frac{6}{5}\right) + \frac{\gamma}{5} = \sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)}{5^k k}$

= $\sum_{k=1}^{\infty}\left(\frac{1}{5k} - \log\left(1+\frac{1}{5k}\right)\right)$

1 .0300784692787049755... ≈ $e\,\Gamma\left(\frac{3}{2},0,1\right) = \frac{e\sqrt{\pi}}{2}erf 1 - 1 = \sum_{k=1}^{\infty}\frac{(2k)!!}{(2k-1)!!k!(2k+1)}$

2 .0300784692787049755... ≈ $\frac{e\sqrt{\pi}}{2}erf 1 = \sum_{k=0}^{\infty}\frac{k!4^k}{(2k+1)!}$

.0301909176355844 4752... ≈ $\zeta(3) - \gamma - \frac{1}{2} - \frac{1}{2}\left(\psi(1+i) + \psi(1+i)\right)$

= $\zeta(3) - \gamma - \frac{1}{2}\left(\psi(2+i) + \psi(2-i)\right)$

= $\sum_{k=2}^{\infty}\frac{1}{k^5+k^3} = \sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(2k+3)-1\right)$

.0302795670706052931 4... ≈ $\frac{\pi^3}{1024} = \int_0^{\infty}\frac{x^2\,dx}{e^{4x}+e^{-4x}}$

1 .0302969550023049038 87... ≈ $\frac{16G}{3} - \frac{\pi^2}{9} - \frac{2\pi}{3}\log\left(2+\sqrt{3}\right) = \int_{-1/2}^{1/2}\frac{\arcsin^2 x\,dx}{x^2}$

.030303030303030303$\underline{03}$ = $\frac{1}{33}$

1 .03034552421621083244... ≈ $\frac{6\zeta(3)}{7}$

1 .03034552421621083244... ≈ $\frac{e^{\pi}}{\pi^{e}}$

.03044845705839327078... ≈ $\frac{\zeta(3)}{4\pi^2} = -\zeta'(-2) = c_2$ Berndt 9.27

1 .03055941076... ≈ j_5 J314

.03060200404263650785... ≈ $\frac{1}{64}\cos\frac{1}{4} + \frac{1}{16}\sin\frac{1}{4} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{(2k)!\,16^k}$

2 .03068839422549073862... ≈ $\frac{\pi^2}{2} - 1 - 2\zeta(3) = \sum_{k=2}^{\infty}\frac{5k^2+5k+2}{k(k+1)^3}$

$= \sum_{k=2}^{\infty}(-1)^k(k^2-1)(\zeta(k)-1)$

.030690333275592300473... ≈ $\frac{\gamma-1}{\pi} - \log\Gamma\left(2+\frac{1}{\pi}\right) = \sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)-1}{\pi^k k}$

.03087121264167682182... ≈ $\frac{\pi^4}{240} - \frac{3}{8} = \int_0^{\infty}\frac{x^3}{e^{2x}\left(e^{2x}-1\right)}dx$

.030952380952380$\underline{95238}$ = $\frac{13}{420} = \sum_{k=0}^{\infty}\frac{1}{(2k+1)(2k+5)(2k+9)}$

.03105385374063061952... ≈ $1 - \frac{\pi^3}{32} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k+1)^3}$

.03111190295912383842... ≈ $\frac{33}{48} - \frac{\pi^2}{48} - \frac{9\zeta(3)}{24} = \sum_{k=1}^{\infty}\frac{(-1)^k}{k(k+2)^3}$

.031125140377033351778... ≈ $\frac{15\zeta(5)}{8} + \frac{45\zeta(3)}{2} + 140\log 2 - 126 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^5(k+1)^5}$

.03117884541829663158... ≈ $\frac{\log 2\pi}{4} - \frac{1+\gamma}{6} + \zeta'(-1) = \sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k(k+1)(k+2)}$

5 .03122263757840562919... ≈ $\sum_{k=1}^{\infty}\frac{2^{2k-1}\zeta(2k)}{(2k-1)!} = \sum_{k=1}^{\infty}\frac{1}{k}\sinh\frac{2}{k}$

.03122842796811705531... ≈ $\frac{\pi^2}{6} + 2\log 2 - 3 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(k+1)(k+2)^2}$

.03124726104771826202... ≈ $\frac{1}{32} - \frac{\pi}{8}\operatorname{csch}4\pi = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2+16}$

.03125002980232238769... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^{5^k}} = -\sum_{k=1}^{\infty}\frac{\mu(5k)}{32^k-1}$

.03125655699572841608... ≈ $2+\frac{\pi^2}{3}-2\log^2 2-8Li_3\left(\frac{1}{2}\right)$

1 .03137872644997944832... ≈ $\sum_{k=1}^{\infty}\frac{1}{(k!)^5}$

.03138298351276753177... ≈ $126-\frac{35\pi^2}{3}-\frac{\pi^4}{9} = \sum_{k=1}^{\infty}\frac{1}{k^5(k+1)^5}$

1 .03141309987957317616... ≈ $\cosh\frac{1}{4} = \sum_{k=0}^{\infty}\frac{1}{(2k)!16^k}$

.03145390200081834392... ≈ $\frac{1}{64}\left(\zeta\left(2,\frac{5}{8}\right)-\zeta\left(2,\frac{9}{8}\right)\right) = \frac{1}{64}\left(\psi^{(1)}\left(\frac{5}{8}\right)-\psi^{(1)}\left(\frac{9}{8}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(4k+1)^2}$

= $\int_1^{\infty}\frac{\log x\ dx}{x^6+x^2}$

.03166788463797585l999... ≈ $\cos 1\sin 1-ci(2) = \int_1^{\infty}\frac{\cos x\sin x}{x^2}dx$

.03202517057518619422... ≈ $1-\log\frac{2\pi}{\sinh\pi}-\frac{\pi}{2}\coth\pi = \sum_{k=2}^{\infty}\left(-\frac{1}{k^2+1}+\log\left(1+\frac{1}{k^2}\right)\right)$

= $\sum_{k=1}^{\infty}(-1)^k\frac{k-1}{k}(\zeta(2k)-1)$

1 .03204910186238365390l... ≈ $\frac{32}{\pi^3} = \sum_{k=0}^{\infty}r(k)^7\left(1+14k+76k^2+168k^3\right)\left(\frac{1}{8}\right)^{2k}$

$where\ r(k)=\Gamma\left(k+\frac{1}{2}\right)\frac{1}{\pi\,\Gamma(k+1)}$ Borwein-Devlin. p. 36

.032067910200377349495... ≈ $\frac{1}{6}+\frac{\pi^2}{64}-\frac{5\log 2}{12} = \int_1^{\infty}\frac{\log x}{(x+1)^4(x-1)}dx$

.032080636429384244827... ≈ $\frac{1}{4096}\left(64\pi^2G+2976\zeta(5)+\pi\left(\psi^{(3)}\left(\frac{3}{4}\right)-\psi^{(3)}\left(\frac{1}{4}\right)\right)\right)$

= $-\int_0^{\pi/4}x^3\log\tan x\,dx$

.032092269954397683... ≈ $\frac{527}{7350}-\frac{2\log 2}{35} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k(k+2)(k+7)}$

.0321500000000000000$\underline{0}$ = $\frac{1}{32} = \int_1^{\infty}\frac{\log^2 x}{x^5}dx$

.03224796396463925040... ≈ $12\log\frac{3}{2}-\frac{29}{6} = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}k}{2^k(2k+6)}$

.03225153443319948918... $\approx \dfrac{1}{\pi^3}$

$.\underline{032258064516129}$ $= \dfrac{1}{31}$

.03230902899166988169 8... $\approx MHS(3,2,1) = 3\zeta^2(3) - \dfrac{203}{48}\zeta(6)$

.03233839744888501383... $\approx \dfrac{\sqrt{\pi}}{2}\left(2-\gamma-2\log 2\right) = \int\limits_0^\infty e^{-x}\sqrt{x}\log x\,dx$

.03235534139396046335 0... $\approx \dfrac{\pi}{8}\sqrt{4-2\sqrt{2}} - \dfrac{\pi}{8} = \int\limits_0^\infty \dfrac{x^2(1-x)}{x^8+1}dx$

.03241199911765660694 7... $\approx \dfrac{1}{6144}\left(8\pi\left(4\pi^2\log 2 - 9\zeta(3) - 24\pi G\right)\right) = -\int_0^{\pi/4} x^2\log\cos x\,dx$

.03241526628556134792... $\approx \zeta(5) + 3\zeta(3) + 2\zeta(4) + 5\zeta(2) - 15$

$= \sum\limits_{k=1}^\infty \dfrac{1}{k^2(k+1)^5}$

.03259889972766034529... $\approx \dfrac{5}{2} - \dfrac{\pi^2}{4} = \sum\limits_{k=1}^\infty \dfrac{1}{k(k+1)^2(k+2)^2}$

1 .03263195574407147268... $\approx \dfrac{\pi\log 2}{4\sqrt{2}} + \dfrac{G}{\sqrt{2}} = \sum\limits_{k=0}^\infty \dbinom{2k}{k}\dfrac{1}{8^k(2k+1)^2}$ Berndt 9.6.13

1 .03266056273537251 92... $\approx \dfrac{242\zeta(3)}{243} = G_5 = 1 + \sum\limits_{k=1}^\infty \dfrac{1}{(3k-1)^5} + \sum\limits_{k=1}^\infty \dfrac{1}{(3k+1)^5}$ J309

1 .03268544015795488313... $\approx \sum\limits_{k=1}^\infty \dfrac{1}{k^{k+3}}$

.03269207045110531343... $\approx \dfrac{\sqrt{3}}{4} - \dfrac{5}{6}$

.03272492347489367957... $\approx \dfrac{\pi}{96} = \int\limits_0^\infty \dfrac{dx}{x^6+64}$

.03283982870224045242... $\approx \dfrac{19}{72} - \dfrac{\log 2}{3} = \sum\limits_{k=1}^\infty \dfrac{(-1)^{k+1}H_k}{(k+2)(k+4)}$

.03289868133696452873... $\approx \dfrac{\pi^2}{300} = \int\limits_1^\infty \dfrac{\log x\,dx}{x^6+x}$

.03302166401177456807... $\approx \dfrac{J_1(4)}{4} = \sum\limits_{k=0}^\infty (-1)^k\dfrac{4^k}{k!(k+1)!}$

.0331678473115512076... $\approx 36\log\dfrac{5}{4} - 8 = \sum\limits_{k=0}^\infty \dfrac{(-1)^{k+1}k}{4^k(k+1)(k+3)}$

1 .03323891093441852714... ≈ $\frac{7\zeta(3)}{16} + \frac{\pi^4}{192} = \sum_{k=1}^{\infty} \frac{k}{(2k-1)^4}$

2 .03326096482301094329... ≈ $\sum_{k=1}^{\infty} \frac{2^k}{k!k^6}$

.033290128624160141531... ≈ $\frac{\gamma}{3} - \frac{3}{2} + \frac{\pi^2}{6} + \frac{1}{6}\left((1-i\sqrt{3})\,\psi\left(\frac{3-i\sqrt{3}}{2}\right) + (1+i\sqrt{3})\,\psi\left(\frac{3+i\sqrt{3}}{2}\right)\right)$

= $\sum_{k=1}^{\infty} (-1)^{k+1}\left(\zeta(3k+2)-1\right) = \sum_{k=2}^{\infty} \frac{1}{k^5+k^2}$

.033333$\underline{3}$ = $\frac{1}{30} = B_2 = B_4$

1 .03348486773424213825... ≈ $\sum_{k=1}^{\infty} \frac{1}{k!k^4}$

1 .03354255600999400585... ≈ $\frac{\pi^3}{30}$

.03365098983432689468... ≈ $\frac{1}{27}\psi^{(1)}\left(\frac{1}{3}\right) - \frac{13\zeta(3)}{81} - \frac{2\pi^3\sqrt{3}}{729}$

= $\frac{1}{162}\left(6\psi^{(1)}\left(\frac{1}{3}\right) + \psi^{(2)}\left(\frac{1}{3}\right)\right) = \sum_{k=1}^{\infty} \frac{k}{(3k+1)^3}$

.033749911073187769232... ≈ $\frac{5\pi}{4} - \frac{\pi}{\sqrt{3}} - 3\log 2 = \int_0^1 \frac{\log(1+x^6)}{(1+x)^2}\,dx$

.03375804113767116028... ≈ $5 - \zeta(2) - \zeta(3) - \zeta(4) - \zeta(5) = \sum_{k=1}^{\infty} \frac{1}{k(k+1)^5}$

= $\sum_{k=2}^{\infty} \frac{1}{k^6 - k^5}$

.03388563588508097179... ≈ $\sum_{k=1}^{\infty} \frac{\mu(k)\,\sigma_0(k)}{2^k - 1}$

1 .033885641474380667... ≈ $\sum_{k=1}^{\infty} \frac{pq(k)}{2^k}$

.03397268964697463845... ≈ $\frac{3}{2} - \frac{\pi^2}{12} - \frac{\log 2}{2} - \frac{\log^2 2}{2} - Li_3\left(\frac{1}{2}\right)$

.03398554396910802706... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(2k+1)-1}{(2k+1)!} = \sum_{k=2}^{\infty}\left(-\frac{1}{k} + \sinh\frac{1}{k}\right)$

.03399571980756843415... ≈ $J_4(2) = \frac{1}{24}\,{}_0F_1(,5,1) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(k+4)!}$ LY 6.117

.03401821139589475518... ≈ $\frac{1}{144}\left(\psi^{(1)}\left(\frac{5}{12}\right) - \psi^{(1)}\left(\frac{11}{12}\right)\right) = \int_1^{\infty} \frac{\log x\,dx}{x^6+1}$

.03407359027997265471... ≈ $\frac{\log 2}{2} - \frac{5}{16} = \sum_{k=1}^{\infty} \frac{1}{2^k (8k+16)} = \int_0^{\pi/4} \sin^4 x \tan x dx$

1 .03421011232779908843... ≈ $HypPFQ\left[\left\{\frac{1}{2},1,1\right\},\{2,2\},\frac{1}{4}\right] = \sum_{k=0}^{\infty} \binom{2k}{k} \frac{1}{16^k (k+1)^2}$

.03421279412205515593... ≈ $\frac{3\log 3}{2} + 2\log 2 - 3 = \sum_{k=1}^{\infty} \frac{1}{36k^3 - k}$ J376

1 .03427468507580717732 1... ≈ $\frac{1}{2}\left(e - \frac{1}{e} + \sqrt{\pi}\left(erf 1 + erfi 1\right)\right) = \int_1^{\infty} \sinh\left(\frac{1}{x^2}\right) dx$

1 .03433631351651708203... ≈ $\frac{\pi}{4}\log\left(2+\sqrt{3}\right) = \sum_{k=0}^{\infty} \frac{(k!)^2 4^k}{(2k)!(2k+1)^2}\left(1 - \frac{3}{4^{k+1}}\right)$ Berndt Ch. 9

1 .03437605526679648295... ≈ $\frac{\pi}{7}\csc\frac{6\pi}{7} = \int_0^{\infty} \frac{dx}{x^7+1}$

.03448275862068965517... ≈ $\frac{1}{29}$

.03454610783810898826... ≈ $\frac{1}{e} - \frac{1}{3} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(k+3)!}$

2 .03458085953975723608 1... ≈ $12\log 2 - 2\pi = \sum_{k=0}^{\infty} \frac{1}{(k+1)(k+3/4)}$

.03462999493452688535 2... ≈ $\frac{3}{2} + \frac{\pi^2}{12} - 2\log 2 - \frac{3\zeta(3)}{4} = \sum_{k=2}^{\infty} \frac{(-1)^k}{k^4 + k^3}$

.03467106568688200291... ≈ $\frac{1}{1024}\left(\psi^{(2)}\left(\frac{7}{8}\right) - \psi^{(2)}\left(\frac{3}{8}\right)\right) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(4k-1)^3}$

1 .03468944145535238591... ≈ $\int_0^1 \frac{\sin x}{\arctan x} dx$

2 .03474083500942906359... ≈ $\frac{(\sinh\pi)\left(\cosh\pi - \cos\pi\sqrt{3}\right)}{2\pi^2}$

= $\frac{\sinh\pi}{4\pi^3}(\cosh\pi - 1)\left(1 + \cos\pi\sqrt{3}\right) + (1+\cosh\pi)\left(1 - \cos\pi\sqrt{3}\right)$

= $\sum_{k=1}^{\infty}\left(1 + \frac{1}{k^6}\right)$ Berndt 4.15.1

1 .03476368168463671401... ≈ $\sum_{k=2}^{\infty}\left(\frac{1-e^{-1/k}}{k} + \frac{1}{k^2 e^{1/k}}\right) = \sum_{k=2}^{\infty}(-1)^k \frac{k^2}{k!}\left(\zeta(k) - 1\right)$

= $-\operatorname{Re}\left\{\sum_{k=1}^{\infty} \frac{\zeta(2k+1)}{i^k}\right\}$

2 .03476368168463671401... ≈ $\sum_{k=1}^{\infty}\left(\frac{1-e^{-1/k}}{k} + \frac{1}{k^2 e^{1/k}}\right) = \sum_{k=2}^{\infty}(-1)^k \frac{k}{(k-1)!}\left(\zeta(k) - 1\right)$

.03492413042327437913... ≈ $Ai(2)$

1 .03495812334779877266... ≈ $2Li_4\left(\frac{1}{2}\right)=\sum_{k=0}^{\infty}\frac{1}{2^k(k+1)^4}=\sum_{k=1}^{\infty}\frac{H^{(4)}{}_k}{2^k} = \sum_{k=1}^{\infty}\frac{H^{(3)}{}_k(k+2)}{2^k k(k+1)}$

.035034887349803758539... ≈ $\gamma-\frac{1}{2}+\frac{1}{4}\left(\psi\left(\frac{1+i}{\sqrt{2}}\right)+\psi\left(\frac{1-i}{\sqrt{2}}\right)+\psi\left(\frac{-1+i}{\sqrt{2}}\right)+\psi\left(\frac{-1-i}{\sqrt{2}}\right)\right)$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(4k+1)-1\right) = \sum_{k=2}^{\infty}\frac{1}{k^5+k}$

.035083836969886080624... ≈ $\frac{1-\gamma}{\pi}+\log\Gamma\left(2-\frac{1}{\pi}\right) = \sum_{k=2}^{\infty}\frac{\zeta(k)-1}{\pi^k k}$

.0352082979998841309... ≈ $\zeta(2)-\zeta(3)-\frac{\zeta(4)}{4}-\frac{3\zeta(6)}{4}+\zeta(2)\zeta(3)+\frac{\zeta^2(3)}{2}-2\zeta(5)$

= $\sum_{k=1}^{\infty}\frac{H_k}{k(k+1)^5}$

1 .0352761804100830494... ≈ $\sqrt{2}\left(\sqrt{3}-1\right) = \csc\frac{5\pi}{12}$ AS 4.3.46, CGF D1

.03532486246595754316... ≈ $\frac{11-\pi^2}{32} = \sum_{k=2}^{\infty}\frac{(-1)^k}{(k^2-1)^3}$

.03536776513153229684... ≈ $\frac{1}{9\pi}$

.03539816339744830962... ≈ $\frac{\pi-3}{4} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k)(2k+1)(2k+2)}$ J244, K ex. 109c

.03544092566737307688... ≈ $-\frac{1}{125}\psi^{(2)}\left(\frac{4}{5}\right)$

.03561265957073754087... ≈ $\frac{\zeta(5)-1}{\zeta(5)}$

.035683076611937861335... ≈ $\frac{\gamma}{\pi}+\frac{1}{2\pi}\left(\psi\left(\frac{\pi+i}{\pi}\right)+\psi\left(\frac{\pi-i}{\pi}\right)\right) = \sum_{k=1}^{\infty}\frac{(-1)^k\zeta(2k+1)}{\pi^{2k+1}}$

$.03571428571428\underline{571428}$ = $\frac{1}{28}$

.035755017483924257133... ≈ $\sum_{p\ prime}\frac{1}{p^5} = \sum_{k=1}^{\infty}\frac{\mu(k)}{k}\log\zeta(5k)$

.03577708763999663514... ≈ $\frac{2}{25\sqrt{5}} = \sum_{k=1}^{\infty}\frac{(-1)^k(2k)!k^2}{(k!)^2} = \sum_{k=1}^{\infty}\frac{(-1)^k(2k)!k^3}{(k!)^2}$

1 .03586852496020436212... ≈ $\sum_{k=1}^{\infty}k\left(\zeta(4k-2)+\zeta(4k-1)+\zeta(4k)+\zeta(4k+1)-4\right)$

.03596284319022298703... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^5+1} = \sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(5k)-1\right)$

.03619475030276797061... ≈ $\frac{\pi^2}{32}+\frac{\pi^4}{384}-\frac{7\zeta(3)}{16} = \sum_{k=1}^{\infty}\frac{k^2}{(2k+1)^4}$

.03619529870877200907... $\approx \frac{2+\pi-7\log 2}{8} = \int\limits_1^\infty \log\left(1+\frac{1}{x^2}\right)\frac{dx}{(x+1)^3}$

.03635239099919388379... $\approx \frac{1}{2} - \arctan\frac{1}{2} = \sum_{k=1}^\infty \frac{(-1)^k}{2^{2k+1}(2k+1)}$

.036382191549123483046... $\approx \frac{1}{\sqrt{2}} - \frac{\pi}{4} + \frac{\pi}{4\sqrt{2}} - \frac{1}{2}\log\left(1+\sqrt{2}\right) = \int\limits_0^\infty \frac{\sin x - x\cos x}{(\sin 2x)^{3/2}}dx$

Prud. 2.5.29.27

.036441086350966020557... $\approx \sum_{k=2}^\infty \frac{k}{k^6+1} = \sum_{k=1}^\infty (-1)^{k+1}\left(\zeta(6k-1)-1\right)$

.03646431135879092524... $\approx \frac{1}{5}\log\sum_{n=1}^\infty \frac{6}{5} = \left(-\frac{1}{5}+\sum_{k=1}^\infty \frac{H^{(n)}{}_k}{6^k}\right)$

.03648997397857652056... $\approx 2-\gamma-2\log 2 = \psi\left(\frac{3}{2}\right) = \psi\left(-\frac{1}{2}\right)$

.036557856669845028621... $\approx \sum_{k=2}^\infty \frac{\zeta(k)-1}{2^k(2k+1)} = \sum_{k=2}^\infty \left(\sqrt{2k}\,\text{arctanh}\frac{1}{\sqrt{2k}} - 1 - \frac{1}{6k}\right)$

.03661654541953101666... $\approx \log\Gamma\left(\frac{4}{5}\right) - \frac{\gamma}{5} = \sum_{k=2}^\infty \frac{\zeta(k)}{5^k k} = -\sum_{k=1}^\infty \left(\log\left(1-\frac{1}{5k}\right)+\frac{1}{5k}\right)$

1 .03662536367637941437... $\approx \frac{1}{36}\psi^{(1)}\left(\frac{1}{6}\right) = \sum_{k=1}^\infty \frac{1}{(6k-5)^2} = \int\limits_0^1 \frac{\log x\,dx}{x^6-1}$

.03663127777746836059... $\approx \frac{2}{e^4} = \sum_{k=0}^\infty \frac{(-1)^k 2^{2k+1}}{k!}$

.036683562016736660896... $\approx \sum_{k=2}^\infty \frac{k^2}{k^7+1} = \sum_{k=1}^\infty (-1)^{k+1}\left(\zeta(7k-2)-1\right)$

.03676084783888546123... $\approx \frac{\pi^3}{128} + \frac{3\pi}{32} - \frac{1}{2} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{(4k^2-1)^3}$

.03681553890925538951... $\approx \frac{3\pi}{256} = \int\limits_0^\infty \frac{dx}{(x^2+4)^3}$

.03686080059124710454... $\approx \frac{1}{4} - \frac{7\zeta(3)}{4\pi^2} = \sum_{k=1}^\infty \frac{\zeta(2k)}{4^k(2k+1)(2k+2)}$

.03690039313500120008... $\approx \frac{1}{5}\left(\frac{\pi}{2} - 2\log 2\right)$

1 .03692775514336992633... $\approx \zeta(5) = \zeta(4)\prod_{p\ prime} \frac{1+p^{-1}+p^{-2}+p^{-3}}{1+p^{-1}+p^{-2}+p^{-3}+p^{-4}}$

$= 2\sum_{k=2}^\infty \frac{(-1)^{k+1}}{k^5\binom{2k}{k}} - \frac{5}{2}\sum_{k=1}^\infty \frac{(-1)^{k+1}}{k^3\binom{2k}{k}}\sum_{j=1}^{k-1}\frac{1}{j^2}$ Borwein-Devlin, p. 42

= $\zeta(4)\prod_{p\ prime}\frac{1+p^{-1}+p^{-2}+p^{-3}}{1+p^{-1}+p^{-2}+p^{-3}+p^{-4}}$

.03698525801019933286... ≈ γ^6

.03703703703703703037 = $\frac{1}{27}$

.03705094729389941980... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^5-k^{-3}} = \sum_{k=1}^{\infty}\left(\zeta(8k-3)-1\right)$

.037087842300593465162... ≈ $\sum_{k=1}^{\infty}\frac{1}{3^{3^k}} = -\sum_{k=1}^{\infty}\frac{\mu(3k)}{27^k+1}$

.03717576492828151482... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^5-k^{-2}} = \sum_{k=1}^{\infty}\left(\zeta(7k-2)-1\right)$

.03717742536040556757... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(6k-1)-1}{k} = -\sum_{k=2}^{\infty}k\log(1-k^{-6})$

2 .03718327157626029784... ≈ $\frac{32}{5\pi} = \binom{3}{½}$

.03722251190098927492... ≈ $-\frac{1}{8192}\psi^{(2)}\left(\frac{3}{16}\right) = \sum_{k=0}^{\infty}\frac{1}{(16k+3)^3}$

.037286142336031613937... ≈ $-\frac{1}{\sqrt{3}}\operatorname{csch}\pi\ \sin\pi\sqrt{3} = \prod_{k=1}^{\infty}\frac{k^2+2k-2}{k^2+2k+2}$

.03729649324336985945... ≈ $\frac{9}{4}\log\frac{3}{2}-\frac{7}{8} = \sum_{k=2}^{\infty}\frac{(-1)^k}{2^k(k^2-1)k} = \sum_{k=1}^{\infty}\frac{1}{3^k(4k+8)}$

1 .03731472072754809588... ≈ $\coth 2 = \operatorname{csch} 2 + \tanh 1 = \frac{e^2+e^{-2}}{e^2-e^{-2}}$

= $\frac{1}{2}\sum_{k=0}^{\infty}\frac{16^k}{(2k)!}B_{2k}$ AS 4.5.67

.03736229369893631474... ≈ $\frac{1}{2}-\frac{3\pi^2}{64} = \sum_{k=1}^{\infty}\frac{1}{(4k^2-1)^3}$ J373, K ex. 109d

.03741043573731790237... ≈ $1-2\zeta(4)+\zeta(3) = \sum_{k=1}^{\infty}\frac{k}{(k+2)^4}$

.03742122104674100704... ≈ $\frac{\pi^3}{1728}+\frac{7\zeta(3)}{432} = -\frac{1}{3456}\psi^{(2)}\left(\frac{1}{4}\right) = \sum_{k=1}^{\infty}\frac{1}{(12k-9)^3}$

.03742970205727879551... ≈ $\frac{\gamma}{3}-\frac{1}{4}+\frac{1}{6}\left(\psi\left(\frac{3+i\sqrt{3}}{2}\right)+\psi\left(\frac{3-i\sqrt{3}}{2}\right)\right)$

= $\sum_{k=2}^{\infty}\frac{1}{k^5-k^{-1}} = \sum_{k=1}^{\infty}\left(\zeta(6k-1)-1\right)$

= $\frac{1}{2}\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(3k+1)-1\right) = \frac{1}{2}\sum_{k=2}^{\infty}\frac{1}{k^4+k}$

.03743018074474338727... $\approx$ $\sum_{k=1}^{\infty}\frac{\zeta(5k)-1}{k!} = \sum_{k=2}^{\infty}\left(e^{k^{-5}}-1\right)$

.03743548339675301485... $\approx$ $\sum_{k=1}^{\infty}\frac{\zeta(5k)-1}{k} = -\sum_{k=2}^{\infty}\log(1-k^{-5})$

.03744053288131686313... $\approx$ $\frac{3\log 2}{4}+\frac{19}{20}-\frac{\pi}{8} = \sum_{k=2}^{\infty}\frac{1}{4k(4k+1)} = \sum_{k=2}^{\infty}(-1)^{k+1}\frac{\zeta(k)-1}{4^k}$

.03756427822373732142... $\approx$ $\frac{\zeta(3)}{32} = \int_1^{\infty}\frac{\log^2 x}{x^5-x}dx = \int_0^{\infty}\frac{x^2}{e^{4x}-1}dx$

.03778748675487953855... $\approx$ $\frac{\pi}{48\sqrt{3}} = 16\sum_{k=1}^{\infty}\frac{1}{\binom{2k}{k}} = \int_0^{\infty}\frac{dx}{(x^2+3)^3}$

27 .03779975589821437814... $\approx$ γ^{-6}

.03780666068310387820... $\approx$ $-\frac{1}{1458}\psi^{(2)}\left(\frac{1}{3}\right) = \sum_{k=1}^{\infty}\frac{1}{(9k-6)^3}$

.037823888735468111... $\approx$ $\frac{16}{\sqrt{e}}-\frac{29}{3} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+4)!2^k}$

.037837038489586332884... $\approx$ $\frac{3\zeta(3)}{4}+2\log 2-\frac{9}{4} = \sum_{k=2}^{\infty}\frac{(-1)^k}{k^5-k^3}$

$= \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k(k+1)^3(k+2)}$

$= \sum_{k=1}^{\infty}\frac{1}{2^k\left(3k^2+9k+6\right)}$

.037901879626703127055... $\approx$ $\frac{1}{2}-\frac{2\log 2}{3} = \sum_{k=1}^{\infty}\frac{(-1)^k}{27k^3-3k}$ [Ramanujan] Berndt Ch. 2

.0379539032344025358... $\approx$ $\sum_{k=2}^{\infty}\frac{1}{k^5-1} = \sum_{k=1}^{\infty}\left(\zeta(5k)-1\right)$

.03797486594036412107... $\approx$ $\sum_{k=1}^{\infty}\frac{\zeta(4k+1)-1}{k} = -\sum_{k=2}^{\infty}\frac{\log(1-k^{-4})}{k}$

.03803647743448792694... $\approx$ $-\frac{1}{7}\cos\pi\sqrt{2} = \prod_{k=1}^{\infty}\left(1-\frac{8}{(2k+1)^2}\right)$

.03804894384475990627... $\approx$ $-\frac{1}{1024}\psi^{(2)}\left(\frac{3}{8}\right) = \sum_{k=1}^{\infty}\frac{1}{(8k-5)^3}$

.03810537240924724234... $\approx$ $\frac{1}{2}-\frac{\gamma}{2}-\frac{\log 2}{4} = \int_0^{\infty}x\,e^{-x}\log x\sin x\,dx$

.038106810383263487993... $\approx$ $\frac{G}{8}-\frac{11}{144} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)^2(2k+5)^2}$ Prud. 5.1.21.42

1 .03810687225724331102... ≈ $\frac{1}{2}\left(\zeta(10) + (\zeta(5))^2\right) = \sum_{k=1}^{\infty}\frac{H^{(5)}{}_k}{k^5}$

1 .0381450173309993166... ≈ $\Gamma\left(\frac{1}{4}\left(9+\sqrt{5}-i\sqrt{2\left(5-\sqrt{5}\right)}\right)\right)\Gamma\left(\frac{1}{4}\left(9+\sqrt{5}+i\sqrt{2\left(5-\sqrt{5}\right)}\right)\right)$

$\times\ \Gamma\left(\frac{1}{4}\left(9-\sqrt{5}-2i\sqrt{\frac{5+\sqrt{5}}{2}}\right)\right)\Gamma\left(\frac{1}{4}\left(9-\sqrt{5}+2i\sqrt{\frac{5+\sqrt{5}}{2}}\right)\right)$

$= \prod_{k=2}^{\infty}\frac{k^5}{k^5-1}$

.0381763098076249602... ≈ $80 - 4\pi^2 - 48\log 2 - 6\zeta(3) = \sum_{k=1}^{\infty}\frac{1}{k^3(2k+1)^3}$

.0383566024300068363 2... ≈ $\frac{1-\log 2}{8} = \sum_{k=1}^{\infty}\frac{1}{32k^2+16k} = \sum_{k=1}^{\infty}\frac{1}{2^k(8k^2+8k)}$

$= \int_0^1 \frac{\log(1+x)}{(2x+2)^2}dx$ GR 4.201.14

1 .0383787451121248920 2... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^{k+1}}{12k-11}$

.0384615384615$\underline{384615}$ = $\frac{1}{26} = \int_0^{\infty}\cos 5x\, e^{-x}\, dx$

.0385018768656984607... ≈ $\cos 2 + \frac{\sin 2}{2} = \cos 2 + \sin 1 \cos 1$

$= \sqrt{\pi}\left(J_{1/2}(2) - J_{3/2}(2)\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}4^k k^2}{(2k)!}$

31 .0385282147330196646 6... ≈ $\pi^3 + \pi^{-3}$

.0386078324507664303... ≈ $\frac{1}{2}\left(\gamma - \frac{1}{2}\right) = \int_0^{\infty}\frac{x\,dx}{(1+x^2)(e^{2\pi x}-1)}$ Andrews p. 87

$= \int_0^1 \frac{\log x}{4\pi^2 + \log^2 x}\frac{dx}{x-1}$ GR 4.282.1

.0386229550505646628 43... ≈ $\frac{1}{108}\left(\psi^{(1)}\left(\frac{1}{6}\right) - \psi^{(1)}\left(\frac{2}{3}\right)\right) - \frac{\log 2}{9} - \frac{\pi}{9\sqrt{3}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(3k+1)^2}$

.0386440776869978928 7... ≈ $\frac{31-3\pi^2}{36} = \int_1^{\infty}\frac{\log x}{x^4(x+1)}dx$

1 .0386631792049704084 6... ≈ $\frac{\pi}{8} + \frac{\pi^3}{48} = \int_0^{\pi/2} x^2 \sin^2 x\, dx$

28 .0387167043140264248 7... ≈ $\frac{7\pi^2}{8} + \frac{\pi^4}{16} + \log 2 + \frac{21\zeta(3)}{2} = \sum_{k=2}^{\infty}\frac{64k^3-20k^2+8k-1}{2k(2k-1)^4}$

$= \sum_{k=2}^{\infty} \frac{k^3 \zeta(k)}{2^k}$

.03876335736944358027... $\approx 2+\zeta(2)-3\zeta(3) = 2\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k}{(k+2)^3}$

$= \int_0^1 \frac{x^2 \log^2 x dx}{(x+1)^2} = \int_1^{\infty} \frac{\log^2 x dx}{x^2 (x+1)^2}$

.038768179602916798941... $\approx \frac{\zeta(3)}{\pi^3}$

.03879365883434284452... $\approx \frac{1}{8\sqrt{2}}\left(\pi + 2\log\left(\sqrt{2}-1\right)\right) = \int_0^1 \frac{1}{\pi^2 + 16\log^2 x} \frac{dx}{x^2+1}$ GR 4.282.9

.03886699365871711031... $\approx 2\arctan\frac{1}{2} + \frac{1}{2}\log\frac{5}{4} - 1 = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^{2k+1} k(2k+1)}$

.03895554778757944443... $\approx \frac{7\zeta(3)}{216} = \sum_{k=1}^{\infty} \frac{1}{(6k-3)^2}$

.038978174603$\underline{174603}$ $= \frac{3929}{100800} = \sum_{k=1}^{\infty} \frac{1}{k(k+5)(k+8)}$

.0390514150076132319... $\approx \sum_{k=1}^{\infty} \frac{\zeta(3k+2)-1}{k!} = \sum_{k=2}^{\infty} \frac{1}{k^2}\left(1+e^{k^{-3}}\right)$

.03906700723799508106... $\approx \frac{1}{8}\left(3 - 4\gamma - 2\psi(-i) - 2\psi(i)\right)$

$= \frac{5}{8} - \frac{\gamma}{2} - \frac{1}{4}\left(\psi(2+i) + \psi(2-i)\right) = \sum_{k=2}^{\infty} \frac{1}{k^5 - k}$

$= \sum_{k=1}^{\infty} \left(\zeta(4k+1) - 1\right)$

.0391481803713593579... $\approx \sum_{k=1}^{\infty} \frac{\zeta(3k+2)-1}{k} = -\sum_{k=2}^{\infty} \frac{\log(1-k^{-3})}{k^2}$

.03915058967111741409... $\approx 1 + \frac{\pi^2}{16} - \frac{21}{16}\zeta(3) = \sum_{k=1}^{\infty} \frac{k}{(2k+3)^3}$

.039155126213126751348... $\approx \frac{1}{32} - \frac{\pi}{48\sqrt{3}} + \frac{\pi^2}{216} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(3k+1)^2 (3k+5)^2}$

.03929439142762194708... $\approx \frac{25\log 5}{2} - 25\log 2 - \frac{11}{4} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{4^k k(k+1)(k+2)}$

$= \sum_{k=1}^{\infty} \frac{1}{5^k (2k+4)}$

.03939972493191508632... $\approx \frac{21-2\pi^2}{32} = \sum_{k=2}^{\infty} \frac{1}{(k^2-1)^3}$

.03945177035019706261... ≈ $\frac{12-\pi^2}{54} = 2\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(3k+3)^2} = \int_0^1 x^2\log(1-x^3)\log x\,dx$

.039459771698800348287... ≈ $2\log\pi - \frac{9}{4} = \sum_{k=2}^{\infty}\frac{k-1}{k+1}\left(\zeta(2k)-1\right)$

= $\sum_{k=2}^{\infty}\left(\frac{k^2-2}{k^2-1}+\frac{2}{k^2}\log(1-k^2)\right)$

1 .03952133779748813524... ≈ $\sqrt{2\cos 1} = \sqrt{e^i + e^{-i}}$

3 .03963550927013314332... ≈ $\frac{30}{\pi^2} = \frac{5}{\zeta(2)}$

1 .03967414409134496043... ≈ $\frac{1}{2}\left(\pi\coth\pi - \pi^2\operatorname{csch}^2\pi - 1\right)$

= $\sum_{k=1}^{\infty}(-1)^{k+1}2k\left(\zeta(2k-1)\right) = \sum_{k=2}^{\infty}\frac{2}{k^2\left(1+k^{-2}\right)^2}$

1 .03972077083991796413... ≈ $\frac{3\log 2}{2} = 3\operatorname{arctanh}\frac{1}{3} = \sum_{k=0}^{\infty}\frac{1}{9^k(2k+1)}$

= $1 + 2\sum_{k=1}^{\infty}\frac{1}{64k^3-4k^2}$ [Ramanujan] Berndt Ch. 2

.03985132614857383264... ≈ $-\frac{1}{250}\psi^{(2)}\left(\frac{3}{5}\right) = \sum_{k=0}^{\infty}\frac{1}{(5k+3)^2}$

.03986813369645287294... ≈ $\frac{\pi^2}{3} - \frac{13}{4} = \sum_{k=1}^{\infty}(-1)^{k+1}k\left(\zeta(k+3)-1\right)$

= $\frac{4}{3}\sum_{k=1}^{\infty}\frac{1}{k^2(k+1)^2(k+2)^2}$

2 .03986813369645287294... ≈ $\frac{\pi^2}{3} - \frac{5}{4} = \int_0^1\frac{(1+x^2)\log x}{x-1}dx$

.0400000000000000000$\underline{0}$ = $\frac{1}{25}$

.0400198661225572484... ≈ $\zeta(3,4) = \zeta(3) - \frac{251}{216} = \frac{1}{2}\int_1^\infty \frac{\log^2 x}{x^5 - x^4}\,dx$

= $\int_0^1\int_0^1\int_0^1 \frac{x^3 y^3 z^3}{1 - xyz}\,dx\,dy\,dz$

.0402471540227731631 4... ≈ $\sum_{k=1}^\infty \frac{\mu(2k-1)}{2^k}$

1 .0403476504088131940 1... ≈ $\frac{\pi}{3\sqrt{10}} = \sqrt{\zeta(4)}$

4 .0405096543107300242 5... ≈ 7γ

.0405368972715197378 29... ≈ $\frac{3\zeta(6)}{4} - \frac{\zeta(3)^2}{2} = MHS(5,1) = \sum_{k=1}^\infty \frac{H_k}{(k+1)^5}$

.0406342516346091785 26... ≈ $\frac{13}{30} - \frac{\pi}{8} = \sum_{k=2}^\infty \frac{1}{16k^2 - 1} = \sum_{k=1}^\infty \frac{\zeta(2k) - 1}{16^k}$

= $\frac{1}{2}\sum_{k=0}^\infty \frac{(-1)^k}{2k+7}$

.0406342576590166422 2... ≈ $\sin\frac{1}{2} - \frac{1}{2}\cos\frac{1}{2} = \sum_{k=1}^\infty \frac{(-1)^k k}{(2k+1)!\,4^k}$

1 .0407700700775575666 3... ≈ $\frac{23\sin 1 - 5\cos 1}{16} = \sum_{k=1}^\infty (-1)^k \frac{k^4}{(2k-1)!}$

.0408269921144456631 1... ≈ $\left(\zeta(3) - 1\right)^2 = \sum_{k=2}^\infty \frac{f_2(k)}{k^2}$ Titchmarsh 1.2.14

= $\sum_{k=2}^\infty \sum_{k=2}^\infty \frac{1}{(jk)^3}$

.0408480265877696780 3... ≈ $1 - \frac{\pi\sqrt{3}}{2} - \frac{\log 3}{4} - \frac{\log 2}{3} = \frac{1}{6}\sum_{k=1}^\infty \frac{1}{6k^2 + k} = \sum_{k=2}^\infty \frac{(-1)^k \zeta(k)}{6^k}$

.0410199491692923326 8... ≈ $168\cos\frac{1}{2} + 92\sin\frac{1}{2} - \frac{383}{2} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{(2k)!4^k(k+2)}$

1 .0410868685817072618 1... ≈ $-Li_1\left(e^{2i}\right) - Li_1\left(e^{-2i}\right)$

.0411378049829478356 2... ≈ $\frac{\pi}{54\sqrt{2}} = \int_0^\infty \frac{dx}{x^4 + 81}$

.04132352701659985... ≈ $\sum_{k=1}^\infty \frac{B_{2k}}{(2k)!2^k}$

.0414238321663628268 1... ≈ $\frac{1}{2} - \frac{1}{2}\tanh\frac{\pi}{2} = \frac{e^{-\pi}}{1 + e^{-\pi}} = \sum_{k=1}^\infty (-1)^{k+1} e^{-\pi k}$ J944

.04142682200263780962... ≈ $\frac{7\zeta(3)}{16} - \frac{\pi^3}{64} = -\frac{1}{128}\psi^{(2)}\left(\frac{3}{4}\right) = \sum_{k=1}^{\infty}\frac{1}{(4k-1)^3}$

1 .04145958644193544755... ≈ $\frac{\pi^2}{6} - \frac{\log^3 3}{2} = 2Li_2\left(\frac{1}{3}\right) - Li_2\left(-\frac{1}{3}\right)$

.041468529380903 5563... ≈ $1 + \frac{1}{3e} - \frac{2\sqrt{e}}{3}\cos\left(\frac{\sqrt{3}}{2} - \frac{\pi}{3}\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(3k+1)!}$

.04151843908482020028 9... ≈ $1 - \frac{\pi^2}{12} + \frac{\pi}{4}\left(\tanh\frac{\pi}{2} - \coth\frac{\pi}{2}\right) = \sum_{k=2}^{\infty}\frac{(-1)^k}{k^4 + k^2}$

.04156008886672620565... ≈ $\frac{26\zeta(3)}{27} + \frac{4\pi^3}{81\sqrt{3}} - 2 = -\frac{1}{27}\psi^{(2)}\left(\frac{4}{3}\right) = \int_1^{\infty}\frac{\log^2 x}{x^5 - x^2}dx$

2 .04156008886672620565... ≈ $\frac{26\zeta(3)}{27} + \frac{4\pi^3}{81\sqrt{3}} = -\frac{1}{27}\psi^{(2)}\left(\frac{1}{3}\right) = 2\sum_{k=1}^{\infty}\frac{1}{(3k-2)^3}$

= $\int_0^1 \frac{\log^2 x}{1 - x^3}dx$

= $\int_0^{\infty}\frac{x^2\,dx}{e^x - e^{-2x}}$

.04156927549039744949... ≈ $1 + \frac{\gamma}{3} - \zeta(2) + \left(\frac{1}{6} + \frac{i}{2\sqrt{3}}\right)\psi\left(\frac{3 - i\sqrt{3}}{2}\right) + \left(\frac{1}{6} - \frac{i}{2\sqrt{3}}\right)\psi\left(\frac{3 + i\sqrt{3}}{2}\right)$

= $\sum_{k=2}^{\infty}\frac{1}{k^5 - k^2}$

.04160256000655360000... ≈ $\sum_{k=1}^{\infty}\frac{1}{5^{2^k}} = -\sum_{k=1}^{\infty}\frac{\mu(2k)}{5^{2k} - 1}$

.04161706975489941139... ≈ $\frac{1}{4\sqrt{2}}\left(\cosh\frac{1}{\sqrt{2}}\sin\frac{1}{\sqrt{2}} - \cos\frac{1}{\sqrt{2}}\sinh\frac{1}{\sqrt{2}}\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(4k)!}$

.04164186716699298379... ≈ $\cos\frac{1}{\sqrt{2}}\cosh\frac{1}{\sqrt{2}} - 1 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(4k)!}$ GR 1.413.2

2 .04166194600313495569... ≈ $\sum_{k=1}^{\infty}\frac{k}{k!\,H_k}$

.0416666666666666666$\underline{6}$ = $\frac{1}{24} = \sum_{k=0}^{\infty}\frac{1}{(3k+1)(3k+4)(3k+7)}$ C132

= $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k}{e^{\pi k} + (-1)^k}$ Prud. 5.3.1.2

= $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^{13}}{e^{\pi k} + (-1)^k}$

1 .0416666666666666666$\underline{6}$ = $\frac{25}{24} = \sum_{k=1}^{\infty}\frac{1}{k({}^{k}\!/_{2} + 2)}$

1 .04166942239863686003... $\approx \sum_{k=1}^{\infty} \frac{1}{(k^2)!}$

1 .04169147034169174794... $\approx \frac{1}{2}\left(\cos 1 + \cosh 1\right) = \frac{\cos 1}{2} + \frac{e}{4} + \frac{1}{4e} = \sum_{k=0}^{\infty} \frac{1}{(4k)!}$

.04169422398598624210... $\approx \sum_{k=2}^{\infty} \frac{\Omega(k)}{k!} = \sum_{k=2}^{\infty}\sum_{j=2}^{\infty} \frac{1}{(k^j)!}$

.04171115653476388938... $\approx \frac{1}{4}\arctan 2 + \frac{\sqrt{2}}{16}\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right) + \frac{\log 3}{8} +$

$+ \frac{\pi(\sqrt{2}-1)}{8} + \frac{\sqrt{2}}{8}\left(\arctan\left(1-\sqrt{2}\right) - \arctan\left(1+\sqrt{2}\right)\right) = \int_2^{\infty} \frac{dx}{x^4 - x^{-4}}$

.04171627610448811878... $\approx \frac{\sinh 1 - \sin 1}{8} = \frac{e}{16} - \frac{1}{16e} - \frac{\sin 1}{8} = \sum_{k=1}^{\infty} \frac{k}{(4k)!}$

1 .04186535509890984463... $\approx \frac{e}{3} - \frac{2}{3\sqrt{e}}\cos\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right) = \sum_{k=0}^{\infty} \frac{1}{(3k+1)!}$ J804

.04188573804681767961... $\approx \frac{1}{6}\log\frac{9}{7} = \int_2^{\infty} \frac{dx}{x^4 - x^{-2}}$

.04189489811963856197... $\approx \sum_{k=2}^{\infty} \frac{(-1)^k}{k^5 \log k}$

721 .04189518147832777266... $\approx \frac{3\pi^6}{4}$

.04201950582536896173... $\approx -\frac{1}{2}\log\left(2 - 2\cos 1\right) = -\log\left(2\sin\frac{1}{2}\right) = \sum_{k=1}^{\infty} \frac{\cos k}{k}$ GR 1.441.2

.04213570832157981 30... $\approx 4\log 2 - \log^2 2 - \frac{9}{4} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} H_k}{(k+2)(k+3)}$

1 .04219061098749472 32... $\approx 2\sinh\frac{1}{2} = \frac{e-1}{\sqrt{e}} = 2(e^{1/2} - e^{-1/2}) = \sum_{k=0}^{\infty} \frac{1}{(2k+1)!4^k}$

$= \prod_{k=1}^{\infty}\left(1 + \frac{1}{4\pi^2 k^2}\right)$ GR 1.431.2

.04221270263816624726... $\approx -\frac{1}{2} - \frac{\pi}{4\sqrt[4]{2}}\left(\cot\frac{\pi}{\sqrt[4]{2}} + \coth\frac{\pi}{\sqrt[4]{2}}\right) = \sum_{k=1}^{\infty} \frac{\zeta(4k)-1}{2^k}$

$= \sum_{k=2}^{\infty} \frac{1}{2k^4 - 1}$

1 .04221270263816624726... $\approx \sum_{k=1}^{\infty} \frac{\zeta(4k)}{2^k} = \sum_{k=1}^{\infty} \frac{1}{2k^4 - 1}$

$= \frac{1}{2} - \frac{\pi}{4\sqrt[4]{2}}\left(\cot\frac{\pi}{\sqrt[4]{2}} + \coth\frac{\pi}{\sqrt[4]{2}}\right)$

1 .042226738809353184449... ≈ $\sum_{k=1}^{\infty}\left(\frac{1}{2^k-1}-\frac{1}{2^{k^2}}\right) = 2\sum_{k=1}^{\infty}\frac{1}{2^{k(k+1)}-2^{k^2}}$

[Ramanujan] Berndt IV, p. 404

2 .04227160072224093168... ≈ $16-\pi^2\sqrt{2} = \sum_{k=1}^{\infty}\left(\frac{(-1)^{k+1}}{(k-1/4)^2}+\frac{(-1)^{k+1}}{(k+1/4)^2}\right)$

.0422873220524388879... ≈ $\sum_{k=0}^{\infty}\frac{B_k 2^k k^2}{k!}$

.04238710124041160909... ≈ $\frac{\log 2}{4}-\frac{\pi}{24} = \sum_{k=1}^{\infty}\frac{1}{(4k-3)(4k-1)(4k+1)(4k+3)}$ J242

.04252787061758480159... ≈ $\frac{1}{\pi}\left(\frac{1}{2}\left(\psi\left(1+\frac{1}{\pi}\right)-\psi\left(1-\frac{1}{\pi}\right)\right)-2\gamma\right)$

= $\sum_{k=1}^{\infty}\frac{1}{k\pi\left(k^2\pi^2-1\right)} = \sum_{k=1}^{\infty}\frac{\zeta(2k+1)}{\pi^{2k+1}}$

.04257821907383586345... ≈ $\frac{8}{125}+\frac{12}{125}\log\frac{4}{3} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2 H_k}{4^k}$

.042704958078479727... ≈ $6-\frac{\pi^2}{4}-2\log 2-\frac{7\zeta(3)}{4} = \sum_{k=1}^{\infty}\frac{1}{k(2k+1)^3}$

.04271767710944303099... ≈ $\frac{1}{2}\cosh\frac{1}{2}-\sinh\frac{1}{2} = \sum_{k=1}^{\infty}\frac{k}{(2k+1)!\,4^k}$

.04282926766662436474... ≈ $\frac{\arctan 2}{4}-\frac{\pi}{4}+\frac{\log 3}{4} = \int_2^{\infty}\frac{dx}{x^4-1}$

.0428984325630382984... ≈ $\frac{\pi^2}{3}\left(1-\frac{\pi^2}{10}\right) = 2\zeta(2)-3\zeta(4)$

1 .042914821466744092886... ≈ $\frac{1}{2}\csc\frac{1}{2} = \int_{-\infty}^{\infty}\frac{e^{-x}\,dx}{1+e^{-2\pi x}}$ Marsden Ex. 4.9

.04299728528618566720... ≈ $16\log\frac{3}{2}-\frac{58}{9} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{2^k(k+2)}$

1 .04310526030709426603... ≈ $3-\pi+5\log 2+\frac{\sqrt{3}}{2}\log\frac{2-\sqrt{3}}{2+\sqrt{3}} = \int_0^1\log\frac{(1+x)^3}{1+x^6}dx$

.0431635415191573980... ≈ $\frac{1}{6}+\frac{\pi}{12\sqrt{3}}-\frac{\log 3}{4} = \sum_{k=1}^{\infty}\frac{1}{(3k-2)(3k-1)3k(3k+1)}$

GR 0.238.3, J252

5 .04316564336002865131... ≈ e^{φ}

.0432018813143121202... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k^4} = \sum_{k=2}^{\infty}\left(Li_4\left(\frac{1}{k}\right)-\frac{1}{k}\right)$

.04321361686294489601... ≈ $\frac{1}{2}\,\frac{\sqrt[4]{2}-1}{\sqrt[4]{2}+1}$ Mel1 4.10.7

.04321391826377225497... ≈ $e^{-\pi} = i^{2i} = (-1)^i = \cos(2\log i) + i\sin(2\log i)$

.0432139182642977983... ≈ $\frac{(2^{1/4}-1)\Gamma(1/4)}{2^{11/4}\pi^{3/4}} = \sum_{k=1}^{\infty} e^{-\pi(2k-1)^2}$ J114

.04321740560665400729... ≈ $\sum_{k=1}^{\infty} e^{-\pi k^2}$

.04324084828357017786... ≈ $\operatorname{arctanh}\frac{1}{e^{\pi}} = -\frac{1}{2}\log\tanh\frac{\pi}{2} = \sum_{k=0}^{\infty}\frac{1}{e^{\pi(2k+1)}(2k+1)}$

1 .04331579784170500416... ≈ $HypPFQ\left[\{1,1,1\},\left\{\frac{2}{3},\frac{4}{3}\right\},\frac{1}{27}\right] = \sum_{k=0}^{\infty}\frac{(k!)^3}{(3k+1)!}$

1 .04338143704975653187... ≈ $-\gamma + \frac{1}{3}\left(-\psi\left(1-2^{-1/3}\right) + \psi\left(1+\frac{1-i\sqrt{3}}{2^{4/3}}\right) + \psi\left(1+\frac{1+i\sqrt{3}}{2^{4/3}}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{\zeta(3k+1)}{2^k} = \sum_{k=1}^{\infty}\frac{1}{2k^4-k}$

.04341079156485192712... ≈ $\frac{23}{4} - \frac{13}{2}\log 2 - \frac{5\log^2 2}{2} = \sum_{k=1}^{\infty}\frac{kH_k}{2^k(k+1(k+2)(k+3)}$

.04344142652643615327 3... ≈ $\frac{2}{3} - \frac{\sqrt{2}}{2}\operatorname{arcsinh}1 = \sum_{k=1}^{\infty}\frac{1}{2^k(2k+1)(2k+3)}$

.04344269231733307978... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(4k)-\zeta(4k+1)\right) = \sum_{k=2}^{\infty}\frac{k-1}{k^5+k}$

.$\underline{0434782608695652173913}$ = $\frac{1}{23}$

.04349162797695096933... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{2^k k^2} = \sum_{k=2}^{\infty}\left(Li_2\left(\frac{2}{k}\right) - \frac{2}{k}\right)$

.04360401198037898637 6... ≈ $\frac{\log 3}{4} - \frac{\log 2}{3} = \sum_{k=1}^{\infty}\frac{1}{27(2k-1)^3-3(2k-1)}$

[Ramanujan] Berndt Ch. 2

1 .04373467400995075724 6... ≈ $\arctan(e-1) = \int_0^1 \frac{1}{e^x+2e^{-x}-2}$

1 .04377882484348362176... ≈ $\frac{\zeta(4)}{\zeta(5)} = \sum_{k=1}^{\infty}\frac{\phi(k)}{k^5}$ Titchmarsh 1.2.13

.0438279703879014939 2... ≈ $\frac{3\log 2}{4} - \frac{\pi}{8} - \frac{1}{12} = \sum_{k=2}^{\infty}\frac{1}{4k(4k-1)} = \sum_{k=2}^{\infty}\frac{\zeta(k)-1}{4^k}$

1 .04386818141942574959... ≈ $\frac{16}{\sqrt{15}}\arcsin\frac{1}{4} = \sum_{k=0}^{\infty}\frac{(k!)^2}{(2k+1)!4^k}$

.04390932788176551020... ≈ $\frac{23}{6\sqrt{2}} - \frac{8}{3} = \int_0^{\pi/4}\sin^3 x\,\tan^2 x\,dx$

1 .04393676209874122833... ≈ $7\pi^3 - 216 = \sum_{k=1}^{\infty}\left(\frac{(-1)^{k+1}}{(k-1/6)^3} - \frac{(-1)^{k+1}}{(k+1/6)^3}\right)$

.04398180468826652940... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k-1)!\zeta(2k)}$ Titchmarsh 14.32.1

1 .04405810996426632590... ≈ $\frac{1}{2} + 2\pi\operatorname{csch}\pi = \sum_{k=0}^{\infty}\frac{(-1)^k}{k^2+1}$

1 .04416123612010169174... ≈ $e^{e^{-\pi}} = \sum_{k=0}^{\infty}\frac{e^{-\pi k}}{k!}$

.04419417382415922028... ≈ $\frac{1}{16\sqrt{2}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{4^k}\binom{2k}{k}$

.044205282387762443861... ≈ $\frac{14}{3} - \log 2 - \zeta(2) - \zeta(3) + \zeta(4) = \sum_{k=2}^{\infty}\frac{1}{k^5-k^4} - \int_2^{\infty}\frac{dx}{x^5-x^4}$

.04427782599695232811... ≈ $2 + 2\log\left(2+\sqrt{3}\right) - \frac{2\pi}{3} + 4\log 2 = \sum_{k=1}^{\infty}\binom{2k}{k}\frac{1}{16^k k(2k+1)}$

.04438324164397169544... ≈ $\frac{1}{4} - \frac{\pi^2}{48} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k+2)^2} = \int_1^{\infty}\frac{dx}{x^5+x^3}$

9 .04441336393990804074... ≈ $\sum_{k=2}^{\infty}\left(\sigma_2(k)-1\right)\left(\zeta(k)-1\right)$

.04444444444444444444$\underline{4}$ = $\frac{2}{45} = \int_1^{\infty}\frac{\operatorname{arccosh}x}{(1+x)^4}$

.0445206260429479365... ≈ $\frac{\zeta(3)}{27} = \sum_{k=1}^{\infty}\frac{1}{(3k)^3}$

.0445243112740431161... ≈ $\frac{3\pi}{32} - \frac{1}{4} = \int_1^{\infty}\frac{dx}{(x^2+1)^3}$

.04456932603301417348... ≈ $9\log 2 - \frac{9\log 3}{2} - \frac{5}{4} = \sum_{k=1}^{\infty}\frac{1}{4^k k(k+1)(k+2)}$

.04462871026284195115... ≈ $\frac{1}{5}\log\frac{5}{4} = \sum_{k=1}^{\infty}\left(Li_k\left(\frac{1}{5}\right) - \frac{1}{5}\right) = \sum_{k=1}^{\infty}\frac{1}{5^{k+1}k}$

.04482164619006956534... ≈ $\frac{3}{8\sqrt{2}} - \frac{\operatorname{arcsinh}1}{4} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{4^k(2k+1)}\binom{2k}{k}$

1 .04501440438616681983... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(k+1)}{2^k k} = -\sum_{k=1}^{\infty}\frac{1}{k}\log\left(1-\frac{1}{2k}\right)$

.04507034144862798539... ≈ $1-\frac{3}{\pi}$

.04510079681832739103... ≈ $144-\frac{391}{2e}-\frac{53e}{2} = \sum_{k=1}^{\infty}\frac{k}{(2k+1)!(k+3)}$

1 .0451009147609597957... ≈ $2\sqrt{2}\,\text{arctanh}\,\frac{1}{2\sqrt{2}}=\sum_{k=0}^{\infty}\frac{1}{8^{k}(2k+1)}$

1 .04516378011749278484... ≈ $li(2)$

2 .0451774444795624753... ≈ $8\log 2-\frac{7}{2} = \sum_{k=1}^{\infty}\frac{H_{k+2}}{2^{k}}$

.04522874755778077232... ≈ $1+\log 2-\frac{3\log 3}{2} = \sum_{k=1}^{\infty}\frac{1}{2^{2k+1}k(2k+1)}$

$.04527529761904\underline{761904}$ = $\frac{1217}{26880} = \sum_{k=1}^{\infty}\frac{1}{k(k+4)(k+8)}$

.04539547856776826518... ≈ $\zeta(4)-\zeta(5)$

1 .04540769073144974624... ≈ $\frac{1}{2\pi}+\frac{\sqrt{\pi}}{2}\coth\pi^{3/2} = \sum_{k=0}^{\infty}\frac{1}{(k^{2}+\pi)}$

.04543145370663020628... ≈ $\frac{2\log 2}{3}-\frac{5}{12}=\sum_{k=2}^{\infty}\frac{(2k-4)!}{(2k)!} = \sum_{k=1}^{\infty}\frac{1}{2^{k}(6k+12)}$

.045452882429679849... ≈ $\sum_{p\ prime}\left(\log\left(1-p^{-2}\right)-p^{-2}\right)$

$.0454545454545454545\underline{45}$ = $\frac{1}{22}$

.04547284088339866736... ≈ $\frac{1}{7\pi}$

.04549015212755020353... ≈ $\frac{7\zeta(4)}{8}-\frac{3\zeta(3)}{4} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(k+1)^{4}}$

.0455584583201640717... ≈ $\frac{17}{8}-3\log 2 = \int_{1}^{\infty}\frac{dx}{e^{x}(e^{x}+1)^{3}}$

.04569261296800628990... ≈ $\frac{\pi^{2}}{216} = \frac{\zeta(2)}{36} = \sum_{k=1}^{\infty}\frac{1}{(6k)^{2}}$

.0458716234174888797... ≈ $\frac{\pi}{32\sqrt{2}}-\frac{1}{16}-\frac{1}{16\sqrt{2}}\log\left(\sqrt{2}-1\right)$

= $\int_{0}^{1}\frac{\log x}{\pi^{2}-16\log^{2}x}\,\frac{dx}{x^{2}-1}$ GR 4.282.10

.04596902017805633200... ≈ $\frac{\gamma}{9}-\frac{\pi}{18\sqrt{3}}+\frac{\log 3}{6}+\frac{\sqrt{3}-3i}{18\sqrt{3}}\,\psi\left(\frac{7-i\sqrt{3}}{6}\right)+\frac{\sqrt{3}+3i}{18\sqrt{3}}\,\psi\left(\frac{7+i\sqrt{3}}{6}\right)$

= $\sum_{k=1}^{\infty}\frac{1}{27k^{3}-1}$

.04599480480499238437... ≈ $\frac{10}{3}\log\frac{3}{2}-\frac{47}{36} = \sum_{k=1}^{\infty}\frac{1}{3^k k(k+1)(k+3)}$

.04603207980357005369... ≈ $\log\left(\Gamma\left(\frac{5}{4}\right)\right)+\frac{\gamma}{4} = \sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)}{4^k k}$ Dingle 3.37

= $\sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)}{4^k k} = \sum_{k=1}^{\infty}\left(\frac{1}{4k}-\log\left(1+\frac{1}{4k}\right)\right)$

.04607329257318598861... ≈ $I_3(2)-\frac{1}{6} = \sum_{k=1}^{\infty}\frac{1}{k!(k+3)!}$

1 .0460779964620999381... ≈ $1-\zeta(2)-\frac{\pi}{\sqrt{2}}\cot\frac{\pi}{\sqrt{2}} = \sum_{k=1}^{\infty}\frac{1}{2k^4-k^2}$

= $\sum_{k=1}^{\infty}\frac{\zeta(2k+2)}{2^k}$

.0461254914187515001... ≈ $\frac{\pi}{8}-\frac{\log 2}{2} = \sum_{k=1}^{\infty}\frac{1}{(4k-2)(4k-1)4k}$ J251

= $\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{1}{(4k-1)^{2j}-1}$ J1120

1 .046250624110635744578...≈ $\frac{1}{2}+\frac{\pi}{\sqrt{5}}\tan\frac{\pi\sqrt{5}}{2} = \sum_{k=1}^{\infty}F_{2k-1}(\zeta(2k)-1)$

.046413566780796640461...≈ $\frac{5}{6}-\frac{4\log 2}{3}+\frac{\pi}{6}\left(\cot\frac{\pi(3+i\sqrt{3})}{4}-\cot\frac{\pi(5-i\sqrt{3})}{4}\right) = \sum_{k=2}^{\infty}\frac{(-1)^k}{k^4+k}$

2 .04646050781571420028... ≈ $\sum_{k=0}^{\infty}\frac{1}{3^k-⅓}$

. 04647598131121680679... ≈ $\frac{3-2G}{8\pi} = \sum_{k=0}^{\infty}\binom{2k}{k}^2\frac{k}{16^k(2k+1)(2k+3)}$

.04655015894144553735... ≈ $\frac{1}{2}-\frac{\pi\sqrt{3}}{12} = \sum_{k=1}^{\infty}\frac{1}{(6k-1)(6k+1)}$

.04659629166272193867... ≈ $-{}_2F_1\left(2,2,\frac{1}{2},-\frac{1}{4}\right) = \sum_{k=1}^{\infty}\frac{(-1)^k(k!)^2}{(2k-2)!}$

2 .04662202447274064617... ≈ $\frac{\pi}{2}\left(\frac{\pi^2}{12}+\log^2 2\right) = \int_0^{\pi/2}(\log\sin x)^2 dx$ GR 4.224.7

= $\int_0^1\frac{\log^2 x\,dx}{\sqrt{1-x^2}}$ GR 4.261.9

.04667385288586553755... ≈ $\frac{11}{e}-4 = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}k^2}{k!(k+2)}$

.04672871759112934131... ≈ $\frac{5}{18}-\frac{\log 2}{3}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{3k+9} = \sum_{k=1}^{\infty}\frac{1}{12k^2+30k+18}$

.04674865014726989243 7... ≈ $-\csc\left((-1)^{1/4}\pi\right)\csc\left((-1)^{3/4}\pi\right)$

.04682145464761832657... ≈ $\sum_{k=1}^{\infty}\frac{H_k{}^2}{(k+1)^5}$

.04684264567028748970 2... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)}{\pi^k k^2}$

1 .04685428575652773249 4... ≈ $\sum_{k=2}^{\infty}\frac{1}{k(k-1)\log k} = \sum_{m=2}^{\infty}\sum_{k=2}^{\infty}\frac{1}{k^m\log k}$

= $\sum_{k=2}^{\infty}\int_k^{\infty}\left(\zeta(x)-1\right)dx$

1 .04685900516324149721... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)}{(k-2)!k!} = \sum_{k=1}^{\infty}\frac{1}{k}I_2\left(\frac{2}{\sqrt{k}}\right)$

.04694369064066946149 7... ≈ $\frac{\pi}{2}-1-2\arctan\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}} = \int_0^{\pi/4}\frac{\sin^3 x}{1+\cos^2 x}dx$

.04704000268662240737... ≈ $\frac{\sin 3}{3} = \binom{0}{3/\pi}$

8 .0471895621705018730... ≈ $5\log 5$

.04719755119659774615... ≈ $\frac{\pi}{3}-1 = \sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!!4^k(2k+1)} = \sum_{k=1}^{\infty}\binom{2k}{k}\frac{1}{16^k(2k+1)}$

1 .04719755119659774615... ≈ $\frac{\pi}{3} = \arccos\frac{1}{2} = \sum_{k=1}^{\infty}(-1)^{k+1}\left(\frac{1}{6k-5}-\frac{1}{6k-1}\right)$ J328

= $\sum_{k=0}^{\infty}\frac{1}{(k+1)(2k+1)(4k+1)}$

= $\sum_{k=0}^{\infty}\binom{2k}{k}\frac{1}{16^k(2k+1)}$

= $\int_0^{\infty}\frac{dx}{x^6+1} = \int_0^{\infty}\frac{x^4\,dx}{x^6+1} = \int_0^{\infty}\frac{x^{1/2}\,dx}{x^3+1}$

.04732516031123848864... ≈ $\frac{1}{4}\left(\psi(i)+\psi(-i)\right)$

1 .04740958101077889502... ≈ $\frac{\pi^4}{93} = \frac{30\zeta(4)}{31} = \sum_{k=1}^{\infty}\frac{a(k)}{k^5}$ Titchmarsh 1.2.13

.04743075727876050808 8... ≈ $\frac{1}{72}\left(2\pi^2-3\pi\sqrt{3}\right) = \sum_{k=1}^{\infty}\frac{k\zeta(2k)}{36^k}$

93648 .04747608302097371669... ≈ π^{10}

5 .04749726737091112617... ≈ $\pi^{\sqrt{2}}$

.04761864123807164496... $\approx \dfrac{\pi^2}{3} + \dfrac{7\pi^4}{90} - 9\zeta(3) = \displaystyle\int_1^\infty \frac{\log^4 x}{(x+1)^4} dx$

.047619047619$\underline{047619}$ $= \dfrac{1}{21}$

.04794309684040571460... $\approx \dfrac{5}{4} - \zeta(3) = \displaystyle\sum_{k=2}^\infty \frac{1}{k^5 - k^3} = \sum_{k=1}^\infty \frac{1}{k(k+3)^3(k+2)}$

.04821311371171887295... $\approx \log 2 - \zeta(2) + 1 = -\displaystyle\sum_{k=2}^\infty \left(\frac{1}{k^2} + \log\left(1 - \frac{1}{k^2}\right)\right) = \sum_{k=2}^\infty \frac{\zeta(2k)-1}{k}$

.04828679513998632735... $\approx \dfrac{\log 2}{4} - \dfrac{1}{8} = \displaystyle\sum_{k-1}^\infty \frac{1}{(4k+2)(4k+4)} = \sum_{k=1}^\infty \frac{1}{2^k(8k+8)} = \sum_{k=2}^\infty \frac{1}{2^k(k^2-1)k}$

.04829894367439111 7963... $\approx \dfrac{3}{2} + \dfrac{\gamma}{4} + \dfrac{19\log 2}{2} - 2\log 3 + 3\zeta'(1) = \displaystyle\sum_{k=2}^\infty \frac{(-1)^k(\zeta(k)-1)}{2^k(k+1)}$

.04831135561607547882... $\approx \dfrac{\pi^2}{18} - \dfrac{1}{2} = \displaystyle\sum_{k=1}^\infty \frac{(k!)^2}{(2k+2)!} = \sum_{k=1}^\infty \frac{4k^2+6k+2}{\binom{2k}{k}}$

.04832930767207351026... $\approx \gamma - \dfrac{1}{5} + \dfrac{1}{2}\left(\psi\left(1 - \dfrac{i}{2}\right) + \psi\left(1 + \dfrac{i}{2}\right)\right) = \displaystyle\sum_{k=2}^\infty \frac{1}{4k^3 + k}$

$= \displaystyle\sum_{k=1}^\infty (-1)^{k+1} \frac{\zeta(2k+1)-1}{4^k}$

1 .04836576852574429 81... $\approx \displaystyle\sum_{k=1}^\infty \frac{1}{(2^k-1)k^3} = \sum_{k=1}^\infty \frac{\sigma_{-3}(k)}{2^k}$

.04848228658358495813... $\approx \dfrac{1}{36}\left(\psi^{(1)}\left(\dfrac{2}{3}\right) - \psi^{(1)}\left(\dfrac{7}{6}\right)\right) = \displaystyle\sum_{k=0}^\infty \frac{(-1)^k}{(3k+4)^2}$

$= \displaystyle\int_1^\infty \frac{\log x \, dx}{x^5 + x^2}$

.04852740547184035013... $\approx \zeta(3) + 3\zeta(5) - 3\zeta(4) - \zeta(6) = \displaystyle\sum_{k=1}^\infty \frac{k^3}{(k+1)^6}$

.04855875496950706743... $\approx \dfrac{24\log 2 + 3\pi^2 - 41}{108} = -\displaystyle\int_0^1 \log(1+x)\, x^2 \log x \, dx$

.04871474579978266535... $\approx \dfrac{3\cos 1 - \sin 1}{16} = \displaystyle\sum_{k=1}^\infty (-1)^{k+1} \frac{k^4}{(2k+1)!}$

.04878973224511449673... $\approx 2\zeta(3) - \dfrac{2035}{864} = \displaystyle\int_1^\infty \frac{\log^2 x}{x^6 - x^5} dx$

1 .04884368944513054097... $\approx \zeta(2) + \zeta(3) + \dfrac{\pi}{\sqrt{3}} \tanh \dfrac{\pi\sqrt{3}}{2} = \displaystyle\sum_{k=1}^\infty \frac{1}{k^5 - k^4 + k^3}$

222 .04891426023005682... $\approx \dfrac{8\pi^3}{3} + 64\pi \log 2 = \displaystyle\int_0^\infty x^{-3/2} Li_2(-x)^2 \, dx$

.04904787316741500727... $\approx \zeta(2)+2\zeta(3)-4 = \sum_{k=1}^{\infty}\frac{k}{(k+1)(k+2)^3}$

$= \sum_{k=1}^{\infty}(-1)^{k+1}k^2\left(\zeta(k+2)-1\right)$

1 .04904787316741500727... $\approx \zeta(2)+2\zeta(3)-3 = \sum_{k=2}^{\infty}\frac{9k^2+11k+4}{k^2(k+1)^3}$

$= \sum_{k=3}^{\infty}(-1)^{k+1}k^2\left(\zeta(k)-1\right)$

3 .04904787316741500727... $\approx \zeta(2)+2\zeta(3)-1 = \sum_{k=1}^{\infty}\frac{H_{k+1}}{k^2}$

4 .04904787316741500727... $\approx \zeta(2)+2\zeta(3) = \sum_{k=2}^{\infty}\frac{k+1}{(k-1)^3} = \sum_{k=1}^{\infty}k^2\left(\zeta(k+1)-1\right)$

$= \sum_{k=1}^{\infty}\frac{H_k H_{k+1}}{k(k+1)}$

.04908738521234051935... $\approx \frac{\pi}{64} = \int_0^{\infty}\frac{x^2\,dx}{(x^4+4)^2}$

5 .04920891519142449724... $\approx \frac{49\sqrt{e}}{16} = \sum_{k=0}^{\infty}\frac{k^4}{k!2^k}$

.0493061443340548457... $\approx \frac{\log 3}{2}-\frac{1}{2} = \sum_{k=1}^{\infty}\frac{\zeta(2k+1)}{3^{2k+1}} = \sum_{k=1}^{\infty}\frac{1}{27k^3-3k}$

$= \int_2^{\infty}\frac{dx}{x^4-x^2}$

.04942814871987317922 9... $\approx \frac{\pi}{4}(-1)^{1/4}\left(\csc\pi(-1)^{1/4}+i\csc\pi(-1)^{3/4}\right) = \sum_{k=2}^{\infty}\frac{(-1)^k}{k^4+1}$

2 .04945101468877368602... $\approx \frac{1}{8}\left(7\pi^2+6\pi\sqrt{3}\log 3+27\log^2 3-12\psi^{(1)}\left(\frac{1}{3}\right)\right)$

$= \sum_{k=1}^{\infty}\frac{H_k}{k(k+1/3)}$

.04947220900479578689 9... $\approx 1-\zeta(4)+\sum_{k=2}^{\infty}\frac{k-1}{k^4\log k} = \sum_{n=1}^{\infty}\frac{1}{(n+1)!}\sum_{k=2}^{\infty}\frac{\log^n k}{k^4}$

.04952247152318661102... $\approx \frac{263}{315}-\frac{\pi}{4} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2k+9}$

.04966396370971660391... $\approx \frac{\operatorname{sech}3}{2} = \frac{1}{e^3+e^{-3}}$

.04971844555217912281... $\approx \frac{9}{128\sqrt{2}} = \sum_{k=1}^{\infty}\frac{(-1)^k k^4}{4^k(2k-1)}\binom{2k}{k}$

$$.04972865765782647927\ldots \approx 3\log\frac{3}{2}-\frac{7}{6} = \sum\frac{(-1)^{k+1}}{k^2+4k+3} = \sum_{k=1}^{\infty}(-1)^{k+1}\left(Li_k\left(\frac{1}{3}\right)-\frac{1}{3}\right)$$

$$.04974894456184419506\ldots \approx \frac{\pi}{\sqrt{2}}-\frac{\pi}{4}-2\log 2 = \int_0^1\frac{\log(1+x^4)}{(1+x)^2}dx$$

$$.04976039488335674102\ldots \approx 1-\frac{\pi^2}{6}+3\log^2\frac{1+\sqrt{5}}{2}$$

$$= \frac{1}{18}HypPFQ\left[\left\{1,\frac{3}{2},2\right\},\left\{\frac{5}{2},\frac{5}{2}\right\},-\frac{1}{4}\right] = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(k!)^2}{(2k+1)!(2k+1)}$$

$$.04978706836786394298\ldots \approx \frac{1}{e^3}$$

$$.04979482660943125711\ldots \approx 14-\frac{23}{\sqrt{e}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{(k+2)!2^k}$$

$$.04984487268884331340\ldots \approx \frac{5\log 2}{6}-\frac{19}{36} = \sum_{k=1}^{\infty}\frac{1}{2^k k(k+2)(k+3)}$$

$$.04987030359909703846\ldots \approx \sum_{k=2}^{\infty}\frac{1}{k^5\log k} = \int_5^{\infty}(\zeta(x)-1)dx$$

$$.04987831118433198057\ldots \approx \frac{\pi^2}{12}+2-4\log 2 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(k+1)^2(k+2)}$$

.0500000000000000000$\underline{0}$ = $\frac{1}{20} = \prod_{k=4}^{\infty}\left(1-\frac{9}{k^2}\right)$

= $\prod_{k=1}^{\infty}\frac{k(k+6)}{(k+3)^2}$ J1061

1 .05007513580866397878... ≈ $2^{1/2}3^{1/4}\pi^{-1/2}$ CFG F5

.05008570429831642856... ≈ $\frac{\zeta(3)}{24}$

.05023803170814624818... ≈ $\frac{51}{48}-\frac{\pi^2}{24}-\frac{\zeta(3)}{2} = \sum_{k=1}^{\infty}\frac{1}{k(k+2)^3}$

.05024420891858541301... ≈ $\zeta(2)-\log 2-\frac{3\zeta(3)}{4} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{(k+1)^3}$

2 .05030233543049796872... ≈ $\prod_{k=2}^{\infty}\left(1+\log\left(\zeta(k)\right)\right)$

.05037557702545246723... ≈ $\frac{\pi^2}{12}\log^2 2-\zeta'(2)\log 2-\frac{\zeta''(2)}{2} = \sum_{k=2}^{\infty}\frac{(-1)^k\log^2 k}{k^2}$

.05040224019441599976... ≈ $\frac{\gamma}{6}-\frac{1}{2}+\left(1+i\sqrt{3}\right)\psi\left(\frac{1-i\sqrt{3}}{4}\right)+\left(1-i\sqrt{3}\right)\psi\left(\frac{1+i\sqrt{3}}{4}\right)$

= $\sum_{k=1}^{\infty}\frac{1}{(2k+1)^3+1}$

.0504298428984897677... ≈ $8-2G-\frac{\pi}{2}-\frac{\pi^2}{4}-3\log 2 = \int_0^1 \log(1-x^4)\log x\,dx$

.05051422578989857135... ≈ $\frac{\zeta(3)-1}{4} = \int_1^{\infty}\frac{\log^2 x}{x^5-x^3}dx = \int_0^{\infty}\frac{x^2\,dx}{e^{2x}(e^{2x}-1)}$

.05066059182116888572... ≈ $\frac{1}{2\pi^2}$

1 .05069508821695116493... ≈ $\frac{1}{25}\psi^{(1)}\left(\frac{1}{5}\right) = \frac{1}{100}\left(\psi^{(1)}\left(\frac{1}{10}\right)+\psi^{(1)}\left(\frac{3}{5}\right)\right) = \sum_{k=1}^{\infty}\frac{1}{(5k-4)^2}$

= $\frac{2\pi^2}{125}-\frac{1}{16}-\frac{1}{25}\psi^{(1)}\left(\frac{9}{5}\right)$

.05072856997918023824... ≈ $I_4(2) = \sum_{k=0}^{\infty}\frac{1}{k!(k+4)!}$ LY 6.114

2 .05095654798327076040... ≈ $\frac{\sqrt{2}}{\pi}\sinh\frac{\pi}{\sqrt{2}} = \prod_{k=1}^{\infty}\left(1+\frac{1}{2k^2}\right)$

1 .05108978836728755075... ≈ $\cosh\frac{1}{\pi} = \frac{e^{1/\pi}+e^{-1/\pi}}{2} = \sum_{k=0}^{\infty}\frac{1}{(2k)!\,\pi^{2k}}$ AS 4.5.63

2 .05120180057137778842... ≈ $\sum_{k=1}^{\infty} \frac{2^{1/k}}{k^5}$

.05129762746913568550 3... ≈ $\frac{1}{128}\left(21\zeta(3) + 4\pi^2 \log 2 - 16\pi G\right)$

= $-\int_0^{\pi/4} x \log \cos x \; dx$

$.0513888888888888888\underline{8}$ = $\frac{37}{720} = \int_1^{\infty} \log\left(1 + \frac{1}{x^2}\right) \frac{dx}{x^{13}}$

.05140418958900707614... ≈ $\frac{\pi^2}{192} = \int_1^{\infty} \frac{\log x \, dx}{x^5 + x}$

.0514373600236584036... ≈ $\sqrt{\frac{\pi}{2}}\left(\frac{3\log 2}{16} + \frac{\gamma}{16} - \frac{1}{8}\right) = -\int_0^{\infty} x^2 e^{-2x^2} \log x \; dx$ GR 4.355.1

.05145657961424741951... ≈ $24 \log \frac{4}{3} - \frac{9}{4} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{3^k \, k(k+1)(k+2)}$

1 .05146222423826721205... ≈ $2\sqrt{\frac{2}{5+\sqrt{5}}} = \csc \frac{3\pi}{5}$

.05153205015748987317... ≈ $\zeta(2) - 6\zeta(3) + 6\zeta(4) - \frac{7}{8} = \sum_{k=2}^{\infty} \frac{k^2 - 4k + 1}{(k+1)^4}$

= $\sum_{k=1}^{\infty} (-1)^{k+1} k^3 \left(\zeta(k+1) - 1\right)$ Berndt 5.8.5

1 .0515363785357774114... ≈ $\sum_{k=1}^{\infty} H^{(3)}{}_k \, (\zeta(k+1) - 1)$

.05160484660389605576... ≈ $\frac{1}{20}\left(\psi^{(1)}\left(\frac{4}{5}\right) - \psi^{(1)}\left(\frac{6}{5}\right)\right) = \sum_{k=1}^{\infty} \frac{k\zeta(2k+1)}{25^k}$

.05165595124019414132... ≈ $5040 - \frac{13700}{e} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k}{k!(k+7)}$

2 .05165596774770009480... ≈ $\frac{\pi^2}{4} \csc^2 \frac{\pi}{\sqrt{2}} + \frac{\pi}{2\sqrt{2}} \cot \frac{\pi}{\sqrt{2}} - 1$

= $\sum_{k=1}^{\infty} \frac{1}{2k^4 - 2k^2 + \frac{1}{2}} = \sum_{k=1}^{\infty} \frac{k\zeta(2k+2)}{2^k}$

1 .05179979026464499973... ≈ $\frac{7\zeta(3)}{8} = \lambda(3) = \sum_{k=0}^{\infty} \frac{1}{(2k+1)^3} = \sum_{k=1}^{\infty} \frac{H_k^{\,2}}{2^k \, k}$

= $\int_0^{\pi/2} x \log \tan x \, dx$

.05181848773650995348... $\approx$ $\frac{178}{225\pi}-\frac{1}{5} = -\sum_{k=1}^{\infty}\left(\frac{(2k-1)!!}{(2k)!!}\right)^2\frac{1}{2k-5}$ J385

.0520833333333333333$\underline{3}$ $=$ $\frac{5}{96} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k(k+2)(k+4)}$

4 .05224025292035358157... $\approx$ $\frac{\pi^3}{3}-2\pi = \int_0^{\pi}\frac{x^2\sin^2 x\,dx}{1-\cos x}$

.05225229129512139667... $\approx$ $\frac{\log 3}{4}+\frac{\log 2}{3}-\frac{\pi}{4\sqrt{3}} = \sum_{k=1}^{\infty}\frac{1}{36k^2-6k} = \sum_{k=2}^{\infty}\frac{\zeta(k)}{6^k}$

$= \int_1^{\infty}\frac{dx}{x^6+x^5+x^4+x^3+x^2+x}$

2 .05226141406559553132... $\approx$ $\frac{\zeta^4(3)}{\zeta(6)} = \sum_{k=1}^{\infty}\frac{\left(\sigma_0(k)\right)^2}{k^3}$ Titchmarsh 1.7.10

.05226342542957716435... $\approx$ $e^{\cos 3}\sin(\sin 3) = \sum_{k=1}^{\infty}\frac{\sin 2k}{k!}$

.05230604277964325414... $\approx$ $\frac{si(1)-\sin 1}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(2k+1)!(2k+1)}$

.05238520628289183021... $\approx$ $\frac{1}{18}HypPFQ\left[\left\{1,\frac{3}{2}\right\},\left\{\frac{5}{2},\frac{5}{2}\right\},-\frac{1}{4}\right] = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k!}{(2k+1)!(2k+1)}$

.05239569649125595197... $\approx$ $\frac{1}{e^3-1} = \sum_{k=1}^{\infty}\frac{1}{e^{3k}} = \sum_{k=1}^{\infty}\frac{1}{\cosh(3k)+\sinh(3k)}$

.05256980729002050897... $\approx$ $\frac{5-6\log 2}{16} = \int_2^{\infty}\log\frac{x}{x-1}\cdot\frac{dx}{x^3}$

4 .05257766636985646658... $\approx$ $\sum_{k=1}^{\infty}\frac{k}{2^k(1-2^{-k})^2} = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{jk}{2^{jk}} = \sum_{k=1}^{\infty}\frac{k\sigma_0(k)}{2^k}$

.$\underline{052631578947368421}$ $=$ $\frac{1}{19}$

.05268025782891315061... $\approx$ $\frac{1}{2}\log\frac{10}{9} = \operatorname{arctanh}\frac{1}{19} = \sum_{k=0}^{\infty}\frac{1}{19^{2k+1}(2k+1)}$ K148

.05274561989207116079... $\approx$ $\frac{\pi}{8}-\frac{1}{6}-\frac{\log 2}{4} = \sum_{k=1}^{\infty}\frac{1}{(4k+2)(4k+3)}$

.052878790640516702047... $\approx$ $\frac{1}{4\pi^2}\left(35\zeta(3)-16\pi G-2\pi^2\log 2\right) = \sum_{k=1}^{\infty}\frac{\zeta(2k)}{2^{4k}(k+1)}$

.05290718550287165801... $\approx$ $\frac{\gamma}{3}+\frac{\log 3}{12}-\frac{\log 2}{3} = -\sum_{k=1}^{\infty}\frac{\psi(2k-1)}{4^k}$

$$.05296102778655728550\ldots \approx 2\log 2 - \frac{4}{3} = \sum_{k=2}^{\infty}\frac{1}{4k^3-k} = \sum_{k=1}^{\infty}\frac{\zeta(2k+1)-1}{4^k}$$

$$= \sum_{k=1}^{\infty}\frac{1}{2^k(4k+12)}$$

$$.05296717050275408242\ldots \approx 1-\frac{7\pi^4}{720} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+2)^4}$$

$$.05298055208215036543\ldots \approx \frac{\pi^3}{32} - G = \int_1^{\infty}\frac{\log^2 x}{(x^2+1)^2}dx$$

$$1 \quad .05302928754551488456 2\ldots \approx \frac{\pi^2}{16\left(2-\sqrt{2}\right)} = \frac{\pi^2}{64}\csc^2\frac{\pi}{8} = \sum_{k=1}^{\infty}\left(\frac{1}{(8k-1)^2}+\frac{1}{(8k-7)^2}\right)$$

.05305164769729844526... $\approx \dfrac{1}{6\pi}$

6 .0530651531362397781... $\approx 2 + 3\sqrt{2}\arcsin\sqrt{\dfrac{2}{3}} \;=\; \sum_{k=1}^{\infty}\dfrac{(2k)!!2^{k}}{(2k-1)!!\,3^{k}}$

.0530853547393914281... $\approx \dfrac{3\zeta(3)}{2} - \dfrac{7}{4} \;=\; 2\sum_{k=2}^{\infty}\dfrac{(-1)^{k}}{(k+1)^{3}}$

$= \int_{1}^{\infty}\dfrac{\log^{2} x\,dx}{x^{4}+x^{3}} \;=\; \int_{0}^{1}\dfrac{x^{2}\log^{2} x\,dx}{1+x}$

1 .053183503816656749... $\approx \sum_{j=3}^{\infty}\sum_{k=1}^{\infty}\left(\zeta(jk-1)-1\right)$

1 .053252407623515317... $\approx 8\log 2 - 2\zeta(2) - \zeta(3) \;=\; \sum_{k=1}^{\infty}\dfrac{1}{2k^{4}-k^{3}} \;=\; \sum_{k=1}^{\infty}\dfrac{\zeta(k+3)}{2^{k}}$

.05330017034343276914... $\approx \dfrac{\gamma}{6} + \dfrac{\log 2}{3} + \dfrac{1-i\sqrt{3}}{12}\psi\left(\dfrac{7-i\sqrt{3}}{4}\right) - \dfrac{1+i\sqrt{3}}{12}\psi\left(\dfrac{7+i\sqrt{3}}{4}\right)$

$= \sum_{k=1}^{\infty}\dfrac{1}{(2k-1)^{3}-1}$

2 .05339577633806633883... $\approx 2\log\pi - \log 2 + 2\log\csc\dfrac{\pi}{\sqrt{2}} \;=\; \sum_{k=1}^{\infty}\dfrac{\zeta(2k)}{2^{k-1}k}$

$= -2\sum_{k=1}^{\infty}\log\left(1-\dfrac{1}{2k^{2}}\right)$

.053500905006153397... $\approx \sum_{k=0}^{\infty}\dfrac{1}{(k+4)!-k!}$

.05352816631052393921... $\approx \dfrac{1}{100}\left(\psi^{(1)}\left(\dfrac{2}{5}\right)-\psi^{(1)}\left(\dfrac{9}{10}\right)\right) \;=\; \int_{1}^{\infty}\dfrac{\log x}{x^{5}+1}dx$

.05357600055545395029... $\approx \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\left(\zeta(jk+5)-1\right)$

.05358069953485240581... $\approx \sum_{k=1}^{\infty}\dfrac{1}{(k+3)!-k!}$

.05361522790552251769... $\approx \dfrac{\pi}{128}\csc^{2}\dfrac{\pi}{4\sqrt{2}}\left(\pi - 2\sqrt{2}\sin\dfrac{\pi}{2\sqrt{2}}\right)$

.05362304129893233991... $\approx \sum_{k=2}^{\infty}\dfrac{\left(\zeta(k)-1\right)^{2}}{k^{3}}$

.05362963066204526106... $\approx \sum_{k=0}^{\infty}\dfrac{k!!}{(k+4)!}$

2 .05363698714066007036... $\approx \sum_{k=1}^{\infty}\dfrac{1}{F_{k}+1}$

5 .0536379309867527384... $\approx \int_{2}^{\infty}\left(x(x-1)\left(\zeta(x)-1\right)\right)dx$

1 .0536825183947192278... ≈ $(1-i)\Gamma\left(\frac{5}{4}\right)-\frac{1}{4}Ei\left(\frac{3}{4},-\frac{1}{4}\right) = {}_1F_1\left(\frac{1}{4},\frac{5}{4},\frac{1}{4}\right) = \sum_{k=0}^{\infty}\frac{1}{k!4^k(4k+1)}$

.0539133428987253767 8... ≈ $\frac{\pi^2-7\zeta(3)}{16}-\frac{1}{27} = \sum_{k=2}^{\infty}\frac{k}{(2k+1)^3}$

3 $.05\underline{395433027011974380}$ = $\frac{43867}{14364} = -\zeta(-17)$

.0539932027501803781... ≈ $\frac{\log 3}{8}-\frac{1}{12} = \int_1^{\infty}\frac{\log x}{(x+2)^3}dx$

1 .0540988310811232864 8... ≈ $\frac{3}{4}\left(1+\log\frac{3}{2}\right) = \sum_{k=1}^{\infty}\frac{kH_k}{3^k}$

1 .0541875185094090759 6... ≈ $-\gamma^2\left(1+\frac{\psi(1-\gamma)}{\gamma}\right) = \sum_{k=2}^{\infty}\gamma^k\zeta(k)$

$.05427579365\underline{0793650}$ = $\frac{5471}{100800} = \sum_{k=1}^{\infty}\frac{1}{k(k+3)(k+8)}$

.05429885502603840429 4...≈ $\frac{\pi}{3\sqrt{3}}+\frac{\pi^2}{18}-\log 3 = \sum_{k=2}^{\infty}\frac{\zeta(k)-\zeta(k+1)}{3^k}$

.05461287175797359580 5...≈ $\frac{\pi^2\log\pi}{4}-\frac{7\zeta(3)}{8\pi^2}-\frac{1}{8} = \sum_{k=1}^{\infty}\frac{\zeta(2k)}{2k(2k+2)4^k}$

.054617753653219488... ≈ $\frac{1}{e}-\log\left(1+\frac{1}{e}\right) = \int_1^{\infty}\frac{dx}{e^x(e^x+1)}$

6 .0546468656033535950... ≈ $3+\frac{\pi}{\sqrt{3}}+2\log 2+\frac{1}{12}\left(\psi^{(1)}\left(\frac{7}{6}\right)-\psi^{(1)}\left(\frac{2}{3}\right)\right)$

= $-\int_0^1\log\left(1+\frac{1}{x^3}\right)\log x\,dx$

1 .0548671006148205789 6... ≈ $\frac{\pi}{\sqrt{\pi^2-1}} = 1+\sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!!\,\pi^{2k}}$ J166

.0549069236133059880 4... ≈ $\frac{\log 2}{2}-\frac{7}{24} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2k+8}$

1 .0549112574742170912 1... ≈ $\zeta(5)\zeta(6) = \sum_{k=1}^{\infty}\frac{\sigma_1(k)}{k^6}$ HW Thm. 290

1 .0550188771985238630 6... ≈ $5Li_2\left(\frac{1}{5}\right) = \sum_{k=0}^{\infty}\frac{1}{5^k(k+1)^2} = \sum_{k=1}^{\infty}\frac{H^{(2)}{}_k}{5^k}$

.055098028649865968 3... ≈ $28\log\frac{4}{3}-8 = \sum_{k=0}^{\infty}\frac{k}{4^k(k+1)(k+3)}$

.05515890003816289835... ≈ $\frac{\log 2}{4\pi} = \int_0^1 \frac{1}{\pi^2 + 4\log^2 x}\frac{dx}{1+x^2}$ GR 4.282.7

.0551766951906664843... ≈ $\sum_{k=2}^{\infty} \frac{\zeta(k)-1}{k!(k+1)!} = \sum_{k=2}^{\infty}\left(\sqrt{k}\, I_1\left(\frac{2}{k}\right) - 1 - \frac{1}{2k}\right)$

1 .05522743812929880804... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(2k)}{(k+1)!} = \sum_{k=1}^{\infty}\left(k^2\left(e^{1/k^2}-1\right)-1\right)$

.05524271728019902534... ≈ $\frac{5}{64\sqrt{2}} = \sum_{k=1}^{\infty} \frac{(-1)^k (2k)!\, k^3}{(k!)^2 4^k}$

.05535964546107901888... ≈ $\frac{\pi^2 - 6\zeta(3)}{48} = \sum_{k=1}^{\infty} \frac{k}{(2k+2)^3}$

9 .0553851381374166266... ≈ $\sqrt{82}$

1 .05538953448087917898... ≈ $\frac{\pi\sqrt{2}}{2}\coth \pi\sqrt{2} - \frac{7}{6} = \sum_{k=2}^{\infty} \frac{2}{k^2+2} = \sum_{k=1}^{\infty} (-1)^{k+1} 2^k \left(\zeta(2k)-1\right)$

2 .05544517187371713576... ≈ $\frac{3\pi^3}{32\sqrt{2}} = \int_0^{\infty} \frac{\log^2 x}{x^4+1}dx = \int_0^{\infty} \frac{\log^2 x}{x^2 + x^{-2}}dx$

= $\int_0^1 \frac{x^2+1}{x^4+1}\log^2 x\, dx$ GR 4.61.7

.0554563517369943079... ≈ $2 - \frac{11}{4\sqrt{2}} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^2}{4^k (k+1)}\binom{2k}{k}$

.0555555555555555555$\underline{5}$ = $\frac{1}{18}$

1 .05563493972360084358... ≈ $\frac{1}{2}\left(\cos\sqrt{2} - \cosh\sqrt{2} + \sqrt{2}\sin\sqrt{2} + \sqrt{2}\sinh\sqrt{2}\right)$

= $\sum_{k=0}^{\infty} \frac{4^k}{(4k)!(2k+1)}$

.05566823333919918611... ≈ $\frac{1}{18} + \frac{2\pi}{9}\operatorname{csch} 3\pi = \sum_{k=0}^{\infty} \frac{(-1)^k}{k^2+9}$

.055785887828552438942... ≈ $\frac{1}{4}\log\frac{5}{4} = \sum_{n=1}^{\infty}\left(-\frac{1}{4} + \sum_{k=1}^{\infty} \frac{H^{(n)}{}_k}{5^k}\right)$

2 .05583811130112521475... ≈ $\sum_{k=1}^{\infty} \frac{H_k}{2^k - 1}$

.0558656982586571151... ≈ $\sum_{k=2}^{\infty} \frac{1}{4^k \zeta(k)}$

38 .05594559842663329504... ≈ $14e$

1 .05607186782993928953... ≈ $\cosh\frac{1}{3}=\frac{1}{2}\left(e^{1/3}+e^{-1/3}\right)=\sum_{k=0}^{\infty}\frac{1}{(2k)!9^k}$

2 .05616758356028304559... ≈ $\frac{5\pi^2}{24} = \int_0^\infty \frac{\log(1+x^2)}{x(1+x)}dx$

.05633047284042820302... ≈ $\sum_{k=2}^{\infty}\frac{k-1}{k^5\log k} = \int_4^5\left(\zeta(x)-1\right)dx$

.05639880925494618472... ≈ $1-8\log^2 2+8\log 2\log 5-2\log^2 5-4Li_2\frac{1}{5} = 1+4Li_2\left(-\frac{1}{4}\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{4^k(k+1)^2}$

= $-\int_0^1\frac{x\log x}{x+4}dx$

.05650534508283039223... ≈ $\frac{\pi}{e^4+1} = \int_0^\infty\frac{x\tan x}{x^2+4}dx$ GR 3.749.1

1 .05661186908589277541... ≈ $\frac{3}{2}\log\frac{3}{2}+\frac{1}{2}\log^2\frac{3}{2}+Li_2\left(\frac{1}{3}\right) = \frac{3}{2}\log\frac{3}{2}-Li_2\left(-\frac{1}{2}\right)$

= $\sum_{k=1}^{\infty}\frac{H_k(k+1)}{3^k k}$

2 .056720205991584933132...≈ $1+\frac{i}{2}-\frac{i\pi}{2}+\frac{i\pi^2}{12}+\log 2-\frac{i}{2}Li_2(e^{2i}) = -\int_0^1\log(x\sin x)\,dx$

73 .05681827550182792733... ≈ $\frac{3\pi^4}{4}$

.05683697542290869392... ≈ $\frac{\pi}{4}\coth\pi+\frac{\pi^2}{4}\operatorname{csch}^2\pi-\frac{3}{4} = \sum_{k=2}^{\infty}\frac{1}{(k^2+1)^2}$

.05685281944005469058... ≈ $\frac{3}{4}-\log 2 = \sum_{k=1}^{\infty}\frac{(-1)^k}{(k+1)(k+3)} = \sum_{k=2}^{\infty}\frac{(2k-3)!}{(2k)!}$

= $\sum_{k=1}^{\infty}\frac{1}{2k(2k+1)(2k+2)}$ J249

= $\sum_{k=1}^{\infty}\frac{k-1}{k}\left(\zeta(2k)-1\right)$

.05696447062846142723... ≈ $3-\frac{8}{e} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{(k+2)!}$

.05699273625446670926... ≈ $\frac{1}{8}-\frac{\pi}{8}\operatorname{csch}\frac{\pi}{2}\operatorname{sech}\frac{\pi}{2} = \sum_{k=2}^{\infty}\frac{(-1)^k}{k^4-1}$

.05712283631136784893... ≈ $\frac{1}{2}-\zeta(2)+\zeta(3) = \sum_{k=2}^{\infty}\frac{1}{k^4+k^3}$

$= \sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(k+3)-1\right) = \sum_{k=1}^{\infty} dz(k,2)$

.05719697698347550546... $\approx \frac{7\pi^4-675}{120} = \int_1^{\infty}\frac{\log^3 x}{x^4+x^3}$

.05724784963607618844... $\approx \frac{G}{16} = \int_0^{\infty}\frac{x\,dx}{e^{4x}+e^{-4x}}$

1 .05725087537572851457... $\approx \frac{1}{2}\left(Ei(1)-Ei(-1)\right) = \sum_{k=0}^{\infty}\frac{1}{(2k+1)!(2k+1)}$ GR 8.432.1

$= SinhIntegral(1) = \int_0^1 \frac{\sinh x}{x}dx = \int_1^{\infty}\sinh\left(\frac{1}{x}\right)\frac{dx}{x}$

5 .0572927945177198187... $\approx \frac{7\zeta^2(3)}{2} = \sum_{k=1}^{\infty}\frac{r(n)}{n^3}$

.057336901410945468 7... $\approx \frac{\zeta(3)}{4} + 2\log 2 + \frac{7\pi^4}{720} + \frac{\pi^2}{12} - 4 = \sum_{k=2}^{\infty}\frac{(-1)^k}{k^5-k^4}$

.05754027657865082526... $\approx \frac{\pi}{e^4} = \int_{-\infty}^{\infty}\frac{\cos 4x}{x^2+1}dx$ GR 3.749.2

.0576131687242798354... $\approx \frac{14}{243} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^5}{2^k}$

1 .05764667095646375314... $\approx \frac{1}{2}\left(Li_3\left(e^{i/3}\right)+Li_3\left(e^{-i/3}\right)\right) = \sum_{k=1}^{\infty}\frac{\cos k/3}{k^3}$

.05770877464577086297... $\approx \frac{\log^4 2}{4} = \int_0^1\frac{\log^2(1+x)}{1+x}dx$

17 .05777785336906047201... $\approx I_0(2\sqrt{5}) = {}_0F_1\left(;1;5\right) = \sum_{k=0}^{\infty}\frac{5^k}{(k!)^2}$

1 .0578799592559688775... $\approx \frac{7\zeta(6)}{4} - \frac{\zeta(3)^2}{2} = \sum_{k=1}^{\infty}\frac{H_k}{k^5}$ Berndt 9.9.5

.05796575782920622441... $\approx \frac{\log 2}{2} - \frac{\gamma}{2} = \sum_{k=1}^{\infty}\frac{\zeta(2k+1)}{2^{2k+1}(2k+1)} = \sum_{k=1}^{\infty}\left(\operatorname{arctanh}\frac{1}{2k} - \frac{1}{2k}\right)$

$= \sum_{k=1}^{\infty}(-1)^{k+1}\psi(k+1)$

$= \int_1^{\infty}\frac{\log\log x\,dx}{(1+x)^2}$ GR 4.325.3

.05800856534682915479... $\approx \sum_{k=2}^{\infty}\frac{\zeta(k)}{k^5} = \zeta(5) - 1 + \sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k^5} = \sum_{k=1}^{\infty}\left(Li_5\left(\frac{1}{k}\right)-\frac{1}{k}\right)$

.05807820472492238243... $\approx \sqrt{\frac{3}{2}} - \frac{7}{6} = \sum_{k=2}^{\infty}\frac{(2k-1)!!}{(2k)!\,3^k}$

.058086078279268574188... ≈ $1-\frac{1}{\sqrt{2}}\left(1+2\operatorname{arccoth}\sqrt{2}-i\arctan\left(1-\frac{1+i}{\sqrt{2}}\right)+i\arctan\left(1-\frac{1-i}{\sqrt{2}}\right)\right)$

= $\int_0^{\pi/4}\frac{\sin^3 x}{1+\sin^2 x}dx$

.05815226940437519841... ≈ $\frac{3\zeta(3)}{2\pi^3}$ Berndt Ch. 5

2 .0581944359406266129... ≈ $\sum_{k=1}^{\infty}\frac{\sigma_{-1}(k)}{k!}$

.058365351918698456359... ≈ $2Li_2\left(-\frac{1}{2}\right)+4\log\frac{3}{2}-\frac{2}{3} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{2^k(k+1)^2}$

1 .05843511582378211213... ≈ ${}_2F_1\left(1,\frac{1}{4},\frac{5}{4},\frac{1}{4}\right) = \sum_{k=0}^{\infty}\frac{1}{4^k(4k+1)}$

.05856382356485817586... ≈ $-\sum_{k=1}^{\infty}\frac{\mu(2k-1)}{2^k+1}$

.05861382625420994135... ≈ $\frac{\pi}{e^4-1} = \int_0^{\infty}\frac{x\cot x}{x^2+4}dx$ GR 3.749.2

4 .05871212641676821819... ≈ $\frac{\pi^4}{24}$, volume of unit 4-sphere

.0588235294117647 = $\frac{1}{17} = \sin^2\arctan\frac{1}{4} = \sum_{k=0}^{\infty}(-1)^k 2^{-4(k+1)} = \int_0^{\infty}\frac{\cos 4x}{e^x}dx$

.05882694094725862925... ≈ $\frac{\zeta'(4)}{\zeta^2(4)} = \sum_{k=1}^{\infty}\frac{\mu(k)\log k}{k^4}$

.05886052973059857964... ≈ $\frac{1}{18}+\frac{1}{9}\log\frac{2}{3}+\frac{1}{9\sqrt{2}}\arctan\frac{1}{\sqrt{2}} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{kH_{2k-1}}{2^k}$

.05889151782819172727... ≈ $\frac{1}{2}\log\frac{9}{8} = \operatorname{arctanh}\frac{1}{17} = \sum_{k=0}^{\infty}\frac{1}{17^{2k+1}(2k+1)}$

1 .058953464852310349... ≈ $\sum_{k=1}^{\infty}\frac{H_k\sin k}{k}$

.05897703250591215633... ≈ $\log\Gamma\left(\frac{3}{4}\right)-\frac{\gamma}{4} = \sum_{k=2}^{\infty}\frac{\zeta(k)}{4^k k} = -\sum_{k=1}^{\infty}\left(\frac{1}{4k}+\log\left(1-\frac{1}{4k}\right)\right)$

3 .05898844426198225439... ≈ $3\zeta(5)+\left(4-\frac{\pi^2}{6}\right)\zeta(3)-6\log^2 2 = \sum_{k=1}^{\infty}\frac{H_k(k+1)}{2k+1}\left(\frac{1}{k^2}+\frac{1}{k^3}+\frac{1}{k^4}\right)$

.05918955184357786985... ≈ $\frac{7\pi^4}{11520} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k)^4} = -\mathrm{Re}\left\{Li_4(i)\right\}$

.05936574836539082147... ≈ $\frac{\pi}{8} - \frac{1}{3} = \sum_{k=1}^{\infty}\frac{1}{(2k+1)(2k+1)(2k+3)}$ J240, J627

= $\sum_{k=1}^{\infty}\frac{1}{16k^2+16k+3}$

1 .05940740534357614454... ≈ $\sqrt{e} - 2 + \sqrt{\frac{e\pi}{2}}erf\left(\frac{1}{\sqrt{2}}\right) = \sum_{k=2}^{\infty}\frac{1}{k!!}$

3 .05940740534357614454... ≈ $\sqrt{e} + \sqrt{\frac{e\pi}{2}}erf\left(\frac{1}{\sqrt{2}}\right) = \sum_{k=0}^{\infty}\frac{1}{k!!} = \sum_{k=1}^{\infty}\frac{k!!}{k!}$

4 .05940740534357614454... ≈ $\sqrt{e} + 1 + \sqrt{\frac{e\pi}{2}}erf\left(\frac{1}{\sqrt{2}}\right) = \sum_{k=0}^{\infty}\frac{k}{k!!}$

.059568942236683030025... ≈ $\frac{\gamma-1}{\pi} - \frac{1}{\pi}\psi\left(2+\frac{1}{\pi}\right) = \sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)-1}{\pi^k}$

1 .059638450439128955916... ≈ $\frac{\pi\sqrt{3}}{4} + \frac{9\log 3}{4} - 4\log 2 = \sum_{k=0}^{\infty}\frac{1}{(k+1)(2k+1)(3k+1)}$

.059705906160195358363... ≈ $\frac{\log 2}{4}\zeta(3) + \frac{3}{4}\zeta'(3) = \sum_{k=1}^{\infty}\frac{(-1)^k}{k^3}\log k$

.059722222222222222222$\underline{2}$ = $\frac{43}{720} = \sum_{k=2}^{\infty}\frac{1}{k(k+1)(k+4)}$

.05973244312050459885... ≈ $\frac{\pi^2}{16}(1+2\log 2-2\log\pi) = -\int_0^{\pi/2}x\log x\,dx$

.05977030443557374251... ≈ $10 - \zeta(2) - \frac{7\zeta(4)}{8} - 8\log 2 - \frac{3\zeta(3)}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2(k+1)^4}$

1 .05997431887... ≈ j_4 J311

$.0600000000000000000\underline{0} = \frac{3}{50}$

4 $.06015693855740995108\ldots \approx e\sqrt{\pi}\, erf\, 1 = \sum_{k=1}^{\infty} \frac{(2k)!!}{(2k-1)!!\,k!}$

$= \int_0^1 \frac{e^x\, dx}{\sqrt{1-x}}$

5 $.06015693855740995108\ldots \approx 1 + e\sqrt{\pi}\, erf\, 1 = -\frac{e}{2}\Gamma\left(-\frac{1}{2},0,1\right) = \sum_{k=1}^{\infty} \frac{k!4^k}{(2k)!}$

$.06017283993600197841\ldots \approx \frac{5}{18} - \frac{1}{2\sqrt{2}}\arctan\frac{1}{\sqrt{2}} = \sum_{k=0}^{\infty} \frac{(-1)^{k+1}k^2}{2^k(2k+1)}$

$.06017573696145124717\ldots \approx \frac{2123}{35280} = \sum_{k=1}^{\infty} \frac{1}{k(k+3)(k+7)}$

$.06034631805191526430\ldots \approx \frac{5\sin 1 - 6\cos 1}{16} = \sum_{k=0}^{\infty} (-1)^k \frac{k^4}{(2k)!}$

$.06041521303050999709\ldots \approx 2 + 6\log\frac{2}{3} + 3\log^2\frac{3}{2} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{H_k}{2^k(k+1)(k+2)}$

1 $.06042024132807165438\ldots \approx \sum_{c=2}^{\infty}\sum_{k=1}^{\infty} \frac{\zeta(ck)-1}{k} = \sum_{c=2}^{\infty}\sum_{n=2}^{\infty} \left(\log\left(\frac{1}{1-n^{-c}}\right) - \frac{1}{n^c}\right)$

$.06053729682182486115\ldots \approx \frac{112 - 9\pi^2 - 24\log 2}{108} = \int_0^1 x^2 \log(1-x^2)\log x\, dx$

$.06055913414121058628\ldots \approx \frac{\pi^3}{512}$

$.06057422948630573216\ldots \approx \sum_{k=1}^{\infty} \frac{|\mu(k)|\log k}{k \cdot \phi(k)} - \frac{315\gamma}{2\pi^4}\zeta(3)$

5 $.06058989694951351915\ldots \approx \sum_{k=1}^{\infty} \frac{k\sigma_0(k)}{2^k - 1}$

1 $.06066017177982128660\ldots \approx \frac{3}{2\sqrt{2}} = 1 + \sum_{k=1}^{\infty} \frac{(2k-1)!!}{(2k)!!9^k}$ CFG B4, J166

$= \prod_{k=0}^{\infty} \left(1 + \frac{(-1)^k}{2k+5}\right)$

$.06093021621632876395 6\ldots \approx 2\log\frac{3}{2} - \frac{3}{4} = \sum_{k=1}^{\infty} \frac{1}{3^k k(k+1)(k+2)}$

$= \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^k(2k+1)}$

$.06097350783144797233\ldots \approx \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\zeta(2k+1)-1}{2k+1} = \sum_{k=2}^{\infty} \left(\frac{1}{k} - \arctan\frac{1}{k}\right)$

.061208719054813641942... ≈ $\sin^2 \frac{1}{4} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k)!\, 2^{2k+1}}$

.061385355812633918024... ≈ $\frac{5}{27} - \frac{4\pi}{81\sqrt{3}} + \frac{4\log 2}{81} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(3k+1)^2 (3k+4)^2}$

.06147271494065079891... ≈ $\frac{4\log 2}{3} - 1 + \frac{\pi}{3} \operatorname{sech} \frac{\pi\sqrt{3}}{2}$

= $\sum_{k=2}^{\infty} \frac{(-1)^k}{k^4 - k}$

.06154053831284500882... ≈ $\frac{1}{3} + \frac{9\pi + 6\pi\sqrt{2} - 24\sqrt{2} - 40}{36 + 24\sqrt{2}} = \int_0^{\pi/4} \frac{\sin^2 x}{(1+\sin x)^2} dx$

.06158863958792450009... ≈ $\frac{1}{4}\left(\pi - \frac{304}{105}\right) = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+9}$

1 .06160803906011000253... ≈ $\frac{\sqrt{2}}{16}\left(\psi^{(1)}\left(1 - \frac{1}{\sqrt{2}}\right) - \psi^{(1)}\left(1 + \frac{1}{\sqrt{2}}\right)\right) = \sum_{k=1}^{\infty} \frac{k}{(2k^2-1)^2}$

= $\sum_{k=1}^{\infty} \frac{k\zeta(2k+1)}{2^{k+1}}$

.06160974642842427642... ≈ $-\frac{3}{4} - \frac{\pi}{4 \cdot 2^{3/4}}\left(\csc\left(\pi 2^{1/4}\right) + \operatorname{csch}\left(\pi 2^{1/4}\right)\right)$

= $\sum_{k=2}^{\infty} \frac{(-1)^k}{k^4 - 2}$

.06163735046020626998... ≈ $9 - 8\cos\frac{1}{2} - 4\sin\frac{1}{2} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k)!\, 4^k (k+1)}$

$.061728395\underline{061728395}$ = $\frac{5}{81} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^4}{5^k}$

.06173140432470719352... ≈ $\sum_{k=0}^{\infty} \frac{(-1)^k}{(k!)^4} = HypPFQ[\{\},\{1,1,1\},-1]$

.06177135864462191091... ≈ $35 - 40\log 2 - 6\zeta(3) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^4 (k+1)^4}$

.06196773353931867016... ≈ $\frac{2}{25}\sqrt{\frac{3}{5}} = \sum_{k=1}^{\infty} (-1)^{k+1} \binom{2k}{k} \frac{k^2}{6^k}$

1 .06202877524308307692... ≈ $\operatorname{arccsch} \frac{\pi}{4}$

.06210770748126123903... ≈ $2 - \frac{\pi^3}{16} = \int_1^{\infty} \frac{\log^2 x}{x^4 + x^2} dx = \int_0^1 \frac{x^2 \log^2 x}{x^2 + 1} dx$

8 .0622577482985496524... ≈ $\sqrt{65}$

.062346338241142771091... ≈ $\frac{1}{16} - \frac{\pi}{4\sqrt{2}} \operatorname{csch} 2\pi\sqrt{2} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2 + 8}$

.06238347847451615361... ≈ $\sum_{k=2}^{\infty}\left(\zeta(k)-1\right)\left(\zeta(k+2)-1\right)$

.062400643626808666895... ≈ $\frac{9450}{\pi^{12}}\left(210\zeta'(8)-\pi^4\zeta'(4)\right) = \sum_{k=2}^{\infty}\frac{|\mu(k)|\log k}{k^4}$

.0625000000000000000$\underline{0}$ = $\frac{1}{16} = \sum_{k=2}^{\infty}\frac{1}{k^5(1-k^{-2})^2} = \sum_{k=1}^{\infty}(k-1)\left(\zeta(2k+1)-1\right)$

= $\sum_{k=1}^{\infty}\frac{k^3}{e^{\pi k}+(-1)^k}$

= $\int_0^{\infty}\frac{x\,dx}{(x^2+2)^3}$

= $\int_1^{\infty}\log\left(1+\frac{1}{x^4}\right)\frac{dx}{x^9}$

= $\int_0^{\infty}\frac{\cos 2\pi x}{e^{2\pi\sqrt{x}}-1}dx$ Prud. 2.5.38.9

1 .06250000000013113727... ≈ $\sum_{k=1}^{\infty}\frac{1}{k^{k^k}}$

.06251525878906250000... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^{2^{2^k}}}$

.062515258789062500054... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^{4^k}}$

1 .06269354038321393057... ≈ $\frac{\pi^2}{12}+\frac{\log^2 2}{2} = \sum_{k=1}^{\infty}\frac{H_k^{\,2}}{2^{k+1}}$

.06269871149984998792... ≈ $\zeta(6)+\frac{11\pi^4}{90}+\frac{28\pi^2}{3}-84-14\zeta(3)-4\zeta(5)$

= $\sum_{k=1}^{\infty}\frac{1}{k^6(k+1)^4}$

1 .062714148932708557698... ≈ $\int_1^2\zeta(x)\log x\,dx$

.062783030996353104594... ≈ $\int_0^1\log(1+x^{12})\,dx$

4 .0628229009806368496... ≈ $\frac{3}{2}+\log 2+\frac{3\sqrt{2}}{4}\log\frac{2+\sqrt{2}}{2-\sqrt{2}} = \sum_{k=1}^{\infty}\frac{kH_{2k-1}}{2^k}$

.06290376269772511922... ≈ $56-\frac{35\pi^2}{6}-\frac{\pi^4}{18}+5\zeta(3)+\zeta(5) = \sum_{k=1}^{\infty}\frac{1}{k^5(k+1)^4}$

.06310625275909953582... ≈ $\frac{\log^2 2}{2}+\gamma\log 2-\gamma = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}\psi(k+1)}{k+1}$

	.0632539690102041...	≈	$\sum_{k=1}^{\infty}\frac{(-1)^{k+1}\log k}{k(k+1)}$	
	.063277280080096903577...	≈	$\frac{\pi}{32}+\frac{G}{4}-\frac{19}{72} = -\int_0^1 \mathrm{x}^3 \operatorname{arccot} x \log x \, dx$	
	.06332780438680511248...	≈	$\frac{\pi^4}{45}+\frac{10\pi^2}{3}-35 = \sum_{k=1}^{\infty}\frac{1}{k^4(k+1)^4}$	
1	.06337350032394316468...	≈	$8+4\sinh\frac{1}{2}-8\cosh\frac{1}{2} = \sum_{k=0}^{\infty}\frac{1}{(2k)!4^k(k+1)}$	
	.0634173769752620034...	≈	$\frac{\pi^4}{1536} = \frac{1}{1536}\psi^{(3)}\left(\frac{1}{2}\right) = \sum_{k=0}^{\infty}\frac{1}{(4k+2)^4}$	
	.0634363430819095293...	≈	$\frac{11}{2}-2e = \sum_{k=1}^{\infty}\frac{k}{(k+3)!} = \sum_{k=0}^{\infty}\frac{1}{k!(k+2)(k+3)(k+4)}$	
1	.06345983311722793077...	≈	$\frac{8G}{3}-\frac{\pi}{3}\log\left(2+\sqrt{3}\right) = \sum_{k=0}^{\infty}\frac{1}{\binom{2k}{k}(2k+1)^2}$	Berndt Ch. 9
	.06348038092346547368...	≈	$\sum_{k=1}^{\infty}\frac{1}{2^{k(k+3)}}$	
1	.06348337074132351926...	≈	$I_0\left(\frac{1}{2}\right) = \sum_{k=0}^{\infty}\frac{1}{(k!)^2 16^k}$	
		=	$\sqrt{e\pi}\sum_{k=0}^{\infty}\frac{(k-\frac{1}{2})!}{(k!)^2}$	
	.06364760900080611621...	≈	$\arctan\frac{1}{2}-\frac{2}{5} = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}k}{4^k(2k+1)}$	
	.06366197723675813431...	≈	$\frac{1}{5\pi}$	
	.0636697649553711265...	≈	$\frac{1}{\zeta(4)}\sum_{k=1}^{\infty}\frac{\log k}{k^4} = -\frac{\zeta'(4)}{\zeta(4)} = \sum_{p\ prime}\frac{\log p}{p^4-1}$	
		=	$\sum_{k=1}^{\infty}\frac{\Lambda(k)}{k^4}$	
	.063827327695777400598...	≈	$2\log 2-\frac{\pi^2}{12}-\frac{1}{2} = \sum_{k=2}^{\infty}\frac{(-1)^k}{k^3+k^2}$	
1	.06387242824454660016...	≈	$\frac{21}{2\pi^2} = \frac{\zeta(4)}{\zeta(6)}$	
3	.06387540935871740999...	≈	$\psi^{(1)}\left(\frac{2}{3}\right)$	
12	.06387540935871740999...	≈	$\psi^{(1)}\left(-\frac{1}{3}\right)$	

1 .06388710376241701175... $\approx \displaystyle\sum_{k=1}^{\infty} \frac{1}{k^{2k}}$

.0639129291353270933... $\approx \displaystyle\frac{\pi^4}{64} - \frac{7\zeta(3)}{4}\log 2 = \sum_{k=1}^{\infty} \frac{H_k}{(2k+1)^3}$

.06400183190906028773... $\approx \displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\sigma_0(k)}{2^k}$

1 .06404394196508864918... $\approx \displaystyle\frac{\zeta(3)\zeta(5)}{\zeta^2(4)}$

.06407528461049067828... $\approx \gamma^5$

.06413507387794809562... $\approx \displaystyle\frac{\gamma}{9}$

.06415002990995841828... $\approx \displaystyle\frac{1}{9\sqrt{3}} = \sum_{k=1}^{\infty} \frac{(-1)^k (2k)!\, k^3}{(k!)^2 2^k}$

.064205801387968452356... $\approx \displaystyle\frac{2}{125} - \frac{28\operatorname{arccsch} 2}{125\sqrt{5}} = \frac{2}{125} - \frac{28\log\phi}{125\sqrt{5}}$

$= \displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{k^3}{\binom{2k}{k}}$

.06422229148740887296... $\approx \displaystyle 20 - \frac{5\pi^2}{3} - \zeta(4) - 2\zeta(3) = \sum_{k=1}^{\infty} \frac{1}{k^3(k+1)^4}$

.064274370716884653601... $\approx \displaystyle 7\log 2 + 2\log\pi - \gamma - \frac{13}{2} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}k^2}{(k+1)(k+2)}\left(\zeta(k+1) - 1\right)$

.06438239351998176981... $\approx \displaystyle\frac{\log 2}{3} - \frac{1}{6} = \sum_{k=0}^{\infty} \frac{(-1)^k}{3k+9}$

$= \displaystyle\int_1^{\infty} \log\left(1 + \frac{1}{x^6}\right)\frac{dx}{x^7}$

.06446776880150635838... $\approx \displaystyle\frac{1}{486}\psi^{(3)}\left(\frac{2}{3}\right) = \sum_{k=0}^{\infty} \frac{1}{(3k+2)^4}$

.064483605572602798069... $\approx \displaystyle\sum_{k=2}^{\infty} (-1)^k \frac{\Omega(k)}{2^k}$

.06451797439929041936... $\approx \displaystyle\frac{1}{e^2 - 1} - \frac{1}{\pi^2 + 1} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\zeta(2k) - 1}{\pi^{2k}}$

.064710682787920926699... $\approx \displaystyle\frac{\pi}{4}\left(\sqrt{4 - 2\sqrt{2}} - 1\right) = \int_0^{\infty} \frac{x^2(1-x)^2}{x^8 + 1}\,dx$

.06471696327987555495... $\approx \displaystyle\frac{4}{e} + 2\gamma - 2Ei(-1) - 3 = -\int_0^1 \frac{x^2 \log x}{e^x}\,dx$

1 .06473417104350337039... $\approx \frac{1}{64}\left(\psi^{(1)}\left(\frac{1}{8}\right)+\psi^{(1)}\left(\frac{3}{8}\right)-\psi^{(1)}\left(\frac{5}{8}\right)-\psi^{(1)}\left(\frac{7}{8}\right)\right)$

$= \frac{1}{32}\left(\psi^{(1)}\left(\frac{1}{8}\right)+\psi^{(1)}\left(\frac{3}{8}\right)\right)-\frac{\pi^2}{8}$

$= \sum_{k=1}^{\infty}\left(\frac{(-1)^{k+1}}{(4k-3)^2}+\frac{(-1)^{k+1}}{(4k-1)^2}\right) = \sum_{k=0}^{\infty}\frac{(-1)^{\lfloor k/2\rfloor}}{(2k+1)^2}$

.06487901723815465273... $\approx \frac{1}{2}+\frac{\pi}{3\sqrt{3}}-\frac{3\log 2}{2} = \int_0^1 \frac{\log(1+x^3)}{(1+x)^2}dx$

.06498017172668905180... $\approx \zeta(4)-\zeta(6) = \frac{\pi^4}{90}-\frac{\pi^6}{945} = \sum_{k=1}^{\infty}\frac{k^2-1}{k^6}$

.06505432440650839036... $\approx \frac{Ei(1)+1}{e}-1 = \sum_{k=1}^{\infty}(-1)^k\,\frac{\psi(k+1)}{(k-1)!}$

.06505816136788066186... $\approx \sum_{k=1}^{\infty}\frac{\log^2 k}{k^4} = \zeta''(4)$

.06506062829357551254... $\approx \frac{11}{12}-\frac{\pi\sqrt{3}}{18}-\frac{\log 3}{2} = \sum_{k=2}^{\infty}\frac{1}{9k^2-3k} = \sum_{k=2}^{\infty}(-1)^k\,\frac{\zeta(k)-1}{3^k}$

.06514255850624644549... $\approx \frac{\log^2 2}{4}\log\frac{3}{5}-\frac{\log 2}{2}Li_2\left(-\frac{2}{3}\right)+\frac{1}{2}Li_3\left(-\frac{2}{3}\right)-\frac{1}{2}Li_3\left(-\frac{1}{3}\right)+\frac{\zeta(3)}{16}$

$= \int_0^1 \frac{\log^2(1+x)}{x(x+4)}dx$

.06519779945532069058... $\approx 5-\frac{\pi^2}{2} = \sum_{k=1}^{\infty}\frac{k}{(k+1)^2(k+2)^2}$

1 .06519779945532069058... $\approx 6-\frac{\pi^2}{3} = \sum_{k=2}^{\infty}\frac{(2k)!!}{(2k-1)!!\,(k^3-k^2)}$

.06528371538988535272... $\approx \frac{G}{2}-\frac{\pi}{8} = \sum_{k=1}^{\infty}\frac{(-1)^k}{(2k+1)^2} = \int_1^{\infty}\frac{\log x}{(x^2+1)^2}dx$

.06529212022186135839... $\approx \frac{1}{2}\log\sec\left(\frac{1}{2}\right) = \sum_{k=1}^{\infty}(-1)^{k+1}\,\frac{\sin^2(k/2)}{k}$ J523

.06530659712633423604... $\approx \frac{10}{\sqrt{e}}-6 = \sum_{k=1}^{\infty}\frac{(-1)^k k}{(k+2)!2^k}$

.06537259256... $\approx \sum_{k=2}^{\infty}\frac{(-1)^k\log^2 k}{k}$

.06544513657702433167... $\approx \frac{241}{144}-\frac{8\log 2}{3}+\frac{\log^2 2}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}H_k}{k+5}$

.06548786238439090235 6... $\approx \frac{1}{4}\left(\psi^{(1)}\left(\frac{\pi}{2}\right)-\psi^{(1)}\left(\frac{1+\pi}{2}\right)\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+\pi)^2}$

.06549676183045346904... $\approx$

$$\frac{1}{2\cdot 3^{11/6}}\left(\left(\sqrt{3}+3i\right)\psi\left(2-\left(-\frac{1}{3}\right)^{1/3}\right)+\left(\sqrt{3}-3i\right)\psi\left(2+\frac{-3^{-1/3}+i3^{1/6}}{2}\right)-2\sqrt{3}\,\psi\left(1+3^{-1/3}\right)\right)$$

$= \quad \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(3k)-1}{3^k}=\sum_{k=2}^{\infty}\frac{1}{3k^3+1}$

.06557201756856993666... ≈ $81-\frac{9\pi\sqrt{3}}{2}-\frac{81\log 3}{2}-\pi^2+\zeta(3)-3\psi^{(1)}\left(\frac{4}{3}\right)$

$= \quad \sum_{k=1}^{\infty}\frac{1}{k^3(3k+1)^2}$

.06567965740448090483... ≈ $\frac{2\log 2}{3}-\frac{19}{36} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k(k+2)(k+3)}$

.06573748689154031248... ≈ $\frac{7\zeta(3)}{128}$

.065774170245478381098... ≈ $\log\frac{(\pi+1)(\pi-1)\csc 1}{\pi^2} = \sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{\pi^{2k}k}$

.06579736267392905746... ≈ $\frac{\pi^2}{150} = \frac{\zeta(2)}{25} = \sum_{k=1}^{\infty}\frac{1}{(5k)^2}$

.0658223444747044061... ≈ $\frac{\pi^2}{36}-\frac{5}{24} = \sum_{k=1}^{\infty}\frac{H_k}{k(k+1)(k+2)(k+3)}$

.06598267284983230587... ≈ $\frac{e^2}{16}-\frac{19}{48} = \sum_{k=0}^{\infty}\frac{2^k}{(k+4)!}$

.06598587683871048216... ≈ $\frac{\log 2}{4}+\frac{\pi}{8}-\frac{1}{2} = \sum_{k=1}^{\infty}\frac{1}{16k^2+12k+2}$

$= \quad \int_1^{\infty}\frac{dx}{x^6+x^5+x^4+x^3}$

.06598803584531253708... ≈ e^{-e}

.06604212644480172199... ≈ $\sqrt{3}\arcsin\frac{1}{\sqrt{3}}-1 = \sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!!3^k(2k+1)}$

.0660913464480888534... ≈ $\frac{\pi^2}{20}\csc\frac{\pi}{\sqrt{5}}+\frac{\pi}{4\sqrt{5}}\cot\frac{\pi}{\sqrt{5}}-\frac{1}{2} = \sum_{k=1}^{\infty}\frac{1}{(5k^2-1)^2}$ J840

.0661733074232325082... ≈ $2\zeta(3)+\zeta(4)+4\zeta(2)-10 = \sum_{k=1}^{\infty}\frac{1}{k^2(k+1)^4}$

.06620909207331664583... ≈ $\frac{\csc 1}{2-2\pi^2}\left((\pi^2-1)\cos 1+(3-\pi^2)\sin 1\right) = \sum_{k=1}^{\infty}\frac{1}{k^2\pi^2-1}$

$= \quad \frac{\pi^2-3}{2(\pi+1)(\pi-1)}-\frac{\cot 1}{2}=\sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{\pi^{2k}}$

.06633268955336798335... ≈ $6+\frac{1}{256}\left(\psi^{(3)}\left(\frac{3}{4}\right)-\psi^{(3)}\left(\frac{1}{4}\right)\right) = \int_1^{\infty}\frac{\log^3 x}{x^4+x^2}$

.06637623973474297119... ≈ $\log\Gamma\left(\frac{9}{10}\right)$

.06640625023283064365... ≈ $\sum_{p\ prime}\frac{1}{2^{2^p}}$

.06642150902189314371... ≈ $\sum_{k=1}^{\infty}\frac{1}{4^{2^k}}=\sum_{k=1}^{\infty}\frac{\mu(2k)}{16^k-1}=-\sum_{k=1}^{\infty}\frac{\mu(4k-2)}{4^{4k-2}-1}$

.06649658092772603273... ≈ $\sqrt{\frac{2}{3}}-\frac{3}{4} = \sum_{k=2}^{\infty}\frac{(-1)^k(2k-1)!!}{(2k)!\,2^k}$

.06660601672682232541... ≈ $\sum_{k=1}^{\infty}\frac{1}{e^{e^k}}$

.06662631248095397825... ≈ $1-\log 2-\frac{\log^2 2}{2}=\int_0^1\frac{\log^2(1+x)}{(1+x)^2}dx$

2 .066646848615237039218... ≈ $\frac{1}{1728}\left(116\pi^3+\psi^{(2)}\left(\frac{5}{12}\right)-\psi^{(2)}\left(\frac{11}{12}\right)\right)$

.0666666666666666666$\underline{6}$ = $\frac{1}{15}$

= $\prod_{k=1}^{\infty}\frac{k(k+6)}{(k+2)(k+4)}$ J1061

1 .0666666666666666666$\underline{6}$ = $\frac{16}{15} = \beta\left(3,\frac{1}{2}\right) = \sum_{k=0}^{\infty}\frac{1}{4^k(k+3)}\binom{2k}{k} = \prod_{k=2}^{\infty}\left(1+\frac{1}{2^{2^k}}\right)$

.06676569631226131157... ≈ $\frac{1}{2}\log\frac{8}{7} = \operatorname{arctanh}\frac{1}{15}$

= $\sum_{k=0}^{\infty}\frac{1}{15^{2k+1}(2k+1)}$ K148

.06678093906442190474... ≈ $\frac{\zeta(3)}{18} = \int_1^{\infty}\frac{\log^2 x}{x^4+x}dx = \int_0^{\infty}\frac{x^2\,dx}{e^{3x}+1}$

1 .06683721243768533963... ≈ $\sum_{k=1}^{\infty}\frac{k\sigma_1(k)}{3^k}$

1 .06687278808178024267... ≈ $\sum_{k=1}^{\infty}\frac{1}{k^{k+2}}$

.06687615866565699453... ≈ $2\log\left(2+\sqrt{3}\right)+4\sqrt{3}+4\log 2-7 = \sum_{k=1}^{\infty}\binom{2k}{k}\frac{1}{16^k k(k+1)}$

3 .06696274638750199199... ≈ $\frac{2\pi^2\log 2}{3}+\frac{2\log^3 2}{3}-4Li_3\left(-\frac{1}{2}\right)-3\zeta(3)$

= $\int_0^1\frac{\log^2 x}{(x+\frac{1}{2})(x+1)}dx$

2 .0670851120199880117... ≈ $\frac{\pi^3}{15}$

.067091633096215553429... ≈ $\frac{\csc^2 1}{4} - \frac{\cot 1}{4} - \frac{\pi^2}{(\pi^2-1)^2} = \sum_{k=1}^{\infty} \frac{k\left(\zeta(2k)-1\right)}{\pi^{2k}}$

1 .06711089410508528045... ≈ $\frac{i}{2}\left(Ei\left(e^{-i}\right) - Ei\left(e^{i}\right)\right) - 1$

= $\pi - 1 - \frac{i}{2}\left(\Gamma\left(0, -e^{-i}\right) - \Gamma\left(0, -e^{i}\right)\right) = \sum_{k=1}^{\infty} \frac{\sin k}{k!k}$

.067519906888675983757... ≈ $\frac{2}{5}\left(arc\cot(1+\sqrt{5}) + arc\coth 2 - \arctan\left(\frac{1+\sqrt{5}}{4}\right)\right)$

= $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} F_k F_{k+1}}{4^k (2k+1)}$

12 .06753397645603743945... ≈ $4G + \pi + \frac{\pi^3}{8} + 2\log 2 = \int_0^1 \log\left(1+\frac{1}{x^2}\right)\log^2 x\,dx$

.067592592592592952$\underline{592}$ = $\frac{169}{2700} = \sum_{k=1}^{\infty} \frac{1}{k(k+3)(k+6)}$

.06759686701139501638... ≈ $\frac{1}{2} - \frac{\pi}{10}\cot\frac{\pi}{5} = \sum_{k=1}^{\infty} \frac{1}{25k^2-1} = \sum_{k=1}^{\infty} \frac{\zeta(2k)}{25^k}$

.0676183320811419985... ≈ $\frac{90}{1331} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^2}{10^k}$

10 .06766199577776584195... ≈ $\cosh 3 = \frac{1}{2}(e^3 + e^{-3}) = \sum_{k=0}^{\infty} \frac{9^k}{(2k)!}$ AS 4.5.63

.067678158936930728162... ≈ $\frac{1}{72}\left(11\pi^2 + 6\left(H\left(-\frac{5}{6}, 2\right) + 9\log 3 + 12\log 2\right) + 3\pi\sqrt{3}(\log 432 - 6)\right)$

= $\sum_{k=1}^{\infty} \frac{H_k}{(6k-1)(6k+1)}$

.06770937508392288924... ≈ $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k\zeta(2k+1)}{16^k} = \sum_{k=1}^{\infty} \frac{16k}{(16k^2+1)^2}$

1 .06771840194669357587... ≈ $\zeta(2) - \gamma = \sum_{k=1}^{\infty} \frac{k^2}{k+1}\left(\zeta(k+1) - 1\right) = \sum_{k=1}^{\infty} \frac{\psi(k+1)}{k(k+1)}$

.06775373784985458697... ≈ $\frac{\pi e^{\pi}}{2e^{2\pi}+2} = \int_0^1 \frac{\cos(2\log x)}{1+x^2}dx$

.06788026407514834638... ≈ $\frac{\pi}{2e^{\pi}} = \int_0^{\infty} \frac{\cos \pi t}{1+t^2}dt$ AS 4.3.146

.06797300991731304833... ≈ $-\frac{1+\gamma}{6} - 2\zeta'(-1) = \sum_{k=2}^{\infty} \frac{\zeta(k)-1}{(k+1)(k+2)}$

Adamchik-Srivastava 2.22

.06804138174397716939... ≈ $\frac{1}{6\sqrt{6}} = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}k^2}{8^k}\binom{2k}{k}$

.06808536722795788835... ≈ $\frac{1}{4}\log\frac{1+e^{-2}}{1-e^{-2}} = \int_1^{\infty}\frac{dx}{e^{2x}-e^{-2x}}$

.06814718055994530941... ≈ $\log 2 - \frac{5}{8} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}H_k}{(k+1)(k+3)} = \int_2^{\infty}\frac{dx}{x^4-x^3}$

$= \int_1^{\infty}\frac{dx}{x(x+1)^3}$

1 .06821393681276108077... ≈ $e^{e^{-e}} = \sum_0^{\infty}\frac{1}{k!e^{ke}}$

2 .06822709436512760885... ≈ $\sum_{k=2}^{\infty}\left(\zeta(k)-1\right)H_{2k-1}$

.06826001937964526234... ≈ $\frac{\pi^2}{6}-\frac{\pi}{2}\coth\pi = \sum_{k=1}^{\infty}\frac{1}{k^4+k^2} = \sum_{k=2}^{\infty}(-1)^k\left(\zeta(2k)-1\right)$

.06830988618379067154... ≈ $\frac{4-\pi}{4\pi} = \int_0^1\frac{dx}{(\pi^2+\log^2 x)(x^2+1)}$ GR 4.282.3

.06834108969889116766... ≈ $\frac{7-3\log 2}{72} = \int_1^2\frac{\log x}{x^4}dx$

.06837797619047619048... ≈ $\frac{919}{13440} = \sum_{k=1}^{\infty}\frac{1}{k(k+2)(k+8)}$

.06838993007841232002... ≈ $\sum_{k=2}^{\infty}\frac{\Omega(k)}{2^k}$

1 .06843424428255624376... ≈ $\cosh\frac{1}{e} = \frac{e^{1/e}+e^{-1/e}}{2} = \sum_{k=0}^{\infty}\frac{1}{(2k)!e^{2k}}$

1 .06849502503075047591... ≈ $\frac{8\zeta(3)}{9} = -Li_3\left(\frac{-1+i\sqrt{3}}{2}\right)-Li_3\left(\frac{-1-i\sqrt{3}}{2}\right)$

.06853308726322481314... ≈ $\log\frac{3}{2}\log 2 + Li_2\left(-\frac{1}{2}\right) - Li_2\left(-\frac{1}{4}\right) = \int_0^1\frac{\log(1+x)}{x+5}$

.0685333821022915429... ≈ $-\frac{\sin\pi\sqrt{3}}{2\pi\sqrt{3}} = \prod_{k=2}^{\infty}\left(1-\frac{3}{k^2}\right) = \prod_{k=1}^{\infty}\frac{k^2+2k-2}{k^2+2k+1}$

2 .0687365065558226299... ≈ $\sum_{k=1}^{\infty}\frac{2^k}{k!k^5} = 2HypPFQ\left[\{1,1,1,1,1,1\},\{2,2,2,2,2,2\},2\right]$

.06891126589612537985... ≈ $-\zeta'(4) = \sum_{k=1}^{\infty}\frac{\log k}{k^4}$

1 .068959332115595113425...≈ $\frac{2\pi}{5}\sqrt{\frac{2}{5-\sqrt{5}}} = \frac{\pi}{5}\csc\frac{4\pi}{5} = \int_0^{\infty}\frac{dx}{x^5+1}$

.0689612184801364854... ≈ $\frac{8\pi^4}{729}-1 = \sum_{k=1}^{\infty}\left(\frac{1}{(3k-1)^4}+\frac{1}{(3k+1)^4}\right)$

1 .0690449676496975387... ≈ $2\sqrt{\frac{2}{7}} = 1+\sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!!8^k}$

1 .06918195449309724743... ≈ $\frac{\pi^3}{29}$

.0691955458920286028... ≈ $\frac{\gamma}{2}-\frac{\pi}{8}+\frac{\log 2}{4} = -\int_0^{\infty}\frac{\log x\,\sin x}{e^x}dx$

.069210168362720747484... ≈ $-\frac{\log 2}{8}\zeta(3)-\frac{7}{8}\zeta'(3) = \sum_{k=0}^{\infty}\frac{\log(2k+1)}{(2k+1)^3}$ Prud. 5.5.1.5

.0693063727312726561... ≈ $\frac{12\log 2+3\pi-14}{54} = -\int_0^1 x^2\arcsin x\,\log x\,dx$

.06934213137376400583... ≈ $\frac{1}{512}\left(\psi^{(2)}\left(\frac{7}{8}\right)-\psi^{(2)}\left(\frac{3}{8}\right)\right) = \int_1^{\infty}\frac{\log^2 x}{x^4+1}$

.06935686230685420356... ≈

$$\frac{1}{2\cdot 3^{11/6}}\left(\left(\sqrt{3}-3i\right)\psi\left(2+\frac{3^{-1/3}-i3^{1/6}}{2}\right)+\left(\sqrt{3}+3i\right)\psi\left(2+\frac{3^{-1/3}+i3^{1/6}}{2}\right)-2\sqrt{3}\left(2-3^{-1/3}\right)\right)$$

= $\sum_{k=1}^{\infty}\frac{\zeta(3k)-1}{3^k} = \sum_{k=2}^{\infty}\frac{1}{3k^3-1}$

.069397608859770623540... ≈ $\sum_{k=1}^{\infty}\frac{1}{k!k^3} = HypPFQ\left[\{1,1,1,1\},\{2,2,2,2\},1\right]$

.06942004590872447261... ≈ $\frac{\pi}{32\sqrt{2}} = \int_0^{\infty}\frac{x^2\,dx}{(x^2+2)^3}$

.0694398829909612926... ≈ $\pi\sqrt{3} + 9\log 3+\frac{\pi^2}{6}-18 + \psi^{(1)}\left(\frac{4}{3}\right) = \sum_{k=1}^{\infty}\frac{1}{k^2(3k+1)^2}$

.06946694693495230181... ≈ $\frac{\pi}{50}\left(\pi+\frac{\pi}{\sqrt{5}}-\frac{1}{2}\sqrt{5(5+2\sqrt{5})}\right) = \sum_{k=1}^{\infty}\frac{k\zeta(2k)}{25^k}$

.06955957164158097295... ≈ $\log^2 2\log 3 - \left(\log 2\right)Li_2\left(\frac{1}{4}\right) - \frac{1}{2}Li_3\left(\frac{1}{4}\right) - \frac{4\log^3 2}{3} + \frac{\zeta(3)}{4}$

= $\int_1^2\frac{\log^2 x}{x+1}dx$

.069628025046703042819... ≈ $\frac{G}{2}+\frac{\pi}{8}+\frac{\pi^3}{32}-\frac{7}{4} = \int_0^1 x\log^2 x\arctan x\,dx$

.06973319205204841124... ≈ $\frac{2\pi\sqrt{3}}{27}-\frac{1}{3} = \int_1^{\infty}\frac{dx}{(x^2+x+1)^2}$

1 .06973319205204841124... ≈ $\frac{2\pi\sqrt{3}}{27} + \frac{2}{3} = \sum_{k=0}^{\infty}\frac{k}{\binom{2k}{k}} = \frac{1}{2}{}_2F_1\left(2,2,\frac{3}{2},\frac{1}{4}\right)$

3 .06998012383946546544... ≈ $\sqrt{3\pi}$

.07009307638669401196... ≈ $\frac{5}{12} - \frac{\log 2}{2} = \sum_{k=0}^{\infty}\frac{(-1)^k}{2k+8} = \sum_{k=2}^{\infty}(-1)^k\,\frac{\zeta(k)-1}{2^{k+1}}$

= $\sum_{k=1}^{\infty}\frac{1}{8k^2+20k+12} = \sum_{k=1}^{\infty}\frac{1}{2^k(3k^2+6k)}$

= $\int_1^{\infty}\log\left(1+\frac{1}{x}\right)\frac{dx}{(x+1)^3}$

.07015057996275895686... ≈ $\frac{7\pi^4}{9720} = \int_1^{\infty}\frac{\log^3 x}{x^4+x}dx$

.07023557414777806495... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)}{(2k)!} = \sum_{k=1}^{\infty}\left(\cosh\sqrt{\frac{1}{k}} - \frac{1}{2k} - 1\right)$

12 .070346316389634502865... ≈ $\frac{e^{\pi}+1}{2} = \int_0^{\pi} e^x \sin x\,dx$

.0704887501485211468... ≈ $\frac{1}{3}\left(\log 3\pi - \log\cosh\frac{\pi\sqrt{3}}{2}\right) = \sum_{k=1}^{\infty}\frac{\zeta(3k)-1}{3k} = -\frac{1}{3}\sum_{k=2}^{\infty}\log\left(1-\frac{1}{k^3}\right)$

.070541352850566718528... ≈ $\frac{1}{3072}\left(192\pi^2 G - \psi^{(3)}\left(\frac{1}{4}\right) + \psi^{(3)}\left(\frac{1}{4}\right)\right) = -\int_0^{\pi/4} x^2 \log\tan x\,dx$

1 .07061055633093042768... ≈ $4Li_2\left(\frac{1}{4}\right) = \sum_{k=0}^{\infty}\frac{1}{4^k(k+1)^2} = 3\sum_{k=1}^{\infty}\frac{H^{(2)}{}_k}{4^k}$

1 .07065009697711308205... ≈ $\frac{e^e}{e^e-1}$

.0706857962810410866... ≈ $4 - \zeta(2) - \zeta(3) - \zeta(4) = \sum_{k=2}^{\infty}\frac{1}{k^5-k^4}$

= $\sum_{k=1}^{\infty}\frac{1}{k(k+1)^4}$

= $\frac{1}{6}\int_0^1 \log(1-x)\log^3 x\,dx$

.070698031526694030437... ≈ $\frac{3\pi^2}{4} - 1 - \frac{9}{2}\log\frac{3}{2}\log 3 + 3(\log 3 - \log 2) + 4Li_2\left(-\frac{1}{2}\right) - \frac{9}{2}Li_2\left(\frac{2}{3}\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}H_k}{2^k k(k+1)(k+2)}$

179 .0707322923047554885... $\approx$ $\left(\pi^2+\log^2 2\right)\frac{3\pi^2+\log^2 2}{18} = \int_0^\infty \frac{\log^5 x\,dx}{(x+2)(x-1)}$ GR 4.264.3

.07073553026306459368... $\approx$ $\frac{2}{9\pi}$

.0707963267948966192... $\approx$ $\frac{\pi-3}{2} = \frac{i}{2}\left(\log\left(1-e^{3i}\right)-\log\left(1-e^{-3i}\right)\right) = \sum_{k=1}^{\infty}\frac{\sin 3k}{k}$

1 .0707963267948966192... $\approx$ $\frac{\pi-1}{2} = \sum_{k=1}^{\infty}\frac{\sin k}{k}$ GR 1.441.1, GR 1.448.1

$= \arctan\frac{\sin 1}{1-\cos 1} = \sum_{k=1}^{\infty}\frac{\sin^2 k}{k^2}$

2 .0707963267948966192... $\approx$ $\frac{\pi+1}{2} = \sum_{k=0}^{\infty}\frac{2^k}{\binom{2k+2}{k}}$

.07084242088030439606... $\approx$ $\frac{1}{1000}\left(\psi^{(2)}\left(\frac{4}{5}\right)-\psi^{(2)}\left(\frac{3}{10}\right)\right) = \int_1^\infty \frac{\log^2 x}{x^4+x^{-1}}$

.07090116891887671352... $\approx$ $\frac{3}{8}+\frac{3}{4}\log\frac{2}{3} = \sum_{k=2}^{\infty}\frac{(-1)^k}{2^k(k^2-1)}$

.07101537321990997509... $\approx$ $\frac{\arctan 2}{5}+\frac{\log 5}{20}-\frac{2\gamma}{5} = \int_0^\infty \frac{\log x\sin^2 x}{e^x}dx$

.07103320989006310636... $\approx$ $\sum_{k=2}^{\infty}(-1)^k F_{k-1}\left(\zeta(2k)-1\right)$

3 .0710678118654752440... $\approx$ $5\sqrt{2}-4 = \sum_{k=0}^{\infty}\frac{k^2}{8^k}\binom{2k+1}{k}$

7 .0710678118654752440... $\approx$ $\sqrt{50} = 5\sqrt{2}$

.07130217810980315986... $\approx$ $120-\frac{326}{e} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!(k+6)}$

.07130874229851250887 4... $\approx$ $\frac{\zeta(3)}{4}-8\log 2+2\log^2 2-\frac{\pi^2}{6}+6 = -\int_0^1 \log^2(1+x)\log x\,dx$

1 .07133434403336294393... $\approx$ $\frac{i}{2}\left(I_0\left(2\sqrt{e^i}\right)-I_0\left(2\sqrt{e^{-i}}\right)\right) = \sum_{k=1}^{\infty}\frac{\sin k}{(k!)^2}$

.07134363852556480380... $\approx$ $\frac{60}{841} = \sum_{k=1}^{\infty}(-1)^{k+1}k\frac{F_{2k}}{4^k}$

.0713495408493620774... $\approx$ $\frac{\pi}{16}-\frac{1}{8} = \sum_{k=1}^{\infty}\frac{1}{(4k-3)(4k-1)(4k+1)}$ J239

$= \int_0^1 \frac{\log x}{\pi^2+4\log^2 x}\,\frac{dx}{x^2-1}$ GR 4.282.8

$.0714285714285\underline{714825}$ = $\frac{1}{14}$

.0717737886118051393... ≈ $\frac{\pi^3}{432} = \int_0^\infty \frac{x^2}{e^{3x}+e^{-3x}}dx$

.071791629712009706036... ≈ $-\frac{1}{2}-\gamma-\frac{1}{2}\left(\psi\left(1-\frac{1}{\sqrt{3}}\right)+\psi\left(1+\frac{1}{\sqrt{3}}\right)\right) = \sum_{k=2}^\infty \frac{1}{3k^3-k}$

.07179676972449082589... ≈ $7-4\sqrt{3} = \sum_{k=1}^\infty \frac{1}{16^k(k+1)}\binom{2k}{k}$

.07189823963991674726... ≈ $\frac{1}{4096}\left(\psi^{(3)}\left(\frac{3}{8}\right)-\psi^{(3)}\left(\frac{7}{8}\right)\right) = \int_1^\infty \frac{\log^3 x}{x^4+1}$

.07190275490382753558... ≈ $\frac{10-3\pi}{8} = \int_0^{\pi/4} \sin^2 x \tan^2 x\, dx$

.07192051811294523186... ≈ $\frac{1}{4}\log\frac{4}{3} = \sum_{k=1}^\infty \left(Li_k\left(\frac{1}{4}\right)-\frac{1}{4}\right)$

.071995909163798022179... ≈ $\sum_{k=2}^\infty \frac{\log k}{k^4-1}$

$.0720000000000000000\underline{0}$ = $\frac{9}{25} = \sum_{k=1}^\infty (-1)^{k+1}\frac{k^2}{9^k}$

.07202849079249295866... ≈ $\sum_{k=2}^\infty \left(\zeta(k)-1\right)^6$

.07215195811269160758... ≈ γ^8

.07246703342411321824... ≈ $\frac{\pi^2-9}{12} = \sum_{k=0}^\infty \frac{(-1)^k}{(k+3)^2} = \sum_{k=2}^\infty \frac{(-1)^k}{k^4-k^2}$

= $\sum_{k=1}^\infty (-1)^{k+1}\frac{\cos^2 k}{k^2}$

= $\int_1^\infty \frac{\log x}{x^3(x+1)}dx$

= $\int_0^\infty \frac{x}{e^{3x}+e^{2x}}dx$ GR 3.411.11

1 .07246703342411321824... ≈ $\frac{\pi^2+3}{12} = \int_0^1 \frac{(1+x^2)\log(1+x)}{x}dx$

.072700105960963691568... ≈ $\frac{3}{8}-\frac{\pi}{6\sqrt{3}} = \sum_{k=1}^\infty \frac{\zeta(2k)-1}{9^k} = \sum_{k=2}^\infty \frac{1}{9k^2-1} = \sum_{k=1}^\infty \frac{1}{9k^2+18k+8}$

.07270478199838776757... ≈ $1-2\arctan\frac{1}{2} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{4^k(2k+1)}$

.07277708725845669284... ≈ $\frac{1}{1000}\left(\psi^{(3)}\left(\frac{3}{10}\right)-\psi^{(3)}\left(\frac{4}{5}\right)\right) = \int_1^\infty \frac{\log^3 x}{x^4+x^{-1}}$

.072815845483677... ≈ $\lim_{n\to\infty}\left(\frac{1}{2}\log^2 n-\sum_{k=1}^n\frac{\log k}{k}\right)$ Berndt 8.17.2

.07284924190995026164... ≈ $-\frac{\gamma}{\pi}-\frac{1}{\pi}\psi\left(1-\frac{1}{\pi}\right)-\frac{1}{\pi(\pi-1)} = \sum_{k=2}^\infty \frac{1}{k^2\pi^2-k\pi}$

= $\frac{1-\gamma}{\pi}-\frac{1}{\pi}\psi\left(2-\frac{1}{\pi}\right) = \sum_{k=2}^\infty\frac{\zeta(k)-1}{\pi^k}$

.072900620536574583049... ≈ $1+Li_2\left(-\frac{1}{3}\right) = -\int_0^1\frac{x\log x}{x+3}dx$

.07296266134931404078... ≈ $1-\sqrt{\frac{\pi}{2}}\,\Gamma\left(\frac{5}{4}\right)\Gamma^{-1}\left(\frac{3}{4}\right) = \sum_{k=1}^\infty\binom{2k}{k}\frac{(-1)^{k+1}}{4^k(4k+1)}$

1 .07318200714936437505... ≈ $\frac{2}{\pi}K\left(\frac{1}{4}\right) = \sum_{k=0}^\infty\binom{2k}{k}^2\frac{1}{64^k}$

.07320598580844476003... ≈ $\frac{1}{4}\left(\psi(1+i)+\psi(1-i)-\psi\left(\frac{1}{2}+i\right)-\psi\left(\frac{1}{2}-i\right)\right)$

= $\int_0^\infty\frac{\cos 2x}{e^x+1}dx$

.0732233047033631189... ≈ $\frac{2-\sqrt{2}}{8} = \int_0^\infty\frac{\cos\pi x}{e^{2\pi\sqrt{x}}-1}dx$ Prud. 2.5.38.9

.07325515198082261749... ≈ $\frac{1}{20736}\left(\psi^{(3)}\left(\frac{1}{4}\right)-\psi^{(3)}\left(\frac{3}{4}\right)\right) = \int_1^\infty \frac{\log^3 x}{x^4+x^{-2}}$

.073262555554936721175... ≈ $\frac{4}{e^4} = \sum_{k=1}^\infty\frac{(-1)^{k+1}4^k k}{k!}$

.07329070292368558994... ≈ $\sum_{k=2}^\infty(-1)^k\frac{\zeta(k)-1}{2^k k} = \sum_{k=2}^\infty \frac{1}{8k^2}\,{}_1F_1\left(1,2,3,-\frac{1}{2k}\right)$

= $\frac{1}{2}\left(\gamma-1+\log\frac{9\pi}{16}\right)$

1 .07330357886990841153... ≈ $120I_6(2)+2670I_7(2)+8520I_8(2)+8000I_9(2)+$
$3025I_{10}(2)+511I_{11}(2)+38I_{12}(2)+I_{13}(2)$

= $\sum_{k=1}^\infty \frac{k^4}{(k!)^2(k+5)}$

755 .07330694419801687855... ≈ $\frac{\pi^7}{4}$

.07344910388202908912... ≈ $\frac{\pi^2}{\cosh^2\pi}$

.07345979246907078119... ≈ $\frac{\pi^2}{18} - \frac{\pi^2}{12\sqrt{3}} = \sum_{k=1}^{\infty}\left(\frac{1}{(12k-7)^2} + \frac{1}{(12k-5)^2}\right)$ J346

1 .07351706431139209227... ≈ $\sum_{k=1}^{\infty}\frac{(k!)^2\zeta(2k)}{(2k)!}$

.07355072789142418039... ≈ $\frac{\pi}{3\sqrt{3}} - \frac{\log 2}{3} - \frac{3}{10} = \sum_{k=0}^{\infty}\frac{(-1)^k}{3k+8}$

.073553956728532... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}H_k}{(2k+1)^2}$

.07363107781851077903... ≈ $\frac{3\pi}{128} = \sum_{k=0}^{\infty}\frac{1}{4^k(2k+1)(2k+3)(2k+5)}\binom{2k}{k}$

.07366791204642548599... ≈ $ci(\pi)$

.07387736114946630558... ≈ $\frac{5}{2} - \frac{4}{\sqrt{e}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k+2)!2^k}$

.07388002973920462501... ≈ $\frac{35}{48\pi^2}$ CFG B5

.07394180231599241053... ≈ $\frac{8}{3} - \frac{11}{3\sqrt{2}} = \int_0^{\pi/4}\sin x\tan^4 x\,dx$

.07399980675447242740... ≈ $\frac{\pi^2}{\sinh^2\pi} = \prod_{k=1}^{\infty}\left(\frac{k^2}{k^2+1}\right)^2$

1 .07399980675447242740... ≈ $1 + \frac{\pi^2}{\sinh^2\pi} = -\psi^{(1)}(i) - \psi^{(1)}(-i)$

.07400348967151741917... ≈ $3 + 6\log\left(\frac{2}{3}\right) - 3\log^2\left(\frac{2}{3}\right) = \sum_{k=1}^{\infty}\frac{H_k}{3^k(k+1)(k+2)}$

.07402868138609661224... ≈ $\frac{\log^2 2\log 3}{2} - \frac{2\log^3 2}{3} - (\log 2)Li_2\left(-\frac{1}{2}\right) - Li_3\left(-\frac{1}{2}\right) - \frac{5\zeta(3)}{8}$

= $\int_0^1\frac{\log^2(1+x)}{x(x+1)(x+2)}$

$.074074074074074074\underline{074}$ = $\frac{2}{27} = \sum_{k=0}^{\infty}(-1)^k\frac{k^2}{2^k} = \sum_{k=0}^{\infty}(-1)^k\frac{k^3}{2^k} = Li_{-2}(-2) = Li_{-3}(-2)$

= $\int_1^{\infty}\frac{\log^2 x}{x^4}\,dx = \int_1^{\infty}\frac{\log^3 x}{x^4}\,dx$

1 .0741508456720383647... ≈ $\frac{\zeta^2(5)}{\zeta(10)} = \sum_{k=1}^{\infty}\frac{2^{\nu(k)}}{k^5}$ Titchmarsh 1.2.8

1 .07418410593764418873... ≈ $\frac{2\log 5}{5} + \frac{1}{\sqrt{5}}\log\frac{1+1/\sqrt{5}}{1-1/\sqrt{5}} = \sum_{k=1}^{\infty}\frac{F_kF_{k+3}}{4^k k}$

.07418593219600418181... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k}{k^3}\left(\zeta(k)-1\right) = \sum_{k=2}^{\infty} Li_3\left(-\frac{1}{k}\right)+\frac{1}{k}$

2 .07422504479637891391... ≈ $\frac{1}{\Gamma\left(-(-1)^{1/5}\right)\Gamma\left((-1)^{2/5}\right)\Gamma\left(-(-1)^{3/5}\right)\Gamma\left((-1)^{4/5}\right)} = \prod_{k=1}^{\infty}\left(1+\frac{1}{k^5}\right)$

5 .074304749728368200293... ≈ $\int_1^{\infty}\frac{x}{\Gamma(x)}\,dx$

.07438118377140325192... ≈ $\frac{1}{3}\log\frac{5}{4} = \sum_{k=1}^{\infty}\frac{1}{3k5^k} = \int_1^{\infty}\frac{dx}{(3x+1)(3x+2)}$

.07442417922462205220... ≈ $\frac{1}{2}+\frac{\sqrt{14}}{7}\operatorname{csch}\pi\sqrt{\frac{7}{2}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{2k^2+7}$

1 .07442638721608040188... ≈ $\frac{\log^3 2}{3}+\frac{7\zeta(3)}{4}-\zeta(2)\log 2$

= $2Li_3\left(\frac{1}{2}\right) = 2\sum_{k=1}^{\infty}\frac{1}{2^k k^3} = \sum_{k=1}^{\infty}\frac{H^{(3)}{}_k}{2^k}$

= $\int_1^{\infty}\frac{\log^2 x}{2x^2-x}dx = \int_0^{\infty}\frac{x^2}{2e^x-1}dx$

1 .07446819773248816683... ≈ $-e^2\log\left(1-e^{-2}\right) = \sum_{k=0}^{\infty}\frac{1}{(k+1)e^{2k}}$

.07454093588033922743... ≈ $\frac{1}{2}-\frac{\operatorname{csch}1}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2\pi^2+1}$

.07473988678897301721... ≈ $\frac{184}{27}-\gamma+\log\frac{4}{9\pi}-\frac{7\zeta(3)}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^3}{2^k(k+1)}\left(\zeta(k+1)-1\right)$

1 .0747946000082483594... ≈ $Li_2\left(\frac{4}{5}\right)$

1 .07483307215669442120... ≈ $\frac{\pi^2}{16}+\frac{G}{2} = \frac{1}{16}\psi^{(1)}\left(\frac{1}{4}\right) = \sum_{k=0}^{\infty}\frac{1}{(4k+1)^2} = \int_0^1\frac{\log x\;dx}{x^4-1}$

3 .07484406769435131887... ≈ $2+\frac{16\pi}{27\sqrt{3}} = \sum_{k=1}^{\infty}\frac{k(k+1)}{\binom{2k}{k}}$

.074850088435720511891... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}k(\zeta(4k)-1) = \sum_{k=2}^{\infty}\frac{1}{k^4(1+k^{-4})^2}$

.074859404114557591023... ≈ $\frac{2\gamma}{3}-\frac{1}{6}+\frac{1}{3}\left(\psi\left(\frac{3+i\sqrt{3}}{2}\right)+\psi\left(\frac{3-i\sqrt{3}}{2}\right)\right)$

= $\frac{2\gamma}{3}-\frac{1}{6}+\frac{1}{3}\left(\psi\left((-1)^{1/3}\right)+\psi\left(-(-1)^{2/3}\right)\right) = \sum_{k=2}^{\infty}\frac{1}{k^4+k}$

$$= \quad 2\sum_{k=2}^{\infty}\frac{1}{k^5-k^{-1}}$$

$$= \quad \sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(3k+1)-1\right)$$

$$.0748775943990545019993\ldots \approx \quad \frac{35-3\pi^2}{72} = \sum_{k=1}^{\infty}\frac{1}{k(k+4)^2} = \int_0^1 x^3\log(1-x)\log x\,dx$$

.07500000000000000000$\underline{0}$ = $\dfrac{3}{40}$

1 .0750476034999202387 2... ≈ $\sqrt{\dfrac{\pi}{e}} \;=\; K_{1/2}\left(\dfrac{1}{2}\right)$

.075075075075075075$\underline{075}$ = $\dfrac{5}{66} \;=\; B_5$

2 .07510976937815191446... ≈ $\displaystyle\sum_{k=0}^{\infty}\frac{1}{(k!!)^4}$

.07512855644747464284... ≈ $\dfrac{\zeta(3)}{16} \;=\; \displaystyle\int_0^1 \frac{\log(1-x^4)\log x}{x}dx$

1 .07521916938666853671... ≈ $\zeta^{\,2}(5) = \displaystyle\sum_{k=1}^{\infty} d_2(k)k^{-5}$ Titchmarsh 1.2.2

.07522047803571148309... ≈ $\dfrac{1}{4}\left(\cos\dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}}\cos\dfrac{1}{\sqrt{2}}\right) \;=\; \displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{(2k+1)!2^k}$

1 .07548231525329211758... ≈ $5\zeta(3) \,-\, 3\zeta(2)$

.0757451952714038687... ≈ $\zeta(2) \,-\, \zeta(3) \,-\, \dfrac{\zeta(4)}{4} \,+\, \zeta(2)\zeta(3) \,-\, 2\zeta(5) \;=\; \displaystyle\sum_{k=1}^{\infty}\frac{H_k}{k(k+1)^4}$

.07580375925340625411... ≈ $1 - \dfrac{4\log 3}{2} \;=\; 2\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^3}{27k^3-3k}$ Berndt 2.5.4

= $\displaystyle\sum_{k=1}^{\infty}\frac{1}{2^k\left(\dfrac{3k^2}{2}+\dfrac{9k}{2}+3\right)}$

.07581633246407917795... ≈ $\dfrac{1}{8\sqrt{e}} \;=\; \displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^3}{k!2^k}$

.07591836734693877551... ≈ $\dfrac{93}{1225} \;=\; \displaystyle\sum_{k=1}^{\infty}\frac{1}{k(k+2)(k+7)}$

.0760156614280973649... ≈ $20e^{-1/4} \,-\, \dfrac{31}{2} \;=\; \displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!4^k(k+2)}$

.07606159707840983298... ≈ $1 - \dfrac{90}{\pi^4} \;=\; \dfrac{\zeta(4)-1}{\zeta(4)}$

.07615901382553683827... ≈ $\dfrac{\sqrt{\pi}}{e}\,erfi\,1 \,-\, 1 \;=\; \displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k!4^k}{(2k)!}$

1 .07615901382553683827... ≈ $\dfrac{\sqrt{\pi}}{e}\,erfi\,1 \;=\; \displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}4^k}{k!\dbinom{2k}{k}} \;=\; \int_0^1\frac{dx}{e^x\sqrt{1-x}}$

.07621074481849448468... ≈ $\displaystyle\sum_{k=1}^{\infty}\frac{\zeta(2k+1)-1}{2k+1} \;=\; \sum_{k=2}^{\infty}\left(\frac{1}{2}\log\frac{k+1}{k-1} - \frac{1}{k}\right)$

= $1-\gamma-\dfrac{\log 2}{2} = \sum_{k=2}^{\infty}\left(\operatorname{arccoth} k-\dfrac{1}{k}\right)$

= $2\int_0^{\infty} x e^{-x}\log x \sin x \, dx$

.07646332214806367238... ≈ $\dfrac{1}{4}\left(\psi\left(1+\dfrac{i}{2}\right)+\psi\left(1-\dfrac{i}{2}\right)-\psi\left(\dfrac{3+i}{2}\right)-\psi\left(\dfrac{3-i}{2}\right)\right)$

$+\ 1-\log 2=\sum_{k=2}^{\infty}\dfrac{(-1)^k}{k^3+k}$

.07648051389327864275... ≈ $\log 2-\dfrac{37}{60} = b(7) = \sum_{k=0}^{\infty}\dfrac{(-1)^k}{k+7}$ GR 8.375

= $\sum_{k=1}^{\infty}\dfrac{1}{4k^2+27k+30}$

= $\int_1^{\infty}\dfrac{dx}{x^8+x^7}$

1 .07667404746858117413... ≈ $\dfrac{\pi}{2}\coth\pi-\dfrac{1}{2} = \dfrac{\pi-1}{2}+\dfrac{\pi}{e^{2\pi}-1}$ J983

= $\sum_{k=1}^{\infty}\dfrac{1}{k^2+1} = \dfrac{1}{2}+\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(2k)-1\right) = \operatorname{Im}\left\{\psi(1+i)\right\}$

= $\int_0^{\infty}\dfrac{\sin x\, dx}{e^x-1}$

= $\int_0^{1}\dfrac{\sin\log x^{-1}}{1-x}dx$

2 .07667404746858117413... ≈ $\dfrac{\pi}{2}\coth\pi+\dfrac{1}{2} = \dfrac{\pi+1}{2}+\dfrac{\pi}{e^{2\pi}-1}$

= $\sum_{k=0}^{\infty}\dfrac{1}{k^2+1} = \dfrac{3}{2}+\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(2k)-1\right) = \operatorname{Im}\left\{\psi(i)\right\}$

.07668525568456209094... ≈ $\dfrac{1}{\pi^{5/2}}\zeta\left(\dfrac{5}{2}\right)$

.07671320486001367265... ≈ $\dfrac{1-\log 2}{4} = \sum_{k=1}^{\infty}\dfrac{1}{16k^2+8k} = \sum_{k=1}^{\infty}\dfrac{1}{2^k(4k^2+4k)}$

= $\int_0^{\pi/2}\dfrac{\sin^3 x\log(\sin x)}{\sqrt{1+\sin^2 x}}dx$ GR 4.386.2

.07681755829903978052... ≈ $\dfrac{56}{729} = \sum_{k=1}^{\infty}(-1)^{k+1}\dfrac{k^2}{8^k}$

$.076923076923\underline{076923}$ = $\dfrac{1}{13}$

.07697262446079313824... ≈ $G-G^2$

$$.07699313976424684494 3... \approx \sum_{p\ prime} p^{-4} = \sum_{k=1}^{\infty} \frac{\mu(k)}{k} \log\left(\zeta(4k)\right)$$

$$.07702272530936605075 2... \approx$$

$$\frac{1}{20(3+\sqrt{5})}\left(\left(19+7\sqrt{5}\right)\arctan\left(\frac{1+\sqrt{5}}{4}\right)-2\left(7+2\sqrt{5}\right)arc\cot\left(1+\sqrt{5}\right)-3\left(3+\sqrt{5}\right)arc\coth 2\right)$$

$$= \sum_{k=1}^{\infty} \frac{(-1)^{k+1} F_k F_{k+1}}{4^k (2k-1)(2k+1)}$$

$$.077056903159594285 4... \approx \zeta(3) - \frac{9}{8} = \zeta(3,3) = \sum_{k=0}^{\infty} \frac{1}{(k+3)^3} = \Phi(1,3,0)$$

$$= \frac{1}{2}\int_0^1 \frac{x^2 \log^2 x}{1-x} dx$$ GR 4.261.12

$$= \int_0^1\int_0^1\int_0^1 \frac{x^2 y^2 z^2}{1-xyz} dx\, dy\, dz$$

$$.07707533991362915215... \approx \frac{1}{2}\log\frac{7}{6} = \text{arctanh}\frac{1}{13} = \sum_{k=0}^{\infty} \frac{1}{13^{2k+1}(2k+1)}$$ K148

$$.07710628438351061421... \approx \frac{\pi^2}{128} = \sum_{k=0}^{\infty} \frac{(2k+1)^2}{(4k+2)^4}$$

$$.07720000000000000000\underline{0} = \frac{139}{1800} = \sum_{k=1}^{\infty} \frac{1}{k(k+3)(k+5)}$$

$$.07720043958926015531... \approx \frac{\pi^4}{16} - \frac{1}{256}\psi^{(3)}\left(\frac{1}{4}\right) = \int_1^{\infty} \frac{\log^3 x}{x^4-1} dx$$

$$.07721566490153286061... \approx \gamma - \frac{1}{2} = \int_0^1 \left(\frac{1}{\log^2 x} - \frac{x}{(1-x)^2}\right) dx$$ GR 4.283.4

$$= 2\int_0^{\infty} \frac{x\, dx}{(1+x^2)(e^{2\pi x}-1)}$$ Andrews p. 87

$$.077254049319369394 9... \approx 1 - \int_0^1 \Gamma(x+1)\, dx$$

$$.07730880240793844225... \approx \frac{\pi^4}{1260} = T(4)$$

$$.077403064078615306111... \approx \frac{9}{16} - \gamma + \log 2 - \frac{\zeta(3)}{2} = \sum_{k=2}^{\infty} \frac{k^2}{k+1}\left(\zeta(2k+1)-1\right)$$

Adamchik-Srivastava 2.24

$$.07743798717441510249... \approx 1 - \sqrt{\pi}\, erf\,\frac{1}{2} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!\,4^k (2k+1)}$$

1 .07746324137541851817... ≈ $\sum_{k=1}^{\infty}\frac{1}{k!!\,k^{3}}$

.07752071017393104727... ≈ $\log(2\cos 1) = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\cos 2k}{k}$ GR 1.441.4

.07755858036092111976... ≈ $\frac{\log 5}{4}-\frac{\pi}{10}\sqrt{1+\frac{2}{\sqrt{5}}}-\frac{1}{4\sqrt{5}}\log\frac{2}{3+\sqrt{5}}$

= $\sum_{k=2}^{\infty}\frac{\zeta(k)}{5^{k}} = \sum_{k=1}^{\infty}\frac{1}{25k^{2}-5k}$

5 .07770625192980658253... ≈ $9\pi^{-1/2}$

.07773398731743422062... ≈ $4\arctan\frac{1}{2}+\log\frac{5}{4}-2 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k(2k+1)4^{k}}$

3 .07775629309491924963... ≈ $\sum_{k=1}^{\infty}\frac{\sigma_0(k)^2}{2^{k}} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\sigma_0(mn)}{2^{mn}}$

$.0777777777777777777\underline{7}$ = $\frac{7}{90}$

.07784610394702787502... ≈ $-\cos\sqrt{e} = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}e^{k}}{k!}$

.07792440345582405854... ≈ $2-2\cos 1-\sin 1 = \sum_{k=1}^{\infty}(-1)^{k}\frac{k}{(k+1)(2k+1)!}$

1 .07792813674185519486... ≈ $\frac{105}{\pi^{4}} = \prod_{p\ prime}\left(1+p^{-4}\right)$ Berndt 5.28

= $\frac{\zeta(4)}{\zeta(8)} = \sum_{k=1}^{\infty}\frac{|\mu(k)|}{k^{4}}$ Titchmarsh 1.2.7

= $\sum_{q\,squarefree} q^{-4}$

.07795867267256012493... ≈ $\frac{8}{9}+2\log\frac{2}{3} = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}k^{2}}{2^{k}(k+1)}$

.07796606266976290757... ≈ $\frac{50G+43}{128\pi}-\frac{1}{7} = \sum_{k=1}^{\infty}\frac{1}{2k+7}\left(\frac{(2k-1)!!}{(2k)!!}\right)^{2}$ J385

2 .0780869212350275376... ≈ $\frac{1}{\log\varphi}$

1 .07818872957581848276... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)}{k!} = \sum_{k=1}^{\infty}\left(e^{1/k}-1-\frac{1}{k}\right)$

.07820534411412707043... ≈ $\frac{\pi}{2e^{3}} = \int_{0}^{\infty}\frac{\cos 3x}{x^{2}+1}dx$

$$.0782055828604531093\ldots \approx \frac{1}{12} - \operatorname{csch}\frac{\pi^2}{2}\operatorname{sech}\frac{\pi^2}{2}\left(\sinh\pi^2 - \pi^2\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2(k^2+\pi^2)}$$

1 $$.0784420108320359272 0\ldots \approx \psi\left(1+\frac{i}{2}\right)+\psi\left(1-\frac{i}{2}\right)-\psi\left(\frac{1+i}{2}\right)-\psi\left(\frac{1-i}{2}\right)$$

$$.0784498410585544626 5\ldots \approx \frac{\pi}{4\sqrt{3}} - \frac{3}{8} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{3^k(2k+1)}$$

$$.0784775796671368383 2\ldots \approx \frac{\pi\left(\sin\pi\sqrt{2}+\sinh\pi\sqrt{2}\right)}{2\sqrt{2}\left(\cosh\pi\sqrt{2}-\cos\pi\sqrt{2}\right)} - 1$$

$$= \sum_{k=2}^{\infty}\frac{1}{k^4+1} = \sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(4k)-1\right)$$

$$.0785\underline{670194003527336860} = \frac{17819}{226800} = \sum_{k=1}^{\infty}\frac{1}{k(k+1)(k+10)}$$

$$.0786791525731399704 97\ldots \approx \frac{2}{3} + \log 2 - \frac{\gamma}{2} + 6\zeta'(1) = \sum_{k=1}^{\infty}\frac{\zeta(2k+1)-1}{k+2}$$

$$= -\sum_{k=2}^{\infty}\left(k^3\log\left(1-\frac{1}{k^2}\right)+k+\frac{1}{2k}\right)$$

$$.0788264185968423446 7\ldots \approx ci(1) - \gamma + \log 2 - \cos 1\log 2 = \int_0^1 \log 2x\,\sin x\,dx$$

$$.0788983500212491810 1\ldots \approx \prod_{k=1}^{\infty}\left(1-\frac{k}{2^k}\right)$$

1 $$.0789155213519426539 8\ldots \approx \frac{2}{3} + \log 2 - \frac{\gamma}{2} + 6\zeta'(-1) = \sum_{k=1}^{\infty}\frac{\zeta(2k-1)-1}{k+2}$$

$$.0789621058600500236 1\ldots \approx \frac{1}{12}\left(4\pi^2 - 24\log^2 2 - 27\right) = \sum_{k=1}^{\infty}\frac{1}{2^k(k+2)^2}$$

$$.0790756439455824958 1\ldots \approx \frac{3\zeta(3)}{4} - \frac{\pi^2}{12} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(k+1)^3}$$

$$.0791098730673356297 65\ldots \approx \log\zeta(4) = \sum_{p\ prime}\log\left(\frac{1}{1-p^{-4}}\right)$$

HW Sec. 17.7

$$.\ 0791295876359811915 9\ldots \approx$$

$$\frac{3\pi G}{32} - \frac{\pi^3}{512}\cot\frac{\pi}{8} + \frac{3\pi^2}{64}\log\left(1+(-1)^{3/4}\right) + \frac{3\pi}{512}(-1)^{1/4}\psi^{(1)}\left(\frac{1}{8}\right) - \frac{3\pi}{512}(-1)^{3/4}\psi^{(1)}\left(\frac{3}{8}\right)$$

$$-\frac{3\pi}{512}(-1)^{1/4}\psi^{(1)}\left(\frac{5}{8}\right) + \frac{3\pi}{512}(-1)^{3/4}\psi^{(1)}\left(\frac{7}{8}\right) + \frac{3}{2}Li_3\left(-(-1)^{3/4}\right) - \frac{3\zeta(3)}{2}$$

$= \int_0^{\pi/8} \frac{x^3}{\sin^2 x}\, dx$

.07921357989350166466... $\approx \log\left(\Gamma\left(\frac{4}{3}\right)\right) + \frac{\gamma}{3} = \sum_{k=2}^{\infty} (-1)^k \frac{\zeta(k)}{3^k k}$ Dingle 3.37

$= \sum_{k=1}^{\infty}\left(\frac{1}{3k} - \log\frac{3k+1}{3k}\right)$

.0792166666666666666$\underline{6}$ $= \frac{7}{96} = \int_1^{\infty} \log\left(1+\frac{1}{x^2}\right)\frac{dx}{x^9}$

.079221397565207165999... $\approx MHS(3,1,2) = \frac{53\zeta(6)}{24} - \frac{3\zeta^2(3)}{2}$

.07924038639496914327... $\approx \frac{1}{2}\left(\cos\frac{1}{\sqrt{2}} - \sqrt{2}\sin\frac{1}{\sqrt{2}}\right) = \sum_{k=1}^{\infty} \frac{(-1)^k k}{(2k+1)!\, 2^k}$

.079332033399540433998... $\approx \int_1^{\infty} \left(\zeta(3x) - 1\right) dx$

.07944154167983592825... $\approx 3\log 2 - 2 = \sum_{k=1}^{\infty} \frac{(-1)^k k}{(k+1)(k+2)}$

$= \sum_{k=1}^{\infty} \frac{1}{16k^3 - k} = \sum_{k=1}^{\infty} \frac{\zeta(2k+1)}{16^k}$

$= \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\binom{2k}{k} 2^k k(2k+1)}$

.07957747154594766788... $\approx \frac{1}{4\pi} = \int_0^{\infty} \frac{\cos(\pi/2x)}{e^{2\pi\sqrt{x}} - 1} dx$

.07970265229714766528... $\approx -\frac{1}{125}\psi^{(2)}\left(\frac{3}{5}\right)$

.07998284307169517701... $\approx 3 - \frac{\pi}{2\sqrt{3}} - \frac{3\log 3}{2} - \frac{1}{3}\psi^{(1)}\left(\frac{4}{3}\right) = \sum_{k=1}^{\infty} \frac{1}{k(3k+1)^2}$

$= \int_0^1 \log(1-x^3)\log x\, dx$

.0800000000000000000$\underline{0}$ $= \frac{2}{25} = \sum_{k=1}^{\infty} \frac{\mu(k)}{10^k + 1}$

.080039732245114496725... $\approx 2\zeta(3) - \frac{251}{108} = -\psi^{(2)}(4) = \int_1^{\infty} \frac{\log^2 x}{x^5 - x^4} dx$

= $\int_0^1 \frac{x^3 \log x}{1-x} dx$

2 .08008382305190411453... ≈ $3^{2/3} = \sqrt[3]{9}$

1 .08012344973464337183... ≈ $\sqrt{\frac{7}{6}} = 1 + \sum_{k=1}^{\infty} \frac{(2k-1)!!}{(2k)!!7^k}$

.08017209138601023641... ≈ $\frac{\pi^4}{1215} = \int_1^{\infty} \frac{\log^3 x}{x^4 - x} dx$

.0803395836283518706... ≈ $\frac{\log 3}{2} - \frac{\pi}{6\sqrt{3}} - \frac{1}{6} = \sum_{k=1}^{\infty} \frac{1}{(3k+2)(3k+3)}$

= $\sum_{k=2}^{\infty} \frac{\zeta(k)-1}{3^k} = \sum_{k=2}^{\infty} \frac{1}{3k(3k-1)} = \int_1^{\infty} \frac{dx}{x^6 + x^5 + x^4}$

.08035760321666974058... ≈ $\log^2 2 - \gamma \log 2 = \sum_{k=2}^{\infty} \frac{(-1)^{k+1} \log k}{k}$

.080362945260742636423... ≈ $-\log\left(\frac{4\pi^2}{\cosh \pi\sqrt{2} - \cos \pi\sqrt{2}}\right) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}\left(\zeta(4k) - 1\right)$

= $\sum_{k=2}^{\infty} \log\left(1 + \frac{1}{k^4}\right)$

.08037423860937130786... ≈ $\sum_{k=2}^{\infty} \frac{1}{k^4 + k^{-1}} = \sum_{k=1}^{\infty} (-1)^{k+1}\left(\zeta(5k-1) - 1\right)$

.080388196756309001668... ≈ $\frac{1}{36}\left(96 \log 2 - 18 \log^2 2 - 55\right) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{H_k}{k+4}$

.0805396774868055356... ≈ $\sum_{k=1}^{\infty} \frac{\sigma_1(k)}{3^k} = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\sigma(mn)}{2^{mn}}$

1 .08056937041937356833... ≈ $\sum_{k=2}^{\infty} \frac{\log^3 k}{6k(k-1)}$

1 .08060461173627943480... ≈ $2 \cos 1 = \frac{1}{2}\left(e^i + e^{-i}\right)$

.08065155633076705272... ≈ $\frac{1}{4} - \frac{1}{2(e-1)^2} = \sum_{k=1}^{\infty} \frac{k B_{2k}}{(2k)!}$

.080836672802165433362... ≈ $\frac{\pi}{10}\left(\sqrt{50 - 10\sqrt{5}} - 5\right) = \int_0^1 \frac{1 - x^2}{x^2} \arctan(x^5)\, dx$

.081045586232111422... ≈ $\sum_{k=2}^{\infty} \frac{(-1)^k}{k^4 \log k}$

.08106146679532725822... ≈ $1 - \log \sqrt{2\pi} = \sum_{k=2}^{\infty} \left(\frac{1}{k+1} - \frac{1}{2k}\right)\left(\zeta(k) - 1\right)$ K ex. 125

= $-\int_1^2 \log\Gamma(x)\,dx$ GR 6.441

= $-\int_0^1 \left(\frac{1}{\log x}+\frac{1}{1-x}-\frac{1}{2}\right)\frac{dx}{\log x}$ GR 4.283.2

.081019302162163287639... ≈ $\frac{1}{5}\log\frac{3}{2} = \frac{1}{5}\sum_{k=1}^{\infty}\frac{1}{3^k k}$

122 .081167438133896765742... ≈ $\frac{8\pi^6}{63} = 120\zeta(6) = \psi^{(5)}(1) = -\int_0^1 \frac{\log^5 x}{1-x}dx$

.08126850326921835705... ≈ $\frac{13}{15}-\frac{\pi}{4} = \sum_{k=0}^{\infty}\frac{(-1)^k}{2k+7} = \int_1^{\infty}\frac{dx}{x^8+x^6} = \int_0^{\pi/4}\tan^6 x\,dx$

.081317489046618966975... ≈ $\int_1^{\infty}\left(\zeta(2k+1)-1\right)dx$

.08134370606024783679... ≈ $\frac{i\pi}{6}\left(\cot\left(\frac{\pi}{2}\left(\sqrt{3}-4+i\right)\right)-\cot\left(\frac{\pi}{2}\left(\sqrt{3}-4-i\right)\right)\right) - \frac{1}{2}-\frac{\coth\pi}{6}$

= $\sum_{k=2}^{\infty}\frac{k^2}{k^6+1} = \sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(6k-2)-1\right)$

.08135442427619763926... ≈ $9\sqrt{2}\arcsin\frac{1}{3} - 1 = \sum_{k=1}^{\infty}\frac{(2k)!!}{(2k-1)!!9^k(2k+1)}$

2 .08136898100560779787... ≈ $\frac{1}{\log^2 2} = \int_0^{\infty}\frac{x\,dx}{2^x}$

.08150589160708211343... ≈ $\frac{1}{2}-\log 2+\frac{\log 3}{4} = \int_1^{\infty}\frac{dx}{e^x(e^x+1)(e^x+2)}$

1 .08163898093058134474... ≈ $\frac{\pi^3+3\zeta(3)}{32} = \sum_{k=0}^{\infty}(-1)^k\left(\frac{1}{(2k+1)^3}+\frac{1}{(2k+2)^3}\right)$

.081812308687234198918... ≈ $\frac{5\pi}{192} = \int_0^{\infty}\frac{(\sin x - x\cos x)^3}{x^7}\,dx$ Prud. 2.5.29.24

.08183283282780650656... ≈ $\sum_{k=2}^{\infty}\frac{k^3}{k^7+1} = \sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(7k-3)-1\right)$

1 .08194757986905618266... ≈ $\frac{4}{\sqrt{\pi}}\sin\frac{1}{2} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!(k+\frac{1}{2})16^k}$

.08197662162246572967... ≈ $15\log\frac{3}{2} - 6 = \sum_{k=1}^{\infty}\frac{k}{3^k(k+1)(k+2)}$

.081976706869326424385... ≈ $\frac{1}{e-1} - \frac{1}{2} = \frac{3-e}{2(e+1)} = \frac{1}{2}\coth\frac{1}{2} - 1 = \sum_{k=1}^{\infty}\frac{B_k}{(2k)!}$

1 .081976706869326424385... ≈ $\frac{1}{2}\coth\frac{1}{2} \;=\; \frac{e^{1/2}+e^{-1/2}}{2\left(e^{1/2}-e^{-1/2}\right)}$

= $\sum_{k=0}^{\infty}\frac{B_{2k}}{(2k)!}$ AS 4.5.67

.0820312500000000000$\underline{0}$ = $\frac{21}{256} \;=\; \sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2}{7^k}$

.08217557586763947333... ≈ $\frac{1}{2}-\frac{\pi}{6\sqrt{3}}-\frac{\log 2}{6} \;=\; \sum_{k=0}^{\infty}\frac{(-1)^k}{6k+8} \;=\; \int_1^{\infty}\frac{dx}{x^9+x^3}$

.08220097694658271483... ≈ $\frac{\log^2 2}{2}-\log 2\log 3+\frac{\log^2 3}{2} \;=\; -\,Li_2\left(\frac{1}{3}\right)-Li_2\left(-\frac{1}{2}\right)$

.0822405264650125059... ≈ $\frac{\pi^2}{12}-\frac{\log^2 2}{2}-\frac{1}{2} = \sum_{k=1}^{\infty}\frac{1}{2^k(2k^2+4k+2)} \;=\; \sum_{k=2}^{\infty}\frac{1}{2^k k^2}$

1 .08232323371113819152... ≈ $\frac{\pi^4}{90} = \zeta(4) \;=\; MHS(2,1,1)$

= $\sum_{k=1}^{\infty}\frac{H^{(3)}{}_k}{k(k+1)}$

= $\frac{36}{17}\sum_{k=1}^{\infty}\frac{1}{k^4\binom{2k}{k}}$ Borwein-Devlin, p. 42

= $3\sum_{k=1}^{\infty}\frac{1}{k^4\binom{2k}{k}} - 9\sum_{k=1}^{\infty}\frac{1}{k^2\binom{2k}{k}}\sum_{j=1}^{k-1}\frac{1}{j^2}$

1 .082392200292393968799... ≈ $\sqrt{4-2\sqrt{2}} \;=\; \prod_{k=0}^{\infty}\left(1+\frac{(-1)^k}{4k+6}\right)$

.0824593807002189801... ≈ $\frac{\gamma}{7}$

.08248290463863016366... ≈ $5\sqrt{\frac{2}{3}}-4 \;=\; \sum_{k=0}^{\infty}\frac{(-1)^k k^2}{8^k}\binom{2k+1}{k} \;=\; \sum_{k=0}^{\infty}\frac{(-1)^k k}{8^k(k+1)}\binom{2k}{k}$

.08257027796642312563... ≈ $-\frac{1}{16}-\frac{\pi}{8}\coth\pi+\frac{\pi\sqrt{2}}{8}\frac{\sin\pi\sqrt{2}+\sinh\pi\sqrt{2}}{\cos\pi\sqrt{2}-\cosh\pi\sqrt{2}}$

= $\sum_{k=1}^{\infty}\left(\zeta(8k-4)-1\right) \;=\; \sum_{k=2}^{\infty}\frac{1}{k^4-k^{-4}}$

.08264462809917355372... ≈ $\frac{10}{121} \;=\; \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{10^k}$

.08268139191081858818... ≈ $4-4\log 2-\log\pi = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(k+1)(k+2)}\left(\zeta(k+1)-1\right)$

1 .08269110766346817971... ≈ $\frac{1}{2}\sum\frac{H_k}{k!} = \sum_{k=1}^{\infty}\frac{H^{e}{}_{k}}{k!}$

.082710571850225464607... ≈ $\frac{1}{24}\left(\gamma+\log 2\pi-1-\frac{6}{\pi^2}\zeta'(2)\right) = -\int_0^{\infty}\frac{x\log x}{e^{2\pi x}-1}dx$

1 .082762193260924580122... ≈ $\frac{6e^{\gamma}}{\pi^2} = \lim_{n\to\infty}\frac{1}{\log n}\prod_{p\ prime\le n}\left(1+p^{-1}\right)$

6 .082762530298219680... ≈ $\sqrt{37}$

.08282126964669066634... ≈ $\sum_{k=1}^{\infty}\left(\zeta(7k-3)-1\right) = \sum_{k=2}^{\infty}\frac{1}{k^4-k^{-3}}$

.08282567572904228772... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(6k-2)-1}{k} = -\sum_{k=2}^{\infty}k^2\log(1-k^6)$

.08285364400527561924... ≈ $-\frac{1}{64}\psi^{(2)}\left(\frac{3}{4}\right) = \frac{7\zeta(3)}{8}-\frac{\pi^3}{32}$

= $\frac{7\zeta(3)}{8}+\frac{i}{2}\left(Li_3(i)-Li_3(-i)\right)$

= $\int_0^1\frac{x^2\log^2 x}{1-x^4}dx = \int_1^{\infty}\frac{\log^2 x}{x^4-1}dx$

.082936658236923116009... ≈ $\frac{1}{16\cdot 2^{1/4}}\left(\cosh\frac{1}{2^{1/4}}\sin\frac{1}{2^{1/4}}-\cos\frac{1}{2^{1/4}}\sinh\frac{1}{2^{1/4}}\right)$

$+\frac{1}{8\sqrt{2}}\sin\frac{1}{2^{1/4}}\sinh\frac{1}{2^{1/4}}$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{2^k k^2}{(4k)!}$

15 .08305708802913518542... ≈ $\frac{41}{e} = \sum_{k=1}^{\infty}(-1)^k\frac{k^7+k^6}{(k-1)!} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{(2k-1)^4}{(k-1)!}$ Berndt 2.9.7

122 .08307674455303501887... ≈ $\sum_{k=2}^{\infty}\frac{\log^5 k}{k(k-1)} = \sum_{m=2}^{\infty}-\zeta^{(5)}(m)$

.0831028751794280558... ≈ $\frac{5}{2}\log^2\frac{5}{6} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}H_k}{5^k(k+1)}$

.0832000000000000000$\underline{0}$ = $\frac{276}{3125} = \sum_{k=1}^{\infty}(-1)^k\frac{k^4}{4^k}$

.08326043967407472341... ≈ $\frac{\pi^2\log 2}{18}-\frac{\log^3 2}{3}-2Li_3\left(-\frac{1}{2}\right)-\frac{15}{36}\zeta(3)$

= $\int_0^1\frac{\log^2(1+x)}{x(x+3)}dx$

	.083331464003213147...	≈	$\sum_{k=2}^{\infty}\frac{\nu(k)}{k^4} = \zeta(4)\sum_{k=1}^{\infty}\frac{\mu(k)}{k}\log\zeta(4k)$	
	$.08333333333333333333\underline{3}$	=	$\frac{1}{12} = \sum_{k=1}^{\infty}\left(\zeta(6k-2)-1\right) = \sum_{k=2}^{\infty}\frac{1}{k^4-k^{-2}}$	
		=	$\int_0^{\infty}\frac{dx}{e^{2\pi\sqrt{x}}-1}$	Prud. 2.5.38.9
2	$.08333333333333333333\underline{3}$	=	$\frac{25}{12} = H_4 = \sum_{k=1}^{\infty}\left(4^k\left(\zeta(2k)-1\right)-1\right)$	
1	.08334160052948288729...	≈	$\sum_{k=1}^{\infty}\frac{k!!}{(k^2)!}$	
	.08334985974162102786...	≈	$\sum_{k=1}^{\infty}\frac{k^3}{(k+3)^5} = \frac{257}{32} + \zeta(2) - 9\zeta(3) + 27\zeta(4) - 27\zeta(5)$	
1	.08334986772601479943...	≈	$\sum_{k=1}^{\infty}\frac{k!}{(k^2)!}$	
	.08335608393974190903...	≈	$\log\frac{5}{3}\log 2 + Li_2\left(-\frac{2}{3}\right) - Li_2\left(-\frac{1}{3}\right) = \int_0^1\frac{\log(1+x)}{x+4}$	
	.08337091074632371852...	≈	$\sum_{k=1}^{\infty}\frac{\zeta(4k)-1}{k^2} = \sum_{k=2}^{\infty}Li_2\left(\frac{1}{k^4}\right)$	
2	.08338294068338349588...	≈	$\cos 1 + \cosh 1 = 2\sum_{k=0}^{\infty}\frac{1}{(4k)!}$	
	.08339856386374852153...	≈	$\prod_{k=1}^{\infty}\left(1-2^{-k}\right)^2$	
1	.08343255638471000236...	≈	$\frac{1}{2}\left(\cos\sqrt[4]{2}+\cosh\sqrt[4]{2}\right) = \sum_{k=0}^{\infty}\frac{2^k}{(4k)!}$	
	.08357849190965680633...	≈	$\sum_{k=2}^{\infty}\frac{\nu(k)}{k^4} = \sum_{p\ prime}\frac{1}{4^p-1}$	
1	.08368031294113097595...	≈	$\frac{\cosh\pi\sqrt{2}-\cos\pi\sqrt{2}}{4\pi^2} = \prod_{k=2}^{\infty}\left(1+\frac{1}{k^4}\right)$	
	.08371538929982973031...	≈	$\frac{47\log 2}{2} + \frac{21\log^2 2}{2} - \frac{85}{4} = \sum_{k=1}^{\infty}\frac{k^2 H_k}{2^k(k+1)(k+2)(k+3)}$	
57	.08391839763994994257...	≈	$21e$	
8103	.08392757538400770999 7...	≈	e^9	
1	.08395487733873059...	≈	$H^{(3)}{}_{3/2}$	
	.0840344058227809849...	≈	$1 - G = \sum_{k=1}^{\infty}\frac{(-1)^k}{(2k+1)^2}$	

$= \sum_{k=1}^{\infty}\frac{k\zeta(2k+1)}{16^k} = \sum_{k=1}^{\infty}\frac{16k}{(16k^2-1)^2}$

$= \frac{1}{8}\sum_{k=1}^{\infty}\frac{k}{2^k}\zeta\left(k+1,\frac{3}{4}\right)$ Adamchik (29)

$= \int_1^{\infty}\frac{\log x}{x^4+x^2}dx$

.08403901165073792345... $\approx -2\log\left(2\sin\frac{1}{2}\right) = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{B_{2k}}{(2k)!k}$ AS 4.3.71

$= 2\sum_{k=1}^{\infty}\frac{\cos k}{k}$

.08406950872765599646l... $\approx \frac{2}{3}+\frac{1}{\sqrt{2\pi}}\zeta\left(\frac{1}{2}\right) = \sum_{k=1}^{\infty}\left(\frac{1}{\sqrt{2\pi k}}-\frac{k^k}{k!e^k}\right)$ Borwein-Devlin, p. 82

3 .08425137534042456838б... $\approx \frac{5\pi^2}{16} = \sum_{k=1}^{\infty}\frac{k^4}{(k^2-1/4)^3}$

.08427394416468169797... $\approx \frac{\pi}{8}-\frac{\pi^2}{32} = \sum_{k=1}^{\infty}\frac{1}{(4k-3)(4k-2)^2(4k-1)}$ J271

.08434693861468627076... $\approx \sum_{k=2}^{\infty}\left(\zeta(k^2)-1\right)$

.08439484441592982708... $\approx \sum_{k=1}^{\infty}\left(\zeta(5k-1)-1\right) = \sum_{k=2}^{\infty}\frac{1}{k^4-k^{-1}}$

.084403571029688132... $\approx \sum_{k=1}^{\infty}\frac{\zeta(4k)-1}{k!} = \sum_{k=2}^{\infty}\left(e^{k^{-4}}-1\right)$

.084410950559573886889... $\approx \frac{\pi}{2} - \cos 1 + si(1) = -\int_1^{\infty}\frac{\cos x}{x^2}dx$

.08444796251617794106... $\approx \log\frac{4\pi}{\sinh\pi} = \sum_{k=1}^{\infty}\frac{\zeta(4k)-1}{k} = -\sum_{k=2}^{\infty}\log(1-k^{-4})$

$= \log 2 + \log\Gamma(2+i) + \log\Gamma(2-i)$

.08452940066786559145... $\approx \sum_{k=1}^{\infty}\frac{\zeta(3k+1)-1}{k^2} = \sum_{k=2}^{\infty}\frac{1}{k}Li_2\left(\frac{1}{k^3}\right)$

.08475613914377404036б... $\approx 1-\frac{1}{2}\cot\frac{1}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}B_{2k}}{(2k)!}$

.0848049724711137773... $\approx e^{-\pi^2/4} = i^{\log i} = e^{\log^2 i}$

.0848054232899400527... $\approx \frac{\pi^4}{15}+\frac{\pi^2}{4}\log^2 2-\frac{\log^4 2}{2}-6Li_4\left(\frac{1}{2}\right)-\frac{21}{4}\zeta(3)\log 2$

$= \int_0^1 \frac{\log^3(1+x)}{x(x+1)}\,dx$

$.0850000000000000000\underline{0} = \frac{17}{200}$

$.08502158796965488786\ldots \approx \sqrt{2}\arctan\frac{1}{\sqrt{2}} - \frac{\pi}{4} = \int_0^{\pi/4} \frac{\sin^2 x}{1+\cos^2 x}\,dx$

4 $.08511300769289273520\ldots \approx \sum_{k=2}^{\infty} \frac{k}{d_k}$

1 $.08514266435747008433\ldots \approx \int_1^2 \frac{dx}{\Gamma(x)}$

$.08515182106984328682\ldots \approx \frac{178}{225\pi} - \frac{1}{6} = \sum_{k=1}^{\infty} \left(\frac{(2k-1)!!}{(2k)!}\right)^2 \frac{1}{2k+6}$ J385

$.08540991615695945401\,5\ldots \approx 12 - \frac{\pi^2}{2} - 4\log 2 - \frac{7\zeta(3)}{2} = -\int_0^1 \log(1-x^2)\log^2 x\,dx$

$.0854160000000000000\underline{0} = \frac{41}{480} = \sum_{k=1}^{\infty} \frac{1}{k(k+2)(k+6)}$

$.08542507002951019287\ldots \approx \pi - \frac{5\pi^2}{24} - 1 = -\int_0^{\infty} \frac{\log(x^2+1)}{x^3(x+1)}\,dx$

1 $.08542507002951019287\ldots \approx \pi - \frac{5\pi^2}{24} = \int_0^{\infty} \frac{\log(x^2+1)}{x^2(x+1)}\,dx$

1 $.08544164127260700187\ldots \approx \sqrt{2}\sinh\frac{1}{\sqrt{2}} = \sum_{k=0}^{\infty} \frac{1}{(2k+1)!\,2^k}$

$= \prod_{k=1}^{\infty} \left(1 + \frac{1}{2\pi^2 k^2}\right)$

1 $.08549529259848404100\,1\ldots \approx \frac{1}{144}\left(16G + 8\pi^2\sqrt{3} + \psi^{(1)}\left(\frac{5}{12}\right) - \psi^{(1)}\left(\frac{11}{12}\right)\right) = \int_1^{\infty} \frac{\log x\,dx}{x^2 - 1 + x^{-2}}$

$.0855033697020790202\ldots \approx \sum_{k=1}^{\infty} \frac{1}{4^k(4^k-1)k}$

$.0855209595315045441\ldots \approx \frac{5}{4} - \frac{1}{3} - \gamma + Ei(-1) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!(k+2)^2}$

20 $.08553692318766774093\ldots \approx e^3$

2 $.08556188259741465263\ldots \approx \sum_{k=1}^{\infty} \frac{1}{2^{\phi(k)}}$

1 $.08558178301061517156\ldots \approx \sum_{k=1}^{\infty} \frac{2^k}{2^{k^2}(2^k-1)}$ Berndt Sec. 4.6

$= \quad \sum_{k=1}^{\infty} \frac{u_2(k)}{2^k}$

$.08559047211209554183\ldots \approx \quad \sum_{k=2}^{\infty} \frac{\log k}{k^4 - k^2}$

$.08562826820107386691 5\ldots \approx \quad \frac{\zeta(3)}{2} - \frac{\pi^2}{24} - \frac{5}{48} = \sum_{k=1}^{\infty} \frac{H_k H_k}{k(k+1)(k+2)(k+3)}$

$.08564884826472105334\ldots \approx \quad \frac{\log 2}{3} + \frac{\pi}{3\sqrt{3}} - \frac{3}{4} = \sum_{k=0}^{\infty} \frac{(-1)^k}{3k+7} = \int_1^{\infty} \frac{dx}{x^8 + x^5}$

$.08569886194979188645\ldots \approx \quad \frac{\log 2}{3} - \frac{1}{6} + \frac{1+i\sqrt{3}}{12}\left(\psi\left(\frac{3-i\sqrt{3}}{4}\right) - \psi\left(\frac{5-i\sqrt{3}}{4}\right)\right)$

$+ \frac{1-i\sqrt{3}}{12}\left(\psi\left(\frac{3+i\sqrt{3}}{4}\right) - \psi\left(\frac{5+i\sqrt{3}}{4}\right)\right)$

$= \quad \sum_{k=2}^{\infty} \frac{(-1)^k}{k^3 + 1}$

$.08580466432218341253\ldots \approx \quad \sum_{k=2}^{\infty} \frac{(-1)^k}{2^k \zeta(k)}$

1 $.08586087978647216963\ldots \approx \quad \gamma + 3\log 2 - \frac{\pi}{2} = -\psi\left(\frac{3}{4}\right)$ GR 8.366.5

$.08597257226217569949\ldots \approx \quad \frac{1}{2}\left(\frac{\sinh 1}{\cosh 1 - \cos 1} - 1\right) = \sum_{k=1}^{\infty} \frac{\cos k}{e^k}$

$.08612854633416616715\ldots \approx \quad \frac{\pi^3}{360} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^3 \sinh \pi k}$

1 $.08616126963048755696\ldots \approx \quad e + \frac{1}{e} - 2 = 2\cosh 1 - 2 = \sum_{k=0}^{\infty} \frac{1}{(2k+1)!(k+1)}$

3 $.08616126963048755696\ldots \approx \quad e + \frac{1}{e} = 2\cosh 1 = 4\sinh^2 \frac{1}{2} = 2\sum_{k=0}^{\infty} \frac{1}{(2k)!}$

$.08621373198939237287 7\ldots \approx \quad \frac{1-\sin 1}{8} \csc^2 \frac{1}{2} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k B_{2k}}{(2k)!}$

$.08621540796174563565\ldots \approx \quad \frac{\pi^3}{48} - \frac{\pi}{8} - \frac{\log 2}{4} + \frac{\log^2 2}{4} = \int_0^{\pi/2} (\log \sin x)^2 \sin^2 x \, dx$

$.08626673833405441469 7\ldots \approx \quad \operatorname{sech} \pi$

1 $.08640577051216805686 5\ldots \approx \quad \frac{7\zeta(3)}{4}(1 - \log 2) + 2\log 2 + \frac{\pi^4}{64} - \frac{\pi^2}{4} = \sum_{k=1}^{\infty} \frac{H_k}{(2k-1)^3}$

$.08641372548729102509 8\ldots \approx \quad \frac{\pi \log 2}{4} - \frac{G}{2} = \int_0^{\pi/4} \frac{\cos x - \sin x}{\cos x + \sin x} x \, dx$

$= \displaystyle\int_0^{\pi/4} \frac{x^2}{(\cos x+\sin x)^2}\,dx$

$= \displaystyle -\int_0^{\pi/4} \log\cos x\,dx$ GR 4.224.5

$.086419753\underline{086419753} = \displaystyle \frac{7}{81} = \sum_{k=1}^{\infty}\frac{\mu(k)}{9^k+1}$

$.08664207052890471645\ldots \approx \displaystyle \int_2^{\infty}\frac{\zeta(x)-1}{2^x}\,dx$

$.08664339756999316367\ldots \approx \displaystyle \frac{\log 2}{8} = \sum_{k=1}^{\infty}\frac{1}{2^{k+3}k} = \sum_{k=1}^{\infty}\frac{1}{32k^2-16k}$

$.08666297626570941293\ldots \approx \displaystyle \frac{7}{8}-\frac{\pi}{4}\coth\pi = \sum_{k=2}^{\infty}\frac{1}{k^4-1} = \sum_{k=1}^{\infty}\zeta(4k)-1$

$.08666887803007007899\ldots \approx \displaystyle \sum_{k=1}^{\infty}\frac{\zeta(3k+1)-1}{k!} = \sum_{k=2}^{\infty}\frac{1}{k}\left(e^{k^{-3}}-1\right)$

1 $.08676647749679035038\ldots \approx \displaystyle 8-\frac{\pi^2}{3}-8\log 2+4\log^2 2 = \sum_{k=0}^{\infty}\binom{2k}{k}\frac{1}{4^k(k+1)^3}$

$.086805555555555555555\underline{5} = \displaystyle \frac{25}{288} = \sum_{k=1}^{\infty}\frac{k^3}{(k+1)(k+2)(k+3)(k+4)(k+5)}$

$.0868345712119972545\ldots \approx \displaystyle \sum_{k=0}^{\infty}\frac{1}{k(k+1)!(k+2)!(k+3)!}$

$.0868515472540698021\ldots \approx \displaystyle \frac{1}{8}+\frac{\pi^2}{32}-\frac{\log 2}{2} = \int_1^{\infty}\frac{\log x}{(x+1)^3(x-1)}\,dx$

$.08686348852519960612\ldots \approx \displaystyle \sum_{k=1}^{\infty}\frac{\zeta(3k+1)-1}{k} = -\sum_{k=2}^{\infty}\frac{\log\left(1-k^{-3}\right)}{k}$

1 $.0868869024422304860\ldots \approx \displaystyle \log\left(-\frac{1}{2}\pi\sqrt{\frac{3}{2}}\csc\pi\sqrt{\frac{3}{2}}\right) = \sum_{k=1}^{\infty}\left(\frac{3}{2}\right)^k\frac{\zeta(2k)-1}{k}$

$.08701174543163461380\ldots \approx \displaystyle \frac{\pi}{64}(2\log 2-1) = -\int_0^{\pi/2}\log(\sin x)\sin^2 x\cos^2 x\,dx$

2 $.08706522863453295984\ldots \approx \displaystyle e^{2/e} = \sum_{k=0}^{\infty}\frac{2^k}{k!e^k}$

$.0871334029164977504\ldots \approx \displaystyle \frac{2\operatorname{arcsinh}1-\sqrt{2}}{4} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{4^k(2k+1)}\binom{2k}{k}$

$.0874405328813168631\ldots \approx \displaystyle 1-\frac{\pi}{8}-\frac{3\log 2}{4} = \frac{1}{4}\sum_{k=1}^{\infty}\frac{1}{4k^2+k} = \sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)}{4^k}$

= $\displaystyle\int_1^\infty \frac{dx}{x^5+x^4+x^3+x^2}$

.08746355685131195353... ≈ $\displaystyle\frac{30}{343} = \sum_{k=1}^\infty (-1)^{k+1}\frac{k^2}{6^k}$

1 .087473569826260952066... ≈ $\displaystyle\zeta(3) - \frac{11}{96} = \sum_{k=1}^\infty \frac{6k+8}{k^5+6k^4+8k^3}$

.08757509762437455880... ≈ $\displaystyle\frac{1}{2}\left(\cosh\frac{1}{\sqrt{2}} - \sqrt{2}\sinh\frac{1}{\sqrt{2}}\right) = \sum_{k=1}^\infty \frac{k}{(2k+1)!2^k}$

1 .087750469214439700994... ≈ $\displaystyle\frac{13\pi^8}{113400} = \frac{13}{12}\zeta(8) = \sum_{k=1}^\infty \frac{H^{(4)}{}_k}{k^4}$

.087764352700090443963... ≈ $\displaystyle\sum_{k=2}^\infty \frac{1}{4^k-1} = \sum_{k=1}^\infty \frac{1}{4^k(4^k-1)} = \sum_{k=2}^\infty \frac{\Omega(4^k)}{4^k}$

.08777477515499598892... ≈ $\displaystyle\frac{\log\pi}{3} - \frac{1}{18} - \frac{3\zeta(3)}{2\pi^2} = \sum_{k=1}^\infty \frac{\zeta(2k)}{4^k k(2k+3)}$

.087776475955337266058... ≈ $-\log G$

.087811230536858587545... ≈ $\displaystyle 2\zeta(2) - \zeta(3) - 2 = \sum_{k=1}^\infty \frac{k}{(k+1)^3(k+2)}$

2 .087811230536858587545... ≈ $\displaystyle 2\zeta(2) - \zeta(3) = \sum_{k=1}^\infty \frac{3k-1}{k^3(2k-1)^2} = \sum_{k=4}^\infty \frac{(k-1)\zeta(k)}{2^{k-2}}$

.087824916296374931495... ≈ $\displaystyle 2 - 2\sqrt{7}\arcsin\frac{1}{2\sqrt{2}} = \sum_{k=1}^\infty \frac{1}{\binom{2k}{k}2^k(2k+1)k}$

.087836323856249096291... ≈ $\displaystyle 24 - \frac{65}{e} = \sum_{k=0}^\infty \frac{(-1)^k}{k!(k+5)} = \int_1^e \frac{\log^4 x}{x^2}dx$

.087981169118280642883... ≈ $\displaystyle\frac{\pi}{6} + \frac{\log 2}{3} - \frac{2}{3} = \int_1^\infty \log\left(1+\frac{1}{x^6}\right)\frac{dx}{x^4}$

1 .0880000000000000000$\underline{0}$ = $\displaystyle\frac{136}{125} = \int_0^\infty \frac{x^2\sin^2 x}{e^x}dx$

.08800306461253316452... ≈ $\displaystyle\frac{J_1\left(2\sqrt{3}\right)}{\sqrt{3}} = \sum_{k=0}^\infty (-1)^k\frac{3^k}{k!(k+1)!}$

.088011138960... ≈ $\displaystyle\sum_{k=1}^\infty \frac{pf(k)}{k^4}$

.08802954295211401722... ≈ $\displaystyle -\frac{1}{6}\cos\frac{\pi\sqrt{7}}{2} = \prod_{k=1}^\infty\left(1-\frac{7}{(2k+1)^2}\right)$

.08806818962515232728... $\approx$ $\frac{\pi^4}{16} - 6 \;=\; \int_1^\infty \frac{\log^3 x}{x^4 - x^2} dx$

6 .08806818962515232728... $\approx$ $\frac{\pi^4}{16} \;=\; \int_0^\infty \frac{\log^3 x \; dx}{x^4 - 1} \;=\; \int_0^1 \frac{\log^3 x \; dx}{x^2 - 1}$

$= \int_0^\infty \frac{x^3 dx}{e^x - e^{-x}}$

1 .08811621992853265180... $\approx$ $\frac{4\pi}{\sinh \pi} \;=\; \prod_{k=2}^\infty \frac{k^4}{k^4 - 1} \;=\; 2\Gamma(2+i)\Gamma(2-i) \;=\; \exp \sum_{k=1}^\infty \frac{\zeta(4k) - 1}{k}$

.08825696421567695798... $\approx$ $Y_0(1)$

.08839538425779600797... $\approx$ $\frac{23}{72} - \frac{\log 2}{3} \;=\; \int_0^1 \log(1+x) \frac{1+x^2}{(1+x)^4} dx$ GR 4.291.23

.088424302496173583342... $\approx$ $\sum_{\omega \in S} \left(\zeta(\omega) - 1\right)$, where S is the set of all non-trivial integer powers

.0884395847555822353... $\approx$ $\sum_{k=2}^\infty \Omega(k)\left(\zeta(k) - 1\right)$

.08848338245436871429... $\approx$ $MHS(4,2) \;=\; \sum_{k > j \geq 1}^\infty \frac{1}{k^4 j^2}$

.088486758123283148089... $\approx$ $\frac{3\log 6}{2} + \log \pi - \gamma - \frac{19}{6} \;=\; \sum_{k=1}^\infty \frac{(-1)^{k+1} 2^k k^2}{(k+1)(k+2)} \left(\zeta(k+1) - 1\right)$

.08854928745850453352... $\approx$ $\frac{1}{2}\left(5\cos 1 \; - 3\sin 1\right) \;=\; \sum_{k=1}^\infty \frac{(-1)^{k+1} k}{(2k)!(2k+3)}$

.08856473461835375506... $\approx$ $\log\left(2\tan\frac{1}{2}\right) \;=\; \sum_{k=1}^\infty (-1)^{k+1} \frac{(2^{2k-1} - 1)B_{2k}}{(2k)!k}$ AS 4.3.73

.08876648328794339088... $\approx$ $\frac{1}{2} - \frac{\pi^2}{24} \;=\; \int_0^1 x \log(1 - x^2) \log x \, dx$

$= -\int_0^1 x \; \log x \; \log(1+x) \; dx$

1 .08879304515180106525... $\approx$ $\frac{\pi \log 2}{2} \;=\; \sum_{k=0}^\infty \binom{2k}{k} \frac{1}{4^k (2k+1)^2}$ Berndt 9.16.3

$= \int_0^1 \frac{\arcsin x}{x} dx \;=\; \int_0^{\pi/2} \log \sin x \, dx$

$= \int_0^1 \frac{x \arcsin x}{1 - x^2} dx$

$= -\int_0^1 \frac{\log x}{\sqrt{1-x^2}}dx$ GR 4.241.7

$= \int_0^1 \log\frac{1+x^2}{x}\frac{dx}{1+x^2}$ GR 4.298.8

$= \int_0^{\pi/2} \log(2\tan x)\,dx$ GR 4.227.3

$= \int_0^{\pi/4} \log(\tan x + \cot x)\,dx$ GR 4.227.15

$= \int_0^{\infty} \log(\coth x)\frac{dx}{\coth x}$ GR 4.375.2

$.0888888888888888888\underline{8} = \frac{4}{45}$

$.08894755261372487720... \approx \sum_{k=2}^{\infty}\frac{\Omega(k)}{k(k+1)} = \sum_{k=2}^{\infty}\sum_{j=2}^{\infty}\frac{1}{k^j\left(k^j+1\right)}$

$= \sum_{k=1}^{\infty}\sum_{m=2}^{\infty}(-1)^k\left(\zeta(mk+m)-1\right)$

$.08904125208589587299... \approx \frac{2\zeta(3)}{27} = \int_1^{\infty}\frac{\log^2 x}{x^4-x}dx = \int_1^{\infty}\frac{x^2\,dx}{e^{3x}-1}$

$= \int_0^1 \frac{x^2\log^2 x}{1-x^3}dx$

$.08904862254808623221... \approx \frac{3\pi}{16}-\frac{1}{2} = \sum_{k=1}^{\infty}\frac{\sin k\cos^3 k}{k} = \sum_{k=1}^{\infty}\frac{\sin k\cos^4 k}{k}$

$.0890738558907803451... \approx \frac{1}{e}+\sqrt{\pi}\left(erf(1)-1\right) = \int_1^{\infty}\frac{e^{-x^2}}{x^2}dx$

$.08917563788757070317 8... \approx \sum_{k=2}^{\infty}\frac{\zeta(k)-1}{(k+1)^2} = \sum_{k=2}^{\infty}\left(k\,Li_2\left(\frac{1}{k}\right)-1\right)$

$.08922988242575037903... \approx \frac{\pi^2}{3}-2-9\log^2 2+3\log 3-2\log 2+3\log 2\log 3-4Li_2\left(\frac{1}{4}\right)$

$= -\int_0^1 \log\left(1-x/2\right)\log(1+x/2)\,dx$

1 $.0892917406337479395... \approx \frac{\sqrt{e}}{2}\left(I_0\left(\frac{1}{2}\right)+I_1\left(\frac{1}{2}\right)\right) = \sum_{k=1}^{\infty}\frac{k}{k!4^k}\binom{2k}{k} = \sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!!\,(k-1)!}$

$.08936964648404897444... \approx \sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)}{2^k k^2} = \sum_{k=1}^{\infty}\left(Li_2\left(-\frac{1}{2k}\right)+\frac{1}{2k}\right)$

1 .08942941322482232241... ≈ $\frac{\sqrt{2}}{3}+\frac{\sqrt{2\pi}}{3}\Gamma\left(\frac{5}{4}\right)\Gamma^{-1}\left(\frac{3}{4}\right) = \frac{\sqrt{3}}{2}+\frac{1}{3}K\left(\frac{1}{\sqrt{2}}\right) = \int_0^1 \sqrt{1+x^4}\,dx$

.08944271909999158786... ≈ $\frac{1}{5\sqrt{5}} = \sum_{k=0}^{\infty}\frac{(-1)^k k}{16^k}\binom{2k}{k}$

.08948841356947412986... ≈ $\int_0^{\infty}\frac{dx}{\Gamma(x)} - \sum_{k=0}^{\infty}\frac{1}{\Gamma(k+1)} = \left(\int_0^{\infty}\frac{dx}{\Gamma(x)}\right) - e$

.08953865022801158500... ≈ $\frac{2\log 2}{7} - \frac{319}{2940} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2+7k}$

.08958558613365280915... ≈ $\frac{1}{3}+\frac{\sqrt{2}}{4}\left(\log(1+\sqrt{2}) - \frac{\pi}{2}\right)$

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{4k+7} = \int_1^{\infty}\frac{dx}{x^8+x^4}$

.089612468468018277509...≈ $\frac{10}{3}+8\log\frac{2}{3} = \sum_{k=0}^{\infty}\frac{(-1)^k k}{2^k(k+2)} = \sum_{k=1}^{\infty}\frac{10}{3^k\left(\frac{15k^2}{2}+\frac{45k}{2}+15\right)}$

1 .08970831204984968123... ≈ $\frac{90}{180-\pi^4} = \frac{1}{2-\zeta(4)} = \sum_{k=1}^{\infty}\frac{f(k)}{k^4}$ Titchmarsh 1.2.15

.089738549735380789530...≈ $\sum_{k=2}^{\infty}\frac{1}{k(k+1)(k+2)\log k}$

1 .08979902432874075413... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k^3} = \sum_{k=2}^{\infty}Li_3\left(\frac{1}{k}\right)-\frac{1}{k}$

7 .0898154036220641092... ≈ $4\sqrt{\pi}$

1 .08997420836724444733... ≈ $\sqrt{\pi}\,erfi\left(\frac{1}{2}\right) = \sum_{k=0}^{\infty}\frac{1}{k!4^k(2k+1)}$

.090000000000000000000$\underline{0}$ = $\dfrac{9}{100} = \sum_{k=1}^{\infty}(-1)^k \dfrac{k}{9^k}$

2 .09002880877233363966... ≈ $\sum_{k=2}^{\infty}\dfrac{\zeta(k)}{2^{k-2}(k-1)} = -\sum_{k=1}^{\infty}\dfrac{2}{k}\log\left(1-\dfrac{1}{2k}\right)$

.09002940262424972530 8... ≈ $\dfrac{1}{2}-\dfrac{\log 2}{2}+\dfrac{1}{8}\left(\psi(i)+\psi(-i)-\psi\left(-\dfrac{1}{2}+i\right)-\psi\left(-\dfrac{1}{2}-i\right)\right)$

= $\int_0^{\infty}\dfrac{\sin^2 x}{e^x\left(e^x+1\right)}dx$

.09005466571394460992... ≈ $\zeta(2)-\log^2 2-2Li_3\left(\dfrac{1}{2}\right) = \sum_{k=1}^{\infty}\dfrac{k}{2^k(k+1)^3}$

24 .09013647349572026364... ≈ $\dfrac{57\pi^5}{512\sqrt{2}} = \int_0^{\infty}\dfrac{\log^4 x}{1+x^4}dx$

.09016994374947424102... ≈ $\dfrac{2}{11+5\sqrt{5}} = \dfrac{1}{\varphi^5}$

3 .09016994374947424102... ≈ $\dfrac{5\left(\sqrt{5}-1\right)}{2} = 10\cos\dfrac{2\pi}{5} = 10\cos\dfrac{\pi}{5}-5 = \sum_{k=0}^{\infty}\dfrac{1}{5^k}\binom{2k+1}{k}$

11 .09016994374947424102... ≈ $\dfrac{11+5\sqrt{5}}{2} = \varphi^5$

.09018615277338802392... ≈ $\dfrac{47}{60}-\log 2 = \sum_{k=0}^{\infty}\dfrac{(-1)^k}{k+6} = \sum_{k=1}^{\infty}\dfrac{1}{4k^2+18k+20}$

= $\int_1^{\infty}\dfrac{dx}{x^7+x^6}$

.09027777777777777777$\underline{7}$ = $\dfrac{13}{144} = \sum_{k=1}^{\infty}\dfrac{1}{k(k+3)(k+4)}$

.09030864473528205964 4... ≈

$$\frac{1}{20\left(1+\sqrt{5}\right)}\left(10\left(1+\sqrt{5}\right)arc\cot 2-2\left(5+2\sqrt{5}\right)arc\coth\left(1+\sqrt{5}\right)+\left(-5+\sqrt{5}\right)\operatorname{arctan}h\frac{1+\sqrt{5}}{4}\right)$$

= $\sum_{k=1}^{\infty}\dfrac{F_k F_k}{4^k(2k-1)(2k+1)}$

1 .09033141072736823003... ≈ $\coth\dfrac{\pi}{2}$

1 .09035488895912495067 6... ≈ $16\log 2-10 = \sum_{k=2}^{\infty}\dfrac{k^2}{2^k(k+2)}$

.09039575736110422609... ≈ $1-{}_2F_1\left(\dfrac{1}{3},\dfrac{1}{2},\dfrac{4}{3},-1\right) = \sum_{k=1}^{\infty}\binom{2k}{k}\dfrac{(-1)^{k+1}}{4^k(3k+1)}$

$.09039730810583037952\ldots \approx \frac{\pi^3}{343} = \left(\frac{\pi}{7}\right)^3$

$.09041166498481915512\,7\ldots \approx -Li^{(1,0)}{}_0\left(-\frac{1}{2}\right) = \sum_{k=1}^{\infty} \frac{(-1)^k}{2^k} \log k$

$.09043060103985748999\,9\ldots \approx \frac{1}{9}\psi^{(1)}\left(\frac{2}{3}\right) - \frac{1}{4} = \sum_{k=2}^{\infty} \frac{1}{(3k-1)^2}$

$= \sum_{k=2}^{\infty} \frac{(k-1)\big(\zeta(k)-1\big)}{3^k}$

$.09045749511556154465\ldots \approx 2 - 4\operatorname{arctanh}\frac{1}{2} + \log\frac{4}{3} = \sum_{k=1}^{\infty} \frac{1}{4^k k(2k+1)}$

$.09049312475579154852\ldots \approx \sum_{k=1}^{\infty} \operatorname{csch}(k\pi)$

$.09060992991398834735\ldots \approx \frac{1-\gamma}{2} + \log\left(\frac{\sqrt{\pi}}{2}\right) = \sum_{k=2}^{\infty} \frac{\zeta(k)-1}{2^k k}$

$= -\sum_{k=2}^{\infty} \left(\log\left(1-\frac{1}{2k}\right) + \frac{1}{2k}\right)$

5 $.090678729317165623\ldots \approx e^2 I_2(2) = {}_1F_1\left(\frac{1}{2}, 2, 4\right) = \sum_{k=0}^{\infty} \frac{1}{(k+1)!}\binom{2k}{k} = \sum_{k=1}^{\infty} \frac{c_k}{k!}$

$.09079510421563956943\ldots \approx 1 - \frac{1}{e} - \log(e-1) = \int_1^{\infty} \frac{dx}{e^x(e^x-1)}$

$.09085774767294840944\,2\ldots \approx \frac{e(e-1)}{(e+1)^3} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^2}{e^k}$

$.090909090909090909\underline{09} = \frac{1}{11}$

1 $.090909090909090909\underline{09} = \frac{12}{11} = \sum_{k=1}^{\infty} \frac{F_{3k}}{6^k}$

$.09094568176679733473\ldots \approx \frac{2}{7\pi}$

$.09095037993576241381\ldots \approx \frac{\pi^2 - 7\zeta(3)}{16} = \sum_{k=1}^{\infty} \frac{k}{(2k+1)^3}$

$.09116077839697731310\,6\ldots \approx \frac{1}{2}\log\frac{6}{5} = \operatorname{arctanh}\frac{1}{11} = \sum_{k=0}^{\infty} \frac{1}{11^{2k+1}(2k+1)}$ K148

1 $.09123025895312295309\,4\ldots \approx \frac{\pi}{\sqrt{21}} \tan\frac{\pi\sqrt{21}}{2} + \frac{1}{5} = \sum_{k=0}^{\infty} \frac{1}{k^2+3k-3}$

$.09128249034461018451\ldots \approx\ \approx \frac{1}{192}\left(15 + 4\pi^2 - 12\pi\coth\pi + 12\pi^2 \csc h^2 \pi\right) = \sum_{k=1}^{\infty} k\big(\zeta(4k)-1\big)$

$= \quad \sum_{k=2}^{\infty}\frac{1}{k^3(1-k^{-4})^2}$

$.09138522593601257980 4\ldots \approx \quad \frac{\pi^2}{108} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(3k+3)^2} = \sum_{k=1}^{\infty}6\frac{1}{\binom{2k}{k}k^2} = \int_1^{\infty}\frac{\log x}{x^4+x}$

$.09140914229522617680\ldots \approx \quad 5e-\frac{27}{2} = \sum_{k=1}^{\infty}\frac{k^2}{(k+3)!}$

1 $.09153795689727323714 4\ldots \approx \quad \frac{1}{8}\sqrt{\frac{5+\sqrt{5}}{2}}\Gamma\left(\frac{1}{5}\right) = \int_0^{\infty}\frac{\sin x^5}{x^5}dx$

$.09158433725626578896\ldots \approx \quad \frac{3\log 2}{4}-\frac{\pi}{8}-\frac{7}{36}+\frac{\pi^2-8G}{16}=\sum_{k=2}^{\infty}\frac{k(\zeta(k)-1)}{4^k}$

$.09159548440788735838\ldots \approx \quad \frac{4\cos 1-1}{17-8\cos 1} = \sum_{k=1}^{\infty}\frac{\cos k}{4^k}$

$.09160199373136531664\ldots \approx \quad \sum_{k=2}^{\infty}\frac{1}{k^4-2}$

1 $.09174406370390610145\ldots \approx \quad \frac{1}{G}$

2 $.09177232570165755748 3\ldots \approx \quad \frac{1}{2}\cosh\frac{\pi\sqrt{7}}{2}\operatorname{sech}\frac{\pi\sqrt{3}}{2} = \prod_{k=1}^{\infty}\frac{k^2+k+2}{k^2+k+1} = \prod_{k=1}^{\infty}\left(1+\frac{1}{k^2+k+1}\right)$

$.09178052520743498628\ldots \approx \quad \frac{\coth\pi}{4}-\frac{1}{2\pi} = \int_0^{\infty}\frac{\sin 2\pi x}{2e^{\pi x}\left(e^{\pi x}-1\right)}$

$.09180726255210907564\ldots \approx \quad \frac{1}{3}\left(2-2\gamma-\psi\left(\frac{3+i\sqrt{3}}{2}\right)-\psi\left(\frac{3-i\sqrt{3}}{2}\right)\right)$

$= \quad \sum_{k=2}^{\infty}\frac{1}{k^4-k} = \sum_{k=1}^{\infty}\left(\zeta(3k+1)-1\right)$

$.09189265634688859583\ldots \approx \quad \frac{1}{8}\left(3\sqrt{2}\sinh\frac{1}{\sqrt{2}}-2\cosh\frac{1}{\sqrt{2}}\right) = \sum_{k=1}^{\infty}\frac{k^2}{(2k+1)!2^k}$

$.09197896550006800954\ldots \approx \quad \frac{1}{25}\psi^{(1)}\left(\frac{4}{5}\right) = \sum_{k=1}^{\infty}\frac{1}{(5k-1)^2} = \int_1^{\infty}\frac{dx}{x^5-1}$

$.09199966835037523246\ldots \approx \quad \frac{1}{\pi^2+1} = \int_0^{\infty}\frac{\cos\pi x}{e^x}$

$.0920000000000000000\underline{0} = \quad \frac{23}{250}$

7 $.\underline{09215686274509803} = \quad \frac{3617}{510} = B_8$

.09216017129775954758... ≈ $\frac{\log 2}{3} - \frac{5}{36} = \int_1^\infty \log\left(1+\frac{1}{x^2}\right)\frac{dx}{x^7}$

.09221212614988646 38... ≈ $24 - \frac{5\pi^5}{64} = \int_1^\infty \frac{\log^4 x}{x^4+x^2}$

.09225098283750300 02... ≈ $\frac{\pi}{4} - \log 2 = \sum_{k=0}^\infty (-1)^k \frac{k}{(k+1)(2k+1)}$

= $\int_0^1 \frac{\log(1+x^2)}{(1+x)^2}dx = \int_1^\infty \log\left(1+\frac{1}{x^2}\right)\frac{dx}{(x+1)^2}$

.09246968585321224373... ≈ $\frac{\log 3}{4} + \frac{\pi\sqrt{3}}{36} - \frac{1}{3} = \sum_{k=1}^\infty \frac{1}{(3k+1)(3k+3)}$

.0925925925925925$\underline{925}$ = $\frac{5}{54} = \sum_{k=1}^\infty (-1)^{k+1}\frac{k^2}{5^k}$

.09259647551900505426... ≈ $\sum_{k=1}^\infty \frac{\mu(2k)k}{2^k}$

1 .09259842732005670329 5... ≈ $\prod_{k=1}^\infty \zeta(3k+1)$

.09262900490322795243... ≈ $\frac{4\pi+8\log 2-17}{12} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{k(2k+1)(k+2)}$

.09264169485372005900 6... ≈ $\gamma - 2\log 2 + 12\log Glaisher - \frac{15}{12} = \sum_{k=1}^\infty \frac{k}{k+2}\left(\zeta(2k+1)-1\right)$

.09266474976763326697... ≈ $\sum_{k=1}^\infty \frac{\zeta(2k+2)-1}{k} = -\sum_{k=2}^\infty \frac{\log(1-k^{-2})}{k^2}$

1 .09267147640207146772... ≈ $\frac{8}{\sqrt{7}}\arcsin\frac{1}{2\sqrt{2}} = \sum_{k=0}^\infty \frac{(k!)^2}{(2k+1)!2^k}$

1 .09268740912914940258... ≈ $\sum_{k=2}^\infty \frac{1}{k^3-7}$

1 .09295817894065074460... ≈ $\frac{16-4\pi}{\pi} = \sum_{k=0}^\infty \binom{2k}{k}^2 \frac{1}{16^k(k+1)^2}$

.09296716692904050529... ≈ $6Li_3\left(\frac{1}{3}\right) - 2 = \int_1^\infty \frac{\log^2 x}{3x^3-x^2}dx$

6 .093047859391469... ≈ $\sum_{k=2}^\infty \frac{k^2\zeta(k)}{k!} = \sum_{k=1}^\infty \left(\frac{e^{1/k}(k+1)}{k^2} - \frac{1}{k}\right)$

.09309836571057463192... ≈ $\frac{1}{2} - \frac{\gamma}{10} - \frac{\pi}{20}\cot\frac{\pi}{5} - \frac{\log 10}{10} + \dots$

= $+\frac{1}{5}\cos\frac{2\pi}{5}\log\sin\frac{\pi}{5}+\frac{1}{5}\cos\frac{4\pi}{5}\log\sin\frac{2\pi}{5}$

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{5k+7} = \int_1^{\infty}\frac{dx}{x^8+x^3}$

.0932390333047333804... ≈ $\frac{I_2(2)}{e^2} = {}_1\mathrm{F}_1(\frac{3}{2},2,-4) = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\binom{2k+1}{k}$

.09328182845904523536... ≈ $e-\frac{21}{8} = \sum_{k=0}^{\infty}\frac{1}{k!(k+2)(k+4)}$

.09331639977950052414... ≈ $\frac{18G+13}{32\pi}-\frac{1}{5} = \sum_{k=1}^{\infty}\left(\frac{(2k-1)!!}{(2k)!}\right)\frac{1}{2k+5}$ J385

.09334770577173107510... ≈ $\frac{22}{e}-8 = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^3}{k!(k+2)}$

2 .09341863762913429294... ≈ $\frac{3e^{1/3}}{2} = \sum_{k=0}^{\infty}\frac{pf(k)}{3^k}$

.09345743518225868261... ≈ $\frac{5-6\log 2}{9} = \sum_{k=1}^{\infty}\frac{1}{k(2k+1)(2k+3)}$

.093750000000000000$\underline{0}$ = $\frac{3}{32} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2}{3^k} = Li_{-2}(-3) = \sum_{k=1}^{\infty}\frac{\mu(k)}{8^k+1}$

.093840726775329790918...≈ $\frac{1}{2}\left(\log 32\pi+\gamma-5\right) = \sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)-1}{k(k+1)}$

2 .09392390426793432431... ≈ $\sum_{k=1}^{\infty}\frac{1}{k}\sin\frac{2}{k} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{2^{2k+1}\zeta(2k)}{(2k-1)!}$

.094033185418749513878...≈ $\frac{11\pi^4}{360}-\frac{\pi^3}{4}+\frac{5\pi^2}{6}-\frac{4\pi}{2}+\frac{5}{6}$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\cos^4 k}{k^4}$

.094129189912233448980...≈ $\frac{9\zeta(3)}{4\pi^2}-\frac{\log 2}{2}+\frac{1}{6} = \sum_{k=1}^{\infty}\frac{\zeta(2k)}{4^k(2k+3)}$

.09415865279831080583... ≈ $1-2\log 2+\log^2 2 = (1-\log 2)^2 = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{H_k}{(k+1)(k+2)}$

.09419755288906060813... ≈ $\frac{1}{2}-\frac{\csc 1}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2\pi^2-1}$

1 .09421980761323831942... ≈ $\frac{\Gamma(1/4)^4}{16\pi^2} = \prod_{k=1}^{\infty}\frac{(4k-1)^2}{(4k-1)^2-1}\frac{(4k+1)^2-1}{(4k+1)^2}$ J1058

.09432397959183673469... ≈ $\frac{1479}{15680} = \sum_{k=1}^{\infty}\frac{1}{k(k+1)(k+8)}$

2	.09439510239319549231...	≈	$\frac{2\pi}{3} = \arccos\left(-\frac{1}{2}\right)$	
	.09453489189183561802...	≈	$\frac{1}{2} + \log\frac{2}{3} = \sum_{k=1}^{\infty}\frac{k-1}{3^k k}$	J145
		=	$\sum_{k=2}^{\infty}\frac{(-1)^k}{2^k k} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^k(2k+2)} = \sum_{k=1}^{\infty}\frac{1}{3^k(2k^2+2k)}$	
2	.09455148154232659148...	≈	real root of Wallis's equation, $x^3 - 2x - 5 = 0$	
	.09457763372636695976...	≈	$\sum_{k=1}^{\infty}\frac{\mu(k)2^k}{3^k - 1}$	
	.094650320622476977272...	≈	$\operatorname{Re}\{\psi(i)\} = \frac{\psi(i)+\psi(-i)}{2}$	
2	.09471254726110129425...	≈	$\operatorname{arccsch}\frac{1}{4}$	
	.094715265430648914224...	≈	$\frac{1}{2} - \frac{4}{\pi^2} = \sum_{k=1}^{\infty}\left(\frac{(2k-1)!!}{(2k)!}\right)^4 \frac{4k+1}{(2k-1)(2k+2)}$	J393
	.094993498849088801...	≈	$\frac{\zeta'(2)}{\pi^2} = \frac{1}{\pi^2}\sum_{k=2}^{\infty}\frac{\log k}{k^2}$	Berndt 9(27.15)
	.09525289856156675428...	≈	$2 - \frac{\pi\sqrt{2}}{2} + 4\log 2 - 2\log\left(2+\sqrt{2}\right) = \sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!!2^k k(2k+1)}$	
	.095413366053212024...	≈	$8\sqrt{2}\,\operatorname{arctanh}\frac{1}{\sqrt{2}} - \frac{1037}{105} = \sum_{k=1}^{\infty}\frac{1}{2^k(2k+7)}$	
1	.09541938988358739798...	≈	$-\zeta\left(\frac{1}{2},\frac{3}{4}\right)$	
1	.095445115010332226914...	≈	$\sqrt{\frac{6}{5}} = \sum_{k=0}^{\infty}\frac{1}{24^k}\binom{2k}{k} = 1 + \sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!!6^k}$	
1	.0955050568967450275...	≈	$\sum_{k=1}^{\infty}\frac{(-1)^k}{\begin{bmatrix}2k\\k\end{bmatrix}}$	
	.095542551491662252...	≈	$e\,Ei(-2) - eEi(-1) - \frac{1}{e} = \int_0^1 \frac{x^2}{e^x(x+1)}dx$	
10	.095597125427094081792...	≈	$\psi^{(1)}\left(\frac{1}{3}\right)$	
11	.095597125427094081792...	≈	$\psi^{(1)}\left(\frac{4}{3}\right)$	
	.09569550305604923590...	≈	$-\sum_{k=1}^{\infty}\frac{\mu(3k)}{2^k+1}$	

.09574501814355474193... $\approx \displaystyle\sum_{k=3}^{\infty}\frac{1}{k^3-8}$

.09587033987177004744... $\approx \displaystyle\frac{5}{3}-\frac{\pi}{2} = \int_1^{\infty}\tanh\log x\,\frac{dx}{x^4}$

.09587469570927995875 8... $\approx \displaystyle\frac{\pi}{2}+2G+\log 2-4 = -\int_0^1 \log(1+x^2)\log x dx$

4 .09587469570927995876... $\approx \displaystyle 2G+\frac{\pi}{2}+\log 2 = -\int_0^1 \log\left(1+\frac{1}{x^2}\right)\log x\,dx$

.09589402415059364248... $\approx \displaystyle\frac{1}{3}\log\sum_{n=1}^{\infty}\frac{4}{3} = \left(-\frac{1}{3}+\sum_{k=1}^{\infty}\frac{H^{(n)}{}_k}{4^k}\right)$

.0960000000000000000$\underline{0}$ $= \displaystyle\frac{12}{125} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2}{4^k}$

.09601182917994165985... $\approx \displaystyle\frac{5\zeta(5)}{54} = \int_1^{\infty}\frac{\log^4 x}{x^4+x}dx$

.09602370915382101293 3... $\approx$

$$\frac{1}{20\left(3+\sqrt{5}\right)}\left(5\left(3+\sqrt{5}\right)arc\cot 2-2\left(5+2\sqrt{5}\right)arc\coth\left(1+\sqrt{5}\right)+\left(-5+\sqrt{5}\right)\arctan h\frac{1+\sqrt{5}}{4}\right)$$

$$= \sum_{k=1}^{\infty}\frac{F_kF_{k+1}}{4^k(2k-1)(2k+1)}$$

.09618807304432171431 2... $\approx \displaystyle\frac{1}{64}\left(\psi^{(1)}\left(\frac{3}{8}\right)-\psi^{(1)}\left(\frac{7}{8}\right)\right) = \int_1^{\infty}\frac{\log x}{1+x^4}dx$

.09620261081692214343 4... $\approx \displaystyle\frac{\gamma}{6}$

1 .09638355658938732799 3... $\approx \sqrt{\zeta(3)}$

.09655115998944373447... $\approx \displaystyle 2\zeta(5)-\zeta(2)\zeta(3) = \sum_{k=1}^{\infty}\frac{H_k}{(k+1)^4}$

$$= MHS(4,1) = MHS(3,1,1) = \sum_{k=2}^{\infty}\frac{1}{k^4 j} = \sum_{k=2}^{\infty}\sum_{j=1}^{k-1}\frac{1}{k^4 j}$$

.09657359027997265470 9... $\approx \displaystyle\frac{\log 2}{2}-\frac{1}{4} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{4k(k+1)}$ J368

$$= \sum_{k=0}^{\infty}\frac{(-1)^k}{2k+6} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+2)(2k+4)}$$

$$= \sum_{k=1}^{\infty}\frac{1}{2^{k+2}(k+1)}$$

= $\sum_{k=1}^{\infty}\frac{1}{2^k k(k+1)(k+2)}$ GR 1.513.7

= $\int_1^{\infty}\frac{dx}{x^7+x^5} = \int_0^{\pi/4}\sin^2 x\,\tan x\,dx = \int_0^{\pi/4}\tan^5 x\,dx$

= $\int_1^{\infty}\frac{\log x\,dx}{(x+1)^3} = -\int_0^1\frac{x\log x\,dx}{(x+1)^3}$

= $\int_1^{\infty}\log\left(1+\frac{1}{x^4}\right)\frac{dx}{x^5}$

= $\int_0^1\frac{\log x\,dx}{(\pi^2+\log^2 x)(1-x^2)}$ GR 4.282.4

.0966149347325... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^k\log k}{k+1}$

1 .09662271123215095765... ≈ $\frac{\pi^2}{9} = \sum_{k=0}^{\infty}\frac{k!}{2^k(k+1)(2k+1)!!} = \sum_{k=0}^{\infty}\frac{2(k!)^2}{(2k+2)!}$ J277

= W_2 J313

= $\sum_{k=1}^{\infty}\left(\frac{1}{6k-5}+\frac{1}{6k-1}\right)$ J338

= $\int_0^{\infty}\frac{\log x\;dx}{x^6-1}$

2 .09703831811430797744... ≈ $\frac{\pi^2\log 2}{3}+\frac{\log^3 2}{3}+2Li_3\left(-\frac{2}{3}\right)-2Li_3\left(-\frac{1}{2}\right)$

= $\int_0^1\frac{\log^2 x}{(x+\frac{1}{2})(x+\frac{3}{2})}dx$

.09720887469821693780 8... ≈ $\frac{e-2}{e^2} = \sum_{k=1}^{\infty}\frac{\mu(k)}{e^k+1}$

.09724912727289673905 9... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{2^k(2^k+1)}$

1 .09726402473266255681... ≈ $\frac{e^2-3}{4} = \sum_{k=1}^{\infty}\frac{2^k}{(k+2)!} = \sum_{k=1}^{\infty}\frac{2^k}{k!(k+5)}$

2 .09726402473266255681... ≈ $\frac{e^2+1}{4} = \sum_{k=0}^{\infty}\frac{2^k}{k!(k+2)}$

1 .09726596902822565952 1... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(k+1)}{\binom{2k}{k}} = 3\log 2-\frac{\pi}{2}+4\sum_{1}^{\infty}\frac{\sqrt{4k-1}}{(4k-1)^2}\arcsin\sqrt{\frac{1}{4k}}$

.0972727780048164... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^4-3}$

.0974619609922506557... ≈ $\frac{1}{37268}\left(\psi^{(4)}\left(\frac{7}{8}\right)-\psi^{(4)}\left(\frac{3}{8}\right)\right) = \int_1^\infty \frac{\log^4 x}{x^4+1}$

.0976835606442167302... ≈ $\frac{5+\pi\sqrt{2}\csc\pi\sqrt{2}}{4} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{k^2+4k+2}$

.097777777777777777777$\underline{7}$ = $\frac{22}{225} = \sum_{k=1}^\infty \frac{1}{k(k+2)(k+5)}$

.0978697894647404515 76... ≈ $9e-\frac{731}{30} = \sum_{k=1}^\infty \frac{1}{(k+1)!(k+6)}$

.0980262093913014 2116... ≈ $-\frac{i}{2}\left(Li_2(e^{3i})-Li_2(e^{-3i})\right) = \sum_{k=1}^\infty \frac{\sin 3k}{k^2}$

.0980914248960508 5351... ≈ $3-\frac{\pi^2}{12}-3\log 2 = \sum_{k=1}^\infty \frac{k-1}{4k^3+2k^2}$

= $\sum_{k=2}^\infty (-1)^k \frac{\zeta(k)-\zeta(k+1)}{2^k}$

.0981747704246810387... ≈ $\frac{\pi}{32} = \sum_{k=1}^\infty \frac{\sin^3 k\cos^3 k}{k} = \int_0^\infty \frac{dx}{(x^2+4)^2}$

1 .0981788360783483 2206... ≈ $\csc(\log\pi) = \frac{2i}{\pi^i-\pi^{-i}}$

.0982718364218131 6146... ≈ $-\log\Gamma\left(\frac{5}{4}\right)$

.0983754225913952 6500... ≈ $\frac{166}{3}+192\log\frac{3}{4} = \sum_{k=1}^\infty \frac{k}{4^k(k+3)}$

.0984573226303042 8595... ≈ $1-\frac{3\zeta(3)}{4} = 1-\eta(3) = \sum_{k=2}^\infty \frac{(-1)^k}{k^3}$

= $\int_0^1\int_0^1\int_0^1 \frac{x\,y\,z}{1+xyz}\,dx\,dy\,dz$

.0984910322058240152... ≈ $-\sum_{k=2}^\infty \frac{\mu(k)}{k^4-k^2}$

1 .098595665055303... ≈ $\sum_{k=1}^\infty \frac{\left(\zeta(2k)-\zeta(2k+1)\right)^2}{2^k}$

1 .0986122886681096914... ≈ $\log 3 = 2\,\text{arctanh}\frac{1}{2} = \sum_{k=0}^\infty \frac{1}{4^k(2k+1)}$

= $1+\sum_{k=1}^\infty \frac{1}{27k^3-3k}$ [Ramanjuan] Berndt Ch. 2, 2.2

= $Li_1\left(\frac{2}{3}\right) = \sum_{k=1}^\infty \frac{2^k}{3^k k}$ J 74

		=	$\int_0^\pi \frac{\sin x}{2+\cos x}dx$	
1	.09863968993119046285...	≈	$3Li_2\left(\frac{1}{3}\right) = \sum_{k=0}^{\infty}\frac{1}{3^k(k+1)^2} = 2\sum_{k=1}^{\infty}\frac{{H^{(2)}}_k}{3^k}$	
1	.098641964394156...	≈	Paris constant	
	.09864675579932628786...	≈	$\frac{1}{2}-\frac{si(2)}{4} = -\int_0^1 \log x \sin^2 x\, dx$	
1	.098685805525187...	≈	Lengyel constant	
	.0987335167120566091...	≈	$\frac{\pi^2}{24}-\frac{5}{16} = \sum_{k=2}^{\infty}\frac{(-1)^k}{(k^2-1)^2}$	J401
	$.098765432\underline{098765432}$	=	$\frac{8}{81} = \sum_{k=1}^{\infty}(-1)^k\frac{k}{8^k} = \int_1^\infty \frac{\log^4 x}{x^4}dx$	
	.0988145063222626381787...	≈	$\frac{\pi}{8}\coth\frac{\pi}{4}-\frac{1}{2} = \sum_{k=1}^{\infty}\frac{1}{16k^2+1}$	
1	.09888976333384898885...	≈	$\sum_{k=2}^{\infty}\zeta(k)\zeta(k+3)-1$	
5	.09901951359278483003...	≈	$\sqrt{26}$	
	.09902102579427790135...	≈	$\frac{\log 2}{7} = \frac{1}{7}\sum_{k=1}^{\infty}\frac{1}{2^k k} = \sum_{k=1}^{\infty}\frac{1}{28k^2-14k}$	
	.099151048958717950329...	≈	$\frac{1}{e}\left(1-H\left(\frac{e-1}{e}\right)\right) = \sum_{k=2}^{\infty}\frac{\zeta(k)-1}{e^k}$	
	.099151855335609679902...	≈	$\frac{\log 2\pi}{4}-\frac{1}{3}-\frac{\gamma}{3}-\zeta'(-1) = \sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k(k+2)}$	
2	.09921712010141008052...	≈	$\frac{4\sqrt{e}}{\pi} = \prod_{k=0}^{\infty}\left(1+\frac{k+2}{k+1}\right)$	
	.09926678198501025408...	≈	$\sum_{k=2}^{\infty}(-1)^k\frac{(\zeta(k)-1)^2}{2^k}$	
7	.0992851788909071140...	≈	$\sum_{k=1}^{\infty}\frac{k^2}{2^k-1} = \sum_{k=1}^{\infty}\frac{\sigma_2(k)}{2^k}$	
29809	.09933344621166650940...	≈	π^9	
	.0993486251243290835...	≈	$\frac{7715}{1728}+\zeta(2)-5\zeta(3) = \sum_{k=1}^{\infty}\frac{k}{(k+5)^3}$	
	.099445877615933685923...	≈	$\sum_{p\ prime}\frac{1}{p^2(p^2-1)}$	

2 .099501138291619401819... ≈ $\log\frac{3\pi\sqrt{3}}{2} = \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} (-1)^k \left(\frac{1}{3k} - \log\frac{3k+1}{3k}\right)$ Prud. 5.5.1.17

.099501341658675... ≈ $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{\zeta(2k+1)-1}{(2k)!}$

1 .0995841579311920324... ≈ $\frac{16}{9} + \frac{2\pi}{3} - 4\log 2 = \sum_{k=1}^{\infty} \frac{(k-\frac{1}{4})!}{(k+\frac{3}{4})!\,k}$

.099586088986234725... ≈ $2\log^2\left(\frac{4}{5}\right) = \sum_{k=1}^{\infty} (-1)^k \frac{H_k}{4^k (k+1)}$

.0996136275967890683... ≈ $\frac{\pi}{4} + \frac{\pi\log 2}{8} - \frac{1}{2} - \frac{G}{2} = \int_0^{\pi/4} x\tan^3 x\,dx$ GR 3.839.2

.0996203229953586595... ≈ $\frac{3}{2} - e - \gamma + Ei(1) = \sum_{k=2}^{\infty} \frac{1}{(k+1)!k}$

.09963420383459667011... ≈ $\frac{1}{2}\operatorname{csch}\pi\,\sinh\frac{\pi}{2} = \prod_{k=0}^{\infty} \frac{4k^2+1}{4k^2+4}$

1 .09975017029461646676... ≈ $\csc 2$

.0997670771886199093... ≈ $2 - \frac{3}{e} - \gamma + Ei(-1) = \sum_{k=0}^{\infty} \frac{(-1)^{k+1}k^2}{(k+1)(k+1)!}$

6 .0999422348144516324... ≈ $e\sqrt{\pi}\,I_0(1) = \sum_{k=0}^{\infty} \frac{(k-\frac{1}{2})!\,2^k}{(k!)^2}$

.09995405375460116541... ≈ $\log\Gamma\left(\frac{\pi-1}{\pi}\right) - \frac{\gamma}{\pi} = \sum_{k=2}^{\infty} \frac{\zeta(k)}{\pi^k k}$

$.10000000000000000000\underline{0} \quad = \quad \frac{1}{10}$

$= \quad \prod_{k=1}^{\infty}\frac{k(k+5)}{(k+2)(k+3)}$ J1061

8 $.100000067076033613307... \approx \quad 1/0.1234567891011121314$, inverse of Champernowne's number

$.10000978265649614187... \quad \approx \quad Li_{10}\left(\frac{1}{10}\right) = \Phi\left(\frac{1}{10},10,0\right) = \sum_{k=1}^{\infty}\frac{1}{10^k k^{10}}$

$.10015243644321545885... \quad \approx \quad \sum_{k=1}^{\infty}\frac{|\mu(k)|(-1)^k}{9^k}$

$.10017140859663285712... \quad \approx \quad \frac{\zeta(3)}{12}$

$.10031730167435392116... \quad \approx \quad \frac{3}{2}-\gamma-\frac{\pi^2}{12} = \sum_{k=3}^{\infty}\frac{\zeta(k)-1}{k} = -\sum_{k=2}^{\infty}\left(\frac{1}{2k^2}+\frac{1}{k}+\log\left(1-\frac{1}{k}\right)\right)$

$.10039121661661862552 6... \approx \quad \frac{93\zeta(5)}{8}-\frac{5\pi^5}{128} = \int_1^{\infty}\frac{\log^4 x}{x^4-1}dx$

$.1004753... \quad \approx \quad \sum_{\substack{\omega \text{ a non-trivial} \\ \text{integer power}}}\frac{1}{\omega^2}$, Goldbach's Theorem

1 $.10055082520425474103... \quad \approx \quad \frac{3}{16}\left(\pi^2-4\right) = \int_0^1\int_0^1\int_0^1\frac{x^2+y^2+z^2}{1-x^2y^2z^2}dx\,dy\,dz$

3 $.10062766802998201755... \quad \approx \quad \frac{\pi^3}{10}$

$.10063775311987115379 9... \approx \quad Li_4\left(\frac{1}{10}\right) = \Phi\left(\frac{1}{10},4,0\right) = \sum_{k=1}^{\infty}\frac{1}{10^k k^4}$

$.10068187878733366234... \quad \approx \quad \frac{\pi}{8}+\frac{G}{2}-\frac{3}{4} = -\int_0^1 x\arctan x\log x\,dx$

$.10085660243000683632... \quad \approx \quad \frac{3-2\log 2}{16} = \int_0^1\frac{\log x}{(x+1)^3}dx$

$.10096171... \quad \approx \quad \sum_{k=2}^{\infty}\frac{\Omega(k)}{k^2+1}$

$.10098700926218576184... \quad \approx \quad \frac{2\log 2}{3}-\frac{13}{36} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k(k+1)(k+3)}$

$.10102051443364380 36... \quad \approx \quad 5-2\sqrt{6} = \sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!\,3^k(k+1)}$

1 $.1010205144336438036... \quad \approx \quad 6-2\sqrt{6} = \sum_{k=0}^{\infty}\frac{1}{12^k(k+1)}\binom{2k}{k}$

.1010284515797971427... ≈ $\frac{\zeta(3)-1}{2}$

.10109738718799412444... ≈ $12\log 2 + 2\log^3 2 - 6\log^2 2 - 6 = \int\limits_0^1 \log^3(1+x)\,dx$

.1011181472985272809... ≈ $\frac{\pi}{64}\left(\pi - \coth\pi + \pi\left(3 - 2\pi\coth\pi\right)\operatorname{csch}^2\pi\right)$

= $\sum_{k=1}^{\infty} k^2\left(\zeta(4k)-1\right) = \sum_{k=2}^{\infty} \frac{k^4(k^4+1)}{(k^4-1)^3}$

7 .10126282473794450598... ≈ $K_3(1)$

.10128868447922298962... ≈ $Li_3\left(\frac{1}{10}\right) = \sum_{k=1}^{\infty} \frac{1}{10^k k^3} = \Phi\left(\frac{1}{10},3,0\right)$

.10131657816350450188 6...≈ $\sum_{k=2}^{\infty} \frac{(-1)^k \log k}{k^2} = \frac{\pi^2}{2} + \frac{\zeta'(2)}{2}$

.10132118364233777144... ≈ π^{-2}

.10148143668599877803... ≈ $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k(2k+1)3^k} = \frac{\pi\sqrt{3}}{3} + \log\frac{4}{3} - 2$

.101560303935709566... ≈ $\sum_{k=1}^{\infty} \frac{1}{11k^3+1} = \sum_{k=1}^{\infty} (-1)^{k+1}\frac{\zeta(3k)}{11^k}$

.10159124502452357539... ≈ $\frac{1}{10} + \frac{2\sqrt{5}}{5}\operatorname{csch}\pi\sqrt{5} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k^2+5}$

.10169508169903635176... ≈ $\sum_{k=2}^{\infty} \log\zeta(2k)$

2 .10175554773379178032... ≈ $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k!}$

.10177395490857989056... ≈ $\frac{G}{9} = \sum_{k=1}^{\infty} \left(\frac{1}{(12k-9)^2} - \frac{1}{(12k-3)^2}\right)$

= $\int\limits_0^{\infty} \frac{x\,dx}{e^{3x}+e^{-3x}} = \int\limits_1^{\infty} \frac{\log x\,dx}{x^4+x^{-2}}$

.10187654235021826540... ≈ $\frac{1}{\sqrt{2}}\arctan\frac{1}{\sqrt{2}} - \frac{1}{3} = \sum_{k=0}^{\infty} \frac{(-1)^{k+1}k}{2^k(2k+1)}$

1 .10198672033465253884... ≈ $\frac{8}{9} + \frac{20}{27}\log\frac{4}{3} = \sum_{k=1}^{\infty} \frac{k^2 H_k}{4^k}$

.10201133481781036474... ≈ $\sum_{k=1}^{\infty} \frac{\mu(k)}{2^k}$

.10204081632653061224 5... = $\frac{5}{49} = \sum_{k=1}^{\infty} \frac{\mu(k)}{7^k + 1}$

.10212913196448488799 8... ≈ $\frac{e^e - e - 1}{e^{e+2}} = \sum_{k=0}^{\infty} \frac{(-1)^k e^k}{k!(k+2)}$

.10213318704652863 03... ≈ $75 - \frac{161\sqrt[3]{e}}{3} = \sum_{k=0}^{\infty} \frac{k^3}{(k+2)!3^k}$

.10218644100153632824... ≈ $\frac{\pi}{28} + \frac{\log 2}{14} - \frac{5}{84} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)(2k+8)}$

.10228427314668489686... ≈ $\frac{1 - \log 2}{3} = \sum_{k=0}^{\infty} \frac{(-1)^k}{3k+6} = \sum_{k=1}^{\infty} \frac{1}{12k^2 + 6k} = \sum_{k=1}^{\infty} \frac{1}{2^k 3(k^2 + k)}$

$= \int_1^{\infty} \frac{dx}{x^7 + x^4}$

1 .10228427314668489686... ≈ $\frac{4 - \log 2}{3} = -\int_0^{\pi/2} \sin(3x) \log\tan x \, dx$

.10246777930717411874... ≈ $\frac{1}{2} - \frac{2\cos 1}{e} = \int_1^e \frac{\log^2 x \sin\log x}{x^2} dx = \int_0^1 \frac{x^2 \sin x}{e^x} dx$

.10253725064595696207... ≈ $\frac{1}{2}\left(\gamma - 2 - 4\log 2 - \log\pi\right) = \sum_{k=2}^{\infty} \frac{k-1}{2^k k}\left(\zeta(k) - 1\right)$

.10253743088359253275... ≈ $\sum_{k=1}^{\infty} \frac{1}{(10k)^k}$

.10261779109939113111... ≈ $Li_2\left(\frac{1}{10}\right) = \sum_{k=1}^{\infty} \frac{1}{10^k k^2} = \Phi\left(\frac{1}{10}, 2, 0\right)$

.1027777777777777777$\underline{7}$ = $\frac{37}{360} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2 + 6k}$

$= \int_0^1 x^5 \log(1+x) \, dx = \int_1^{\infty} \log\left(1 + \frac{1}{x}\right) \frac{dx}{x^7}$

.10280837917801415228... ≈ $\frac{\pi^2}{96} = \frac{\zeta(2)}{16} = \sum_{k=1}^{\infty} \frac{1}{(4k)^2}$

.10295335196822332547 5... ≈

$$\frac{1}{6144}\left(8\pi\left(24\pi G + 4\pi^2 \log 2 - \frac{3\pi\zeta(3)}{256}\right) + \psi^{(3)}\left(\frac{3}{4}\right) - \psi^{(3)}\left(\frac{1}{4}\right)\right)$$

$= -\int_0^{\pi/4} x^2 \log\sin x \, dx$

.103171586152707595114... ≈ $1 + Li_2\left(-\frac{1}{2}\right) = -\int_0^1 \frac{x\log x}{x+2}dx$

1 .10317951782010706548... ≈ $\frac{1}{4\sqrt{2}}\left(H\left(\frac{i}{2^{1/4}}\right) - H\left(\frac{1}{2^{1/4}}\right) + H\left(-\frac{i}{2^{1/4}}\right) - H\left(-\frac{1}{2^{1/4}}\right)\right)$

$= \sum_{k=1}^{\infty}\frac{\zeta(4k-1)}{2^k} = \sum_{k=1}^{\infty}\frac{k}{2k^4-1}$

1 .10322736392868822057... ≈ $\pi G + \frac{1}{16}\left(2\pi^2\log 2 - 35\zeta(3)\right) = \sum_{k=1}^{\infty}\frac{2^k}{\binom{2k}{k}k^3}$

2 .10329797038480241946... ≈ $\sum_{k=2}^{\infty}\frac{5}{k^3-5} = \sum_{k=1}^{\infty}5^k\left(\zeta(3k)-1\right)$

.10330865572984108688... ≈ $\frac{106}{15} - 8\sqrt{2}\arctan\frac{1}{\sqrt{2}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{2^k(2k+7)}$

1 .10358810559827853091... ≈ $\prod_{k=1}^{\infty}\left(1+\log\left(\zeta(2k)\right)\right)$

.10359958052928999945... ≈ $\frac{\zeta(3)}{4} - 2 = 2\lambda(3) - 2 = \int_1^{\infty}\frac{\log^2 x}{x^4-x^2}dx = \int_1^{\infty}\frac{x^2\,dx}{e^{3x}-e^x}$

$= \int_0^1 \frac{x^2\log^2 x}{1-x^2}dx$ GR 4.262.13

2 .10359958052928999945... ≈ $\frac{7\zeta(3)}{4} = 2\lambda(3) = 2\sum_{k=0}^{\infty}\frac{1}{(2k+1)^3}$

$= \int_0^1 \frac{\log^2 x}{1-x^2}dx$ GR 4.261.13

$= \int_1^{\infty}\frac{x^2\,dx}{e^x - e^{-x}}$

.10363832351432696479... ≈ $\frac{3}{e} - 1 = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2}{(k+1)!} = \sum_{k=0}^{\infty}(-1)^k\frac{k^5}{(k+1)!}$

1 .10363832351432696479... ≈ $\frac{3}{e} = \sum_{k=1}^{\infty}(-1)^k\frac{(2k-1)^2}{(k-1)!}$

.10365376431822690417... ≈ $\frac{\pi}{2e^e} = \int_0^{\infty}\frac{\cos ex}{1+x^2}dx$ AS 4.3.146

.10367765131532296838... ≈ $\frac{10}{9\pi} - \frac{1}{4} = \sum_{k=1}^{\infty}\left(\frac{(2k-1)!!}{(2k)!!}\right)^2\frac{1}{2k+4}$

.10370336342207529206... ≈ $\frac{\sqrt{\pi}}{2}\left(\frac{\log^2 2}{2} + \pi^2 - \log 2 + \frac{\gamma\log 2}{2} + \frac{\gamma^2}{8} - \frac{\gamma}{4}\right)$

$= \displaystyle\int_0^\infty x^2 e^{-x^2} \log^2 x \, dx$

.10387370626474633199... $\approx \displaystyle\frac{11}{24} - \frac{\pi\sqrt{2}\left(\cot \pi\sqrt{2} + \coth \pi\sqrt{2}\right)}{16} = \sum_{k=2}^\infty \frac{1}{k^4 - 4}$

$= \displaystyle\sum_{k=1}^\infty 4^{k-1}\left(\zeta(4k) - 1\right)$

.1039321458392100935... $\approx \displaystyle\frac{5 - 8\log 2 + 2\log^2 2}{4} = \sum_{k=1}^\infty (-1)^{k+1} \frac{H_k}{k+3}$

.10399139854430475376... $\approx \displaystyle\frac{G}{4} - \frac{1}{8} = \sum_{k=1}^\infty \frac{(-1)^k k}{(4k^2-1)^2}$

.10403647111890759328... $\approx \displaystyle\int_1^\infty \frac{dx}{x^6 + x^3 + 1}$

1 .1040997522396467546... $\approx \displaystyle\sum_{k=1}^\infty \frac{\zeta(4k)}{k(k+1)}$

.10413006886308670891... $\approx \displaystyle\frac{3\pi}{64\sqrt{2}} = \int_0^\infty \frac{dx}{(x^2+2)^3}$

1 .10421860013157296289... $\approx \displaystyle\sum_{k=0}^\infty \frac{1}{3^k(3k+1)}$

.10438069727200191854... $\approx \displaystyle\frac{\pi^2}{24} + \log 2 - 1 = \sum_{k=1}^\infty \frac{1}{4k^2(2k+1)} = \sum_{k=3}^\infty (-1)^{k+1} \frac{\zeta(k)}{2^k}$

$= \displaystyle\sum_{k=1}^\infty \frac{H_k}{2^k k(k+1)(k+2)}$

15 .10441257307551529523... $\approx \log 10!$

.10452866617695729446... $\approx \displaystyle\frac{\pi\sqrt{2}}{\sinh \pi\sqrt{2}} = \prod_{k=1}^\infty \frac{k^2}{k^2+2}$

.10459370928947691247... $\approx \displaystyle\frac{\gamma}{2} + \frac{\pi}{8} + \frac{\log 2}{4} - \frac{3}{4} = -\int_0^\infty \frac{x^2 \log x \sin x}{e^x} dx$

.10459978807807261686... $\approx \displaystyle\frac{\pi}{3\sqrt{3}} - \frac{1}{2} = \sum_{k=1}^\infty \frac{k!(k+1)!}{(2k+2)!} = \sum_{k=1}^\infty \frac{1}{\binom{2k}{k}(4k+2)}$

$= \displaystyle\int_1^\infty \frac{dx}{x^5 + x^4 + x^3}$

.1046120855592865083... $\approx si(1) - \sin 1 = \displaystyle 2\sum_{k=1}^\infty \frac{(-1)^{k+1} k}{(2k+1)!(2k+1)}$

$= \quad -\int_0^1 x\log x\sin x dx$

.10464479648887394501... $\approx \quad \frac{\pi}{2}\left({}_1F_1\left(\frac{1}{2},2,\frac{1}{4}\right)-1\right) \ = \ \frac{\pi}{2}\left(e^{1/8}\left(I_0\left(\frac{1}{8}\right)-I_1\left(\frac{1}{8}\right)\right)-1\right)$

$= \quad \sum_{k=1}^{\infty}\frac{(k-\frac{1}{2})!(k+\frac{1}{2})!}{(2k+1)!(k+1)!}$

.10469603169456307070... $\approx \quad \frac{8}{9}\log\frac{9}{8} \ = \ \sum_{k=1}^{\infty}(-1)^{k+1}\frac{H_k}{8^k}$

1 .1047379393598043171... $\approx \quad \frac{\sqrt{\pi}}{4}\left(erf\,1+erfi\,1\right) \ = \ \int_1^{\infty}\cosh\left(\frac{1}{x^2}\right)\frac{dx}{x^2}$

$= \quad 2\int_1^{\infty}\cosh\left(\frac{1}{x^4}\right)\frac{dx}{x^3}$

.104742177484448455438... $\approx \quad \frac{1}{256}\left(4\pi^2\sqrt{20-14\sqrt{2}}+\psi^{(1)}\left(\frac{5}{16}\right)-\psi^{(1)}\left(\frac{13}{16}\right)\right)$

$= \quad \int_1^{\infty}\frac{\log x\,dx}{x^4+x^{-4}}$

3814279.104760220592209219 6... $\approx \quad e^{e^e}$

1 .104780232756763784223... $\approx \quad \sum_{k=2}^{\infty}\frac{H_k}{k^2\log k}$

.10492617049176940466... $\approx \quad \sum_{k=1}^{\infty}(-1)^{k+1}k^3\left(\zeta(3k)-1\right) \ = \ \sum_{k=2}^{\infty}\frac{k^3(k^6-4k^3+1)}{(k^3+1)^4}$

.10493197278911564626... $\approx \quad \frac{617}{5880} \ = \ \sum_{k=1}^{\infty}\frac{1}{k(k+1)(k+7)}$

.10498535989448968551... $\approx \quad 71-43\sqrt{e} \ = \ \sum_{k=1}^{\infty}\frac{k^2}{(k+1)!2^k(k+3)}$

.105005729382340648409... $\approx \quad \sum_{k=2}^{\infty}\frac{\zeta(k)}{(k+1)^3} \ = \ \sum_{k=1}^{\infty}\left(k\,Li_3\frac{1}{k}-1\right)$

.10500911500948221002... $\approx \quad \log\frac{\pi}{2\sqrt{2}} \ = \ \sum_{k=1}^{\infty}\frac{\zeta(2k)}{16^k k} \ = \ -\sum_{k=1}^{\infty}\log\left(1-\frac{1}{16k^2}\right)$

$= \quad -\int_0^1\frac{1-x}{1+x}\,\frac{x^2}{1+x^2}\,\frac{dx}{\log x}$ GR 4.267.5

1 .10503487957029846576... $\approx \quad \prod_{k=1}^{\infty}\left(1+\frac{1}{\pi^2 k(k+1)}\right)$

.10506593315177356353... ≈ $\dfrac{7}{4}-\zeta(2) = \sum_{k=2}^{\infty}\dfrac{1}{k^4-k^2} = \sum_{k=2}^{\infty}\dfrac{1}{k(k+1)^2}$

= $\sum_{k=2}^{\infty}\left(\zeta(2k)-1\right) = \sum_{k=1}^{\infty}(-1)^{k+1}k\left(\zeta(k+2)-1\right)$

= $\sum_{\substack{\omega \text{ a non-trivial} \\ \text{integer power}}} \dfrac{1}{\omega^2-1} = \sum_{k=2}^{\infty}\dfrac{\Omega(k)}{k^2}$

5 .1050880620834140125... ≈ $\dfrac{13\pi}{8}$

.10513093186547389584... ≈ $\sum_{k=1}^{\infty}\dfrac{\left(\zeta(k)-1\right)^2}{(k!)^2}$

.10513961458004101794... ≈ $\dfrac{5}{8}-\dfrac{3\log 2}{4} = \sum_{k=2}^{\infty}\dfrac{1}{2^k(k^2-1)}$

.10516633568168574612... ≈ $\psi'(10)$

1 .10517091807564762481... ≈ $e^{1/10}$

.10518387310098706333... ≈ $\dfrac{\sin 1}{8} = \sum_{k=1}^{\infty}\dfrac{(-1)^{k+1}k^3}{(2k+1)!}$

.1052150134619642098... ≈ $\gamma - ci\left(\dfrac{\pi}{2}\right) = -\int_0^{\pi/2}\sin x\log x\,dx$

$.\underline{105263157894736842}$ = $\dfrac{2}{19}$

.10536051565782630123... ≈ $\log 10-\log 9 = \sum_{k=1}^{\infty}\dfrac{1}{10^k k} = \Phi\left(\dfrac{1}{10},1,0\right)$

= $2\operatorname{arctanh}\dfrac{1}{19} = \sum_{k=0}^{\infty}\dfrac{1}{19^{2k+1}(2k+1)}$ K148

1 .1053671693677152639... ≈ $\sum_{k=1}^{\infty}\dfrac{1}{(2^k-1)k^2} = \sum_{k=1}^{\infty}\dfrac{\sigma_{-2}(k)}{2^k}$

1 .10550574317015055093... ≈ $\zeta(3)+\zeta(2)\zeta(3)-2\zeta(5) = \sum_{k=1}^{\infty}\dfrac{k(k+2)H_k}{(k+1)^4}$

.1055728090000841214... ≈ $1-\dfrac{2}{\sqrt{5}} = \sum_{k=1}^{\infty}\dfrac{(-1)^{k+1}(2k-1)!!}{(2k)!\,4^k}$

.1056316660907490692... ≈ $\sum_{k=1}^{\infty}\dfrac{\zeta(3k)-1}{2^k} = \sum_{k=2}^{\infty}\dfrac{k}{2k^3-1}$

.10569608377461678485... ≈ $\dfrac{1-\gamma}{4} = \int_0^{\infty}x^3e^{-x^2}\log x\,dx$

.105762192887320219473... ≈ $\dfrac{i}{6}\left(\psi^{(3)}(1+i)-\psi^{(3)}(1-i)\right) = i\sum_{k=1}^{\infty}\left(\dfrac{1}{(k-i)^4}-\dfrac{1}{(k+i)^4}\right)$

$.10590811817894415833... \approx \frac{2\log 3}{3} + 2\log 2\log 3 - \log^2 2 - \frac{2\log 2}{3} - \log^2 3$

$= \sum_{k=1}^{\infty} \frac{(-1)^{k+1} kH_k}{2^k (k+1)}$

$.10592205557311457100... \approx 4\log 2 - \frac{8}{3} = \sum_{k=1}^{\infty} \frac{1}{2^k (2k+6)}$

1 $.10602621952710291551... \approx \frac{4}{\pi}\sinh\frac{\pi}{4} = \prod_{k=1}^{\infty}\left(1+\frac{1}{16k^2}\right)$

$.10610329539459689051... \approx \frac{1}{3\pi}$

$.10612203480430171906... \approx \frac{\pi}{10} + \frac{\log 2}{5} - \frac{26}{75} = \int_1^{\infty} \log\left(1+\frac{1}{x^2}\right)\frac{dx}{x^6}$

$.10618300668850053432... \approx \sum_{k=2}^{\infty} (-1)^{k+1} \frac{\zeta(k+1)-1}{k^2}$

$.10620077643952524149... \approx \sum_{k=2}^{\infty} \frac{1}{k^4 \log k} = -\int_4^{\infty} \left(\zeta(s)-1\right) ds$

$.10629208289690908211... \approx \frac{\pi}{4e^2} = \int_0^{\infty} \frac{\cos x}{x^2+4} dx$

$= -\int_0^{\pi/2} \cos(2\tan x)\sin^2 x\, dx$ GR 3.716.4

2 $.10632856926531259962... \approx \sum_{k=2}^{\infty} k\left(2^{\zeta(k)-1} - 1\right)$

$.10634708332087194237... \approx \frac{\log 2}{2} - \frac{\log^2 2}{2}$

$= \sum_{k=1}^{\infty} \frac{(-1)^k kH_k}{k+1}$

$.10640018741773440887... \approx \log^2 2 - \frac{\pi^2}{12} - Li_2\left(-\frac{1}{2}\right) = \int_0^1 \frac{\log(1+x)}{x+3}$

$.10642015279284592346... \approx \frac{1}{8}\left(i\psi^{(1)}(2+i) - i\psi^{(1)}(2-i) - \psi^{(2)}(2+i) - \psi^{(2)}(2-i)\right)$

$= \sum_{k=1}^{\infty} (-1)^{k+1} k^2 \left(\zeta(2k+1)-1\right) = \sum_{k=2}^{\infty} \frac{k(k^2-1)}{(k^2+1)^3}$

$.10649228722599689650... \approx 24\zeta(5) + 14\zeta(3) - \frac{2\pi^4}{5} - \zeta(2) - 1 = \sum_{k=2}^{\infty} \frac{1-11k+11k^2-k^3}{(k+1)^5}$

$= \sum_{k=1}^{\infty} (-1)^k k^4 \left(\zeta(k+1)-1\right)$

.106666666666666666$\underline{6}$ = $\frac{8}{75} = \int_0^1 x^4 \arccos x \, dx$

.10670678914431444686... ≈ $\frac{1}{6} - \frac{\pi\sqrt{3}}{12}\operatorname{csch}\frac{\pi\sqrt{3}}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{4k^2+3}$

.106843054835371727673... ≈ $\frac{\log\left(2-\sqrt{3}\right)}{\sqrt{3}} + \frac{1}{6}\log\left(26-15\sqrt{3}\right)\log\left(2-\sqrt{3}\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}2^k k}{\binom{2k}{k}(k+1)(2k+1)}$

.106855714568623758486... ≈ $\sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)-1}{2k+1} = \sum_{k=1}^{\infty}\left(\sqrt{k}\arctan\frac{1}{\sqrt{k}} - 1 + \frac{1}{3k}\right)$

.10691675656662222620... ≈ $\frac{1}{10} + \frac{\pi\sqrt{10}}{20}\operatorname{csch}\pi\sqrt{\frac{5}{2}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{2k^2+5}$

1 .10714871779409050301... ≈ $\arctan 2 = \arcsin\frac{2}{\sqrt{5}} = \sum_{k=0}^{\infty}(-1)^k\frac{2^{2k+1}}{2k+1}$

1 .1071585896687866549... ≈ $\int_2^{\infty} H_x\left(\zeta(x)-1\right)$

.10730091830127584519... ≈ $\frac{1}{2} - \frac{\pi}{8} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(4k^2-1)^2}$ GR 0.237.2

= $\sum_{k=1}^{\infty}\frac{1}{16k^2-1} = \sum_{k=1}^{\infty}\frac{\zeta(2k)}{16^k}$ GR 0.232.1

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{4k+6}$

= $\int_1^{\infty}\frac{1}{x^7+x^3}$

1 .10736702429642214912... ≈ $\frac{\pi^3}{28}$

.10742579474316097693... ≈ $1 + 4\log\frac{4}{5} = \sum_{k=1}^{\infty}\frac{1}{5^k k(k+1)} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{4^k(k+1)}$

.1076539192264845... ≈ one-ninth constant

.10767237579923111189... ≈ $10 - 6\sqrt{e} = \sum_{k=1}^{\infty}\frac{k}{(k+2)!\,2^k}$

3 .1077987001308638468... ≈ $\sqrt{e\pi}\,I_0\left(\frac{1}{2}\right) = \sum_{k=0}^{\infty}\frac{(k-\frac{1}{2})!}{(k!)^2}$

.10801497980775036754... $\approx$ $\frac{\pi}{4}\coth\frac{\pi}{2} - \gamma - \frac{1}{2}\left(1 + \psi\left(1+\frac{i}{2}\right) + \psi\left(1-\frac{i}{2}\right)\right)$

$= \sum_{k=1}^{\infty}\frac{k-1}{4k^3+k} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{4^k}\big(\zeta(2k)-\zeta(2k+1)\big)$

.10808308959542341351... $\approx$ $\frac{1-e^2}{8} = \frac{e\sinh 1}{4} = \sum_{k=0}^{\infty}\frac{(-1)^k 2^k}{(k+3)!}$

.10819766216224657297... $\approx$ $\frac{3}{2}\log\frac{3}{2} - \frac{1}{2} = \sum_{k=2}^{\infty}\frac{(-1)^k}{2^k(k^2-k)}$

.1082531754730548308... $\approx$ $\frac{\sqrt{3}}{16} = \sum_{k=0}^{\infty}\frac{(-1)^k k}{12^k}\binom{2k}{k}$

.10830682115332050466... $\approx$ $4G - \frac{32}{9} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k+3/2)^2}$

1 .10835994182903472933... $\approx$ $\sum_{k=1}^{\infty}\frac{1}{4^k\big(\zeta(k+1)-1\big)}$

1 .10854136010382960099... $\approx$ $\sum_{k=1}^{\infty}\frac{H_k}{2^k+1}$

.10874938913908651895... $\approx$ $\frac{\sqrt{\pi}}{16}\left(1-\frac{1}{e^4}\right) = \int_0^{\infty} e^{-x^2}\sin^2 x\cos^2 x\,dx$

.10900953585926706785... $\approx$ $\sum_{k=2}^{\infty}\frac{(\zeta(k)-1)^2}{k^2}$

.10905015894144553735... $\approx$ $\frac{27-4\pi\sqrt{3}}{48} = \int_1^{\infty}\log\left(1+\frac{1}{x^3}\right)\frac{dx}{x^5}$

.109096051791331928583...$\approx$ $\frac{71-6\pi^2}{108} = \sum_{k=1}^{\infty}\frac{1}{k(k+3)^2} = \int_0^1 x^2\log(1-x)\log x\,dx$

.10922574351594719161... $\approx$ $\frac{\log\pi}{2} - \frac{1}{4} - \frac{7\zeta(3)}{4\pi^2} = \sum_{k=1}^{\infty}\frac{\zeta(2k)}{4^k k(2k+2)}$

.10928875430471840710... $\approx$ $\int_0^1\frac{x^6\,dx}{1+x^{12}}$

.1093750000000000000$\underline{0}$ = $\frac{7}{64} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{k}{7^k}$

1 .10938951007484345298... $\approx$ $\sum_{k=1}^{\infty}\frac{1}{k!}\sin\frac{k}{2} = e^{\cos(1/2)}\sin\left(\sin\frac{1}{2}\right)$

.10946549987757265695... $\approx$ $\frac{1}{12} + \frac{\pi\sqrt{2}}{4}\operatorname{csch}\pi\sqrt{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2+2k+3}$

.10947570824873003253... $\approx$ $\frac{4\sqrt{2}-5}{6} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{4^k(k+2)}\binom{2k}{k}$

.10955986393152561224... $\approx \displaystyle\sum_{k=2}^{\infty}\frac{(\zeta(k)-1)^2}{2^k}$

.109575787818588751201... $\approx \displaystyle\frac{\pi^2}{432}-\frac{\pi}{12\sqrt{3}}+\frac{1}{144}\left(\psi^{(1)}\left(\frac{1}{6}\right)-\psi^{(1)}\left(\frac{2}{3}\right)\right)$

$= \displaystyle\sum_{k=0}^{\infty}\frac{(-1)^k}{(3k+1)^2(3k+3)^2}$

.10960862136441250153... $\approx 2+4Li_3\left(-\frac{1}{2}\right) = \displaystyle\int_1^{\infty}\frac{\log^2 x}{2x^3+x^2}dx$

.10965469252512710826... $\approx \displaystyle\frac{1}{2}\left(Ei(-e^2)-Ei(1)\right) = \int_0^1 e^{-e^{2x}}dx$

.10967200... $\approx \displaystyle\sum_{k=2}^{\infty}\frac{\Omega(k)}{k^2-1}$

2 .1097428012368919745... $\approx \displaystyle\sum_{k=2}^{\infty}\frac{1}{k\log^2 k}$

$.1097560975609756\underline{09756}$ $= \displaystyle\frac{9}{82} = \frac{1}{2\cosh\log 9} = \sum_{k=0}^{\infty}(-1)^k e^{-2(2k+1)\log 3}$ J943

.10981384722661197608... $\approx \displaystyle\log 2-\frac{7}{12}=\sum_{k=0}^{\infty}\frac{(-1)^k}{k+5}=\sum_{k=1}^{\infty}\frac{1}{4k^2+14k+12} = \int_1^{\infty}\frac{dx}{x^6+x^5}$

.1098873045414083466... $\approx \displaystyle\frac{\pi}{4}-\frac{\arctan\sqrt{2}}{\sqrt{2}} = \int_0^{\pi/4}\frac{\sin^2 x}{1+\sin^2 x}dx$

.109944999939506899848... $\approx$

$$\frac{1}{5\left(1+\sqrt{5}\right)}\left(\sqrt{5}\log\left(512\left(11+\sqrt{5}\right)\right)+11\log\left(11+\sqrt{5}\right)-2\left(7+2\sqrt{5}\right)\log\left(11-\sqrt{5}\right)-3\sqrt{5}\log\frac{4}{3}\right)$$

$= \displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}F_k F_{k+1}}{4^k k(k+1)}$

	.11005944310440489081...	≈	$\sum_{k=2}^{\infty}\frac{1}{k^3}\log\frac{k}{k-1}$	
	.11008536730149237917...	≈	$\left(3\sqrt{2}-2\right)\frac{\pi}{64}$	
	.11030407191369955112...	≈	$\frac{3\zeta(3)}{4}+\frac{\pi^2}{12}+2\log 2-3 = \sum_{k=2}^{\infty}\frac{(-1)^k}{k^4-k^3}$	
	.11037723272581821425...	≈	$\frac{1}{256}\left(\psi^{(1)}\left(\frac{1}{8}\right)+\psi^{(1)}\left(\frac{3}{8}\right)-\psi^{(1)}\left(\frac{5}{8}\right)-\psi^{(1)}\left(\frac{7}{8}\right)\right)-\frac{\log(1+\sqrt{2})}{4\sqrt{2}}$	
		=	$\sum_{k=0}^{\infty}\frac{(-1)^k}{(4k+1)^2(4k+3)^2}$	
9	.1104335791442988819...	≈	$\sqrt{83}$	
1	.11062653532614811718...	≈	$\frac{\zeta(3)}{\zeta(4)} = \sum_{k=1}^{\infty}\frac{\phi(k)}{k^4}$	Titchmarsh 1.2.13
1	.11072073453959156175...	≈	$\frac{\pi}{2\sqrt{2}} = 1+\sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!!2^k(2k+1)} = \sum_{k=1}^{\infty}\binom{2k}{k}\frac{1}{8^k(2k+1)}$	
		=	$\sum_{k=1}^{\infty}\left(\frac{(-1)^{k+1}}{4k-3}+\frac{(-1)^{k+1}}{4k-1}\right)$	J76, K ex. 108c
		=	$\sum_{k=0}^{\infty}\frac{(-1)^{\lfloor k/2\rfloor}}{2k+1}$	Prud. 5.1.4.3
		=	$\prod_{k=0}^{\infty}\left(1+\frac{(-1)^k}{2k+3}\right)$	
		=	$\int_0^{\infty}\frac{dx}{x^4+1}$	Seaborn p. 136
		=	$\int_0^{\infty}\frac{dx}{x^2+2} = \int_0^{\infty}\frac{dx}{2x^2+1} = \int_0^{\infty}\frac{x^2\,dx}{x^4+1}$	
		=	$\int_0^{1}\frac{dx}{(1+x^2)\sqrt{1-x^2}}$	
		=	$\int_0^{\pi/2}\frac{d\theta}{1+\sin^2\theta}$	
	.11074505351367928181...	≈	$\log\Gamma\left(\frac{2}{3}\right)-\frac{\gamma}{3} = \sum_{k=2}^{\infty}\frac{\zeta(k)}{3^k k} = -\sum_{k=1}^{\infty}\log\left(1-\frac{1}{3k}\right)-\frac{1}{k}$	
	.11083778744223266...	≈	$\sum_{k=0}^{\infty}\frac{(-1)^k}{k^3+4}$	
	.11094333469148877202...	≈	$\sum_{k=1}^{\infty}\frac{1}{10k^3+1} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(3k)}{10^k}$	

.11100175129090444718... $\approx \quad 9\zeta(4) - 6\zeta(3) + \zeta(2) - \dfrac{65}{16} = \displaystyle\sum_{k=1}^{\infty} \frac{k^2}{(k+3)^4}$

.11100752891282199021... $\approx \quad \gamma^4$

.11100821732964315991... $\approx \quad \dfrac{\log^3 2}{3}$

.11100942712945975694... $\approx \quad \displaystyle\sum_{k=2}^{\infty} \frac{\zeta(k)}{k!\,k^3}$

.11105930793590622477... $\approx \quad \dfrac{\gamma^2}{3}$

1 .11110935160523173201... $\approx \quad e\,Li_2\left(\dfrac{1}{e}\right) = \displaystyle\sum_{k=0}^{\infty} \frac{1}{e^k (k+1)^2}$

$.1111111111111111111\underline{1}$ $= \quad \dfrac{1}{9} = \displaystyle\int_1^{\infty} \frac{\log x}{x^4} = -\int_0^{\pi/2} \log(\sin x)\,\sin^2 x \cos x\, dx$

$= \quad \displaystyle\sum_{k=1}^{\infty} \frac{\mu(k)}{3^k + 1} = \sum_{k=1}^{\infty} \frac{\mu(k)}{6^k + 1} = \sum_{k=1}^{\infty} \frac{\mu(k)}{9^k - 1}$

.11118875955726352102... $\approx \quad \dfrac{1}{64}\left(16\pi G - 2\pi^2 \log 2 - 21\zeta(3)\right) = \displaystyle\int_0^{\pi/4} x^2 \tan x\, dx$

.1112477908543131171... $\approx \quad \dfrac{1}{8}\left(4\log 2\pi - 5 - \dfrac{12\zeta(3)}{\pi^2}\right) = \displaystyle\sum_{k=1}^{\infty} \frac{\zeta(2k) - 1}{k(k+1)(k+2)}$

.11138797835071256452 6... $\approx \quad \gamma + \dfrac{\pi^2}{4} + \log\dfrac{9\pi}{4} - \dfrac{44}{9} = \displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^2}{2^k (k+1)}\left(\zeta(k+1) - 1\right)$

.1114472084426055567... $\approx \quad -\gamma - \dfrac{1}{2}\left(\psi\left(1 + \dfrac{1}{\sqrt{2}}\right) + \psi\left(1 - \dfrac{1}{\sqrt{2}}\right)\right) - 1$

$= \quad \dfrac{1}{\sqrt{2}} - 1 - \gamma - \dfrac{\pi}{2}\cot\dfrac{\pi}{\sqrt{2}} - \psi\left(1 + \dfrac{1}{\sqrt{2}}\right)$

$= \quad -\dfrac{1}{2}\left(H\left(\dfrac{1}{\sqrt{2}}\right) + H\left(-\dfrac{1}{\sqrt{3}}\right)\right)$

$= \quad \displaystyle\sum_{k=2}^{\infty} \frac{1}{2k^3 - k} = \sum_{k=1}^{\infty} \frac{\zeta(2k+1) - 1}{2^k}$

1 .1114472084426055567... $\approx \quad -\gamma - \dfrac{1}{2}\left(\psi\left(1 + \dfrac{1}{\sqrt{2}}\right) + \psi\left(1 - \dfrac{1}{\sqrt{2}}\right)\right)$

$= \quad \displaystyle\sum_{k=1}^{\infty} \frac{1}{2k^3 - k} = \sum_{k=1}^{\infty} \frac{\zeta(2k+1)}{2^k}$

.11145269365447544048... $\approx \quad \dfrac{\pi^2}{6} - \dfrac{\log^3 2}{3} - \dfrac{\pi^2 \log 2}{3} + 2Li_3\left(-\dfrac{1}{2}\right) + \dfrac{3}{2}\zeta(3)$

$= \int_1^\infty \frac{\log^2 x}{(x+2)(x+1)^2}$

.11157177565710487788... $\approx \frac{1}{2}\log\frac{5}{4} = \operatorname{arctanh}\frac{1}{9} = \sum_{k=0}^\infty \frac{1}{9^{2k+1}(2k+1)}$

$= \operatorname{Im}\left\{ i\log\left(1-\frac{i}{2}\right)\right\}$

.11157214582108777725... $\approx \frac{i}{12}\left(\psi^{(1)}\left(1+\frac{i}{3}\right)-\psi^{(1)}\left(1-\frac{i}{3}\right)\right) = \sum_{k=1}^\infty \frac{(-1)^{k+1}k}{9^k}\left(\zeta(2k+1)\right)$

.11163962923836248965... $\approx \sum_{k=1}^\infty \frac{1}{k^4-5}$

1 .1116439382240662949... $\approx \frac{1}{3\left(\sqrt{3}-3i\right)}\left(3i\gamma-\gamma\sqrt{3}+\left(3i+\sqrt{3}\right)\psi\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+\psi\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\right)$

$= \frac{1}{3}\left(1-\gamma-\left(1+(-1)^{2/3}\right)\psi\left((-1)^{1/3}\right)+\left(-1+(-1)^{1/3}\right)\psi\left(-(-1)^{2/3}\right)\right)$

$= \sum_{k=1}^\infty \frac{1}{k^2+k^{-1}} = \frac{1}{2}+\sum_{k=1}^\infty (-1)^{k+1}\left(\zeta(3k-1)-1\right)$

$= 1+\sum_{k=1}^\infty (-1)^{k+1}\left(\zeta(3k)-\zeta(3k+1))\right)$

.11168642734821136196... $\approx \sum_{k=0}^\infty \frac{(-1)^k}{5k+6} = \int_1^\infty \frac{dx}{x^7+x^2}$

.1119002751525975723... $\approx Li_4\left(\frac{1}{9}\right) = \sum_{k=1}^\infty \frac{1}{9^k 4^k}$

.11191813909204296853... $\approx \sum_{k=2}^\infty \left(\zeta(k)-1\right)^5$

.11196542976354232023... $\approx \sum_{k=2}^\infty \frac{2}{k^3(1-k^{-2})^2} = \sum_{k=1}^\infty \frac{\zeta(2k+1)-1}{2k}$

.11207559668468037943... $\approx \frac{\pi(\pi-2)}{32} = \sum_{k=1}^\infty \frac{k\zeta(2k)}{16^k} = \sum_{k=1}^\infty \frac{16k^2}{(16k^2-1)^2}$

.11210169134154575458... $\approx \gamma+\frac{1}{2}\left(\psi\left(1+\frac{i}{\pi}\right)+\psi\left(1-\frac{i}{\pi}\right)\right) = \sum_{k=1}^\infty \frac{1}{k(1+k^2\pi^2)}$

.11211354880511828217... $\approx \sum_{k=1}^\infty \frac{\mu(k)}{2^{k+1}-1}$

1 .11225990481734629821... $\approx \sum_{k=1}^\infty H_k^{(2)}\left(\zeta(k+1)-1\right)$

6 .1123246470082041... $\approx \sum_{k=1}^\infty \frac{e^{1/k}}{F_k}$

4 .11233516712056609118... ≈ $\frac{5\pi^2}{12} = \frac{\zeta^{3}(2)}{\zeta(4)} = \sum_{k=1}^{\infty} \frac{d(k^2)}{k^2}$ Titchmarsh 1.2.9

= $\prod_{p} \frac{1+p^{-2}}{(1-p^{-2})^2}$

.11237685045624666687... ≈ $\frac{8}{2e} - \frac{e}{2} = \int_1^{\infty} \sinh\left(\frac{1}{x^2}\right) \frac{dx}{x^9} = \frac{1}{2}\int_1^{\infty} \sinh\left(\frac{1}{x}\right) \frac{dx}{x^5}$

.11239173638994824403... ≈ $\frac{1}{24}\left(\pi^2 - 21 - 3\pi\sqrt{2}\csc\pi\sqrt{2}\right)$

= $\frac{1}{48}\left(\csc\frac{\pi}{\sqrt{2}}\sec\frac{\pi}{\sqrt{2}}\right)\left(\pi^2\sin\pi\sqrt{2} - 3\pi\sqrt{2} - 21\sin\pi\sqrt{2}\right)$

= $\sum_{k=2}^{\infty} \frac{(-1)^k}{k^4 - 2k^2}$

2 .11259134098889446139... ≈ $\sum_{k=1}^{\infty} \frac{2^{1/k}}{k^4}$

.11269283467121196426... ≈ $\frac{3\zeta(3)}{32} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+2)^3}$

= $-\frac{1}{2}\left(Li_3(i) + Li_3(-i)\right) = -\sum_{k=1}^{\infty} \frac{1}{k}\cos\frac{k\pi}{2}$

= $\int_0^1\int_0^1\int_0^1 \frac{xyz}{1+x^2y^2z^2}\,dx\,dy\,dz$

.11270765259874453157... ≈ $Li_3\left(\frac{1}{9}\right) = \sum_{k=1}^{\infty} \frac{1}{9^k k^3}$

.112741986560723906051...≈ $\frac{\pi^2}{16} - \frac{\pi}{8} + \frac{\log 2}{2} - \frac{G}{2} = \sum_{k=0}^{\infty} \frac{1}{(2k+1(4k+1)(4k+3)^2}$

.11281706418194086693... ≈ $\sum_{k=2}^{\infty} \frac{\zeta(k+2)-1}{\zeta(k)}$

.11281950399381730995... ≈ $\frac{1}{2}\log(2e-1) - 1 + \frac{1}{e} = \int_0^1 \frac{\left(1-e^{-x}\right)^2}{2-e^{-x}}\,dx$

. 11292216109598113029... ≈ $\frac{1}{4} - \frac{\pi^2}{72} = \sum_{k=1}^{\infty} \frac{H_k H_{k+1}}{k(k+1)(k+2)(k+3)}$

4 .11292518301340488139... ≈ $\frac{1}{\pi}\Phi\left(\frac{1}{\pi},-\pi,1\right) = \sum_{k=1}^{\infty} \frac{k^{\pi}}{\pi^k}$

.11304621735534278232... ≈ $\frac{3}{4} - 3\log\frac{4}{3} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}k}{3^k(k+1)}$

1 .1131069910407629915... ≈ $\int_{3/2}^{\infty} \left(\zeta(x) - 1\right) dx$

1 .11318390185276958041... ≈ $\sum_{k=0}^{\infty}\frac{1}{k^8+k^7+k^6+k^5+k^4+k^3+k^2+k+1}$

.11319164174034262221... ≈ $-\log\Gamma\left(\frac{4}{3}\right)$

.11324478557577296923... ≈ $\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\left(\zeta(jk+4)-1\right)$

4 .11325037878292751717... ≈ $e^{\sqrt{2}}$

1 .113289355783182401858... ≈ $\sum_{k=4}^{\infty}\frac{(-1)^k}{S1(k,4)}$

.113513747770728380036... ≈ $2\log 2+\frac{3\log 3}{2}-\frac{\pi\sqrt{3}}{2}-\frac{1}{5} = \sum_{k=1}^{\infty}\frac{\zeta(k+1)-1}{6^k}$

.11356645044528407969... ≈ $\frac{5}{36}\left(1+\log\frac{5}{6}\right) = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{H_k k}{5^k}$

.11357028943968722004... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k}{k^{k+1}}$

.11360253356370630128... ≈ $\sum_{k=2}^{\infty}\frac{\log k}{k!2^k}$

.11370563888010938117... ≈ $\frac{3}{2}-2\log 2 = \sum_{k=1}^{\infty}\frac{1}{k(4k^2-1)^2}$ J104, K ex. 110e

= $\sum_{k=1}^{\infty}\frac{1}{k(2k+1)(2k+2)} = \sum_{k=1}^{\infty}\frac{1}{2^k(k+1)(k+2)}$

= $\sum_{k=1}^{\infty}\frac{k-1}{4k^3-k} = \sum_{k=1}^{\infty}\frac{\zeta(2k)-\zeta(2k+1)}{4^k}$

= $\sum_{k=2}^{\infty}\frac{(-1)^k}{k^2+k}$

= $\int_1^{\infty}\frac{dx}{(x^2+x)^2} = \int_1^{\infty}\frac{dx}{e^x(e^x+1)^2}$

.1138888888888986028... ≈ $-\sum_{k=1}^{\infty}\frac{|\mu(k)|(-1)^k}{9^k-1}$

.11392894125692285447... ≈ $6-\frac{16}{e} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!(k+4)} = \int_1^e\frac{\log^3 x}{x^2}dx$

.1139565939558891... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k}{k(2k+1)\log k}$

.11422322878784232903... ≈ $-\frac{9}{112}-\frac{7\cdot 2^{3/4}\pi}{224}\left(\csc\left(\pi 2^{3/4}\right)+\operatorname{csch}\left(\pi 2^{3/4}\right)\right)$

= $\sum_{k=2}^{\infty}\frac{(-1)^k}{k^4-8}$

.11427266790258497564... ≈ $4\left(8\log 2-8\log 2\log 3+2\log^2 3-3\log 3-1\right)$

= $\sum_{k=1}^{\infty}\frac{H_k}{4^k(k+2)}$

.11436020697851003 06... ≈ $Li_2\left(\frac{1}{9}\right) = \sum_{k=1}^{\infty}\frac{1}{9^k k^2}$

= $6Li_2\left(\frac{1}{3}\right)-\frac{\pi^2}{3}+\log^2 3$

2 .11450175075145702914... ≈ $Ei(1)-Ei(-1) = 2\sum_{k=0}^{\infty}\frac{1}{(2k+1)!(2k+1)}$

= $2\,SinhIntegral(1) = -2isi(i) = 2HypPFQ\left[\left\{\frac{1}{2}\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},\frac{1}{4}\right]$

.11458333333333333333$\underline{3}$ = $\frac{11}{96} = \sum_{k=1}^{\infty}\frac{1}{k(k+2)(k+4)}$

.11466451051968119045... ≈ $4\log^2 2+4\log 2-\frac{2\pi^2}{3}+2 = \sum_{k=1}^{\infty}\frac{k}{2^k(k+2)^2}$

.11467552038971033281... ≈ $\frac{1}{2}-\frac{\pi}{2\sqrt{6}}\operatorname{csch}\frac{\pi}{\sqrt{6}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{6k^2+1}$

.11468222811783944732... ≈ $2\log\left(2+\sqrt{5}\right)-4\log 2 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(2k-1)!!}{(2k)!\,4^k k}$

.11477710244367366979... ≈ $\frac{1}{16}\cos\frac{1}{2}+\frac{1}{8}\sin\frac{1}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{(2k)!\,4^k}$

3 .11481544930980446101... ≈ $2\tan 1$

3 .11489208086538667920... ≈ $\frac{1}{2}\left(Ei(e)-Ei\left(\frac{1}{e}\right)\right)-1 = \sum_{k=1}^{\infty}\frac{\sinh k}{k!\,k}$

.11490348493190048047... ≈ $J_2(1)$

.11508905442240150010... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k}{k^3-1}$

.11512212943741039107... ≈ $\frac{1}{10}\left(\pi\cot\frac{3\pi}{5}-\pi\cot\frac{4\pi}{5}-4\cos\frac{6\pi}{5}\log\sin\frac{\pi}{5}\right)$

$+\frac{1}{10}\left(4\cos\frac{8\pi}{5}\log\sin\frac{\pi}{5}-4\cos\frac{12\pi}{5}\log\sin\frac{2\pi}{5}\right)$

$+\frac{1}{10}\left(4\cos\frac{16\pi}{5}\log\sin\frac{2\pi}{5}\right)$

= $\psi\left(\frac{2}{5}\right)+\psi\left(\frac{9}{10}\right)-\psi\left(\frac{3}{10}\right)-\psi\left(\frac{4}{5}\right)$

= $\sum_{k=1}^{\infty} \frac{1}{(5k-1)(5k-2)}$

.11524253454462680448... ≈ $\frac{\log(e+1)-1}{e} = -\int_0^1 \frac{\log x}{(x+e)^2}\,dx$

.11544313298030657212... ≈ $\frac{\gamma}{5}$

.1154773270741587992... ≈ $\frac{\log^3 2}{3} - \frac{1}{2}\log^2 2 \log 3 + (\log 2)\, Li_2\left(-\frac{1}{2}\right) + Li_3\left(-\frac{1}{2}\right) + \frac{7\zeta(3)}{8}$

= $\int_0^1 \frac{\log^2(1+x)}{x(x+2)} = \int_1^2 \frac{\log^2 x}{x^2-1}dx$

.11552453009332421824... ≈ $\frac{\log 2}{6} = \sum_{k=1}^{\infty} \frac{1}{24k^2-12k} = \sum_{k=1}^{\infty} \frac{1}{6\cdot 2^k k}$

= $\int_0^{\infty} \frac{dx}{(x+1)(x+2)(x+4)}$

.11565942657526592131... ≈ $\frac{3\pi^2}{256}$

.11577782305257759047... ≈ $\pi + \log 2 - 6 + \frac{\sqrt{3}}{2}\log\frac{2+\sqrt{3}}{2-\sqrt{3}} = \int_1^{\infty} \log\left(1+\frac{1}{x^6}\right)\frac{dx}{x^2}$

= $\int_0^1 \log(1+x^6)\,dx = \int_1^{\infty} \log\left(1+\frac{1}{x^6}\right)\frac{dx}{x^2}$

.1157824102885971962... ≈ $\frac{1}{2}\log^2\left(\frac{1+\sqrt{5}}{2}\right) = \frac{1}{2}\,\text{arccsch}^2 2$

= $\sum_{k=0}^{\infty} \frac{(-1)^k}{2^{2k+2}} \frac{(2k)!!}{(2k+1)!!} \frac{1}{2k+2} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{4\binom{2k}{k}k^2}$ J143

.11591190225020152905... ≈ $\frac{1}{4}\arctan\frac{1}{2} = \sum_{k=1}^{\infty} \frac{(-1)^k}{2^{2k+1}(2k-1)}$

.11593151565841244881... ≈ $\log 2 - \gamma = \sum_{k=1}^{\infty} \frac{\psi(k)}{2^k} = \sum_{k=1}^{\infty} \frac{\zeta(2k+1)-1}{k+1}$

= $\sum_{k=1}^{\infty} \frac{\zeta(2k+1)}{4^k(2k+1)}$

= $-\sum_{k=2}^{\infty} \left(k\log\left(1-\frac{1}{k^2}\right) + \frac{1}{k}\right)$

= $\int_0^{\infty} \left(\cos x - \frac{1}{1+2x}\right)\frac{dx}{x}$

1 .1159761639... ≈ j_3 J311

.11621872996764452288... ≈ $\frac{\pi^2}{2} - \frac{9\pi}{4} + \frac{27}{12} = \sum_{k=1}^{\infty} \frac{\sin 3k}{k^3}$

= $\frac{i}{2}\left(Li_3(e^{-3i}) - Li_3(e^{3i})\right)$ GR 1.443.5

1 .1165422623587819774... ≈ $\frac{1}{3}\Gamma\left(\frac{1}{4}\right)\cos\frac{\pi}{8} = \int_0^{\infty} \frac{\sin(x^4)}{x^4} dx$

.116565609128780240753... ≈ $\frac{7\zeta(3)}{4\pi^2} - \frac{\log 2}{2} + \frac{1}{4} = \sum_{k=1}^{\infty} \frac{\zeta(2k)}{4^k(2k+2)}$

.1165733601435639903... ≈ $\frac{\operatorname{arcsinh}\sqrt{2}}{2\sqrt{2}} - \frac{1}{2\sqrt{3}} = \sum_{k=1}^{\infty} (-1)^{k+1} \binom{2k}{k} \frac{k}{2^k(2k+1)}$

.11683996854645729525... ≈ $\frac{7}{8}\log\frac{8}{7} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} H_k}{7^k}$

.11685027506808491368... ≈ $\frac{\pi^2}{16} - \frac{1}{2} = \sum_{k=1}^{\infty} \frac{1}{((2k-1)(2k+1))^2} = \sum_{k=1}^{\infty} \frac{1}{(4k^2-1)^2}$ J247, J373

.117128467834480403863... ≈ $\frac{1}{\zeta^3(4)}\left(6\zeta(3)\zeta'(3)\zeta''(3) - 6\left(\zeta'(3)\right)^3 - \zeta^2(3)\zeta^{(3)}(3)\right)$

= $\sum_{k=2}^{\infty} \frac{\mu(k)\log^3 k}{k^3}$

.11718390123864249466... ≈ $\zeta(3) - 24\log 2 - \frac{5\pi^2}{3} + 32 = \sum_{k=1}^{\infty} \frac{1}{k^3(2k+1)^2}$

.11718750000000000000$\underline{0}$ = $\frac{15}{128} = \sum_{k=1}^{\infty} \frac{(-1)^k k^4}{3^k}$

.11735494335470499209... ≈ $\frac{\pi}{12} - \frac{13}{90} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)(2k+7)}$

.11736052233261182097... ≈ $\frac{2}{3} - \operatorname{arctanh}\frac{1}{2} = \sum_{k=0}^{\infty} \frac{1}{4^k(2k+1)}$

.11738089122341846026... ≈ $\frac{1}{6}\left(8 + \pi\sqrt{3}\csc\pi\sqrt{3}\right) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2+4k+1}$

.11738251378938078257... ≈ $\frac{2G+1}{2\pi} - \frac{1}{3} = \sum_{k=1}^{\infty} \left(\frac{(2k-1)!!}{(2k)!!}\right)^2 \frac{1}{2k+3}$ J385

8 .11742425283353643637... ≈ $\frac{\pi^4}{12} = 4\zeta(2)L(2) = \sum_{k=1}^{\infty} \frac{r(k)}{k^2}$

2 .11744314664488689721... ≈ $2^{\zeta(4)} = \prod_{k=1}^{\infty} 2^{1/k^4}$

.117571778435605270... ≈ $\zeta(3) + \zeta(2) + \frac{\zeta(4)}{4} - 3 = \sum_{k=1}^{\infty} \frac{H_k}{(k+2)^2}$

.$\underline{1176470588235294}$ = $\frac{2}{17}$

.11778303565638345454... ≈ $\log\frac{9}{8} = \sum_{k=1}^{\infty}\frac{1}{9^k k} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{8^k k} = \Phi\left(\frac{1}{9},1,0\right)$

= $2\operatorname{arctanh}\frac{1}{17} = 2\sum_{k=0}^{\infty}\frac{1}{17^{2k+1}(2k+1)}$ K148

1 .11779030404890075888... ≈ $\frac{1}{2}\sum_{k=1}^{\infty}\frac{(k+1)(k+2)}{k}\left(\zeta(k+2)-1\right)$

.11785113019775792073... ≈ $\frac{1}{6\sqrt{2}} = \int_0^{\pi/4}\sin^2 x\cos x\,dx\int_0^{\pi/4}\frac{\sin^3 x}{\tan x}dx$

.117950148843131726911... ≈ $-\frac{1}{2}Li_2\left(-\frac{1}{4}\right) = -Li_2\left(\frac{i}{2}\right) - Li_2\left(-\frac{i}{2}\right)$

1 .1180339887498948482... ≈ $\frac{\sqrt{5}}{2} = \varphi - \frac{1}{2} = \sum_{k=0}^{\infty}\binom{2k}{k}\frac{1}{20^k}$

= $\sum_{k=1}^{\infty}\frac{1}{1+F_{2k+1}}$

.1182267093239637759... ≈ $\frac{3}{2}-\cos 1-\sin 1 = \sum_{k=1}^{\infty}\frac{(-1)^k}{(2k)!(2k+2)}$

.11827223029970254620... ≈ $1-\frac{1}{2}\left((1-\pi)\sin 1 + (\cos 1-1)\log(2-2\cos 1)\right)$

= $\sum_{k=1}^{\infty}\frac{\cos k}{k(k+1)}$

.11827410889083245278... ≈ $\frac{G}{\pi} - \frac{\log 2}{4} = \sum_{k=1}^{\infty}\frac{(4^k-1)\zeta(2k)}{16^k(2k+1)}$

= $\sum_{k=1}^{\infty}2k\left(\operatorname{arctanh}\frac{1}{2k} - 2\operatorname{arctanh}\frac{1}{4k}\right)$

.1183333333333333333$\underline{3}$ = $\frac{71}{600} = \sum_{k=1}^{\infty}\frac{1}{k(k+1)(k+6)}$

2 .11836707979750799405... ≈ $\sum_{k=1}^{\infty}\frac{1}{k}\log\frac{k+2}{k}$

.118379103687155739697... ≈ $\frac{7\pi^4}{5760} = -Li_4(i) - Li_4(-i)$

.118438778425057529626... ≈ $\frac{5\pi^3}{162\sqrt{3}} - \frac{13\zeta(3)}{36} = \frac{1}{432}\left(\psi^{(2)}\left(\frac{5}{6}\right)-\psi^{(2)}\left(\frac{1}{3}\right)\right)$

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{(3k+2)^3}$

.118598363315566157860... ≈

$$\frac{1}{5\left(3+\sqrt{5}\right)}\left(\left(14+4\sqrt{5}\right)\log\frac{7+2\sqrt{5}}{(1+\sqrt{5})^2} - \left(11+\sqrt{5}\right)\log\frac{11+\sqrt{5}}{8} + \log\frac{64}{27} - 6\left(3+\sqrt{5}\right)\right)$$

$= \quad \sum_{k=1}^{\infty} \frac{(-1)^{k+1} F_k F_k}{4^k k(k+1)}$

.11862116356640599538... $\approx \quad \Gamma(1-e,1) = \int_1^{\infty} x^{-e} e^{-x}\, dx$

.11862641298045697477... $\approx \quad 1-\log\left(1+\sqrt{2}\right) = 1-\operatorname{arcsinh} 1 = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(2k-1)!!}{(2k)!\,(2k+1)}$ J389

$= \quad \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{4^k (2k+1)} \binom{2k}{k}$

.11865060567398120259... $\approx \quad \sum_{k=2}^{\infty} H_{k-1}(\zeta(2k)-1) = \sum_{k=2}^{\infty} \frac{\log\left(1-k^{-2}\right)}{1-k^2}$

1 .11866150546368657586... $\approx \quad 2e + \gamma - 3 - Ei(1) = \sum_{k=0}^{\infty} \frac{k^3}{k!(k+1)^3}$

4 .11871837492687201437... $\approx \quad \frac{\sqrt{2\pi}}{8} \Gamma^2\left(\frac{1}{4}\right) = -\int_0^1 \frac{\log x}{\sqrt{x(1-x^2)}} dx$ GR 4.241.11

.11873149673078164295... $\approx \quad \frac{\pi}{4} - \frac{2}{3} = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+5} = \int_1^{\infty} \frac{dx}{x^6+x^4} = \int_0^{\pi/4} \tan^4 x\, dx$

.11877185156792292102... $\approx \quad \left(\frac{\pi}{8}+\frac{1}{2}\right)\log 2 - \frac{1}{2} = \int_0^1 \frac{x^2 \arctan x}{x+1} dx$

5 .11881481068515228908... $\approx \quad \sum_{k=0}^{\infty} \frac{k^2}{(k+1)!!}$

7 .11881481068515228908... $\approx \quad \sum_{k=0}^{\infty} \frac{(k+1)!!}{k!}$

.1189394022668291491... $\approx \quad \frac{\pi^4}{15} - \frac{51}{8} = \psi^{(3)}(3) = \int_1^{\infty} \frac{\log^3 x}{x^4 - x^3} dx = \int_0^{\infty} \frac{x^3}{e^{2x}\left(e^x - 1\right)} dx$

.11899805009931065100... $\approx \quad \int_1^{\infty} \left(\zeta(2x+1)-1\right) dx$

.11910825533882119347... $\approx \quad \frac{\pi+5}{36} - \frac{\pi^2}{144} - \frac{\log 2}{18} = -\int_0^1 \mathrm{x}^2 \operatorname{arccot} x \log x\, dx$

.11911975130622439689... $\approx \quad \frac{2\sin 1}{e} - \frac{1}{2} = \int_1^e \frac{\log^2 x \cos\log x}{x^2} dx$

.11920292202211755594... $\approx \quad \frac{1}{e^2+1} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{e^{2k}} = \int_0^{\infty} \frac{\cos(ex)}{e^x} dx$

.11925098885450811785... $\approx \quad \zeta(4) + \zeta(5) - 2$

.119366207318921501827... ≈ $\dfrac{3}{8\pi}$

.1193689967602449311... ≈ $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\sigma_1(k)}{2^k-1}$

9 .11940551591183183145... ≈ $\gamma^4+\gamma^{-4}$

.1194258969062993801... ≈ $\displaystyle\frac{\pi^2+8G}{144} = \sum_{k=1}^{\infty}\frac{1}{(12k-9)^2}$

.11956681346419146407... ≈ $\displaystyle\cos 1-\frac{\sin 1}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(2k-2)!}$

.11973366944845609388... ≈ $\displaystyle\zeta(3)-\zeta(4) = \sum_{k=1}^{\infty}\frac{k}{(k+1)^4}$

.11985638465105075007... ≈ $\displaystyle\frac{1}{4}\sin\frac{1}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(2k)!4^k}$

.11995921833353320816... ≈ $\displaystyle\frac{Ei(1)}{e}-\gamma = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}\,\psi(k+1)}{k!}$

$.1200000000000000000\underline{0}$ = $\displaystyle\frac{3}{25} = \sum_{k=1}^{\infty}\frac{\mu(k)}{5^k+1}$

1 $.1200000000000000000\underline{0}$ = $\displaystyle\frac{28}{25} = \sum_{k=1}^{\infty}\frac{F_kF_{k+2}}{4^k}$

.12025202884329818022... ≈ $\displaystyle\frac{1}{2}\arctan\frac{1}{2}+\frac{1}{2}\log\frac{4}{5} = \sum_{k=1}^{\infty}\frac{(-1)^k}{2^{2k+1}k(2k-1)}$

.12025650515289120796... ≈ $\displaystyle\frac{7-3\sqrt{3}}{15} = \sum_{k=1}^{\infty}\binom{2k}{k}\frac{(-1)^{k+1}}{2^k(k+3)}$

.12041013026451893284... ≈ $\displaystyle\zeta(3)+\frac{\pi^2}{12}+\frac{1}{2} = \sum_{k=1}^{\infty}\frac{H_k}{k(k+1)^2(k+2)}$

.12044213230101764656... ≈ $\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^k}{(k!)^3} = HypPFQ\left[\{\},\{1,1,\},-1\right]$

.1204749741031959545... ≈ $\displaystyle e\log(e+1)-e-\frac{e}{e+1} = \sum_{k=0}^{\infty}\frac{(-1)^k k}{e^k(k+1)}$

1 .12049744137026227297... ≈ $\displaystyle\sum_{k=2}^{\infty}\frac{H_{k-1}}{k^2+1} = \frac{\gamma\pi}{2}\coth\left(\pi(1+i)\right)-\frac{\gamma}{2}+$

= $\displaystyle +\frac{i}{4}\left(\left(\psi(1-i)\right)^2-\left(\psi(1+i)\right)^2-\psi^{(1)}(1-i)+\psi^{(1)}(1+i)\right)$

.1205008204107738885... ≈ $\displaystyle\frac{\pi(1-\log 2)}{8} = -\int_0^1 x\arcsin x\log x\,dx$

.1205922055573114571... ≈ $\displaystyle\frac{2\log 2}{5}-\frac{47}{300} = \sum_{k=0}^{\infty}\frac{1}{(k+1)(k+5)} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2+5k}$

= $\int_1^\infty \log\left(1+\frac{1}{x}\right)\frac{dx}{x^6} = \int_0^1 x^4 \log(x+1)\,dx$

1 .12065218367427254118... ≈ $\frac{\pi}{\sqrt{3}} - \log 2 = \sum_{k=0}^\infty \frac{(-1)^k}{k+\frac{2}{3}}$

1 .1207109299781110955... ≈ $\frac{\pi}{4\sqrt{2}}\tan\frac{\pi}{2\sqrt{2}} = \sum_{k=1}^\infty \frac{1}{8k^2-8k+1} = \sum_{k=1}^\infty \frac{(4^k-1)\zeta(2k)}{8^k}$

= $\sum_{k=1}^\infty \left(\frac{1}{2k^2-1} - \frac{1}{8k^2-1}\right) = \sum_{k=1}^\infty \frac{6k^2}{(2k^2-1)(8k^2-1)}$

.12078223763524522235... ≈ $-\log\left(\frac{\sqrt{\pi}}{2}\right) = \frac{1}{2}\log\frac{4}{\pi} = -\log\Gamma\left(\frac{3}{2}\right)$

= $-\int_0^\infty \left(\frac{1}{2} - \frac{1}{e^x+1}\right)\frac{dx}{e^{2x}x}$ GR 3.427.3

= $\int_0^\infty \sinh^2\left(\frac{x}{2}\right)\frac{dx}{xe^x\cosh x}$ GR 3.553.2

.1208515214587351411... ≈ $\frac{3\zeta(3)}{2} + 12\log 2 - 10 = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{k^3(k+1)^3}$

.12092539655805535367... ≈ $\sum_{k=2}^\infty \frac{1}{k^4-6}$

.12111826828242117256... ≈ $\frac{\pi^3}{256} = \sum_{k=0}^\infty \frac{(-1)^k}{(4k+2)^3}$

.12112420800258050246... ≈ $\sum_{k=-\infty}^\infty \frac{(-1)^k}{2^{k^2}} = \prod_{k=1}^\infty \frac{\left(1-2^{-k}\right)}{\left(1+2^{-k}\right)}$

.12113906570177660197... ≈ $\frac{1}{6} + \frac{1}{2\pi^4} - \frac{\coth\pi^2}{2\pi^2} = \sum_{k=1}^\infty \frac{1}{k^2(k^2+\pi^2)}$

.12114363133110502303... ≈ $\log\Gamma\left(\frac{5}{6}\right)$

.1213203435596425732... ≈ $\frac{3}{\sqrt{2}} - 2 = \sum_{k=0}^\infty \frac{(-1)^{k+1}k}{4^k(k+1)}\binom{2k}{k}$

= $\int_0^{\pi/4} \sin x \tan^2 x\,dx$

.1216063600734577727098...≈ $\sum_{k=1}^\infty \frac{\mu(2k-1)}{4^k+1}$

1 .12173301393634378687... ≈ $\frac{4\zeta(2)}{9} + \frac{g_2}{2} = \frac{1}{9}\psi^{(1)}\left(\frac{1}{3}\right)$

= $\int_0^1 \frac{\log x}{x^3-1}dx = \int_1^\infty \frac{x\log x}{x^3-1}dx$

.12179382823357308312... $\approx \dfrac{\zeta(3)}{\pi^2}$

.12186043243265752791... $\approx 4\log\dfrac{3}{2} - \dfrac{3}{2} = \dfrac{1}{2}\Phi\left(-\dfrac{1}{2},1,3\right) = \displaystyle\sum_{k=0}^{\infty}\dfrac{(-1)^k}{2^k(2k+6)}$

$= \displaystyle\sum_{k=1}^{\infty}\dfrac{(-1)^{k+1}}{2^k(k+2)} = \sum_{k=1}^{\infty}\dfrac{1}{3^k\left(\dfrac{k^3}{3}+\dfrac{3k^2}{2}+k\right)}$

1 .12191977628228799971... $\approx \dfrac{14}{15}\zeta(3) = \displaystyle\sum_{k=1}^{\infty}\dfrac{a(k)}{k^4}$ Titchmarsh 1.2.13

.12200602389514317644... $\approx \gamma + \dfrac{1}{2}\left(\psi\left(1+\dfrac{i}{3}\right)+\psi\left(1-\dfrac{i}{3}\right)\right) = \dfrac{1}{2}\left(H\left(\dfrac{i}{3}\right)+H\left(-\dfrac{i}{3}\right)\right)$

$= \displaystyle\sum_{k=1}^{\infty}\dfrac{(-1)^{k+1}\zeta(2k+1)}{9^k}$

.12202746647028717052... $\approx \dfrac{\pi\sqrt{3}}{6} + 6\log 2 - \dfrac{9\log 3}{2} = \displaystyle\int_0^{\infty}\log\left(1+\dfrac{1}{(x+2)^3}\right)dx$

.12206661758682452713... $\approx \dfrac{1}{8} - \dfrac{\pi}{4}\operatorname{csch} 2\pi = \displaystyle\sum_{k=1}^{\infty}\dfrac{(-1)^{k+1}}{k^2+4}$

$= \displaystyle\int_0^{\infty}\dfrac{\sin x\cos x}{e^x+1}dx$

55 .12212239940160755246... $\approx 26\zeta(3)+\dfrac{4\pi^3\sqrt{3}}{9} = -\psi^{(2)}\left(\dfrac{1}{3}\right) = 2\displaystyle\sum_{k=0}^{\infty}\dfrac{1}{(k+\frac{1}{3})^3}$

.122239373141852827246...$\approx \displaystyle\sum_{k=1}^{\infty}\dfrac{(-1)^{k+1}\zeta(3k)}{9^k} = \sum_{k=1}^{\infty}\dfrac{1}{9k^3+1}$

1 .12229100107160344571... $\approx \zeta(4)\zeta(5) = \displaystyle\sum_{k=1}^{\infty}\dfrac{\sigma_1(k)}{k^5}$ HW Thm. 290

$= \displaystyle\sum_{k=1}^{\infty}\dfrac{\sigma_{-1}(k)}{k^4}$

.12235085376504869019... $\approx 1 - \dfrac{\pi}{2} + \log 2 = \displaystyle\sum_{k=2}^{\infty}\dfrac{(-1)^k}{2k^2-k}$

2 .12240935830266022225... $\approx \displaystyle\sum_{k=2}^{\infty}\dfrac{\zeta(k)}{k^{k-2}}$

.12241743810962728388... $\approx 2\sin^2\dfrac{1}{4} = \displaystyle\sum_{k=1}^{\infty}\dfrac{(-1)^{k+1}}{(2k)!4^k}$

.12244897959183673469... $\approx \dfrac{6}{49} = \displaystyle\sum_{k=1}^{\infty}\dfrac{(-1)^{k+1}k}{6^k}$

1 .122462048309372981433...$\approx 2^{1/6} = \displaystyle\prod_{k=1}^{\infty}\left(1+\dfrac{(-1)^{k+1}}{6k-1}\right)$

.122479299181446442339... ≈ $2\operatorname{arcsinh}^2\left(\frac{1}{4}\right) = 2\log^2\frac{1+\sqrt{17}}{4} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{\binom{2k}{k}4^k k^2}$

.12248933156343207709... ≈ $\frac{1}{2}\arctan\frac{1}{4} = \frac{1}{8}\sum_{k=0}^{\infty}\frac{(-1)^k}{16^k(2k+1)} = \int_2^{\infty}\frac{dx}{x^3+x^{-1}}$

.12263282904358114150... ≈ $\frac{\pi^2+8\log 2-11}{36} = -\int_0^1 x^2\log\left(1+\frac{1}{x}\right)\log x\,dx$

= $\int_1^{\infty}\frac{\log(x+1)\log x}{x^4}dx$

.1228573089942169826... ≈ $\frac{7}{\sqrt{e}} - 2 = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}k^2}{(k+1)!2^k}$

.12287969597930143319... ≈ $\log 2 - 1 + \frac{\log^2 2}{2} - \frac{\log^3 2}{3} + \frac{\zeta(3)}{4} = \int_0^1\frac{\log^2(1+x)}{x(x+1)^2}dx$

.1229028434255558374... ≈ $\frac{\sin 2\sqrt{2}}{\sqrt{2\pi}} = \sum_{k=0}^{\infty}\frac{(-1)^k 2^k}{k!(k+\frac{1}{2})}$

.1229495015331819518... ≈ $32 - \frac{5\pi^5}{48} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k+\frac{1}{2})^5}$

$.1230769230769\underline{230769}$ = $\frac{8}{65} = \frac{1}{2\cosh\log 8} = \sum_{k=1}^{\infty}(-1)^k e^{-3(2k+1)\log 2}$ J943

4 .12310562561766054982... ≈ $\sqrt{17}$

1 .12316448278630278663... ≈ $\frac{\sqrt{3\pi}}{2}\mathit{erfi}\frac{1}{\sqrt{3}} = \sum_{k=0}^{\infty}\frac{1}{k!3^k(2k+1)}$

2 .12321607812022000506... ≈ $\frac{1}{4\sqrt{2}}\left(\psi\left(-\frac{1}{\sqrt{2}}\right)-\psi\left(\frac{1}{\sqrt{2}}\right)\right) = \sum_{k=1}^{\infty}\frac{k\zeta(2k+1)}{2^k} = \sum_{k=1}^{\infty}\frac{2k}{(2k^2-1)^2}$

.123456789101112131415... ≈ Champernowne number, integers written in sequence

$.123456790\underline{123456790}$ = $\frac{10}{81} = \sum_{k=1}^{\infty}\frac{k}{10^k} = \sum_{k=1}^{\infty}\frac{(-1)^k k^4}{2^k}$

.12346308879239152315... ≈ $\frac{37\pi^6}{22680} - \zeta^2(3)$ 18 MI 4, p. 15

= $\frac{2}{3}\zeta(6) + \frac{1}{3}\zeta(2)\zeta(4) + \frac{1}{3}\zeta^3(2) - \zeta^2(2) = \sum_{k=1}^{\infty}\frac{H_k^{\;2}}{(k+1)^4}$

.12360922914430632778... ≈ $\sum_{k=1}^{\infty}\frac{1}{3^{2^k}} = -\sum_{k=1}^{\infty}\frac{\mu(2k)}{9^k-1}$

.12370512629640090505... ≈ $2\left(\gamma - ci\left(\frac{1}{2}\right) - \log 2\right) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k)!4^k(k)}$

.12374349124688633191... ≈ $\frac{355}{108} + \zeta(2) - 4\zeta(3) = \sum_{k=1}^{\infty} \frac{k}{(k+4)^3}$

1 .123912033326422... ≈ $\int_1^{\infty} \left(1 - \frac{1}{\zeta(x)}\right) dx$

8 .1240384046359603605... ≈ $\sqrt{66}$

.12414146221522759619... ≈ $\frac{3}{2}\log^2\frac{4}{3} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}H_k}{3^k(k+1)}$

.12416465383603675513... ≈ $\frac{\gamma}{2} + \frac{1}{4}\left(\psi\left(1+\frac{i}{2}\right) + \psi\left(1-\frac{i}{2}\right)\right) = \frac{1}{4}\left(H\left(\frac{i}{2}\right) + H\left(-\frac{i}{2}\right)\right)$

= $\sum_{k=1}^{\infty} \frac{1}{2k(4k^2+1)}$

= $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\zeta(2k+1)}{2^{2k+1}}$

529 $.1242424242424242424\underline{24}$ = $\frac{174611}{330} = -B_{20}$

.12427001649629550601... ≈ $\frac{\gamma}{2} + \frac{\pi}{4}\coth\pi - 1 + \frac{1}{4}\left(\psi(i) + \psi(-i)\right)$

= $\frac{1}{4}\left((1+i)H(1-i) + (1-i)H(1+i) - 3\right)$

= $\sum_{k=2}^{\infty} \frac{1}{k^3+k^2+k+1} = \sum_{k=2}^{\infty} \frac{1}{k^3-k^{-1}} - \sum_{k=2}^{\infty} \frac{1}{k^4-1}$

.12429703178972643908... ≈ $\frac{4}{25}\left(1 + \log\frac{4}{5}\right) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}kH_k}{4^k}$

.12448261123279340605... ≈ $\frac{5}{2}\log^2\frac{5}{4} = \sum_{k=1}^{\infty} \frac{H_k}{5^k(k+1)}$

.124511599540703343791... ≈ $\sum_{k=1}^{\infty} (-1)^{k+1}\frac{\mu(k)}{8^k+1}$

.12451690929714139346... ≈ $\frac{1}{9}\left(\frac{\pi}{\sqrt{3}} - \log 2\right) = \sum_{k=1}^{\infty} \frac{1}{(6k-4)(6k-1)}$ J263

1 .124558536969386481587... ≈ $\frac{\pi^3}{4}\log 2 - \frac{9\pi}{8}\zeta(3) = \int_{-1}^{1} \frac{\arcsin^3 x}{x} dx$

.1246189861593984036... ≈ $11\log 2 - \frac{15}{2} = \sum_{k=0}^{\infty} \frac{k}{2^k(k+1)(k+3)}$

1 .124913711872468221... $\approx$ $8\log 2+\frac{2\pi^2}{3}-12=\sum_{k=1}^{\infty}\frac{1}{k^2(2k+1)^2}$

.12493815628119643158... $\approx$ $\frac{1}{512}\Phi\left(-\frac{1}{16},3,\frac{1}{4}\right)=\int_0^1\frac{\log^2 x}{x^4+16}dx$

.12497309494932548316... $\approx$ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{2^k+1}$

.12497829888022485654... $\approx$ $\sinh\frac{\sqrt{2}}{4}\sin\frac{\sqrt{2}}{4} = \frac{i}{2}\left(\cos\frac{(-1)^{1/4}}{2}-\cosh\frac{(-1)^{1/4}}{2}\right)$

$= \sum_{k=1}^{\infty}\frac{\sin k\pi/2}{(2k)!4^k}$

$.1250000000000000000\underline{0}$ = $\frac{1}{8} = \sum_{k=1}^{\infty} \frac{k}{(4k^2-1)^2}$ J364

= $\sum_{k=1}^{\infty} \frac{H_k}{(k+1)(k+2)(k+3)}$

= $\sum_{k=1}^{\infty} \frac{k^5}{e^{\pi k} - (-1)^k}$

= $\sum_{k=1}^{\infty} \frac{\mu(k)}{4^k + 1}$

= $\int_0^1 x^3 \arcsin x \arccos x \, dx$

= $\int_1^{\infty} \log\left(1 + \frac{1}{x^2}\right) \frac{dx}{x^5}$

1 $.12500000000000000000\underline{0}$ = $\frac{9}{8} = H^{(3)}{}_2$

4 $.12500000000000000000\underline{0}$ = $\frac{33}{8} = \sum_{k=1}^{\infty} \frac{k^3}{3^k}$

.1250494112773596116... ≈ $1 + \zeta(2) + 12\zeta(4) - 6\zeta(3) - 8\zeta(5) = \sum_{k=1}^{\infty} \frac{k^3}{(k+2)^5}$

.125116312213159864814... ≈ $\frac{\pi^2}{6} - \frac{15}{\pi^2} = \zeta(2) - \frac{\zeta(2)}{\zeta(4)} = \sum_{k=1}^{\infty} \frac{(1 - |\mu(k)|)}{k^2} = \sum_{n \text{ not squarefree}} \frac{1}{n^2}$

.12519689192003630759... ≈ $\frac{1}{6} - \frac{4\pi}{9\sqrt{3}} + \frac{\pi}{2} = \int_0^{\pi/2} \frac{(1-\sin x)^2}{(2-\sin x)^2} dx$

.12521403053199898469... ≈ $-\frac{1}{8192} \psi^{(2)}\left(\frac{1}{8}\right) = \sum_{k=0}^{\infty} \frac{1}{(16k+2)^3}$

.12521985470651613593... ≈ $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(k-1)! 2^k \zeta(2k)}$ Titchmarsh 14.32.1

1 .12522166627417648991... ≈ $\frac{\pi^2}{3} - \frac{\pi^4}{45} = 2(\zeta(2) - \zeta(4)) = \int_0^{\infty} \frac{x^4}{\sinh^4 x} dx$

2 .125353851428293758868... ≈

$$\frac{9\zeta(5)}{2} - \frac{\pi^2}{96}(17 + 24\zeta(3)) - 2\zeta(3)^2 + \frac{9\zeta(3)}{4} + \frac{97\pi^6}{22680} - \frac{5\pi^4}{288} - \frac{1}{16}$$

= $\sum_{k=1}^{\infty} \frac{H_k H_{k+2}}{k^4}$

1 .1253860830832697192... ≈ $\frac{1}{e}(Ei(2) - Ei(1)) = \int_0^1 \frac{e^x \, dx}{(1+x)}$

2 .12538708076642786114... ≈ $\frac{\pi^2}{6} + \log^2 2 = \sum_{k=1}^{\infty} \frac{H_k^{\ 2}}{2^k}$

.12546068941849408713... ≈ $-\frac{1}{3456}\psi^{(2)}\left(\frac{1}{6}\right) = \sum_{k=1}^{\infty} \frac{1}{(12k-10)^3}$

1 .12547234373397543081... ≈ $\frac{\csc 3}{6} - \frac{1}{18} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2\pi^2 - 9}$

.12549268916822826564... ≈ $\frac{\sqrt{3}}{6}\arctan\frac{5}{\sqrt{3}} + \frac{1}{12}\log\frac{7}{3} - \frac{\pi\sqrt{3}}{18} = \int_2^{\infty} \frac{dx}{x^3 - x^{-3}}$

.12552511502545031168... ≈ $\sum_{k=2}^{\infty} \frac{\zeta(k)}{k^4} = \sum_{k=1}^{\infty} Li_4\left(\frac{1}{k}\right) - \frac{1}{k}$

.1255356315950551333... ≈ $1 + \sum_{k=2}^{\infty} \mu(k)\left(\zeta(k) - 1\right) = 1 + \sum_{k=2}^{\infty}\sum_{n=2}^{\infty} \frac{\mu(k)}{n^k}$

= $\sum_{\omega \in S} \frac{1}{\omega(\omega - 1)}$, where S is the set of all non-trivial integer powers

= $1 - \sum_{\substack{\omega\, a\, nontrivial \\ \text{int}\, eger\ power}} \frac{1}{\omega}$

3 .125552388163504646336...≈ $\frac{3\zeta(3)}{2} + \frac{\pi^2}{12} + \frac{1}{2} = \sum_{k=1}^{\infty} \frac{H_k H_k}{k(k+2)}$

.12556728472287967689... ≈ $\frac{1}{2^2} F_1\left(2,2,\frac{7}{2},-\frac{1}{4}\right) - \frac{1}{3^2} F_1\left(3,3,\frac{5}{2},-\frac{1}{4}\right)$

= $\frac{4}{25} - \frac{4\sqrt{5}}{125}\text{arccsch}2 = \frac{4}{25}\left(1 - \frac{1}{\sqrt{5}}\log\frac{1+\sqrt{5}}{2}\right)$

= $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^2}{\binom{2k}{k}}$

.12560247419757510714... ≈ $\frac{7\pi^2}{2} + \frac{\pi^4}{15} + \frac{\pi^6}{945} - 9\zeta(3) - 3\zeta(5) - 28 = \sum_{k=1}^{\infty} \frac{1}{k^6(k+1)^3}$

.12573215444196347523... ≈ $\sum_{k=2}^{\infty} \frac{\zeta(k)}{2^k k^2} = \sum_{k=1}^{\infty} Li_2\left(\frac{1}{2k}\right) - \frac{1}{2k}$

2 .12594525836597494819... ≈ $\frac{\sqrt{2}}{9\pi}\sinh \pi\sqrt{2} = \prod_{k=1}^{\infty}\left(1 + \frac{2}{(k+2)^2}\right)$

.12596596230599036339... ≈ $-\frac{1}{1458}\psi^{(2)}\left(\frac{2}{9}\right) = \sum_{k=1}^{\infty} \frac{1}{(9k-7)^3}$

.12600168081459039851... ≈ $Li_4\left(\frac{1}{8}\right) = \sum_{k=1}^{\infty} \frac{1}{8^k k^4}$

$$.12613687638920210969\ldots \approx \frac{1}{20}\left(\log 31+\sqrt{5}\log\left(\frac{12}{6+\sqrt{5}}-1\right)-2\pi\sqrt{5-2\sqrt{5}}\right)$$

$$+\frac{1}{10}\left(\sqrt{2\left(5+\sqrt{5}\right)}\arctan\sqrt{13+\frac{22}{\sqrt{5}}}-\sqrt{10-2\sqrt{5}}\arctan\sqrt{13-\frac{22}{\sqrt{5}}}\right)$$

$$= \int_2^\infty \frac{dx}{x^3-x^{-2}}$$

$$.1262315670845302317\ldots \approx 21-\frac{5\pi^2}{2}-\frac{\pi^4}{30}+5\zeta(3)+\zeta(5)=\sum_{k=1}^\infty \frac{1}{k^5(k+1)^3}$$

$$.12629662103275089876\ldots \approx \frac{\pi^3}{512}+\frac{7\zeta(3)}{128} = -\frac{1}{1024}\psi^{(2)}\left(\frac{1}{4}\right) = \sum_{k=1}^\infty \frac{1}{(8k-6)^3}$$

$$.12632148170620903636 5\ldots \approx -\frac{\pi}{\sqrt{3}}\log\sqrt[3]{2\pi}\,\frac{\Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{1}{3}\right)} = -\int_0^1 \frac{1}{1+x+x^2}\log\log\frac{1}{x}\,dx \qquad \text{GR 4.325.5}$$

$$.1264492721085758196\ldots \approx \frac{1}{2}-\frac{\pi}{3\sqrt{3}}+\frac{\log 2}{3} = \sum_{k=0}^\infty \frac{(-1)^k}{3k+5} = \int_1^\infty \frac{dx}{x^6+x^3}dx$$

$$.12657458790630216782\ldots \approx 1+3\zeta(2)-4\zeta(3) = \sum_{k=2}^\infty (-1)^k k^2\left(\zeta(k)-\zeta(k+1)\right)$$

$$= \sum_{k=1}^\infty \frac{4k^3-k^2-2k-1}{k^2(k+1)^3}$$

11 $$.12666675920208825192\ldots \approx \frac{11\pi\left(\sqrt{3}+1\right)}{6\sqrt{2}} = \int_0^\infty \frac{dx}{1+x^{12/11}}$$

1 $$.12673386731705664643\ldots \approx \zeta\left(\frac{7}{2}\right)$$

$$.12687268616381905856\ldots \approx 11-4e$$

2 $$.12692801104297249644\ldots \approx \log\left(1+e^2\right)$$

$$.1269531324505805969\ldots \approx \sum_{k=1}^\infty \frac{1}{2^{3^k}} = -\sum_{k=1}^\infty \frac{\mu(3k)}{8^k-1}$$

1 $$.12700904300677231423\ldots \approx \frac{\Gamma\left(\frac{4}{3}\right)^2}{\Gamma\left(\frac{4}{3}+\frac{i}{3}\right)\Gamma\left(\frac{4}{3}-\frac{i}{3}\right)} = \prod_{k=1}^\infty \left(1+\frac{1}{(3k+1)^2}\right)$$

$$.12702954097934859904\ldots \approx Li_3\left(\frac{1}{8}\right) = \sum_{k=1}^\infty \frac{1}{8^k k^3}$$

$$.1271613037212348273\ldots \approx \frac{3\log 2}{4}-\frac{\pi}{8} = \sum_{k=1}^\infty \frac{1}{16k^2-4k} = \sum_{k=2}^\infty \frac{\zeta(k)}{4^k}$$

= $\int_1^\infty \frac{dx}{x^4+x^3+x^2+x}$

.12732395447351626862... ≈ $\frac{2}{5\pi}$

3 .12733564569914135311... ≈ $2^{\zeta(2)} = \prod_{k=1}^{\infty} 2^{1/k^2}$

.127494719314267653131... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(2k+1)}{\pi^{2k}k}$

.12755009587421398544... ≈ $\zeta(4) + 10\zeta(2) - 2\zeta(3) - 15 = \sum_{k=1}^{\infty} \frac{1}{k^4(k+1)^3}$

.12759750555417038785... ≈ $\frac{13\zeta(3)}{216} + \frac{4\pi^3}{1296\sqrt{3}} = -\frac{1}{432}\psi^{(2)}\left(\frac{1}{3}\right) = \sum_{k=1}^{\infty} \frac{1}{(6k-4)^3}$

1 .12762596520638078523... ≈ $\cosh\frac{1}{2} = \frac{e^{1/2}+e^{-1/2}}{2} = \sum_{k=0}^{\infty} \frac{1}{(2k)!4^k}$ AS 4.5.63

= $\prod_{k=0}^{\infty}\left(1+\frac{1}{\pi^2(2k+1)^2}\right)$ J1079

.1276340777968094125... ≈ $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^{k+1}+1} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^k\left(2^k+1\right)}$

.12764596870103231042... ≈ $\frac{\zeta(4)-1}{\zeta(2)-1}$

1 .1276546553915548012... ≈ $4\log 2 - \frac{\pi^2}{6} = \sum_{k=1}^{\infty} \frac{1}{2k^3-k^2} = \sum_{k=1}^{\infty} \frac{\zeta(k+1)}{2^k}$

= $\int_0^1 Li_2(x^2)\frac{dx}{x^2}$

.12769462267733193161... ≈ $2\arcsin^2\left(\frac{1}{4}\right) = \sum_{k=1}^{\infty} \frac{(k-1)!(k-1)!}{(2k)!4^k}$ K ex. 123

.12770640594149767080... ≈ $\frac{1}{4}\log\frac{5}{3} = \int_2^\infty \frac{x\,dx}{x^4-1}$

.12807218723612184102... ≈ $\prod_{k=1}^{\infty}\left(1-\frac{\sigma_0(k)}{2^k}\right)$

.12812726991641121051... ≈ $\pi\left(\frac{1}{4}-\frac{\sqrt{2}}{4}+\frac{\sqrt{3}}{12}\right) = \int_0^\infty \frac{dx}{(x^2+1)(x^2+2)(x^2+3)}$

28 .12820766517479018967... ≈ $\sum_{k=1}^{\infty} \frac{k^3}{2^k-1} = \sum_{k=1}^{\infty} \frac{\sigma_3(k)}{2^k}$

1 .1283791670955125739... ≈ $\frac{2}{\sqrt{\pi}} = \prod_{k=1}^{\infty}\left(1+\frac{k+1}{k+2}\right)$

.12850549763598564086... ≈ $\sin 1\; ci(1) - \cos 1\; si(1) + (1-\gamma)\sin 1$

= $-\int_0^1 x \log x \, \sin(1-x) \, dx$

1 .12852792472431008541... ≈ $(-1)^{1/4}\frac{\pi}{4}\left(i\cot\left(\pi(-1)^{1/4}\right)+\cot\left(\pi(-1)^{3/4}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{k^2}{k^4+1} = \frac{1}{2}+\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(4k-2)-1\right)$

.12876478703996353961... ≈ $\frac{2\log 2}{3} - \frac{1}{3} = \sum_{k=1}^{\infty}\frac{1}{6k^2+9k+3} = \sum_{k=1}^{\infty}\frac{1}{2^k(3k+3)}$

= $\int_1^{\infty}\log\left(1+\frac{1}{x^3}\right)\frac{dx}{x^4}$

1 .12878702990812596126... ≈ $\Gamma\left(\frac{5}{6}\right)$

.12893682721187715867... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^{k(k+2)}}$

.1289432494744020511... ≈ $J_3(2) = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!(k+3)!} = {}_0F_1\left(;4;-1\right)$ LY 6.117

.12895651449431342780... ≈ $\frac{1}{250}\psi^{(2)}\left(\frac{2}{5}\right) = \sum_{k=0}^{\infty}\frac{1}{(5k+2)^3}$

.12897955148524914959... ≈ $\sum_{k=1}^{\infty}\frac{1}{(8k)^k}$

.12904071920074026864... ≈ $8-\frac{88}{5\sqrt{5}} = \sum_{k=1}^{\infty}(-1)^{k+1}\binom{2k+1}{k}\frac{k}{16^k}$

.129111538257100692534... ≈ $\frac{1}{2}+\frac{1}{8}Li_3(-4) = \int_1^{\infty}\frac{\log^2 x}{x^3+4x^2}dx$

.12913986010995340567... ≈ $Li_2\left(\frac{1}{8}\right) = \sum_{k=1}^{\infty}\frac{1}{8^k k^2}$

1 .12917677626041007740... ≈ $\sum_{k=1}^{\infty}\frac{1}{k^{2k-1}}$

.12920899385988654843... ≈ $\frac{27}{2}\left(\log 3-1\right)-\zeta(3) = \sum_{k=1}^{\infty}\frac{1}{9k^5-k^3}$

1 .12928728700635855478... ≈ $\sum_{k=0}^{\infty}\frac{1}{k^7+k^6+k^5+k^4+k^3+k^2+k+1}$

.12931033647404047771... ≈ $-\frac{1}{4}\left(1+\sqrt{3}\right)\left(24+\left(1+\sqrt{3}\right)\pi-\log 1728+\sqrt{3}\left(3\log 3-24\right)\right)$

$$-\frac{\sqrt{3}}{4}\left(1+\sqrt{3}\right)\left(\log 64(2-\sqrt{3})-\log(2+\sqrt{3})\right)-\frac{3+3\sqrt{3}}{2}\left(2+\sqrt{3}\right)$$

$= \displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\zeta(k+1)}{12^k}$

.12931985286416790888... $\approx \displaystyle\frac{\pi^2}{12} - \log 2 = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k}{(k+1)^2}$ J347

$= \displaystyle\sum_{k=1}^{\infty} \frac{k-1}{4k^3 - 2k^2} = \sum_{k=2}^{\infty} \frac{\zeta(k) - \zeta(k+1)}{2^k}$

$= \displaystyle\int_1^{\infty} \frac{\log^2 x}{(x+1)^3} dx = \int_0^1 \frac{x \log^2 x}{(x+1)^3} dx = -\int_0^1 \frac{x \log x}{(x+1)^2} dx$

.12950217839233689628... $\approx \displaystyle -e + \sum_{k=1}^{\infty} \frac{e^{1/k}}{k^4}$

.12958986973548106716... $\approx \displaystyle \zeta(3) - \frac{\pi^2}{12} - \frac{1}{4} = \sum_{k=2}^{\infty} \frac{2k-1}{k^6 + k^5 - k^4 - k^3}$

$= \displaystyle\sum_{k=2}^{\infty} k\left(\zeta(2k) - \zeta(2k+1)\right)$

2 .12970254898330641813... $\approx \displaystyle\sum_{k=0}^{\infty} \frac{1}{(k!)^3} = HypPFQ\left[\{\},\{1,1\},1\right]$

3 .12988103563175856528... $\approx \displaystyle \pi \tanh \pi = i\left(\psi\left(\frac{1}{2} - i\right) - \psi\left(\frac{1}{2} + i\right)\right)$

.12988213255715796275... $\approx \displaystyle\sum_{k=2}^{\infty} \frac{1}{k^3 + k^2 + k} = \sum_{k=1}^{\infty} \left(\zeta(3k) - \zeta(3k+1)\right)$

.12994946687227935132... $\approx \displaystyle\frac{1}{\pi\sqrt{6}}$ CFG A28

1 .13004260498008966987... $\approx \displaystyle \pi G - \frac{\pi}{2} + \frac{\pi^2}{8} - \frac{7\zeta(3)}{4} + \log 2 = \int_0^1 \frac{\arccos^2 x}{(1-x^2)(1+x)} dx$

.13010096960687982790... $\approx \displaystyle 1 + \frac{\pi^2}{8} - \frac{7\zeta(3)}{4} = \sum_{k=0}^{\infty} \frac{2k+1}{(2k+3)^3}$

1 .13020115950688002... $\approx \displaystyle\sum_{k=1}^{\infty} \frac{H_k{}^3}{3^k}$

.13027382637343684041... $\approx \displaystyle\frac{1}{4}\sinh\frac{1}{2} = \sum_{k=0}^{\infty} \frac{k}{(2k)!4^k}$

.130287258904574144837... $\approx \displaystyle\frac{1}{1+\sqrt{5}}\left(2\psi^{(1)}\left(\frac{5-\sqrt{5}}{2}\right) - \left(3+\sqrt{5}\right)\psi^{(1)}\left(\frac{5+\sqrt{5}}{2}\right)\right)$

$= \displaystyle\sum_{k=1}^{\infty} (-1)^k L_k k \left(\zeta(k+1) - 1\right)$

.13030594417711116272... $\approx \displaystyle\frac{\pi}{4\sqrt{2}}\left(\sqrt{2-\sqrt{2}} - 1\right) = \int_0^{\infty} \frac{x(1-x)}{1+x^8} dx$

.13033070075390631148... $\approx \frac{1}{2}\left(\log 2\pi - 1 - \gamma\right) = \sum_{k=2}^{\infty} \frac{\zeta(k)-1}{k^2+k}$

$= \sum_{k=2}^{\infty} \left(1 - \frac{1}{2k} + (k-1)\log\frac{k-1}{k}\right)$

.13039559891064138117... $\approx 10 - \pi^2 = \sum_{k=1}^{\infty} \frac{1}{k^3(k+1)^3}$ K ex. 127

.13039644878526240217... $\approx \sum_{k=2}^{\infty} \frac{\Omega(k)}{k(k-1)} = \sum_{k=2}^{\infty}\sum_{j=2}^{\infty} \frac{1}{k^j(k^j-1)} = \sum_{k=1}^{\infty}\sum_{m=2}^{\infty}\left(\zeta(mk+m)-1\right)$

1 .13039644878526240217... $\approx \sum_{k=2}^{\infty} \left(\sigma_0(k)-1\right)\left(\zeta(k)-1\right) = \sum_{j=2}^{\infty}\sum_{k=1}^{\infty}\left(\zeta(jk)-1\right)$

$= \sum_{k=2}^{\infty}\sum_{j=2}^{\infty} \frac{1}{k^j-1} = \sum_{k=2}^{\infty} \frac{\Omega(k)+1}{k(k-1)} = \sum_{k=2}^{\infty} \frac{\Omega(k)}{k-1}$

$= \sum_{m=1}^{\infty}\sum_{k=2}^{\infty} \frac{\Omega(k)}{k^m}$

2 .13039644878526240217... $\approx \sum_{k=2}^{\infty} \sigma_0(k)\left(\zeta(k)-1\right)$

.13039943558343319807... $\approx \frac{\pi}{4} - \frac{\pi^2}{32} - \frac{\log 2}{2} = \int_0^{\pi/4} x \tan^2 x \, dx$ GR 3.839.1

1 .13046409806996953523... $\approx \sum_{k=2}^{\infty} \frac{\log k}{k^2(1-k^{-2})^2} = \sum_{k=2}^{\infty} \frac{(1+\Omega(k))\log k}{k^2}$

$= \sum_{j=1}^{\infty}\sum_{k=2}^{\infty} \frac{j\log k}{k^{2j}}$

2 .13068123163714450692... $\approx \frac{3e}{4} + \frac{1}{4e} = \sum_{k=0}^{\infty} \frac{k+1}{(2k)!}$

.13078948972335204627... $\approx \frac{1}{6}\left(3\log 2 - 3\log\pi + 2\gamma - 12\zeta'(1) - 1\right)$

$= \sum_{k=2}^{\infty} (-1)^k \frac{\zeta(k)-1}{k+2} = \sum_{k=2}^{\infty}\left(\frac{1}{3k} - \frac{1}{2} + k - k^2 \log\left(1+\frac{1}{k}\right)\right)$

.13086510517299719078... $\approx \frac{8}{49} + \frac{\pi}{28}(1+\sqrt{2}) - \frac{4\log 2}{7} - \frac{\sqrt{2}}{7}\log(1+\sqrt{2})$

$= \sum_{k=1}^{\infty} \frac{1}{k(8k+7)}$ Prud. 5.1.7.24

.13086676482635094575... $\approx \frac{1}{8} + \frac{\pi}{2}\operatorname{csch} 2\pi = \sum_{k=0}^{\infty} \frac{(-1)^k}{k^2+4}$

1 .13107417077142445833 6... $\approx \frac{1}{\pi^3}\cos\left(\frac{\pi\sqrt{3-2i\sqrt{3}}}{2}\right)\cos\left(\frac{\pi\sqrt{3+2i\sqrt{3}}}{2}\right)\cosh\frac{\pi\sqrt{3}}{2}$

$= \displaystyle\sum_{k=1}^{\infty}\left(1+\frac{1}{k^3(k+1)^3}\right)$

1 .13114454049046657278... $\approx \dfrac{\cos\pi^{1/4}+\cosh\pi^{1/4}}{2} = \displaystyle\sum_{k=0}^{\infty}\frac{\pi^k}{(4k)!}$

.131263476981215714927... $\approx \dfrac{\pi^2}{4}+4G-6 = -\displaystyle\int_0^1 \arcsin^2 x\log x\,dx$

1 .131279000668548355421... $\approx -i(-1)^{3/4}\pi\cos\left((-1)^{1/4}\pi\right)-i(-1)^{1/4}\pi\cos\left((-1)^{3/4}\pi\right)$

.13129209787766019855... $\approx \zeta(3)+Li_3\left(\dfrac{1}{8}\right)-Li_3\left(\dfrac{7}{8}\right)-\dfrac{1}{2}\log\dfrac{8}{7}\left(\log\dfrac{8}{7}\log 8+2Li_2\left(\dfrac{7}{8}\right)\right)$

$= \displaystyle\sum_{k=1}^{\infty}\frac{H_k}{8^k k^2}$

1 .13133829660062637151... $\approx \displaystyle\sum_{k=1}^{\infty}\frac{1}{k^k k!}$

.131447068412064828263... $\approx -\dfrac{1}{2}\left(\psi^{(2)}(2-i)+\psi^{(2)}(2+i)\right) = \zeta(3,2+i)+\zeta(3,2-i)$

$= \displaystyle\int_0^{\infty}\frac{x^2\cos x}{e^x(e^x-1)}dx$

.131474973783080624966... $\approx \dfrac{7\zeta(3)}{64} = \displaystyle\sum_{k=1}^{\infty}\frac{1}{(4k+3)^3}$

.13150733113734147474... $\approx \displaystyle\sum_{k=2}^{\infty}\frac{(-1)^k}{k}\left(\zeta(k)-1\right)^3$

4 .1315925305995249344... $\approx \displaystyle\prod_{k=2}^{\infty}\frac{1}{2-\zeta(k)}$

.13178753240877183809... $\approx \dfrac{\log 7}{6}-\dfrac{\pi\sqrt{3}}{6}-\dfrac{\sqrt{3}}{3}\arctan\dfrac{5}{\sqrt{3}} = \displaystyle\int_2^{\infty}\frac{dx}{x^3-1}$

.13179532375909606051... $\approx \displaystyle\sum_{k=2}^{\infty}\frac{k-1}{k^4\log k} = \sum_{k=2}^{\infty}\left(\frac{1}{k^3\log k}-\frac{1}{k^4\log k}\right) = \int_3^4\left(\zeta(s)-1\right)ds$

.131928586401657639217... $\approx \dfrac{\pi^2}{4}-4G+\dfrac{1}{384}\left(\psi^{(3)}\left(\dfrac{3}{4}\right)-\psi^{(3)}\left(\dfrac{1}{4}\right)\right) = \displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{(k+1/2)^4}$

.13197175367742096432... $\approx \dfrac{\pi}{4}+\dfrac{\log 2}{2}-1 = \displaystyle\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+2)(2k+3)}$

$= \displaystyle\sum_{k=1}^{\infty}\frac{1}{8k^2+6k+1}$

$= \displaystyle\sum_{k=2}^{\infty}\left(1-\beta(k)\right)$

$= \displaystyle\int_0^{1/2}\log(1+4x^2)\,dx = \int_1^{\infty}\log\left(1+\frac{1}{x^4}\right)\frac{dx}{x^3}$

1	.13197175367742096432...	$\approx$	$\frac{\pi}{4}+\frac{\log 2}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2k(2k+1)}$	J154
		$=$	$\sum_{k=1}^{\infty}\frac{1}{8k^2-10k+3}$	
		$=$	$\sum_{k=1}^{\infty}\frac{(-1)^{\lfloor k/2\rfloor}}{k+1}$	Prud. 5.1.2.5
		$=$	$\int_0^1 x\log\left(1+\frac{1}{x^4}\right)dx$	
		$=$	$\int_0^{\infty}\log\left(1+\frac{1}{2x(x+1)}\right)dx$	
		$=$	$\int_1^{\infty}\frac{\arctan x}{x^2}dx$	
	.13197325257243247...	$\approx$	$-\frac{ie^{-i}}{2}\left(Li_3(e^i)-e^{2i}Li_3(e^{-i})\right) = \sum_{k=1}^{\infty}\frac{\sin k}{(k+1)^3}$	
3	.13203378002080632299...	$\approx$	$\gamma+\frac{3\log 3}{2}+\frac{\pi}{2\sqrt{3}} = -\psi\left(\frac{1}{3}\right) = -\frac{\Gamma'(\frac{1}{3})}{\Gamma(\frac{1}{3})}$	GR 8.336.3
	.1321188648648523596...	$\approx$	$\frac{2}{9}\left(1+\log\frac{2}{3}\right) = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{kH_k}{2^k}$	
	.1321205588285576784...	$\approx$	$\frac{1}{2}-\frac{1}{e} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k+2)!}$	
	.13212915413764997511...	$\approx$	$\frac{6}{7}\log\frac{7}{6} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}H_k}{6^k}$	
	.13223624503284413822...	$\approx$	$\sum_{k=2}^{\infty}\frac{1}{k^4-7}$	
	.13225425564190408112...	$\approx$	$-\sum_{k=1}^{\infty}\frac{\mu(5k)}{2^k}$	
	.1323184679680565842...	$\approx$	$\frac{5}{8}(i-\pi^2)-\frac{1}{4}\left(Li_3(-e^{3i})+3Li_3(-e^i)\right) = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\cos^3 k}{k^3}$	
11	.13235425497340275579...	$\approx$	$\frac{5815}{729}\sqrt[3]{e} = \sum_{k=0}^{\infty}\frac{k^6}{k!3^k}$	
	.13242435197214638289...	$\approx$	$\log(\pi-2)$	
	.13250071473061817597...	$\approx$	$\frac{\pi\sqrt{3}}{21}-\frac{2\log 2}{7}+\frac{1}{14} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+3)(3k+2)}$	
25	.13274122871834590770...	$\approx$	8π	
1	.132843018043786287416...	$\approx$	$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\mu(k)}{2^k-1}$	

.13290111441703984606... ≈ $\frac{1}{e^2+e^{-2}} = \frac{1}{2\cosh 2} = \sum_{k=0}^{\infty}(-1)^k e^{-2(2k+1)}$ J943

1 .133003096319346347478... ≈ $\Gamma\left(\frac{9}{4}\right)$

.13301594051492813654... ≈ $2\log 2+\frac{\pi^2}{4}-\frac{3\log^2 2}{2}-3 = \sum_{k=1}^{\infty}\frac{1}{2^k k^2(k+1)^2}$

.13302701266008896243... ≈ $1-\frac{\pi}{16}\left(\cot\frac{\pi}{8}+\cot\frac{5\pi}{8}\right)+\frac{\sqrt{2}}{4}\left(\log\sin\frac{\pi}{8}-\log\sin\frac{3\pi}{8}\right)$

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{4k+5} = \int_1^{\infty}\frac{dx}{x^6+x^2}$

1 .13309003545679845241... ≈ $\frac{\pi}{4\log 2} = \int_0^{\infty}\frac{dx}{2^x+2^{-x}}$

.13313701469403142513... ≈ $\psi^{(1)}(8)$

1 .13314845306682631683... ≈ $\sqrt[8]{e} = \sum_{k=0}^{\infty}\frac{1}{k!8^k} = \sum_{k=0}^{\infty}\frac{1}{k!!2^k}$

.13314972493191508633... ≈ $\frac{1}{16}\left(12-\pi^2\right) = -\int_0^1 x\log x\ \mathrm{arctan}h\ x\,dx$

.13316890350812216272... ≈ $\frac{1}{2}-\frac{\pi}{2\sqrt{5}}\operatorname{csch}\frac{\pi}{\sqrt{5}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{5k^2+1}$

.1332526249619079565... ≈ $2-2\log 2-\log^2 2 = \sum_{k=1}^{\infty}\frac{H_k}{2^k(k+1)(k+2)}$

1 .1334789151328136608... ≈ $3\zeta(5)-\zeta(2)\zeta(3) = \sum_{k=1}^{\infty}\frac{H_k}{k^4}$

.13348855233523269955... ≈ $\frac{5}{12}-\frac{\sqrt{2}}{3}-\log 2+\log\left(1+\sqrt{2}\right) = \sum_{k=1}^{\infty}\binom{2k}{k}\frac{(-1)^{k+1}}{4^k k(k+2)}$

1 .13350690717842253363... ≈ $\sum_{k=1}^{\infty}\left(\zeta(2k)\zeta(2k+1)-1\right)$

.13353139262452262315... ≈ $\log 8-\log 7 = Li_1\left(\frac{1}{8}\right) = \sum_{k=1}^{\infty}\frac{1}{8^k k}$

= $2\operatorname{arctanh}\frac{1}{15} = 2\sum_{k=1}\frac{1}{15^{2k+1}(2k+1)}$ K148

.13355961141529107611... ≈ $\frac{3}{16}\left(1+\log\frac{3}{4}\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}kH_k}{3^k}$

.13356187812884380949... ≈ $\frac{\zeta(3)}{9} = \int_0^1\frac{\log(1-x^3)\log x}{x}dx$

6 .13357217028888628968... ≈ $\sum_{k=1}^{\infty}\binom{2k}{k}\frac{1}{k!k}$

.13360524590502163581... $\approx -\gamma - \frac{1}{2}\psi\left(1-\frac{1}{\pi}\right) - \frac{1}{2}\psi\left(1+\frac{1}{\pi}\right) = \sum_{k=1}^{\infty}\frac{\zeta(2k+1)}{\pi^{2k}}$

$= \sum_{k=1}^{\infty}\frac{1}{k(\pi^2k^2-1)}$

.133790899699404867961... $\approx \frac{8}{3} - \gamma + \log\frac{4}{9\pi} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(k+1)(k+2)}\left(\zeta(k+1)-1\right)$

1 .13379961485058323867... $\approx \frac{\sqrt{7}}{2}\arcsin\frac{2}{\sqrt{7}} = \sum_{k=0}^{\infty}\binom{2k}{k}\frac{1}{7^k(2k+1)}$

.1338487269990037604... $\approx \frac{2}{5} + \frac{\cos 2 - 2\sin 2 - 5}{10e} = \int_0^1 \frac{\sin^2 x dx}{e^x}$

.1339745962155613532... $\approx 1 - \frac{\sqrt{3}}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(2k-1)!!}{(2k)!\,3^k}$

.13402867288711745264... $\approx \sum_{k=2}^{\infty}\frac{1}{(k+1)^2}\log\frac{k}{k-1}$

.13429612527404824389... $\approx \frac{1}{16}\left(2\pi^3\coth\pi\operatorname{csch}^2\pi + 3\pi^2\operatorname{csch}^2\pi + 3\pi\coth\pi - 8\right)$

$= \sum_{k=1}^{\infty}\frac{1}{(k^2+1)^3}$

.13432868018188702397... $\approx \frac{\coth 2}{4} - \frac{1}{8} = \frac{e^4+3}{8(e^4-1)} = \sum_{k=1}^{\infty}\frac{1}{k^2\pi^2+4}$ J951

.134355508461793914859... $\approx \frac{2\pi}{27\sqrt{3}} = \int_0^{\infty}\frac{dx}{x^3+27}$

.13451799537444075968... $\approx \arctan\frac{1}{e^2} = \int_2^{\infty}\frac{dx}{e^x+e^{-x}}$

.13458167809871830285... $\approx \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{3^k+2}$

1 .13459265710651098406... $\approx \frac{1}{\operatorname{arcsinh}1} = \prod_{k=1}^{\infty}\left(1+\frac{\operatorname{arcsinh}^2 1}{\pi^2k^2}\right)$

1 .13476822975405664508... $\approx \sum_{k=1}^{\infty}\frac{1}{k^k 2^{k-1}} = \int_0^1 x^{-x/2}dx$ Prud. 2.3.18.1

2 .13493355566839176637... $\approx \frac{e\pi}{4} = -\int_0^{\infty}\frac{e^{-x}}{1+x^4}dx$

.13496703342411321824... $\approx \frac{\pi^2}{12} - \frac{11}{16} = \sum_{k=1}^{\infty}\frac{1}{(k^2-1)^2}$ J400

$= \sum_{k=1}^{\infty}\frac{1}{k^2(k+2)^2} = \sum_{k=2}^{\infty}\frac{1}{k^4(1-k^{-2})^2}$

= $\sum_{k=2}^{\infty}(k-1)\left(\zeta(2k)-1\right)$

1 .13500636856055755855... ≈ $\sum_{k=1}^{\infty}\frac{H_k{}^2}{\binom{2k}{k}}$

.13507557646703492935... ≈ $\frac{\cos 1}{4} = \sum_{k=1}^{\infty}\frac{(-1)^k k^2}{(2k+1)!}$

.13515503603605479399... ≈ $\frac{1}{3}\log\frac{3}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}H^E{}_k}{2^k} = \sum_{k=1}^{\infty}\left(Li_k\left(\frac{1}{3}\right)-\frac{1}{3}\right)$

1 .1352918229484613014... ≈ $\frac{2\pi^2}{3}-\frac{49}{9} = -\int_0^1\frac{x\log x}{1-\sqrt{x}}dx$

.13529241631288141552... ≈ $Ai(1)$

20 .13532399155553168391... ≈ $2\cosh 3 = e^3 + e^{-3}$

.1353352832366126919... ≈ $\frac{1}{e^2} = \sum_{k=0}^{\infty}\frac{(-1)^k 2^k}{k!} = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}2^k(k+1)}{k!} = \sum_{k=0}^{\infty}\frac{(-1)^k 2^k(k+3)}{k!}$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k2^k}{(k+2)!}$

= $-\int_0^{\infty}\frac{\log^3 x}{(x+e)^3}dx$

1 .1353359783187... ≈ $\sum_{k=2}^{\infty}\frac{|\mu(k)|}{\phi^4(k)}$

.13550747569970917394... ≈ $\frac{\pi}{2\cosh\pi} = \frac{\pi}{e^{\pi}+e^{-\pi}} = \sum_{k=1}^{\infty}(-1)^{k+1}2^{2k-1}\beta(2k-1)$

= $\int_0^{\infty}\frac{\cos 2x}{\cosh x}dx$ GR 2.981.3

.135511632809070287634...≈ $\frac{1}{4\pi}\left(\psi\left(1-\frac{1}{4\pi}\right)-\psi\left(\frac{1}{2}-\frac{1}{4\pi}\right)\right) = \int_{-\infty}^{0}\frac{e^{-x}dx}{1+e^{-2\pi x}}$

1 .13563527673789986838... ≈ $\sqrt[9]{\pi}$

.13574766976703828118... ≈ $I_2(1)$

.13577015870381094462... ≈ $\frac{\cosh 1-1}{4} = \frac{1}{2}\sinh^2\frac{1}{2} = \frac{1}{4}\sinh 1\tanh\frac{1}{2} = \int_1^{\infty}\sinh\left(\frac{1}{x^4}\right)\frac{dx}{x^5}$

.13579007871686364792... ≈ $\prod_{k=0}^{\infty}\frac{k!}{k!+1}$

.1358333333333333333$\underline{3}$ = $\frac{163}{1200} = \sum_{k=1}^{\infty}\frac{1}{k(k+1)(k+5)}$

5 .13583989702753762283... ≈ $\sum_{k=1}^{\infty} k^3\left(\zeta(k)-1\right)^2$

.13587558784724491212... ≈ $\frac{5}{2}+\log\frac{1}{6\sqrt{\pi}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}2^k k}{(k+1)(k+2)}\left(\zeta(k+1)-1\right)$

.13593397770140961534... ≈ $11\sqrt{e}-18 = \sum_{k=1}^{\infty}\frac{k^2}{(k+2)!2^k}$

.13601452749106658148... ≈ $\frac{\pi}{2\sinh\pi} = \Gamma\left(-1+i\right)\Gamma\left(-1-i\right)$

= $\sum_{k=2}^{\infty}\frac{(-1)^k}{k^2+1} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2+2k+2}$

= $\int_0^1 \frac{\cos(\log x)}{(1+x)^2}dx$ GR 3.883.1

= $\int_1^{\infty} \frac{\cos(\log x)}{(1+x)^2}dx$

= $\int_0^{\infty} \frac{\sin x}{e^x\left(e^x+1\right)}dx$

.13607105874307714553... ≈ $\frac{16}{e}-\frac{23}{4} = \sum_{k=1}^{\infty}\frac{(-1)^k}{k!(k+4)}$

1 .13607213441296927732... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)}{k^2} = \sum_{k=2}^{\infty} Li_2\left(\frac{1}{k}\right)$

3 .13607554308348927218... ≈ $2\zeta(4)-2\,Li_4\left(-\frac{1}{2}\right) = \int_0^1 \frac{\log^3 x}{(x+2)(x-1)}dx$

.13608276348795433879... ≈ $\frac{1}{3\sqrt{6}} = \sum_{k=0}^{\infty}\frac{(-1)^k k}{8^k}\binom{2k}{k}$

.13610110214420788621... ≈ $\sum_{k=1}^{\infty}\frac{1}{8k^3+1} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(3k)}{8^k}$

= $\frac{\gamma}{6}-\frac{1}{3}+\frac{\log 2}{3}+\frac{2\sqrt{3}}{6(\sqrt{3}-3i)}\psi\left(\frac{3-i\sqrt{3}}{4}\right)-\frac{\sqrt{3}+3i}{6(\sqrt{3}-3i)}\psi\left(\frac{3+i\sqrt{3}}{4}\right)$

1 .13610166675096593629... ≈ $4\left(e^{1/4}-1\right) = \sum_{k=1}^{\infty}\frac{1}{(k+1)!4^k}$

1 .136257878761295049973...≈ $\frac{16}{15}+\frac{16}{15\sqrt{15}}\arcsin\frac{1}{4} = \sum_{k=0}^{\infty}\frac{k!k!}{(2k)!4^k} = \sum_{k=0}^{\infty}\frac{1}{4^k\binom{2k}{k}}$

.13629436111989061883... ≈ $2\log 2-\frac{5}{4} = \sum_{k=2}^{\infty}\frac{(-1)^k}{k^3-k} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+1)(k+2)(k+3)}$ GR 1.513.7

= $\sum_{k=2}^{\infty}\frac{1}{2^k(k+1)} = \sum_{k=1}^{\infty}(-1)^{k+1}\left(Li_k\left(\frac{1}{2}\right)-\frac{1}{2}\right)$

.13642582141524984691... ≈ $\frac{1}{\pi}\left(\gamma+\psi\left(1+\frac{1}{\pi}\right)\right)=\sum_{k=1}^{\infty}\frac{1}{k^2\pi^2+k\pi} = \sum_{k=2}^{\infty}\frac{(-1)^k\zeta(k)}{\pi^k}$

2 .13649393614881149501... ≈ $\sum_{k=1}^{\infty}\sigma_1(k)\left(\zeta(k+1)-1\right)$

.13661977236758134308... ≈ $\frac{2}{\pi}-\frac{1}{2} = \sum_{k=1}^{\infty}\left(\frac{(2k-1)!!}{(2k)!!}\right)^2\frac{1}{2k+2}$ J385

= $\sum_{k=1}^{\infty}(-1)^{k+1}\left(\frac{(2k-1)!!}{(2k)!!}\right)^3\frac{4k+1}{(2k-1)(2k+2)}$ J391

.13675623268328324608... ≈ $\frac{13\zeta(3)}{27}-\frac{2\pi^3}{81\sqrt{3}} = -\frac{1}{54}\psi^{(2)}\left(\frac{2}{3}\right) = \sum_{k=0}^{\infty}\frac{1}{(3k+2)^3}$

.13678737055089295255... ≈ $\frac{5\pi}{4}\coth\frac{\pi}{2}-\frac{\pi^2}{6}-\frac{5}{2} = \sum_{k=1}^{\infty}\frac{k^2-1}{4k^4+k^2}$

= $\sum_{k=1}^{\infty}\frac{(-1)^k}{4^k}\left(\zeta(2k)-\zeta(2k+2)\right)$

1 .13682347004754040317... ≈ $\sum_{k=1}^{\infty}\frac{k^3}{k^5+1} = \frac{1}{2}+\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(5k-3)-1\right)$

.13685910370427359482... ≈ $\zeta(3)+\frac{\pi^2}{2}-6 = \sum_{k=1}^{\infty}\frac{1}{k^2(k+1)^3}$

.13695378264465721768... ≈ $1+3\log\frac{3}{4} = \sum_{k=1}^{\infty}\frac{1}{4^k k(k+1)}$ J149

.13706676420458308572... ≈ $-\frac{\sin\pi\sqrt{3}}{\pi\sqrt{3}} = \prod_{k=2}^{\infty}\left(1-\frac{2}{k^2-1}\right)$

.13707783890401886971... ≈ $\frac{\pi^2}{72} = \sum_{k=1}^{\infty}\frac{1}{(6k-3)^2}$

= $\sum_{k=1}^{\infty}\left(\frac{1}{(12k-3)^2}+\frac{1}{(12k-9)^2}\right)$

= $\sum_{k=1}^{\infty}\frac{1}{4\binom{2k}{k}k^2}$

= $\int_1^{\infty}\log\left(1+\frac{1}{x^6}\right)\frac{dx}{x}$

.13711720445599746947... ≈ $-\frac{\zeta'(3)}{\zeta^2(3)}=\sum_{k=1}^{\infty}\frac{\mu(k)\log k}{k^3}$

14 .13716694115406957308... ≈ $\frac{9\pi}{2}$

1 .137185713031541409889... ≈ $\frac{\pi^2}{2}+\frac{\log^2 2}{2}-\frac{\pi^2\log 2}{2}-8\log 2+\frac{7\zeta(3)}{2} = \sum_{k=1}^{\infty}\frac{H_k}{k(2k-1)^2}$

.13737215566266794947... ≈ $\frac{\zeta(3)-\zeta(5)}{\zeta(3)}$

.13744801974139900405... ≈ $\frac{2}{9}+\frac{\pi}{12}-\frac{\log 2}{2} = \sum_{k=1}^{\infty}\frac{1}{k(8k+6)}$

.13756632099228274... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k}{k^3-2}$

.13786028238589160388... ≈ $\frac{1}{2\sinh 2} = \sum_{k=0}^{\infty}e^{-2(2k+1)}$ AS 4.5.62, J942

2 .13791866423119022685... ≈ $\frac{4\pi}{5}\sqrt{\frac{2}{5-\sqrt{5}}} = \frac{2\pi}{5}\csc\frac{4\pi}{5} = \int_0^{\infty}\frac{x\,dx}{1+x^{5/2}}$

.13793103448275862069... ≈ $\frac{4}{29} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}F_{2k}}{4^k}$

1 .13793789723437361776... ≈ $\frac{e^{1/4}\sqrt{\pi}}{2} = \int_0^{\infty}e^{-x^2}\cosh x\,dx$

12 .13818191912955082028... ≈ $\pi\sqrt{2}\left(1+\sqrt{3}\right) = \int_0^{\infty}\log\left(1+x^{-12}\right)dx$

.13822531568397836679... ≈ $\log\Gamma\left(1-e^{-1}\right)-\frac{\gamma}{e} = -\sum_{k=1}^{\infty}\left(\frac{1}{ek}+\log\left(1-\frac{1}{ek}\right)\right)$

$= \sum_{k=1}^{\infty}\frac{\zeta(k)}{e^k k}$

1 .13830174381437745969... ≈ $\frac{I_2\left(2\sqrt{e}\right)}{e} = \frac{1}{2}\,{}_0F_1\left(;3,e\right) = \sum_{k=0}^{\infty}\frac{e^k}{k!(k+2)!}$

1 .13838999497166186097... ≈ $\sum_{k=1}^{\infty}\frac{1}{k^{k+1}}$

1 .13842023421981077330... ≈ $\frac{5\pi^3}{81\sqrt{3}}+\frac{\zeta(3)}{36} = \sum_{k=0}^{\infty}(-1)^k\left(\frac{1}{(3k+1)^3}+\frac{1}{(3k+2)^3}+\frac{1}{(3k+3)^3}\right)$

.1384389944122896311... ≈ $2\log^2 2-\frac{\pi^2}{12} = \int_0^1\left(K(k')-\log\frac{4}{k}\right)\frac{dk}{k}$ GR 8.145

1 .13847187367241658231... ≈ $\frac{9+\pi\sqrt{3}-9\log 3}{4} = \sum_{k=1}^{\infty}\frac{(k-\frac{1}{3})!}{(k+\frac{2}{3})!\,k}$

.13862943611198906188... ≈ $\frac{\log 2}{5}$

.13867292839014782042... ≈ $4\log 2-2\log(2+\sqrt{3}) = \sum_{k=1}^{\infty}\frac{1}{16^k k}\binom{2k}{k}$

= $\sum_{k=1}^{\infty} \frac{(2k-1)!!}{(2k)!\,4^k k}$

1 .13878400779449272069... ≈ $2G - \log 2 = \sum_{k=1}^{\infty} \frac{(-1)^k (12k^2-1)}{k(4k^2-1)^2}$

.1388400918174489452... ≈ $\frac{\pi}{16\sqrt{2}} = \int_0^{\infty} \frac{dx}{x^4+16}$

$.13888888888888888888\underline{8}$ = $\frac{5}{36} = \sum_{k=1}^{\infty} \frac{1}{k(k+2)(k+3)} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}k}{5^k}$

= $\sum_{k=1}^{\infty} \frac{k^2}{(k+1)(k+2)(k+3)(k+4)}$

2 .13912111551783417702... ≈ $2e - 2\sqrt{e} = \sum_{k=0}^{\infty} \frac{e - pf(k)}{2^k}$

.13918211807199192222... ≈ $1 - \frac{4}{\sqrt{5}} \operatorname{arcsinh} \frac{1}{2} = \sum_{k=1}^{\infty} \frac{20k^2-5}{5^k} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k!k!}{(2k+1)!}$

.1391999869582814821... ≈ $2\operatorname{arcsinh} 1 + 2\log\left(1+\sqrt{2}\right) - 2\log 2 - 2$

= $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}(2k-1)!!}{(2k)!\,k(2k+1)}$

.13920767329150225896... ≈ $\frac{1}{3}\left(1 - \frac{\pi \log 2}{4} - \frac{\pi}{2} + \frac{\pi^2}{16} + G\right) = \int_0^{\pi/4} \frac{x^2 \tan^2 x}{\cos^2 x} dx$ GR 3.839.4

.13929766616936680005... ≈ $\frac{1}{6} - \frac{\pi}{2\sqrt{6}} \operatorname{csch} \pi\sqrt{\frac{3}{2}} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2k^2+3}$

.13940279264033098825... ≈ $\frac{\sqrt{\pi}}{2}\left(1 - erf(1)\right) = \int_1^{\infty} e^{-x^2} dx$

.1394197585790642108... ≈ $\frac{e}{4} + \frac{5}{4e} - 1 = \int_1^{\infty} \sinh\left(\frac{1}{x^2}\right) \frac{dx}{x^7} = \frac{1}{2}\int_1^{\infty} \sinh\left(\frac{1}{x}\right) \frac{dx}{x^4}$

.13943035409132289... ≈ $\zeta\left(\frac{5}{2}\right) - \zeta(3)$

2 .13946638410409682249... ≈ $\frac{4\pi}{9\sqrt{3}} + \frac{4}{3} = {}_2F_1\left(2,2,\frac{3}{2},\frac{1}{4}\right)$

= $\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k-1)!} = \sum_{k=1}^{\infty} \frac{k}{\binom{2k-1}{k}} = \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$

1 .13949392732454912231... ≈ $\sec \frac{1}{2} = \sum_{k=0}^{\infty} (-1)^k \frac{E_{2k}}{(2k)!4^k}$

.13949803613097262886... ≈ $\frac{\log 2}{3} - \frac{\log 3}{12} = \sum_{k=2}^{\infty} \frac{H_{2k-2}}{4^k}$

3 .13972346501305816133... ≈ $\log\left(e^{\pi}-e^{-\pi}\right)$

.1398233212546408378... ≈ $\frac{1}{5}-\frac{\cos 1+2\sin 1}{5e^{2}}=\int\limits_{0}^{1}\frac{\sin x\,dx}{e^{2x}}$

.1400000000000000000$\underline{0}$ = $\frac{7}{50}$

= $\frac{1}{2\cosh\,\log 7} = \sum_{k=0}^{\infty}(-1)^{k}e^{-(2k+1)\log 7}$ J943

.14002410170685231710... ≈ $\frac{1}{128}\left(2\pi^{4}\log 2-18\pi^{2}\zeta(3)+93\zeta(5)\right)=\int\limits_{0}^{\pi/2}x^{3}\log\sin x\,dx$

.14002478837788934398... ≈ $\frac{1}{\pi+4}$

.14004960899154477194.. ≈ $\sum_{k=1}^{\infty}\frac{1}{e^{2k}+1}$ Berndt 6.14

.14018615277338802392... ≈ $\frac{5}{6}-\log 2 = \sum_{k=1}^{\infty}\frac{1}{2k(2k+1)} = \sum_{k=0}^{\infty}\frac{(-1)^{k}}{k+4}$

= $\sum_{k=1}^{\infty}\frac{1}{4k^{2}+10k+6}$

= $\sum_{k=2}^{\infty}(-1)^{k}\,\frac{\zeta(k)-1}{2^{k}}$

= $\int\limits_{1}^{\infty}\frac{dx}{x^{5}+x^{4}}$

1 .1405189944514195213... ≈ $\sqrt{3}\operatorname{arctanh}\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{2}\log(2+\sqrt{3}) = \sum_{k=0}^{\infty}\frac{1}{3^{k}(2k+1)}$

.1406250000000000000$\underline{0}$ = $\frac{9}{64} = \sum_{k=1}^{\infty}\frac{k}{9^{k}}$

2 .14064147795560999654... ≈ $\psi(9)$

23 .14069263277926900573... ≈ $e^{\pi} = i^{-2i}$ Shown transcendental by Gelfond in 1934

18 .140717361492126458627...≈ $\frac{\pi}{2}\sinh\pi = \sum_{k=1}^{\infty}\frac{\pi^{2k}k}{(2k)!}$

.14095448355614340534... ≈ $\frac{1}{2048}\left(8\pi\left(24\pi G+2\pi^{2}\log 2-9\zeta(3)\right)+\psi^{(3)}\left(\frac{3}{4}\right)-\psi^{(3)}\left(\frac{1}{4}\right)\right)$

= $\int\limits_{0}^{\pi/4}\frac{x^{3}}{\tan x}dx$

2 .14106616355751237475... ≈ $e-\gamma$

.14111423493159236387... ≈ $\sum_{k=1}^{\infty} \frac{1}{2^k 2^{2^k}}$

.14114512672306050551... ≈ $\frac{1}{8}\log\left(\left(1-e^{i}\right)^3\left(1-e^{-i}\right)^3\left(1-e^{3i}\right)\left(1-e^{-3i}\right)\right) = -\sum_{k=1}^{\infty} \frac{\cos^3 k}{k}$

1 .14135809459005578983... ≈ $\sum_{k=2}^{\infty}\left(Li_2\left(\frac{2}{k}\right) - \frac{2}{k}\right) = \sum_{k=2}^{\infty} \frac{2^k\left(\zeta(k)-1\right)}{k^2}$

7 .1414284285428499980... ≈ $\sqrt{51}$

10 .141434622207162551... ≈ $\frac{2\pi^2}{3} + 2\log^2 2 + 4\log 3 + 4Li_2\left(-\frac{1}{2}\right) = \int_0^1 \frac{\log^2 x}{(x+\frac{1}{2})^3}\,dx$

. 14147106052612918735... ≈ $\frac{4}{9\pi} = \sum_{k=0}^{\infty} \binom{2k}{k}^2 \frac{k}{16^k (k+1)(k+2)}$

2 .14148606390327757201... ≈ $3^{\log 2} = 2^{\log 3} = \prod_{k=1}^{\infty} 3^{(-1)^{k+1}/k}$

.14155012612792688996... ≈ $32\log\frac{3}{2} - \frac{77}{6} = \sum_{k=0}^{\infty} \frac{(-1)^k}{2^k (k+5)}$

.1415926535798932384... ≈ $\pi - 3 = \sum_{k=1}^{\infty} (-1)^k \frac{\sin 6k}{k}$

1 .1415926535798932384... ≈ $\pi - 2 = \sum_{k=1}^{\infty} (-1)^k \frac{\sin 4k}{k}$

= $\int_0^1 \arccos^2 x\,dx$

= $\int_0^{\pi/2} x^2 \sin x\,dx$

3 .1415926535798932384... ≈ π

= $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$

= $48 \arctan\left(\frac{1}{49}\right) + 128 \arctan\left(\frac{1}{57}\right) - 20 \arctan\left(\frac{1}{239}\right) + 48 \arctan\left(\frac{1}{110443}\right)$

Borwein-Devlin p. 74

= $176 \arctan\left(\frac{1}{57}\right) + 28 \arctan\left(\frac{1}{239}\right) - 48 \arctan\left(\frac{1}{682}\right) + 96 \arctan\left(\frac{1}{12943}\right)$

Borwein-Devlin p. 74

$$= \sum_{k=0}^{\infty} \frac{50k-6}{2^k \binom{3k}{k}}$$ Borwein-Devlin p. 94

$$= \sum_{k=1}^{\infty} \frac{3^k-1}{4^k}\zeta(k+1) = \sum_{k=1}^{\infty} \frac{8}{(4k-1)(4k-3)}$$ Vardi, p. 158

$$= \int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \int_{-\infty}^{\infty} \frac{e^{x/2}dx}{e^x+1}$$

$$= \int_0^{\pi} \frac{x}{1+\sin x}dx$$

$$= \int_0^{\pi} \frac{dx}{\sqrt{2}+\cos 2x}$$

$$= \int_0^{\infty} \log\left(1+x^{-2}\right)dx = \int_0^{\infty} \log\left(1+\frac{3}{x^2+1}\right)dx$$

$$= \int_0^{\infty} \log(1+x^2)\frac{dx}{x^2}$$ GR 4.295.3

$$= \int_0^{\infty} \frac{\log\left(x^2+(e-1)^2\right)}{x^2+1}dx$$

$$= \int_0^{1} \log^2\left(\frac{1-x^2}{x}\right)\sqrt{1-x^2}\,dx$$ GR 4.298.20

$$= -\int_0^{\infty} \log x \log\left(1+\frac{1}{x^2}\right)dx$$

$$= \int_0^{\infty} \frac{\log(x^2+(e-1)^2)}{x^2+1}dx$$

$$= \int_0^{1} \operatorname{arccsc} x \log^2 x dx$$

6 .1415926535798932384... $\approx$ $\pi+3 = {}_2F_1\left(2,2,\frac{3}{2},\frac{1}{2}\right) = \sum_{k=1}^{\infty} \frac{2^k k}{\binom{2k}{k}}$

7 .1415926535798932384... $\approx$ $\pi+4 = \sum_{k=1}^{\infty} \frac{2^{k+1}}{\binom{2k}{k}}$

12 .1416960000000000000$\underline{0}$ $=$ $12\,\frac{2214}{15625} = \sum_{k=1}^{\infty} \frac{k^6}{6^k}$

.141749006226296033507... ≈ $-\int_0^1 \log^2(1-x)\log^2 x\,dx$

.1417988257045170 6824... ≈ $\frac{\log 2}{2}+\frac{\pi^2}{32}-\frac{\pi}{4}+\frac{\pi\log 2}{8}$

= $\int_0^{\pi/4}\left(\frac{\pi}{4}-x\tan x\right)\tan x\,dx$ GR 3.797.1

.14189224816475113638... ≈ $\frac{\pi}{e^{\pi}-1} = \sum_{k=0}^{\infty}\frac{B_k\pi^k}{k!}$ J152

.14189705460416392281... ≈ $\arctan\frac{1}{7} = \sum_{k=0}^{\infty}\frac{(-1)^k}{7^{2k+1}(2k+1)}$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\arctan\left(\frac{2}{(k+2)^2}\right)$

[Ramanujan] Berndt Ch. 2, Eq. 7.5

= $\sum_{k=1}^{\infty}\arctan\left(\frac{1}{2(k+3)^2}\right)$

[Ramanujan] Berndt Ch. 2, Eq. 7.6

1 .142001361603259308316... ≈ $\frac{7\zeta(3)}{2}+\frac{\pi^2}{2}-8 = \sum_{k=2}^{\infty}\frac{k(k-1)(\zeta(k)-1)}{2^{k-1}}$

.142023809668304780503... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\mu(k)}{7^k+1}$

.1421792525356509979... ≈ $4-\sqrt{\pi}\,\Gamma\left(\frac{1}{4}\right)\Gamma^{-1}\left(\frac{3}{4}\right)+2\log 2 = \sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!k(4k+1)}$

1 .14239732857810663217... ≈ $\frac{4\pi}{11}$

.142514197935710915709... ≈ $6\zeta(4)-\frac{\pi^2\log^3 2}{4}-\frac{\log^4 2}{4}-6Li_4\left(\frac{1}{2}\right)-\frac{21\zeta(3)\log 2}{4}$

= $\int_0^1\frac{\log^3(1+x)}{x}dx = \int_1^2\frac{\log^3 x}{x-1}dx$

.14260686462899218614... ≈ $-\sum_{k=1}^{\infty}\frac{\phi(k)\mu(k)}{2^k}$

1 .14267405371701917447... ≈ $\frac{\pi^2}{25}\csc^2\frac{\pi}{5} = \sum_{k=1}^{\infty}\left(\frac{1}{(5k-1)^2}+\frac{1}{(5k-4)^2}\right)$

.14269908169872415481... ≈ $\frac{\pi}{8}-\frac{1}{4} = \int_1^{\infty}\frac{dx}{(x^2+1)^2}$

.142805053720382885 3... ≈ $\frac{14}{81}+\frac{2}{27}\log\frac{2}{3} = \sum_{k=1}^{\infty}(-1)^k\frac{H_k k^3}{2^k}$

.14280817593690705451... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k\left(\zeta(k)-1\right)}{k^2} = \sum_{k=2}^{\infty}\left(\frac{1}{k}+Li_2\left(-\frac{1}{k}\right)\right)$

$.142857142857\underline{142857}$ = $\frac{1}{7}$

1 .14327171004415288088... ≈ $\frac{\pi}{3G}$

3 .1434583548180913141... ≈ $\log\left(e^{\pi}+e^{-\pi}\right)$

.14354757722361027859... ≈ $\frac{\pi^3}{216}$

.14359649906328252459... ≈ $\sum_{k=3}^{\infty}\frac{\zeta(k)-1}{k-1} = -\sum_{k=2}^{\infty}\left(\frac{1}{k^2}+\frac{1}{k}\log\left(1-\frac{1}{k}\right)\right)$

3 .14376428513040493119.... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)}{F_k}$

1 .14383564379164032590 7... ≈ $e^{\cos 1}\cos(\sin 1) = \frac{1}{2}\left(e^{e^{i}}+e^{e^{-i}}\right) = \sum_{k=0}^{\infty}\frac{\cos k}{k!}$ GR 1.449.1

= $\cos(\sin 1)\left(\cosh(\cos 1)+\sinh(\cos 1)\right)$

.14384103622589046372... ≈ $\frac{1}{2}\log\frac{4}{3} = \operatorname{arctanh}\frac{1}{7} = \sum_{k=0}^{\infty}\frac{1}{7^{2k+1}(2k+1)}$ K146

= $\int_2^{\infty}\frac{dx}{x^3-x} = \int_0^{\infty}\frac{dx}{(x+1)(x+2)(x+3)}$

.14385069144662706314... ≈ $\frac{5\pi}{2e^4} = \int_{-\infty}^{\infty}\frac{\cos 4x}{(1+x^2)^2}dx$

.14395417475074672624 3... ≈

$$\frac{1}{5\left(3+\sqrt{5}\right)}\left(10\left(3+\sqrt{5}\right)\log\frac{5}{4}-\left(5-\sqrt{5}\right)\log\left(2+\frac{2}{\sqrt{5}}\right)+\left(25+11\sqrt{5}\right)\log\frac{5+\sqrt{5}}{8}\right)$$

= $\sum_{k=1}^{\infty}\frac{F_k F_k}{4^k k(k+1)}$

1 .14407812827660760234... ≈ $Li_2\left(\frac{5}{6}\right)$

1 .14411512680284093362... ≈ $3\log 2-2G-\frac{\pi}{2}+\frac{\pi^2}{4} = \sum_{k=1}^{\infty}\frac{8k-1}{k(4k-1)^2}$

= $\sum_{k=1}^{\infty}\frac{(k+1)\zeta(k+1)}{4^k}$

.14417037552999332259... ≈ $Li_4\left(\frac{1}{7}\right) = \sum_{k=1}^{\infty}\frac{1}{7^k k^4}$

.14426354954966209729... ≈ $2\log\frac{3}{2}-\frac{2}{3} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{2^k(k+1)}$

.14430391622538321515... ≈ $\frac{\gamma}{4} = \int_0^{\infty}\left(e^{-x^4}-e^{-x^2}\right)\frac{dx}{x}$ GR 3.469.3

= $-\int_0^{\infty} e^{-x^2}x\log x\,dx$

3 .14439095907643773669... ≈ $\sum_{k=2}^{\infty}\left(\zeta^2(k)-1\right)^2$

.14448481337205342354... ≈ $e\left(e^{-1/e}-1\right)-e^{-1/e} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(k+1)!e^k}$

.14449574963534399955... ≈ $\sum_{k=1}^{\infty}\frac{H^{(3)}{}_k}{3^k(2k+1)}$

2 .14457270072683366581... ≈ $\sum_{k=1}^{\infty}\frac{2^k}{k!k^4}$

.14471321009242743159 9... ≈ $\frac{2}{9}\left(3+Li_3(-3)\right) = \int_1^{\infty}\frac{\log^2 x\,dx}{x^3+3x^2}$

.14472988584940017414 3... ≈ $-1+\log\pi = \log\frac{\pi}{e} = \sum_{k=1}^{\infty}\frac{\zeta(2k)}{4^k(2k^2+k)}$ AMM 74, 80–81 (1967)

1 .14472988584940017414 3... ≈ $\log\pi = \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty}(-1)^k\left(\frac{1}{2k}-\log\frac{k+1/2}{k}\right)$ Prud. 5.5.1.17

.14476040944446771627... ≈ $31\zeta(5)-32 = \zeta\left(5,\frac{3}{2}\right) = \sum_{k=1}^{\infty}\frac{1}{(k+\frac{1}{2})^5}$

1 .14479174504811776643... ≈ $\sum_{k=1}^{\infty}\frac{1}{k^2}\cos\frac{1}{k} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}\zeta(2k)}{(2k-2)!}$

.14490138006499325868... ≈ $\frac{8}{25}-\frac{\pi}{10}(\sqrt{2}-1)-\frac{4\log 2}{5}+\frac{\sqrt{2}}{5}\log(1+\sqrt{2})$

= $\sum_{k=1}^{\infty}\frac{1}{k(8k+5)}$ Prud. 5.1.7.23

.14493406684822643647... ≈ $\frac{\pi^2}{6}-\frac{3}{2} = \sum_{k=2}^{\infty}\frac{1}{k^3+k^2} = \sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(k+2)-1\right)$

= $\sum_{k=2}^{\infty}\frac{H_k}{k^2+k} = \sum_{k=1}^{\infty}\frac{\zeta(2k)-\zeta(2k+2)}{4^k}$

= $\int_0^{\infty}\frac{4x}{(1+x^2)^2(e^{2\pi x}-1)}dx$

= $\int_0^1 \log(1+x^2)\frac{dx}{x}$ GR 4.29.1

1 .14493406684822643647... ≈ $\frac{\pi^2}{6} - \frac{1}{2} = \sum_{k=2}^{\infty}(-1)^k\left(\zeta(k)+\zeta(k+1)-2\right)$

2 .14502939711102560008... ≈ $\pi^{2/3}$

.14540466392318244170... ≈ $\frac{381}{2187} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^7}{2^k}$

.14541345786885905697... ≈ $2 - \log(e^2-1) = \sum_{k=1}^{\infty}\frac{1}{ke^{2k}}$ GR 1.513.3

.14544838683609430704... ≈ $\frac{1}{25}\psi^{(1)}\left(\frac{3}{5}\right) = 8\pi^2 - 25 - \frac{8\pi^2\sqrt{5}}{5} - \frac{1}{25}\psi^{(1)}\left(\frac{7}{5}\right)$

= $\sum_{k=1}^{\infty}\frac{1}{(5k-2)^2}$

.14552316993048935367... ≈ $Li_3\left(\frac{1}{7}\right) = \Phi\left(\frac{1}{7},3,0\right) = \sum_{k=1}^{\infty}\frac{1}{7^k k^3}$

1 .145624268262139279409...≈ $\frac{\pi^2}{12} + \frac{\pi^3}{32} - \frac{\pi}{18} - \left(\frac{3G}{2} + \frac{1}{3} + \frac{3\pi^2}{16}\right)\log 2 + G\left(\frac{2}{3} - \frac{\pi}{4}\right) + \frac{7\zeta(3)}{4}$

= $\sum_{k=1}^{\infty}\frac{H_k}{(4k-3)^2}$

.14573634739282131233... ≈ $\frac{11}{25} + \frac{1}{100}\left(3\log 5 - 8\arctan 2 - 44\gamma\right) = \int_0^{\infty} x e^{-x}\log x\cos^2 x\, dx$

.14581354982799554765... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)}{k!(k+1)!} = \sum_{k=1}^{\infty}\left(\sqrt{k}\, I_1\left(\frac{2}{\sqrt{k}}\right) - 1 - \frac{1}{2k}\right)$

.1458333333333333333$\underline{3}$ = $\frac{7}{48} = \sum_{k=1}^{\infty}\frac{1}{k^2+4k} = \sum_{k=0}^{\infty}\frac{1}{(k+1)(k+5)} = \sum_{k=3}^{\infty}\frac{(-1)^{k+1}}{k^2-4}$

= $\int_0^1 x^3\log(x+1)\, dx$

= $\int_1^{\infty}\log\left(1+\frac{1}{x}\right)\frac{dx}{x^5}$

.14583694321462489756... ≈ $\frac{53}{24} - \log\pi + 56\zeta'(-6) + 140\zeta'(-4) + 56\zeta'(-2)$

= $\sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{k+4}$

.14585774315725853880... ≈ $\frac{\log 2}{2}+\frac{1}{4}-\frac{2G+1}{2\pi} = \sum_{k=2}^{\infty}\left(\frac{(2k-2)!!}{(2k)!!}\right)^2 \frac{1}{2k-2}$ J385

.14588643502776689782... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{2^k}\,\zeta(2k+1)$

.14589803375031545539... ≈ $\frac{7-3\sqrt{5}}{2} = \varphi^{-4}$

.14606785416078990650... ≈ $2\log 2 - \frac{\log^2 2}{2} - 1 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}H_k}{k+2}$

3 .1462643699419723423... ≈ $\sqrt{2} + \sqrt{3}$

.1463045386077697265... ≈ $4 - \frac{\pi^2}{4} - 2\log 2 = \sum_{k=1}^{\infty}\frac{1}{k(2k+1)^2}$ GR 0.236.7

= $\int_0^1 \log(1-x^2)\log x\,dx$

.14630461084237077693... ≈ $2\sqrt{2}\arctan\frac{1}{\sqrt{2}} + \log\frac{3}{2} - 2 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k(2k+1)2^k}$

.14632994687169923278... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^4-8}$

.14641197161688952006... ≈ $\int_0^{\infty}\frac{\cos x}{e^{x/2}+1}dx$

.146443311828333259614... ≈ $\frac{\pi}{4}\left(\pi\cosh\pi - \sinh\pi\right)\operatorname{csch}^2\pi = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{(k^2+1)^2}$

.1464466094067262378... ≈ $\frac{1}{2} - \frac{1}{2\sqrt{2}} = \frac{2-\sqrt{2}}{4}$

.1464539518270309601... ≈ $\frac{1}{2} - \frac{2\cos 1 + \sin 1}{2e} = \int_1^e \frac{\log x\,\sin\log x}{x^2}dx$

= $\int_0^1 xe^{-x}\sin x\,dx$

1 .14649907252864280790... ≈ $PFQ\left[\{1,1,1\},\{2,2,2\},1\right] = \sum_{k=1}^{\infty}\frac{1}{k!k^2} = \frac{1}{2}\int_0^1 e^x\log^2 x\,dx$

.146581405847371179872... ≈ $\gamma - 1 + \log\frac{9\pi}{16} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^k(k+1)}\left(\zeta(k+1)-1\right)$

2 .146592370069285194862... ≈ $\frac{6\pi}{\sqrt{5}} - 2\pi = -\int_0^{2\pi}\frac{\cos x}{3/2+\cos x}dx$

.14665032755625354074... ≈ $24 - 13\cos 1 - 20\sin 1 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(2k)!(k+2)}$

1 .14673299472862373219... ≈ $\sum_{k=1}^{\infty}\frac{1}{e^{k/2}+1}$ Berndt 6.14.1

.146997759396759645699... ≈ $\frac{1}{4\pi\sqrt{2}}\zeta\left(\frac{3}{2}\right) = \int_0^{\infty}\frac{\sqrt{x}\,dx}{e^{2\pi x}-1}$

2 .147043391355541844907... ≈ $\frac{\pi^3}{32}+\frac{3\pi}{8} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^4}{(k^2-1/4)^3}$

.14711677137965943279... ≈ $\frac{i}{4}\left(\psi^{(1)}(2+i)-\psi^{(1)}(2-i)\right) = \frac{i}{4}\left(\zeta(2,2+i)-\zeta(2,2-i)\right)$

= $\sum_{k=1}^{\infty}(-1)^{k+1}k\left(\zeta(2k+1)-1\right) = \sum_{k=2}^{\infty}\frac{k}{\left(k^2+1\right)^2}$

= $\frac{1}{2}\int_0^{\infty}\frac{x\sin x}{e^x(e^x-1)}dx$

.1471503722317380396... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^k+2}$

.14718776495032468057... ≈ $\frac{9\log 3}{40}-\frac{1}{10} = \int_0^{\infty}\frac{(\sin x - x\cos x)^3}{x^6}dx$ Prud. 5.2.29.24

.14722067695924125830... ≈ $-\frac{\log^2 2}{2}-\frac{\pi^2}{12}+\log 2\log 3 - Li_2\left(-\frac{1}{2}\right)$

= $\log 2\log 3 - \log^2 2 - \frac{1}{2}Li_2\left(\frac{1}{4}\right)$

= $\int_0^1\frac{\log(1+x)}{x+2}dx$

12 .147239059010648715201... ≈ $\frac{90}{\pi^4-90} = \frac{1}{\zeta(4)-1}$

.14726215563702155805... ≈ $\frac{3\pi}{64} = \int_0^1 x^3\arccos x dx$

.14729145183287003209... ≈ $1-\frac{Ei(1)+1-\gamma}{e} = \sum_{k=1}^{\infty}(-1)^k\frac{H_k}{(k-1)!}$

= $-\int_0^1 e^{x-1}x\log x\,dx$

.14747016658015036982... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}k^2\left(\zeta(3k)-1\right) = \sum_{k=2}^{\infty}\frac{k^3(k^3-1)}{(k^3+1)^3}$

.14758361765043327418... ≈ $\frac{1}{\pi}\arctan\left(\frac{1}{2}\right)$

Plouffe’s constant, proved transcendental by Barbara Margolius

.14766767192387686404... ≈ $\sum_{k=1}^{\infty}\frac{1}{9k^3-1} = \sum_{k=1}^{\infty}\frac{\zeta(3k)}{9^k}$

.14775472229893261117... ≈ $5-\frac{8}{\sqrt{e}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{2^k(k+3)!}$

.14779357469631903702... ≈ $\frac{\sqrt{2}}{2}+\frac{\operatorname{arcsinh}1}{2}-1$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}(2k-1)!!}{(2k)!\,(4k^2-1)}$

1 .14779357469631903702... ≈ $\frac{\sqrt{2}}{2}+\frac{\operatorname{arcsinh}1}{2} = \frac{\sqrt{2}}{2}+\frac{1}{2}\log\left(1+\sqrt{2}\right)$

= $\int_0^1\sqrt{1+x^2}\,dx$

.147822241805417... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k\log k}{k^2-1}$

.14791843300216453709... ≈ $\frac{3}{2}\left(\log 3-1\right) = \sum_{k=1}^{\infty}\frac{1}{9k^3-k}$ J375

= $\sum_{k=1}^{\infty}\frac{\zeta(2k+1)}{9^k}$

.14792374993677554778... ≈ $4-3e^{1/4} = \sum_{k=0}^{\infty}\frac{k}{(k+1)!\,4^k}$

1 .14795487733873058... ≈ $H_{5/2}^{(3)}$

.14797291388012184113... ≈ $\frac{\sqrt{2}}{4}\coth\sqrt{2}-\frac{1}{4} = \sum_{k=1}^{\infty}\frac{1}{k^2\pi^2+2}$

1 .14812009937000665077... ≈ $\frac{1}{7}+\frac{\pi\tan\pi\sqrt{2}}{8\sqrt{2}} = \sum_{k=1}^{\infty}\frac{1}{4k^2+4k-7}$

1 .148135649546457344478... ≈ $\sum_{k=1}^{\infty}\frac{\Phi(k)}{2^k k}$

$.148148148148148148\underline{148}$ = $\frac{4}{27} = \sum_{k=0}^{\infty}\frac{(-1)^k(k+1)}{2^k}$

3 .14821354898637004864... ≈ $\sum_{k=0}^{\infty}\frac{\zeta(k+2)}{(k!)^2} = \sum_{k=1}^{\infty}\frac{1}{k^2}I_0\left(2\sqrt{\frac{1}{k}}\right)$

.14831179749879259272... ≈ $Li_2\left(\frac{1}{7}\right)=\Phi\left(\frac{1}{7},2,0\right)=\sum_{k=1}^{\infty}\frac{1}{7^k k^2}$

.14834567213507945656... ≈ $4\sqrt{2}\arctan\left(\frac{1}{\sqrt{2}}\right)-\frac{10}{3} = \sum_{k=0}^{\infty}\frac{1}{2^k(2k+5)}$

1 .14838061778888222872... ≈ $\frac{\pi^3}{27}$

.14839396162690884587... ≈ $1-\frac{\pi\sqrt{3}}{18}-\frac{\log 3}{2} = \frac{1}{3}hg\left(\frac{1}{3}\right) = \frac{1}{3}\sum_{k=1}^{\infty}\frac{1}{3k^2+k}$

= $\sum_{k=2}^{\infty}\frac{(-1)^k\zeta(k)}{3^k}$

= $\int_1^{\infty}\frac{dx}{x^4+x^3+x^2}$

.14849853757254048108... ≈ $\frac{1}{4}-\frac{3}{4e^2} = \sum_{k=0}^{\infty}\frac{(-1)^k 2^k}{k!(k+2)}$

1 .1486983549970350068... ≈ $2^{1/5}$

1 .14874831559185321424... ≈ $16(7-4\sqrt{3}) = \sum_{k=0}^{\infty}\frac{1}{16^k(k+1)}\binom{2k+2}{k}$

.14885277443216080376... ≈ $4Li_3\left(\frac{1}{2}\right)-2 = \int_1^{\infty}\frac{\log^2 x}{2x^3-x^2}dx$

.14900914406163310702... ≈ $2\log\left(\frac{1}{2}+\frac{1}{\sqrt{3}}\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(2k-1)!!}{(2k)!\,3^k k}$

.14901768840433479264... ≈ $\sum_{k=2}^{\infty}\frac{\log k}{k^3+k^2}$

8 .14912752141674121819... ≈ $\frac{4e^3-7}{9} = \sum_{k=1}^{\infty}\frac{3^k k^2}{(k+2)!}$

.14918597297347943780... ≈ $9\log\frac{3}{2}-\frac{7}{2} = \sum_{k=0}^{\infty}\frac{(-1)^k}{2^k(k+1)(k+2)(k+3)}$

= $-\frac{1}{2}+\sum_{k=0}^{\infty}\frac{1}{3^k(k+2)}$

.14919664819825141009... ≈ $\frac{\sqrt{\pi}}{8e^{9/4}}\left(e^2-1\right) = \int_0^{\infty}e^{-x^2}\sin^2 x\cos x\,dx$

.14936120510359182894... ≈ $\frac{3}{e^3} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{3^k k}{k!}$

6 .14968813538870263774... ≈ $2\ {}_2F_1\left(2,3,\frac{3}{2},\frac{1}{4}\right) = \sum_{k=1}^{\infty}\frac{k!(k+1)!}{(2k-1)!}$

.149762131525267452517... ≈ $\frac{19}{5}-\frac{\pi}{2}-3\log 2 = \sum_{k=2}^{\infty}\frac{1}{k(4k+1)} = \sum_{k=2}^{\infty}\frac{(-1)^k(\zeta(k)-1)}{4^{k-1}}$

1 .14997961547450537807... ≈ $\sum_{k=2}^{\infty}2^k\left(\zeta(k)-1\right)^3$

$$.1500000000000000000\underline{0} \quad = \quad \frac{3}{20}$$

$$.150076728375356585727\ldots \approx \quad \sum_{k=2}^{\infty}(-1)^k \frac{\zeta(k)-1}{2^k(k-1)} = \sum_{k=2}^{\infty}\frac{1}{2k}\log\left(1+\frac{1}{2k}\right)$$

$$.15017325550213874759\ldots \quad \approx \quad \frac{2}{\pi\sqrt{3}}\sin\frac{\pi\sqrt{3}}{2} = \prod_{k=1}^{\infty}\left(1-\frac{3}{4k^2}\right)$$

$$.15025711289494928567\,5\ldots \approx \quad \frac{\zeta(3)}{8} = \sum_{k=1}^{\infty}\frac{1}{(2k)^3} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}H_k}{(k+1)^2}$$

$$= \quad \int_0^1 \frac{\log^2(1+x^2)}{x}dx = -\int_0^1 \frac{\log(1+x)\log x}{1+x}dx$$

$$= \quad \int_0^1\int_0^1 \frac{\log(1+xy)}{1+xy}dx\,dy$$

1 $$.15050842410886528915\ldots \quad \approx \quad \frac{1}{16}\left(\psi^{(1)}\left(\frac{1}{8}\right)-\psi^{(1)}\left(\frac{5}{8}\right)\right)-\frac{\pi}{2\sqrt{2}}\log(1+\sqrt{2})-\frac{\pi^2}{4\sqrt{2}}$$

Berndt 3.3.2

$$= \quad i\sqrt{2}\left(Li_2\left(i(1-\sqrt{2}\right)-Li_2\left(i(\sqrt{2}-1\right)\right)$$

$$= \quad \sum_{k=0}^{\infty}\frac{2^k(k!)^2}{(2k)!(2k+1)^2}$$

$$= \quad 2\int_0^{\pi/2}\frac{x\sin x}{2-\cos^2 x}dx$$

$$.15058433946987839463\ldots \quad \approx \quad \frac{1}{2}(\sin 1-\cos 1) = \sum_{k=1}^{\infty}\frac{(-1)^k k}{(2k+1)!}$$

$$= \quad \int_1^{\infty}\sin\left(\frac{1}{x^2}\right)\frac{dx}{x^5}$$

$$.15068795010189670188\ldots \quad \approx \quad \frac{3\zeta(3)}{4}-\frac{\pi^4}{15}-\frac{\pi^2\log^2 2}{4}-2\log^3 2+\frac{\log^4 2}{4}$$

$$+\,6Li_4\left(\frac{1}{2}\right)+\frac{21}{4}\zeta(3)\log 2$$

$$= \quad \int_0^1 \frac{\log^3(1+x)}{x^2(x+1)}dx$$

$$.1507282898071237098\ldots \quad \approx \quad 8\log 2-4\log 3-1 = \sum_{k=1}^{\infty}\frac{(-1)^k}{3^k k(k+1)}$$

$$.150770165868941637996\ldots \approx \quad \frac{1}{4}-\frac{\pi}{2\sqrt{6}}\operatorname{csch}\pi\sqrt{\frac{2}{3}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{3k^2+2}$$

$$.1508174952896963158 8\ldots \quad \approx \quad \sum_{k=2}^{\infty}\left(\zeta(k)-1\right)\left(\zeta(k+1)-1\right)$$

.15084550274572283253... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^k}{k^3+3}$

.1508916323731138546... ≈ $\frac{110}{729} = \Phi\left(\frac{1}{10},-2,0\right) = Li_{-2}\left(\frac{1}{10}\right) = \sum_{k=1}^{\infty}\frac{k^2}{10^k}$

.15097924480756398406... ≈ $\frac{\pi^2}{3}-\frac{113}{36} = \sum_{k=1}^{\infty}\frac{k}{(k+2)(k+4)^2}$

1 .15098236809467638636... ≈ $\frac{\pi\sqrt{3}}{10}+\frac{3\log 3}{10}+\frac{2\log 2}{5} = \sum_{k=1}^{\infty}\frac{1}{6k^2-5k} = \sum_{k=0}^{\infty}\frac{1}{(k+1)(6k+1)}$

= $-\int_0^1 \frac{\log(1-x^6)}{x^6}dx$

.15114994701951815422... ≈ $\frac{\pi}{12\sqrt{3}} = \frac{1}{4}\sum_{k=1}^{\infty}\frac{1}{9k^2-9k+2} = \sum_{k=1}^{\infty}\frac{1}{4\binom{2k}{k}} = \int_0^{\infty}\frac{dx}{(x^2+3)^2}$

.15116440861650701737... ≈ $\zeta(3)+Li_3\left(\frac{1}{7}\right)-Li_3\left(\frac{6}{7}\right)-\frac{1}{2}\log\frac{7}{6}\left(\log\frac{7}{6}\log 7+2Li_2\left(\frac{6}{7}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{H_k}{7^k k^2}$

.1511911868771830003... ≈ $-\sum_{k=1}^{\infty}\frac{\mu(4k-1)}{2^{4k-1}-1}$

81 .15123429216781268058... ≈ γ^{-8}

.15128172040710543908... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\sigma_0(k)}{3^k}$

1 .15129254649702284201... ≈ $\frac{1}{2}\log 10$

.15129773663885396750... ≈ $\sum_{k=1}^{\infty}\frac{1}{F_k k^3}$

.151632664928158355901...≈ $\frac{1}{4\sqrt{e}} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2}{k!2^k}$

1 .15163948007840140449... ≈ $\frac{\zeta(3)\zeta(5)}{\zeta(4)}$

2 .15168401595131957998... ≈ $\sum_{k=1}^{\infty}\frac{\sqrt{k+1}}{k^{5/2}} = \sum_{k=2}^{\infty}\frac{\sqrt{k}}{(k+1)^{5/2}}$

.151696880108669610301...≈ $\frac{\pi}{8}\coth\frac{\pi}{2}-\frac{\pi^2}{16}\operatorname{csch}^2\frac{\pi}{2}-\frac{4}{25}$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)-1}{4^k} = \sum_{k=2}^{\infty}\frac{4k^2}{(4k^2+1)^2}$

.15169744087717637782... ≈ $\frac{\pi}{8}(2\log 2-1) = -\int\limits_0^{\pi/2}\log(\sin x)\sin^2 x\,dx$ GR 4.384.9

.151822325947027200439... ≈ $\log\frac{2}{e-1}$

.15189472903279400784... ≈ $\frac{4}{9}-\frac{\pi}{6}+\frac{\log 2}{3} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2k^2+3k}$

= $\int\limits_1^{\infty}\log\left(1+\frac{1}{x^2}\right)\frac{dx}{x^4}$

1 .151912873455946838843... ≈ $\sum_{k=0}^{\infty}\frac{1}{k^6+k^5+k^4+k^3+k^2+k+1}$

.1519346306616288552... ≈ $\frac{5}{6}\log\frac{6}{5} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{H_k}{5^k}$

.152020846736913608124... ≈ $\sum_{k=2}^{\infty}\frac{\log\zeta(k)}{k^2}$

.15204470482002019445... ≈ $2\sqrt{5}\log\frac{1+\sqrt{5}}{2}-2 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{\binom{2k}{k}(2k+1)k}$

.152185252101046135100... ≈ $-\frac{1}{5}\cos\pi\sqrt{\frac{3}{2}} = \prod_{k=1}^{\infty}\left(1-\frac{6}{(2k+1)^2}\right)$

1 .15220558708386827494... ≈ $\frac{\pi^2}{18}+\frac{4\log 2}{3}-\frac{2\log^2 2}{3} = \sum_{k=1}^{\infty}\frac{H_k}{(k+1)(2k-1)}$

.1523809523809$\underline{523809}$ = $\frac{16}{105} = \sum_{k=0}^{\infty}\binom{2k}{k}\frac{1}{4^k(k+3)(k+4)}$

.152607305856597283596... ≈ $\frac{8}{7}\log\frac{8}{7} = \sum_{k=1}^{\infty}\frac{H_k}{8^k}$

.15273597526733744319... ≈ $\frac{1}{2}-\frac{e^2}{4} = \sum_{k=0}^{\infty}\frac{2^k}{k!(k+4)(k+7)}$

17 .152789708268945095760... ≈ $(\pi+1)^2$

1 .15281481352532739811... ≈ $\frac{1}{2}+\frac{3\sqrt{2}}{4}\arcsin\frac{1}{\sqrt{3}} = \sum_{k=1}^{\infty}\frac{(2k)!!}{(2k-1)!!3^k}$

.15289600209100167943... ≈ $\sum_{k=2}^{\infty}\frac{k-1}{k^{k+1}}$

.15289756126777578495... ≈ $\frac{e(e+1)}{(e-1)^3}-\frac{5}{e} = \sum_{k=1}^{\infty}\frac{k^2}{e^k} - \int\limits_1^{\infty}\frac{x^2}{e^x}dx$

.15300902999217927813... ≈ $3-\zeta(2)-\zeta(3) = \sum_{k=2}^{\infty}\frac{1}{k^4-k^3} = \sum_{k=1}^{\infty}\frac{1}{k(k+1)^3}$

= $\sum_{k=1}^{\infty}\left(\zeta(k+3)-1\right)$

1 .15300902999217927813... ≈ $4-\zeta(2)-\zeta(3) = \sum_{k=2}^{\infty}\left(\zeta(k)+\zeta(k+2)-2\right)$

.15303055291394703953 6... ≈ $\frac{1}{2}+\gamma-\frac{\pi}{2\sqrt{3}}\cot\frac{\pi}{\sqrt{3}}+\frac{1}{2}\left(\psi\left(\frac{1}{\sqrt{3}}\right)+\psi\left(-\frac{1}{\sqrt{3}}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{k-1}{3k^3-k} = \sum_{k=1}^{\infty}\frac{\zeta(2k)-\zeta(2k+1)}{3^k}$

.15309938731499271977... ≈ $\frac{\log\zeta(3)}{\zeta(3)}$

1 .15318817864647891814... ≈ $\frac{\pi}{2}si\left(\frac{\pi}{2}\right)-1 = \int_0^1\frac{\arcsin x}{\arccos x}dx$

1 . 15326598908047301786... ≈ $\frac{\pi^4}{16}-\frac{\pi^2}{2}$

= $\frac{\pi^2}{2}\int_0^1\frac{x^2\log x}{x^2-1}\,dx$ Borwein-Devlin, p. 54

= $\int_0^{\infty}\int_y^{\infty}\frac{(x-y)^2}{x\,y\sinh(x+y)}\log\frac{x+y}{x-y}dx\,dy$ Borwein-Devlin, p. 53

2 .15334809493716234826 8... ≈ $\pi\coth\pi-1 = i\left(\psi(1-i)-\psi(1+i)\right) = 2\,\mathrm{Im}\{\psi(1+i)\}$

= $\sum_{k=1}^{\infty}\left(\frac{1}{k-i}-\frac{1}{k+i}\right)$

3 .15334809493716234827... ≈ $\pi\coth\pi = 1+\sum_{k=1}^{\infty}\frac{2}{k^2+1}$ J947

4 .15334809493716234827... ≈ $\pi\coth\pi+1 = i\left(\psi(-i)-\psi(i)\right) = 2\,\mathrm{Im}\left\{\psi(i)\right\}$

.153426409720027345 3... ≈ $\frac{1-\log 2}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k-1)2k(2k+1)}$ GR 0.238.2, J237

= $\sum_{k=1}^{\infty}\left(k\log\left(\frac{2k+1}{2k-1}\right)-1\right)$ J128

= $\sum_{k=1}^{\infty}\frac{1}{k(8k+4)}$

= $\sum_{k=2}^{\infty}\frac{1}{2^k(k^2-k)}$

= $\sum_{k=2}^{\infty} \frac{(2k-3)!k}{(2k)!}$

= $\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+4}$ K Ex. 109b

= $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{8k^3-2k}$

= $\sum_{k=1}^{\infty} \frac{\zeta(2k)}{4^k(2k+1)} = -\sum_{k=1}^{\infty}\left(1+4k^2\log\left(1-\frac{1}{4k^2}\right)\right)$

= $\int_1^{\infty} \frac{dx}{x^5+x^3} = \int_0^{\pi/4} \tan^3 x\, dx$

= $-\int_0^{\pi} \frac{\sin^3 x}{\sqrt{1+\sin^2 x}} \log\sin x\, dx$ Prud. 2.6.34.44

= $\int_1^2 \frac{\log x}{x^2} dx = \int_2^{\infty} \log\frac{x}{x-1}\cdot\frac{dx}{x}$

= $\int_1^{\infty} \log\left(1+\frac{1}{x}\right)\frac{dx}{(x+1)^2} = \int_0^1 \frac{\log(1+x)}{(1+x)^2} dx$ GR 4.291.14

= $\int_0^1 (1-x)\arctan x\, dx$

= $-\int_0^1 \left(\frac{1}{1-x^2}+\frac{1}{2\log x}-\frac{1}{2}\right)\frac{dx}{\log x}$ GR 4.283.5

= $-\int_0^{\infty} \left\{\left(\frac{1}{2}+\frac{1}{x}\right)e^{-x}-\frac{e^{-x/2}}{x}\right\}\frac{dx}{x}$ GR 3.438.1

= $\int_0^{\infty} \frac{dx}{e^{2x}(e^{2x}+1)}$

1 .15344961551374677440... ≈ $2J_1(2) = 3J_1(2)-J_2(2)-J_0(2) = \sum_{k=0}^{\infty}(-1)^k \frac{k^4}{(k!)^2}$

= $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k-1)!}\binom{2k}{k}$

$.153486153486\underline{153486}$ = $\frac{2}{13}$

.1535170865068480981... ≈ $\frac{1}{6\cdot 7^{1/3}}\left(\left(1-i\sqrt{3}\right)\psi\left(\frac{7+(-7)^{2/3}}{7}\right)+\left(1+i\sqrt{3}\right)\psi\left(1-\left(-\frac{1}{7}\right)^{1/3}\right)-2\psi\left(1+\frac{1}{7^{1/3}}\right)\right)$

= $\sum_{k=1}^{\infty} \frac{1}{7k^3+1} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(3k)}{7^k}$

.15354072519537154850... ≈ $\prod_{k=1}^{\infty}\frac{k^2}{k^2+1+k^{-1}}$

.15354517795933754758... ≈ $\psi^{(1)}(7) = \zeta(2) - H^{(2)}{}_6 = \zeta(2) - \frac{5369}{3600}$

1 .15356499489510775346... ≈ $e^{1/7}$

1 .15383506784998943054... ≈ $\pi^{1/8}$

.15386843320506566025... ≈ $\frac{181}{90} - \frac{\gamma}{5} - \frac{\log 2\pi}{2} + 4\zeta'(-3) + 6\zeta'(-2) + 360\zeta'(-1)$

= $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k+4}$

.15409235403694969975... ≈ $\frac{\pi}{48}\left(\pi - \sqrt{3}\sin\frac{\pi}{\sqrt{3}}\right)\csc^2\frac{\pi}{2\sqrt{3}} = \sum_{k=1}^{\infty}\frac{k\zeta(2k)}{12^k} = \sum_{k=1}^{\infty}\frac{12k^2}{(12k^2-1)^2}$

.154113806319188570 8... ≈ $2\zeta(3) - \frac{9}{4} = \sum_{k=3}^{\infty}(-1)^{k+1}(k-1)(k-2)(\zeta(k)-1) = \sum_{k=2}^{\infty}\frac{2}{(k+1)^3}$

= $-\psi^{(2)}(3)$

= $\int_1^{\infty}\frac{\log^2 x}{x^4-x^3}$

= $\int_0^1\frac{x^2\log x}{1-x}dx$

= $\int_0^{\infty}\frac{x^2}{e^{2x}(e^x-1)}dx$

.154142969550249868... ≈ $\sum_{k=1}^{\infty}\frac{H^{(3)}{}_k}{4^k(k+1)}$

.15415067982725830429... ≈ $\log 7 - \log 6 = Li_1\left(\frac{1}{7}\right) = \sum_{k=1}^{\infty}\frac{1}{7^k k}$

= $2\operatorname{arctanh}\frac{1}{13} = 2\sum_{k=0}^{\infty}\frac{1}{13^{2k+1}(2k+1)}$ K148

.15421256876702122842... ≈ $\frac{\pi^2}{64} = \sum_{k=1}^{\infty}\frac{1}{((2k-1)^2-4)^2}$ J383

15 .15426224147926418976... ≈ $e^e = \sum_{k=0}^{\infty}\frac{e^k}{k!}$ Not known to be transcendental

= $\prod_{k=0}^{\infty}e^{1/k!}$

$.15443132980306572121... \approx 2\gamma - 1 = \sum_{k=2}^{\infty} \frac{k-2}{k}\left(\zeta(k)-1\right) = \sum_{k=2}^{\infty}\left(2\log\left(1-\frac{1}{k}\right) + \frac{2k-1}{k(k-1)}\right)$

1 $.15443132980306572121... \approx 2\gamma = -\sum_{k=1}^{\infty} \frac{\mu(k)\log^2 k}{k}$

2 $.15443469003188372176... \approx \sqrt[3]{10}$

$.15447598997928881635 0... \approx \sum_{k=1}^{\infty} (-1)^{k+1} \log\zeta(2k+1)$

1 $.1545763107479915715... \approx \frac{40}{9} - \frac{\pi^2}{3} = H^{(2)}{}_{3/2}$

$.1545789367644481 1... \approx \sum_{k=2}^{\infty} \frac{(-1)^k}{k^3 \log k}$

1 $.1545804352581151025... \approx \sum_{k=1}^{\infty} \frac{1}{S_2(2k,k)}$

$.15467960838455727096... \approx \frac{7}{32\sqrt{2}} = \sum_{k=1}^{\infty} \frac{(-1)^k k^3}{4^k(2k-1)}\binom{2k}{k}$

$.15468248282360638137... \approx \sum_{k=1}^{\infty} \frac{1}{(3k-1)^3 - 1}$

$= \frac{\gamma}{9} + \frac{\pi}{18\sqrt{3}} + \frac{\log 3}{6} + \frac{1}{18\left(\sqrt{3}+3i\right)}\left(4\sqrt{3}\psi\left(\frac{5-i\sqrt{3}}{6}\right) - 2\left(\sqrt{3}-3i\right)\psi\left(\frac{5+i\sqrt{3}}{6}\right)\right)$

$.1547005383792515290... \approx \frac{2}{\sqrt{3}} - 1 = \sum_{k=0}^{\infty} \frac{(-1)^{k+1} k}{2^k(k+1)}\binom{2k}{k}$

$= \sum_{k=0}^{\infty} \frac{(2k)!}{(k!)^2 4^{2k}}$

$= \sum_{k=1}^{\infty} \frac{(2k-1)!!}{(2k)!\, 4^k}$

1 $.1547005383792515290 2... \approx \frac{2}{\sqrt{3}} = \csc\frac{\pi}{3} = \sum_{k=0}^{\infty} \frac{1}{16^k}\binom{2k}{k}$ AS 4.3.46, CFG B4

$= 1 + \sum_{k=1}^{\infty} \frac{(2k-1)!!}{(2k)!!2^{2k}}$ J166

$.15476190476190\underline{476190} = \frac{13}{84} = \sum_{k=0}^{\infty} \frac{1}{(3k+1)(3k+10)}$ K134

2 $.15484548537713570608... \approx 3e - 6 = \int_0^1 e^{x^{1/3}}\,dx$

7 .15484548537713570608... ≈ $3e - 1 = \sum_{k=1}^{\infty}\frac{k^4}{(k+1)!}$

8 .15484548537713570608... ≈ $3e = \sum_{k=1}^{\infty}\frac{k(k+1)}{k!}$

.15491933384829667541... ≈ $\frac{1}{5}\sqrt{\frac{3}{5}} = \sum_{k=1}^{\infty}(-1)^{k+1}\binom{2k}{k}\frac{k}{6^k}$

.15494982830181068512... ≈ $-\operatorname{Re}\{\Gamma(i)\}$

.154951797129101438393... ≈ $\frac{3\pi+12}{8}\log 2 - \frac{\pi}{4} - G = \sum_{k=1}^{\infty}\frac{H_k}{(4k-1)(4k+1)}$

.15519690003711989154... ≈ $-\zeta\left(-\frac{2}{3}\right)$

.15535275300432320321... ≈ $\gamma + \psi\left(1+\pi^2\right) = H_{\pi^2} = \sum_{k=1}^{\infty}\frac{1}{k^2\pi^2+k}$

.15555973599994086446... ≈ $\frac{\zeta(2)-\zeta(3)}{\zeta(2)+\zeta(3)}$

.15562624502848612111... ≈ $216 - 18\pi\sqrt{3} - \pi^2 - 54\log 3 - 72\log 2 + \zeta(3)$

= $\sum_{k=1}^{\infty}\frac{1}{6k^4+k^3} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(k+3)}{6^k}$

1 .15572734979092171791... ≈ $\frac{\pi}{e} = \int_{-\infty}^{\infty}\frac{\cos x}{(1+x^2)^2}dx = \int_{0}^{\infty}\frac{2\cos t}{1+t^2}dt$

.155761215939713984946... ≈ $\sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)-1}{(k!)^2} = \sum_{k=2}^{\infty}\left(\frac{1}{k} - 1 + J_0\left(2\sqrt{\frac{1}{k}}\right)\right)$

.15580498523890464859... ≈ $4 - 4\sqrt{2}\arctan\frac{1}{\sqrt{2}} + 2\log\frac{2}{3} - Li_2\left(-\frac{1}{2}\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^k k^2(2k+1)}$

.1559436947653744735... ≈ $\cos\sqrt{2} = \sum_{k=0}^{\infty}(-1)^k\frac{2^k}{(2k)!}$ GR 1.411.3

.15609765541856722578... ≈ $\int_{1}^{\infty}\frac{dx}{x^4+x^3+x}$

.1561951536544784133... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k^2+1} = \sum_{k=1}^{\infty}\frac{1}{4k}\left(1+2k-2\log\left(1-\frac{1}{k}\right)+2k^2\log\left(1-\frac{1}{k}\right)\right)$

.15622977483540679645... ≈ $-\sum_{k=1}^{\infty}\frac{H_k\mu(k)}{2^k}$

.156344287479823877804... ≈ $-\frac{7}{10}+\frac{\pi}{4}\coth\frac{\pi}{2} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)-1}{4^k}$

$= \sum_{k=2}^{\infty}\frac{1}{4k^2+1} = -\mathrm{Re}\left\{\sum_{k=2}^{\infty}\left(\zeta(k)-1\right)\left(\frac{i}{2}\right)^k\right\}$

.15641068822825414085... ≈ $\frac{\pi}{e^3} = \int_0^{\infty}\frac{\cos 3x}{(1+x^2)^2}dx = \int_{-\infty}^{\infty}\frac{\cos 3x}{x^2+1}dx$

.15651764274966565182... ≈ $\frac{1}{e^2-1} = \frac{\coth 1-1}{2} = \sum_{k=1}^{\infty}\frac{1}{e^{2k}} = \sum_{k=1}^{\infty}\frac{1}{k^2\pi^2+1} = \sum_{k=0}^{\infty}\frac{B_k 2^{k-1}}{k!}$

1 .15651764274966565182... ≈ $\frac{e^2}{e^2-1} = \frac{1+\coth 1}{2} = \sum_{k=0}^{\infty}\frac{1}{e^{2k}} = \sum_{k=0}^{\infty}\frac{1}{k^2\pi^2+1}$ J951

.15652368328780337093... ≈ $6\cos 1+10\sin 1-\frac{23}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k)!(k+2)}$

1 .15654977606425149905... ≈ $\frac{1}{2}\left(e-e^{\cos 2}\cos(\sin 2)\right) = \frac{e}{2}-\frac{e^{e^{2i}}+e^{e^{-2i}}}{4} = \sum_{k=1}^{\infty}\frac{\sin^2 k}{k!}$

.156643842100387720666... ≈ $\log^3 2-\gamma\log^2 2-2\gamma_1\log 2 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\log^2 k$

γ_1 = Stieltjes const.

12 .1567207587610607447... ≈ $\pi^3-6\pi = \int_0^{\pi}x^3\sin x\,dx$

1 .15688147304141093867... ≈ $\frac{2+\log\pi}{e} = -\int_0^{\infty}\log x\log\left(1+\frac{1}{\pi^2e^2x^2}\right)dx$

.156899682117108925297... ≈ $\frac{\pi\sqrt{3}}{6}-\frac{3}{4} = \int_1^{\infty}\log\left(1+\frac{1}{x^3}\right)\frac{dx}{x^3}$

.15707963267948966192... ≈ $\frac{\pi}{20} = \int_0^{\infty}\frac{dx}{e^{5x}+e^{-5x}}$

.157187089473767855916... ≈ $\frac{3}{e^3-1}$

3 .157465672184835229312... ≈ $\pi\log(1+\sqrt{3}) = \int_0^{\infty}\frac{\log(x^2+3)}{x^2+1}dx$

1 .15753627711664634890... ≈ $\frac{26\zeta(3)}{27} = G_3 = 1+\sum_{k=1}^{\infty}\frac{1}{(3k-1)^3}+\sum_{k=1}^{\infty}\frac{1}{(3k+1)^3}$

1 .15757868669705850021... ≈ $\frac{\sqrt{\pi}}{4}\zeta\left(\frac{3}{2}\right) = \int_0^{\infty}\frac{x^2\,dx}{e^{x^2}-1}$

$.157660149167832330391\ldots \approx \frac{1}{9}\left(\frac{2\pi^2}{6}-\psi^{(1)}\left(\frac{1}{3}\right)\right) = \int_1^\infty \frac{x\log x}{1+x+x^2}dx$ GR 4.233.3

$= \left(-\frac{1}{1+(-1)^{1/3}}\right)\left((-1)^{1/3}Li_2((-1)^{1/3}) + Li_2(-1)^{2/3}\right)$

$.1577286052509934237\ldots \approx \cos^3 1 = \frac{1}{4}\sum_{k=0}^\infty (-1)^k \frac{3^{2k}+3}{(2k)!}$ GR 1.412.4

$.15779670004249836201\ldots \approx \prod_{k=2}^\infty \left(1-\frac{k}{2^k}\right)$

$.15783266728161021181\ldots \approx \frac{\pi-1}{\pi^2 e^{1/\pi}} = \sum_{k=1}^\infty (-1)^{k+1}\frac{k^2}{k!\pi^k}$

9 $.157839086795201393147\ldots \approx 8\log\pi$

$.\underline{157894736842105263} = \frac{3}{19}$

$.15803013970713941960\ldots \approx \frac{1}{4}-\frac{1}{4e} = \int_1^\infty \cosh\left(\frac{1}{x^4}\right)\frac{dx}{x^9}$

$.1580762\ldots \approx \sum_{\substack{\omega \text{ a non-trivial}\\ \text{integer power}}} \frac{1}{(\omega-1)^2}$

$.158151287891164991627\ldots \approx \frac{3\zeta(3)}{2}-\zeta(2) = \int_0^1 \frac{x\log^2 x}{(1+x)^2}dx = \int_1^\infty \frac{\log^2 x}{x(1+x)^2}$

$.15822957412289865433\ldots \approx \sum_{k=1}^\infty \frac{H_k}{3^k(2k+1)}$

$.158299797338769411097\ldots \approx \sum_{k=0}^\infty \frac{k!}{S_1(2k,k)}$

$.15871527483457111423\ldots \approx \frac{1}{2}-\frac{\pi}{4}\operatorname{csch}\frac{\pi}{2} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{4k^2+1}$

$.158747447059348972898\ldots \approx -\log\Gamma\left(2+\frac{i}{2}\right)-\log\Gamma\left(2-\frac{i}{2}\right) = \sum_{k=1}^\infty (-1)^{k+1}\frac{\zeta(2k)-1}{4^k k}$

$.158806563699494307872\ldots \approx \frac{1}{6}-\frac{\pi\sqrt{3}}{6}\operatorname{csch}\pi\sqrt{3} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{k^2+3}$ J125

$.15886747797947540615\ldots \approx \frac{\pi^2}{16}-\frac{G}{2} = \sum_{k=1}^\infty \frac{1}{(4k-1)^2} = \int_1^\infty \frac{\log x\,dx}{x^4-1}$

$.15888308335967185650\ldots \approx 6\log 2 - 4 = \sum_{k=1}^\infty \frac{k}{2^k(k+1)(k+2)}$

4 $.15888308335967185650\ldots \approx 6\log 2 = \sum_{k=0}^\infty \frac{(-1)^k}{(k+\frac{1}{3})(k+\frac{2}{3})}$

12 .15888308335967185650... ≈ $8+6\log 2 = \sum_{k=1}^{\infty}\frac{k^2 H_k}{2^k}$

40 .15890107530112852685... ≈ $\sum_{k=1}^{\infty}\frac{k^2}{F_k}$

1 .1589416532550922853... ≈ $\sum_{k=0}^{\infty}\frac{B_{2k}}{(2k)!}\binom{2k}{k}$

3 .1589836751001364405 3... ≈ $\sum_{k=1}^{\infty}\frac{t_4(k)}{2^k} = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\frac{\sigma_0(k)}{2^{kj}-1} = \sum_{1=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{1}{2^{ijk}-1} = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\frac{1}{2^{iijkl}}$

7 .1589965368043850938 2... ≈ $I_0(2\sqrt{3}) = \sum_{k=0}^{\infty}\frac{3^k}{(k!)^2} = {}_oF_1(;1;3)$

.1590933528074342109 1... ≈ $\frac{\pi}{14}-\frac{16}{245} = \int_0^1 x^6 \arcsin x\,dx$ GR 4.523.1

.15910554477104412896 6... ≈ $\frac{1}{5+\sqrt{5}}\left(2\log 5+\left(1+\sqrt{5}\right)\log\left(5-2\sqrt{5}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{F_k F_{k+1}}{4^k k(k+1)} = \sum_{k=1}^{\infty}\frac{F_k L_k}{4^k(2k+1)}$

.1591190225020152905 4... ≈ $\frac{5}{2}\arctan\frac{1}{2} - 1 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^{2k+1}(2k-1)(2k+1)}$

.15913709258673958744 4... ≈ $\frac{13-16\log 2}{12} = \sum_{k=1}^{\infty}\frac{1}{k(2k+1)(k+2)}$

.1591484528128319195... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{H_{2k}}{8^k} = \frac{1}{9}\left(4\log\frac{9}{8}+2\sqrt{2}\arctan\frac{1}{2\sqrt{2}}\right)$

.15915494309189533576 9... ≈ $\frac{1}{2\pi}$

1 .1592484598827166306 4... ≈ $\frac{\zeta(3)}{\zeta(5)}$

.15934368985917586135 4... ≈ $\sum_{k=1}^{\infty}\frac{\left(\zeta(2k)-\zeta(2k+2)\right)^2}{2^k}$

.15943612687526747080 1... ≈ $\frac{\pi^2}{32}-\frac{5}{12}+\frac{2\log 2}{3}-\frac{\log^2 2}{3} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}H_k}{k(k+3)}$

13 .15947253478581149177 9... ≈ $\frac{4\pi^2}{3} = \psi^{(1)}\left(\frac{1}{3}\right)+\psi^{(1)}\left(\frac{2}{3}\right)$

.1595628100828400101 1... ≈ $\frac{\pi^2}{24}+\frac{\pi}{2}-1 = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\cos^4 k}{k^2}$

$.1597222222222222222\underline{2} \quad = \quad \frac{23}{144} = \sum_{k=1}^{\infty} \frac{1}{k(k+1)(k+4)}$

1 $.15973454203768787427... \quad \approx \quad \sqrt{e}\,\mathrm{arccoth}\sqrt{e} = \frac{\sqrt{e}}{2} \log \coth \frac{1}{4} = \sum_{k=0}^{\infty} \frac{1}{e^k (2k+1)}$

$.15986890374243097175 7... \approx \quad \gamma \log 2 - \frac{1}{2} \log^2 2 = \sum_{k=2}^{\infty} (-1)^k \frac{\log k}{k} = -\sum_{k=1}^{\infty} \frac{\psi(k)}{2^k k}$

2 $.15990451200777555816... \quad \approx \quad \psi^{(1)}\left(\frac{5}{6}\right)$

$.16000000000000000000\underline{0} \quad = \quad \frac{4}{25} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k}{4^k}$

1 $.160014413116984775294\ldots \approx \quad \frac{\sqrt{3}}{4}\Gamma\left(\frac{1}{3}\right) = \int_0^\infty \frac{\sin^3 x}{x^3}\, dx$

$.1600718432431485233\ldots \quad \approx \quad \frac{1}{5}(\gamma - \log 4 + \log 5) = \sum_{k=1}^{\infty} (-1)^k \frac{\psi(k)}{4^k}$

2 $.1601271206667850515\ldots \quad \approx \quad \sum_{k=1}^{\infty} \frac{\log k \log(k+1)}{k^2}$

$.16019301354439554287\ldots \quad \approx \quad -Li_2\left(-\frac{1}{6}\right) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{6^k k^2} = \sum_{k=1}^{\infty} \frac{H_k}{7^k k}$

$.16043435647123872247\ldots \quad \approx \quad \frac{16}{9} - \frac{7\log 2}{3} = \sum_{k=1}^{\infty} \frac{1}{2^k k(k+3)}$

$.16044978576935580042\ldots \quad \approx \quad \frac{\pi}{2\sqrt{3}e^{\sqrt{3}}} = \int_0^\infty \frac{\cos x}{x^2+3}\, dx$

$.1605922055573114571\ldots \quad \approx \quad \frac{24\log 2 - 7}{60} = \sum_{k=1}^{\infty} \frac{1}{2k^2 + 11k + 12} = \int_0^1 x^4 \log\left(1 + \frac{1}{x}\right) dx$

$.1606027941427883920\ldots \quad \approx \quad 2 - \frac{5}{e} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(k+3)} = \int_0^1 x^2 e^{-x}\, dx = \int_1^\infty \frac{\log^2 x}{x^2}\, dx$

$= \quad \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)! + 2k!}$

$.160889766020364044721\ldots \approx \quad \frac{\pi \sin 1}{4} - \frac{1}{2} = -\sum_{k=1}^{\infty} \frac{\cos 2k}{(2k-1)(2k+1)}$ GR 1.444.7

$.1610391299195894597\ldots \quad \approx \quad \frac{\pi\sqrt{2}}{2} + \log 2 - 4 + \frac{\sqrt{2}}{2}\log\frac{2+\sqrt{2}}{2-\sqrt{2}}$

$= \quad \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{4k^2 + k}$

$= \quad \int_0^1 \log(1+x^4)\, dx = \int_1^\infty \log\left(1 + \frac{1}{x^4}\right)\frac{dx}{x^2}$

4 $.1610391299195894597\ldots \quad \approx \quad \frac{\pi\sqrt{2}}{2} + \log 2 + \frac{\sqrt{2}}{2}\log\frac{2+\sqrt{2}}{2-\sqrt{2}} = \int_0^1 \log\left(1 + \frac{1}{x^4}\right) dx$

$.16126033258846573748\ldots \quad \approx \quad \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}\left(Li_k\left(\frac{1}{2}\right) - \frac{1}{2}\right) = -\frac{\log 2}{2} + \sum_{k=1}^{\infty} \frac{\log k}{2^k}$

$.1613630697302135426\ldots \quad \approx \quad \frac{1}{\sqrt{2}}\coth\frac{1}{\sqrt{2}} - 1 = \sum_{k=1}^{\infty} \frac{1}{k^2\pi^2 + \frac{1}{2}} = \sum_{k=1}^{\infty} \frac{B_{2k} 2^k}{(2k)!}$

.16137647245029392651... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{(2k)!(2k)!} = \frac{1}{2}\sum\left(I_0\left(\frac{2}{\sqrt{k}}\right)+J_0\left(\frac{2}{\sqrt{k}}\right)-2\right)$

.161439361571195633610... ≈ $\log\sinh 1 = \log\left(\frac{e^2-1}{2e}\right) = \sum_{k=1}^{\infty}\frac{2^{2k-1}B_{2k}}{(2k)!k}$

= $\sum_{k=1}^{\infty}\frac{2^{2k-1}B_{2k}}{(2k)!k}$

= $-\log\left(\Gamma\left(1+\frac{i}{\pi}\right)\Gamma\left(1-\frac{i}{\pi}\right)\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}\zeta(2k)}{\pi^{2k}k}$

1 .161439361571195633610... ≈ $1+\log\sinh 1 = \log\left(\frac{e^2-1}{2}\right) = \sum_{k=1}^{\infty}\frac{(-1)^k 2^k B_k}{k!k}$ Berndt 5.8.5

.16149102437656156341... ≈ $\frac{\pi^3}{192} = \int_0^1\frac{\arctan^2 x\,dx}{1+x^2}$

.161745915929683120084... ≈ $\frac{1}{12\sqrt{2}}\left(\log\left(17-12\sqrt{2}\right)+\sqrt{3}\operatorname{arctan}h\frac{2\sqrt{6}}{5}+\pi\left(-1+\sqrt{3}\right)\right)$

= $\int_1^{\infty}\frac{1}{x^6+x^{-6}}dx$

2 .16196647795827451054... ≈ $\sum_{k=1}^{\infty}\frac{k^k}{\left\{\begin{matrix}2k\\ k\end{matrix}\right\}}$

1 .162037037037037037037 = $\frac{251}{216} = H^{(3)}{}_3$

1 .16204051536831577300... ≈ $\sum_{k=2}^{\infty}\zeta(k)\left(\zeta(2k-1)-1\right)$

.16208817230500686559... ≈ $\cos\left(\frac{1}{\sqrt{3}}\right)-1 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k)!3^k}$

8 .16209713905398032767... ≈ $\frac{3\pi\sqrt{3}}{2} = \sum_{k=0}^{\infty}\frac{1}{(k+\frac{1}{6})(k+\frac{5}{6})}$

.162162162162162162162 = $\frac{6}{37} = \frac{1}{2\cosh\log 6} = \sum_{k=0}^{\infty}(-1)^k e^{-(2k+1)\log 6}$ J943

= $\int_0^{\infty}\frac{\sin 6x}{e^x}dx$

2 .1622134783048980717... ≈ $\frac{\pi^2}{2}-4\log 2 = 2\sum_{k=1}^{\infty}\frac{1}{k(2k-1)^2} = \sum_{k=1}^{\infty}\frac{k\zeta(k+2)}{2^k}$

3 .162277660168379331999... ≈ $\sqrt{10}$

1 .16231908520772565705... ≈ $e(erf1)-\frac{2}{\sqrt{\pi}} = \frac{2e}{\sqrt{\pi}}\Gamma\left(\frac{3}{2},0,1\right)=\sum_{k=1}^{\infty}\frac{1}{(k+\frac{1}{2})!}$

.16233623003241194031 6... ≈ $13e-\frac{71}{2} = \sum_{k=1}^{\infty}\frac{k^3}{(k+3)!}$

.16246830094483901460 4... ≈ $\frac{\pi^4-93\zeta(5)}{6} = \sum_{k=1}^{\infty}\frac{k}{(k+\frac{1}{2})^5}$

.16265177907411370337 2... ≈ $\sum_{k=3}^{\infty}(-1)^{k+1}\frac{\zeta(k)}{k!} = \sum_{k=1}^{\infty}\frac{1}{2k^2}\left(1-2k-2k^2e^{-1/k}+2k^2\right)$

.16266128557107162607... ≈ $\left(\sqrt{2}-1\right)\frac{\pi}{8} = \sum_{k=1}^{\infty}\frac{(-1)^k}{27(2k-1)^3-2(2k-1)}$

= $\int_0^1 \frac{x\arcsin x}{(1+x^2)^2}dx$ GR 4.512.7

.16268207245178092744... ≈ $2\log 2-\log 3-\frac{1}{8} = \int_1^2 \frac{dx}{x^4+x^3}$

.16277007036916213982... ≈ $2-3\arctan\frac{1}{2}+2\log\frac{4}{5} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^{2k+1}(2k-1)k(2k+1)}$

.16282421456517386178 3... ≈ $\frac{1}{\pi+3}$

.16286500591778933036 3... ≈ $\frac{11\pi^2}{96}-\frac{\log^2 2}{8} = \int_0^1 \frac{x\log(1+x)}{1+x^2}dx$

1 .16292745855318123597... ≈ $\frac{\pi^3+3\zeta(3)}{256}+\frac{1}{1024}\left(\psi^{(2)}\left(\frac{7}{8}\right)+\psi^{(2)}\left(\frac{5}{8}\right)-\psi^{(2)}\left(\frac{3}{8}\right)-\psi^{(2)}\left(\frac{1}{8}\right)\right)$

= $\sum_{k=0}^{\infty}(-1)^k\left(\frac{1}{(4k+1)^3}+\frac{1}{(4k+2)^3}+\frac{1}{(4k+3)^3}+\frac{1}{(4k+4)^3}\right)$

.16301233304332304576... ≈ $\frac{\sqrt{\pi}}{4e} = \int_0^{\infty} x e^{-x^2}\sin x\cos x\, dx$

.16304881167003232259 8... ≈ $\sum_{k=1}^{\infty}\frac{\mu(k)}{k}\log\left(\zeta(2k+1)\right)$

.1630566660772681888... ≈ $7-3e-\gamma+Ei(1) = \sum_{k=1}^{\infty}\frac{k^2}{(k+2)!(k+1)}$

.163224793... ≈ $\sum_{k=2}^{\infty}\frac{\Omega(k)}{(k-1)^2}$

.16329442747435331717... ≈ $\frac{8}{9}-\frac{\pi}{6}(\sqrt{2}+1)-\frac{4\log 2}{3}+\frac{\sqrt{2}}{3}\log(1+\sqrt{2})$

= $\sum_{k=1}^{\infty}\frac{1}{k(8k+3)}$ Prud. 5.1.7.22

.16326530612244897959... $\approx \frac{8}{49} = \sum_{k=1}^{\infty} \frac{k}{8^k}$

.16345258536674495499... $\approx \frac{\pi^2}{6} - \frac{40}{27} = \int_0^{\pi/2} x^2 \cos^3 x \, dx$

.16376335736944358027 3... $\approx \frac{17}{8} + \zeta(2) - 3\zeta(3) = \sum_{k=1}^{\infty} \frac{k}{(k+3)^3}$

.16389714773777593436... $\approx \frac{1}{9e} - \frac{2\sqrt{e}}{9} \cos\left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k}{(3k)!}$

.16390063283767393728 7... $\approx \log \frac{3\pi}{8} = \log\left(\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{5}{2}\right)\right) = \sum_{k=1}^{\infty} \frac{\zeta(2k)-1}{4^k k}$

.16395341373865284877... $\approx \frac{3-e}{e-1} = \sum_{k=1}^{\infty} \frac{1}{\pi^2 k^2 + 1/4}$

2 .16395341373865284877... $\approx \coth \frac{1}{2} = \frac{e+1}{e-1} = \sum_{k=0}^{\infty} \frac{B_{2k}}{(2k)!}$

4 .16395341373865284877... $\approx \frac{3e-1}{e-1} = \sum_{k=0}^{\infty} \frac{1}{\pi^2 k^2 + 1/4}$

.16402319032763527735... $\approx \frac{1}{12}\left(\psi^{(1)}\left(\frac{2}{3}\right) - \psi^{(1)}\left(\frac{4}{3}\right)\right) = \frac{3}{4} + \frac{\pi^2}{9} - \frac{1}{6}\psi^{(1)}\left(\frac{1}{3}\right)$

$= \sum_{k=1}^{\infty} \frac{9k}{(9k^2-1)^2}$

2 .16408863181124968788... $\approx \sum_{k=2}^{\infty} \frac{\zeta(k)}{k^2-3}$

.16421356237309504880... $\approx \sqrt{2} - \frac{5}{4} = \sum_{k=2}^{\infty} \frac{(2k-1)!!}{(2k)!\, 2^k}$

1 .16422971372530337364... $\approx \Gamma\left(\frac{4}{5}\right)$

.16425953179193084498... $\approx \log 2 - \frac{\pi^2}{24} - \frac{\pi}{4} - \frac{\log^2 2}{2} + Li_2\left(\frac{1-i}{2}\right) + Li_2\left(\frac{1+i}{2}\right)$

$= \int_0^1 \frac{\log(1+x^2)}{x(1+x)^2} dx$

1 .16429638328046168107... $\approx \sum_{k=1}^{\infty} \frac{1}{(k!!)^3}$

.16435115173527894666 3... $\approx 1 - \frac{\pi}{3\sqrt{3}} - \frac{\log 2}{3} = \sum_{k=0}^{\infty} \frac{(-1)^k}{3k+4} = \int_1^{\infty} \frac{dx}{x^5+x^2}$

.16440195389316542965 3... $\approx \log^2 \frac{3}{2} = \sum_{k=1}^{\infty} \frac{(-1)^k H_k}{2^k (k+1)}$

6 .16441400296897645025... ≈ $\sqrt{38}$

.16447904046849545588... ≈ $\frac{1}{e} + \gamma - Ei(-1) - 1 = \sum_{k==0}^{\infty} \frac{(-1)^{k+1} k}{(k+1)!(k+1)} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(k+2)^2}$

= $-\int_0^1 \frac{x \log x}{e^x} dx$

1 .16448105293002501181... ≈ $\frac{\pi^2}{6} - \log^2 2 = 2Li_2\left(\frac{1}{2}\right) = \sum_{k=1}^{\infty} \frac{1}{2^k k^2} = \sum_{k=0}^{\infty} \frac{H^{(2)}{}_k}{2^k}$

.16449340668482264365... ≈ $\frac{\pi^2}{60} = \int_1^{\infty} \log\left(1 + \frac{1}{x^5}\right) \frac{dx}{x}$

2 .16464646742227638303... ≈ $\frac{\pi^4}{45} = 2\zeta(4) = -\int_0^1 \frac{\log(1-x)\log^2 x}{x}$

.164707267714841884974...≈ $\frac{7\zeta(3)}{8}(1 + \log 2) - \frac{\pi^2 \log 2}{8} - \frac{\pi^4}{128} = \sum_{k=1}^{\infty} \frac{kH_k}{(2k+1)^3}$

.16482268215827724019... ≈ $-\frac{\zeta'(3)}{\zeta(3)} = \frac{1}{\zeta(3)} \sum_{k=2}^{\infty} \frac{\log k}{k^3} = \sum_{p\ prime} \frac{\log p}{p^3 - 1}$

= $\sum_{k=1}^{\infty} \frac{\Lambda(k)}{k^3}$

.164895120905594654651...≈ $-\frac{\zeta'(3)}{\zeta(6)} - \frac{1786050\zeta 3)\zeta'(6)}{\pi^{12}} = \sum_{k=1}^{\infty} \frac{|\mu(k)| \log k}{k^2}$

1 .16494809158137192362... ≈ $\frac{1}{4 - \pi}$

.165122266041052985705...≈ $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{\mu(k)}{6^k + 1}$

.165129148016224359068...≈ $\zeta(3) - \zeta(5)$

9 .1651513899116800132... ≈ $\sqrt{84} = 2\sqrt{21}$

2 .16522134231822621051... ≈ $\sum_{k=2}^{\infty} \frac{k+1}{k(k^2 - k - 1)} = \sum_{k=2}^{\infty} F_k(\zeta(k) - 1)$

= $\frac{3}{2} - \frac{\sqrt{5}}{2} - \gamma - \psi\left(\frac{\sqrt{5} - 1}{2}\right) + \frac{\pi}{10}(5 + \sqrt{5})\tan\frac{\pi\sqrt{5}}{2}$

.16528053142278903778... ≈ $1 - \frac{1}{3e} - \frac{2}{3}\sqrt{e}\cos\frac{\sqrt{3}}{2} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(3k)!}$

2 .1653822153269363594... ≈ $e(\gamma - Ei(-1)) = \sum_{k=1}^{\infty} \frac{H_k}{k!}$

.16542114370045092921... ≈ $-\zeta'(-1)$

.16546878552437427... ≈ $1-\zeta(3)+\sum_{k=2}^{\infty}\frac{k-1}{k^3\log k} = \sum_{n=1}^{\infty}\frac{1}{(n+1)!}\sum_{k=2}^{\infty}\frac{\log^n k}{k^3}$

1 .1654880934812284345 5... ≈ $-\frac{\pi}{2}\left(I_0(1)+L_0(1)-e\right) = \int_0^1 e^x \arcsin x\,dx$

.165521949620303461 6... ≈ $2\left(\log^2 3-4\log 2\log 3+4\log^2 2\right) = \sum_{k=1}^{\infty}\frac{H_k}{4^k(k+1)}$

1 .1655711615479214833 8... ≈ $\frac{\pi}{4}+\frac{\sqrt{3}}{6}\log\left(2+\sqrt{3}\right) = \sum_{k=0}^{\infty}\left(\frac{(-1)^k}{6k+1}+\frac{(-1)^k}{6k+3}\right)$

= $\int_1^{\infty}\frac{dx}{x^2+x^{-2}-1}$

.165672052444307559... ≈ $\frac{1}{10}\left(2\gamma-\arctan 2+\log 5\right) = -\int_0^{\infty} e^{-x}\sin x\cos x\log x\,dx$

3 .1656732488943091781 42... ≈ $\sqrt{6}\,\csc\pi\sqrt{e}\,\sin\pi\sqrt{2} = \prod_{k=1}^{\infty}\left(1+\frac{1}{k^2+2k-2}\right)$

.1658583180167143939 8... ≈ $1-\frac{\pi^2\log 2}{12}-\frac{\log^3 2}{12}+\frac{1}{2}Li_3\left(-\frac{1}{2}\right) = 1+\frac{1}{2}Li_3(-2)$

= $\int_1^{\infty}\frac{\log^2 x\,dx}{x^3+2x^2}$

.1658734166578102966 42... ≈ $\frac{1}{4}\left(\cosh 1\sin 1-\cos 1\sinh 1\right) = \frac{\cos 1+\sin 1+e^2(\sin 1-\cos 1)}{8e}$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}4^k k}{(4k)!}$

.1659090634758519607 1... ≈ $\frac{9\log 3}{50}+\frac{\pi^2}{30}-\frac{\pi\sqrt{3}}{50}-\frac{63}{250} = \sum_{k=1}^{\infty}\frac{1}{3k^3+5k^2}$

16 .1659674921921150416 7... ≈ $-\frac{1}{8}\psi^{(2)}\left(\frac{1}{4}\right) = \int_1^{\infty}\frac{\log^2 x\,dx}{x^{3/2}-x^{-1/2}}$

.1659993890544020609 4... ≈ $\zeta(2)-4\zeta(3)+4\zeta(4)-1 = \sum_{k=1}^{\infty}\frac{k^2}{(k+2)^4}$

.1662436161232751205 53... ≈ $\frac{4G}{\pi}-1 = \sum_{k=1}^{\infty}\left(\frac{(2k-1)!!}{(2k)!!}\right)^2\frac{1}{(2k+1)}$ J385

1 .1662436161232751205 53... ≈ $\frac{4G}{\pi} = \sum_{k=0}^{\infty}\binom{2k}{k}^2\frac{1}{16^k(2k+1)}$

.1662699748688509511 16... ≈ $1-\cos 1\cosh 1 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}4^k}{(4k)!}$ GR 1.413.2

.16651232599446473986... $\approx \dfrac{\log^3 2}{2}$

.16652097674818765092... $\approx \zeta(2) + \dfrac{\zeta(3)}{2} - 3\log 2 \;=\; \sum_{k=1}^{\infty} \dfrac{k^2-1}{4k^4-2k^3}$

$= \sum_{k=2}^{\infty} \dfrac{\zeta(k)-\zeta(k+2)}{2^k}$

.16658896190385933716... $\approx \dfrac{\gamma^2}{2}$

.16662631182952538859... $\approx \dfrac{\pi}{4} - \dfrac{\log 2}{2} - \dfrac{\pi \log 2}{8} \;=\; \int_0^{\pi/4} \dfrac{x \sin x}{(\sin x + \cos x)\cos^2 x}\,dx$ GR 3.811.5

.1666666666666666666$\underline{6}$ $= \dfrac{1}{6} \;=\; \sum_{k=3}^{\infty} \dfrac{k}{(k+1)!} \;=\; \sum_{k=1}^{\infty} \dfrac{1}{k^2\pi^2} \;=\; \prod_{k=3}^{\infty}\left(1-\dfrac{4}{k^2}\right)$

$= \sum_{k=1}^{\infty} \dfrac{\zeta(2k)-1}{4^k}$

$= \sum_{k=2}^{\infty} \dfrac{1}{k^3+k-1/k} \;=\; \sum_{k=1}^{\infty} \dfrac{k}{(k+2)(k+3)(k+4)}$

$= \sum_{k=0}^{\infty} \dfrac{(-1)^k}{(2k+1)(2k+5)}$

$= \sum_{k=2}^{\infty} \dfrac{1}{k!(k+2)} \;=\; \sum_{k=1}^{\infty} \dfrac{(-1)^{k+1}}{2^k(k+2)}\dbinom{2k}{k}$

$= \prod_{k=3}^{\infty}\left(1-\dfrac{4}{k^2}\right)$

$= \prod_{k=1}^{\infty} \dfrac{k(k+4)}{(k+2)(k+2)} \;=\; \prod_{k=1}^{\infty} \dfrac{k(k+6)}{(k+5)(k+1)}$ J1061

$= \int_1^{\infty} \dfrac{\operatorname{arccosh} x}{(1+x)^3}\,dx$

1 .1666666666666666666$\underline{6}$ $= \dfrac{7}{6} \;=\; \dfrac{\zeta^2(4)}{\zeta(8)} \;=\; \sum_{k=1}^{\infty} \dfrac{2^{\omega(k)}}{k^4}$ HW Thm. 301

$= \prod_{p\ prime}\left(\dfrac{1+p^{-4}}{1-p^{-4}}\right)$ Berndt Ch. 5

.16686510441795247511... $\approx \dfrac{\sinh 1 - \sin 1}{2} \;=\; \dfrac{e}{4} - \dfrac{1}{4e} - \dfrac{\sin 1}{2} = \sum_{k=0}^{\infty} \dfrac{1}{(4k+3)!}$

.16699172896087386807... $\approx \sum_{k=2}^{\infty} \dfrac{\zeta(k)-1}{k!k!} \;=\; \sum_{k=2}^{\infty}\left(I_0\!\left(\dfrac{2}{\sqrt{k}}\right)-1-\dfrac{1}{k}\right)$

1 .16706362568697266872... $\approx \dfrac{1}{2}\left(\cos\sqrt{2} + \cosh\sqrt{2}\right) \;=\; \cos\left((-1)^{1/4}\right)\cosh\left((-1)^{1/4}\right) = \sum_{k=0}^{\infty} \dfrac{4^k}{(4k)!}$

22 .167168296791950681691... ≈ $2e^2 = \sum_{k=0}^{\infty} \frac{2^k (k+1)}{k!}$ GR 1.212

1 .16723171987003124525... ≈ $\arccos\frac{\pi}{8}$

2 .16736062588226195190... ≈ $\frac{\cosh\pi\sqrt{2} - \cos\pi\sqrt{2}}{2\pi^2}$

= $-\frac{\sin\left((-1)^{1/4}\pi\right)\sin\left((-1)^{3/4}\pi\right)}{\pi^2} = \prod_{k=1}^{\infty}\left(1+\frac{1}{k^4}\right)$

.167407484127008886452... ≈ $\pi - \log 2 + \sqrt{3}\log\frac{\sqrt{3}-1}{\sqrt{3}+1} = \psi\left(\frac{11}{12}\right) - \psi\left(\frac{5}{6}\right)$

= $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{6k^2 - k}$

.16745771316555894862... ≈ $\prod_{k=1}^{\infty}\left(1 - \frac{1}{2(k-1)!}\right)$

5 .16771278004997002925... ≈ $\frac{\pi^3}{6}$, volume of the unit sphere in R^6

.16782559481552120796... ≈ $\frac{\gamma + \log\pi}{2} - \log 2 = \sum_{k=2}^{\infty} (-1)^k \frac{\zeta(k)}{2^k k}$ Dingle 3.37

= $\sum_{k=1}^{\infty}\left(\frac{1}{2k} - \log\frac{2k+1}{2k}\right)$

.16791444558408471119... ≈ $\frac{9}{2} - \frac{3}{2}\cot\frac{1}{3} = \sum_{k=1}^{\infty} \frac{1}{k^2\pi^2 - 1/9}$

.16805750730968383857... ≈ $\sum_{k=1}^{\infty} \frac{\nu(k)}{3^k} = \sum_{p\ prime} \frac{1}{3^p - 1}$

1 .16805831337591852552... ≈ $\frac{1}{3}\left(e + \frac{2}{\sqrt{e}}\cos\frac{\sqrt{3}}{2}\right) = \sum_{k=0}^{\infty} \frac{1}{(3k)!}$ J803

.16807806319774282939... ≈ $\sum_{k=1}^{\infty} \frac{((k-1)!)^3}{(3k)!}$

.16809262741929253132... ≈ $\frac{\zeta(3) - 1}{\zeta(3)}$

1 .168156588029466646... ≈ $\sum_{k=2}^{\infty} \frac{\log^2 k}{2k(k-1)}$

1 .16823412933514314651... ≈ $\sum_{k=1}^{\infty}\left(\zeta^4(3k) - 1\right)$

.16823611831060646525... ≈ $\operatorname{arctanh}\frac{1}{6} = \sum_{k=0}^{\infty}\frac{1}{6^{2k+1}(2k+1)}$ AS 4.5.64, J941

.16838922476583426925... ≈ $\frac{\gamma}{6} + \frac{\log 2}{3} + 2\frac{\psi\left(\frac{5-i\sqrt{3}}{4}\right)}{\sqrt{3}\left(3i+\sqrt{3}\right)} + \left(\frac{1}{6} - \frac{\psi\left(\frac{5+i\sqrt{3}}{4}\right)}{\sqrt{3}\left(3i+\sqrt{3}\right)}\right)$

= $\sum_{k=1}^{\infty}\frac{1}{(2k)^3-1} = \sum_{k=1}^{\infty}\frac{\zeta(3k)}{8^k}$

1 .168437795256228934... ≈ $\sum_{k=2}^{\infty}\frac{2k}{k^3+2} = \sum_{k=1}^{\infty}(-1)^{k+1}2^k\left(\zeta(3k-1)-1\right)$

.16846317173057183421... ≈ $Li_4\left(\frac{1}{6}\right) = \sum_{k=1}^{\infty}\frac{1}{6^k k^4}$

1 .1685237539993828436... ≈ $\frac{3\log 3}{2} - 2\log 2 + \frac{\pi}{2\sqrt{3}} = \psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{3}\right)$

= $\int_1^{\infty}\log\frac{1-x^2}{1+x+x^2}dx = \int_0^1\log\frac{x^2+x+1}{1-x^2}\,dx$

.168547888329363395939... ≈ $\frac{\pi}{4} - \frac{\pi^2}{16} = \sum_{k=1}^{\infty}\frac{1}{8(2k-1)^4-2(2k-1)^2}$

.168609905834157724449... ≈ $\frac{1}{4}\left(2\gamma + \log 216\pi - 7\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}2^k}{(k+1)(k+2)}\left(\zeta(k+1)-1\right)$

.168655097090202230721... ≈ $4\sqrt{2}\log\left(\sqrt{2}-1\right) - \frac{\pi^2}{4} - 2\pi\left(\sqrt{2}-1\right) + 8\log 2 - \frac{7\zeta(3)}{16}$

= $\sum_{k=1}^{\infty}\frac{1}{8(2k-1)^4-2(2k-1)^3}$

.16870806500348277326... ≈ $-\gamma - \frac{1}{2}\psi\left(1-\frac{1}{2\sqrt{2}}\right) - \frac{1}{2}\psi\left(1-\frac{4+\sqrt{2}}{4}\right) = \sum_{k=1}^{\infty}\frac{\zeta(2k+1)}{8^k}$

= $\sum_{k=1}^{\infty}\frac{1}{8k^3-k}$

.16905087983986064158... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k^k}$

1 .16936160473579879151... ≈ $\binom{1}{1/5} = \binom{1}{4/5}$

.16944667603360116353... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k}{k^3-3}$

.16945266811965260313... ≈ $\frac{14}{27} - \frac{\pi}{9} = \int_0^1 x^2\arcsin x\arccos x\,dx$

.16945271999473895451... ≈ $\frac{1}{9}\left(e-\frac{2}{\sqrt{e}}\cos\left(\frac{\sqrt{3}}{2}-\frac{\pi}{3}\right)\right)$

.16948029848741747267... ≈ $\sum_{k=2}^{\infty}\frac{\Omega(k^4)}{k^4}$

= $\sum_{k=2}^{\infty}\frac{1}{k^4-1} + \sum_{k=2}^{\infty}\frac{1}{k^4-k^{-4}} + \sum_{k=2}^{\infty}\frac{1}{k^{12}-k^{-4}} + \sum_{k=2}^{\infty}\frac{1}{k^{20}-k^4}$

= $\sum_{k=1}^{\infty}\left(\zeta(4k)-1\right) + \sum_{k=1}^{\infty}\left(\zeta(8k-4)-1\right) + \sum_{k=1}^{\infty}\left(\zeta(16k-4)-1\right) + \sum_{k=1}^{\infty}\left(\zeta(16k+4)-1\right)$

.16951227828754808073... ≈ $2-\cot\frac{1}{2} = \sum_{k=1}^{\infty}\frac{1}{k^2\pi^2-1/4}$

= $\sum_{k=1}^{\infty}\frac{1}{2^k}\tan\frac{1}{2^{k+1}}$ Berndt ch. 31

3 .16992500144231236295... ≈ $\log_2 9$

11 .1699273161019477314... ≈ $12-22\gamma+6\gamma^2+\pi^2=\int_0^{\infty}\frac{x^3\log^2 x\,dx}{e^x}$

1 .17001912860314990390... ≈ $\sqrt{\frac{3}{2}}\arcsin\sqrt{\frac{3}{2}} = \sum_{k=0}^{\infty}\binom{2k}{k}\frac{1}{6^k(2k+1)}$

.17005969112949392838... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^2}\log\frac{k+1}{k}$

.17032355274830491796... ≈ $Li_3\left(\frac{1}{6}\right) = \sum_{k=1}^{\infty}\frac{1}{6^k k^3}$

.170557349502438204366...≈ $1+\frac{\pi^2}{12}-\log(e+1)+Li_2\left(-\frac{1}{e}\right) = \int_0^1\frac{x\,dx}{e^x+1}$

2 .17063941571244840982... ≈ $\sum_{k=2}^{\infty}\sqrt{\zeta(k)-1}$

1 .17063957497359386223... ≈ $\frac{23}{14}-\frac{\pi}{2^{5/4}}\left(\cot\pi 2^{3/4}+\coth\pi 2^{3/4}\right)$

= $\sum_{k=2}^{\infty}\frac{8}{k^4-8} = \sum_{k=1}^{\infty}8^k\left(\zeta(4k)-1\right)$

.1706661896898962359... ≈ $\frac{\pi}{6}\coth\frac{\pi}{3}-\frac{1}{2} = \sum_{k=1}^{\infty}\frac{1}{9k^2+1} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)}{9^k}$

.1707701846682093606... ≈ $\frac{\log 3}{2}-\frac{\pi}{6\sqrt{3}}-\frac{5}{12}+\frac{1}{9}\psi^{(1)}\left(\frac{2}{3}\right) = \sum_{k=1}^{\infty}\frac{6k-1}{3k(k-1)^2}$

= $\sum_{k=1}^{\infty}\frac{k(\zeta(k)-1)}{3^k}$

.17083144025442980463... ≈ $\sum\frac{F_k{}^3}{8^k}$

.1710822479183635679... ≈ $\sum_{k=1}^{\infty}\frac{\mu(k)}{4^k}$

.1711785303644720138... ≈ $\sum\left(\frac{1}{8}-\frac{\zeta(3k+1)-1}{\zeta(k)-1}\right)$

2 .17132436809105815141... ≈ $\mathrm{Re}\left\{\psi(-i)-i\psi(i)\right\}$

1 .1713480439548652889... ≈ $\cosh\frac{1}{\sqrt{3}} = \sum_{k=0}^{\infty}\frac{1}{3^k(2k)!}$

.17138335479471051464... ≈ $\sum(-1)^{k+1}\left(\zeta(k+1)-1\right)^4$

1 .17139260811917011128... ≈ $\sum_{k=2}^{\infty}\sigma_{-1}(k-1)\left(\zeta(k)-1\right)$

1 .1714235822309350626... ≈ $\frac{\pi^8}{8100} = \zeta^2(4) = \sum_{k=1}^{\infty}d(k)k^{-4}$ Titchmarsh 1.2.1

.171458091265465835370... ≈ $-\frac{1}{2}\log\left(\frac{5}{4}-\cos 1\right) = \sum_{k=1}^{\infty}\frac{\cos k}{2^k k}$

.1715728752538099024... ≈ $3-2\sqrt{2} = \sum_{k=0}^{\infty}\binom{2k}{k}\frac{1}{8^k(k+1)} = \sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!2^k(k+1)}$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}(2k-1)!!}{(2k)!(k+1)} = \sum_{k=1}^{\infty}\binom{2k}{k}\frac{(-1)^{k+1}}{4^k\,(k+1)}$

= $\frac{1}{2}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}(k+2)$

.17166142139812197607... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{(k+1)^2} = \sum_{k=2}^{\infty}\left(k^2 Li_2\left(\frac{1}{k^2}\right)-1\right)$

.17172241473708489791... ≈ $\frac{\zeta(3)}{7}$

.17173815835609831453... ≈ $6-3\cos 1-5\sin 1 = \frac{1}{2}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!(k+2)}$

= $\int_1^e\frac{\log^3 x\cos\log x}{x}dx$

.17186598552400983788... ≈ $\gamma-1+\frac{1}{2}\left(\psi(2+i)+\psi(2-i)\right)$

= $\gamma+\frac{1}{2}\left(\psi(i)+\psi(-i)-1\right) = \gamma+\frac{1}{2}\left(\psi(1+i)+\psi(1-i)-1\right)$

= $\sum_{k=2}^{\infty}\frac{1}{k^3+k} = \sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(2k+1)-1\right)$

= $\sum_{k=1}^{\infty}\left(\zeta(4k-1)-\zeta(4k+1)\right)$

= $\int_0^{\infty}\frac{1-\cos x}{e^x\left(e^x-1\right)}dx$

.17190806304093687756... ≈ $\int_1^{\infty}\frac{dx}{x^4+x^2+x}$

1 .1719536193447294453... ≈ $-\frac{i}{\sqrt{3}}\left(Li_2\left((-1)^{1/3}\right)-Li_2\left(-(-1)^{2/3}\right)\right)=\int_0^{\infty}\frac{xdx}{e^x+e^{-x}-1}$ GR 3.418.1

= $\frac{1}{3}\left(\psi^{(1)}\left(\frac{1}{3}\right)-\frac{2\pi^2}{3}\right) = -\int_0^1\frac{\log x}{1-x+x^2}dx$ GR 4.23.2

5 .172029132381426797... ≈ $\frac{\pi^2+9\zeta(3)}{4} = -\int_0^1\frac{\log^3 x}{(x+1)^3}dx$

1 .1720419066693079021... ≈ $\frac{8}{5\pi}\sinh\frac{\pi}{2} = \prod_{k=1}^{\infty}\left(1+\frac{1}{4(k+1)^3}\right)$

1 .1720663568851251981... ≈ $\sum_{k=2}^{\infty}\frac{\log k}{k^2-2}$

.17210289868366378217... ≈ $\frac{1}{2}-\frac{1}{2}\tanh\frac{\pi}{4} = \sum_{k=1}^{\infty}(-1)^{k+1}e^{-\pi k/2}$ J944

3 .1722189581254505277... ≈ $e^{2\gamma}$

.17224705723145852332... ≈ $\sum_{k=1}^{\infty}\frac{k^2}{(3k)!}$

.17229635526084760319... ≈ $\zeta(2)-\zeta(3)-\frac{\zeta(4)}{4} = \sum_{k=1}^{\infty}\frac{H_k}{k(k+1)^3}$

.17240583062364046923... ≈ $\frac{\pi^2}{16}-\frac{4}{9} = \sum_{k=1}^{\infty}\frac{k}{4^k}\left(\zeta(2k)-1\right)$

.17250070367941164573... ≈ $1-\frac{1}{\sqrt{2}}\cot\frac{1}{\sqrt{2}} = \sum_{k=1}^{\infty}\frac{1}{k^2\pi^2-\tfrac{1}{2}}$

.172504053920943245495...≈ $\frac{1}{10}\left(3\log 3-\frac{\pi}{2}\right) = \int_0^1 li\left(\frac{1}{x}\right)\sin(3\log x)\,dx$ GR 6.213.1

.17254456941296968757... ≈ $\sum_{k=2}^{\infty}\frac{k^3}{(k^3+1)^2} = \sum_{k=1}^{\infty}(-1)^{k+1}k\left(\zeta(3k)-1\right)$

2 .17258661797937013053... ≈ $\sum_{k=1}^{\infty}\frac{1}{F_k^{\,3}}$

.17259944116823072530... ≈ $\frac{3}{2}\left(3\log^2\frac{3}{2}+4\log\frac{3}{2}-2\right) = \sum_{k=1}^{\infty}\frac{H_k}{3^k(k+2)}$

.17260374626909167851... ≈ $-\log\sin 1 = \sum_{k=1}^{\infty}\frac{\zeta(2k)}{\pi^{2k}k} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}B_{2k}4^{k}}{(2k)!2k}$

AS 4.3.71, Berndt Ch. 5

2 .17263205728576287162 8... ≈ $\zeta(\zeta(2))$

.17264325482630846776... ≈ $\sum_{k=2}^{\infty}(-1)^{k}\frac{\zeta(k)}{k^{3}} = \sum_{k=1}^{\infty}\left(Li_{2}\left(-\frac{1}{k}\right)+\frac{1}{k}\right)$

.17273053919054494131... ≈ $\frac{5}{e}-\frac{5}{3} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!(k+3)}$

.1728274509745820502... ≈ $\frac{\pi\log 2}{2}-G = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}H_{k}}{2k+1}$ Adamchik (24)

= $\int_{0}^{1}\frac{\log(1+x^{2})}{1+x^{2}}dx$ GR 4.295.5

.17285567305508899909 8... ≈ $4e^{-\pi} = \int_{-\infty}^{\infty}\frac{\cos 4x}{x^{2}+\pi^{2}/16}$

.17289680030426476... ≈ $\sum_{k=2}^{\infty}\frac{\log k}{2^{k}k}$

1 .17303552576131315974... ≈ $\sum_{k=1}^{\infty}\zeta(2k)\big(\zeta(2k)-1\big) = \sum_{k=2}^{\infty}\left(\frac{1}{2}-\frac{\pi}{2k}\cot\frac{\pi}{k}\right)$

= $\sum_{k=1}^{\infty}\sum_{n=2}^{\infty}\frac{\zeta(2k)}{n^{2k}} = \sum_{m=1}^{\infty}\sum_{k=1}^{\infty}\frac{1}{m^{2}k^{2}-1}$

= $\frac{3}{4}+\sum_{k=2}^{\infty}\left(\frac{k^{2}-3}{2(k^{2}-1)}-\frac{\pi}{k}\cot\frac{\pi}{k}\right)$

.17305861469432215834... ≈ $\gamma-\frac{2\gamma}{e}+Ei(-1)+\frac{2Ei(1)}{e} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}kH_{k}}{(k+1)!}$

.17311810051999912821... ≈ $\int_{2}^{\infty}\big(\zeta(x)-1\big)^{2}dx$

2 .17325431251955413824... ≈ $\frac{\zeta(3/2)}{\zeta(3)} = \lim_{n\to\infty}\frac{1}{\sqrt{n}}\left(\sum_{m=1}^{n}h(m)-n\right)$

$h(m)$ = lowest exponent in prime factorization of m.

.17328078030779429331 9... ≈ $1-\gamma-\frac{1}{2}\left(\psi\left(2-e^{i}\right)+\psi\left(2-e^{-i}\right)\right)$

= $\sum_{k=1}^{\infty}\big(\zeta(k+1)-1\big)\cos k$

.17328679513998632735... ≈ $\frac{\log 2}{4} = \sum_{k=1}^{\infty}\frac{1}{(4k-3)(4k-2)(4k-1)}$ J253

$= \displaystyle\sum_{k=0}^{\infty}\frac{(-1)^k}{4k+4}$

$= \displaystyle\sum_{r=1}^{\infty}\sum_{s=1}^{\infty}\frac{1}{(4s-1)^{2r}}$ J1124

$= \displaystyle\sum_{k=1}^{\infty}\frac{1}{8(2k-1)^3-2(2k-1)}$ [Ramanujan] Berndt Ch. 2

$= \displaystyle\int_1^{\infty}\frac{dx}{x^5+x}$

$= \displaystyle -\int_0^1\frac{x\log x}{(1+x^2)^2}\,dx$ GR 4.234.2

1 .1733466294835716297... $\approx \displaystyle\sum_{k=1}^{\infty}\frac{1}{k!!k^2}$

.17338224470333561885... $\approx \displaystyle\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k!k}$

3 . 17343648530607134219... $\approx 4\log\left(2+\sqrt{3}\right)-\dfrac{2\pi}{3}$

$= \displaystyle\int_{-1}^{1}\int_{-1}^{1}\frac{dx\,dy}{\sqrt{1+x^2+y^2}}$ Borwein-Devlin, p. 53

.17352520779833572367... $\approx \displaystyle\sum_{k=0}^{\infty}\frac{(-1)^k}{2k^3+3}$

2 .173532954993580700771... $\approx \displaystyle\int_0^1\frac{\log^2(1-x)}{\sqrt{x}}\,dx$

1 .17356302722472693495... $\approx \gamma - eEi(-1) = \displaystyle\sum_{k=1}^{\infty}\frac{\psi(k+1)}{k!}$

.173749320202926737911... $\approx \displaystyle -\sum_{k=1}^{\infty}\frac{BernoulliB(2k)}{EulerE(2k)}$

.17378563457299202316... $\approx \displaystyle\sum\frac{1}{(6k)^k}$

.17402964843834081341... $\approx \dfrac{\zeta(4)}{4}+\zeta(2)\zeta(3)-2\zeta(5) = \displaystyle\sum_{k=1}^{\infty}\frac{kH_k}{(k+1)^4}$

.17412827332774280224... $\approx \displaystyle\sum_{k=1}^{\infty}\frac{1}{3^k(3^k-1)k}$

.17417956274165000788... $\approx Li_2\left(\dfrac{1}{6}\right) = \displaystyle\sum_{k=1}^{\infty}\frac{1}{6^k k^2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}H_k}{5^k k}$

.17426680583299550083... $\approx \operatorname{arcsinh}1-\dfrac{1}{\sqrt{2}} = \log\left(1+\sqrt{2}\right)-\dfrac{\sqrt{2}}{2}$

$= \quad \int_0^1 \frac{x^2\,dx}{(x^2+1)^{3/2}}$

.17440510968851802738... $\approx \quad \frac{\pi^2}{6} + 3\pi\sqrt{3} + 9\log 3 + 6\log 4 - 36 = \sum_{k=1}^{\infty} \frac{1}{6k^3+k^2}$

$= \quad \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\zeta(k+2)}{6^k}$

2 .17454635174140... $\approx \quad \sum_{k=1}^{\infty} (-1)^{k+1} \frac{e^{1/k}}{k}$

.17464425873159243712 6... $\approx \quad \prod_{k=1}^{\infty} \left(1 - \frac{1}{k^2+1/2}\right)$

.17472077024896732623... $\approx \quad \sum_{k=1}^{\infty} (\zeta(k)-1)^4$

.17476263929944353642... $\approx \quad \sum_{p\ prime} \frac{1}{p^3} = \sum_{k=1}^{\infty} \frac{\mu(k)}{k} \log\zeta(3k)$

.17488106576263372626... $\approx \quad 2 - \frac{\pi}{4} - \frac{3\log 2}{2} = \sum_{k=1}^{\infty} \frac{1}{k(8k+2)}$

.1750000000000000000$\underline{0}$ = $\frac{7}{40}$

1 .1752011936438014569... ≈ $\sinh 1 = \frac{e-e^{-1}}{2} = 2\cosh\frac{1}{2}\sinh\frac{1}{2} = -i\sin i$

= $\sum_{k=0}^{\infty}\frac{1}{(2k+1)!}$ AS 4.5.62, GR 1.411.2

= $\prod_{k=1}^{\infty}\left(1+\frac{1}{\pi^2 k^2}\right)$ GR 1.431.2

= $\int_1^{\infty}\cosh\left(\frac{1}{x}\right)\frac{dx}{x^2}$

.175311881551605975687... ≈ $3\log 2-\frac{\pi}{2}-\frac{1}{3} = \sum_{k=1}^{\infty}\frac{\zeta(k+1)-1}{4^k}$

.17542341873682316960... ≈ $\frac{1+i}{4}\left({}_2F_1(1,1-i,2-i,-2)-i\ {}_2F_1(1,1+i,2+i,-2)\right)$

= $\int_0^{\infty}\frac{\cos x}{e^x+2}dx$

.175497994056966649393... ≈ $\sum_{k=1}^{\infty}\frac{\log k}{(k+1)!}$

.175510690849480992305... ≈ $\sum_{k=1}^{\infty}\frac{1}{e^{2^k}-1}$

.175639364649935926576... ≈ $1-\frac{\sqrt{e}}{2} = \sum_{k=2}^{\infty}\frac{k-1}{k!2^k} = \sum_{n=1}^{\infty}\frac{1}{n!}\sum_{k=2}^{\infty}\frac{\log^n k}{k!2^k}$

.17575073121372975946... ≈ $8+\frac{3}{8e^2} = \int_0^1\frac{x\sinh x}{e^x}dx$

.1757812500000000000$\underline{0}$ = $\frac{45}{256} = \sum_{k=1}^{\infty}\frac{k^2}{9^k}$

.175886808246818344795... ≈ $\log^2\frac{3}{2}\log 3+Li_2\left(\frac{2}{3}\right)\log\frac{9}{4}-2\left(Li_3\left(-\frac{1}{2}\right)+Li_3\left(\frac{1}{3}\right)-Li_3\left(\frac{2}{3}\right)+\zeta(3)\right)$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{H^{(2)}{}_k}{2^k(k+1)}$

.17592265828206878168... ≈ $\frac{J_2(2\sqrt{3})}{e} = \frac{1}{2}{}_0F_1(;3;e) = \sum_{k=0}^{\infty}\frac{(-1)^k e^k}{k!(k+2)!}$

.17593361191311075521... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{2^k(k-1)} = -\sum_{k=2}^{\infty}\frac{1}{2k}\log\left(1-\frac{1}{2k}\right)$

1 .1759460993017423069... ≈ $\frac{i}{4}\left(\psi^{(1)}\left(\frac{1+i}{2}\right)-\psi^{(1)}\left(\frac{1-i}{2}\right)\right) = \int_0^{\infty}\frac{x\sin x}{\sinh x}dx$

.17606065374881092046... ≈ $\sum_{k=1}^{\infty}\frac{1}{6k^3+1} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(3k)}{6^k}$

.176081477370773117343... ≈ $\frac{1}{2\sqrt{3}}\sinh\frac{1}{\sqrt{3}} = \sum_{k=1}^{\infty}\frac{k}{(2k)!3^k}$

12 .1761363792503046546... ≈ $\frac{\pi^4}{8} = \int_0^{\infty}\frac{\log^3 x\,dx}{x^2-1} = \int_0^{\infty}\frac{x^3\,dx}{\sinh x}$

1 .17624173838258275887... ≈ $\zeta(\pi)$

1 .176270818259917... ≈ smallest known Salem number

1 .1762808182599175065... ≈ larger real root of Lehmer’s polynomial

$z^{10}+z^9-z^7-z^6-z^5-z^4-z^3+z+1$ Borwein-Devlin. p. 35

.176394350735484192865... ≈ $\frac{\pi}{20}-\frac{1}{20}+\frac{\log 2}{10} = \int_1^{\infty}\frac{\arctan x}{x^6}dx$

1 .176434685750034331624... ≈ $\prod_{k=1}^{\infty}\left(1+\frac{(-1)^{k+1}}{2^{2^k}}\right)$

.176522118686148379106... ≈ $\frac{3}{2}+\frac{\gamma}{2}-\frac{\log 8\pi}{2} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(k+1)-1}{k+2}$

= $\sum_{k=2}^{\infty}\left(\frac{1}{2k}+k\log\left(1+\frac{1}{k}\right)-1\right)$

.176528539860746230765... ≈ $\frac{3}{4\sqrt{2}}\log\frac{1+\sqrt{1/2}}{1-\sqrt{1/2}}-\log 2-1$

= $\sum_{k=0}^{\infty}\frac{1}{2^k(k+1)(k+2)(k+3)}$ J279

.17659040885772532883... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{3^k+1}$

.1767766952966368811... ≈ $\frac{1}{4\sqrt{2}} = \sum_{k=0}^{\infty}\frac{(-1)^k k}{4^k}\binom{2k}{k}$

.176849761028064425527... ≈ $e^{-1/\gamma}$

.17709857491700906705... ≈ $5\cos 1-3\sin 1 = \sum_{k=1}^{\infty}\frac{(-1)^k}{(2k-1)!(2k+3)}$

= $\int_1^{e}\frac{\log^3 x\,\sin\log x}{x}dx = \int_1^{\infty}\sin\left(\frac{1}{x}\right)\frac{dx}{x^5}$

1 .177286727908419879284... ≈ $\sum_{k=2}^{\infty}\left(\zeta^2(k)-1\right)\gamma^{k-1}$

.17729989403903630843... ≈ $\frac{\pi}{6\sqrt{3}} - \frac{1}{8} = \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{(3k+1)(3k+5)}$

1 .17730832347023263216... ≈ $\frac{27}{4}\log 3 - 9\log 2 = \sum_{k=0}^{\infty} \frac{1}{(2k+1)[(2k+1)^2 - 1/9]}$

.17733145610396423590... ≈ $\frac{\gamma}{12}\left(24\log 2 - 9\zeta(3) + \pi^2 - 12\right) = \sum_{k=1}^{\infty} \frac{H_k H_{k+1}}{k(k+1)^2}$

1 .17743789377685370624... ≈ $4I_2(2) = {}_0F_1\left(;3;\frac{1}{2}\right) = 2\sum_{k=0}^{\infty} \frac{1}{k!(k+2)!2^k}$

.177532966575886781764... ≈ $1 - \frac{\pi^2}{12} = \sum_{k=2}^{\infty} \frac{(-1)^k}{k^2} = \sum_{k=1}^{\infty} \frac{1}{k(k+2)^2}$

$= \frac{1}{2}\left(Li_2\left(-e^{2i}\right) + Li_2\left(-e^{-2i}\right)\right) = \sum_{k=1}^{\infty} (-1)^k \frac{\cos 2k}{k^2}$

$= \int_0^1 x\log(1-x)\log x\,dx = \int_1^{\infty} \frac{\log x}{x^2(x+1)}dx = \int_0^{\infty} \frac{x\,dx}{e^{2x}+e^x}$ GR 3.411.10

$= \int_0^{\infty} \frac{x}{e^x\left(e^x+1\right)}dx$

$= -\int_0^1 \frac{x\log x}{1+x}dx$

$= \int_0^1\int_0^1 \frac{x\,y}{1+xy}\,dx\,dy$

.17756041552698560584... ≈ $\frac{1}{8}\left(2\sqrt{3}\log(2+\sqrt{3}) - \pi\right) = \sum_{k=1}^{\infty} \frac{\cos\frac{k\pi}{3}}{2k-1}$ J513

2 .1775860903036021305... ≈ $\pi\log 2$

$= \int_{-1}^{1} \frac{\log(1+x)}{\sqrt{1-x^2}}dx$

$= \int_0^{\infty} \frac{\log(x^2+1)}{x^2+1}dx = \int_0^{\infty} \frac{\log(x^2+4)}{x^2+4}dx$

$= \int_0^{\infty} \frac{\log(1-x^2)}{1+x^2}dx$ GR 4.295.15

$= \int_0^{\infty} \log\left(\frac{1+x^2}{x}\right)\frac{dx}{1+x^2}$ GR 4.298.9

$= -\int_0^{\pi} x\tan x\,dx$

$= \int_0^{\pi/2} \frac{x^2}{\sin^2 x}\,dx$ GR 3.837.1

$= \int_{-1}^{1} \frac{\arcsin x}{x}\,dx = \int_0^{\infty} \frac{\arctan x}{x^2}\,dx$

$= -\int_0^{\pi} \log\sin\left(\frac{x}{2}\right)dx = \int_0^{\pi/2} \log(4\tan x)\,dx$

1 .177662037037037$\underline{037}$ $= \frac{2035}{1728} = H^{(3)}{}_4$

1 .17766403002319739668... $\approx \sqrt[7]{\pi}$

.17778407880661290134... $\approx -ci\left(\frac{1}{2}\right) = -\gamma + \log 2 - \sum_{k=0}^{\infty} \frac{1}{(2k+1)!2^{2k+1}(2k+1)}$

.17785231624655055289... $\approx \sum_{k=1}^{\infty} \frac{k^3}{(3k)!}$

.177860854725449691202... $\approx \frac{\zeta^2(3) - \zeta(6)}{2\zeta(3)} = \sum \frac{1}{n^3}$, where *n* has an odd number of prime factors

[Ramanujan] Berndt Ch. 5

.17802570195060925877... $\approx \frac{\log 2}{2} + \frac{\pi^2}{16} - \frac{\pi}{4} = \int_0^{\pi/4} \frac{x^2 \tan x}{\cos^2 x}\,dx$ GR 3.839.3

.17804799789057856097... $\approx \sum_{k=1}^{\infty} \frac{1}{e^{2k} - 1}$ Berndt 6.14.4

3 .17805383034794561965... $\approx \log 4!$

1 .17809724509617246442... $\approx \frac{3\pi}{8} = \arctan\left(1 + \sqrt{2}\right)$

$= \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k-1)^2} \sin\frac{6k-3}{2}$ J523

$= \prod_{k=1}^{\infty} \frac{(k+1)^2}{(k+\frac{1}{2})(k+\frac{3}{2})} = \prod_{k=1}^{\infty} \frac{4k^2+8k+4}{4k^2+8k+3}$

$= \int_0^{\infty} \frac{dx}{(x^2+1)^3}$

$= \int_0^1 \frac{x^{3/2}}{\sqrt{1-x}}\,dx$ GR 3.226.2

$= \int_0^\infty \frac{\sin^3 x}{x}dx$ GR 3.827.4

.1781397802902698674... $\approx$ $1-\gamma+\frac{\gamma^2}{2}-\frac{\pi^2}{24} = \int_0^\infty \frac{\log x \sin x}{x^2}dx$

.17814029797808795269... $\approx$ $\sum_{k=1}^\infty \frac{H_k}{6^k k^2}$

.1781631171987300031... $\approx$ $\frac{\zeta(2)}{2}-\frac{\gamma}{4}-\frac{1}{2} = \sum_{k=1}^\infty \frac{\psi(k+1)}{k(k+1)(k+2)}$

2 .1781835566085708640... $\approx$ $\cosh\sqrt{2} = \frac{e^{\sqrt{2}}+e^{-\sqrt{2}}}{2} = \sum_{k=0}^\infty \frac{2^k}{(2k)!}$ GR 1.411.2

10 .17822221602772843362... $\approx$ $\sum_{k=2}^\infty \frac{k^2}{k!!}$

11 .17822221602772843362... $\approx$ $3\sqrt{e}+3\sqrt{\frac{\pi e}{2}}erf\left(\frac{1}{\sqrt{2}}\right)+2 = \sum_{k=1}^\infty \frac{k^2}{k!!}$

.1782494456033578066... $\approx$ $\frac{1}{2}+\frac{\pi}{4}-\arctan 2 = \int_1^2 \frac{dx}{x^4+x^2}$

.17836449302910417474... $\approx$ $\frac{\pi^2}{24}+\frac{\pi^4}{144}-\frac{\zeta(3)}{4} = \sum_{k=0}^\infty \frac{(2k+1)^2}{(2k+2)^4}$

.178403258246176718301... $\approx$ $\sum_{k=1}^\infty \frac{k!k!k!}{(3k)!}$

.17843151565841244881... $\approx$ $\frac{1}{16}-\gamma+\log 2 = \sum_{k=1}^\infty \frac{k^2}{k+1}\left(\zeta(2k+1)-1\right)$

.178487847956197627252... $\approx$ $\frac{25}{3}-3e = \sum_{k=0}^\infty \frac{1}{k!(k+2)(k+5)}$

.1785148410513678046... $\approx$ $\frac{4}{5}\log\frac{5}{4} = \sum_{k=1}^\infty (-1)^{k+1}\frac{\mathrm{H}_k}{4^k}$

.1787967688915270398... $\approx$ $\frac{\pi}{4\sqrt{3}}-\frac{\log 3}{4} = \sum_{k=1}^\infty \frac{1}{3k(3k-1)(3k-2)}$ J250

$= \int_1^\infty \frac{dx}{x^4+x^2+1}$

.17881507892743738987... $\approx$ $\sqrt{2}\arctan\frac{\tan 1}{\sqrt{2}}-1 = \int_0^1 \frac{\sin^2 x}{1+\cos^2 x}dx$

.17888543819998317571... $\approx$ $\frac{2}{5\sqrt{5}} = \sum_{k=1}^\infty \frac{(-1)^{k+1}(2k)!k}{(k!)^2}$

2 .17894802393254433150... $\approx \sum_{k=2}^{\infty} 2^k \left(\zeta(k)-1\right)^2$

.17895369203283464849 7... $\approx \frac{1}{2}-\frac{\cot 1}{2} = \sum_{k=1}^{\infty} \frac{1}{k^2\pi^2-1}$ GKP eq. 6.88

$= \sum_{k=1}^{\infty} \frac{\zeta(2k)}{\pi^{2k}} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}2^{2k-1}B_{2k}}{(2k)!}$

.17895690327368864585... $\approx \frac{\sqrt{\pi}}{8}\left(erfi 1 - erf 1\right) = \int_1^{\infty} \sinh\left(\frac{1}{x^4}\right)\frac{dx}{x^3}$

1 .17897974447216727023 2... $\approx \frac{2si(\pi)}{\pi} = \frac{2}{\pi}\int_0^{\pi} \frac{\sin x}{x}dx$

.17905419982836822413... $\approx \left(5-2\sqrt{6}\right)\sqrt{\pi} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(k-\frac{1}{2})!}{(k+1)!\,2^k}$

.17922493693603655502... $\approx \frac{20}{3}-16\log\frac{3}{2} = \sum_{k=0}^{\infty} \frac{(-1)^k}{2^k(k+4)}$

.179268385730264188 6... $\approx \frac{2-\log 3}{4}-\frac{1}{2e}+\frac{1}{4}\log\left(1+\frac{2}{e}\right) = \int_0^1 \frac{dx}{e^x(e^x+2)}$

.17928754997141122017... $\approx \sum_{k=2}^{\infty} (-1)^k \frac{\zeta(k)-1}{\zeta(k)+1}$

.17937407873401718196 2... $\approx e^{1-e} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}e^k}{(k-1)!}$

.17938430648086754802... $\approx 1+(\pi-1)\left(\log(\pi-1)-\log\pi\right) = \sum_{k=1}^{\infty} \frac{1}{\pi^k k(k+1)}$ J149

376 .17943461425965006202 9... $\approx \frac{145739620510}{387420489} = \sum_{k=1}^{\infty} \frac{k^{10}}{10^k}$

.1795075337635633909... $\approx \frac{\gamma^2}{4}-\frac{\gamma}{2}+\frac{\pi^2}{24}-\frac{\log 2}{2}+\frac{\gamma\log 2}{2}+\frac{\log^2 2}{4} = \int_0^{\infty} \frac{x\log^2 x dx}{e^{2x}}$

5 .17961063184875140986 7... $\approx \pi\sqrt{e}$

1 .17963464938133757842... $\approx \frac{1}{6}-\frac{\pi}{4\sqrt{3}}\cot\frac{\pi\sqrt{3}}{2} = \sum_{k=1}^{\infty} \frac{1}{4k^2-3}$

.17974050277429023935... $\approx \frac{3}{4}-\frac{\sqrt{3}}{2}\operatorname{arctanh}\frac{1}{\sqrt{3}} = \frac{3}{4}-\frac{\sqrt{3}}{2}\log(2-\sqrt{3})$

$= \sum_{k=0}^{\infty} \frac{k}{3^k(2k+1)}$

.1797480451435278353... ≈ $3-\sqrt{\pi}\,\Gamma\left(\frac{1}{3}\right)\Gamma^{-1}\left(\frac{5}{6}\right)+\;2\log 2 \;=\; \sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!k(3k+1)}$

.179842459798468021675... ≈ $\frac{7}{6}\log\frac{7}{6} \;=\; \sum_{k=1}^{\infty}\frac{H_k}{7^k}$

.18000000000000000000$\underline{0}$ = $\frac{9}{50}$

1 .18011660505096216475... ≈ $\frac{5\pi\zeta(3)}{16} \;=\; \int_0^1 \frac{\arcsin x \arccos^2 x}{x}dx$

.180267715063095577924... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!4^k\zeta(2k+1)}$ Titchmarsh 14.32.3

. 18028136579451194155... ≈ $4-\frac{12}{\pi} \;=\; \sum_{k=0}^{\infty}\binom{2k}{k}^2\frac{k}{16^k(k+1)^2}$

1 .18034059901609622604... ≈ $\frac{\sqrt{\pi}}{\Gamma^2\left(\frac{3}{4}\right)} \;=\; \sum_{k=0}^{\infty}\binom{2k}{k}^2\frac{1}{32^k}$

.1804080208620997292... ≈ $2-\frac{3}{\sqrt{e}} \;=\; \sum_{k=0}^{\infty}\frac{(-1)^k k}{(k+1)!2^k}$

.18064575946380647538... ≈ $\frac{\pi}{50}\sqrt{10\left(5+\sqrt{5}\right)} \;+\; \frac{1}{20}\left((1-\sqrt{5})\operatorname{arccsch}2-\sqrt{5}\operatorname{arcsinh}2\right)$

$+\log\frac{\sqrt{5}-1}{32}$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{5k-1}$

5 .180668317897115748417... ≈ $e^{\zeta(2)}$

.18067126259065494279... ≈ $\frac{\pi^2}{16\left(2+\sqrt{2}\right)} \;=\; \frac{\pi^2}{64}\csc^2\frac{3\pi}{8} \;=\; \sum_{k=1}^{\infty}\left(\frac{1}{(8k-3)^2}+\frac{1}{(8k-5)^2}\right)$

= $-\int_0^1\frac{x^2\log x}{(1-x^2)(1+x^4)}dx$ GR 4.234.5, Prud. 2.6.8.7

.1809494085014393275... ≈ $\frac{\pi}{8\sqrt{3}}-\frac{\log 3}{24} = \int_1^{\infty}\frac{dx}{x^3+8}$

.18102687827174792616... ≈ $\frac{\cosh 1-1}{3} \;=\; \int_1^{\infty}\sinh\left(\frac{1}{x^3}\right)\frac{dx}{x^4}$

.181097823860873992248... ≈ $\prod_{k=2}^{\infty}\left(1-\frac{2}{2^k-1}\right)$

.18121985982797669470... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(k+1)-1}{2^k(k+1)} = \sum_{k=2}^{\infty}\left(-\frac{1}{k}-2\log\left(1-\frac{1}{2k}\right)\right)$

3 .18127370927465812677... ≈ $2I_1(2) = \sum_{k=1}^{\infty}\frac{1}{(2k-1)!}\binom{2k}{k}$

.18132295573711532536... ≈ $\psi^{(1)}(6)$

1 .18136041286564598031... ≈ $e^{1/6}$

1 .1813918433423787507... ≈ $\int_1^{\infty}\frac{dx}{x!}$

1 .18143677605944287322... ≈ $2^{1/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3},\frac{4}{3},-1\right) = \sum_{k=0}^{\infty}\frac{1}{2^k(3k+1)}$

4 .18156333442222918673... ≈ $\frac{-1}{5e\left(1+\sqrt{5}\right)}\left(e^{(5-\sqrt{5})/2}\left(2-e^{\sqrt{5}}\left(3+\sqrt{5}\right)\right)+1+\sqrt{5}\right) = \sum_{k=1}^{\infty}\frac{F_kF_{k+1}}{k!}$

1 .18156494901025691257... ≈ $\prod_{p\ prime}\left(1+p^{-3}\right)$ Berndt 5.28

= $\frac{\zeta(3)}{\zeta(6)} = \sum_{k=1}^{\infty}\frac{|\mu(k)|}{k^3}$ Titchmarsh 1.2.7

= $\sum_{q\ squarefree} q^{-3}$

1 .18163590060367735153... ≈ $\frac{2\sqrt{\pi}}{3} = \int_0^{\infty}\frac{\sin^2(x^2)}{x^4}dx$ GR 3.852.3

.18174808646696599422... ≈ $\frac{\pi^2}{12} + 4\sqrt{2}\,\mathrm{arcsinh}1 - 2\log 2 - \frac{\log^2 2}{2} - 4$

= $\sum_{k=1}^{\infty}\frac{1}{2^k k^2(2k+1)}$

.1817655842134115276... ≈ $\frac{3}{2} - \gamma + \frac{\pi}{2\sqrt{3}} - \frac{3\log 3}{2} = \psi\left(\frac{5}{3}\right)$

.18178077665207622952 0... ≈ $125 - \frac{5\pi^2}{6} - 25\log 10 + \zeta(3) - \frac{25\pi}{2}\cot\frac{\pi}{5}$

$+ 50\left(\cos\frac{2\pi}{5}\log\sin\frac{\pi}{5} + \cos\frac{4\pi}{5}\log\sin\frac{2\pi}{5}\right)$

= $\sum_{k=1}^{\infty}\frac{1}{5k^4+k^3} = \sum(-1)^{k+1}\frac{\zeta(k+3)}{5^k}$

.181783302972344801352... ≈ $\frac{\pi^2}{18} + \frac{3\log 2}{4} - \frac{2\pi}{9} - \frac{16}{27} = \sum_{k=1}^{\infty}\frac{1}{4k^3+3k^2}$

.181818181818181818$\underline{18}$ = $\frac{2}{11}$

.1819778713379401005... ≈ $6\pi - \frac{\pi^2}{12} - 6\log 2 + 6\sqrt{3}\log\frac{\sqrt{3}-1}{\sqrt{3}+1} = \sum_{k=1}^{\infty}\frac{(-1)^k}{6k^3-k^2}$

.18199538670263388783... ≈ $\sum_{k=1}^{\infty}\frac{\mu(k)}{3^k}$

1 .18211814116655673118... ≈ $HypPFQ\left[\left\{\frac{1}{2},1,1\right\},\left\{\frac{1}{4},\frac{3}{4}\right\},\frac{1}{16}\right] = \sum_{k=0}^{\infty}\frac{1}{\binom{4k}{2k}}$

.18214511576348082181... ≈ $\frac{\pi^2}{12} - \gamma\log 2 - \frac{\log^2 2}{2} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\psi(k+1)}{k}$

.18215350260523806676 1... ≈ $\sum_{k=2}^{\infty}\frac{1}{3^k-1} = \sum_{k=1}^{\infty}\frac{1}{3^k(3^k-1)} = \sum_{k=2}^{\infty}\frac{\Omega(3^k)}{3^k}$

.18232155679395462621 2... ≈ $\log\frac{6}{5} = Li_1\left(\frac{1}{6}\right) = \Phi\left(\frac{1}{6},1,0\right) = \sum_{k=1}^{\infty}\frac{1}{6^k k}$

= $2\operatorname{arctanh} 11 = 2\sum_{k=0}^{\infty}\frac{1}{11^{2k+1}(2k+1)}$ J248

.18237763854617013873 7... ≈ $\frac{1}{4}\,{}_2F_1\left(2,2,\frac{3}{2},-\frac{1}{8}\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{\binom{2k}{k}2^k}$

.18248208162674477641 9... ≈ $\frac{11}{6} - \frac{\gamma}{4} - \frac{\log 2\pi}{2} + 3\zeta'(-1) + 3\zeta'(-2) = \sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k+3}$

.18248783058761050080 2... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^3+3}$

.18254472613276952213... ≈ $\zeta(3) - 1 - \frac{1}{4}\left(\psi^{(2)}(2-i) + \psi^{(2)}(2+i)\right)$

= $\zeta(3) - 1 + \frac{1}{2}\left(\zeta(3,2+2i) + \zeta(3,2-2i)\right)$

= $\int_0^{\infty}\frac{x^2\cos^2 x}{e^x(e^x-1)}dx$

.18277045187202515960 8... ≈ $\frac{\pi^2}{54} = \frac{\zeta(2)}{9} = \sum_{k=1}^{\infty}\frac{1}{(3k)^2} = \int_1^{\infty}\frac{\log x}{x^4-x}$

27 .18281828459045235360 3... ≈ $10e$

.18316463254842946724... ≈ $\frac{1}{4}\log(1+2\cos 1) = \sum_{k=1}^{\infty}\frac{\sin^2 k\cos k}{k}$

.183203987462730633284... $\approx \frac{5}{2} - \frac{\pi}{2^{7/4}}\left(\cot 2^{1/4}\pi + \coth 2^{1/4}\pi\right) = \sum_{k=2}^{\infty} \frac{2}{k^4 - 2}$

$= \sum_{k=1}^{\infty} 2^k \left(\zeta(4k) - 1\right)$

.18331367237300160091... $\approx$

$\frac{1}{512}\left(\psi^{(3)}\left(\frac{3}{4}\right) - \psi^{(3)}\left(\frac{1}{4}\right) - 2\pi\left(\pi^3 - 8\pi^2 \log 2 - 96\pi G + 36\zeta(3)\right)\right)$

$= \int_0^1 \frac{\arctan^4 x}{x^2}\,dx$

.18334824604139334840... $\approx \sum_{k=2}^{\infty} \frac{(-1)^k\left(\zeta(k) - 1\right)}{2k - 1} = \sum_{k=2}^{\infty}\left(\frac{1}{k} - \sqrt{\frac{1}{k}}\arctan\sqrt{\frac{1}{k}}\right)$

.183392750954659037507... $\approx \frac{1}{2}\left(\psi\left(\frac{\pi+1}{2}\right) - \psi\left(\frac{\pi}{2}\right)\right) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+\pi}$

1 .18345229451243828094... $\approx \frac{\sqrt{\pi}}{2}\left(\zeta\left(\frac{1}{2},\frac{1}{4}\right) - \zeta\left(\frac{1}{2},\frac{3}{4}\right)\right) = \sqrt{\pi}\sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{2k+1}}$

$= \int_0^{\pi/4} \frac{dx}{\sqrt{\log \cot x}}$ GR 3.511

2 .183488127407812202908... $\approx \frac{2}{G}$

.1835034190722739673... $\approx 1 - \sqrt{\frac{2}{3}} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(2k-1)!!}{(2k)!\,2^k}$

.183547497347006560022... $\approx \sum_{k=2}^{\infty} \frac{(-1)^k \nu(k)}{2^k}$

.18356105041486997256... $\approx \frac{\coth \pi}{2} - \frac{1}{\pi} = \int_0^{\infty} \frac{\sin \pi x}{e^{\pi x}\left(e^{\pi x} - 1\right)}\,dx$

.183578490978106856292... $\approx -\sum_{k=1}^{\infty} \frac{\mu(2k)}{2^k + 1} = -\sum_{k=1}^{\infty} \frac{\mu(2k)}{4^k + 1} = \frac{1}{2}\sum \frac{(-1)^k \mu(k)}{2^k + 1}$

.18359374976716935640... $\approx \sum_{k=1}^{\infty} \frac{\mu(k)}{2^{2^k}}$

.18373345259830798076... $\approx \frac{\gamma}{\pi}$

488 .183816543814241755771... $\approx \psi^{(3)}\left(\frac{1}{3}\right)$

23 .183906551043041255504... $\approx e^{\pi} + e^{-\pi} = 2\cosh \pi$

.1839397205857211608... ≈ $\frac{1}{2e} = \sum_{k=1}^{\infty} \frac{k}{(2k+1)!}$

= $\int_1^{\infty} \frac{x\,dx}{e^{x^2}}$

= $\int_1^{\infty} \sinh\left(\frac{1}{x^2}\right)\frac{dx}{x^5} = \frac{1}{2}\int_1^{\infty} \sinh\left(\frac{1}{x}\right)\frac{dx}{x^3}$

.18394974527683870489 9... ≈ $\frac{\pi\sqrt{5}}{20}\tanh\frac{\pi\sqrt{5}}{2} - \frac{1}{6} = \sum_{k=1}^{\infty} \frac{1}{4k^2+4k+6}$

1 .1840000000000000000$\underline{0}$ = $\frac{148}{125} = \sum_{k=1}^{\infty} \frac{F_k^{\,2}k}{4^k}$

2 .18400947026785195289 5... ≈ $\sum_{k=2}^{\infty} \frac{1}{k^{3/2}-k^{1/2}} = \sum_{k=1}^{\infty}\left(\zeta\left(2k+1\right)/2\right)-1\right)$

3 .18400947026785195289 5... ≈ $\sum_{k=2}^{\infty} \frac{1}{k^{3/2}-k} = \sum_{k=3}^{\infty}\left(\zeta(k/2)-1\right)$

.1840341753914914215... ≈ $\log\zeta(3) = \sum_{p\ prime}\log\left(\frac{1}{1-p^{-3}}\right)$ HW Sec. 17.7

= $\sum_{k=2}^{\infty} \frac{\Lambda(k)}{k^3\log k}$ Titchmarsh 1.1.9

3 .18416722563191043326... ≈ $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{(k-1)^2}$

.18424139854155154160... ≈ $\log_{16}\frac{5}{3} = \int_1^{\infty} \frac{dx}{4^x-4^{-x}}$

.18430873967674565649... ≈ $\frac{\pi\sqrt{3}}{4} + \frac{3\log 3}{4} - 2 = \sum_{k=1}^{\infty} \frac{(k-\frac{1}{2})!}{(k+\frac{1}{2})!(6k+1)}$

.18432034259551909517... ≈ $\frac{2\log 2}{3} - \frac{5}{18} = \sum_{1}^{\infty} \frac{(-1)^{k+1}}{k^2+3k} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)(k+4)}$

= $\int_0^1 x^2\log(1+x)\,dx = \int_1^{\infty} \log\left(1+\frac{1}{x}\right)\frac{dx}{x^4}$

.18447121328067061916 8... ≈ $\frac{8\zeta(3)}{9} - \frac{4\pi^3}{81\sqrt{3}}$

= $\left(-\frac{2}{1+(-1)^{1/3}}\right)\left((-1)^{1/3}Li_3(-(-1)^{1/3}) + Li_3((-1)^{2/3})\right)$

= $\int_0^1 \frac{x\log^2 x\,dx}{x^2+x+1}$

.18453467186138662977... ≈ $\frac{3\pi^2}{32}-\frac{20}{27} = \sum_{k=1}^{\infty}\frac{k^2(\zeta(2k)-1)}{4^k} = \sum_{k=2}^{\infty}\frac{4k^2(4k^2+1)}{(4k^2-1)^3}$

1 .18459307293865315132... ≈ $e^{1/4}\sqrt{\pi}\, erf\left(\frac{1}{2}\right)=\sum_{k=0}^{\infty}\frac{k!}{(2k+1)!}=\sum_{k=0}^{\infty}\frac{1}{(2k+1)!!2^k}$

= $\int_0^1 \frac{e^x}{e^{x^2}}dx$

1 .184626477442252628916... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}2^{2k-1}(\zeta(2k)-1)}{(2k-1)!} = \sum_{k=2}^{\infty}\frac{1}{k}\sin\frac{2}{k}$

.184713841175145145685... ≈ $\zeta(3)-\zeta(6)$

1 .184860269128474... ≈ $\zeta^{-1}(6)$

.184904842471782293011... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}(\zeta(2k+1)-1)}{k!} = \sum_{k=2}^{\infty}\frac{1}{k}\left(1-e^{-1/k^2}\right)$

.184914711864907675754... ≈ $\frac{1}{e(1-1/e)^2}-\frac{2}{e} = \sum_{k=1}^{\infty}\frac{k}{e^k}-\int_1^{\infty}\frac{x}{e^x}dx$

.185066349816248591534... ≈ $\frac{8}{49}\left(1+\log\frac{8}{7}\right) = \sum_{k=1}^{\infty}\frac{kH_k}{8^k}$

2 .18528545178748245... ≈ $\zeta^{-1}(3/2)$

.18533014860121959977... ≈ $\int_1^{\infty}\sin\left(\frac{1}{x^3}\right)\frac{dx}{x^3}$

8 .1853527718724499700... ≈ $\sqrt{67}$

24 .18541655363135590105... ≈ $e^e\left(1+\gamma-Ei(-e)\right) = \sum_{k=1}^{\infty}\frac{e^kH_k}{k!}$

.18544423087144965373... ≈ $-\frac{1}{16}Li_3(-4) = \frac{\pi^2\log 2}{48}+\frac{\log^3 2}{12}-\frac{1}{16}Li_3\left(-\frac{1}{4}\right)$

= $\int_1^{\infty}\frac{\log^2 x}{x^3+4x}$

1 .18559746638186798649... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^{2k-1}-1} = \sum_{k=1}^{\infty}\frac{2^k}{4^k-1} = \sum_{k=1}^{\infty}\frac{1}{2^k-2^{-k}} = \frac{1}{4}\sum_{k=1}^{\infty}\frac{\nu(k)}{2^k}$

1 .185662037037037037037... ≈ $\frac{256103}{261000} = H^{(3)}{}_5$

1 .1856670077645066173... ≈ $\frac{\pi}{32}\csc^2\frac{\pi}{\sqrt{2}}\left(2\pi-\sqrt{2}\sin\pi\sqrt{2}\right) = \sum_{k=1}^{\infty}\frac{k^2}{(2k^2-1)^2}$

$.1857845358006592 41215... \approx \frac{G}{2} - \frac{\pi \log 2}{8} = \int_0^{\pi/4} x \tan x \, dx$ GR 3.747.6

$.18595599358672051939... \approx \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\zeta(2k+1)-1}{k} = \sum_{k=2}^{\infty} \frac{1}{k} \log\left(1 + \frac{1}{k^2}\right)$

2 $.18604814112086736 2993... \approx \frac{1}{2}\left(\log 2 - 1 - \log(3 - 2\sqrt{2})\right) = \sum_{k=1}^{\infty} \frac{H_{21+1}}{2^k}$

4 $.1860529265117195 9669... \approx \sum_{k=2}^{\infty} \frac{3^k(\zeta(k)-1)}{k!} = \sum_{k=2}^{\infty} \left(e^{3/k} - 1\right)$

$.18605586311714536 2029... \approx \frac{\pi^2}{6} - \frac{1}{3}\left(8 - 8\log 2 + 4\log^2 2\right) = \sum_{k=1}^{\infty} \frac{H_k}{k(k+1)(2k+3)}$

$.18618296110159529376... \approx \sum_{k=0}^{\infty} \frac{(-1)^k}{2^k + 2}$

$.18620063576578214941... \approx 2 - \frac{\pi\sqrt{3}}{3} = \sum_{k=1}^{\infty} \frac{1}{\binom{2k}{k}(2k+1)k}$

$= \int_0^1 \log \frac{1+x}{1+x^3} \, dx$

$.18623228252032220 4432... \approx \frac{\sinh \pi}{2\pi^3} = \sum_{k=0}^{\infty} \frac{\pi^{4k}}{(4k+3)!}$

$.1864467513294867736... \approx \sum_{k=1}^{\infty} \frac{H^{(2)}{}_k}{6^k k}$

$.18645414143259205 7975... \approx \frac{\pi}{4} \log \frac{2\sqrt{3}}{1+\sqrt{3}} = \frac{\pi}{8}\left(\log 6 - 2 \operatorname{arcsinh} \frac{1}{\sqrt{2}}\right)$

$= \int_0^1 \arcsin x \, \frac{x}{1+2x^2} \, dx$ GR 4.521.3

$.18646310636181362051... \approx -\sum_{k=1}^{\infty} \frac{\mu(3k)}{4^k}$

$.18647806660946676075... \approx \frac{2I_2(2)}{e^2} = \sum_{k=1}^{\infty} (-1)^{k+1} \binom{2k}{k} \frac{k}{k!}$

$.18650334233862388596... \approx \sum_{k=2}^{\infty} \frac{1}{k^3+1} = \sum_{k=1}^{\infty} (-1)^{k+1}\left(\zeta(3k) - 1\right)$

$= \frac{\gamma}{3} - \frac{5}{6} + \frac{1}{6}\left(\left(1 + i\sqrt{3}\right)\psi\left(\frac{1 - i\sqrt{3}}{2}\right) + \left(1 - i\sqrt{3}\right)\psi\left(\frac{1 + i\sqrt{3}}{2}\right)\right)$

1 $.18650334233862388596... \approx \sum_{k=0}^{\infty} \frac{1}{k^5 + k^4 + k^3 + k^2 + k + 1} = 1 + \sum_{k=1}^{\infty} (-1)^{k+1}\left(\zeta(3k) - 1\right)$

$= \quad \frac{1}{2} + \sum_{k=2}^{\infty} \frac{k}{k^3 - 1}$

$= \quad \frac{\gamma}{3} - \frac{\pi}{2} + \frac{1}{6}\left(\left(1 + i\sqrt{3}\right)\psi\left(\frac{1 - i\sqrt{3}}{2}\right) + \left(1 - i\sqrt{3}\right)\psi\left(\frac{1 + i\sqrt{3}}{2}\right)\right)$

$= \quad \frac{1}{6}\left(-3 + 2\gamma + 2\left(1 + (-1)^{2/3}\right)\psi\left(2 - (-1)^{1/3}\right) - 2(-1)^{2/3}\psi\left(2 + (-1)^{2/3}\right)\right)$

1 .1866007335148928206... $\approx \sum_{k=1}^{\infty} \frac{k}{e^k - 1}$

.1866931403397504774... $\approx \frac{1}{2}\left(1 - {}_1\mathrm{F}_1\left(\frac{1}{2}, 3, -4\right)\right) = \frac{1}{2} - \frac{2}{3e^2} I_0(2) - \frac{1}{2e^2} I_1(2)$

$= \sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{(k+2)!}\binom{2k}{k}$

.1867444291836768034O4... $\approx \frac{1}{\sqrt{2\cos(1/2)}} \sin\frac{1}{4} = \sum_{k=0}^{\infty} \frac{(-1)^k (2k)!}{(k!)^2 4^k} \sin\frac{2k+1}{2}$

.18677536394571209O196... $\approx \frac{\pi}{6\sqrt{3}} - \frac{\log 2}{6} = \int_1^{\infty} \frac{dx}{x^5 + x^{-1}}$

1 .1868273377200538821 6... $\approx \frac{\pi}{\sqrt{7}} \tanh\frac{\pi\sqrt{7}}{2} = \sum_{k=1}^{\infty} \frac{1}{k^2 - k + 2}$

1 .18683727525826858588... $\approx 3\left(\sqrt[3]{3} - 1\right) = \sum_{k=0}^{\infty} \frac{1}{(k+1)!3^k}$

.1869453483802580252 66... $\approx \frac{1}{16}\left(6\zeta(3) - \pi^2 + 6\pi\sqrt{3} - 27\right) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^3(3k+2)}$

.18704390591656489823... $\approx \frac{2}{7}\sqrt{\frac{3}{7}} = \sum_{k=0}^{\infty} (-1)^k \binom{2k}{k} \frac{k}{3^k}$

.18716578302492737369... $\approx \sum_{k=2}^{\infty} \frac{\zeta(k+1) - 1}{2^k(\zeta(k) - 1)}$

1 .18723136493137661467... $\approx \frac{1}{11} + \frac{12}{121}\sqrt{11} \arcsin\frac{1}{2\sqrt{3}} = \sum_{k=1}^{\infty} \frac{1}{3^k \binom{2k}{k}}$

1 .18741041172372594879... $\approx \frac{\pi}{\sqrt{7}}$ K Ex. 108d

.187426422828231080265... ≈ $\frac{\pi^2 \log 2}{24} - \frac{3\pi\zeta(3)}{16} = -\int_0^{\pi/2} x^2 \log\sin x \; dx$

.1875000000000000000$\underline{0}$ = $\frac{3}{16} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k}{3^k}$

= $\sum_{\substack{k=1 \\ k\neq 2}}^{\infty} \frac{(-1)^{k+1}}{(k+2)(k-2)}$ GR 0.237.5

= $\sum_{k=1}^{\infty} \frac{\mu(k) 4^k}{16^k - 1}$

1 .187538169020838240502... ≈ $1 + \frac{i}{2} - \frac{i\pi^2}{24} + \log 2 - \frac{i}{2} Li_2(-e^{2i}) = -\int_0^1 \log(x\cos x)\, dx$

.188016647161420393823... ≈ $\sum_{k=2}^{\infty} \frac{\zeta(k) - 1}{(k+2)(k-1)}$

.18806319451591876232... ≈ $\frac{1}{\sqrt{9\pi}}$

1 .188163855465169281367... ≈ $\frac{\pi \log 2}{8} + G = \int_1^{\infty} \frac{\log(1+x)}{1+x^2} dx$

= $\int_0^{\pi/4} \log(1 + \cot x)\, dx$ GR 4.227.13

1 .18834174768091853673... ≈ $\sum_{k=2}^{\infty} (-1)^k \frac{\zeta(k)}{(k-1)!} = \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{ke^{1/k}} \right)$

.1883873799298847433... ≈ $\frac{1}{2} - \frac{1}{2\sqrt{2}} \log\left(1 + \sqrt{2}\right)$

= $\frac{1}{4}\left(2 - \sqrt{2}\,\mathrm{arcsinh} 1\right) = \frac{1}{2}\left(1 - \frac{1}{\sqrt{2}} \mathrm{arctanh} \frac{1}{\sqrt{2}}\right)$

= $\sum_{k=1}^{\infty} \frac{1}{2^k (2k-1)(2k+1)}$

1 .18839510577812121626... ≈ $\csc 1 = \sum_{k=0}^{\infty} \frac{(-1)^k \left(2 - 4^k\right) B_{2k}}{(2k)!}$ [Ramanujan] Berndt Ch. 5

.188399266485106896861... ≈ $\frac{11 - \pi^2}{6} = \sum_{k=1}^{\infty} \frac{k}{(k+4)(k+2)^2}$

.18842230702002753786... ≈ $\sum_{k=2}^{\infty} \frac{1}{k^4 - 10}$

.18872300160567799280... ≈ $2\log(1+\sqrt{2}) - 4\log 2 + 8 - \frac{\pi(1+\sqrt{2})}{2}$

= $8 - \frac{\pi}{2}\cot\frac{\pi}{8} - 4\log 2 + \sqrt{2}\left(\log\sin\frac{\pi}{8} - \log\sin\frac{3\pi}{8}\right)$

= $\sum_{k=1}^{\infty}\frac{1}{8k^2+k} = \sum(-1)^{k+1}\frac{\zeta(k+1)}{8^k}$ Prud. 5.1.5.21

= $-\int_0^1 \log(1-x^6)\,dx$

.1887270467095280645... ≈ $\frac{4}{9}\left(1-\log\frac{16}{9}\right) = \sum_{k=1}^{\infty}\frac{(k-1)H_k}{4^k}$

.1887703343907271768... ≈ $\frac{1}{2(1+\sqrt{e})}$ J147

4 .18879020478639098461 7... ≈ $\frac{4\pi}{3}$, volume of the unit sphere

3 .1889178875489624223... ≈ $\frac{\sinh\pi\sqrt{2}}{3\pi\sqrt{2}} = \prod_{k=2}^{\infty}\left(1+\frac{2}{k^2}\right)$

.18894844769873820405 5... ≈ $I_2(2) - \frac{1}{2} = \sum_{k=1}^{\infty}\frac{1}{k!(k+2)!}$

.18906978378367123604 4... ≈ $1 + 2\log\frac{2}{3} = \sum_{k=1}^{\infty}\frac{1}{3^k k(k+1)}$ J149

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^k(k+1)}$

1 .18920711500272106672... ≈ $\sqrt[4]{2} = \prod_{k=0}^{\infty}\left(1+\frac{(-1)^k}{4k+3}\right)$

.18930064124495395454... ≈ $\psi(i) + \psi(-i) = \psi(1+i) + \psi(1-i)$

1 .18930064124495395454... ≈ $\psi(2+i) + \psi(2-i)$

.1893037484509927148... ≈ $\frac{1}{\pi} - \frac{4}{\pi^3} = \int_0^1 x^2 \sin\pi x\,dx$

1 .18934087784832795825... ≈ $\sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)}{(k-2)!} = \sum_{k=1}^{\infty}e^{-1/k}\left(\frac{2}{k^2}-\frac{1}{k}\right)$

.1893415020203205152.. ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}H_k H_k}{2^k}$

.18947234582049235190 2... ≈ $\frac{\sqrt{\pi}erf1}{4} - \frac{1}{2e} = \int_1^{\infty}x^2 e^{-x^2}\,dx$

.189503010523246254898... ≈ $1-\frac{\sqrt{2}\operatorname{arcsinh}\sqrt{2}}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^k(2k+1)}\binom{2k}{k}$

= $\sum_{k=1}^{\infty}\frac{H_k}{2^k(k+1)^2}$

2 .18950347926669754176... ≈ $\sum_{k=2}^{\infty}k^2\left(\zeta(k)-1\right)^2$

.18950600846025541144... ≈ $\frac{\zeta(3)}{4}-\frac{\log^3 2}{3}=\int_0^1\frac{\log^2(1+x)}{x(x+1)}dx = \int_1^2\frac{\log^2 x}{x^2-x}dx$

.189727372555663311541... ≈ $\frac{1}{8\sqrt{2}}\left(\psi\left(1-\frac{1}{2\sqrt{2}}\right)-\psi\left(1+\frac{1}{\sqrt{2}}\right)\right)=\sum_{k=1}^{\infty}\frac{k\zeta(2k+1)}{4^k}=\sum_{k=1}^{\infty}\frac{8k}{(8k^2-1)^2}$

406 .189869040564760332... ≈ $\frac{1}{4}\left(e^{e^2}+e^{e^{-2}}\right)+\frac{e}{2} = \sum_{k=0}^{\infty}\frac{\cosh^2 k}{k!}$

.1898789722210629262... ≈ $\frac{1}{2}-\frac{\sqrt{\pi}\operatorname{csch}\sqrt{\pi}}{2}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2\pi+1}$

.189957907718062725272... ≈ $\frac{\pi}{2\sqrt{5}}\csc\pi\sqrt{5}-\frac{17}{20} = \sum_{k=3}^{\infty}\frac{(-1)^{k+1}}{k^2-5}$

.18995863340718094647... ≈ $\log\frac{2\pi}{3\sqrt{3}} = \sum_{k=1}^{\infty}\frac{\zeta(2k)}{9^k k} = -\sum_{k=1}^{\infty}\log\left(1-\frac{1}{9k^2}\right)$

.190028484997676054144... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^{k+1}}{k}\left(\zeta(k)-\zeta(k+1)\right) = \sum_{k=2}^{\infty}\frac{k-1}{k}\log\left(1+\frac{1}{k}\right)$

.190086499075236586883... ≈ $\log\frac{\sqrt{3}+1}{\sqrt{3}-1} = \sum_{k=1}^{\infty}\frac{1}{3^k(4k-2)}$

1 .19020822799902279358... ≈ $\prod_{k=2}^{\infty}\frac{1}{1-3^{-k}}$

1 .19029166666666666666$\underline{6}$ = $\frac{28567}{24000} = H^{(3)}{}_6$

.19042323918287231446... ≈ $\frac{7\pi}{2}-4\sqrt{2}\,E\left(-1\right) = \sum_{k=1}^{\infty}\frac{(k-½)!(k+½)!}{(k+1)!(k+1)!2^k}$

.190598923241496942... ≈ $\frac{15-8\sqrt{3}}{6}=\sum_{k=1}^{\infty}\frac{1}{6^k(k+2)}\binom{2k}{k}$

.190632130643561206769... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k}{2^k-1}\left(\zeta(k)-1\right)$

1 .19063934875899894829... ≈ $\Gamma\left(\frac{7}{3}\right)$

.1906471826647054863... ≈ $1-\frac{\arctan\left(\sqrt{2}\tan 1\right)}{\sqrt{2}} = \int_0^1 \frac{\sin^2 x}{1+\sin^2 x}dx$

1 .190661478044776833140... ≈ $\prod_{k=0}^{\infty}\left(1+\frac{(-1)^k}{2^{2^k}}\right)$

.1906692472681567121... ≈ $\frac{9}{4}-e-\frac{\gamma}{2}+\frac{Ei(1)}{2} = \sum_{k=1}^{\infty}\frac{1}{(k+2)!k}$

= $\sum_{k=3}^{\infty}\frac{1}{(k+1)!-3k!}$

.190800137777535619037... ≈ $\frac{1}{2}\left(\log^2 5-2\log 5\log 6+\log^2 6+2Li_2\left(\frac{1}{6}\right)\right) = \sum_{k=1}^{\infty}\frac{H_k}{6^k k}$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{5^k k^2}$

.190856278271211720658... ≈ $\frac{i}{16}\left(\psi^{(1)}\left(\frac{1+i}{2}\right)+\psi^{(1)}\left(\frac{2-i}{2}\right)-\psi^{(1)}\left(\frac{2+i}{2}\right)-\psi^{(1)}\left(\frac{1-i}{2}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(k^2+1)^2}$

.190985931710274402923... ≈ $\frac{3}{5\pi}$

.1910642658527378865... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k^2} = \sum_{k=2}^{\infty}\left(Li_2\left(\frac{1}{k}\right)-\frac{1}{k}\right)$

.19123512945396982066... ≈ $\frac{\pi\tanh\pi}{8}-\frac{1}{5} = \sum_{k=1}^{\infty}\frac{1}{4k^2+4k+5}$

.1913909914437681436... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^{2^k}}$

3 .191538243211461423520... ≈ $\frac{8}{\sqrt{2\pi}}$

.19166717873220686446... ≈ $-\sum_{k=1}^{\infty}\frac{\mu(2k)}{5^k}$

.19178804830118728496... ≈ $\frac{4\log 2}{3}-\frac{2\log 3}{3} = \int_1^2 \frac{dx}{x^4+x}$

3 .19184376659888928138... ≈ $\frac{\zeta(2)-1}{\zeta(3)-1}$

.191899085506264827981... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k+1)}{(2k+1)!} = \sum_{k=1}^{\infty}\left(\frac{1}{k}-\sin\frac{1}{k}\right)$

1 .19190510063271858384... ≈ G^{-2}

.192046874791755813246... ≈ $\frac{e}{e^e-1}$

.19209280551924242045... ≈ $\sum_{k=1}^{\infty}\frac{\log k}{e^k}$

.192127760367036391091... ≈ $\frac{1}{\sqrt{2}}\sinh\pi\ \operatorname{csch}\pi\sqrt{2} = \prod_{k=0}^{\infty}\frac{k^2+1}{k^2+2}$

.1923076923076$\underline{923076}$ = $\frac{5}{26} = \int_0^{\infty}\frac{\sin 5x}{e^x}dx$

= $\frac{1}{2\cosh\log 5} = \sum_{k=0}^{\infty}(-1)^k e^{-(\log 5)/(2k+1)}$ J943

.192315516821184589663... ≈ γ^3

.192405221633844286869... ≈ $\frac{\gamma}{3}$

.192406280693940317174... ≈ $\sum_{k=1}^{\infty}\frac{{H_k}^3}{8^k}$

.19245008972987525484... ≈ $\frac{1}{3\sqrt{3}} = \sum_{k=0}^{\infty}\frac{(-1)^k k}{2^k}\binom{2k}{k}$

.19247498022682262243... ≈ $2\log 6 - 2\log\left(3+\sqrt{6}\right) = \sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!\,3^k k}$

.192547577875765302901... ≈ $\frac{\csc^2 1}{8}\left(2-\sin 2\right) = \sum_{k=1}^{\infty}\frac{k\zeta(2k)}{\pi^{2k}}$

1 .192549103088454622134... ≈ $\frac{\pi^3}{26}$

.192694724646388148682... ≈ $-1-2e\,Ei(-1) = \int_0^{\infty}\frac{x}{e^x(x+1)^2}dx$

.192901316796912429363... ≈ $\frac{\pi\gamma}{4} + \pi\log\Gamma\left(\frac{3}{4}\right) - \frac{\pi\log\pi}{4} = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}\log(2k+1)}{2k+1}$

[Ramanujan] Berndt Ch. 8

1 .193057650962194448202... ≈ $\frac{3}{\pi}\sinh\frac{\pi}{3} = \prod_{k=1}^{\infty}\left(1+\frac{1}{9k^2}\right)$

.193147180559945309417... ≈ $\log 2 - \frac{1}{2} = \sum_{k=2}^{\infty}\frac{1}{4k^2-2k} = \sum_{k=2}^{\infty}\frac{1}{2k(2k-1)}$

= $\sum_{k=1}^{\infty}\frac{1}{(2k-1)2k(2k+1)}$ J236

= $\sum_{k=1}^{\infty}\frac{1}{8k^3-2k}$ [Ramanujan] Berndt Ch2, 0.1

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{k+3} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{4k^5+k}$ K Ex. 107f

$= \sum_{k=2}^{\infty}\frac{1}{2^k k}$

$= \sum_{k=0}^{\infty}\frac{1}{2^k(k+1)(k+2)(k+3)}$ J279

$= \sum_{k=2}^{\infty}\frac{\zeta(k)-1}{2^k} = \sum_{k=1}^{\infty}\frac{\zeta(2k+1)}{2^{2k+1}}$

$= \int_2^{\infty}\frac{dx}{x^3-x^2} = \int_1^{\infty}\frac{dx}{x^4+x^3}$

$= \int_1^{\infty}\frac{dx}{x(x+1)^2} = \int_1^{\infty}\log\left(1+\frac{1}{x^2}\right)\frac{dx}{x^3}$

$= \int_0^{\infty}\frac{x\,dx}{(1+x^2)\sinh\pi x}$ GR 3.522

$= \int_0^{\infty}\frac{dx}{e^{2x}\left(e^x-1\right)}$

1 .193207118561710398445... ≈ $\frac{9822481}{8232000} = H^{(3)}{}_7$

2 .193245422464301915297... ≈ $\frac{2\pi^2}{9} = \sum_{k=1}^{\infty}\frac{3^k}{\binom{2k}{k}k^2}$

2 .19328005073801545656... ≈ $e^{\pi/4} = i^{-i/2}$

.193497800269717788082... ≈ $\frac{\pi G}{4} - \frac{7\zeta(3)}{16} = -\int_0^{\pi/4}x\log\tan x\ dx$

.193509206599196935107... ≈ $\frac{6}{\pi^3} = \int_0^{\infty}\frac{dx}{e^{\pi x^{1/3}}}$

1 .193546496554454968807... ≈ $\sum_{k=1}^{\infty}\frac{1}{k!(2^k-1)} = \sum_{k=1}^{\infty}\left(e^{1/2^k}-1\right)$

41 .193555674716123563188... ≈ $e^{e+1} = \sum_{k=1}^{\infty}\frac{e^k}{(k-1)!}$

.19370762577767145856... ≈ $\zeta(3)-\zeta(7)$

.193710100392006734616... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(3k)-1}{k!} = -\sum_{k=2}^{\infty}\frac{1}{k}\log\left(e^{-k^{-3}}-1\right)$

.193998857662600941460... ≈ $-\log 2 - \log\Gamma\left(\frac{1+i\sqrt{3}}{2}\right) - \log\Gamma\left(\frac{1-i\sqrt{3}}{2}\right)$

$= \quad \log\left(\frac{1}{2\Gamma\left(-(-1)^{1/3}\right)\Gamma\left((-1)^{2/3}\right)}\right)$

$= \quad \sum_{k=2}^{\infty}\log\left(1+\frac{1}{k^3}\right) = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(3k)-1}{k}$

.19403566716396653285... ≈ $\frac{1}{6}+\frac{\sqrt{6}}{3}\operatorname{csch}\pi\sqrt{\frac{3}{2}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{2k^2+3}$

.19403879384913797074... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k\left(\zeta(k)-1\right)}{k^2-1} = \sum_{k=2}^{\infty}\left(\frac{1-k^2}{2k}\log\left(1+\frac{1}{k}\right)+\frac{1}{2}-\frac{1}{4k}\right)$

.194169602671133044121... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{H^{(3)}{}_k}{4^k}$

.194173022150715234759... ≈ $\frac{i}{4}\left(\psi\left(1-(-1)^{1/4}\right)+\psi\left(1+(-1)^{1/4}\right)\right)-\frac{1}{2}$

$-\frac{i}{4}\left(\psi\left(1-(-1)^{3/4}\right)+\psi\left(1+(-1)^{3/4}\right)\right)$

$= \quad \sum_{k=2}^{\infty}\frac{k}{k^4+1} = \sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(4k-1)-1\right)$

.194182706187011851678... ≈ $\frac{1}{4}-\frac{\cot\sqrt{2}}{2\sqrt{2}} = \sum_{k=1}^{\infty}\frac{1}{k^2\pi^2-2}$

.1944444444444444444$\underline{4}$ = $\frac{7}{36} = \sum_{k=1}^{\infty}\frac{k}{7^k} = \Phi\left(\frac{1}{7},-1,0\right) = \sum_{k=1}^{\infty}\frac{1}{k(k+1)(k+3)}$

.19449226482417135531... ≈ $\frac{1}{\pi+2}$

2 .194528049465325113615... ≈ $\frac{e^2-3}{2} = \sum_{k=1}^{\infty}\frac{2^k k}{(k+2)!}$

3 .194528049465325113615... ≈ $\frac{e^2-1}{2} = \sum_{k=0}^{\infty}\frac{2^k}{(k+1)!}$

4 .194528049465325113615... ≈ $\frac{e^2+1}{2} = \sum_{k=1}^{\infty}\frac{2^k k}{(k+1)!}$

57 .194577266401621023664... ≈ $2e^2\left(I_0(2)+I_1(2)\right) = \sum_{k=1}^{\infty}\binom{2k}{k}\frac{1}{(k-1)!}$

.194700195767851217061... ≈ $\frac{1}{4e^{1/4}} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{k}{k!4^k}$

5 .194940984113795745459... ≈ 9γ

1 .194957661910227628163... ≈ $\sqrt{\frac{\pi}{2}}\,erfi\frac{1}{\sqrt{2}} = \sum_{k=0}^{\infty}\frac{1}{k!2^k(2k+1)}$

.195090322016128267848... ≈ $\sin\frac{\pi}{16} = \cos\frac{7\pi}{16}$

.195262145875634983733... ≈ $\frac{2-\sqrt{2}}{3} = \int_0^{\pi/4}\frac{\sin^3 x}{\cos^4 x}dx$

.195289701596312945328... ≈ $\zeta(2)+\frac{\pi}{\sqrt{3}}\cot\frac{\pi}{\sqrt{3}}-1 = \sum_{k=2}^{\infty}\frac{k^2-1}{3k^4-k^2} = \sum_{k=1}^{\infty}\frac{\zeta(2k)-\zeta(2k+1)}{3^k}$

.19541428041665277483... ≈ $\sum_{k=1}^{\infty}\binom{2k}{k}\left(\zeta(4k)-1\right)$

.19545006939687776935... ≈ $\int_1^{\infty}x\left(\zeta(2x+1)-1\right)dx$

.195527927242921852033... ≈ $-\sum_{k=1}^{\infty}\frac{B_k k}{(k+1)!}$

.195686394433340358651... ≈ $\gamma^2+\frac{\pi^2}{6}-2HypPFQ[\{1,1,1\},\{2,2,2\},-1] = \int_1^{\infty}\frac{\log^2 x}{e^x}dx$

1 .195726097034987200905... ≈ $-\frac{\pi}{2}\cot\frac{\pi}{\sqrt{2}}$

.19573254621533971315... ≈ $-\frac{1}{12}Li_3(-3) = \frac{\pi^2\log 3}{72}+\frac{\log^3 3}{72}-\frac{1}{12}Li_3\left(-\frac{1}{3}\right)$

= $\int_1^{\infty}\frac{\log^2 x}{x^3+3x}dx$

.19582440721127770263.... ≈ $\sum_{k=2}^{\infty}(-1)^k\frac{\left(\zeta(k)-1\right)^2}{k}$

1 .195901614544554380728... ≈ $e^{(\cos 1)/2}\cos\left(\frac{\sin 1}{2}\right) = \sum_{k=0}^{\infty}\frac{\cos k}{k!2^k}$ GR 1.463.1

= $\frac{1}{2}\left(e^{e^i/2}+e^{e^{-i}/2}\right)$

2 .196152422706631880582... ≈ $3\left(\sqrt{3}-1\right) = \sum_{k=0}^{\infty}\frac{1}{6^k}\binom{2k}{k}$

5 .196152422706631880582... ≈ $\sqrt{27}$

.19621530110394897370.... ≈ $\frac{1}{2}-\frac{\pi}{2\sqrt{3}}\text{csch}\frac{\pi}{\sqrt{3}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{3k^2+1}$

.196218053911705438694... ≈ $\frac{9\log 3}{8}-\frac{3\log 2}{2} = \sum_{k=1}^{\infty}\frac{H_{2k}}{9^k}$

1 .19630930268377512457... ≈ $\frac{\pi}{2}\tanh 1 = \int_0^{\infty}\sin\frac{2x}{\pi}\cdot\frac{dx}{\sinh x}$

.19634954084936207740... ≈ $\frac{\pi}{16}$

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)\left((2k+1)^4+4\right)}$ K Ex. 107e

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k-1)^2}\sin\frac{2k-1}{4}$ J523

= $\sum_{k=1}^{\infty}\frac{\sin^3 k\cos k}{k} = \sum_{k=1}^{\infty}\frac{\sin^3 k\cos^2 k}{k}$

= $\int_0^{\infty}\frac{x^2dx}{(x^2+1)^3} = \int_{-\infty}^{\infty}\frac{x^2\,dx}{(x^2+4)^2}$

= $\int_0^{\infty}\frac{dx}{e^{4x}+e^{-4x}}$

1 .1964255215679256229... ≈ $\sum_{k=1}^{\infty}H_{2k}(\zeta(2k)-1)$

1 .1964612764365364055... ≈ $4G-\frac{\pi^2}{4} = \int_0^1\frac{\arcsin^2 x\,dx}{x^2}$

= $\int_0^{\pi/2}\frac{x^2\cos x}{\sin^2 x}dx$ GR 3.837.5

= $\int_0^{\infty}\frac{\arctan^3 x}{x^2\sqrt{1+x^2}}dx$ GR 4.534

4 .19650915062661921078... ≈ $\sum_{k=1}^{\infty}\frac{1}{(k!)^2}\binom{2k}{k} = HypPFQ\left[\left\{\frac{1}{2}\right\},\{1,1\},4\right]-1$

.196516308968177764468... ≈ $\frac{13\zeta(3)}{9}+\frac{2\pi^3}{27\sqrt{3}}-\left(\frac{\pi}{18\sqrt{3}}+\frac{\log 3}{6}\right)\psi^{(1)}\left(\frac{1}{3}\right)$

= $\sum_{k=1}^{\infty}\frac{H_k}{(3k+1)^2}$

.196548095270468200041... ≈ $-J_0\left(2\sqrt{2}\right)=\sum_{k=0}^{\infty}(-1)^{k+1}\frac{2^k}{(k!)^2}$

.196611933241481852537... ≈ $\frac{e}{(e+1)^2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{e^k}$

.1967948799868928... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\mu(k)}{5^k+1}$

1 .19682684120429803382... ≈ $\frac{3}{\sqrt{2\pi}}$

.1968464433591154... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k}{k(2k-1)\log k}$

.19691464526060857190... ≈ $2-\frac{3\zeta(3)}{2} = \int_0^\infty \frac{x^2}{e^x\left(e^x+1\right)}dx$

= $\int_1^\infty \frac{dx}{x^3+x^2} = \int_0^\infty \frac{x^2dx}{e^{3x}+e^{2x}}$

.19693931676727955858\!9... ≈ $\frac{\sqrt{\pi}}{9}$

.19696383943103328429\!2... ≈ $\sum_{k=2}^\infty \frac{(-1)^k}{k!\zeta(k)}$

2 .19710775308636668118... ≈ $\frac{25\cosh 1}{32}+\frac{27\sinh 1}{32} = \frac{13e}{16}-\frac{1}{32e} = \sum_{k=1}^\infty \frac{k^5}{(2k)!}$

2 .19722457733621938279... ≈ $2\log 3$

.19729351159865257571... ≈ $1-\frac{si(2)}{2} = \sum_{k=1}^\infty \frac{(-1)^{k+1}4^k}{(2k+1)!(2k+1)}$

1 .19732915450711073927... ≈ $\pi^2+8G-16 = \zeta\left(2,\frac{5}{4}\right)$

17 .19732915450711073927... ≈ $\pi^2+8G = \psi^{(1)}\left(\frac{1}{4}\right) = \sum_{k=1}^\infty \frac{1}{(k+1/4)^2}$

.19739555984988075837... ≈ $\arctan\frac{1}{5} = \sum_{k=0}^\infty \frac{(-1)^k}{5^{2k+1}(2k+1)}$

= $\sum_{k=1}^\infty \arctan\left(\frac{1}{2(k+2)^2}\right)$ [Ramanujan] Berndt Ch. 2, Eq. 7.6

.19739555984988075837\!0... ≈ $\arctan\frac{1}{5} = \sum_{k=0}^\infty \frac{(-1)^k}{5^{2k+1}(2k+1)}$

1 .19746703342411321823... ≈ $\frac{3}{8}+\frac{\pi^2}{12} = \sum_{k=1}^\infty k(\zeta(2k)+\zeta(2k+1)-2)$

.19749121692001552262... ≈ $2-\cos 1-\frac{3\sin 1}{2} = \sum_{k=1}^\infty (-1)^{k+1}\frac{k^2}{(2k)!(k+1)}$

1 .19762901264525276357\!4... ≈ $\frac{\pi}{4}\coth\frac{\pi}{4} = 1+\sum_{k=1}^\infty \frac{2}{16k^2+1}$

.19770010596096369156\!8... ≈ $\frac{1}{2}-\frac{\pi}{6\sqrt{3}} = \sum_{k=1}^\infty \frac{1}{(3k+1)(3k-1)} = \sum_{k=1}^\infty \frac{\zeta(2k)}{9^k}$

.19778622897495995348\!4... ≈ $\frac{1}{25}\left(4+3\operatorname{arccot}2-2\log 5+4\log 2\right) = \sum_{k=1}^\infty (-1)^{k+1}\frac{kH_{2k}}{4^k}$

.19779977390100482207\!4... ≈ $60-22e = \sum_{k=1}^\infty \frac{k}{k!(2k+10)}$

.19783868142621756223... ≈ $\frac{\gamma^2}{4}+\frac{\gamma\log 2}{2}+\frac{\log^2 2}{4}-\frac{\pi^2}{48} = -\int_0^\infty \frac{\log x\sin^2 x}{x}dx$

3 .19784701747655329989... ≈ $\frac{19\pi^3}{12\sqrt{3}}-27+\frac{13\zeta(3)}{8}+\frac{1}{16}\psi^{(2)}\left(\frac{7}{6}\right) = \frac{5\pi^3}{6\sqrt{3}}-\frac{39\zeta(3)}{4}$

= $\sum_{k=1}^\infty \frac{(-1)^{k+1}}{(k-1/3)^3}$

.19809855862344313982... ≈ $\sum_{k=2}^\infty \frac{1}{k^3+k^{-2}} = \sum_{k=1}^\infty (-1)^{k+1}\left(\zeta(5k-2)-1\right)$

.19812624288563685333... ≈ $-\zeta'(3) = -\sum_{k=1}^\infty \frac{\log k}{k^3}$

1 .19814023473559220744... ≈ $2\left(E(-1)-K(-1)\right) = \frac{\sqrt{\pi}}{2}\Gamma\left(\frac{3}{4}\right)\Gamma^{-1}\left(\frac{5}{4}\right)$

= $\int_0^\pi \frac{\sin^2 x}{\sqrt{1+\sin^2 x}}dx = \int_0^{\pi/2}\sqrt{\sin x}\,dx$

.198286366972179591534... ≈ $\sum_{k=1}^\infty \left(\zeta(2k)-\zeta(2k+1)\right)^2$

.198457336201944398935... ≈ $H_1(1) = \frac{2}{\pi}\sum_{k=0}^\infty \frac{(-1)^k}{((2k+1)!!)^2(2k+3)}$

.198533563970020508153... ≈ $\sum_{k=1}^\infty (-1)^{k+1}\frac{(\zeta(k+1)-1)^2}{2^k}$

41 .198612524858179308583... ≈ $\frac{3e^4+1}{4} = \sum_{k=1}^\infty \frac{4^k k}{(k+1)!}$

.198751655234615030482... ≈ $\sum_{k=0}^\infty \frac{(-1)^k}{3^k+2}$

.198773136847982861838... ≈ $-\frac{\pi}{4\sqrt{3}}\csc 2\pi\sqrt{3}-\frac{17}{66} = \sum_{k=4}^\infty \frac{(-1)^k}{k^2-12}$

.19912432950340708603... ≈ $\sum_{k=0}^\infty \frac{(-1)^k}{k!k!!}$

.199240645990717318999... ≈ $3-2\gamma-2e+2Ei(1) = -\int_0^1 e^x x^2\log x\,dx$

.199270073726951610183... ≈ $\sum_{k=1}^\infty (-1)^{k+1}\frac{\mu(k)}{k}\log\left(\zeta(2k+1)\right)$

.199376013451697954050... ≈ $\sum_{k=1}^\infty \frac{\log k}{\binom{2k}{k}}$

	.1994452414109950646 98...	≈	$\frac{40}{147}+\frac{\pi}{14}-\frac{\log 8}{7} = \sum_{k=1}^{\infty}\frac{1}{k(4k+7)}$	
	.19947114020071633897...	≈	$\frac{1}{\sqrt{8\pi}}$	
1	.19948511645445344506...	≈	$\frac{\pi^2}{6}+\log 2\log 3-\log^2 3 = Li_2\left(\frac{1}{3}\right)+Li_2\left(\frac{2}{3}\right)$	
2	.19950034058923293351 3...	≈	$\frac{2}{\sin 2} = \prod_{k=0}^{\infty}\left(2^k\tan\frac{1}{2^k}\right)^{2^k}$	Berndt ch. 31
	.19951049429709192344 5...	≈	$\frac{\pi}{4}\cos 2+\frac{1}{2}CosIntegral(2)\sin 2-\frac{1}{2}SinIntegral(1)\cos 1$	
		=	$\int_0^{\infty}\frac{dx}{e^x\left(x^2+4\right)}$	
	.19957749438845398324 1...	≈	$\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{k^3+2}$	
1	.199678640257733...	≈	root of $(x-1)e^x = (x+1)e^{-x}$	
5	.19978843376326363955 3...	≈	γ^{-3}	

.2000000000000000000$\underline{0}$ $\quad = \quad \frac{1}{5} = \sin^2 \arctan\frac{1}{7} = \sum_{k=1}^{\infty}\frac{1}{6^k} = \sum_{k=1}^{\infty}\frac{\mu(k)}{5^k-1}$

$= \int_0^{\infty}\frac{\cos 2x dx}{e^x} = \int_1^{e}\frac{\log^4 x dx}{x}$

$= \prod_{k=1}^{\infty}\frac{k(k+5)}{(k+4)(k+1)}$ J1061

.2000786289729195969 7... $\quad \approx \quad \frac{1}{6}\big(\psi(2+i)+\psi(2-i)\big) + (-1+i\sqrt{3})\psi\left(\frac{4-i-\sqrt{3}}{2}\right)$

$+ (-1-i\sqrt{3})\psi\left(\frac{4+i-\sqrt{3}}{2}\right) + (-1-i\sqrt{3})\psi\left(\frac{4-i+\sqrt{3}}{2}\right)$

$+ (-1+i\sqrt{3})\psi\left(\frac{4+i+\sqrt{3}}{2}\right)$

$= \sum_{k=2}^{\infty}\frac{k^3}{k^6+1} \;=\; \sum_{k=1}^{\infty}(-1)^{k+1}(\zeta(6k-3)-1)$

.2000937253541084694... $\quad \approx \quad \frac{\pi}{\sqrt{3}} + 2\log 2 - 3 \;=\; \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{3k^2+k}$

$= \int_0^1 \log(1+x^3)\,dx \;=\; \int_1^{\infty}\log\left(1+\frac{1}{x^3}\right)\frac{dx}{x^2}$

3 .2000937253541084694... $\quad \approx \quad \frac{\pi}{\sqrt{3}} + 2\log 2 = \psi\left(\frac{1}{3}\right) - \psi\left(\frac{1}{6}\right) \;=\; \int_0^1 \log\left(1+\frac{1}{x^3}\right)dx$

.2002462199990232558... $\quad \approx \quad 2-2\sqrt{2}\log(1+\sqrt{2}) + \log 2 \;=\; \sum_{k=1}^{\infty}\frac{1}{2^k(2k+1)k}$

$= 2 - 2\sqrt{2}\operatorname{arcsinh}1 + \log 2$

.2003428171932657142... $\quad \approx \quad \frac{\zeta(3)}{6}$

1 .2003838570989443866 5... $\quad \approx \quad 2\pi^2 - 2\log 3\log\frac{729}{64} \;+\; 8Li_2\left(-\frac{1}{2}\right) - 12Li_2\left(\frac{2}{3}\right)$

$= \sum_{k=3}^{\infty}\frac{(-1)^{k+1}}{StirlingS1(k,3)}$

1 .2004217548761414261... $\quad \approx \quad \binom{1}{1/4} \;=\; \binom{1}{3/4} \;=\; \frac{1}{(1/4)!(3/4)!}$

.2006138946204895637... $\quad \approx \quad \frac{1-\log 2}{2} + \frac{1+i}{8} + \left(i\psi\left(1+\frac{i}{2}\right) - \psi\left(1-\frac{i}{2}\right) + \psi\left(\frac{3}{2}-\frac{i}{2}\right) - i\psi\left(\frac{3}{2}+\frac{i}{2}\right)\right)$

$$= \frac{1}{8}\left(\psi\left(\frac{1}{2}+\frac{i}{2}\right)+\psi\left(\frac{1}{2}-\frac{i}{2}\right)-\psi\left(1-\frac{i}{2}\right)-\psi\left(1+\frac{i}{2}\right)+\pi\tanh\frac{\pi}{2}\right)$$

$$+\frac{3}{4}-\frac{\log 2}{2}-\frac{\pi}{8}\coth\frac{\pi}{2}$$

$$= \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^3+k^2+k+1}$$

$.2006514648222101678\ldots \approx \mathrm{Ei}(-e)-\mathrm{Ei}(-1) = \int_0^1 e^{-e^x}dx$

$.20122400855211050189 7\ldots \approx \sin 2-\sin^2 1 = \sum_{k=0}^{\infty}\frac{(-1)^k 4^k}{(2k)!(k+1)}$

6 $.20125533605996403509 5\ldots \approx \frac{\pi^3}{5}$

$.2013679876336687216\ldots \approx \frac{9-e^2}{8} = \sum_{k=0}^{\infty}\frac{2^k k}{(k+3)!}$

$.2014368968853534813 2\ldots \approx \sum_{k=2}^{\infty}(-1)^k\frac{(\zeta(k)-1)^2}{k!}$

$.2015139603997051899\ldots \approx \frac{1}{6}+\frac{\sqrt{6}}{3}\operatorname{csch}\pi\sqrt{\frac{3}{2}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{2k^2+3}$

$.20165752411061940806 7\ldots \approx \frac{\pi}{2}-\sqrt{\pi} = \int_0^{\infty}\frac{\cos x-e^{-x^2}}{x^2}dx$

1 $.2017266059898424532 6\ldots \approx \sum_{k=1}^{\infty}\frac{1}{3^k-2}$

$.20181127608389805796 4\ldots \approx \frac{\pi^2}{2}-3\log^2 3+3\log 2\log 3+2Li_2\left(-\frac{1}{2}\right)-3Li_2\left(\frac{2}{3}\right)$

$$= \sum_{k=1}^{\infty}\frac{(-1)^{k+1}H_k}{2^k k(k+1)}$$

$.2018304381180897838 2\ldots \approx \frac{e}{8}-\frac{3}{8e} = \sum_{k=1}^{\infty}\frac{k^2}{(2k+1)!}$

$.20194822765801286993 5\ldots \approx \frac{e\sin 1}{1+e^2+2e\cos 1} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\sin k}{e^k}$

$.2019690652581689489 3\ldots \approx 16\gamma-4\zeta(3)+\zeta(5)+8\left(\psi\left(1+\frac{i}{2}\right)+\psi\left(1-\frac{i}{2}\right)\right)$

$$= \sum_{k=1}^{\infty}\frac{1}{4k^7+k^5} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k+5)}{4^k}$$

1 $.2020224917616113171 5\ldots \approx \frac{1}{2G-1}$

.2020569031595942854... ≈ $\zeta(3)-1 = \sum_{k=2}^{\infty}\frac{1}{k^3} = \frac{1}{2}\int_0^1 \frac{x\log^2 x}{1-x}dx$ GR 4.26.12

= $\int_0^\infty \frac{6x-2x^3}{(1+x^2)^3}\frac{dx}{e^{2\pi x}-1}$ Henrici, p. 274

= $\int_0^1\int_0^1\int_0^1 \frac{xyz}{1-xyz}dx\,dy\,dz$

1 .2020569031595942854... ≈ $\zeta(3) = \sum_{k=1}^{\infty}\frac{1}{k^3} = \sum_{k=1}^{\infty}\frac{3k^2+3k+1}{k^2(k+1)^3}$

= $MHS(2,1)$

= $\sum_{k=1}^{\infty}\frac{H_k}{(k+1)^2}$ Berndt 9.9.4

= $\sum_{k=1}^{\infty}\frac{H_k(2k+1)}{k^2(k+1)^2}$

= $\sum_{k=1}^{\infty}\frac{H^{(2)}{}_k}{k(k+1)}$

= $\frac{5}{2}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{\binom{2k}{k}k^3} = \frac{5}{2}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}(k!)^2}{(2k)!k^3}$ Croft/Guy F17

Originally Apéry, Asterisque, Soc. Math. de France 61 (1979) 11-13

= $\sum(-1)^{k+1}\frac{(k!)^{10}(205k^2+250k+77)}{64((2k+1)!)^5}$ Elec. J. Comb. 4(2) (1997)

= $\frac{7\pi^3}{180}-2\sum_{k=1}^{\infty}\frac{1}{k^3\left(e^{2\pi k}-1\right)} = \frac{7\pi^3}{180}-2\sum_{k=1}^{\infty}\frac{\sigma_{-3}(k)}{e^{2\pi k}}$ Grosswald

Nachr. der Akad. Wiss. Göttingen, Math. Phys. Kl. II (1970) 9-13

= $\frac{2\pi^3}{45}-4\sum_{k=1}^{\infty}\frac{\sigma_{-3}(k)}{e^{2\pi k}}\left(2\pi^2k^2+\pi k+\frac{1}{2}\right)$ Terras

Acta Arith. XXIX (1976) 181-189

= $\frac{4\pi^2}{7}\sum_{k=0}^{\infty}\frac{\zeta(2k)}{4^k(2k+2)} + \frac{2\pi^2}{7}\log 2 - \frac{\pi^2}{7}$ Yue 1993

= $\frac{4\pi^2}{9}\sum_{k=0}^{\infty}\frac{\zeta(2k)}{4^k(2k+3)} + \frac{2\pi^2}{9}\log 2 - \frac{2\pi^2}{27}$ Yue 1993

= $-2\pi^2\sum_{k=0}^{\infty}\frac{\zeta(2k)}{4^k\left(2k+2)(2k+3\right)}$ Yue 1993

$$= -\frac{4\pi^2}{7}\sum_{k=0}^{\infty}\frac{\zeta(2k)}{4^k(2k+1)(2k+2)}$$

Ewell, AMM 97 (1990) 209-210

$$= -\frac{8\pi^2}{9}\sum_{k=0}^{\infty}\frac{\zeta(2k)}{4^k(2k+1)(2k+3)}$$

Yue 1993

$$= -\frac{\pi^2}{3}\sum_{k=0}^{\infty}\frac{(2k+5)\zeta(2k)}{4^k(2k+1)(2k+2)(2k+3)}$$

Cvijović 1997

$$= \frac{4\pi^2}{3}\left(1-3\log\frac{3}{2}\right)+\frac{\pi^2}{3}\sum_{k=0}^{\infty}\frac{(2k+5)(1-\zeta(2k))}{4^k(2k+1)(2k+2)(2k+3)}$$

Cvijović 1997

$$= \frac{\pi^2}{14}\left(\frac{3}{2}+\log\frac{4}{\pi}-4\sum_{k=0}^{\infty}\frac{(2^{2k-1}-1)\zeta(2k)}{16^k k(2k+1)(2k+2)}\right)$$

Ewell, Rocky Mtn. J. Math. 25, 3(1995) 1003-1012

$$= \zeta(6)\prod_{p\ prime}\left(1+p^{-4}\right)$$

$$= \zeta(4)\prod_{p\ prime}\frac{1+p^{-1}+p^{-2}+p^{-3}}{1+p^{-1}+p^{-2}}$$

$$= \frac{5}{4}HypPFQ[\{1,1,1,1\},\{\frac{3}{2},2,2\},-\frac{1}{4}]$$

$$= \frac{1}{2}\int_0^1\frac{\log^2 x}{1-x}dx$$

GR 4.26.12

$$= \int_1^{\infty}\frac{\log^2 x}{x^2-x}dx \;=\; -\int_0^{\infty}x\log\left(1-e^x\right)dx$$

$$= \int_0^1\frac{\log^2(1-x)}{x}dx$$

$$= \int_0^1\log(1-x)\log x\frac{dx}{x}$$

GR 4.315.3

$$= \frac{4}{3}\int_0^{\pi/4}\log(\cos 2x)\tan x\,dx$$

GR 4.391.4

$$= \int_0^{\pi/4}\frac{\sec^2 x\ (\log\tan x)^2}{1-\tan x}dx$$

$$= -\frac{16}{3}\int_0^1\arctan x\ \log x(\log x+2)dx$$

GR 4.594

$$= \frac{1}{2}\int_0^{\infty}\frac{x^2dx}{e^x-1} \;=\; \frac{1}{2}\int_{-\infty}^{0}\frac{x^2e^x}{1-e^x}dx$$

$$= \int_0^\infty \frac{x^5 dx}{e^{x^2}-1}$$

$$= \frac{2}{3}\int_{-\infty}^\infty \frac{e^{-3x}}{e^{e^{-x}}+1}dx$$ GR 3.333.2

$$= \frac{1}{2}\int_{-\infty}^\infty \frac{e^{-3x}}{e^{e^{-x}}-1}dx$$ GR 3.333.1

$$= \int_{-\infty}^0 Li_2\left(e^x\right)dx$$

2 .2020569031595942854... ≈ $\zeta(3)+1 \ = \sum_{k=2}^\infty k(\zeta(k)-\zeta(k+2)) \ = \sum_{k=2}^\infty \frac{2k^2+k-1}{k^3(k-1)}$

.20205881140141513388... ≈ $\sum_{k=2}^\infty \frac{1}{k^3-k^{-13}} = \sum_{k=1}^\infty (\zeta(16k-13)-1)$

.20206072056927833979... ≈ $\sum_{k=2}^\infty \frac{1}{k^3-k^{-14}} = \sum_{k=1}^\infty (\zeta(15k-12)-1)$

.2020626180169407781... ≈ $\sum_{k=2}^\infty (-1)^k \frac{\log\zeta(k)}{k}$

.2020645408229235151... ≈ $\sum_{k=2}^\infty \frac{1}{k^3-k^{-11}} = \sum_{k=1}^\infty (\zeta(14k-11)-1)$

.20207218728189006223... ≈ $\sum_{k=2}^\infty \frac{1}{k^3-k^{-10}} = \sum_{k=1}^\infty (\zeta(13k-10)-1)$

.20208749884843252958... ≈ $\sum_{k=2}^\infty \frac{1}{k^3-k^{-9}} = \sum_{k=1}^\infty (\zeta(12k-9)-1)$

.20211818111271553567... ≈ $\sum_{k=2}^\infty \frac{1}{k^3-k^{-8}} = \sum_{k=1}^\infty (\zeta(11k-8)-1)$

.20217973584362803956... ≈ $\sum_{k=2}^\infty \frac{1}{k^3-k^{-7}} = \sum_{k=1}^\infty (\zeta(10k-7)-1)$

.20218183904523934454... ≈ $\sum_{k=2}^\infty (-1)^k \frac{2^k\zeta(k)}{k!} \ = \ \sum_{k=1}^\infty \left(e^{-2k}-1-\frac{2}{k}\right)$

.20230346757903910324... ≈ $\sum_{k=2}^\infty \frac{1}{k^3-k^{-6}} = \sum_{k=1}^\infty (\zeta(9k-6)-1)$

.2025530074563600768... ≈ $\frac{\gamma}{4}-\frac{7}{16}+\frac{i}{4\sqrt{2}}+\frac{i\pi}{8}\left(\cot\left((-1)^{1/4}\pi\right)-\cot\left((-1)^{3/4}\pi\right)\right)$

$$+\frac{1}{8}\left(\psi(2-i)+\psi(2+i)\right)+\frac{i}{4}\left(\psi\left((-1)^{1/4}\right)-\psi\left((-1)^{3/4}\right)\right)$$

$$= \sum_{k=2}^\infty \frac{1}{k^3-k^{-5}} \ = \ \sum_{k=1}^\infty (\zeta(8k-5)-1)$$

.2026055828608337918... ≈ $\Phi\left(\frac{1}{5},4,0\right) = Li_4\left(\frac{1}{5}\right) = \sum_{k=1}^{\infty}\frac{1}{5^k k^4}$

.20261336470055239403... ≈ $\frac{3\cos 1}{8} = \sum_{k=1}^{\infty}(-1)^k\frac{k^3}{(2k)!}$

.20264236728467554289... ≈ $\frac{2}{\pi^2} = -\sum_{k=1}^{\infty}\frac{\mu(2k)}{(2k)^2}$

$= -\int_0^1 x\cos\pi x\,dx = -\int_0^1 x^2\cos\pi x\,dx$

$= \int_0^{\infty}\frac{dx}{e^{\pi x^{1/2}}}$

.20273255405408219099... ≈ $\frac{\log 3-\log 2}{2} = \operatorname{arctanh}\frac{1}{5}$ AS 4.5.64, J941, K148

$= \sum_{k=0}^{\infty}\frac{1}{5^{2k+1}(2k+1)} = \frac{1}{2}\sum_{k=1}^{\infty}\frac{1}{3^k k} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^{k+1}k}$

$= \sum_{k=1}^{\infty}\frac{H_{k-1}}{3^k} = \sum_{n=1}^{\infty}\left(-\frac{1}{2}+\sum_{k=1}^{\infty}\frac{H^{(n)}{}_k}{3^k}\right)$

$= \int_1^{\infty}\frac{\log x\,dx}{(2x+1)^2}$

$= \int_0^1 \frac{\psi(x)\sin\pi x\sin 5\pi x}{x}dx$ GR 1.513.7

.20278149888418782515... ≈ $-\frac{i}{4}\left(\psi\left(2-e^{-1/2}\right)+\psi\left(2+e^{-1/2}\right)-\psi\left(2-e^{1/2}\right)-\psi\left(2+e^{1/2}\right)\right)$

$= \sum_{k=1}^{\infty}\left(\zeta(2k+1)-1\right)\sin k$

.2030591755625688533... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^3-k^{-4}} = \sum_{k=1}^{\infty}(\zeta(7k-4)-1)$

.2030714163944594964... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(6k-3)-1}{k} = -\sum_{k=2}^{\infty}k^3\log(1-k^{-6})$

3 .203171468376931... ≈ root of $\psi(x)=1$

2 .2032805943696585702... ≈ $\tan(\log\pi) = \frac{i\pi^{-i}-i\pi^{i}}{\pi^{i}+\pi^{-i}}$

.2032809514312953715... ≈ $\log\Gamma\left(\frac{3}{4}\right)$

.2034004007029468657... ≈ $1-\gamma+Ei(-1) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!(k+1)^2} = \sum_{k=2}^{\infty}\frac{(-1)^{k+1}}{(k+1)!-k!}$

.2034098247917199633... ≈ $\zeta(2)-3\zeta(3)+2\zeta(4)$

.2034498410585544626... ≈ $\frac{\pi\sqrt{3}-3}{12} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+2)(3k+2)}$

.20351594808527823401... ≈ $\sum_{k=1}^{\infty}\log\zeta(3k)$

.2037037037037037$\underline{037}$ = $\frac{11}{54} = \sum_{k=1}^{\infty}\frac{1}{k(3k+9)}$

.20378322349200082864... ≈ $\zeta(2)\ -\ 2\zeta(3)\ +\ \frac{26}{27} = \sum_{k=2}^{\infty}\frac{k}{(k+2)^3}$

2 .20385659643... ≈ $\prod_{p\ prime}\left(1+\frac{p}{\left(p^2-1\right)\left(p-1\right)}\right) = \frac{\pi^2}{6}\prod_{p\ prime1}\left(1+\frac{1}{p^2(p-1)}\right)$

.20387066303430163678... ≈ $1024-128\pi-\frac{32\pi^2}{3}-\frac{2\pi^4}{45}+768\log 2+16\zeta(3)+\zeta(5)$

= $\sum_{k=1}^{\infty}\frac{1}{4k^6+k^5} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(k+5)}{4^k}$

61 .20393695705629065255... ≈ $\frac{\pi^5}{5}$

.2040552253015036112... ≈ $\frac{\pi}{6}\left(\pi+3\sqrt{5\left(5+2\sqrt{5}\right)}\right)+\frac{5}{4}\left(5\log 5-20+\sqrt{5}\log\frac{3+\sqrt{5}}{2}\right)$

= $\sum_{k=1}^{\infty}\frac{1}{5k^3+k^2} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(k+2)}{5^k}$

.2040955659954144905... ≈ $\zeta(4)-4\zeta(2)+4\pi\coth\frac{\pi}{2}-8$

= $\sum_{k=1}^{\infty}\frac{1}{4k^6+k^4} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k+4)}{4^k}$

.20409636872394546218... ≈ $\frac{\gamma}{3}-\frac{1}{12}+\frac{1}{6}\left(\psi\left(\frac{3+i\sqrt{3}}{2}\right)+\psi\left(\frac{3-i\sqrt{3}}{2}\right)\right)$

= $\sum_{k=2}^{\infty}\frac{1}{k^3-k^{-3}} = \sum_{k=1}^{\infty}(\zeta(6k-3)-1)$

.204137464513048262896... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(5k-2)-1}{k} = -\sum_{k=2}^{\infty}k^2\log(1-k^{-5})$

.20420112391462942986... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(4k-1)-1}{k^2} = \sum_{k=2}^{\infty}k\,Li_2\left(\frac{1}{k^4}\right)$

.20438826329796590155... ≈ $\sum_{k=1}^{\infty}\frac{H^{(3)}{}_k}{6^k}$

2 .204389986991009810573... ≈ $\log\left(\frac{2\pi}{\log 2}\right)$ [Ramanujan] Berndt ch. 22

1 .20450727367468051428... ≈ $\frac{e\sqrt{\pi}}{4} = \int_0^\infty x^2 e^{1-x^2}\, dx$

5 .204869916193387826218... ≈ $\sqrt{2}\sinh \pi\sqrt{2}\ \csc h\pi = \prod_{k=0}^\infty \left(\frac{k^2+2}{k^2+1}\right)$

.20487993766538552923... ≈ $2\log\left(1+\sqrt{2}\right) + 2\sqrt{2} - 2\log 2 - 3 = \sum_{k=1}^\infty \binom{2k}{k} \frac{(-1)^{k+1}}{4^k k(k+1)}$

.205074501291913924132... ≈ $4\log 2 - \log\pi + \gamma - 2 = \sum_{k=1}^\infty \frac{k}{2^k (k+1)}\left(\zeta(k+1)-1\right)$

.205324195733310319... ≈ $\Phi\left(\frac{1}{5},3,0\right) = Li_3\left(\frac{1}{5}\right) = \sum_{k=1}^\infty \frac{1}{5^k k^3}$

.205616758356028305... ≈ $\frac{\pi^2}{48} = \frac{\zeta(2)}{8} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{4k^2} = -\sum_{k=1}^\infty \frac{\cos\frac{\pi k}{2}}{k^2}$

= $-\frac{1}{2}\left(Li_2(i) + Li_2(-i)\right)$

= $\sum_{k=1}^\infty \frac{H_k}{k(k+1)}$

= $\int_1^\infty \frac{\log x\, dx}{x^3+x}$

= $-\int_0^1 \frac{x\log x\, dx}{x^2+1}$

= $\int_1^\infty \log\left(1+\frac{1}{x^4}\right)\frac{dx}{x}$

.20569925553652392085... ≈ $\sum_{k=1}^\infty \frac{(-1)^{k+1}}{2^k+1}$

1 .2057408843136237278... ≈ $\prod_{k=1}^\infty \left(1+\frac{1}{6^k}\right)$

.2057996486783263402... ≈ $\frac{7\pi^3}{180} = \sum_{k=1}^\infty \frac{\coth k\pi}{k^3}$ Ramanujan

.205904604818110652966... ≈ $\frac{\left(\sqrt{3}-1\right)}{2^{5/3}}\Gamma\left(\frac{4}{3}\right) = \int_0^\infty e^{-x^3} \sin x^3\, dx$

.20602208432520564821... ≈ $-\frac{2}{3} Li_2\left(-\frac{1}{3}\right) = \int_0^1 \frac{\log^2 x}{(x+3)^2}\, dx$

.2061001222524821839... $\approx \frac{\pi^2}{36} - \frac{49}{720} \; = \; \sum_{k=1}^{\infty} \frac{1}{k^2(k+6)}$

.2061649072311420998... $\approx \sum_{k=2}^{\infty} \zeta(k^2-1) - 1$

.206183195886038352118... $\approx \frac{1}{5\sqrt{5}} \left(6\, arc \coth \frac{13}{\sqrt{5}} + \log \frac{21+8\sqrt{5}}{11} \right) = \sum_{k=1}^{\infty} \frac{F_k F_k F_k}{6^k k}$

.206260493108447712133... $\approx \frac{i}{8} \left(\psi^{(1)}\left(1+\frac{i}{2}\right) - \psi^{(1)}\left(1-\frac{i}{2}\right) \right) \; = \; \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k}{4^k} \zeta(2k+1)$

.2062609130638129393... $\approx \sum_{k=2}^{\infty} \frac{1}{k^3 - k^{-2}} \; = \; \sum_{k=1}^{\infty} (\zeta(5k-2) - 1) \; = \; -\operatorname{Re}\left\{ \operatorname{Li}_2(i) \right\}$

.20627022192421495449... $\approx \sum_{k=1}^{\infty} \frac{\zeta(3k+2)}{6^k} \; = \; \sum_{k=1}^{\infty} \frac{1}{6k^5 - k^2}$

.2063431723054151902... $\approx 2 - 2\log^2 2 + 4 \log 2 \log 3 - 2\log^2 3 - 4 \operatorname{Li}_2\left(\frac{1}{3}\right)$

$= \sum_{k=0}^{\infty} \frac{(-1)^k}{2^k (k+2)^2}$

.20640000777321563927... $\approx \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\zeta(3k)}{5^k} \; = \; \sum_{k=1}^{\infty} \frac{1}{5k^3+1}$

.20640432108289179934... $\approx \sum_{k=1}^{\infty} \frac{\zeta(4k-1)-1}{k} \; = \; -\sum_{k=2}^{\infty} k \log(1-k^{-4})$

.2064267754028415969... $\approx \frac{\pi\sqrt{3}}{6} \coth \frac{\pi}{\sqrt{3}} - \frac{3}{4} \; = \; \sum_{k=2}^{\infty} \frac{1}{3k^2+1}$

$= \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\zeta(2k)-1}{3^k}$

.2064919501357334736... $\approx \sum_{k=1}^{\infty} H_k \left(\zeta(k+3) - 1\right) \; = \; \sum_{k=2}^{\infty} \frac{1}{k^3 - k^2} \log \frac{k}{k-1}$

1 .206541445572400796162... $\approx \frac{1}{2} \log \pi + \frac{3}{2}(1-\gamma) \; = \; \sum_{k=2}^{\infty} \frac{\zeta(k)-1}{k} \left(\frac{3}{2}\right)^k$ Srivastava

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.2066325441561950286... $\approx \sum_{k=1}^{\infty} \frac{\zeta(3k)-1}{k^2} \; = \; \sum_{k=2}^{\infty} Li_2\left(\frac{1}{k^3}\right)$

.2070019212239866979... $\approx \frac{I_0(-4)}{e^4} \; = \; \sum_{k=0}^{\infty} (-1)^k \binom{2k}{k} \frac{2^k}{k!}$

1 .20702166335531798225... $\approx 1 - e + erfi(1)\sqrt{\pi} = \sum_{k=1}^{\infty} \frac{1}{k!(2k-1)}$

$= \int_0^1 \frac{e^{x^2} - 1}{x^2} dx$ GR 3.466.3

5 .2070620835894383024... ≈ $\frac{e^e - 1}{e} = \sum_{k=0}^{\infty} \frac{e^k}{(k+1)!}$

3 .2071862410262113746... ≈ $\frac{\zeta(3)}{2} + \frac{25\pi^2}{288} + \frac{3023}{1728} = \sum_{k=1}^{\infty} \frac{H_k H_{k+4}}{k(k+4)}$

4 .2071991610585799989... ≈ $\frac{7\zeta(3)}{2} = \int_0^{\infty} \frac{x^2 dx}{\sinh x} = -\int_{-1}^{0} \frac{\log^2(1+x)}{(1+x)\sinh(\log(1+x))} dx$

1 .207248417124939789... ≈ $-\frac{1}{4}\left(\sqrt{\frac{\pi}{e}}\, erfi(\sqrt{e}) - \sqrt{e\pi}\, erfi\left(\frac{1}{\sqrt{e}}\right)\right) = \sum_{k=0}^{\infty} \frac{\sinh k}{k!(2k+1)}$

.2073855510286739853... ≈ $\frac{\zeta(5)}{5}$

.20747371684548153956... ≈ $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{\zeta(2k) - 1}{k(2k+1)}$

.20749259869231265718... ≈ $\frac{\pi}{10} - \frac{8}{75} = \int_0^1 x^4 \arcsin x dx$ GR 4.523.1

.207546365655439172084... ≈ $\frac{1}{2}\left(\log 2\pi + \gamma - 2\right) = \sum_{k=2}^{\infty} (-1)^k \frac{\zeta(k)}{k(k+1)}$

2 .2077177285358922614... ≈ $\sum_{k=2}^{\infty} (\zeta^3(k) - \zeta^2(k))$

.2078795763507619085... ≈ $i^i = e^{-\pi/2}$ (principal value) $= e^{i \log i}$

= $\cos(\log i) + i \sin(\log i) = \cosh\left(\frac{\pi}{2}\right) - \sinh\left(\frac{\pi}{2}\right)$

.20788622497735456602... ≈ $-\zeta\left(-\frac{1}{2}\right)$

3 .207897324931068989... ≈ $\frac{e^e + e^{2+1/e} - e^2 - 1}{2e} = \sum_{k=0}^{\infty} \frac{\cosh k}{(k+1)!}$

1 .208150778890... ≈ $\sum_{k=2}^{\infty} \frac{1}{\phi^4(k)}$

.2082601377261734183... ≈ $\frac{3\pi}{32\sqrt{2}} = \int_{-\infty}^{\infty} \frac{dx}{(x^2+2)^3} = \int_0^{\pi/2} \sqrt{\sin^5 x \cos^3 x}\, dx$

.2082642730272670724... ≈ $32\pi + \frac{8\pi^2}{3} + \zeta(4) + 192 \log 2 - 4\zeta(3) - 256$

= $\sum_{k=1}^{\infty} \frac{1}{4k^5 + k^4} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\zeta(k+4)}{4^k}$

$.2083333333333333333\underline{3}$ = $\frac{5}{24} = \int_1^{\infty} \frac{\log(1+x)}{x^5} dx$

.2084273952291510642... ≈ $e(Ei(-2)-Ei(-1))-\frac{\log 2}{e} = \int_0^1 \frac{\log(1+x)}{e^x}dx$

.20853542049582140151... ≈ $-\frac{1}{8}Li_3(-2) = \int_1^\infty \frac{\log^2 x}{x^3+2x}dx$

.2087121525220799967... ≈ $\frac{5-\sqrt{21}}{2} = \sum_{k=1}^\infty \frac{1}{7^k(k+1)}\binom{2k}{k}$

= $1+\sum_{k=0}^\infty \frac{(-1)^{k+1}}{3^k(k+1)}\binom{2k}{k}$

1 .2087325662825996745... ≈ $e^{1+1/e}-e = \sum_{k=0}^\infty \frac{1}{(k+1)!e^k}$

.20873967247130024435... ≈ $\zeta(3)-4\gamma-2\psi\left(1+\frac{i}{2}\right)-2\psi\left(1-\frac{i}{2}\right) = \mathrm{Re}\left\{\sum_{k=1}^\infty \frac{\zeta(k+3)}{(2i)^k}\right\}$

= $\sum_{k=1}^\infty \frac{1}{4k^5+k^3} = \sum_{k=1}^\infty (-1)^{k+1}\frac{\zeta(2k+3)}{4^k}$

6 .208758035711110196... ≈ $\pi\left(I_0(1)+StruveL_0(1)\right) = \int_0^\pi e^{\sin x}dx$

.2087613945440038371... ≈ $\frac{\pi^2}{12}+2\log 2-2 = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{k(k+1)^2}$ J365

= $\sum_{k=2}^\infty \frac{(-1)^k}{k^3-k^2} = -\mathrm{Re}\left\{\sum_{k=1}^\infty \frac{\zeta(k+3)}{(2i)^k}\right\}$

= $-\int_0^1 \log x\log(1+x)dx$ GR 4.221.2

1 .2087613945440038371... ≈ $\frac{\pi^2}{12}+2\log 2-1 = \int_0^1 \frac{(1+x)\log(1+x)}{x}dx$

2 .2087613945440038371... ≈ $\frac{\pi^2}{12}+2\log 2 = \sum_{k=1}^\infty \frac{H_{2k}}{k(k+1)}$

= $-\int_0^1 \log\left(1+\frac{1}{x}\right)\log x\,dx$

= $\int_1^\infty \frac{\log(1+x)\log x}{x^2}dx$

1 .20888446534786304666... ≈ $\sum_{k=2}^\infty (\zeta(k)-1)^{k-2}$

.208891435466871385503... ≈ $-(1-\gamma)\cos 1-\frac{1}{2}\left(\log\Gamma\left(2-e^{-i}\right)+\log\Gamma\left(2-e^{i}\right)\right)$

= $-\sum_{k=2}^\infty \frac{\cos k}{k}\left(\zeta(k)-1\right)$

.20901547528998000945... ≈ $\frac{6}{5}Li_2\left(\frac{1}{6}\right)-\frac{1}{5} = \frac{1}{5}\sum_{k=1}^{\infty}\frac{1}{6^k(k+1)^2} = \sum_{k=1}^{\infty}\frac{H^{(2)}{}_k}{6^k}$

.2091083022560555467... ≈ $\sum_{k=1}^{\infty}(-1)^k\frac{\zeta(4k+1)}{4^k} = \sum_{k=1}^{\infty}\frac{1}{4k^5+k}$

$= \gamma-\frac{1}{2}+\frac{1}{2}\left(\psi\left(\frac{3-i}{2}\right)+\psi\left(\frac{3+i}{2}\right)\right)$

.2091146336814109663... ≈ $\frac{\pi\sqrt{2}}{8}\tanh\frac{\pi}{\sqrt{2}}-\frac{1}{3} = \sum_{k=1}^{\infty}\frac{1}{4k^2+4k+3}$

.2091867376129094839... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k+1)}{(2k+1)!} = \sum_{k=1}^{\infty}\left(\sinh\frac{1}{k}-\frac{1}{k}\right)$

.2091995761561452337... ≈ $\frac{2\pi}{3\sqrt{3}}-1 = \int_0^{\pi/2}\frac{(1-\sin x)^2}{2-\sin x}dx$

1 .2091995761561452337... ≈ $\frac{2\pi}{3\sqrt{3}} = \sum_{k=0}^{\infty}\frac{k!}{(2k+1)!!2^k} = \sum_{k=0}^{\infty}\frac{(k!)^2}{(2k+1)!}$ CFG D11, J261, J265

$= \sum_{k=0}^{\infty}\frac{1}{\binom{2k}{k}(2k+1)}$

$= \prod_{k=1}^{\infty}\frac{9k^2}{(3k-1)(3k+1)}$

$= \int_0^{\infty}\frac{dx}{x^3+1} = \int_0^{\infty}\frac{dx}{e^x+e^{-x}-1} = \int_0^{\pi}\frac{dx}{2+\sin x}$

$= \int_0^{\infty}\frac{x\,dx}{x^3+1}$ GR 2.145.3

$= \int_0^{\infty}\frac{dx}{x^2+x+1} = \int_1^{\infty}\frac{dx}{x^2-x+1}$

.20920040041194212307 3...≈

$$\frac{1}{5+5\sqrt{5}}\left(2\sqrt{5}\arctan h\frac{13+4\sqrt{5}}{89}+\sqrt{5}\log\frac{4}{3}+\log\frac{14311+4955\sqrt{5}}{5046}\right)$$

$= \sum_{k=2}^{\infty}(\zeta(k)-1)^{k-2}$

1 .2093019812040210 8873... ≈ $\frac{81}{2}-\frac{9\pi\sqrt{3}}{2}-\frac{3\pi^2}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k^2-1/9)^2}$

1 .2093450108726071602 8... = $\frac{\pi^2\log 2}{4\sqrt{2}} = -\int_0^{\infty}\frac{\log^2 x\,dx}{2x^2-1}$

$.20943951023931954923... \approx \frac{\pi}{15} = \int \frac{(\sin x - x\cos x)^2}{x^6}$ Prud. 2.5.29.24

$.20947938452318754957... \approx \log 2 - \frac{1}{2} + \sum_{k=2}^{\infty} \frac{\Omega(k)}{2^k k} = \sum_{k=2}^{\infty}\sum_{j=1}^{\infty} \frac{1}{k^j 2^{k^j}}$

$.209523809523\underline{8095238} = \frac{22}{105} = \sum_{k=0}^{\infty} \frac{1}{(2k+1)(2k+9)} = \sum_{k=0}^{\infty} \frac{1}{4^k (k+2)(k+4)} \binom{2k}{k}$

$.20965704024101087349... \approx \log \frac{(\pi-1)\csc\sqrt{\pi}}{\sqrt{\pi}} = \sum_{k=1}^{\infty} \frac{\zeta(2k)-1}{\pi^k k}$

$.2099125364431486880... \approx \frac{72}{343} = \Phi\left(\frac{1}{8}, -2, 0\right) = Li_{-2}\left(\frac{1}{8}\right) = \sum_{k=1}^{\infty} \frac{k^2}{8^k}$

$.2099703838980124359... \approx \sum_{k=1}^{\infty} (-1)^k \frac{\log k}{k!}$

2 $.2100595293751996419... \approx \frac{10\pi^3}{81\sqrt{3}} = \int_0^1 \frac{\log^2 x dx}{x^2 - x + 1} = \int_0^{\infty} \frac{\log^2 x dx}{x^3 + 1} = \int_0^{\infty} \frac{x \log^2 x dx}{x^3 + 1}$ GR 4.261.2

$= \int_0^{\infty} \frac{\log^2 x}{x^2 + x^{-1}} dx$

1 $.2100728623371011609... \approx \frac{\pi^2\sqrt{2}}{8} - \frac{1}{8}\Phi\left(\frac{1}{2}, 2, \frac{1}{2}\right) = \frac{13\pi^2}{48\sqrt{2}} - \frac{\log^2 2}{8\sqrt{2}} - \frac{1}{\sqrt{2}} Li_2\left(\frac{1}{\sqrt{2}}\right)$

$= \int_0^1 \frac{\log x dx}{2x^2 - 1}$

$.21007463698428630555... \approx \zeta(3) \sum_{k=1}^{\infty} \frac{\mu(k)}{k} \log \zeta(3k) = \sum_{k=1}^{\infty} \frac{\nu(k)}{k^3}$ Titchmarsh 1.6.2

$.21010491829804817083... \approx \frac{4}{\sqrt{\pi}}\left(1 - \frac{\pi\sqrt{3}}{6}\right) = \sum_{k=1}^{\infty} \frac{(k-1)!}{(k+\frac{1}{2})! 4^k}$ Dingle p. 70

$.2103026500837349292... \approx \sum_{k=1}^{\infty} \frac{1}{(5k)^k}$

$.2103677462019741266... \approx \frac{\sin 1}{4} = \sum_{k=0}^{\infty} \frac{(-1)^k}{4(2k+1)!} = \sum_{k=1}^{\infty} \frac{(-1)^k k^2}{(2k-1)!(2k+1)}$

2 $.2104061805415171122... \approx \zeta(3) + \zeta(7)$

$.\underline{210526315789473684} = \frac{4}{19}$

$.2105684264267640696... \approx 6(\log 3 - \log 2) - \frac{20}{9} = \sum_{k=0}^{\infty} \frac{(-1)^k}{2^k (k+1)(k+4)}$

8 $.2105966657212352544... \approx \frac{\pi^2}{\zeta(3)}$

.21065725122580698811... ≈ $\frac{\pi}{12}+\frac{\log 2}{6}-\frac{1}{6} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)(2k+4)}$

= $\int_0^1 x^2 \arctan x\, dx$

.21070685294285255947... ≈ $\frac{1}{3}-\frac{1}{3e} = \int_1^{\infty}\cosh\left(\frac{1}{x^3}\right)\frac{dx}{x^7}$

1 .21070728769546454499... ≈ $\sum_{k=0}^{\infty}\left(\log(1+e^k)-k\right)$

1 .21072413030105918014... ≈ $\prod_{k=1}^{\infty}\left(1-\frac{(-1)^k}{2k}\right)$

46 .21079108380376900112... ≈ $17e$

.2109329927620049189... ≈ $\frac{1}{8}\left(2\psi(i)+2\psi(-i)+4\gamma-1\right)$

= $\frac{\gamma}{2}-\frac{3}{8}-\frac{1}{4}\left(\psi(2+i)-\psi(2-i)\right) = \sum_{k=2}^{\infty}\frac{k}{k^4-1}$

= $\frac{\gamma}{2}-\frac{1}{8}-\frac{1}{4}\left(\psi(i)+\psi(-i)\right)$

= $\sum_{k=1}^{\infty}(\zeta(4k-1)-1)$

.210957913030417776977...≈ $2+3e\,Ei(-1) = \int_0^{\infty}\frac{x^2}{e^x(x+1)^2}dx$

.21100377542970477... ≈ $\Phi\left(\frac{1}{5},2,0\right) = \mathrm{Li}_2(\frac{1}{5}) = \sum_{k=1}^{\infty}\frac{1}{5^k k^2}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}H_k}{4^k k}$

= $\zeta(2) + \log 4 \log 5 - \log^2 5 - Li_2\left(\frac{4}{5}\right)$

2 .211047296015498988... ≈ $2^{\log \pi} = \pi^{\log 2} = \prod_{k=1}^{\infty}\pi^{(-1)^{k+1}/k}$

1 .2110560275684595248... ≈ E(¾)

.21107368019578121657... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(3k)-1}{k!} = \sum_{k=2}^{\infty}\left(e^{1/k^3}-1\right)$

7 .2111025509279785862... ≈ $\sqrt{52} = 2\sqrt{13}$

1 .21117389623631662052... ≈ $\sqrt{\frac{\pi}{\pi-1}} = 1 + \sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!!\,\pi^k}$ J166

.2113553412158423477... ≈ $-\sum_{k=1}^{\infty}\frac{\mu(3k)}{2^k}$

.2113921675492335697... $\approx \frac{1}{2}(1-\gamma)$ Berndt 7.14.2

.2114662504455634405... $\approx \sum_{k=1}^{\infty}\frac{\zeta(3k)-1}{k} = \sum_{k=1}^{\infty}\sum_{m=2}^{\infty}\frac{1}{km^{3k}} = -\sum_{m=2}^{\infty}\log\left(1-m^{-3}\right)$

$= -\log\left(\frac{1}{3\pi}\cosh\frac{\pi\sqrt{3}}{2}\right) = \log 3\pi - \log\cosh\frac{\pi\sqrt{3}}{2}$

$= \log\Gamma\left(\frac{5-i\sqrt{3}}{2}\right)+\log\Gamma\left(\frac{5+i\sqrt{3}}{2}\right)$

.2116554125653740016... $\approx \frac{\pi-e}{2} = \sum_{k=1}^{\infty}\frac{\sin ke}{k}$

.2116629762657094129... $\approx 1-\frac{\pi}{4}\coth\pi$

8 .21168165538361560419... $\approx Ei(e)$

.211724802568414922304... $\approx \frac{\pi}{2}+\frac{\pi^2}{4}-\frac{\pi^3}{32}+\left(\frac{3G}{2}-3-\frac{3\pi^2}{16}\right)\log 2 - G\left(2+\frac{\pi}{4}\right)+\frac{7\zeta(3)}{4}$

$= \sum_{k=1}^{\infty}\frac{H_k}{(4k-1)^2}$

.21180346509178047528... $\approx \sum_{k=3}^{\infty}\frac{(-1)^{k+!}}{k!-2}$

.21184411114622914200... $\approx 8\log 2-\frac{16}{3} = \sum_{k=0}^{\infty}\frac{1}{2^k(2k+8)} = \sum_{k=1}^{\infty}\frac{1}{2^k(k+3)}$

.211845504171653293581... $\approx \frac{1}{4}\left(\gamma+\log 2 - ci(2)\right) = -\int_0^1 \log x\sin x\cos x\,dx$

.2118668325155665206... $\approx 2-e+(e-1)\log(e-1) = \sum_{k=1}^{\infty}\frac{1}{e^k k(k+1)}$ J149

.21192220832860146172... $\approx \frac{\pi^3}{8}-4G = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(k+1/2)^3}$

1 .21218980184258542413... $\approx \sum_{k=0}^{\infty}\frac{1}{3^k+1/3}$

.21220659078919378103... $\approx \frac{2}{3\pi} = -\binom{0}{3/2} = \int_0^1 x\sin^3\pi x\,dx$

.21231792754821907256... $\approx \log 3+\frac{1}{2}-2\log 2 = \int_1^2\frac{dx}{x^3+x^2}$

7 .2123414189575657124... $\approx 6\zeta(3) = \int_0^{\infty}\frac{x^3\,dx}{e^x+e^{-x}-2}$

= $\displaystyle\int_0^\infty \frac{dx}{e^{x^{1/3}}-1}$

.2123457762393784225... ≈ $\displaystyle\frac{\gamma}{e}$

.21236601338915846716... ≈ $\displaystyle\sum_{k=1}^\infty \frac{\zeta(2k+1)-1}{k^2} = \sum_{k=2}^\infty \frac{1}{k} Li_k\left(\frac{1}{k^2}\right)$

3 .21241741387051884997... ≈ $\displaystyle\sum_{k=2}^\infty \frac{H_k \log k}{k(k-1)}$

.21243489082496738756... ≈ $\displaystyle -\sum_{k=1}^\infty \frac{\mu(k)k}{2^k-1}$

.2125841657938186422... ≈ $\displaystyle\frac{\pi}{2e^2} = \int_0^\infty \frac{\cos 2x\,dx}{x^2+1} = \int_{-\infty}^\infty \frac{\cos x\,dx}{x^2+4}$ AS 4.3.146, Seaborn Ex. 8.3

= $\displaystyle\int_0^{\pi/2} \sin(2\tan x)\tan x\,dx$ GR 3.716.6

= $\displaystyle\int_0^\infty \cos(2\tan x)\sin x\frac{dx}{x}$ GR 3.881.4

.2125900290236663752... ≈ $\displaystyle\sum_{k=1}^\infty \frac{\log k}{k^3-1}$

.2126941666417438848... ≈ $\displaystyle\log 2 - \log^2 2 = \frac{1}{2}\sum_{k=1}^\infty \frac{H_k}{4k^3-k}$

.21271556360953137436... ≈ $\displaystyle\frac{10+3\pi^2-24\log 2}{108} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{k^3+3k^2}$

.2127399592398526553... ≈ $\displaystyle I_3(2) = \frac{1}{6}\,{}_0\mathrm{F}_1(;4;1) = \sum_{k=0}^\infty \frac{1}{k!(k+3)!}$ LY 6.114

.21299012788134175955... ≈ $\displaystyle 2\log\left(2+\sqrt{6}\right) - 4\log 2 = \sum_{k=1}^\infty \frac{(-1)^{k+1}(2k-1)!!}{(2k)!\,2^k k}$

= $\displaystyle\sum_{k=1}^\infty \frac{(-1)^{k+1}}{8^k k}\binom{2k}{k}$

.2130254214972640800... ≈ $\displaystyle\sum_{k=2}^\infty (\zeta(k)-\zeta(k+1))^2$

.2130613194252668472... ≈ $\displaystyle\frac{2}{\sqrt{e}} - 1 = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{2^k(k+1)!}$

.2131391994087528955... ≈ $\displaystyle\frac{7\zeta(3)}{4\pi^2} = -\sum_{k=0}^\infty \frac{\zeta(2k)}{4^k(2k+1)(2k+2)}$

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.2131713636468552304... $\approx$ $\dfrac{\sqrt{\pi}}{8e^{9/4}}\left(e^2 erfi\dfrac{1}{2} + erfi\dfrac{3}{2}\right) = \displaystyle\int_0^\infty e^{-x^2}\sin x\cos^2 x\,dx$

.2132354226771896621... $\approx$ $hg(\dfrac{1}{7}) = \displaystyle\sum_{k=1}^\infty \frac{1}{7k^2+k} = \sum_{k=1}^\infty (-1)^{k+1}\frac{\zeta(k+1)}{7^k}$

.21324361862292308081... $\approx$ $\dfrac{1}{\sqrt{7\pi}}$

1 .21339030483058352767... $\approx$ $\displaystyle\sum_{k=1}^\infty \frac{1}{2^k - ½} = 2\sum_{k=1}^\infty \frac{1}{2^{k+1}-1}$

3 .21339030483058352767... $\approx$ $\displaystyle\sum_{k=1}^\infty \frac{\sigma_0(k)}{2^{k-1}} = \sum_{k=1}^\infty \frac{D(k)}{2^k}, \qquad D(n) = \sum_{k=1}^\infty \sigma_0(k)$

.21339538425779600797... $\approx$ $\dfrac{4}{9} - \dfrac{\log 2}{3} = \displaystyle\sum_{k=1}^\infty \frac{1}{4k^2+6k}$

$= -\displaystyle\int_0^1 x\sqrt{1-x^2}\log x\,dx$ GR 4.241.10

$= -\displaystyle\int_0^{\pi/2} \log(\sin x)\sin x\cos^2 x\,dx$ GR 4.384.11

.2134900423278414108... $\approx$ $-\dfrac{\sin 4}{2\sqrt{\pi}} = \displaystyle\sum_{k=0}^\infty \frac{(-1)^{k+1}4^k}{k!(k+½)}$

.21355314682832332797... $\approx$ $\dfrac{2\log}{3} - \dfrac{1879}{7560} = \displaystyle\sum_{k=1}^\infty \frac{(-1)^{k+1}}{k(3+k/3)}$

.21359992335049570832... $\approx$ $\dfrac{1}{8}(2\cos\sqrt{2} + \sqrt{2}\sin\sqrt{2}) = \displaystyle\sum_{k=1}^\infty \frac{(-1)^{k+1}2^k k^2}{(2k+1)!}$

$= \displaystyle\sum_{k=1}^\infty \frac{(-1)^{k+1}2^{k-1}k^2}{(2k)!}$

.2136245483189690209... $\approx$ $\dfrac{1}{6} - \dfrac{\cot\sqrt{3}}{2\sqrt{3}} = \displaystyle\sum_{k=1}^\infty \frac{1}{k^2\pi^2 - 3}$

.2136285768839217172... $\approx$ $\dfrac{1}{11}\,{}_1F_1\left(\dfrac{11}{2}, \dfrac{13}{2}, 1\right) = \displaystyle\sum_{k=1}^\infty \frac{k}{k!(2k+9)}$

$= \dfrac{511e}{32} - \dfrac{945\sqrt{\pi}}{64} erfi 1$

.2137995823051770829... $\approx$ $\dfrac{1}{9} + \dfrac{8}{27}\operatorname{arcsinh}\dfrac{1}{2\sqrt{2}} = \displaystyle\sum_{k=1}^\infty \frac{(-1)^{k+1}}{2^k\binom{2k}{k}}$

.21395716453136275079... ≈ $\sum_{k=1}^{\infty}\frac{1}{6k^4-k} = \sum_{k=1}^{\infty}\frac{\zeta(3k+1)}{6^k}$

.2140332924621754206... ≈ $\sum_{k=1}^{\infty}5^k(\zeta(5k)-1) = \sum_{k=2}^{\infty}\frac{5}{k^5-5}$

1 .2140948960494351644... ≈ $\frac{1}{2\pi}\cosh\frac{\pi\sqrt{3}}{2} = \frac{1}{2\Gamma(-(-1)^{1/3})\Gamma((-1)^{2/3})} = \prod_{k=2}^{\infty}\left(1+\frac{1}{k^3}\right)$

.2140972656978841028... ≈ $\frac{1}{e-1}+\frac{1}{e} = \sum_{k=1}^{\infty}\frac{1}{e^k}-\int_1^{\infty}\frac{dx}{e^x}$

1 .21414333416888383841559... ≈ $-\sum_{k=2}^{\infty}\phi(k)\mu(k)\left(\zeta(k)-1\right)$

.21425280887200263 7721... ≈ $\frac{1}{81}\left(\psi^{(1)}\left(\frac{4}{9}\right)-\psi^{(1)}\left(\frac{7}{9}\right)+\pi^2 Root\left(64-48\#1+\#1^2\&,2\right)\right)$

= $\int_1^{\infty}\frac{\log x\,dx}{x^3+1+x^{-3}}$

.2142857142857$\underline{142857}$ = $\frac{3}{14}$

1 .2143665571615205518... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{\log k}$

.21439095672453216 4999... ≈ $\frac{\pi^2}{27}-\frac{\pi\sqrt{3}}{36} = \sum_{k=1}^{\infty}\frac{k\zeta(2k)}{9^k} = \sum_{k=1}^{\infty}\frac{9k^2}{(9k^2-1)^2}$

.21446148570677665005... ≈ $\int_2^{\infty}\frac{\zeta(x)-1}{x}dx = \int_1^{\infty}\frac{\zeta(2x)-1}{x}dx$

.2146010386577196351... ≈ $-\frac{1-\log 2}{2}+\frac{1}{8}\left(\psi\left(\frac{3}{2}+\frac{1}{\sqrt{2}}\right)+\psi\left(\frac{3}{2}-\frac{1}{\sqrt{2}}\right)-\psi\left(\frac{1}{\sqrt{2}}\right)-\psi\left(-\frac{1}{\sqrt{2}}\right)\right)$

= $\sum_{k=2}^{\infty}\frac{(-1)^k}{k^3-2k}$

.2146018366025516903 8... ≈ $1-\frac{\pi}{4} = \sum_{k=0}^{\infty}\frac{(-1)^k}{2k+3}$

= $\sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!\,(4k^2-1)}$ J387

= $\int_1^{\infty}\frac{dx}{x^4+x^2} = \int_0^{\pi/4}\frac{\sin^2 x\,dx}{\cos^2 x} = \int_0^1\frac{\arcsin x}{(1+x)^2}dx$

2 .21473404859284429825... ≈ $\frac{\pi^3}{14}$

1 .2147753992090018159... ≈ $\log\sqrt{2\pi}+3\log 3-3 = \int_3^4\log\Gamma(x)dx$ GR 6.441.1

.2148028473931230436... ≈ $\log\pi - \frac{3\log 2}{2} + \log\csc\frac{\pi}{2\sqrt{2}}$

$= \sum_{k=1}^{\infty}\frac{\zeta(2k)}{8^k k} = -\sum_{k=1}^{\infty}\log\left(1-\frac{1}{8k^2}\right)$

.21485158948632195387... ≈ $8\log 4 - 8\log 5 + 2 = \sum_{k=0}^{\infty}\frac{(-1)^k}{4^k(2k+4)}$

.21506864959361451798... ≈ $\sum_{k=2}^{\infty}\frac{(\zeta(k)-1)^2}{k!}$

.215232513719719453473... ≈ $\sum_{p\ prime}\frac{(-1)^{k+1}}{2^p-1}$, where p is the kth prime

.215392508466695431 7... ≈ $\sum_{k=2}^{\infty}\zeta(k)(\zeta(k+2)-1)$

.2154822031355754126... ≈ $\frac{4-\sqrt{2}}{12} = \frac{1}{3} - \frac{1}{6\sqrt{2}} = \int_0^{\pi/4}\sin x\cos^2 x\,dx = \int_0^{\pi/4}\frac{\sin^3 x}{\tan^2 x}dx$

.2156418779165612598... ≈ $24 - 6\pi - \frac{\pi^2}{2} = \int_0^1 \arcsin^2 x\ \arccos^2 x\ dx$

.21565337159490150711... ≈ $\frac{\pi^2 - 7\zeta(3)}{4} - \frac{4}{27} = \sum_{k=2}^{\infty}\frac{4k}{(2k+1)^3}$

$= \sum_{k=2}^{\infty}(-1)^k k(k-1)\frac{\zeta(k)-1}{2^k}$

.215739918811204054 1... ≈ $7\zeta(3) + \frac{31\zeta(5)}{4} - \frac{\pi^4}{6} = \sum_{k=1}^{\infty}\frac{k^2}{(k+1/2)^5}$

.2157615543388356956... ≈ $\frac{3}{4}\log\frac{4}{3} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\mathrm{H}_k}{3^k}$

1 .2158542037080532573... ≈ $\frac{12}{\pi^2} = \frac{2}{\zeta(2)}$

2 .21587501645954320319... ≈ $\sum_{k=2}^{\infty}\left(\frac{\zeta(k)-1}{\zeta(k+1)-1} - 2\right)$

.21590570178361070... ≈ $\frac{1}{2}\left(\psi\left(\frac{1+e}{2}\right) - \psi\left(\frac{e}{2}\right)\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{k+e}$

1 .21615887964567006081... ≈ $\sum_{k=1}^{\infty}\phi(k)\big(\zeta(k+1)-1\big)$

.216166179190846827... ≈ $\frac{1-e^2}{4} = \frac{\sinh 1}{4e} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{2^k}{(k+2)!}$

1 .2163163809651677137... ≈ $\cos\frac{1}{4} + \sin\frac{1}{4} = \sqrt{1+\sin\frac{1}{2}} = \prod_0^{\infty}\left(1 + \frac{(-1)^k}{(2k+1)\pi}\right)$

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{(4k)!!4^k}+\sum_{k=0}^{\infty}\frac{(-1)^k}{(4k+2)!!2\bullet 4^k}$

.21634854253072016398... ≈ $4+2G+\frac{\pi^2}{4}-\frac{\pi^3}{8}-\frac{7\zeta(3)}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2\zeta(k+1)}{4^k}$

13 .2163690571131573733... ≈ $\sum_{k=2}^{\infty}k^4(\zeta(k)-1)^2$

.2163953243244931459... ≈ $3\log 3-3\log 2-1=\sum_{k=1}^{\infty}\frac{1}{3^k(k+1)}$ GR 1.513.5

= $\sum_{k=1}^{\infty}\frac{(-1)^k}{2^k k(k+1)}$

= $\int_0^1 \log\left(1+\frac{x}{2}\right)$

2 .21643971276034212127... ≈ $\prod_{k=1}^{\infty}\frac{k^2+3}{k^2+2} = \prod_{k=1}^{\infty}\left(1+\frac{1}{k^2+2}\right)$

.2167432468349777594... ≈ $\frac{\sqrt{2}}{32}\left(\pi+\log(3+2\sqrt{2})\right)=\sum_{k=1}^{\infty}\frac{1}{(8k-7)(8k-3)}$ J266

1 .2167459561582441825... ≈ $\log 2+\frac{\pi}{6} = \sum_{k=1}^{\infty}\frac{1}{4k^2-3k} = \sum_{k=0}^{\infty}\frac{1}{(k+1)(4k+1)}$

.216823416815804965275... ≈ $\frac{1}{2}-\frac{\log 2}{2}+\frac{1}{8}\left(-\psi(i)-\psi(-i)+\psi\left(-\frac{1}{2}+i\right)+\psi\left(-\frac{1}{2}-i\right)\right)$

= $\int_0^{\infty}\frac{\cos^2 x}{e^x\left(e^x+1\right)}dx$

.216872352716516197139... ≈ $\zeta(3)\ -\ 2\log^2 2\log 5+2\log 2\log^2 5-\frac{\log^3 5}{2}+Li_2\left(\frac{4}{5}\right)\log\frac{4}{5}$

$+Li_3\left(\frac{1}{5}\right)-Li_3\left(\frac{4}{5}\right)$

= $\sum_{k=1}^{\infty}\frac{H_k}{5^k k^2}$

.2169542943774763694... ≈ $\frac{-\sin\pi\sqrt{2}}{\pi\sqrt{2}}=\prod_{2}^{\infty}\left(1-\frac{2}{k^2}\right)$

48 .2171406141667015744... = $\frac{93\zeta(5)}{2} = \int_0^{\infty}\frac{x^4\,dx}{\sinh x}$

.21740110027233965471... ≈ $\frac{\pi^2-9}{4}=\sum_{k=1}^{\infty}\frac{k}{(k+1)(k+3)^2}$

1 .2174856795003257177... ≈ $24I_4(2)={}_0F_1(;5;1)=24\sum_{k=0}^{\infty}\frac{1}{k!(k+4)!}$

	.21754216068060015891...	≈	$\frac{1}{4}\left(\cosh 2\pi-\pi\sinh\pi-\pi^2\cosh\pi-1\right)\operatorname{csch}^2\pi \;=\; \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k^2+1)^2}$	
	.2178566498449504...	≈	$\sum_{k=2}^{\infty}\frac{(-1)^k}{k^3-4}$	
	.218086336152111299701...	≈	$\frac{1}{2}+\frac{\pi^2}{24}-\log 2 \;=\; \sum_{k=1}^{\infty}\frac{(-1)^{k+1}H_k}{k(k+2)}$	
	.218116796249920922292...	≈	$-\frac{i}{4}\left(\psi\left(2-e^{-2i}\right)-\psi\left(2-e^{2i}\right)\right)=\sum_{k=1}^{\infty}\sin k\cos k\left(\zeta(k+1)-1\right)$	
2	.2181595437576882231...	≈	$\Gamma\left(\frac{2}{5}\right)$	
1	.21816975871035137...	≈	$j_2=2-g_2$	J311
	.2182818284590452354...	≈	$e-\frac{5}{2}=\sum_{k=1}^{\infty}\frac{1}{(k+2)!}=\sum_{k=0}^{\infty}\frac{1}{k!(k+2)(k+4)}$	
1	.218282905017277621...	≈	$\arctan e \prec \sum_{k=0}^{\infty}\frac{(-1)^k e^{2k+1}}{(2k+1)}$	
	.2185147401709677798...	≈	$\zeta(3)-48\log 2-\frac{2\pi^2}{3}-8\pi+64=\sum_{k=1}^{\infty}\frac{1}{4k^4+k^3}$	
		=	$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(k+3)}{4^k}$	
37	.2185600000000000000$\underline{0}$	=	$\frac{116308}{3125}=\sum_{k=1}^{\infty}\frac{F_k{}^2k^3}{4^k}$	
1	.2186792518257414537...	≈	$\frac{2\sqrt{2}}{\pi}\sinh\frac{\pi}{2\sqrt{2}}$	
2	.21869835298393806209...	≈	$\sum_{k=1}^{\infty}\arcsin\left(\frac{1}{k^2}\right)$	
	.218709374152227347877...	≈	$\sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)}{k(2k-1)} \;=\; \sum_{k=1}^{\infty}\left(\frac{1}{k}-\frac{2}{\sqrt{k}}\arctan\frac{1}{\sqrt{k}}+\log\left(1+\frac{1}{k}\right)\right)$	
	.2187858681527455514...	≈	$\sum_{k=1}^{\infty}\frac{H_k}{6^k} \;=\; \frac{6}{5}\log\frac{6}{5}$	
3	.2188758248682007492...	≈	$2\log 5$	
	.218977308611084812924...	≈	$\frac{136}{625}-\frac{4\operatorname{arccsch}2}{625\sqrt{5}} \;=\; \sum_{k=1}^{\infty}(-1)^k\frac{k^4}{\binom{2k}{k}}$	

.21917134772771973069... ≈ $\frac{\cosh 1}{2} - \frac{\sqrt{\pi}}{8}\left(erf\,(1) + erfi(1)\right) = \int\limits_1^\infty \sinh\left(\frac{1}{x^2}\right)\frac{dx}{x^4}$

.219235991877121... ≈ $\sum_{k=1}^\infty \frac{(-1)^{k+1}}{k+k^{-3}}$

1 .21925158248756821... ≈ $8\log 2 - 4\log^2 2 - 2\zeta(3) = \sum_{k=1}^\infty \frac{H_k}{k^2(2k-1)}$

.2193839343955202737... ≈ $-Ei(-1) = -\gamma - \sum_{k=1}^\infty \frac{(-1)^k}{k!k}$ LY 6.423

= $\int\limits_1^\infty \frac{\log dx}{e^x} = -\gamma - \int\limits_0^1 \frac{1-e^x}{x}dx$

= $\int\limits_0^\infty \frac{dx}{e^{e^x}}$

= $\int\limits_0^\infty xe^{x-e^x}dx$

2 .2193999651440434251... ≈ $\zeta(3)+\zeta(6)$

.2194513564886152673... ≈ $\sum_{k=1}^\infty \frac{1}{k^2}\left(Li_k\left(\frac{1}{2}\right) - \frac{1}{2}\right)$

9 .2195444572928873100... ≈ $\sqrt{85}$

.21955691692893092525... ≈ $2+\zeta(2) - \pi\coth\frac{\pi}{2} = \sum_{k=1}^\infty \frac{1}{4k^4+k^2}$

= $\sum_{k=1}^\infty (-1)^{k+1}\frac{\zeta(2k+2)}{4^k}$

= $-\operatorname{Re}\left\{\sum_{k=1}^\infty \frac{\zeta(k+2)}{(2i)^k}\right\}$

.21972115565045840685... ≈ $\frac{e}{4} - \frac{5}{4e} = \int\limits_1^\infty \cosh\left(\frac{1}{x^2}\right)\frac{dx}{x^7} = \frac{1}{2}\int\limits_1^\infty \cosh\left(\frac{1}{x}\right)\frac{dx}{x^4}$

.2197492662513467923... ≈ $\sum_{k=1}^\infty (-1)^{k+1}\frac{\sigma_0(k)}{k!}$

.2198859521312955567... ≈ $1 + ci(1) - \cos 1 - \gamma = -\int\limits_0^1 x\log x\cos x\,dx$

.219897978495778103... ≈ $\sum_{k=1}^\infty (\zeta(k^2+2)-1)$

2265 .21992083594050635655... ≈ $\frac{3\pi^7}{4}$

.2199376456133308016... ≈ $\sum_{k=1}^\infty \frac{(-1)^{k+1}H^{(3)}{}_k}{4^k k}$

.21995972094619680320... ≈ $\frac{\gamma^3}{6}-\frac{\gamma\pi^2}{24}+\frac{\gamma^2\log 2}{2}-\frac{\pi^2\log 2}{24}+\frac{\gamma\log^2 2}{2}+\frac{\log^3 2}{6}+\frac{\zeta(3)}{3}$

= $\int\limits_0^\infty \frac{\log^2 x\sin^2 x}{x}dx$

.22002529287246675798... ≈ $\frac{1}{2}J_1(1) = \sum_{k=1}^\infty \frac{(-1)^{k+1}k}{(k!)^2 4^k}$

.220026086659111379573... ≈ $\log 2-\frac{1}{4}\left(\psi\left(1-\frac{i}{2\sqrt{3}}\right)+\psi\left(1+\frac{i}{2\sqrt{3}}\right)\right)$

$+\frac{1}{4}\left(\psi\left(\frac{3-i\sqrt{3}}{6}\right)+\psi\left(\frac{3+i\sqrt{3}}{6}\right)\right)$

= $\sum_{k=1}^\infty \frac{(-1)^{k+1}}{3k^3+k}$

.2200507449966154983... ≈ $2\log 2-\frac{4G}{\pi}=\sum_{k=1}^\infty\left(\frac{(2k-1)!!}{(2k)!!}\right)^2\frac{1}{2k}$ J385

.220087400543110224086... ≈ $\sum_{k=2}^\infty \frac{\zeta(2k-1)}{2^k k} = -\sum_{k=1}^\infty\left(k\log\left(1-\frac{1}{2k^2}\right)+\frac{1}{2k}\right)$

.2203297599887572963... ≈ $\frac{\pi}{2}\coth\pi-\frac{\pi}{4}\coth\frac{\pi}{2}-\frac{1}{2}=\frac{\pi}{4}\tanh\frac{\pi}{2}-\frac{1}{2}$

= $\sum_{k=1}^\infty \frac{1}{4k^2+4k+2} = -\operatorname{Re}\left\{\sum_{k=1}^\infty \frac{\zeta(2k)}{(2i)^k}\right\}$

= $\sum_{k=1}^\infty (-1)^k\frac{\zeta(4k)}{4^k} = \sum_{k=1}^\infty \frac{1}{4k^4+1}$

.22037068922591731767... ≈ $\frac{\gamma}{4}-\frac{7}{64}+\frac{1}{8}\left(\psi(2-i)+\psi(2+i)\right)+\frac{i}{16}\left(\psi^{(1)}(2+i)-\psi^{(1)}(2-i)\right)$

= $\sum_{k=2}^\infty \frac{1}{k^3(1-k^{-4})^2}$

1 .2204070660909404377... ≈ $\zeta(3)^{\zeta(4)}$

.2204359059283144034... ≈ $\frac{1}{6}\psi^{(1)}\left(\frac{1}{3}\right)-\frac{4\pi^2}{27}$

= $-\frac{5\pi^2}{108}-\frac{(-1)^{2/3}}{2}Li_2\left((-1)^{1/3}\right)+\frac{1}{3}Li_2\left(-(-1)^{2/3}\right)$

$-\frac{(-1)^{2/3}}{4}Li_2\left((-1)^{2/3}\right)$

= $\frac{1}{36}\left(\psi'\left(\frac{1}{3}\right)-\psi'\left(\frac{5}{6}\right)\right)=\sum_{k=0}^\infty\frac{(-1)^k}{(3k+2)^2}$

= $\displaystyle\int_1^\infty \frac{\log x dx}{x^3+1} = -\int_0^1 \frac{x\log x}{x^3+1}dx = \int_0^\infty \frac{x dx}{e^{2x}+e^{-x}}$

.220496639095139468548... ≈ $\displaystyle\frac{3}{8\pi\sqrt{2}}\zeta\left(\frac{3}{2}\right) = \int_0^\infty \frac{dx}{e^{2\pi x^{2/3}}-1}$

.22056218167484024814... ≈ $\displaystyle\frac{1}{2\sqrt{e}}\left(\gamma - 2 - Ei\left(\frac{1}{2}\right) - \log 2\right) + 1 = \sum_{k=1}^\infty \frac{(-1)^{k+1} kH_k}{k!2^k}$

.2205840407496980887... ≈ $\displaystyle\cos\frac{\pi^2}{2} = \text{Re}\left\{i^\pi\right\}$

.220608143827399102241... ≈ $\displaystyle\frac{3\zeta(3)}{2} + 4\log 2 - \frac{\pi^2}{3} - 6 = -\int_0^1 \log(1+x)\log^2 x\,dx$

6 .220608143827399102241... ≈ $\displaystyle\zeta(2) + 4\log 2 + \frac{3\zeta(3)}{2} = \int_0^1 \log\left(1+\frac{1}{x}\right)\log^2 x\,dx$

.2206356001526515934... ≈ $\displaystyle\frac{\log 2}{\pi} = \int_0^1 x\tan\pi x\,dx = \int_0^1 \left(\frac{1}{2} - x\right)\tan\pi x\,dx$

2 .220671308254797711926... ≈ $\displaystyle\frac{\arctan\sqrt{2}}{12\sqrt{2}}\left(4\pi^2 - 4\arctan^2\sqrt{2} + 3\log^2 3\right) = \int_0 \frac{\log^{\infty 2} x}{x^2+2x+3}dx$

1 .2210113690345504761... ≈ $\displaystyle\sum_{k=2}^\infty \left(1 - \frac{1}{\zeta(k)^2}\right)$

.2211818066979397521... ≈ $\displaystyle\frac{1}{16}(27 + 3\pi\sqrt{3} - 2\pi^2 - 27\log 3 + 8\zeta(3))$

= $\displaystyle\sum_{k=1}^\infty \frac{1}{k^3(3k+2)}$

.2213229557371153254... ≈ $\displaystyle\frac{\pi^2}{6} - \frac{205}{144} = \psi^{(1)}(5) = \Phi(1,2,5) = \zeta(2,5)$

1 .2214027581601698339... ≈ $\sqrt[5]{e}$

2 .22144146907918312350... ≈ $\displaystyle\frac{\pi}{\sqrt{2}} = \int_{-\infty}^\infty \frac{dx}{x^4+1}$ Marsden p. 231

= $\displaystyle\int_{-\infty}^\infty \frac{x^2 dx}{x^4+1} = \int_0^{\pi/2} \sqrt{\tan x}\,dx$

= $\displaystyle\int_0^\pi \frac{d\theta}{1+\sin^2\theta}$ Marsden p. 259

= $\displaystyle\int_0^\pi \frac{\log\left(x^2+\tfrac{1}{2}\right)}{x^2}dx = \int_0^\infty \frac{\cosh\left(\tfrac{x}{2}\right)}{\cosh x}dx$

= $\int\limits_0^\infty \log\left(1+\frac{1}{2x^4}\right)dx$

.22148156265041945009... ≈ $-\frac{\gamma}{\pi}-\frac{1}{\pi}\psi\left(1-\frac{1}{\pi}\right)=\sum_{k=2}^\infty\frac{\zeta(k)}{\pi^k}=\sum_{k=1}^\infty\frac{1}{\pi^2k^2-\pi k}$

.22155673136318950341... ≈ $\frac{\sqrt{\pi}}{8}$

.221565357507408085262... ≈

$$\frac{1}{10\left(3+\sqrt{5}\right)}\left(\left(7+3\sqrt{5}\right)\arctan\frac{1+\sqrt{5}}{4}+\left(3+\sqrt{5}\right)arc\coth 2-2arc\cot\left(1+\sqrt{5}\right)\right)$$

= $\sum_{k=1}^\infty\frac{(-1)^{k+1}F_kF_{k+1}}{4^k\,(2k-1)}$

6 .221566530860219558... ≈ $6\zeta(5)=\int\limits_0^1\frac{\log(1-x)\log^3 x}{x}dx$

.22156908018641904867... ≈ $\zeta(3)-1+\frac{1}{4}\left(\psi^{(2)}(2-i)+\psi^{(2)}(2+i)\right)$

= $\int\limits_0^\infty\frac{x^2\sin^2 x}{e^x(e^x-1)}dx$

.22157359027997265471... ≈ $\frac{\log 2}{2}-\frac{1}{8}=\sum_{k=2}^\infty\frac{(2k-4)!k^2}{(2k)!}$

= $\sum_{k=2}^\infty\frac{(2k-1)!!}{(2k)!\,(k^2-1)}$

.221689395109267039... ≈ $\sum_{k=2}^\infty\frac{1}{k^3-1}=\sum_{k=1}^\infty(\zeta(3k)-1)=\zeta(3)-1+\sum_{k=2}^\infty\frac{1}{k^6-k^3}$

= $\frac{1}{3}\left(\gamma+1+\frac{1+i\sqrt{3}}{2}\psi\left(\frac{5+i\sqrt{3}}{2}\right)+\frac{1-i\sqrt{3}}{2}\psi\left(\frac{5-i\sqrt{3}}{2}\right)\right)$

= $\frac{\gamma+1}{3}+\frac{1}{6}\left((1-i\sqrt{3})\psi\left(\frac{3-i\sqrt{3}}{2}\right)+(1+i\sqrt{3})\psi\left(\frac{3+i\sqrt{3}}{2}\right)\right)$

= $\frac{\gamma}{3}+\frac{2}{\sqrt{3}\left(3i+\sqrt{3}\right)}\psi\left(\frac{5-i\sqrt{3}}{2}\right)+\left(\frac{1}{3}-\frac{2}{\sqrt{3}\left(3i+\sqrt{3}\right)}\psi\left(\frac{5+i\sqrt{3}}{2}\right)\right)$

= $\frac{1}{3}\left(\gamma+\left(1+(-1)^{2/3}\right)\psi\left(2+(-1)^{1/3}\right)-(-1)^{2/3}\psi\left(2-(-1)^{2/3}\right)\right)$

.2217458866641557648... ≈ $\frac{1}{6}(\log 3+\log 4-2\gamma)=\sum_{k=1}^\infty\frac{\psi(2k)}{4^k}$

.22181330818986560703... ≈ $2\log 2+\log^2 2-\frac{\pi^2}{6}=\sum_{k=0}^\infty\frac{k}{2^k(k+1)^2}$

1 .2218797259653540443... ≈ $\sum_{k=2}^{\infty}\frac{\log\frac{k}{k-1}}{k^2+1}$

1 .2218799453198801385 19... ≈ $\frac{97\pi^6}{22680}-2\left(\zeta(3)\right)^2=\sum_{k=1}^{\infty}\frac{H_kH_k}{k^4}$

.221976971159108564... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^4-11}$

.2220003049334945764 1... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k+1)-1}{k!}=\sum_{k=2}^{\infty}\left(\frac{e^{1/k^2}}{k}-\frac{1}{k}\right)$

.2222222222222222222$\underline{2}$ = $\frac{2}{9}=\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k}{2^k}=\int_0^1 x^2\arccos x\,dx$

= $\sum_{k=1}^{\infty}\frac{\mu(k)3^k}{9^k-1}$

.2224953658877517235 82... ≈ $\sum_{k=1}^{\infty}\frac{B_k}{(k+1)!}$

.2225623991043403355... ≈ $\frac{1}{6}\log\frac{2}{3}+\frac{\sqrt{2}}{3}\arctan\frac{1}{\sqrt{2}}=\sum_{k=1}^{\infty}(-1)^{k+1}\frac{H_{2k-1}}{2^k}$

.2226404109678132491 86... ≈ $\frac{\csc^3 1}{32}\left(\cos 3+12\sin 1-9\right)=\sum_{k=1}^{\infty}\frac{k^2\zeta(2k)}{\pi^{2k}}$

2 .2226510919300050873... ≈ $\frac{3e}{4}+\frac{1}{2e}=\sum_{k=0}^{\infty}\frac{k^2+1}{(2k)!}$

.2227305519627176711... ≈ $\frac{21\zeta(3)}{4}-\frac{\pi^4}{16}=\sum_{k=1}^{\infty}\frac{12k}{(2k+1)^4}$

= $\sum_{k=3}^{\infty}(-1)^{k+1}\frac{k(k-1)(k-2)\zeta(k)}{2^k}$

.22281841361727370564... ≈ $G-\log 2$

2 .22289441688521111339... ≈ $-8\gamma-4\left(\psi\left(1-\frac{1}{2\sqrt{2}}\right)-\psi\left(1+\frac{\sqrt{2}}{4}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{8}{8k^3-k}=\sum_{k=1}^{\infty}\frac{\zeta(2k+1)}{2^{k-1}}$

8 .22290568860880978... ≈ $\sum_{k=1}^{\infty}\frac{2^k\zeta(k+1)}{k!}=\sum_{k=1}^{\infty}\frac{e^{2/k}-1}{k}$

2 .2230562526551668354... ≈ $\frac{1}{2}(9\log 3-\pi\sqrt{3})=\sum_{k=1}^{\infty}\frac{3}{3k^2-k}=\sum_{k=2}^{\infty}\frac{\zeta(k)}{3^{k-2}}$

= $\sum_{k=0}^{\infty}\frac{1}{(k+1)(k+2/3)}$

.2231435513142097558... ≈ $\log 5-\log 4=\Phi\left(\frac{1}{5},1,0\right)=\mathrm{Li}_1\left(\frac{1}{5}\right)=\sum_{k=1}^{\infty}\frac{1}{5^k k}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{4^k k}$

= $2\sum_{k=0}^{\infty}\frac{1}{(2k+1)\,9^{2k+1}} = 2\operatorname{arctanh}\frac{1}{9} = \int_0^{\infty}\frac{dx}{4e^x+1}$ K148

.2231443845875105813... ≈ $\sum_{k=1}^{\infty}\frac{H^{(3)}{}_k}{3^k(k+1)}$

.2232442754839327307... ≈ $2\sin 1+\cos 1-2=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(2k)!(k+1)}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k-1)!(2k+2)}$

= $\int_0^1 x^2\sin x\,dx$

= $\int_1^e \frac{\log^2 x\,\sin\log x}{x^2}dx = \int_1^{\infty}\sin\left(\frac{1}{x}\right)\frac{dx}{x^4}$

.22334260293720331337... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)-\zeta(2k+1)}{(2k)!}$

.2234226145863756302... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^k}{k!+1}$

.22360872607835041217... ≈ $\sum_{k=2}^{\infty}\frac{(\zeta(k)-1)^2}{k}$

98 .22381079275099866005... ≈ $\frac{267}{e}=\sum_{k=0}^{\infty}\frac{(-1)^k k^9}{k!}$

.223867833455760676... ≈ $\frac{1}{4}-\frac{\pi}{\sqrt{2}}\frac{1}{e^{\pi\sqrt{2}}+e^{-\pi\sqrt{2}}} = \frac{1}{4}-\frac{\pi}{\sqrt{2}}\operatorname{csch}\pi\sqrt{2}$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2+2}$ J125

.2238907791412356681... ≈ $J_0(2) = {}_0\mathrm{F}_1(;1;-1)=\sum_{k=0}^{\infty}\frac{(-1)^k}{(k!)^2}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{(k!)^2}$ LY 6.115

= $\sum_{k=0}^{\infty}(-1)^k\frac{1}{(2k)!}\binom{2k}{k} = \sum_{k=0}^{\infty}(-1)^{k+1}\frac{k^2}{(2k)!}\binom{2k}{k}$

1 .223917650911311696437...≈ $e^{\zeta(3)-1}$

.22393085952708464047... ≈ $\sum_{k=2}^{\infty}\frac{4}{k^3(1-k^{-2})^2} = \sum_{k=1}^{\infty}\frac{\zeta(2k+1)-1}{k}=\sum_{k=2}^{\infty}\frac{\log(1-k^{-2})}{k}$

3 .2241045595332964672... ≈ $\sum_{k=2}^{\infty}(-1)^k\frac{k^3\zeta(k)}{k!} = \sum_{k=2}^{\infty}\left(\frac{1}{k}+\frac{3k-1-k^2}{e^{1/k}k^3}\right)$

$$.22417142752923610239 5\ldots \approx -3 + \log 8\pi = \sum_{k=1}^{\infty} \frac{\zeta(2k)-1}{k(2k+1)}$$

$$.22439722516960958821 4\ldots \approx \sum_{k=2}^{\infty} (-1)^k \frac{\zeta(2k-1)}{k^2} = \sum_{k=1}^{\infty} \left(\frac{1}{k} + kLi_2\left(-\frac{1}{k^2} \right) \right)$$

$$.2244181877441891147\ldots \approx \frac{7}{36}(1+\log\frac{7}{6}) = \sum_{k=1}^{\infty} \frac{k\,H_k}{7^k}$$

$$.2244806244272477796\ldots \approx \frac{5}{3} - \frac{\gamma}{3} - \frac{\log 2\pi}{2} + 2\zeta'(-1)$$

$$= \sum_{k=1}^{\infty} \frac{\zeta(k+1)-1}{k+3} = \sum_{k=2}^{\infty} \left(-\frac{1}{3k} - \frac{1}{2} - k - k^2 \log\left(1 - \frac{1}{k}\right) \right)$$

$$.22451725198323206267\ldots \approx \gamma^e$$

$$.2247448713915890491\ldots \approx \sqrt{\frac{3}{2}} - 1 = \sum_{k=1}^{\infty} \frac{(2k-1)!!}{(2k)!\,3^k}$$

$$1\quad .2247448713915890491\ldots \approx \sqrt{\frac{3}{2}} = \sum_{k=0}^{\infty} \frac{1}{12^k} \binom{2k}{k}$$

$$= \prod_{k=0}^{\infty} \left(1 + \frac{(-1)^k}{6k+3} \right)$$

$$.22475370091249333740\ldots \approx \frac{8}{e} - e = \int_1^{\infty} \sinh\left(\frac{1}{x}\right) \frac{dx}{x^5}$$

$$1\quad .2247643131279778005\ldots \approx \frac{1}{18} - \frac{\cot 3}{6} = \sum_{k=1}^{\infty} \frac{1}{k^2\pi^2 - 9}$$

$$.2247951016770520918\ldots \approx \frac{1}{2} - \frac{\pi}{4\sqrt{2}} \cot\frac{\pi}{2\sqrt{2}} = \sum_{k=1}^{\infty} \frac{1}{8k^2-1} = \sum_{k=1}^{\infty} \frac{\zeta(2k)}{8^k}$$

$$.22482218262595674557\ldots \approx -\frac{\pi}{2\sqrt{3}} \cot\frac{\pi}{\sqrt{3}} = \sum_{k=2}^{\infty} \frac{1}{3k^2-1} = \sum_{k=1}^{\infty} \frac{\zeta(2k)-1}{3^k}$$

$$.224829516649410894\ldots \approx \frac{1}{81}(32 - 3\pi^2 - 24\log 2 + 27\zeta(3)) = \sum_{k=1}^{\infty} \frac{1}{k^3(2k+3)}$$

$.2250000000000000\underline{0}$ = $\frac{9}{40}$

.22507207156031203903... ≈ $\frac{\pi}{4} - \frac{\pi}{2\sqrt{3}} + \frac{\log 2}{2} = \int_0^1 \arctan x^3 dx$

.225079079039276517... ≈ $\frac{1}{\pi\sqrt{2}} = \Gamma^{-1}\left(\frac{1}{4}\right)\Gamma^{-1}\left(\frac{3}{4}\right)$

.225141424845734406... ≈ $2G + \frac{\pi}{2} - \pi \log 2 - 1 = -\frac{1}{2}\int_0^1 \left(E(k) - \frac{\pi}{2}\right)\frac{dk}{k}$ GR 6.149.1

.225191570994199437... ≈ $\sum_{k=1}^{\infty} (-1)^1 \frac{2^k}{(k+1)!!}$

.225385669342423928 5... ≈ $\frac{3\zeta(3)}{16} = -\frac{1}{4}Li_3(-1) = -Li_3(i) - Li_3(-i)$

= $\int_1^{\infty} \frac{\log^2 x dx}{x^3 + x} = -\int_0^1 \frac{x \log^2 x dx}{x^2+1}$

= $\int_0^{\infty} \frac{x^2 dx}{e^{2x}+1}$

1 .225399673560564079... ≈ $\frac{2e}{2e-1} = \sum_{k=0}^{\infty} \frac{1}{(2e)^k}$

1 .22541670246517764513... ≈ $\Gamma\left(\frac{3}{4}\right) = \pi\sqrt{2}\ \Gamma^{-1}\left(\frac{1}{4}\right)$

.22542240623577277232... ≈ $\sum_{k=1}^{\infty} \frac{\mu(k)}{k(k+1)} = -\sum_{k=1}^{\infty} \frac{\mu(k)}{k+1}$

1 .22548919991903600533... ≈ $\frac{\pi^2+1}{\pi^2-1} = \coth(\log \pi)$

.22552719639726524630 7...≈ $\frac{1}{18}\left(4 + \sqrt{2}\,\text{arccot}\,2 - 2\log 3 + 2\log 2\right) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{kH_{2k}}{2^k}$

.225603783608435750 3... ≈ $\sum_{k=1}^{\infty} \frac{H_{2k}}{8^k} = \frac{1}{7}\left(4\log\frac{8}{7} + \sqrt{2}\log\frac{4+\sqrt{2}}{4-\sqrt{2}}\right)$

1 .225704705128497409 5... ≈ $\prod_{k=1}^{\infty} \zeta(3k)$

.225791352644727432 4... ≈ $\log\sqrt{\frac{\pi}{2}} = \sum_{k=1}^{\infty} \frac{\zeta(2k)}{2^{2k+1}k} \prec \sum_{k=2}^{\infty} (-1)^k \log k$ K ex. 207

.22603241503205748814 1...≈ $\frac{\pi}{8} - \frac{1}{6} = \int_1^{\infty} \frac{\arctan x}{x^5} dx$

3 .22606699662600489292... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}(\zeta^3(2k)-1)$

8 .22631388275361511237... ≈ $\frac{\sqrt{\pi}}{4} erfi\, 2 \;=\; \sum_{k=0}^{\infty}\frac{4^k}{k!(2k+1)}$

.2265234857049204362... ≈ $\frac{e}{12}$

2 .226562500000000000$\underline{0}$ = $\frac{285}{128} \;=\; \Phi\left(\frac{1}{5},-4,0\right) \;=\; \mathrm{Li}_{-4}\left(\frac{1}{5}\right) \;=\; \sum_{k=1}^{\infty}\frac{k^4}{5^k}$

.226589292402242872682... ≈

$$\frac{1}{5\left(1+\sqrt{5}\right)}\left(\left(3+\sqrt{5}\right)Li_2\left(\frac{-3-\sqrt{5}}{8}\right)-\left(1+\sqrt{5}\right)Li_2\left(\frac{1}{4}\right)-2Li_2\left(-\frac{1}{\left(1+\sqrt{5}\right)^2}\right)\right)$$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}F_kF_{k+1}}{4^k k^2}$

.226600619263869269496... ≈ $\frac{1}{4}\Gamma\left(\frac{5}{4}\right) \;=\; \int_0^{\infty} x^4 e^{-x^4}dx$

.226724920529277231... ≈ $\frac{\pi}{8\sqrt{3}} \;=\; \sum_{k=1}^{\infty}\frac{1}{36k^2-36k+5}$

.22679965131342898... ≈ $\frac{287}{4}-\frac{153}{2}\log 2-\frac{77}{2}\log^2 2 = \sum_{k=1}^{\infty}\frac{k^3\mathrm{H}_k}{2^k(k+1)(k+2)(k+3)}$

.226801406646162522... ≈ $\frac{\pi}{2}\cot\frac{7\pi}{8} \;+\; 4\log 2 \;+\; \sqrt{2}\log\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}$

= $\sum_{k=1}^{\infty}\frac{1}{8k^2-k}$

.2268159262423682842... ≈ $\frac{1}{9}\left(72\log^2 2+168\log 2-149\right) = \sum_{k=1}^{\infty}\frac{\mathrm{H}_k}{2^k(k+4)}$

1 .2269368084163... ≈ root of $\zeta(x)=5$

.22728072574031781053... ≈ $\frac{2}{27}\Phi\left(-2,3,\frac{2}{3}\right) \;=\; \int_1^{\infty}\frac{\log^2 x}{x^3+2}dx$

.227350210732224060137... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(k+1)-1}{3^k k} \;=\; -\sum_{k=1}^{\infty}\frac{1}{k}\log\left(1-\frac{1}{3k}\right)$

.22740742820168557... ≈ Ai(-2)

.22741127776021876... ≈ $3-4\log 2 = \sum_{k=0}^{\infty}\frac{1}{2^k(k+2)(k+3)} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2(k+1)^2}$

= $\sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!\,(k+1)^2}$

1 .22741127776021876... ≈ $4-4\log 2 = \sum_{k=0}^{\infty}\frac{1}{4^k(k+1)^2}\binom{2k}{k}$

= $\sum_{k=1}^{\infty}\frac{1}{k(k+½)} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k\left(k^2-1/4\right)} = \sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)}{2^{k-2}}$

= $\sum_{k=1}^{\infty}\frac{(k-\frac{1}{2})!}{k(k+\frac{1}{2})!}$

1 .2274299244886647836... ≈ $2\operatorname{arcsinh}\frac{\sqrt{3}}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}3^k}{k^2\binom{2k}{k}}$

4 .2274535333762165408... ≈ $\gamma+\frac{\pi}{2}+3\log 2 = -\frac{\Gamma'(1/4)}{\Gamma(1/4)} = -\psi\left(\frac{1}{4}\right)$

1 .2274801249330395378... ≈ $3\zeta(5)+\left(6-\frac{\pi^2}{6}\right)\zeta(3)-\frac{\pi^4}{72}-12\log^2 2 = \sum_{k=1}^{\infty}\frac{H_k}{2k+1}\left(\frac{1}{k^2}+\frac{1}{k^3}+\frac{1}{k^4}\right)$

1 .227558761008398655271... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)\left(\zeta(k)-1\right)}{k-1}$

1 .22774325454718653743... ≈ $\sum_{k=1}^{\infty}\frac{H_k}{k!k^2}$

1 .2278740270406456302... ≈ $\sum_{k=1}^{\infty}\frac{1}{k!\zeta(k+1)}$

1 .227947177299515679... ≈ $\log(2+\sqrt{2}) = \Phi\left(\frac{1}{\sqrt{2}},1,0\right)$

.228062398169704735632... ≈ $\sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)}{2k+1} = \sum_{k=1}^{\infty}\left(\sqrt{k}\arctan\frac{1}{\sqrt{k}}-\frac{3k-1}{3k}\right)$

2 .2281692032865347008... ≈ $\frac{7}{\pi}$

16 .2284953398496993218... ≈ $e^2\left(1+\sqrt{\frac{\pi}{2}}erf\sqrt{2}\right) = \sum_{k=0}^{\infty}\frac{2^k}{k!!}$

$.2285714\underline{285714}$ = $\frac{8}{35} = \prod_{p\ prime}\frac{\left(1-p^{-2}\right)^2}{1+p^{-2}+p^{-4}}$

2 .22869694419946472103... ≈ $\sum_{k=2}^{\infty}\left(\frac{\zeta^2(k)}{\zeta(2k)}-1\right) = \sum_{s=2}^{\infty}\sum_{k=2}^{\infty}\frac{2^{\omega(k)}}{k^s} = \sum_{k=2}^{\infty}\frac{2^{\omega(k)}}{k(k-1)}$

.22881039760335375977... ≈ $\frac{\pi^2}{2}\zeta(3)-\frac{11\zeta(5)}{2} = MHS(3,2) = \sum_{k>j\geq 1}^{\infty}\frac{1}{k^3j^2} = MHS(2,2,1)$

.22886074213595258118... ≈ $\frac{3\log^2 2}{4} - \frac{\pi}{2} + \frac{7\pi^2}{48} = \int_1^\infty \frac{\log(1+x^2)}{x^4+x^3}dx$

.22899139854430475 38... ≈ $\frac{G}{4} = \sum_{k=0}^\infty \frac{(-1)^k}{(4k+2)^2}$

= $\sum_{k=1}^\infty \left(\frac{1}{(8k-6)^2} - \frac{1}{(8k-2)^2} \right)$

= $\int_0^\infty \frac{x\,dx}{e^{2x}+e^{-2x}} = \int_1^\infty \frac{\log x\,dx}{x^3+x^{-1}}$

= $\int_1^\infty \frac{x\log x}{1+x^4}dx$

.2290126440163087258... ≈ $\frac{e}{e+1}(\log(e+1)-1) = \sum_{k=1}^\infty \frac{(-1)^{k+1}H_k}{e^k}$

.22931201355922031619... ≈ $\sum_{k=1}^\infty \frac{H^{(2)}{}_k}{5^k k}$

.22933275684053178149... ≈ $\frac{7\zeta(3)}{8} - \frac{\pi^2}{12} = \int_1^\infty \frac{\log^2 x}{(x+1)^2(x-1)}dx$

.229337572693540946... ≈ $-\frac{1}{2}\log\left(1-\frac{1}{e}\right) = \sum_{k=1}^\infty \frac{1}{2e^k k}$

.2293956465190967515... ≈ $I_0(2\sqrt{e}) = {}_0\mathrm{F}_1(;1;e) = \sum_{k=0}^\infty \frac{e^k}{(k!)^2}$

.22953715453852182998... ≈ $\frac{3}{2} - \gamma - \log 2 = \sum_{k=2}^\infty (-1)^k \frac{k-1}{k}\big(\zeta(k)-1\big)$

= $\sum_{k=2}^\infty \left(\log\left(1+\frac{1}{k}\right) - \frac{1}{k+1} \right)$

.22955420488734733958... ≈ $\frac{1}{8}\cos\frac{1}{2} + \frac{1}{4}\sin\frac{1}{2} = \sum_{k=1}^\infty \frac{(-1)^{k+1}k}{(2k-1)!\,4^k}$

.229637154538521829976...≈ $\frac{3}{2} - \gamma - \log 2 = \sum_{k=2}^\infty (-1)^k \frac{k-1}{k}\big(\zeta(k)-1\big)$

.22968134246639210945... ≈ $\frac{1}{2\sqrt{2}}\sin\frac{1}{\sqrt{2}} = \sum_{k=1}^\infty \frac{(-1)^{k+1}k}{(2k)!2^k}$

.2298488470659301413... ≈ $\sin^2\frac{1}{2} = \frac{1-\cos 1}{2} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{2(2k)!}$ GR 1.412.1

= $\int_1^\infty \sin\left(\frac{1}{x^2}\right)\frac{dx}{x^3}$

2 .2299046338314288180... ≈ $\sum_{k=1}^\infty \frac{\zeta(2k)}{(2k-2)!} = \sum_{k=1}^\infty \frac{1}{k^2}\cosh\frac{1}{k}$

.2299524263050534905... $\approx \frac{24}{25}+\frac{3\log 2}{5}-\frac{\pi}{10} = \sum_{k=1}^{\infty}\frac{1}{k(4k+5)}$

.2300377961276525287... $\approx \frac{\pi}{4}\frac{\sqrt{2}-1}{\sqrt{2}} = \int_0^1 \frac{x\arccos x\,dx}{(1+x^2)^2}$ GR 4.512.8

.2300810595330690538... $\approx -\frac{1}{4}\cos\frac{\pi\sqrt{5}}{2} = \prod_{k=1}^{\infty}\left(1-\frac{5}{(2k+1)^2}\right)$

.23017214130974243... $\approx \frac{1}{2e^{1/8}}\sqrt{\frac{\pi}{2}}erfi\left(\frac{1}{2\sqrt{2}}\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!\binom{2k}{k}2^k}$

.230202847471530986301... $\approx 16-\frac{26}{\sqrt{e}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!2^k(k+3)}$

.2302585092994045684... $\approx \frac{\log 10}{10}$

3 .2302781623234310862... $\approx \frac{1}{2}+2\pi\operatorname{csch}\frac{\pi}{2} = \sum_{k=0}^{\infty}\frac{(-1)^k}{4k^2+1}$

.23027799638651103535... $\approx \sum_{k=1}^{\infty}\frac{1}{4^k\zeta(2k)}$

.23030742859234663119... $\approx \sum_{s=2}^{\infty}\sum_{k=0}^{\infty}\sum_{j=1}^{\infty}\frac{1}{\left(2^k3^j\right)^s}$

.23032766854168419192... $\approx \frac{5-\sqrt{5}}{12} \prec \sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{k+2}\binom{2k}{k}$

.23032943298089031951... $\approx \frac{1}{\sqrt{6\pi}}$

.230389497291991018199... $\approx \frac{1}{4}\left(\psi\left(\frac{-1-i}{2}\right)+\psi\left(\frac{-1+i}{2}\right)-\psi\left(\frac{i}{2}\right)-\psi\left(-\frac{i}{2}\right)\right)$

$= \int_0^{\infty}\frac{\cos x}{e^x\left(e^x+1\right)}dx$

.230636794909050220202... $\approx G\sqrt{2}-\frac{\pi^2}{8}+\frac{1}{32}\left(\psi^{(1)}\left(\frac{1}{8}\right)+\psi^{(1)}\left(\frac{3}{8}\right)\right) = \int_0^{\pi/4}\frac{x\,dx}{\sin x+\cos x}$

.2306420746215602059... $\approx \gamma-\frac{\log 2}{2} = \int_0^{\infty}\left(\frac{1}{1+2x^2}-\cos x\right)\frac{dx}{x}$

$.230769230769\underline{230769}$ $= \frac{3}{13} = \int_0^{\infty}\frac{\sin^5 x\,dx}{e^x}$

9 .2309553591249981798... $\approx \sum_{k=1}^{\infty}\frac{k^4}{k^k}$

.2310490601866484365... ≈ $\frac{\log 2}{3} = \sum_{k=0}^{\infty} \frac{(-1)^k}{3k+3} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\binom{2k}{k} 2^k k}$ J292

= $\int_1^{\infty} \frac{dx}{x^4+x} = \int_0^{\infty} \frac{dx}{(3x+1)(3x+2)} = \int_0^{\infty} \frac{dx}{e^{2x}+3}$

1 .23106491129307991381... ≈ $\cosh\left(\cosh\left(\frac{\pi}{2}\right) - \sinh\left(\frac{\pi}{2}\right)\right) + \sinh\left(\cosh\left(\frac{\pi}{2}\right) - \sinh\left(\frac{\pi}{2}\right)\right)$

= $e^{i^i} = e^{e^{-\pi/2}}$

.231085022619546563... ≈ $\sum_{k=2}^{\infty} \frac{\zeta(k)}{k!k^2}$

1 .231141930209041686814... ≈ $\frac{\pi^6}{1890} + \frac{(\zeta(3))^2}{2} = \sum_{k=1}^{\infty} \frac{H^{(3)}{}_k}{k^3}$

.231274970101998413257... ≈ $\sum_{k=1}^{\infty} \log(\zeta(2k+1))$

.2312995750804091368... ≈ $\sum_{k=0}^{\infty} \frac{k!!}{(k+3)!}$

1 .23145031894093929237... ≈ $\frac{\pi^2}{12} + 2\log 2 - \gamma(1+\log 2) = \sum_{k=1}^{\infty} \frac{(k+1)H_k}{2^k k}$

.23153140126682770631... ≈ $\frac{1}{\pi e^{1/\pi}} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}k}{k!\pi^k}$

.2316936064808334898... ≈ $Ai\left(\frac{1}{2}\right)$

.23172006924572925... ≈ $\sum_{k=2}^{\infty} \frac{k-1}{k^2}(\zeta(k)-1) = -\sum_{k=2}^{\infty}\left(\log\left(1-\frac{1}{k}\right) + Li_2\left(\frac{1}{k}\right)\right)$

2 .23180901896315164115... ≈ $\sum_{k=2}^{\infty}\left(\frac{1}{2-\zeta(k)} - 1\right)$

.2318630313168248976... ≈ $2\log 2 - 2\gamma = \sum_{k=0}^{\infty} \frac{\psi(k+1)}{2^k}$

1 .2318630313168248976... ≈ $2\log 2 - 2\gamma + 1 = \sum_{k=1}^{\infty} \frac{\psi(k)k}{2^k}$

2 .2318630313168248976... ≈ $2 + 2\log 2 - 2\gamma = \sum_{k=0}^{\infty} \frac{k\,\psi(k+1)}{2^k}$

1 .23192438217929238150... ≈ $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{\sigma_1(k)\mu(k)}{2^k - 1}$

.2319647590645935870... ≈ $\frac{9e}{2}-12 = \sum_{k=0}^{\infty}\frac{k}{k!(2k+8)}$

.23225904734361889414... ≈ $\frac{\pi}{6}-\frac{2}{9}K(-1) = \int_0^1 x^2\arcsin(x^2)\,dx$

.2322682280657035591... ≈ $\frac{351-14\pi\sqrt{3}-126\log 3}{588} = \sum_{k=1}^{\infty}\frac{1}{k(3k+7)}$

.2323905146006299... ≈ $\sum_{k=2}^{\infty}(-1)^k\frac{\log k}{k(k-1)}$

.232609077617500793647... ≈ $\frac{6\zeta(3)}{\pi^3} = \int_0^{\infty}\frac{dx}{e^{\pi x^{1/3}}-1}$

1 .2329104505535085664... ≈ $16\zeta(3)-18 = \int_0^{\infty}\frac{x^2\,dx}{e^x\left(e^{x/2}-1\right)}$

3 .2329104505535085664... ≈ $16(\zeta(3)-1) = \int_0^1\frac{\log^2 x\,dx}{1-\sqrt{x}}$ GR 4.512.8

= $16\zeta(3)-16 = \int_0^{\infty}\frac{x^2\,dx}{e^{x/2}\left(e^{x/2}-1\right)}$

.23300810595330690538... ≈ $-\frac{1}{4}\cos\frac{\pi\sqrt{5}}{2} = \prod_{k=1}^{\infty}\left(1-\frac{5}{(2k+1)^2}\right)$

4 .23309653956994697203... ≈ $\frac{\left(2\sqrt{2}-1\right)\sqrt{\pi}}{2}\zeta\left(\frac{3}{2}\right) = i\psi^{(1/2)}\left(\frac{1}{2}\right)$

.2331107569002249049... ≈ $\zeta(3)-\frac{\pi^3}{32}$

.233207814761447350348... ≈ $\frac{\pi}{4}-\frac{\pi^2}{48}-\frac{\log 2}{2} = -\int_0^1 \arctan x\log x\,dx$ GR 4.593.1

.23333714989369979863... ≈ $-\sum_{k=1}^{\infty}\frac{\mu(2k)}{4^k}$

1 .2333471496549337876... ≈ $1+\pi^3\coth\pi\,\csc h^2\pi = -\operatorname{Im}\left\{\psi^{(2)}(i)\right\}$

1 .2334031175112170571... ≈ $\operatorname{arcsinh}\frac{\pi}{2}$

1 .233527691640341023900... ≈ $\sum_{k=0}^{\infty}I_k(1)\,J_k(1)$

.2337005501361698274... ≈ $\frac{\pi^2}{8}-1 = \sum_{k=2}^{\infty}\frac{1}{(2k-1)^2} = \sum_{k=2}^{\infty}\frac{(k-1)(\zeta(k)-1)}{2^k}$

$= \quad \sum_{k=1}^{\infty} \frac{(k-1)!(k-1)!2^k}{(2k)!}$

$= \quad -\int_0^1 \frac{x^2 \log x}{1-x^2} dx$

$= \quad \int_1^{\infty} \frac{\log x}{x^4 - x^2} dx$

1 .2337005501361698274... $\approx \quad \frac{\pi^2}{8} = \sum_{k=2}^{\infty} \frac{k^2(k^2+1)}{(k^2-1)^3} = \sum_{k=0}^{\infty} \frac{k!}{(2k+1)!!(k+1)}$ J276

$= \quad Li_2(1+i) + Li_2(1-i)$

$= \quad \lambda(2) = \frac{3\zeta(2)}{4} = \sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} = \sum_{k=1}^{\infty} k^2(\zeta(2k)-1)$

$= \quad \sum_{k=1}^{\infty} \left(\frac{1}{(4k-1)^2} + \frac{1}{(4k-3)^2} \right)$

$= \quad \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k} k^2}$

$= \quad \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} \frac{\zeta(k+j)}{2^{k+j}}$

$= \quad \int_0^1 \frac{\log x \, dx}{x^2 - 1}$ GR 4.231.13

$= \quad \int_0^{\infty} \frac{\log x \, dx}{x^4 - 1}$

$= \quad \int_0^{\infty} \frac{\arctan x \, dx}{1+x^2}$

$= \quad \int_0^{\infty} \frac{\arctan x^2 \, dx}{1+x^2}$ GR 4.538.1

$= \quad \int_0^{\infty} \frac{\arcsin x \, dx}{\sqrt{1-x^2}}$

$= \quad \int_0^1 K(k) \frac{dk}{k+1}$ GR 6.144

$= \quad \int_0^1 E(k') \, dk$ GR 6.148.2

.2337719376128496402... $\approx \quad \sum_{k=2}^{\infty} \frac{\zeta(k)}{k! 2^k} = \sum_{k=1}^{\infty} \frac{\zeta(k)}{(2k)!!} = \sum_{k=1}^{\infty} \left(e^{1/2k} - 1 - \frac{1}{2k} \right)$

$$.23384524559381660957\ldots \approx \frac{1}{4}\,PFQ\left[\left\{\frac{2}{3}\right\},\left\{\frac{3}{2},\frac{5}{3}\right\},-\frac{1}{4}\right] = \int_1^\infty \sin\left(\frac{1}{x^3}\right)\frac{dx}{x^2}$$

$$.23405882321255763058\ldots \approx \frac{1}{2}+\gamma-\frac{\pi}{2\sqrt{2}}\cot\frac{\pi}{\sqrt{2}}+\frac{1}{2}\left(\psi\left(1+\frac{1}{\sqrt{2}}\right)+\psi\left(1-\frac{1}{\sqrt{2}}\right)\right)$$

$$= \sum_{k=1}^\infty \frac{k-1}{2k^3-k} = \sum_{k=1}^\infty \frac{\zeta(2k)-\zeta(2k+1)}{2^k}$$

$$.2341491301348092065\ldots \approx \sum_{k=1}^\infty \frac{1}{6^k-1} = \sum_{k=1}^\infty \frac{\sigma_0(k)}{6^k}$$

$$.23416331197556167758 6\ldots \approx \frac{2}{9}\left(\psi^{(1)}\left(\frac{1}{3}\right)-\psi^{(1)}\left(\frac{2}{3}\right)-\pi\sqrt{3}\log 3\right) = \sum_{k=1}^\infty \frac{H_k}{\binom{2k}{k}(2k+1)}$$

$$.23416739426215481494\ldots \approx \sum_{k=1}^\infty \frac{\zeta(3k)}{6^k} = \sum_{k=1}^\infty \frac{1}{36k^6-6k^3}$$

$$.23437505960464477539\ldots \approx \sum_{k=0}^\infty \frac{(-1)^k}{2^{k!}}$$

$$.23448787651535460351\ldots \approx \frac{3}{2}-\log 2\sqrt{\pi} = \sum_{k=1}^\infty \frac{\zeta(2k)-1}{2k+1} = \sum_{k=2}^\infty \left(k\ \mathrm{arctanh}\frac{1}{k}-1\right)$$

$$.23460816051643033475\ldots \approx \sum_{k=2}^\infty \frac{\log k}{k^3-k}$$

$$.23474219154782710282\ldots \approx -\gamma-\frac{\zeta(3)}{6}-\frac{1}{2}\left(\psi\left(1+\frac{1}{\sqrt{6}}\right)+\psi\left(1-\frac{1}{\sqrt{6}}\right)\right)$$

$$= \sum_{k=1}^\infty \frac{1}{36k^5-6k^3} = \sum_{k=1}^\infty \frac{\zeta(2k+1)}{6^k}$$

$$.2348023134420334486\ldots \approx 1-J_0(1) = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{(k!)^2\,4^k}$$

$$.2348485056670728727\ldots \approx \frac{\pi^4}{16}-16 = \zeta\left(4,\frac{3}{2}\right) = \sum_{k=1}^\infty \frac{1}{(k+\frac{1}{2})^4}$$

$$.23489547178539975478 2\ldots \approx \frac{1}{729}\left(\psi^{(2)}\left(\frac{4}{9}\right)-\psi^{(2)}\left(\frac{7}{9}\right)+16\pi^3\left(\cos\frac{\pi}{18}-2\sqrt{3}\sin\frac{\pi}{18}\right)\right)$$

$$= \int_1^\infty \frac{\log^2 x\ dx}{x^3+1+x^{-3}}$$

$$.23500181462286777683\ldots \approx \frac{3\log 2}{2}-\frac{\log 5}{3} = \int_1^2 \frac{dx}{x^3+x}$$

$$.23500807155578588862\ldots \approx \frac{2\sin 1}{5+4\cos 1} = \sum_{k=1}^\infty (-1)^{k+1}\frac{\sin k}{2^k}$$

.235028714066989... ≈ $\frac{1}{36}\left(\pi^2-12+\pi\sqrt{3}+2\pi\sin\sqrt{3}\right)-\csc\frac{\pi\sqrt{3}}{2}\sec\frac{\pi\sqrt{3}}{2}$

= $\sum_{k=2}^{\infty}\frac{(-1)^k}{k^4-3k^2}$

.2352941176470588 = $\frac{4}{17}=\int_0^{\infty}\frac{\sin 4x\,dx}{e^x}$

= $\frac{1}{2\cosh(\log 4)}=\sum_{k=0}^{\infty}(-1)^k e^{-(2k+1)2\log 2}$ J943

1 .2354882677465134772... ≈ $3\pi\,\mathrm{sech}\left(\frac{\pi\sqrt{3}}{2}\right)=\prod_{k=2}^{\infty}\frac{k^3}{k^3-1}=\exp\sum_{k=1}^{\infty}\frac{\zeta(3k)-1}{k}$

= $\exp\sum_{k=2}^{\infty}\log\frac{k^3}{k^3-1}$

= $\sum_{k=1}^{\infty}\frac{z(k)}{k^3}$

= $3\Gamma\left((-1)^{1/3}\right)\Gamma\left(-(-1)^{2/3}\right)=\Gamma\left(\frac{5+i\sqrt{3}}{2}\right)\Gamma\left(\frac{5-i\sqrt{3}}{2}\right)$

.23549337339183675461... ≈ $\frac{3\zeta(3)}{4}-2\log^3 2=\int_0^1\frac{\log^3(1+x)}{x^2}dx$

.23550021965155579081... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^k(2^k+1)}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^{k+1}-1}$

.2357588823428846432... ≈ $\frac{2}{e}-\frac{1}{2}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!(k+2)}=\sum_{k=2}^{\infty}\frac{(-1)^k k}{(k+1)!}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k+1)!+k!}$

.23584952830141509528... ≈ $\frac{\sqrt{89}}{40}$ CFG D1

.235900297686263453821... ≈ $Li_2\left(-\frac{1}{4}\right)=\frac{1}{2}\left(4\log^2 2-4\log 2\log 5+\log^2 5+Li_2\left(\frac{1}{5}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{H_k}{5^k k}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{4^k k^2}$

= $-\int_0^1\frac{\log x}{x+4}dx$

.2359352947271664591... ≈ $(1-e)e^{1/e}+e=\sum_{k=1}^{\infty}\frac{k}{(k+1)!e^k}$

5 .23598775598298873077... ≈ $\frac{5\pi}{3}=\int_0^{\infty}\frac{dx}{1+x^{6/5}}$

.23605508940591339349... ≈ $1-\log\left(2(\sqrt{2}-1)\right)-\frac{2+\log 2}{2\sqrt{2}}=-\int_0^{\pi/4}\sin x\log(\sin x)dx$

.236067977499789696... ≈ $\sqrt{5} - 2 = \frac{1}{\varphi^3}$

1 .236067977499789696... ≈ $\sqrt{5} - 1 \prec \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2k-1} \binom{2k}{k}$

= $\sum_{k=0}^{\infty} \frac{\Gamma(3k+1)}{\Gamma(2k+2)k!8^k}$ Berndt Ch. 3, Eq. 15.6

2 .236067977499789696... ≈ $\sqrt{5} = 2\varphi - 1 = \varphi^3 - 2 = \sum_{k=0}^{\infty} \frac{1}{5^k} \binom{2k}{k}$

4 .236067977499789696... ≈ $\varphi^3 = 2\varphi + 1$

.2361484197761438437... ≈ $12 \log 2 + 9 \log 3 - 3\pi\sqrt{3} - \frac{\pi^2}{6} = \sum_{k=1}^{\infty} \frac{1}{6k^3 - k^2}$

2 .236204051641727403... ≈ $-\psi^{(2)}\left(\frac{5}{2}\right) = 2\left(-\frac{224}{27} + 7\zeta(3)\right)$

1 .2363225572368495083... ≈ $\frac{1}{4} - \frac{\pi}{2\sqrt{6}} \cot \pi \sqrt{\frac{2}{3}} = \sum_{k=1}^{\infty} \frac{1}{3k^2 - 2}$

.236352644292107892846... ≈ $\frac{1}{32} \Phi\left(-2, 3, \frac{1}{2}\right) = \int_1^{\infty} \frac{\log^2 x}{x^3 + 2x^{-1}}$

.23645341864792755189... ≈ $3 - 2\cos 1 - 2\sin 1 = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k)!(k+1)}$

1 .2365409530250961101... ≈ $\frac{3\sqrt{e}}{4} = \sum_{k=0}^{\infty} \frac{k^2}{k!2^k} = \sum_{k=0}^{\infty} \frac{k^2}{(2k)!!}$

1 .236583454845086541925... ≈ $\frac{\csc 3}{6} + \frac{1}{18} = \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{k^2\pi^2 - 9}$

.236585624515848405511... ≈ $2G + \frac{\pi^2}{12} - \pi \log 2 - \frac{\log^2 2}{2} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} H_k}{k(2k+1)}$

.2368775568501150593... ≈ $\frac{5\pi^3}{81\sqrt{3}} - \frac{13\zeta(3)}{18} = \frac{1}{216}\left(\psi^{(2)}\left(\frac{5}{6}\right) - \psi^{(2)}\left(\frac{1}{3}\right)\right)$

= $2\sum_{k=0}^{\infty} \frac{(-1)^k}{(3k+2)^3} = \int_0^{\infty} \frac{x^2 x dx}{e^{2x} + e^{-x}}$

= $\int_1^{\infty} \frac{\log^2 x}{x^3 + 1} dx$

.2369260389502474311... ≈ $\frac{25}{64\sqrt{e}} = \sum_{k=0}^{\infty} \frac{(-1)^{k+1} k^6}{k!2^k}$

2 .2369962865801349333... ≈ $\sum_{k=1}^{\infty}\frac{4^k}{\binom{2k}{k}k^4}$

.23722693183674748744... ≈ $\frac{9\zeta(3)-\pi^2}{4} = \int_1^{\infty}\frac{\log^3 x}{(x+1)^3} = -\int_0^1\frac{x\log^3 x}{(x+1)^3}$

.2374007861516191461... ≈ $\log(3-\sqrt{3})$

.23746299346156328589 8... ≈ $\frac{\pi}{2}-\frac{4}{3} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k+5/2}$

= $\int_0^{\pi/2}\frac{\sin^2 x}{(1+\sin x)^2}dx$

.2375599012791608147 5... ≈ $\frac{\pi^2}{24}-\frac{3}{4}\log^2\left(\frac{\sqrt{5}-1}{2}\right) = \chi_2\left(\sqrt{5}-2\right)$ Berndt Ch. 9

= $\frac{1}{4}PFQ\left[\left\{\frac{1}{2},1,1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},-\frac{1}{4}\right] = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{\binom{2k}{k}(4k^2+2k)}$

9 .2376043070340122321... ≈ $\frac{16\sqrt{3}}{3}$ CFG D11

17 .2376296213703045782... ≈ $\sum_{k=1}^{\infty}\frac{k^3}{(k+1)!!}$

.2376534800363119539 6... ≈ $\frac{\pi^2}{30}-\frac{137}{1500} = \sum_{k=1}^{\infty}\frac{1}{k^2(k+5)}$

.2376943865253026097... ≈ $\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}(\zeta(jk+3)-1)$

98 .2378571746915510161 4... ≈ $24\pi\left(\log^2 2+\frac{\pi^2}{12}\right) = \int_0^{\infty}\frac{e^x x^3\,dx}{\sqrt{(e^x-1)^3}}$ GR 3.455.2

.2378794283541037605... ≈ $\frac{1}{144}\left(\psi^{(1)}\left(\frac{1}{6}\right)-\psi^{(1)}\left(\frac{2}{3}\right)\right) = \int_1^{\infty}\frac{\log x\,dx}{x^3+x^{-3}}$

.2379275450005479544... ≈ $\zeta(2)-\zeta(3)+\gamma\zeta(2)-2\gamma = \sum_{k=1}^{\infty}\frac{\psi(k+1)}{k(k+1)^2}$

.2379961001986213019 9... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^3\log k} = \int_3^{\infty}\left(\zeta(s)-1\right)ds$

.2380351360576801492... ≈ $\sqrt{e}-\sqrt{\frac{e\pi}{2}}erf\left(\frac{1}{\sqrt{2}}\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!!}$

.238270508910354881... ≈ $8\sqrt{2}\operatorname{arctanh}\frac{1}{\sqrt{2}}-\frac{146}{15} = \sum_{k=0}^{\infty}\frac{1}{2^k(2k+7)}$

6 .238324625039507785... ≈ $9\log 2$

.2384058440442351119... ≈ $\frac{2}{e^2+1}$

9 .23851916587804425989... = $\frac{\pi^2(2\pi^2+3\log^2 2)}{16\sqrt{2}} = \int_0^\infty \frac{\log^3 x\,dx}{2x^2-1}$

1 .2386215874332069455... ≈ $\sum_{k=1}^\infty (-1)^{k+1}\frac{\zeta(2k)}{k!} = \sum_{k=1}^\infty \left(1-e^{-1/k^2}\right)$

.2386897959988965136... ≈ $\sum_{k=0}^\infty \frac{(-1)^k}{k^2-k+1}$

.23873241463784300365... ≈ $\frac{3}{4\pi}$

.238859029709734187498... ≈ $\frac{\pi}{32}\left(2\coth\frac{\pi}{2}+\pi\left(\pi\coth\frac{\pi}{2}-3\right)\operatorname{csc}h^2\frac{\pi}{2}\right) = \sum_{k=1}^\infty \frac{(-1)^{k+1}k^2\zeta(2k)}{4^k}$

.2388700094983357625... ≈ $\frac{1}{16}(e+\frac{3}{e}) = \sum_{k=1}^\infty \frac{k^3}{(2k+1)!}$

2 .2389074401461054137... ≈ $\sum_{k=1}^\infty \frac{k\,H^{(3)}{}_k}{2^k}$

2 .23898465830296421173... ≈ $\zeta(3)+\zeta(5)$

.2391336269283829281... ≈ $2\cos 1-\sin 1 = \sum_{k=0}^\infty \frac{(-1)^k}{(2k)!(2k+3)} = \sum_{k=0}^\infty (-1)^k\frac{(2k+2)}{(2k)!}$

= $\int_1^e \frac{\log^2 x\,\cos\log x}{x}\,dx$

.2392912109915616173... ≈ $Ei(-1)-\log(e-1)+1 = \gamma_{1/ke^k}$

.23930966947430038807... ≈ $\sum_{k=2}^\infty \frac{\Omega(k^3)}{k^3}$

= $\sum_{k=2}^\infty \frac{1}{k^3-1} + \sum_{k=2}^\infty \frac{1}{k^6-k^{-3}} + \sum_{k=2}^\infty \frac{1}{k^{12}-k^3}$

= $\sum_{k=1}^\infty \left(\zeta(3k)-1\right) + \sum_{k=1}^\infty \left(\zeta(9k-3)-1\right) + \sum_{k=1}^\infty \left(\zeta(9k+3)-1\right)$

.2394143885900714409... ≈ $\frac{1}{8}-\frac{\cot 2}{4} = \sum_{k=1}^\infty \frac{1}{\pi^2k^2-4}$

2 .2394673388969121878... ≈ $2\zeta(3)-2\zeta(4)$

.239560747340741949878... ≈ $\frac{1}{6}\left(2-\pi\cot\frac{\pi(3-i\sqrt{3})}{4}-\pi\cot\frac{\pi(3+i\sqrt{3})}{4}-2\log 2\right)$

= $\sum_{k=1}^\infty \frac{(-1)^k k^2}{k^3+1}$

.2396049490072432625... ≈ $\log\frac{\sqrt{e}}{2(\sqrt{e}-1)}$ J157

.2396540036990536524 7... ≈ $1-\frac{2\sqrt{3}}{3}\operatorname{arcsinh}\frac{1}{\sqrt{2}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}2^k}{\binom{2k}{k}(2k+1)}$

.2396880802440039427... ≈ $\sum_{k=1}^{\infty}\frac{1}{k^3+8}$

.2397127693021015001... ≈ $\frac{1}{2}\sin\frac{1}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{4^k(2k-1)!}$

1 .23971953814622741196 5... ≈ $\sum_{k=1}^{\infty}\frac{1}{\sigma_k(k)}$

.2397469173053871842... ≈ $\zeta''(3) = \sum_{k=1}^{\infty}\frac{\log^2 k}{k^3}$

.2397554029243698487... ≈ $2\sin^2\frac{1}{2\sqrt{2}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k)!2^k}$

.2397640410755053514 2... ≈ $\frac{1}{2}J_2(2\sqrt{2}) = \frac{1}{2}\,{}_0\mathrm{F}_1(;3;2) = \sum_{k=0}^{\infty}\frac{(-1)^k 2^k}{k!(k+2)!}$

.2398117420005647259... ≈ $\gamma - ci(1) = -\int_0^1 \log x\,\sin x\,dx$ GR 4.381.1

= $\sum_{k=1}^{\infty}\frac{(-1)^k}{(2k!)2k}$

.2398886294498805764 94... ≈ $\frac{7}{4}-\log\pi+12\zeta'(-2) = \frac{7}{4}-\log\pi+\frac{3\zeta(3)}{\pi^2}$

= $\sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{k+2}$

= $-\sum_{k=2}^{\infty}\left(k^4\log\left(1-\frac{1}{k^2}\right)+k^2+\frac{1}{2}\right)$

1 .2399427971695997118 1... ≈ $\frac{1}{3}+\frac{\pi^2}{30}+\frac{5\log 2}{6} = \int_0^1\frac{\log^2 x}{(x+1)^6}dx$

.2399635244956309553... ≈ $\zeta\left(\frac{1}{2},\frac{1}{4}\right)$

.240000000000000000$\underline{0}$ = $\frac{6}{25}=\Phi\left(\frac{1}{6},-1,0\right) = \sum_{k=1}^{\infty}\frac{k}{6^k}$

.2402265069591007123 3... ≈ $\frac{1}{2}\log^2 2 = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{H_k}{k+1} = \sum_{k=1}^{\infty}\frac{H_k}{2^{k+1}(k+1)}$

$= \quad \displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\binom{2k}{k} 2^k k^2}$

$= \quad \displaystyle\int_0^1 \frac{\log(1+x)}{1+x} dx$ GR 4.791.6

$= \quad \displaystyle\int_0^1 \frac{\log(1-x)}{1+x} dx = \int_1^2 \frac{\log x}{x} dx = -\int_0^{\infty} \frac{\log x}{e^x+1}$

.2402290139165550493... $\approx$ $\displaystyle 2\log(1+e) - 2\log 2 - 1 = \int_0^1 \frac{e^x-1}{e^x+1} dx$

1 .24025106721199280702... $\approx$ $\displaystyle \frac{\pi^3}{25} = \frac{1}{100}\left(\psi^{(2)}\left(\frac{1}{4}\right) - \psi^{(2)}\left(\frac{3}{4}\right)\right)$

.2404113806319188571... $\approx$ $\displaystyle \frac{\zeta(3)}{5}$

.2404882110038654635... $\approx$ $\displaystyle \sum_{k=1}^{\infty} (-1)^{k+1} \frac{H^{(3)}{}_k}{3^k}$

1 .2404900146990321114... $\approx$ $\displaystyle \binom{1}{1/3} = \binom{1}{2/3}$

.24060591252980172375... $\approx$ $\displaystyle \frac{1}{2}\operatorname{arcsinh}\frac{1}{2} = \log 2 - \frac{\log\left(24 - 8\sqrt{5}\right)}{4} = \sum_{k=0}^{\infty} \frac{(-1)^k}{4^{2k+1}(2k+1)} \binom{2k}{k}$

.2408202605290378657... $\approx$ $\displaystyle 1 + \zeta(2) - 2\zeta(3) = \sum_{k=1}^{\infty} \frac{k}{(k+2)^3}$

$= \quad \displaystyle \sum_{k=1}^{\infty} (-1)^{k+1} k^2 (\zeta(k+1) - 1)$ Berndt 5.8.5

2 .240903291691171216051... $\approx$ $\displaystyle \sum_{k=1}^{\infty} \frac{1}{(k-1)!\zeta(2k)}$ Titchmarsh 14.32.1

.241018750885055611796... $\approx$ $\displaystyle \frac{2\log 3}{3} - \frac{\pi\sqrt{3}}{6} - \frac{1}{2} = \sum_{k=1}^{\infty} \frac{\zeta(k+1) - 1}{3^k}$

.241029960180470772184... $\approx$ $\displaystyle \frac{\pi}{2} - \frac{\pi^2}{24} - \log 2 - \frac{3\zeta(3)}{16} = \int_0^1 \arctan x \log^2 x\, dx$

.241172868732950643445... $\approx$ $\displaystyle 2 - 2\sqrt{2} - \log 2 + 2\operatorname{arcsinh} 1 = -\int_0^1 \operatorname{arcsinh} x \log x\, dx$

.24120041818608921979... $\approx$ $\displaystyle \frac{1}{6}\sqrt{\frac{2\pi}{3}} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(k - \frac{1}{2})!}{(k-1)!\, 2^k}$

13 .2412927053104041794... $\approx$ $\displaystyle \frac{257\sqrt{e}}{32} = \sum_{k=1}^{\infty} \frac{k^5}{k!2^k}$

.241325348385993028026... ≈ $\frac{\pi^2\gamma}{12} - \frac{\gamma\log^2 2}{2} + \frac{\log^3 2}{6} - \frac{\zeta(3)}{8} = \sum_{k=1}^{\infty}\frac{\psi(k)}{2^k k^2}$

.2414339756999316368... ≈ $\frac{5\log 2}{4} - \frac{5}{8} = \sum_{k=2}^{\infty}\frac{k}{2^k(k^2-1)}$

.24145300700522385466... ≈ $\frac{1}{\pi+1}$

.241564475270490445... ≈ $\log\frac{4}{\pi} = -\int_0^1 \frac{(1-x)^2}{1+x^2}\frac{dx}{\log x}$ GR 4.267.2

= $\int_0^1 \frac{x\log x - x + 1}{x\log^2 x}\log(1+x)dx$ GR 4.314.3

.24160256000655360000... ≈ $-\sum_{k=1}^{\infty}\frac{\mu(2k)}{5^k-1} = \sum_{k=1}^{\infty}\frac{1}{(\sqrt{5})^{2^k}}$

1 .24200357980307846214... ≈ $\sqrt{\frac{15-\sqrt{33}}{6}}$ Associated with a sailing curve. Mel1 p.153

1 .24203498624037301976... ≈ $\frac{\pi^2(1+e)}{e(1+\pi^2)} = \int_0^{\pi} e^{-x/\pi}\sin x\,dx$

.24203742082351294482... ≈ $\prod_{k=2}^{\infty}(2-\zeta(k))$

1 .2420620948124149458... ≈ $\sum_{k=1}^{\infty}\frac{1}{(2^k-1)k} = \sum_{k=1}^{\infty}\log\frac{2^k}{2^k-1} = -\sum_{k=`}^{\infty}\log(1-2^{-k}) = \sum_{k=1}^{\infty}\frac{\sigma_{-1}(k)}{2^k}$

= $\sum_{k=1}^{\infty}\frac{\sigma_1(k)}{2^k k}$

.242080571455143661373... ≈ $\log\frac{3^{27/8}}{32} = \int_0^{\infty}\frac{\sin^8 x}{x^5}dx$

.2420958985825988417 8... ≈ $\frac{\pi^2}{6}(\gamma-1) - \frac{\zeta'(2)}{6} = -\int_0^{\infty}\frac{x\log x}{e^x-1}dx$

.2421491872872994446 1... ≈ $\frac{112-9\pi^2-24\log 2}{27} = \int_0^1 \sqrt{x}\log(1-x)\log x\,dx$

.242156981956213712584... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\mu(k)}{4^k+1}$

.2422365365648423451... ≈ $\frac{\pi^3}{128} = \int_0^{\infty}\frac{x^2dx}{e^{2x}+e^{-2x}} = \int_1^{\infty}\frac{\log^2 x\,dx}{x^3+x^{-1}}$

.24230006367431704... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^k}{k^3+2}$

.242345452227360949... ≈ $c_2 = \frac{1}{6}(3\gamma\zeta(2) - \zeta(3) - \gamma^3)$ Patterson Ex. A4.2

.24241455349784382316... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}H^{e}{}_{k}}{k!}$

3 .24260941092524821060... ≈ $\sum_{k=2}^{\infty}\frac{1}{(\log k)^{k}}$

.24264068711928514641... ≈ $3\sqrt{2}-4=\sum_{k=1}^{\infty}\frac{k}{8^{k}(k+1)}\binom{2k}{k}$

4 .24264068711928514641... ≈ $\sqrt{18} = 3\sqrt{2}$

.24270570106525549741... ≈ $\sum_{k=1}^{\infty}\left(Li_{k}\left(\frac{k-1}{k}\right)-\frac{k-1}{k}\right)$

2 .24286466001463290864... ≈ $8+\pi^{2}-13\zeta(3)=\sum_{k=1}^{\infty}\frac{8k^{4}-3k^{3}-k^{2}-3k-1}{k^{3}(k+1)^{3}}$

= $\sum_{k=1}^{\infty}(-1)^{k}k^{3}(\zeta(k)-\zeta(k+2))$

.24288389527403994851... ≈ $\zeta^{2}(3)-\zeta(3)$

.243003037439321231363... ≈ $\frac{\pi}{\sqrt{3}}-\frac{\pi}{2}=\sum_{k=1}^{\infty}\frac{(k-\frac{1}{2})!(k+\frac{1}{2})!}{(2k+1)!}$

= $\int_{0}^{1}\frac{1-x^{2}}{x^{2}}\arctan(x^{3})dx$

.24314542372036488714... ≈ $\frac{1126}{9^{2}\cdot 7\cdot 5}-\frac{2\log 2}{9} = \sum_{k=1}^{\infty}\frac{1}{k(2k+9)}$

2 .2432472337755517756... ≈ $\frac{\pi^{4}}{120}+\frac{7\pi^{2}}{48}-\frac{1}{128}$

= $\sum_{k=1}^{\infty}k^{3}(\zeta(2k)-1) = \sum_{k=2}^{\infty}\frac{k^{2}(k^{4}+k^{2}+1)}{(k^{2}+1)^{4}}$

.2432798195308605298... ≈ $\frac{e-2}{(e-1)^{2}}=\sum_{k=0}^{\infty}\frac{(k+1)\mathrm{B}_{k}}{k!}$

.24346227069171501162... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-\zeta(k+1)}{k!} = \sum_{k=1}^{\infty}\left(e^{1/k}-\frac{e^{1/k}}{k}+\frac{1}{k^{2}}-1\right)$

.24360635350064073424... ≈ $5\sqrt{e}-8=\sum_{k=1}^{\infty}\frac{k}{k!2^{k}(k+2)}$

8 .2436903475949711355... ≈ $9G$

.2437208648653150558... ≈ $8\log\frac{3}{2}-3 = \Phi\left(-\frac{1}{2},1,3\right) = \sum_{k=0}^{\infty}\frac{(-1)^{k}}{2^{k}(k+3)}$

.2437477471996805242... ≈ $\frac{\pi}{4\sqrt{2}}-\frac{\log(1+\sqrt{2})}{2\sqrt{2}} = \sum_{k=0}^{\infty}\frac{(-1)^{k}}{4k+3} = \int_{1}^{\infty}\frac{dx}{x^{4}+1} = \int_{0}^{1}\frac{x^{2}dx}{x^{4}+1}$

= $\displaystyle\int_0^\infty \frac{dx}{3e^x+e^{-x}}$

.243831369344327582636... ≈ $\displaystyle\frac{7}{2}-\frac{1}{2}\pi\sqrt{\frac{3}{2}}\cot\pi\sqrt{\frac{3}{2}} = \sum_{k=1}^\infty \left(\frac{3}{2}\right)^k \bigl(\zeta(2k)-1\bigr)$

.24393229000971240854... ≈ $\displaystyle\sum_{k=1}^\infty k(\zeta(3k)-1) = \sum_{k=2}^\infty \frac{1}{k^3\left(1-k^{-3}\right)^2}$

.2440377410938141863... ≈ $\gamma(1-\gamma)$

5 .24411510858423962093... ≈ $\displaystyle\frac{1}{\sqrt{2\pi}}\Gamma^2\left(\frac{1}{4}\right)$, related to the lemniscate constant

.2441332351736490543... ≈ $\displaystyle\frac{1}{4}-\frac{\pi}{e^{2\pi}-e^{-2\pi}} = \frac{1}{4}-\frac{\pi\operatorname{csch}2\pi}{2} = 2\sum_{k=1}^\infty \frac{(-1)^{k+1}}{k^2+4}$

= $\displaystyle\int_0^\infty \frac{\sin 2x\,dx}{e^x+1}$

.24413606414846882031... ≈ $\displaystyle\frac{\log 9}{9} = \frac{2\log 3}{9}$ J137

.24433865716447323985... ≈ $\displaystyle\sum_{k=1}^\infty \frac{\psi\left(k-\tfrac{1}{2}\right)}{(2k)^2}$

.24470170753009738037... ≈ $\displaystyle 2-\pi+2\log 2 = \sum_{k=1}^\infty \frac{(2k-1)!!}{(2k)!\,k(2k+1)}$

.244795427738853473585... ≈ $\displaystyle\frac{\pi G}{8}+\frac{\pi^2\log 2}{32}-\frac{35\zeta(3)}{128} = -\int_0^{\pi/4} x\log\sin x\,dx$

.24483487621925456777... ≈ $\displaystyle 4\sin^2\frac{1}{4} = \sum_{k=1}^\infty \frac{1}{4^k(2k-1)k}$

.2449186624037091293... ≈ $\displaystyle\frac{\sqrt{e}-1}{\sqrt{e}+1}$ J148

.24497866312686415417... ≈ $\displaystyle\arctan\frac{1}{4} = \sum_{k=0}^\infty \frac{(-1)^k}{4^{2k+1}(2k+1)}$

6 .24499799839839820585... ≈ $\sqrt{39}$

.245010117125076991397... ≈ $\displaystyle\frac{\zeta(3)}{4}-\frac{\log^3 2}{6} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{\binom{2k}{k}2^k k^3}$

.2450881595266180682... ≈ $\displaystyle 6-\frac{\pi\sqrt{3}}{2}-\frac{3\log 3}{2}-2\log 2 = hg\left(\frac{1}{6}\right) = \sum_{k=1}^\infty \frac{1}{6k^2+k}$

= $\displaystyle\sum_{k=1}^\infty (-1)^{k+1}\frac{\zeta(k+1)}{6^k}$

= $-\int_0^1 \log(1-x^6)\ dx$

.24528120346676643... ≈ $\frac{\pi^2}{16}+\frac{\pi}{4}\log 2 - G = \int_0^{\pi/4} \frac{x^2 dx}{\cos^2 x}$ GR 3.857.3

= $\int_0^1 \arctan^2 x\, dx$

.2454369260617025968... ≈ $\frac{5\pi}{64} = \int_0^1 x^3 \arcsin x\, dx$

2 .245667095372459229031... ≈ $\frac{1}{72}\left(\pi^2+12\right)\left(\pi^2-12\zeta(3)\right)+\frac{7\zeta(5)}{2} = \sum_{k=1}^{\infty}\frac{H_k H_{k+1}}{k^3}$

.2457259411499884910 2... ≈ $\frac{\sin 1}{4}-\frac{1}{2}\sin 1\,\log\left(2-2\cos 1\right) = \sum_{k=2}^{\infty}\frac{\sin k}{k^2-1}$

1 .245730939615517326... ≈ $\sqrt[5]{3}$

2 .245762562305203038... ≈ $\frac{1}{2}-\frac{3}{4e^2}+\frac{e^2}{4} = \frac{1}{2}-\frac{\cosh 2}{2}+\sinh 2 = \sum_{k=0}^{\infty}\frac{4^k}{(2k)!(k+1)}$

.2458370070002374305... ≈ $\frac{1}{2}-\frac{\sin 1+\cos 1}{2e} = \int_0^1 \frac{\sin x\, dx}{e^x}$

= $\int_1^e \frac{\sin\log x}{x^2}dx$

.2458565798473464061 5... ≈ $\int_1^{\infty}\frac{dx}{e^x(1+\log x)}$

.2458855407471566264... ≈ $\zeta(2)+2\pi-16+12\log 2 = \sum_{k=1}^{\infty}\frac{1}{4k^3+k^2}$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(k+2)}{4^k}$

2 .2459952948352307... ≈ root of $\psi^{(1)}(x)=\frac{1}{2}$

.2460937497671693563... ≈ $-\sum_{k=1}^{\infty}\frac{\mu(2k)}{2^{2^k}}$

1 .2461220320130387106 6... ≈ $\frac{\pi^2}{6}+2(\log 3-2\log 2)\log 2 = Li_2\left(\frac{1}{4}\right)+Li_2\left(\frac{3}{4}\right)$

8 .2462112512353210996... ≈ $\sqrt{68} = 2\sqrt{17}$

.246272788760729064 4... ≈ $-\Phi\left(-\frac{1}{4},4,0\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{4^k k^4}$

.24644094118152729128... ≈ $\sum_{k=1}^{\infty}\frac{1}{k^4+k^3+k^2+k+1}$

1 .24645048028046102679... ≈ $\sqrt{2}\log(1+\sqrt{2})=\sqrt{2}\,\mathrm{arcsinh}\,1=\sqrt{2}\,\mathrm{arctanh}\,\frac{1}{\sqrt{2}}$

= $\frac{1}{\sqrt{2}}\log\frac{1+\sqrt{2}}{\sqrt{2}-1}$

= $\sum_{k=0}^{\infty}\frac{1}{2^k(2k+1)}=\sum_{k=1}^{\infty}\frac{H^{O}{}_k}{2^k}$

= $1+\sum_{k=1}^{\infty}(-1)^k\frac{(2k)!!}{(2k+3)!!}$ J141

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}(2k)!!}{(2k-1)!!k}$

= $\int_0^{\infty}\frac{dx}{(x+1)\sqrt{1+x^2}}$

= $\int_0^{1}\frac{dx}{(x+1)\sqrt{1-x}}$

= $\int_0^{\pi/2}\frac{dx}{\cos x+\sin x}$

.24656042443351259912... ≈ $\frac{\pi}{32}\left(\pi-\sqrt{2}\sin\frac{\pi}{\sqrt{2}}\right)=\sum_{k=1}^{\infty}\frac{k\zeta(2k)}{8^k}=\sum_{k=1}^{\infty}\frac{8k^2}{(8k^2-1)^2}$

.2465775113946629846... ≈ $-\sum_{k=1}^{\infty}\frac{\mu(2k)}{2^{k+1}-1}=\sum_{k=1}^{\infty}\frac{\mu(2k-1)}{4^k-1}$

.2466029308397481445... ≈ $\frac{3}{2}\log^2\frac{3}{2}=\sum_{k=1}^{\infty}\frac{H_k}{3^k(k+1)}$

.24664774656563557283... ≈ $\frac{5\pi^3}{648\sqrt{3}}+\frac{13\zeta(3)}{144}=\frac{1}{1728}\left(\psi^{(2)}\left(\frac{2}{3}\right)-\psi^{(2)}\left(\frac{1}{6}\right)\right)=\int_1^{\infty}\frac{\log^2 x}{x^3+x^{-3}}dx$

1 .2468083128715153704... ≈ $e-e\log(e-1)=\sum_{k=0}^{\infty}\frac{1}{e^k(k+1)}$

1 .2468502198629158993... ≈ $\mathrm{arcsec}\,\pi$

.2469158097729950883... ≈ $\sum_{k=1}^{\infty}\frac{1}{e^{k^2}}-\int_1^{\infty}\frac{dx}{e^{x^2}}$

3 .24696970113341457455... ≈ $\frac{\pi^4}{30}=-\int_0^1\int_0^1\int_0^1\frac{\log xyz}{1-xyz}dx\,dy\,dz$

.2470062502950185373... ≈ $\frac{\log 3}{2}-\frac{\pi}{6\sqrt{3}}=\sum_{k=1}^{\infty}\frac{1}{9k^2-3k}=\sum_{k=2}^{\infty}\frac{\zeta(k)}{3^k}=\int_1^{\infty}\frac{dx}{x^3+x^2+x}$

3 .2472180759044068717... ≈ $3\zeta(4)+\frac{\zeta(6)}{4096}=\frac{\pi^4}{30}+\frac{\pi^6}{3870720}=\sum_{k=1}^{\infty}\frac{1}{a(k)^6}$,

. where $a(k)$ is the nearest integer to $\sqrt[3]{k}$. AMM 101, 6, p. 579

.24734587314487935615... ≈ $\frac{1}{4\pi}(\gamma+\log 4\pi)=\int_0^1 \log\Gamma(x)\sin 4\pi x\,dx$ GR 6.443.1

.24739755275181306469... ≈ $\frac{1}{6\cdot 2^{2/3}}\left(\left(1+i\sqrt{3}\right)\psi\left(2+(-2)^{1/3}\right)+\left(1-i\sqrt{3}\right)\psi\left(2-(-1)^{2/3}2^{1/3}\right)-2\psi\left(2-2^{1/3}\right)\right)$

= $\sum_{k=2}^{\infty}\frac{1}{k^3-2}$

.2474039592545229296... ≈ $\sin\frac{1}{4}=\operatorname{Im}\left\{(-1)^{1/12}\right\}=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!4^{2k+1}}=\sum_{k=0}^{\infty}\frac{(-1)^k}{2(4k+2)!!\,4^k}$

.2474724535468611637... ≈ $\psi\left(\frac{7}{4}\right)=\frac{4}{3}-\gamma+\frac{\pi}{2}-3\log 2$

.24777401584773147... ≈ $\sum_{k=2}^{\infty}\frac{2^{\nu(k)}}{2^k-1}$

1 .24789678253165704516... ≈ $3G-\frac{3}{2}=\sum_{k=1}^{\infty}\frac{(-1)^k(12k)}{(4k^2-1)^2}$

2 .24789678253165704516... ≈ $3G-\frac{1}{2}=\sum_{k=1}^{\infty}\frac{(-1)^k k^3}{(4k^2-1/4)^3}$

.24792943928640702505... ≈ $\sum_{k=2}^{\infty}\mu(2k)\left(\zeta(k)-1\right)$

.24803846578347406657... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k^2-1}=\sum_{k=2}^{\infty}\frac{1}{2}+\frac{1}{4k}+\left(2k-\frac{1}{2k}\right)\log\left(1-\frac{1}{j}\right)$

.24805021344239856140... ≈ $\frac{\pi^3}{125}$

.24813666619074161415... ≈ $\frac{\pi}{2\sqrt{6}}\coth\frac{\pi}{\sqrt{6}}-\frac{1}{2}=\sum_{k=1}^{\infty}\frac{1}{6k^2+1}$

.24832930767207351026... ≈ $\gamma+\frac{1}{2}\left(\psi\left(1+\frac{i}{2}\right)+\psi\left(1-\frac{i}{2}\right)\right)=\frac{1}{2}\left(H\left(\frac{i}{2}\right)+H\left(-\frac{i}{2}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{1}{4k^3+k}$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k+1)}{4^k}=-\operatorname{Re}\left\{\sum_{k=1}^{\infty}\frac{\zeta(k+1)}{(2i)^k}\right\}$

.24843082992504495613... ≈ $8-\frac{\pi^3}{4}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k+1/2)^3}$

.24873117470336123904... ≈ $\frac{\pi}{2}\log\left(\frac{2\sqrt{2}}{1+2\sqrt{2}}\right)=\int_0^1\left(\frac{x}{x^2+1}\right)\arcsin x\,dx$ GR 4.521.3

.248754477033784262547... ≈ $\log A_1$, the log of Glaisher's constant

= $\frac{1}{12}\left(\gamma+\log 2\pi\right)-\frac{\zeta'(2)}{2\pi^2}$

.2488053033594882285... ≈ $\zeta(2)+3\zeta(4)-3\zeta(3)-\zeta(5)=\sum_{k=1}^{\infty}\frac{k^3}{(k+1)^5}$

2 .248887103715128928098... ≈ $\sinh^2\pi\ \csc\left(\pi\sqrt{\sqrt{2}-1}\right)\operatorname{csc}h\left(\pi\sqrt{\sqrt{2}+1}\right)=\prod_{k=1}^{\infty}\frac{k^4+2k^2+1}{k^4+2k^2-1}$

.24913159214626539868... ≈ $\frac{7\pi^4}{360}-\frac{\pi^2}{6}=\int_0^{\infty}\frac{x^4}{\cosh^4 x}dx$

1 .249367050523975265... ≈ $\sinh\frac{\pi}{3}$

.24944135064668743718... ≈ $\int_1^{\infty}\frac{\log x}{e^x-1}dx$

.2494607332457750217... ≈ $\frac{1}{12\cdot 2^{2/3}}\left(\left(1+i\sqrt{3}\right)\psi\left(1-\left(-\frac{1}{2}\right)^{1/3}\right)+\left(1-i\sqrt{3}\right)\psi\left(\frac{1}{2}\left(2+(-2)^{2/3}\right)\right)-2\psi\left(1+2^{-1/3}\right)\right.$

= $\sum_{k=1}^{\infty}\frac{1}{4k^3+1}=\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(3k)}{2^{2k}}$

.2498263975004615315... ≈ $\sin\frac{1}{2}\sinh\frac{1}{2}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(4k-2)!2^{2k-1}}$ GR 1.413.1

.2499045736175649326... ≈ $\int_3^{\infty}\left(e^{\zeta(x)-1}-1\right)dx$

$$.25000000000000000000\underline{0} = \frac{1}{4} = \sum_{k=1}^{\infty}\frac{1}{3k^2+6k} = \sum_{k=1}^{\infty}\frac{1}{4k^2+4k}$$

$$= \sum_{k=2}^{\infty}\frac{1}{(k-1)k(k+1)}$$ J268, K153

$$= \sum_{k=2}^{\infty}\frac{1}{k^3-k} = \sum_{k=1}^{\infty}(\zeta(2k)-1)$$

$$= \sum_{k=1}^{\infty}\frac{k}{4k^4+1}$$

$$= \sum_{k=2}^{\infty}\frac{2k+1}{k^2(k+1)^2} = \sum_{k=3}^{\infty}(-1)^{k+1}(k-1)(\zeta(k)-1)$$

$$= \sum_{k=2}^{\infty}\frac{(-1)^k}{k^2-1} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+1)(k+3)} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2+2k} = \sum_{k=3}^{\infty}\frac{(-1)^{k+1}}{k^2-2k}$$

$$= \sum_{k=1}^{\infty}(-1)^k\,\frac{\zeta(4k-1)}{4^k} = \sum_{k=1}^{\infty}\frac{k}{4k^4+1}$$

$$= \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{3^k} = \sum_{k=1}^{\infty}\frac{\mu(k)}{4^k-1} = \sum_{k=1}^{\infty}\frac{\mu(k)2^k}{4^k-1}$$

$$= \int_1^{\infty}\frac{dx}{x^5} = \int_0^{\infty}\frac{x^3\,dx}{(x+1)^5} = \int_1^{\infty}\frac{\log x\,dx}{x^3} = \int_1^{\infty}\frac{\log^2 x\,dx}{x^3}$$

$$= \int_0^1 x\arcsin x\arccos x\,dx$$

$$= -\int_0^1 x\log x\,dx$$

$$= -\int_0^{\pi/2}\log(\sin x)\sin x\cos x\,dx = \int_0^{\pi/2}\left(\log\sin x\right)^2\sin x\,\cos x\,dx$$

$$= \int_0^{\infty}\frac{(\sin x - x\cos x)^2}{x^5}\,dx$$ Prud. 2.5.29.24

$$= \int_1^{\infty}\log\left(1+\frac{1}{x}\right)\frac{dx}{x^3}$$

$$= \int_0^{\infty}\frac{x^3\,dx}{e^{x^4}} = \int\frac{x^7\,dx}{e^{x^4}}$$

$$1\quad .25000000000000000000\underline{0} = \frac{5}{4} = H_2^{(2)}$$

$$2\quad .25000000000000000000\underline{0} = \frac{9}{4} = \sum_{k=1}^{\infty}\frac{k}{3^{k-1}} = \sum_{k=1}^{\infty}\frac{F_kF_{k+1}}{3^k}$$

$$68\quad .2500000000000000000\underline{0} = \frac{273}{4} = \Phi\left(\frac{1}{3},-5,0\right) = \sum_{k=1}^{\infty}\frac{k^5}{3^k}$$

.250204424109600389291... ≈ $-\zeta''(-1)$

.25088497033571728323... ≈ $\sum_{k=2}^{\infty} \frac{\zeta(k)-1}{2^k-1}$

1 .25164759779046301759... ≈ $\lim_{n\to\infty} \lg^{(n)} p_n \ , \ \ p_1 = 2; \ p_n = \text{smallest prime} > 2^{p_{n-1}}$

Bertrand's number b such that all floor(2^2^…^b) are prime.

1 .25173804073865146774... ≈ $\frac{1}{2}\left(I_0(2) - J_0(2)\right) = \sum_{k=0}^{\infty} \frac{1}{(2k)!(2k)!}$

.2517516771329061417... ≈ $\frac{1}{4}\text{HypPFQ}\left(\{1,1,1,1\},\{\frac{3}{2},\frac{3}{2},2,2\},\frac{1}{16}\right) = \sum_{k=1}^{\infty} \frac{((k-1)!)^3}{((2k)!)^2}$

2 .2517525890667211077... ≈ $\psi(10)$

.2518661728632815153... ≈ $\frac{\pi}{4\sqrt{5}} \coth \frac{\pi\sqrt{5}}{2} - \frac{1}{10} = \sum_{k=1}^{\infty} \frac{1}{4k^2+5}$

.251997590741547646964... ≈ $\frac{3}{\zeta(2)} - \frac{3}{\zeta(3)} + \frac{1}{\zeta(4)} = -\sum_{k=1}^{\infty} \frac{\mu(k)}{k}\left(1-\frac{1}{k}\right)^3$

6 .25218484381226192605... ≈ $7\zeta(3) + 4\log 2 - \frac{\pi^2}{2} = \sum_{k=1}^{\infty} \frac{2(2k+1)}{k(2k-1)^3}$

= $\sum_{k=1}^{\infty} \frac{k^2\zeta(k+2)}{2^k}$

.252235065786835203... ≈ $1 - \frac{1}{e} - \log 2 + \log\left(1+\frac{1}{e}\right) = \int_0^1 \frac{dx}{e^x(e^x+1)}$

.2523132522020160048... ≈ $\frac{1}{\sqrt{5\pi}}$

4 .2523508795026238253... ≈ $I_0(2\sqrt{2}) = {}_0\text{F}_1(;1;2) = \sum_{k=0}^{\infty} \frac{2^k}{(k!)^2}$

1 .252504033125214762308... ≈ $\pi \,\text{sech} \frac{\pi}{2} = \Gamma\left(\frac{1+i}{2}\right)\Gamma\left(\frac{1-i}{2}\right)$

.25257519204461276085... ≈ $\zeta(3) - \frac{\pi^2\gamma}{6} = \sum_{k=1}^{\infty} \frac{\psi(k)}{k^2}$

.2526802514207865349... ≈ $\text{arccsc}\, 4$

1 .2527629684953679957... ≈ $Li_1\left(\frac{5}{7}\right)$

$.25311355311355\underline{311355}$ = $\frac{691}{2730} = B_6$

.25314917581260433564... ≈ $\pi^2 - 8\zeta(3)$

2 .25317537822610367757... $\approx \sum_{k=1}^{\infty}\frac{k!!}{(k^2)!!}$

.2531758671875729746... $= 1-\frac{erf(1)\sqrt{\pi}}{2}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!(2k+1)}$

.253207692986502289614... $\approx \frac{\sqrt{\pi}}{7}$

1 .25322219586794307564... $\approx \frac{1}{2-\zeta(3)}=\sum_{k=1}^{\infty}\frac{f(k)}{k^3}$ Titchmarsh 1.2.15

.2532310965879049271... $\approx \frac{9\log 3-9-\pi\sqrt{3}}{8}+\frac{\pi^2}{12} = \sum_{k=1}^{\infty}\frac{1}{3k^3+2k^2}$

.25326501580752209885... $\approx \frac{\pi^3}{48}-\frac{\pi}{8} = \int_0^{\pi/2}x^2\cos^2 x\,dx$

1 .25331413731550025121... $\approx \sqrt{\frac{\pi}{2}} = -i\sqrt{i\log i}$

.25347801172011391904... $\approx \frac{1}{4}\left(I_1(2)-J_1(2)\right) = \sum_{k=0}^{\infty}\frac{k}{(2k)!(2k)!}$

1 .25349875569995347164... $\approx \sum_{k=2}^{\infty}\frac{1}{k!-1}$

2 .25356105078342759598... $\approx \frac{\pi\log 12}{2\sqrt{3}} = \int_0^{\infty}\frac{\log(x^2+3)}{x^2+3}$

.25358069953485240581... $\approx \sum_{k=0}^{\infty}\frac{1}{(k+3)!-k!}$

.25363533035541760758... $\approx \frac{1}{\sqrt{\pi}}\left(\sqrt{\frac{2\pi}{e}}erfi\frac{1}{\sqrt{2}}-1\right) = \frac{2}{3\sqrt{\pi}}\,{}_1F_1\left(2,\frac{5}{2},-\frac{1}{2}\right)=\sum_{k=0}^{\infty}\frac{(-1)^{k+1}k}{(k+\frac{1}{2})!2^k}$

2 .25374537232763811712... $\approx 24-8e = \int_0^1 e^{x^{1/4}}dx$

.25375156554939945296... $\approx \psi\left(\frac{4}{5}\right)-\psi\left(\frac{3}{10}\right) = \int_1^{\infty}\frac{dx}{x^4+x^{-1}}=\int_0^{\infty}\frac{dx}{e^{3x}+e^{-2x}}$

.2537816629587442171... $\approx 6-\frac{5\pi^2}{6}-\frac{\pi^4}{45}+3\zeta(3)+\zeta(5)=\sum_{k=1}^{\infty}\frac{1}{k^5(k+1)^2}$

2 .2539064884185793236... $\approx \sum_{k=1}^{\infty}\frac{2^{2k}}{2^{2^k}}$

.25391006493009715683... $\approx \sum_{k=1}^{\infty}\frac{1}{4^{k^2}}$

.2539169614342191961... $\approx \sum_{k=2}^{\infty}\frac{1}{2^k}\log\frac{k}{k-1} = \sum_{k=1}^{\infty}\frac{1}{k}\left(Li_k\left(\frac{1}{2}\right)-\frac{1}{2}\right)$

.25392774317526456667... ≈ $\sum_{k=1}^{\infty} \frac{1}{(2k)^{2k}}$

.254023674895814278282... ≈ $\sum_{k=1}^{\infty} \frac{1}{k(2k+1)\log k}$

.2541161907463435341... ≈ $\Phi\left(\frac{1}{4},4,0\right) = Li_4\left(\frac{1}{4}\right) = \sum_{k=1}^{\infty} \frac{1}{4^k k^4}$

.254162992999762569536... ≈ $\int_1^{\infty} \frac{\sin x}{e^x} dx$

.2543330950302498178... ≈ $\frac{2}{4+\sqrt{2}+\sqrt{6}}$ CFG D1

.25435288196373948719... ≈ $\frac{\pi}{6\sqrt{3}} - \frac{\log 2}{3} + \frac{\log 3}{6} = \int_1^2 \frac{dx}{x^3+1}$

.25455829718791131122... ≈ $\frac{5}{32}\left(21\sqrt{\pi}\, erfi(1) - 22e\right) = \frac{1}{9}\, {}_1F_1\left(\frac{9}{2},\frac{11}{2},1\right) = \sum_{k=0}^{\infty} \frac{1}{k!(2k+7)}$

.25457360529167341532... ≈ $\frac{\pi}{16}\left(\sqrt{2}\coth\frac{\pi}{\sqrt{2}} - \pi\,\mathrm{csch}^2\frac{\pi}{\sqrt{2}}\right)$

= $\sum_{k=1}^{\infty} \frac{k^2}{(2k^2+1)^2} = \sum_{k=1}^{\infty} \frac{1}{(2k+k^{-1})^2} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k\zeta(2k)}{2^{k+1}}$

.2545892393290283910... ≈ $\frac{1}{2}\cosh\frac{\pi}{2} - 1 = \frac{e^{\pi/2}+e^{-\pi/2}}{4} = \sum_{k=0}^{\infty} \frac{\pi^{4k}}{(4k)!16^k}$

1 .2545892393290283910... ≈ $\frac{1}{2}\cosh\frac{\pi}{2} = \prod_{k=1}^{\infty}\left(1+\frac{1}{(2k+1)^2}\right) = \sum_{k=1}^{\infty} \frac{\pi^{4k}}{(4k)!16^k}$

= $\int_0^{\pi/2} \sin x \sinh x \, dx$

23 .2547075102248651316... ≈ $\frac{3\pi^3}{4}$

7 .2551974569368714024... ≈ $\frac{4\pi}{\sqrt{3}} = \int_0^{\infty} \log\left(1+\frac{26}{x^3+1}\right) dx$

11 .2551974569368714024... ≈ $4\left(1+\frac{\pi}{\sqrt{3}}\right) = \sum_{k=0}^{\infty} \frac{3^k}{\binom{2k}{k}}$

2 .25525193041276157045... ≈ $\frac{e+1}{\sqrt{e}} = 2\cosh\frac{1}{2}$

.25541281188299534160... ≈ $\frac{1}{2}\log\frac{5}{4} = \operatorname{arctanh}\frac{1}{4} = \sum_{k=0}^{\infty} \frac{1}{4^{2k+1}(2k+1)}$ AS 4.5.64, J941

4	.25548971297818640064...	≈	$\frac{7}{\zeta(2)}=\frac{42}{\pi^2}$	
	.2555555555555555555$\underline{5}$	=	$\frac{23}{90}=\sum_{k=0}^{\infty}\frac{1}{(2k+1)(2k+7)}$	K133
	.25567284722879676889...	≈	$\frac{1}{25}(15-8\sqrt{5}\operatorname{arcsinh}\frac{1}{2}) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k!(k+1)!}{(2k+1)!}$	
1	.25589486222017979491...	≈	$\sum_{k=0}^{\infty}\frac{1}{(k+2)!-k!}$	
	.25611766843180047273...	≈	$\psi(4)-1=\frac{5}{6}-\gamma$	
	.2561953953354862697...	≈	$4\log 2-\log^2 2+\pi+\frac{\pi^2}{12}-6=-\int_0^1 \arcsin x\log^2 x dx$	
	.2562113149146646868...	≈	$\frac{1}{2}-\frac{\pi\sqrt{2}}{4}\operatorname{csch}\frac{\pi}{\sqrt{2}}$	
		=	$\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2k^2+1}$	
	.2564289918675422335...	≈	$\frac{7e^{1/3}}{3}-3=\sum_{k=1}^{\infty}\frac{k^2}{(k+1)!3^k}$	
	.25651051462943384518...	≈	$\frac{\pi^2}{16}-\frac{3\log^2 2}{4} = \int_0^1\frac{\log(1+x^2)}{x(1+x)}dx$	
60	.25661076956300322279...	≈	$3e^3=\sum_{k=0}^{\infty}\frac{3^k k}{k!}$	
1	.2566370614359172954...	≈	$\frac{2\pi}{5}$	
	.2566552446666378985...	≈	$\sum_{k=1}^{\infty}\frac{H^{(3)}{}_k}{5^k}$	
2	.2567583341910251478...	≈	$\frac{4}{\sqrt{\pi}}$	
1	.25676579620140483035...	≈	$\csc\frac{\pi}{\sqrt{2}}$	
	.25696181539935394992...	≈	$\frac{1}{8}\left(I_0(2)-J_0(2)\right) = \sum_{k=0}^{\infty}\frac{k^2}{(2k)!(2k)!}$	
1	.2570142442930860605...	≈	$\mathrm{HypPFQ}\left(\{1\},\{\frac{1}{2},\frac{1}{2}\},\frac{1}{16}\right) = \sum_{k=0}^{\infty}\frac{1}{(2k)!\binom{2k}{k}}$	
2	.25702155562579161794...	≈	$16\log 2-\frac{53}{6}=\sum_{k=1}^{\infty}\frac{H_{k+3}}{2^k}$	

	.25712309488167048602...	≈	$\frac{4-\pi}{8\Gamma\left(\frac{5}{4}\right)}\sqrt{\pi}\Gamma\left(\frac{3}{4}\right) = -\int_0^{\pi}\frac{\sin^2 x}{\sqrt{1+\sin^2 x}}\log\sin x\,dx$
1	.25727411566918505938...	≈	$\sqrt[5]{\pi}$
	.2574778574166709171...	≈	$\gamma - 2\gamma\log 2 + \log^2 2 = \sum_{k=1}^{\infty}\frac{\psi(k+1)}{2^k(k+1)}$
	.25766368334716467709...	≈	$\frac{\pi^2}{48}-\frac{\log^2 2}{8}-\frac{1}{4}Li_2\left(-\frac{1}{2}\right) = \int_0^1\frac{\log x}{(x+2)(x-2)}dx$
	.25769993632568293344...	≈	$\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^3+2}$
1	.25774688694436963001...	≈	$-\sum_{k=2}^{\infty}\zeta'(k) = \sum_{k=2}^{\infty}\frac{\log k}{k^2-k} = \sum_{m=1}^{\infty}\sum_{k=2}^{\infty}\frac{\log k}{k^m}$
		=	$\sum_{k=1}^{\infty}\frac{1}{k}\log\frac{k+1}{k} = \sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)}{k-1} = \sum_{k=1}^{\infty}H_k(\zeta(k+1)-1)$
	.257766634383410892217...	≈	$\sum_{k=1}^{\infty}\frac{H^{(3)}{}_k}{2^k(2k+1)}$
	.2578491922439319876...	≈	$\frac{1}{e}\left(I_0(1)-I_1(1)\right) = {}_1\mathrm{F}_1\left(\frac{3}{2},2,-2\right) = \sum_{k=0}^{\infty}\frac{(-1)^k k}{k!2^k}\binom{2k}{k}$
	.25791302898862685560...	≈	$-\frac{1}{125}\psi^{(2)}\left(\frac{2}{5}\right) = \int_0^{\infty}\frac{x^2\,dx}{e^{2x}-e^{-3x}}$
	.25794569478485536661...	≈	$\zeta(4)-2\zeta(3)+\frac{2\pi^2}{3}-5 = \sum_{k=1}^{\infty}\frac{1}{k^4(k+1)^2}$
2	.25824371511171325474...	≈	$2^{\sinh 1} = \prod_{k=0}^{\infty}2^{1/(2k+1)!}$
404	.25826782999151312012...	≈	$\frac{1}{4}\left(e^{e^2}+e^{e^{-2}}\right) = \sum_{k=0}^{\infty}\frac{\sinh k\,\cosh k}{k!}$
	.25839365571170619449...	≈	$\sum_{k=2}^{\infty}\frac{\zeta(k+1)-1}{\zeta(k)}$
	.25846139579657330529...	=	$Li_3\left(\frac{1}{4}\right) = \Phi\left(\frac{1}{4},3,0\right) = \sum_{k=1}^{\infty}\frac{1}{4^k k^3}$
		=	$\int_0^1\frac{\log(1-x/4)\log x}{x}dx$
	.2586397057283358176...	≈	$\frac{\pi^2}{6}-2\log 2 = \sum_{k=2}^{\infty}\frac{\zeta(k)-\zeta(k+1)}{2^{k-1}} = \sum_{k=2}^{\infty}\frac{k-1}{k^2(2k-1)}$

.258742781782167056... $\approx$ $\frac{3\pi}{4}\left(\log 2-\frac{7}{12}\right)=\int\limits_0^\infty \frac{xe^{-2x}dx}{\sqrt{e^x-1}}$ GR 3.42.0

3 .2587558259772099036... $\approx$ $\gamma^2+\frac{\pi^2}{6}+2\gamma\log 2+\log^2 2=\int\limits_0^\infty \frac{\log^2\frac{x}{2}dx}{e^x}$

.25881904510252076235... $\approx$ $\frac{\sqrt{2-\sqrt{3}}}{2} = \frac{1}{\sqrt{6}-\sqrt{2}}$ CFG C1

$=$ $\sin\frac{\pi}{12}=\frac{\sqrt{2}}{4}(\sqrt{3}-1)$ AS 4.3.46, CFG D1

.25916049726579360505... $\approx$ $2-2\sqrt{2}\arctan\frac{1}{\sqrt{2}}=\sum_{k=0}^\infty \frac{(-1)^k}{2^k(2k+3)}$

.259259259259259259$\underline{259}$ $=$ $\frac{7}{27} = \Phi\left(\frac{1}{7},-2,0\right) = \sum_{k=1}^\infty \frac{k^2}{7^k}$

.25938244878246085531... $\approx$ $\prod_{k=1}^\infty \left(1-\frac{k}{\binom{2k}{k}}\right)$

1 .2599210498948731647... $\approx$ $\sqrt[3]{2} = \prod_{k=0}^\infty\left(1+\frac{(-1)^k}{3k+2}\right)$

.2599301927099794910... $\approx$ $\frac{\log 8}{8}$ J137

.2602019392137596558... $\approx$ $2+\frac{\pi^2}{4}-\frac{7\zeta(3)}{2}$

$=$ $\sum_{k=1}^\infty (-1)^{k+1}\frac{k^2\zeta(k+1)}{2^k} = \sum_{k=1}^\infty \frac{1}{2k^2}\left(1-\frac{1}{2k}\right)\left(1+\frac{1}{2k}\right)^{-3}$

1 .2602057107052417107 7... $\approx$ $\prod_{k=1}^\infty \zeta(2k+1)$

.26022951451104364006... $\approx$ $\frac{-i}{4}\left(e^{e^{2i}}-e^{e^{-2i}}\right) = \frac{1}{2}e^{\cos 2}\sin(\sin 2) = \sum_{k=1}^\infty \frac{\sin k\cos k}{k!}$

.26039505099275673875... $\approx$ $\log 2\left(\sqrt{e}-1\right) = \sum_{k=1}^\infty (-1)^k\frac{B_k}{k!2^k k}$ Berndt 5.8.5

.260442806300988445... $\approx$ $-\frac{\pi}{2}\log\frac{\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{1}{4}\right)} = -\int\limits_0^1 \log\log\left(\frac{1}{x}\right)\frac{dx}{x^2+1}$ GR 4.325.4

= $\int_0^\infty \frac{\log x\,dx}{e^x+e^{-x}}$

.2605008067101075324... ≈ $\prod_{k=1}^\infty \left(1+\frac{1}{5^k}\right)$

.26051672044433666221... ≈ $\sum_{k=2}^\infty (-1)^k(\zeta(k)-1)^3$

.26054765274687368081... ≈ $\frac{1}{2}\sinh\frac{1}{2} = \sum_{k=1}^\infty \frac{1}{(2k-1)!4^k}$

1 .260591836521356119... ≈ $\cosh\frac{1}{\sqrt{2}} = \sum_{k=0}^\infty \frac{1}{(2k)!2^k}$

.2606009559118338926... ≈ $\frac{\pi^2}{12}\csc\frac{\pi}{\sqrt{3}}+\frac{\pi}{4\sqrt{3}}\cot\frac{\pi}{\sqrt{3}}-\frac{1}{2} = \sum_{k=1}^\infty \frac{1}{(3k^2-1)^2}$ J840

.2606482340867767481... ≈ $\frac{81\log 3}{2}+\zeta(4)+\frac{3\pi^2}{2}+\frac{9\pi\sqrt{3}}{2}-81 = \sum_{k=1}^\infty \frac{1}{3k^5+k^4}$

= $\sum_{k=1}^\infty \frac{(-1)^{k+1}\zeta(k+4)}{3^k}$

.26069875393561620639... ≈ $\sum_{k=0}^\infty \frac{\phi(k)}{5^k}$

.2607962396687288864... ≈ $\sum_{k=0}^\infty \frac{S_2(2k,k)}{k!S_1(2k,k)}$

.2609764038171073234... ≈ $\frac{\pi^2-3\zeta(3)}{24} = \sum_{k=1}^\infty \frac{2k-1}{(2k)^3}$

.26116848088744543358... ≈ $-2\log(\cos\frac{1}{2}) = \sum_{k=1}^\infty (-1)^{k+1}\frac{(2^{2k}-1)B_{2k}}{(2k)!k}$ AS 4.3.72

.26117239648121182407... ≈ $2\arcsin^2\frac{1}{2\sqrt{2}} = \sum_{k=1}^\infty \frac{(k-1)!(k-1)!}{(2k)!2^k}$

1 .26136274645318147699... ≈ $\sum_{k=1}^\infty \frac{1}{k^3-k^2+k}$

.26145030431082465654... ≈ $\zeta(3)-3\gamma-\frac{3}{2}\left(\psi\left(1+\frac{i}{\sqrt{3}}\right)+\psi\left(1-\frac{i}{\sqrt{3}}\right)\right)$

.261497212847642783755...≈ $\gamma+\sum_{p\ prime}\left(\log\left(1-\frac{1}{p}\right)+\frac{1}{p}\right)$ Mertens constant

= $\lim_{n\to\infty}\left(-\log\log n+\sum_{p\ prime<\ n}\frac{1}{p}\right)$

4 .2614996683698700833... ≈ $\frac{742}{343}e^{1/3} = \sum_{k=1}^\infty \frac{k^5}{k!3^k}$

.26162407188227391826... ≈ $\log\frac{3\sqrt{3}}{4} = \operatorname{arctanh}\frac{1}{2}+\log\frac{3}{4}$

= $\sum_{k=2}^{\infty}\left(Li_k\left(\frac{1}{2}\right)+Li_k\left(-\frac{1}{2}\right)\right) = \sum_{k=1}^{\infty}\frac{1}{4^k k(2k-1)}$

1 .261758473394486609961... ≈ $\frac{\pi}{eG}$

1 .2617986109848068386... ≈ $\prod_{k=1}^{\infty}\left(1+\frac{1}{2^k(2k+1)}\right)$

= $\int_1^{\infty}\frac{\operatorname{arccosh} x}{x^4}dx = \int_0^{\infty}\frac{dx}{e^{3x}+e^{-3x}}$

.26179938779914943654... ≈ $\frac{\pi}{12} = \sum_{k=0}^{\infty}\frac{(-1)^k}{6k+3} = \sum_{k=0}^{\infty}\frac{1}{4^{2k+1}(2k+1)}\binom{2k}{k}$

= $\int_0^{\infty}\frac{x^5\,dx}{1+x^{12}}$

= $\int_0^{\infty}\frac{\sin x^3+x^3\cos x^3}{x^7}dx$

.26182548658308238562... ≈ $1-\frac{\pi^2(1+\log 2)}{12}-\log 2+\frac{\log^2 2}{2}+\frac{\log^3 2}{6}+\frac{7\zeta(3)}{8}$

= $1-\log 2-Li_2\left(\frac{1}{2}\right)+Li_3\left(\frac{1}{2}\right) = \sum_{k=1}^{\infty}\frac{1}{2^k k^3(k+1)}$

1 .26185950714291487420... ≈ $\log_3 4$

.26199914181620333205... ≈ $\sum_{k=1}^{\infty}\frac{\mu(4k-3)}{4^k-1}$

.2623178579609159738... ≈ $16\zeta(3)-\frac{256103}{13500} = \int_0^1\frac{x^2\log^2 x}{1-\sqrt{x}}dx$

.262326598980425730577... ≈ $\frac{\pi^2}{4}+\log\frac{\pi}{16}-\gamma = \sum_{k=1}^{\infty}\frac{k^2}{2^k(k+1)}\left(\zeta(k+1)-1\right)$

1 .2624343094110320122... ≈ $\frac{e^{\pi/2}}{e^{\pi/2}-1} = \frac{1}{1-i^i} = \sum_{k=0}^{\infty}i^{ik}$

.2624491973164381282... ≈ $\sum_{k=1}^{\infty}\frac{|\mu(k)|(-1)^{k+1}}{3^k}$

.2625896447527351375... ≈ $-\sum_{k=1}^{\infty}\frac{\mu(k)\sigma_0(k)}{2^k}$

.262600198699885171294... ≈ $\frac{\pi^2}{12}+\frac{\pi^4}{180}-\frac{\zeta(3)}{2}-\frac{1}{2} = \sum_{k=1}^{\infty}\frac{H^{(3)}{}_k}{k(k+1)(k+2)}$

1 .2626272556789116834... ≈ $\arctan\pi$

.2627243708971271954... ≈ $\sum_{k=2}^{\infty}\frac{\log\frac{k}{k-1}}{k^2}$

.262745097806385042621... ≈ $\sum_{k=1}^{\infty}\frac{\mu(k)}{2^{2^k}-1}$

.26277559179844674016... ≈ $\frac{\pi^2}{8}-\log 2-\frac{5}{18}=\sum_{k=2}^{\infty}\frac{4k+1}{2k(2k+1)^2}$

= $\sum_{k=2}^{\infty}(-1)^k k\frac{\zeta(k)-1}{2^k}$

.2629441382869882525... ≈ $\prod_{k=2}^{\infty}\left(1-\frac{1}{k!!}\right)$

.26294994756616124993... ≈ $\frac{7\zeta(3)}{32}=\int_0^1\frac{x\log^2 x}{1-x^4}dx=\int_0^{\infty}\frac{t^2\,dt}{e^{2t}-e^{-2t}}$

.2631123379580001512... ≈ $\frac{1}{2}-\frac{1}{2e}+\frac{1}{4}\left(Ei\left(-i\sqrt{2},1\right)+Ei\left(-i\sqrt{2},1\right)-\Gamma\left(1+i\sqrt{2}\right)-\Gamma\left(1-i\sqrt{2}\right)\right)$

= $\frac{1}{4}\left(2+i\sqrt{2}\left(\Gamma(-i\sqrt{2},0,1)-\Gamma(i\sqrt{2},0,1)\right)\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!(k^2+2)}$

$.\underline{263157894736842105}$ = $\frac{5}{19}$

.263189450695716229836... ≈ $\frac{2\pi^2}{75}=\prod_{p\ prime}\frac{1-p^{-2}}{\left(1+p^{-2}\right)^2}$

.2632337919139127016... ≈ $\sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)-1}{k!!}$

.263300183272010131698... ≈ $\int_0^{\infty}e^{-e^{x^2}}dx$

.26360014128128492209... ≈ $\frac{2}{3}-\frac{2\pi\sqrt{3}}{27}=\frac{1}{6}\,{}_2F_1\left(2,2,\frac{5}{2},\frac{1}{4}\right)=\sum_{k=1}^{\infty}\frac{k}{\binom{2k}{k}(2k+1)}$

3 .26365763667448738854... ≈ $\frac{\pi^2}{6}+\zeta(3)+\frac{5}{12}=\sum_{k=1}^{\infty}\frac{H_kH_{k+2}}{k(k+3)}$

.26375471929963096576... ≈ $\sum_{k=1}^{\infty}\frac{H^{(2)}{}_k}{5^k}=\frac{1}{4}\sum_{k=0}^{\infty}\frac{1}{5^k(k+1)^2}=\frac{5}{4}Li_2\left(\frac{1}{5}\right)$

.26378417660568080153... ≈ $\frac{\pi^2}{18}+\frac{4}{9}\log 2-\frac{16}{27} = \sum_{k=1}^{\infty}\frac{1}{2k^3+3k^2}$

5 .263789013914324596712... ≈ $\frac{8\pi^2}{15}$, the volume of the unit sphere in R^5

.263820220500669832561... ≈ $Li_2(e)-\frac{\pi^2}{6}-1+i\pi = \sum_{k=1}^{\infty}\frac{(-1)^k B_k}{(k+1)!k}$ [Ramanujan] Berndt Ch. 9

.2639435073548419286... ≈ $\frac{\pi}{2}+\log 2-2 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2k^2+k}$

= $\sum_{k=1}^{\infty}\frac{(k-\frac{1}{2})!}{(k+\frac{1}{2})!(4k+1)}$

= $\int_0^1 \log(1+x^2)\,dx = \int_0^{\infty}\log\left(1+\frac{1}{x^2}\right)\frac{dx}{x^2}$

= $-\int_0^1 \arcsin x\log x\,dx$ GR 4.591

2 .2639435073548419286... ≈ $\frac{\pi}{2}+\log 2 = \psi\left(\frac{1}{2}\right)-\psi\left(\frac{1}{4}\right)$

= $\int_0^1 \log\left(1+\frac{1}{x^2}\right)dx = \int_0^{\infty}\log\left(1+\frac{2}{x(x+2)}\right)dx$

.26415921800062670474... ≈ $\sum_{k=2}^{\infty}(-1)^k\frac{\Omega(k)k}{2^k}$

1 .2641811503891615965... ≈ $\sum_{k=1}^{\infty}\frac{k}{(k!)^3}$

.26424111765711535681... ≈ $1-\frac{2}{e} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!(k+2)} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(k+1)!} = \sum_{k=0}^{\infty}\frac{(-1)^k k^4}{(k+1)!}$

= $\sum_{k=1}^{\infty}\frac{1}{(2k)!(k+1)} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+1)!+k!}$

= $\sum_{k=1}^{\infty}(-1)^k\frac{k^2H_k}{k!} = \sum_{n=1}^{\infty}\frac{1}{n!}\sum_{k=2}^{\infty}(-1)^k\frac{\log^n k}{k!}$

= $\int_0^1 xe^{-x}\,dx = \int_1^e \frac{\log x}{x^2}dx$

.264247851441806613... ≈ $(\sqrt{3}-1)\frac{\pi}{16}-\frac{\sqrt{3}-1}{4}\log\frac{\sqrt{3}-1}{\sqrt{2}} = \sum_{k=0}^{\infty}(-1)^k\frac{(2-\sqrt{3})^{2k+1}}{4k+1}$

.264261279935529928... ≈ $-\frac{2\pi}{25}\csc\frac{7\pi}{5} = \int_0^{\infty}\frac{x^2}{(x^5+1)^2}$

1 .2644997803484442092... ≈ $\sum_{k=0}^{\infty}\frac{1}{2^k+1} = \sum_{k=1}^{\infty}\frac{\mu(k)-(-1)^k}{2^k-1}$

.2645364561314071182... ≈ $9e-\frac{121}{5}=\sum_{k=0}^{\infty}\frac{1}{k!(k+1)(k+6)}$

.26516504294495532165... ≈ $\frac{3}{8\sqrt{2}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{4^k(2k-1)}\binom{2k}{k}$

4 .26531129233226693864... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)}{F_{k-1}}$

.26549889859872509761... ≈ $\frac{4\sin 1}{17-8\cos 1} = \sum_{k=1}^{\infty}\frac{\sin k}{4^k}$

1 .26551212348464539649... ≈ $\log 2\sqrt{\pi} = \log\left(-\Gamma\left(-\frac{1}{2}\right)\right)$

4744 .2655508035287003564... ≈ $\frac{\pi^8}{2}$

1 .2656250596046447754... ≈ $\sum_{k=0}^{\infty}\frac{1}{2^{k!}}$

.2656288146972656805... ≈ $-\sum_{k=1}^{\infty}\frac{\mu(3k)}{4^k-1}=\sum_{k=1}^{\infty}\frac{1}{(\sqrt[3]{4})^{3^k}}$

1 .2656974916168336867... ≈ $\frac{1}{2e}\Big((e^2-1)\gamma-4e+4e\cosh 1+(2+2e^2)(si(1)-ci(1))$

$+(1-e^2)\log 2+(e^2-1)ci(2)-(e^2+1)si(2)\Big)$

= $e+\frac{1}{e}-2+\gamma\sinh 1-eEi(-1)+\frac{1}{2}\left(eEi(-2)+\frac{\log 2+2Ei(1)-Ei(2)}{e}-e\log 2\right)$

= $\sum_{k=1}^{\infty}\frac{H_k}{(2k-1)!}$

.26580222883407969212... ≈ $\operatorname{sech} 2 = \frac{1}{\cosh 2} = \frac{2}{e^2+e^{-2}}$

.265827997639478656858...≈

$$\frac{1}{2\sqrt{5}}\left(H\left(1-i\sqrt{\frac{\sqrt{5}-1}{2}}\right)+H\left(1+i\sqrt{\frac{\sqrt{5}-1}{2}}\right)-H\left(1-\sqrt{\frac{1+\sqrt{5}}{2}}\right)-H\left(1+\sqrt{\frac{1+\sqrt{5}}{2}}\right)\right)$$

= $\sum_{k=1}^{\infty}F_k\left(\zeta(2k+1)-1\right)$

.2658649582793069827... ≈ $\frac{2G}{3}+\frac{\pi}{12}\log(2-\sqrt{3})$ Berndt 9.18

.2658700952308663684... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^{k(k+1)}}$

4 .2660590852787117341... ≈ $\frac{1}{e}\Phi\left(\frac{1}{e},-e,1\right) = \sum_{k=1}^{\infty}\frac{k^e}{e^k}$

1 .2660658777520083356... ≈ $I_o(1)$

$= \displaystyle\sum_{k=0}^{\infty} \frac{J_k(1)}{k!} = \frac{1}{e\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(k-\frac{1}{2})!2^k}{(k!)^2}$

$= \displaystyle\int_0^1 e^{\cos \pi x} dx$

.26607684664517036909... $\approx \displaystyle\frac{9}{4} + \frac{1}{4}\left(\psi^{(1)}\left(\frac{5}{6}\right) - \psi^{(1)}\left(\frac{1}{3}\right)\right) = \sum_{k=1}^{\infty} \frac{(-1)^{k+}}{(k+2/3)^2}$

118 .2661309556922124918... $\approx \displaystyle\frac{31\pi^6}{252} = \frac{465\zeta(6)}{4} = -\int_0^1 (\log^5 x) \frac{dx}{1+x}$ GR 4.264.1

$\approx \displaystyle\int_0^{\infty} \frac{x^5\, dx}{1+e^x}$

.2662553420414154886... $\approx -\cos(\pi\sqrt{2})$

1 .2663099938221493948... $\approx \displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{\mathrm{F}_k}$

5 .26636677634645847824... $\approx \displaystyle\prod_{k=2}^{\infty} \zeta^2(k)$

2 .26653450769984883507... $\approx \displaystyle\int_1^{\infty} \frac{dx}{\Gamma(x)}$

23 .266574141643676834793... $\approx \displaystyle\frac{32\pi^5}{243\sqrt{3}} = \int_0^1 \frac{\log^4 x}{x^2+x+1} dx$

.26665285034506621240... $\approx \displaystyle\frac{1}{2} - \pi^3 \coth\pi \operatorname{csch}^2\pi = i\left(\zeta(3,2+i) - \zeta(3,2-i)\right)$

$= \displaystyle\int_0^{\infty} \frac{x^2 \sin x}{e^x\left(e^x-1\right)} dx$

.26666285196940009798... $\approx \displaystyle\sum_{k=1}^{\infty} \frac{|\mu(2k)|}{4^k}$

.2666666666666666666$\underline{6}$ $= \displaystyle\frac{4}{15} = \int_0^{\infty} \frac{dx}{e^{\pi x^{1/4}} - 1}$

1 .266741049904... $\approx \displaystyle\prod_{k=1}^{\infty}\left(1 + \frac{1}{k(k+1)(k+2)}\right)$

1 .26677747056299951115... $\approx 8\left(2\log 2 - \log(2+\sqrt{2})\right) = \displaystyle\sum_{k=0}^{\infty} \frac{1}{(\mathrm{k}+1)8^{\mathrm{k}}} \binom{2k+1}{k}$

.26693825063518343798... $\approx \displaystyle\frac{1-\log(e-1)}{e-1} = -\int_0^1 \frac{\log x}{(x+e-1)^2} dx$

2 .266991747212676201917... $\approx \displaystyle\frac{11\sqrt{e}}{8} = \sum_{k=1}^{\infty} \frac{k^3}{k!2^k}$

.2673999983697851853... ≈ $-\frac{1}{2}\log\left(2-\sqrt{2}\right)$

= $\log\left(\cos\frac{\pi}{8}+\sin\frac{\pi}{8}\right) = \log\left(\frac{1}{2}\left(\sqrt{2+\sqrt{2}}+\sqrt{2+\sqrt{2}}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{1}{k}\cos\left(\frac{k\pi}{4}\right)$

= $\left(-\frac{1}{4}+\frac{i}{4}\right)(-1)^{1/4}\sqrt{2}\left(\log(1-(-1)^{1/4})+\log(1+(-1)^{3/4})\right)$

.26765263908273260692... ≈ $Li_2\left(\frac{1}{4}\right) = \frac{\pi^2}{6}+2\left(\log 3-2\log 2\right)\log 2 - Li_2\left(\frac{3}{4}\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}H_k}{3^k k} = \int_0^1 \frac{\log(1-x/4)}{x}dx$

5 .267778605597073091897... ≈ $2\pi^2\log 2 - 7\zeta(3) = \int_0^{\pi}\frac{x^2\sin x\,dx}{1-\cos x}$

.267793401721690887137... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^k 2^k}{(k-1)!\zeta(2k)}$ Titchmarsh 14.32.1

.26794919243112270647... ≈ $2-\sqrt{3} = \sum_{k=1}^{\infty}\frac{1}{6^k(k+1)}\binom{2k}{k}$

1 .26794919243112270647... ≈ $3-\sqrt{3} = \sum_{k=0}^{\infty}\frac{1}{6^k(k+1)}\binom{2k}{k}$

4 .2681148649088081629... ≈ $\frac{160\pi}{27}-\frac{8\pi^3}{27}-\frac{64\pi}{27}\log 2 = \int_0^{\infty}x^{-5/2}Li_2(-x)^2\,dx$

.268253843893107859... ≈ $\frac{\pi^6}{216}-\frac{\pi^4}{12}+\frac{\pi^2}{2}-1=(\zeta(2)-1)^3=\sum_{k=1}^{\infty}\frac{\mathrm{f}_3(k)}{k^2}$ Titchmarsh 1.2.14

.268941421369995120749... ≈ $\frac{1}{e+1} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{e^k}$

.26903975345638420957... ≈ $\frac{\sqrt{\pi}\,erfi\,1}{4e} = \int_0^{\infty}e^{-x^2}\sin x\cos x\,dx$

.269205039384214394948... ≈ $\sum_{k=1}^{\infty}(-1)^k\frac{k}{F_k}$

.2696105027080089818... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{k^2+1} = \int_0^{\infty}\frac{\cos x}{1+e^x}dx = \int_0^1\frac{\cos(\log x)}{1+x}dx$

= $\frac{1}{4}\left[\psi\left(1+\frac{i}{2}\right)+\psi\left(1-\frac{i}{2}\right)-\psi\left(\frac{1}{2}+\frac{i}{2}\right)-\psi\left(\frac{1}{2}-\frac{i}{2}\right)\right]$

		=	$\frac{1}{4}\left[\psi\left(\frac{i}{2}\right)+\psi\left(-\frac{i}{2}\right)-\psi\left(\frac{1+i}{2}\right)-\psi\left(\frac{1-i}{2}\right)\right]$	
4	.2698671113367835...	≈	$\frac{\pi e}{2} = \int_0^\infty e^{\cos x}\sin(\sin x + x)\frac{dx}{x}$	GR 3.973.4
	.270034797849637221...	≈	$\frac{\pi}{2e^{\sqrt{2}}\sqrt{2}} = \int_0^\infty \frac{\cos x dx}{x^2+2}$	
	.27007167349302089432...	≈	$\frac{3\zeta(3)}{\pi^2}+2\log\pi+2\log 2+12\log(Glaisher)+\gamma-\frac{22}{3}$	
		=	$\sum_{k=1}^\infty \frac{k}{k+4}\left(\zeta(k+1)-1\right)$	
	.27008820585226910892...	≈	$\frac{16\sqrt{\pi}}{105}$	
	.27015115293406985870...	≈	$\frac{\cos 1}{2}=\sum_{k=0}^\infty \frac{(-1)^k}{2(2k)!}=\sum_{k=1}^\infty\frac{(-1)^k k}{(2k-1)!(2k+1)}$	
	.27020975013529207772...	≈	$\frac{\pi}{8}-\frac{1}{2}\arctan\frac{1}{4} = \int_1^2\frac{xdx}{x^4+1}$	
	.270222741100238847013...	≈	$\frac{6\zeta(3)}{\pi^2}+2\log\pi-\frac{11}{4}=\sum_{k=1}^\infty\frac{k}{k+2}\left(\zeta(2k)-1\right)$	
	.27027668478811225897...	≈	$\frac{\pi^2}{16}-\frac{\log 2}{2}=\int_1^\infty\frac{\log x}{(x+1)^2(x-1)}dx$	
	.270310072072109588...	≈	$\frac{2}{3}\log\frac{3}{2} = \sum_{k=1}^\infty(-1)^{k+1}\frac{H_k}{2^k}$	J 137
	.2703628454614781700...	≈	$\log 2+\gamma-1=\sum_{k=2}^\infty\frac{(-1)^k}{k}(\zeta(k)-1)$	
		=	$\sum_{k=2}^\infty\left(\frac{1}{k}+\log\frac{1}{1+1/k}\right)$	K 124h
1	.2703628454614781700...	≈	$\log 2+\gamma$	
		=	$\int_1^2 H(x)\,dx$	
		=	$\int_0^\infty\left(\frac{1}{1+x^2}-\cos x\right)\frac{dx}{x}$	
	.2704365764717983238...	≈	$1+\frac{\pi}{4\sqrt{2}}\tan\frac{\pi}{\sqrt{2}}=\sum_{k=1}^\infty\frac{1}{4k^2+4k-1}$	
4	.27050983124842272307...	≈	$\frac{5}{2}(3\sqrt{5}-5) = \sum_{k=0}^\infty\frac{1}{5^k}\binom{2k+2}{k}$	

.27051653806731302836... ≈ $5-\pi+\log 2+\frac{\sqrt{3}}{2}\log\frac{2-\sqrt{3}}{2+\sqrt{3}} = \int_0^1 \log\frac{1+x}{1+x^6}\,dx$

.270580808427784548... ≈ $\frac{\pi^4}{360}=\frac{\zeta(4)}{4}=\sum_{k=1}^{\infty}\frac{H_k}{(k+1)^3} = MHS(3,1)$

.270670566473225383788... ≈ $\frac{2}{e^2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}2^k}{(k-1)!} = \sum_{k=1}^{\infty}\frac{(-1)^k k^2 2^k}{k!} = \sum_{k=1}^{\infty}\frac{(-1)^k k^3 2^k}{k!}$

3 .27072717208067888495... ≈ $2\pi\log 2-\frac{\pi^3}{12}+\frac{\pi^3\log 2}{3}-\frac{3\pi\zeta(3)}{2} = \int_0^{\pi/2}\frac{x^4}{\sin^4 x}dx$

1 .27074704126839914207... ≈ $\frac{\sqrt{e}}{2\left(\sqrt{e}-1\right)} = \sum_{k=0}^{\infty}\frac{(-1)^k B_k}{2^k k!}$

1 .2709398238358032492... ≈ $\frac{\pi^2}{2}-4G = \sum_{k=1}^{\infty}\frac{k}{2^k}\zeta\left(k+1,\frac{5}{4}\right)$

.27101495139941834789... ≈ $\frac{\pi}{\cosh\pi} = \Gamma\left(\frac{1}{2}+i\right)\Gamma\left(\frac{1}{2}-i\right)$

= $\sum_{k=1}^{\infty}(-1)^{k+1}2^{2k}\beta(2k-1)$

.27103467023440152719... ≈ $\frac{\pi(\sqrt{3}-1)}{6\sqrt{2}} = \frac{\pi}{3\sqrt{2}\left(\sqrt{3}+1\right)} = \int_0^{\infty}\frac{x^4}{x^{12}+1}dx = \int_0^{\infty}\frac{x^6}{x^{12}+1}dx$

.2711198115330838692... ≈ $\frac{\log 2}{2}+\frac{1}{8}\left(\psi\left(\frac{1}{2}-\frac{i}{\sqrt{2}}\right)+\psi\left(\frac{1}{2}+\frac{i}{\sqrt{2}}\right)-\psi\left(1-\frac{i}{\sqrt{2}}\right)-\psi\left(1+\frac{i}{\sqrt{2}}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^3+2k}$

.27122707202423443856... ≈ $\frac{\sqrt{e}}{16}+\frac{1}{16\sqrt{e}}+\frac{1}{4}\sinh\frac{1}{2} = \sum_{k=1}^{\infty}\frac{k}{(2k-1)!\,4^k}$

1 .27128710490414662707... ≈ $-\frac{(-1)^{3/4}}{4}\left(4\Gamma\left(\frac{5}{4}\right)-\Gamma\left(\frac{1}{4},-1\right)\right) = {}_1F_1\left(\frac{1}{4},\frac{5}{4},1\right) = \sum_{k=0}^{\infty}\frac{1}{k!(4k+1)}$

.271360410318151750467... ≈ $\frac{1}{2\sqrt{2}}\sinh\frac{1}{\sqrt{2}} = \sum_{k=1}^{\infty}\frac{k}{(2k)!2^k}$

.27154031740762188924... ≈ $\frac{\cosh 1-1}{2} = \sinh^2\frac{1}{2} = \int_1^{\infty}\sinh\left(\frac{1}{x^2}\right)\frac{dx}{x^3}$

.2716916193741975876... ≈ $2-\frac{\pi^2}{12}-2\log 2+\log^2 2 = \int_0^{\pi/2}\left(\log\sin x\right)^2\sin x\,dx$

1 .2717234563121371107... ≈ ${}_0F_1\left(;2;\frac{1}{2}\right)=\sqrt{2}I_1(\sqrt{2})=\sum_{k=0}^{\infty}\frac{1}{2^k\,k!(k+1)!}$

.2717962498229888776... $\approx \dfrac{63+5\pi\sqrt{3}-45\log 3}{150} = \sum_{k=1}^{\infty}\dfrac{1}{k(3k+5)}$

.2718281828549045235... $\approx \dfrac{e}{10}$

1 .2720196459514068964 2... $\approx \sqrt{\phi}$

.2720290549821331629 5... $\approx \dfrac{\pi}{\sinh \pi} = \prod_{k=1}^{\infty}\left(1-\dfrac{1}{k^2+1}\right) = \prod_{k=2}^{\infty}\left(1-\dfrac{2}{k^2+1}\right)$

$= \Gamma(i)\Gamma(-i) = \dfrac{\Gamma(3+i)\Gamma(3-i)}{10} = \Gamma(1+i)\Gamma(1-i)$

$= \dfrac{1}{2}\Gamma(2+i)\Gamma(2-i) = 10\Gamma(-2+i)\Gamma(-2-i)$

1 .2721655269759086776... $\approx \dfrac{\sqrt{6}}{9}$

.2721982612879502663 1... $\approx \dfrac{\pi\log 2}{8} = G - \dfrac{i}{2}\left(Li_2\left(\dfrac{1-i}{2}\right) - Li_2\left(\dfrac{1+i}{2}\right)\right)$

$= \int_0^1 \dfrac{\log(1+x)}{1+x^2}dx$ GR 4.291.8

$= \int_1^{\infty} \dfrac{\log(x-1)}{1+x^2}dx$ GR 4.291.11

$= -\int_0^1 \dfrac{\log x\, dx}{\sqrt{1-x^4}}$ GR 4.243

$= \int_0^{\log 2} \dfrac{x\, dx}{e^x+2e^{-x}-2}$ GR 3.418.3

$= -\int_0^{\pi/2} \log(\sin x)\dfrac{\sin x\, dx}{\sqrt{1+\sin^2 x}}$ GR 4.386.1

$= \int_0^1 \log\left(\dfrac{1-x}{x}\right)\dfrac{dx}{1+x^2}$ GR 4.297.3

$= \int_0^{\pi/4} \dfrac{x\, dx}{(\cos x+\sin x)\cos x}$

$= -\int_0^1 x \arccos x \log x\, dx$

$= \int_0^1 \dfrac{\arctan x}{x+1}\, dx$

$= \int_0^{\pi/4} \log(1+\tan x)\, dx$ GR 4.227.9

= $\displaystyle\int_0^{\pi/4} \log((\cot x) - 1)\, dx$ GR 4.227.14

.272204279827698971... ≈ $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k^4 - 12}$

15 .272386255090690069056... ≈ $\displaystyle\sum_{k=1}^{\infty} \frac{2^k}{F_k F_k}$

.2725887222397812377... ≈ $\displaystyle 4\log 2 - \frac{5}{2} = \sum_{k=1}^{\infty} \frac{1}{2^k (k+2)}$

6 .2726176910987667721... ≈ $\displaystyle 8 - 2\cot\frac{1}{4} = \sum_{k=1}^{\infty} \frac{1}{k^2\pi^2 - 1/16}$

.27270887912132973503... ≈ $\displaystyle 3 - 2\sqrt{e} - \gamma + \log 2 + Ei(\frac{1}{2}) = \sum_{k=1}^{\infty} \frac{k}{(k+1)!2^k k}$

.272727272727272727$\underline{27}$ = $\displaystyle\frac{3}{11}$

.27292258601186271514... ≈ $\displaystyle\psi^{(1)}(1+\pi) = \sum_{k=1}^{\infty} \frac{1}{(\pi + k)^2}$

1 .27312531205531787229... ≈ $\displaystyle 1 + \frac{\pi}{2\sqrt{5}} \tan\frac{\pi\sqrt{5}}{2} = \sum_{k=2}^{\infty} \frac{k^2}{k^4 - 3k^2 + 1} = \sum_{k=1}^{\infty} F_{2k}\left(\zeta(2k) - 1\right)$

.273167869100517867... ≈ $\displaystyle\frac{2}{\sqrt{7}} \arcsin\frac{1}{2\sqrt{2}} = \sum_{k=1}^{\infty} \frac{1}{2^k \binom{2k}{k} k}$

1 .2732395447351626862... ≈ $\displaystyle\frac{4}{\pi} = \frac{\Gamma(2)}{\Gamma(3/2)^2} = \binom{1}{1/2} = \frac{5}{4} + \sum_{k=2}^{\infty} \left(\frac{(2k-3)!!}{(2k)!!}\right)^2$ J274

= $\displaystyle\sum_{k=0}^{\infty} \binom{2k}{k} \frac{1}{16^k (k+1)}$

= $\displaystyle\sum_{k=0}^{\infty} \frac{1}{2^k} \tan\frac{\pi}{2^{k+2}}$ K ex. 105

= $\displaystyle\prod_{k=1}^{\infty} \frac{(2k+1)^2}{2k(2k+2)}$

.2733618190819584304... ≈ $\displaystyle\sum_{k=1}^{\infty} \frac{1}{2^{k+2} - 1}$

.27351246536656649216... ≈ $\displaystyle\frac{26\zeta(3)}{27} - \frac{4\pi^3}{81\sqrt{3}} = -\frac{1}{27}\psi^{(2)}\left(\frac{2}{3}\right)$

= $\displaystyle\int_1^{\infty} \frac{\log^2 x}{x^3 - 1} dx = \int_1^{\infty} \frac{x^2}{e^{2x} - e^{-x}} dx$

9 .2736184954957037525... ≈ $\sqrt{86}$

1 .27380620491960053093... $\approx$ $\pi\log\frac{3}{2}=\int\limits_0^1\log(1+3x^2)\frac{dx}{\sqrt{1-x^2}}$ GR 4.295.38

.2738423195924228646... $\approx$ $\frac{1}{2}\left(8\log^2 2+14\log 2-13\right)=\sum_{k=1}^{\infty}\frac{H_k}{2^k(k+3)}$

.27389166654145381... $\approx$ $\frac{1}{2}\left(54-3\pi\sqrt{3}-\pi^2-27\log 3+2\zeta(3)\right)$

$=$ $\sum_{k=1}^{\infty}\frac{1}{3k^4+k^3}=\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(k+3)}{3^k}$

.2738982192085186132... $\approx$

$$\frac{1}{8}\left(\left(\pi-2\,\text{arctanh}\left(\sin\frac{\pi}{8}\right)\right)\cos\frac{\pi}{8}-\left(\pi-2\,\text{arctanh}\left(\cos\frac{\pi}{8}\right)\right)\sin\frac{\pi}{8}\right)$$

$=$ $\int\limits_1^{\infty}\frac{dx}{x^4+x^{-4}}$

.27395754917283546268... $\approx$ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{2k-1}=\sum_{k=2}^{\infty}\left(\frac{1}{\sqrt{k}}\text{arctanh}\frac{1}{\sqrt{k}}-\frac{1}{k^2}\right)$

.27413352840916617145... $\approx$ $\frac{-2\sin\pi\sqrt{3}}{\pi\sqrt{3}}=\prod_{k=1}^{\infty}\left(1-\frac{3}{(k+2)^2}\right)$

.2741556778080377394... $\approx$ $\frac{\pi^2}{36}=\sum_{k=1}^{\infty}\frac{1}{2\binom{2k}{k}k^2}=\sum_{k=1}^{\infty}\frac{\cos\frac{\pi k}{3}}{k^2}$

$=$ $\int\limits_1^{\infty}\log\left(1+\frac{1}{x^3}\right)\frac{dx}{x}=-\int\limits_0^1\frac{\log(1-x^6)}{x}dx$

2 .2741699796952078083... $\approx$ $32(5\sqrt{2}-7)=\sum_{k=0}^{\infty}\frac{1}{8^k}\binom{2k+3}{k}$

1 .2743205342359344929... $\approx$ $\frac{\pi}{\sqrt{2}}\coth\frac{\pi}{\sqrt{2}}-1=\sum_{k=1}^{\infty}\frac{1}{(k^2+1/2)}$ J274

$=$ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)}{2^{k-1}}$

28 .2743338230813914616... $\approx$ 9π

.27443271527712032311... $\approx$ $\frac{6}{25}+\frac{\pi^2}{6}+\frac{4\sqrt{5}\,\log\phi}{125}=\frac{1}{2}\,{}_2F_1\left(2,2,\frac{3}{2},-\frac{1}{4}\right)$

		=	$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k}{\binom{2k}{k}}$
	.2745343040797586252...	≈	$\frac{83711}{304920}=\sum_{k=1}^{\infty}\frac{1}{k(k+11)}$
	.27454031009933117475...	≈	$\sum_{k=1}^{\infty}\frac{\sqrt{k}}{5^k} = Li_{-1/2}\left(\frac{1}{5}\right) = \Phi(\frac{1}{5},-\frac{1}{2},0)$
	.2746530721670274228...	≈	$\frac{\log 3}{4} = \frac{1}{2}\operatorname{arctanh}\frac{1}{2} = \sum_{k=1}^{\infty}\frac{1}{4^k(2k-1)}$
		=	$\int_0^{\infty}\frac{dx}{e^{2x}+2}$
	.2747164641260063488...	≈	$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{H_{2k}}{4^k}=\frac{1}{5}\left(\pi-2\arctan 2+2\log\frac{5}{4}\right)$
7452	.274833361552916627...	≈	$\frac{\pi^9}{4}$
	.27489603948279800810...	≈	$\frac{4}{9}+\frac{\pi}{6}-\log 2=\sum_{k=1}^{\infty}\frac{1}{k(4k+3)}$
1	.27498151558114209935...	≈	$\sum_{k=1}^{\infty}\frac{1}{3^k-2^k}$

.2750000000000000000$\underline{0}$ = $\frac{11}{40}$

.27509195393450010741... ≈ $\sum_{k=1}^{\infty}\frac{\sin k}{(k+1)^2} = -\frac{i}{2}e^{-i}\left(\mathrm{Li}_2(e^i) - e^{2i}\mathrm{Li}_2(e^{-i})\right)$

3 .27523219111171830436... ≈ $\frac{3}{G}$

.27538770702495781532... ≈ $\frac{\pi^2}{12} + \log 2 - \frac{\log^2 2}{2} - 1 = \log 2 + Li_2\left(\frac{1}{2}\right) - 1$

= $\sum_{k=1}^{\infty}\frac{1}{2^k k^2 (k+1)}$

.27539847458111520468... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!\zeta(2k)}$

.27543695159824347151... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^k}{k^3+k^3+k+1}$

= $\frac{1}{4} + 2\pi\,\mathrm{csch}\,\pi + \frac{\log 2}{2} + \frac{1}{8}\left(\psi(1+\frac{i}{2}) + \psi(1-\frac{i}{2}) - \psi(\frac{1}{2}+\frac{i}{2}) - \psi(\frac{1}{2}-\frac{i}{2})\right)$

.27557534443399966272... ≈ $\gamma - \frac{\pi}{2} + \frac{i}{2}\left(\log\Gamma(1-i) - \log\Gamma(1+i)\right)$

= $\sum_{k=1}^{\infty}\left(\frac{1}{k} - \arctan\frac{1}{k}\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}\zeta(2k+1)}{(2k+1)}$

.27568727380043716389... ≈ $-\int_0^1 \log x \tan x\, dx$

.27572056477178320776... ≈ $\mathrm{csch}\,2 = \frac{1}{e^2 - e^{-2}} = \frac{1}{2} - \sum_{k=1}^{\infty}\frac{2^{2k}(2^{2k-1}-1)\mathrm{B}_{2k}}{(2k)!}$ J132, AS 4.5.65

.27574430680044962269... ≈ $\log 2 + \frac{\log(\cos 1)}{2} + \frac{\log(-\cos 2)}{8} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\cos^4 k}{k}$

.275806840982172055143... ≈ $\frac{\pi G}{4} - \frac{\zeta(3)}{2} + \frac{\pi^2}{16}\log(1+i) + \frac{1}{2}Li_3(-i)$

= $\frac{1}{64}\left(16\pi G + 2\pi^2\log 2 - 35\zeta(3)\right) = \int_0^{\pi/4} x^2 \cot x\, dx$

3 .275822918721811... ≈ Levy constant

4 .2758373284623804537... ≈ $\frac{8\pi}{5}\sqrt{\frac{2}{5-\sqrt{5}}} = \frac{4\pi}{5}\csc\frac{4\pi}{5} = \int_0^{\infty}\frac{x\,dx}{1+x^{5/2}4}$

.27591672059822730077... ≈ $\sum_{k=2}^{\infty}(-1)^k\left(1 - \frac{1}{\zeta(k)}\right) = \sum_{k=2}^{\infty}\frac{\mu(k)}{k(k+1)}$

.27613216654423932362... ≈ $\frac{1}{4}+\frac{\pi}{2\sqrt{2}}\operatorname{csch}\pi\sqrt{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2-2k+3}$

.27614994701951815422... ≈ $\frac{1}{8}+\frac{\pi}{12\sqrt{3}} = \sum_{k=1}^{\infty}\frac{1}{9k^2-4}$

.27625744117189995523... ≈ $\frac{5\pi^3}{324\sqrt{3}} = \int_0^{\infty}\frac{\log^2 x}{x^3+x^{-3}}\,dx$

1 .27627201552085350313... ≈ $\frac{13\pi}{32} = \int_0^{\infty}\frac{x^3-\sin^3 x}{x^5}dx$ GR 3.787.3

.276326390168236933... ≈ $Erf\left(\frac{1}{4}\right) = \frac{2}{\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!(4^{2k+1})(2k+1)}$

.2763586311608143417... ≈ $\frac{\zeta(3)}{\pi}+\frac{\gamma}{\pi^3}-\frac{1}{6}+\frac{\psi(1+\pi)}{\pi^3} = \sum_{k=1}^{\infty}\frac{1}{k^3(k+\pi)}$

.27648051389327864275... ≈ $\log 2-\frac{5}{12} = \sum_{k=1}^{\infty}\binom{2k}{k}\frac{1}{4^k k(k+2)}$

1 .27650248066272008091... ≈ $4\pi-\frac{\pi^2}{3}-8 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2(k^2-1/4)}$

.27657738541362436498... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!+1}$

2 .27660348762875491939... ≈ $\frac{2}{e}I_2(2\sqrt{e}) = {}_0\mathrm{F}_1(;3;e) = 2\sum_{k=0}^{\infty}\frac{e^k}{k!(k+2)!}$

1 .276631920058325092360...≈ $\frac{1}{8}\zeta\left(\frac{3}{2},\frac{1}{4}\right) = \sum_{k=0}^{\infty}\frac{1}{(4k+1)^{3/2}} = \frac{\pi}{2}\left(\frac{1}{2}+\sum_{0}^{\infty}\frac{(-1)^{k(k+1)/2}}{\sqrt{k}+\sqrt{k+2}}\right)$

[Ramanujan] Berndt IV p. 405

.276837783997013443598...≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{\binom{3k}{k}}$

.276877988824579262196...≈ $4\log^2 2-\frac{\pi^2}{6} = \sum_{k=1}^{\infty}\frac{H_k}{k(k+1)(2k+1)}$

.27700075399818548102... ≈ $-\frac{43}{234}-\frac{9\pi\sqrt{13}}{234}\csc\pi\sqrt{13}$

= $\sum_{k=4}^{\infty}\frac{(-1)^k}{k^2-13}$

.2770211831173581170... ≈ $\frac{\pi^2-\pi}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(k-\frac{1}{2})!\,(\pi-1)^k}{(k-1)!\,\pi^k}$

.27704798770564582706... $\approx$ $\frac{14}{25} + \frac{1}{100}\left(8\arctan 2 - 3\log 5 - 56\gamma\right) = \int\limits_0^\infty x e^{-x} \log x \sin^2 x \, dx$

.27708998545725868233... $\approx$ $\sum_{k=2}^\infty \frac{1}{4^k(\zeta(k)-1)}$

.2771173658778438152... $\approx$ $\sum_{k=2}^\infty (\zeta(k)-1)^3$

4 .27714550058094978113... $\approx$ $\Phi\left(\frac{1}{2}, 2, \frac{1}{2}\right)$

.27723885095508739363... $\approx$ $\zeta(3) - \log 2 \log^2 3 + 4\log 3 \log^2 2 - 4\log 2 - \log\frac{4}{3} Li_2\left(\frac{3}{4}\right)$

$+ Li_3\left(\frac{1}{4}\right) - Li_3\left(\frac{3}{4}\right)$

$= \sum_{k=1}^\infty \frac{H_k}{4^k k^2} = \sum_{k=1}^\infty \frac{(-1)^{k+1} H^{(2)}{}_k}{3^k k}$

.27732500588274005705... $\approx$ $\gamma \log^2 2$

.27734304784012952697... $\approx$ $-\zeta\left(-\frac{1}{3}\right)$

1 .27739019782838851219... $\approx$ $\int\limits_0^\infty \frac{x \log(1+x)}{e^x - 1} dx$

1 .277409057559636731195... $\approx$ $\frac{\pi\sqrt{3}}{12} + \frac{3\log 3}{4} = \sum_{k=1}^\infty \frac{1}{3k^2 - 2k}$

$= -\int\limits_0^1 \frac{\log(1-x^3)}{x^3} dx$

.27750463411224827642... $\approx$ $-Li_2(1-e) - 1 = \sum_{k=1}^\infty \frac{(-1)^k B_k}{(k+1)!}$

1 .27750463411224827642... $\approx$ $-Li_2(1-e) = \sum_{k=0}^\infty \frac{(-1)^k B_k}{(k+1)!}$

.2776801836348978904... $\approx$ $\frac{\pi}{8\sqrt{2}} = \int\limits_0^\infty \frac{dx}{(x^2+2)^2} = \int\limits_0^\infty \frac{x^2\,dx}{(x^4+1)^2}$

192 .27783871506088740604... $\approx$ $\frac{\pi^6}{5}$

1 .27792255262726960230... $\approx$ $2\sin\log 2 = -i(2^i - 2^{-i})$

.2779871641507590436... $\approx$ $\frac{\log 7}{7}$

1 .2779927307151103419... $\approx \sum_{k=2}^{\infty} \frac{1_k}{(k^2-k)\log^2 k}$

.278114315443049617352... $\approx \frac{5\zeta(3)}{8} + 2\pi\log 2 + \log^2 2 - \frac{\pi^2}{6} - 4G$

$= \sum_{k=1}^{\infty} \frac{(-1)^{k+1} H_k}{k^2(2k+1)}$

.27818226241059482875... $\approx 1 - \frac{\text{arcsinh} 2}{2} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2k+1}\binom{2k}{k}$

2 .27820645668385604647... $\approx \sum_{k=1}^{\infty} \frac{\zeta(2k)}{k!(k-1)!} = \sum_{k=1}^{\infty} \frac{1}{k} I_1\left(\frac{2}{k}\right)$

7 .278557014709636999... $\approx \Phi\left(-4, 3, \frac{1}{2}\right) = \int_0^1 \frac{\log^2 x}{x^2+1/4}$

$= \frac{\pi^3}{4} + \pi\log^2 2 + 2i\left(Li_3\left(\frac{i}{2}\right) - Li_3\left(-\frac{i}{2}\right)\right)$

.27865247955551829632... $\approx 1 - \frac{1}{2\log 2} = \sum_{k=1}^{\infty} \frac{1}{2^k} - \int_1^{\infty} \frac{dx}{2^x}$

17 .27875959474386281154... $\approx \frac{11\pi}{2}$

.2788055852806619765... $\approx \sqrt{\pi}(1 - erf 1) = \Gamma\left(\frac{1}{2}, 1\right) = \int_1^{\infty} \frac{dx}{e^x\sqrt{x}}$

.27883951715812842167... $\approx \frac{e}{2} + \frac{5}{2e} - 2 = \int_1^{\infty} \sinh\left(\frac{1}{x}\right)\frac{dx}{x^4}$

.27892943914276219471... $\approx \frac{5}{4}\log\frac{5}{4} = \sum_{k=1}^{\infty} \frac{H_k}{5^k} = \frac{1}{4}\sum_{k=1}^{\infty} \frac{1}{5^{k-1}k}$

2 .27899152293407918396... $\approx \sum_{k=2}^{\infty} k\log\zeta(k)$

.27918783603518461481... $\approx \sum_{k=2}^{\infty} \frac{\zeta(k)-\zeta(k+1)}{k} = \sum_{k=1}^{\infty} \frac{k-1}{k^2}\left(1 + k\log\left(1 - \frac{1}{k}\right)\right)$

.27930095364867322411... $\approx 3 - \frac{\pi\sqrt{3}}{2} = \sum_{k=0}^{\infty} \frac{1}{6k^2 - 1/6} = \sum_{k=0}^{\infty} \frac{(-1)^k}{3^k(2k+3)}$

.27937855536096096724... $\approx \sum_{k=1}^{\infty} \frac{S_2(2k,k)}{(2k)^{2k}}$

.27937884849256930308... $\approx \frac{1}{2}\left(\zeta\left(\frac{1}{2}, 1\right) - \zeta\left(\frac{1}{2}, \frac{3}{2}\right)\right) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{2k+2}}$

$= \frac{1}{2}\left(\sqrt{2} + (2-\sqrt{2})\zeta\left(\frac{1}{2}\right)\right)$

.2794002624059601442... ≈ $\sum_{k=1}^{\infty}\frac{1}{4^k+1}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{4^k-1}$

.27956707220948995874... ≈ $\frac{1}{2}J_0(\sqrt{2}) = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}k^2}{(2k)!2^k}\binom{2k}{k}$

2 .2795853023360672674... ≈ $I_0(2) = \sum_{k=0}^{\infty}\frac{1}{(k!)^2} = \sum_{k=0}^{\infty}\frac{k^2}{(k!)^2}$ LY 6.112

= $_0F_1(;1;1) = \frac{1}{2\pi}\int_0^{2\pi} e^{2\cos\theta}d\theta$ Marsden p. 203

= $\sum_{k=0}^{\infty}\frac{1}{(2k)!}\binom{2k}{k} = \sum_{k=0}^{\infty}\frac{k^2}{(2k)!}\binom{2k}{k} = e^2\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\binom{2k}{k}$

.2797954925972408684... ≈ $\frac{1}{10}\left(2\gamma+\psi(1+i\sqrt{5})+\psi(1-i\sqrt{5})\right) = \sum_{k=1}^{\infty}\frac{1}{k^3+5k}$

1 .2798830013730224939... ≈ $e^{\cos 1}\sin(\sin 1) = -\frac{i}{2}\left(e^{e^i} - e^{e^{-i}}\right) = \sum_{k=1}^{\infty}\frac{\sin k}{k!}$ GR 1.449.1

1 .27997614913081785... ≈ $\frac{\sqrt{2\pi}\,\mathrm{erfi}(\sqrt{2})}{e^2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}8^k}{k!\binom{2k}{k}}$

.280000000000000000$\underline{0}$ = $\frac{7}{25} = \prod_{p\ prime}\frac{1-p^{-6}}{\left(1+p^{-2}\right)^3}$

.28010112968305596106... ≈ $\frac{\sqrt{\pi}}{4e}(e-1) = \int_0^{\infty}\frac{\sin^2 x}{e^{x^2}}dx$

7 .2801098892805182711... ≈ $\sqrt{53}$

.28015988490015307012... ≈ $\sum_{k=1}^{\infty}\frac{\mu(k)}{\binom{2k}{k}}$

.2802481474936587141... ≈ $\frac{\gamma}{3} + \frac{1}{6}\left(\psi\left(1+i\sqrt{\frac{3}{2}}\right)+\psi\left(1-i\sqrt{\frac{3}{2}}\right)\right) = \sum_{k=1}^{\infty}\frac{1}{2k^3+3k}$

.2802730522345168582... ≈ $\frac{1}{6}\left(\pi\sqrt{3} - 9\log 3 + 2\psi^{(1)}(\frac{2}{3})\right) = \sum_{k=1}^{\infty}\frac{1}{k(3k-1)^2}$

.28037230554677604783... ≈ $\frac{5}{3} - 2\log 2 = hg\left(\frac{3}{2}\right) = \sum_{k=2}^{\infty}\frac{1}{2k^2+k}$

= $\sum_{k=2}^{\infty}\frac{(-1)^k(\zeta(k)-1)}{2^{k-1}}$

1 .28037230554677604783... ≈ $\frac{8}{3} - 2\log 2 = \frac{3}{2}\sum_{k=2}^{\infty}\frac{1}{k(k+3/2)}$

.28043325348408594672... ≈ $\frac{1}{36}\psi^{(1)}\left(\frac{1}{3}\right) = \sum_{k=1}^{\infty}\frac{1}{(6k-4)^2}$

1 .28049886910873569104... ≈ $\sum_{k=0}^{\infty}\frac{1}{\binom{2k}{k}^2} = \text{HypPFQ}\left(\{1,1,1\},\{\frac{1}{2},\frac{1}{2}\},\frac{1}{16}\right)$

.280536386394339161571... ≈ $(1-\gamma)\sin 1 + \frac{i}{2}\left(\log\Gamma\left(2-e^{-i}\right) - \log\Gamma\left(2-e^{i}\right)\right)$

= $\sum_{k=2}^{\infty}\frac{\sin k}{k}\left(\zeta(k)-1\right)$

6 .28063130354983230561... ≈ $\sum_{k=2}^{\infty}k(\zeta^2(k)-1)$

.2806438353212647928... ≈ $2\gamma\log 2 + \log^2 2 = l\left(-\frac{1}{2}\right)$ Berndt 8.17.8

.28086951303729453745... ≈ $\frac{352}{735} - \frac{2\log 2}{7} = \sum_{k=1}^{\infty}\frac{1}{k(2k+7)}$

.28092980362016137146... ≈ $\log\frac{\sqrt{e}+1}{2} = \sum_{k=1}^{\infty}\frac{(1-2^k)\zeta(1-k)}{k!2^k}$

= $\sum_{k=1}^{\infty}\frac{(-1)^k(2^k-1)B_k}{k!2^k\,k}$ [Ramanujan] Berndt Ch. 5

.2809462377984494453... ≈ $-\frac{64}{27} - \frac{1}{2}\psi^{(2)}\left(\frac{3}{4}\right) = 28\zeta(3) - \pi^3 - \frac{64}{27} = \sum_{k=1}^{\infty}\frac{1}{(k+3/4)^3}$

.2810251833787232758... ≈ $\frac{\pi^2}{24} - \frac{25}{192} = \sum_{k=1}^{\infty}\frac{1}{k^3+4k^2}$

.28103798890283904259... ≈ $2\sqrt{3}\log\left(\sqrt{\frac{3}{2}}+\frac{1}{\sqrt{2}}\right) - 2 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}2^k}{\binom{2k}{k}(2k+1)k}$

2 .281037988902839042593... ≈ $\sqrt{3}\log\frac{\sqrt{3}+1}{\sqrt{3}-1}$

.28128147005811129632... ≈ $\frac{1}{2}\log(2\cos\frac{1}{2}) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}\cos k}{k}$ GR 1.441.4

.28171817154095476464... ≈ $3-e = \sum_{k=0}^{\infty}\frac{k}{(k+2)!} = \sum_{k=2}^{\infty}\frac{1}{k!k(k-1)}$

= $\sum_{k=0}^{\infty}\frac{k}{k!(2k+6)} = \sum_{k=0}^{\infty}\frac{1}{k!(k+2)(k+3)}$

1 .28171817154095476464... ≈ $4-e = \sum_{k=0}^{\infty}\frac{k^2}{k!(k+2)}$

.28173321065145428066... ≈ $\frac{e-1}{2e} + \frac{\cos(\sin 2) - e^{\cos 2}}{2e^{\cos 2}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}\sin^2 k}{k!}$

.28184380823877678730... ≈ $\frac{\pi}{4}\left(\cot\frac{3\pi}{8} - \cot\frac{7\pi}{8}\right) - \log 2 + \sqrt{2}\left(\log\sin\frac{\pi}{8} - \log\sin\frac{3\pi}{8}\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{4k^2-k}$

.281864748315611781912... ≈ $\frac{1}{8}(2\log 2\pi + 2\gamma - \pi)\left(\zeta\left(\frac{1}{2},\frac{1}{4}\right) - \zeta\left(\frac{1}{2},\frac{3}{4}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}\log(2k+1)}{\sqrt{2k+1}}$

.2819136638478887003... ≈ $\log 2 - \frac{\pi^2}{24} = \sum_{k=1}^{\infty}\frac{3k+1}{2k^2(2k+1)^2}$

= $\sum_{k=3}^{\infty}\frac{(-1)^{k+1}k\zeta(k)}{2^k}$

= $\int_0^1 \operatorname{arctanh} x \log x \, dx$

.28209479177387814347... ≈ $\frac{1}{\sqrt{4\pi}}$

.28230880383984003308... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}H^{(3)}{}_k}{3^k k}$

$.28\underline{2352941176470588}$ = $\frac{24}{85} = \int_0^{\infty}\frac{\sin^4 x}{e^x}$

1 .282427129100622636875... ≈ $\exp\left(\frac{1}{12} - \zeta'(-1)\right)$, Glaisher's constant

1 .28251500934298169528... ≈ $\pi\sqrt{3} - 6\log 2 = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+1)(k+2/3)}$

.28253095179252723739... ≈ $\frac{\pi^2\log 2}{3} + \frac{8\log^3 2}{3} - 2\log^2 2\log 3 + 4\log 2\, Li_2\left(\frac{1}{4}\right) - 3\zeta(3)$

$+ 4Li_3\left(\frac{1}{4}\right)$

= $\int_0^1 \frac{\log^2(1+x)}{x(x+1/2)}dx$

1 .28254983016186409554... ≈ $\frac{\pi}{\sqrt{6}} = \sqrt{\zeta(2)}$

	.2825795519962425136...	≈	$\frac{1}{2}I_1(1)=\sum_{k=0}^{\infty}\frac{k}{(k!)^2 4^k}$
	.28267861238222538196...	≈	$\sum_{k=1}^{\infty}\frac{1}{k^3+6}$
	.282698133451692176803...	≈	$\frac{1}{12}\left(\pi^2+2H\left(-\frac{2}{3},2\right)+\pi\sqrt{3}\left(\log 3-1\right)+9\log 3\right)$
		=	$\sum_{k=1}^{\infty}\frac{H_k}{(3k-1)(3k+1)}$
	.2827674723312838187...	≈	$\sum_{k=2}^{\infty}\frac{1}{k^3-3}$
	.282823346997933107054...	≈	$\sum_{k=1}^{\infty}\frac{1}{\binom{2k}{k}k(k+1)}$
	.2829799868805425028...	≈	${}_0\mathrm{F}_1(\,;2;-2)\;=\;\frac{J_1(2\sqrt{2})}{\sqrt{2}}\;=\sum_{k=0}^{\infty}\frac{(-1)^k 2^k}{k!(k+1)!}$
	.2831421008321609417...	≈	$\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k\,\zeta(k)}$
	.28318530717958648...	≈	$2\pi-6\;=\int_0^1\arcsin^2 x\arccos x\,dx$
6	.28318530717958648...	≈	$2\pi\;=\;\Gamma\left(\frac{1}{6}\right)\Gamma\left(\frac{5}{6}\right)$
		=	$\sum_{k=0}^{\infty}\frac{1}{(k+\frac{1}{4})(k+\frac{3}{4})}$
		=	$\int_0^{\infty}\log\left(1+x^{-6}\right)dx$
		=	$\int_0^{2\pi}\frac{d\theta}{\sqrt{2}+\cos\theta}\;=\;\int_{-\infty}^{\infty}\frac{e^{x/6}dx}{e^x+1}$
6	.2832291305689209135...	≈	$2\pi\coth 2\pi$
2	.2833333333333333333$\underline{3}$	=	$\frac{137}{60}\;=\;H_5$
	.2833796840775488247...	≈	$\sum_{k=1}^{\infty}\frac{\zeta(2k)}{(2k+1)!}\;=\;\sum_{k=1}^{\infty}\left(k\sinh\frac{1}{k}-1\right)$
	.2834686894262107496...	≈	$1-e^{-1/3}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!3^k}$
5	.28350800118212351862...	≈	$\pi 2^{3/4}\;=\int_0^{\infty}\log\left(1+2x^{-4}\right)dx$

.28360467567550685407... ≈ $3\log 2-3\log 3+\frac{3}{2} = \sum_{k=1}^{\infty}\frac{k}{3^k(k+1)} = \sum_{k=0}^{\infty}\frac{(-1)^k}{2^k(k+1)(k+3)}$

.2837571104739336568... ≈ $\log\sqrt{\pi}-\frac{\gamma}{2} = \sum_{k=2}^{\infty}\frac{\zeta(k)}{2^k k} = -\sum_{k=1}^{\infty}\left(\log\left(1-\frac{1}{2k}\right)+\frac{1}{2k}\right)$

.28375717363054911029... ≈ $\frac{6}{25}\left(1+\log\frac{6}{5}\right) = \sum_{k=1}^{\infty}\frac{k\,H_k}{6^k}$

.28382295573711532536... ≈ $\frac{\pi^2}{6}-\frac{49}{36} = \sum_{k=4}^{\infty}\frac{1}{k^2} = \psi^{(1)}(4) = \zeta(2,4) = \Phi(1,2,4)$

.2838338208091531730... ≈ $\frac{1}{4}\left(1+\frac{1}{e^2}\right) = \sum_{k=0}^{\infty}\frac{(-1)^k 2^k}{(k+2)!} = \int_0^1\frac{\sinh x}{e^x}dx = \int_0^1\frac{1}{1+\coth x}dx$

1 .28402541668774148073... ≈ $\sqrt[4]{e}$

.28403142497970661726... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)-\zeta(2k+2)}{(2k)!}$

= $\sum_{k=2}^{\infty}\left(\frac{1}{k^2}-1+\left(1-\frac{1}{k^2}\right)\cosh\frac{1}{k}\right)$

.28422698551241120133... ≈ $ci(1)\sin 1-ci(2)\sin 1-\mathrm{si}(1)\cos 1+\mathrm{si}(2)\cos(1)$

= $\int_0^1\frac{\sin x\,dx}{1+x}$

6 .2842701641260997176... ≈ $\sum_{k=1}^{\infty}\frac{k\,H_k{}^2}{2^k}$

.28427535596883239397... ≈ $\frac{\pi^2}{12}-\log^2 2-\frac{\log^2 3}{6}+\frac{\log^2 2\log 3}{4}-Li_2\left(-\frac{1}{2}\right)\log 2$

$-\frac{1}{2}Li_3\left(-\frac{1}{2}\right)-\frac{21}{48}\zeta(3)$

= $\int_0^1\frac{\log^2(1+x)}{x^2(x+2)}dx$

2 .28438013687073247692... ≈ $\zeta(3)+\zeta(4)$

.28439044848526304562... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)}{(2k-1)!} = \sum_{k=1}^{\infty}\left(\sqrt{\frac{1}{k}}\sinh\sqrt{\frac{1}{k}}-\frac{1}{k}\right)$

2 .28479465715621326434... ≈ $\frac{8\pi}{11}$

3 .28492699492757736265... ≈ $\sum_{k=1}^{\infty}\frac{F_k}{k!!}$

.28504535266071318937... ≈ $\frac{\pi^2\log 2}{24}$

1 .28515906306284057434... ≈ $\sum_{k=1}^{\infty}\frac{1}{k!\zeta(2k)}$

.28518527799578963 0197... ≈ $\frac{\sqrt{\pi}\sqrt{2-\sqrt{2}}}{2^{9/4}} = \int_0^\infty e^{-x^2}\sin x^2\,dx$

.28539816339744830962... ≈ $\frac{\pi}{4}-\frac{1}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{4k^2-1}$ J366, J606

= $\sum_{k=1}^{\infty}\frac{\sin k\cos k}{k} = \sum_{k=1}^{\infty}\frac{\sin k\cos^2 k}{k}$

= $\int_0^1 x\arctan x\,dx$

.2855993321445266580... ≈ $\frac{\pi}{11}$

1 .2856908396267850266... ≈ $\frac{15e}{32}+\frac{1}{32e} = \sum_{k=0}^{\infty}\frac{k^4}{(2k)!}$

$.285714285714\underline{285714}$ = $\frac{2}{7}$

1 . 2857644965889154964... ≈ $\frac{\pi}{\Gamma^2\left(\frac{5}{8}\right)\Gamma^2\left(\frac{7}{8}\right)} = \sum_{k=0}^{\infty}\frac{(4k)!}{4^{4k}(k!)^4}$

.285816417082407503426... ≈ $\frac{\zeta(3)}{3}-\frac{\pi^2}{54}+\frac{11}{162} = \sum_{k=1}^{\infty}\frac{1}{k^3(k+3)}$

1 .2858945918269595525... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2}{k!!}$

.28602878170071023341... ≈ $\frac{\pi^2-2\pi}{16}+\frac{3\log 2}{4}-\frac{G}{2} = \sum_{k=1}^{\infty}\frac{8k-1}{4k(4k-1)^2}$

= $\sum_{k=2}^{\infty}\frac{k\,\zeta(k)}{4^k}$

.2860913089823752704... ≈ $\zeta(3)-\boldsymbol{G}$

.28623351671205660912... ≈ $\frac{\pi^2}{24}-\frac{1}{8} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^3+2k^2}$

.2866116523516815594... ≈ $\frac{6-\sqrt{2}}{16}$ CFG D1

.2868382595494097246... ≈ $\frac{1}{4}\mathrm{HypPFQ}\left(\{1,1,1,1\},\{\frac{1}{2},2,2\},\frac{1}{4}\right) = \sum_{k=0}^{\infty}\frac{(k!)^2}{(2k)!(2k+2)^2}$

.286924988361124608425... ≈ $\frac{\pi}{4\sqrt{2}}\coth\frac{\pi}{\sqrt{2}}-\frac{\pi^2}{8}\operatorname{csch}^2\frac{\pi}{\sqrt{2}}-\frac{2}{9} = \sum_{k=2}^{\infty}\frac{2k^2}{(2k^2+1)}$

= $\sum_{k=1}^{\infty}(-1)^{k+1}k\frac{\zeta(2k)-1}{2^k}$

1 .28701743034606835519... ≈ $e^{1/32}\left(1+\sqrt{\frac{\pi}{2}}\,erf\,\frac{1}{4\sqrt{2}}\right)=\sum_{k=1}^{\infty}\frac{1}{k!!\,4^k}$

1 .287044548647559922... ≈ $\prod_{k=2}^{\infty}\left(1+\frac{1}{(k!)^2}\right)$

1 .287159051356654205862... ≈ $\sum_{k=2}^{\infty}\left(e^{\zeta(k)-1}-1\right)$

1 .2872818803541853045... ≈ $\frac{2e^3-17}{18}=\sum_{k=1}^{\infty}\frac{3^k k^2}{(k+3)!}$

.28735299049400502... ≈ $-c_4$ Patterson Ex. 4.4.2

$=\frac{1}{120}(\gamma^5-10\gamma^3\zeta(2)+20\gamma^2\zeta(3)+15\gamma\zeta^2(2)-30\gamma\zeta(4)+24\zeta(5))$

2 .2873552871788423912... ≈ $e\sin 1=\sum_{k=1}^{\infty}\frac{2^{k/2}\sin\frac{\pi k}{4}}{k!}$

1 .287534665778537497484... ≈ $2G-\frac{\pi\log 2}{4}\;=\;i\left(Li_2\left(\frac{1-i}{2}\right)-Li_2\left(\frac{1+i}{2}\right)\right)$

2 .28767128758328065181... ≈ $2\,e^{\cos 1}\cos(\sin 1)\;=\;e^{e^{i}}+e^{e^{-i}}$

3 .2876820724517809274... ≈ $2\log 2-\log 3\;=\;\Phi\left(\frac{1}{4},1,1\right)\;=\;\sum_{k=1}^{\infty}\frac{1}{4^k k}$ J117

= $\int_1^2\frac{dx}{x^2+x}\;=\;\int_1^{\infty}\frac{dx}{3x^2+x}\;=\;\int_0^{\log 2}\frac{dx}{e^x+1}\;=\int_0^{\infty}\frac{dx}{3e^x+1}$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{3^k k}\;=\;\mathrm{Li}_1\left(\frac{1}{4}\right)\;=\;2\sum_{k=0}^{\infty}\frac{1}{7^{2k+1}(2k+1)}\;=\;2\operatorname{arctanh}\frac{1}{7}$

10 .287788753390262846... ≈ $\sum_{k=0}^{\infty}\frac{k^e}{k!}$

1 .28802252469807745737... ≈ $\log\Gamma\left(\frac{1}{4}\right)$

2 .288037795340032418... ≈ $\Gamma(\pi)$

.2881757683093445651... ≈ $\psi\left(\frac{6}{5}\right)+\gamma\;=\;hg\left(\frac{1}{5}\right)$

= $5\;-\;\frac{\pi}{2}\sqrt{1+\frac{2}{\sqrt{5}}}\;-\;\frac{5\log 5}{4}\;+\;\frac{\sqrt{5}}{4}\log\frac{2}{3+\sqrt{5}}$

= $\sum_{k=1}^{\infty}\frac{1}{5k^2+k}\;=\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(k+1)}{5^k}$

3 .28836238184562492552... ≈ $\sum_{k=1}^{\infty}\frac{t_3(k)}{k!}$

.28838543541673038149... ≈ $\sum_{k=2}^{\infty}\frac{\phi(k)}{k^2(k+1)} = \sum_{k=2}^{\infty}(-1)^k\left(\frac{\zeta(k)}{\zeta(k+1)}-1\right)$

.28852888971289910742... ≈ $\frac{i}{6}\left(\psi\left(\frac{2}{3}-\frac{i}{3}\right)-\psi\left(\frac{2}{3}+\frac{i}{3}\right)\right)=\sum_{k=0}^{\infty}\frac{1}{(3k+2)^2+1}$

.2886078324507664303... ≈ $\frac{\gamma}{2}$

= $\int_0^{\infty}(e^{-x^2}-e^{-x})\frac{dx}{x}$ GR 3.463

= $-\int_0^{\infty}(e^{-x^2}-\frac{1}{x^2+1})\frac{dx}{x}$ GR 3.467

= $\int_0^{\infty}\frac{e^{-x^2}-\cos x}{x}\,dx$

.2886294361119890619... ≈ $\frac{\log 2}{5}+\frac{3}{20} = \sum_{k=0}^{\infty}\frac{1}{(2k+1)(2k+6)}$

.2887880950866024213... ≈ $\prod_{k=1}^{\infty}\left(1-\frac{1}{2^k}\right)$

= $1+\sum_{k=1}^{\infty}(-1)^k\left(\frac{1}{2^{(3k^2+k)/2}}+\frac{1}{2^{(3k^2-k)/2}}\right)$ Hall Thm. 4.1.3

.28881854182630927207... ≈ $\sum_{k=2}^{\infty}\frac{\log\zeta(k)}{k(k-1)}$

1 .288863135... ≈ $\prod_{p\ prime}\left(1+\frac{1}{p^2(p-1)^2}\right)$

.28893183744773042948... ≈ $\frac{\pi}{4e}=\int_0^{\pi/2}\sin(\tan x)\sin^2 x\tan x\,dx$ GR 3.716.8

.2889466641286552456... ≈ $\sum_{k=1}^{\infty}\frac{1}{4^k\zeta(2k+1)}$

.2890254822222362424... ≈ $\frac{\pi}{\pi^2+1}=\int_0^{\infty}\frac{\sin\pi x}{e^x}dx$ GR 3.463

.289025491920818... ≈ $\frac{1}{\text{one}-\text{ninth constant}}$

1 .28903527533251028409... ≈ $\frac{4}{3}\sum_{k=0}^{\infty}\frac{(-1)^k}{3^k(2k+1)^2} = \sum_{k=0}^{\infty}\frac{(k!)^2 3^k}{(2k)!(2k+1)^2}$

= $-\frac{\pi}{3\sqrt{3}}\log 3-\frac{10\pi^2}{27}+5\sum_{k=0}^{\infty}\frac{1}{(3k+1)^2}$ Berndt 32.7

= $-\frac{\pi}{3\sqrt{3}}\log 3-\frac{10\pi^2}{27}+\frac{5}{9}\psi^{(1)}\left(\frac{1}{3}\right)$

= $\frac{1}{3}\Phi\left(-\frac{1}{3},2,\frac{1}{2}\right) = \text{HypPFQ}\left(\{1,1,1,1\},\left\{\frac{1}{2},2,2\right\},\frac{3}{4}\right)$

5 .28903989659218829555... ≈ $-\psi\left(\frac{1}{5}\right)$

.2891098726196906... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k}{k^2\log k}$

.28914464857067158311 2... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{F_k}$

.28922492052927723132... ≈ $\frac{2\pi\sqrt{3}+3}{48} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+3)(3k+1)}$

2 .2894597716988003483... ≈ $2\log\pi$

.28972503617945007235 9... ≈ $\frac{1001}{3455} = \prod_{p\ prime}\frac{1-p^{-2}+p^{-4}}{\left(1+p^{-2}\right)^2}$

2 .2898200986307827102... ≈ $li(\pi)$

.2898681336964528729... ≈ $\frac{\pi^2}{3}-3 = \sum_{k=1}^{\infty}\frac{1}{k^2(k+1)^2} = \sum_{k=1}^{\infty}\frac{k}{(k+1)(k+2)^2}$ K Ex. 111

= $\sum_{k=1}^{\infty}k(\zeta(k+3)-1)$

1 .2898681336964528729... ≈ $\frac{\pi^2}{3}-2 = 2\zeta(2)-2 = \sum_{k=2}^{\infty}(k\zeta(k)-(k+1)\zeta(k+1)+1)$

= $1+\sum_{k=1}^{\infty}\frac{1}{k^2(k+1)^2}$

2 .2898681336964528729... ≈ $\frac{\pi^2}{3}-1 = \sum_{k=1}^{\infty}k\left(\zeta(k+1)+\zeta(k+2)-2\right)$

= $\int_0^{\infty}\frac{1+e^{-x}}{e^x-1}dx$ GR 3.411.25

= $-\int_0^1\frac{1+x}{1-x}\log x\,dx$ GR 4.231.4

3 .2898681336964528729... ≈ $\frac{\pi^2}{3} = 2\zeta(2) = \sum_{k=3}^{\infty}\frac{(k-1)\zeta(k)}{2^{k-2}}$

= $\int_{-\infty}^{\infty}\frac{x^2dx}{\sinh^2 x} = \int_0^{\infty}\frac{\log^2 x\,dx}{(1+x)^2} = \int_1^{\infty}\frac{\log^2 x\,dx}{(x-1)^2}$ GR 4.231.4

= $\int_0^1\frac{\log^2(1-x)}{x^2}dx = -\int_0^1\frac{\log(1-\sqrt{x})}{x}dx$

= $\int_0^{\infty}\frac{x^2\,dx}{e^x+e^{-x}-2}$

= $\int_0^\infty \frac{dx}{e^{x^{1/3}}-1}$

.28989794855663561964... ≈ $\frac{\sqrt{6}-1}{5}$ CFG C1

.2899379892228521445... ≈ $\left(1-\frac{1}{e}\right)(1-\log(e-1)) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} H_k}{(e-1)^k}$

57 .289961630759424687... ≈ $\cot 1^\circ = \cot \frac{\pi}{180}$

.2900000000000000000$\underline{0}$ = $\frac{29}{100} = \sum_{k=1}^{\infty} \frac{1}{k^2+7k+6}$

.29013991712236773249... ≈ $\frac{\sqrt{2}}{3} \arctan \frac{1}{\sqrt{2}} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} H^{O}{}_{k}}{2^k}$

1 .29045464908758548549... ≈ $2^{1/e} = \prod_{k=0}^{\infty} 2^{(-1)^k/k!}$

.29051419476144759919... ≈ $\int_0^\infty \frac{\sin x dx}{e^x+2}$

= $\frac{1}{4}\left({}_2F_1\left(i,1,1+i,-\frac{1}{2}\right) + {}_2F_1\left(-i,1,1-i,-\frac{1}{2}\right) - 2\pi \operatorname{csch}(\pi) \cos(\log(2)) \right)$

.29053073339766362911... ≈ $\frac{1}{4}\left(3-\log 2\pi\right) = \sum_{k=1}^{\infty} \frac{2^k}{k+2} \left(\zeta(k+1)-1\right)$

2 .29069825230323823095... ≈ $e \, erf(1) = \sum_{k=0}^{\infty} \frac{1}{(k+\frac{1}{2})!}$

.2907302564411782374... ≈ $\frac{\pi}{4\sqrt{3}} \coth \frac{\pi\sqrt{3}}{2} - \frac{1}{6} = \sum_{k=1}^{\infty} \frac{1}{4k^2+3}$

.29081912799355107029... ≈ $4 - 8 \arctan \frac{1}{2} = \sum_{k=0}^{\infty} \frac{(-1)^k}{4^k (2k+3)}$

.29098835343466321219... ≈ $\frac{1}{2e-2} = \int_0^1 \frac{\log(1+(e-1)x)}{1+(e-1)x} dx$

.291120263232506253... ≈ $\frac{\pi^2}{24} - \frac{\log^2 2}{4} = \sum_{k=1}^{\infty} \frac{1}{2^{k+1} k^2}$ GR 4.295.17

= $\int_0^1 \frac{\log(1+x^2) dx}{x(1+x^2)}$ GR 4.295.17

= $-\int_0^1 \frac{\log(1-x^2/2)}{x} dx$

.29112157403119441224... ≈ $\sum_{k=1}^{\infty} \frac{1}{5k^3-1} = \sum_{k=1}^{\infty} \frac{\zeta(3k)}{5^k}$

.2912006131960343334... ≈ $2-2\cos\sqrt{2}-\sqrt{2}\sin\sqrt{2}=\sum_{k=1}^{\infty}(-1)^{k+1}\frac{2^{k+1}k}{(2k+1)!(k+1)}$

.2912758840711328645... ≈ $\gamma_{1/(k^2+1)}=\frac{1}{4}\left(2\pi\coth\pi-\pi-2\right)=\sum_{k=1}^{\infty}\frac{1}{k^2+1}-\int_1^{\infty}\frac{dx}{x^2+1}$

1 .29128195012492507311 5... ≈ $\frac{\pi^3}{24}$

1 .2912859970626635404 1... ≈ $\sum_{k=1}^{\infty}\frac{1}{k^k}=\int_0^1\frac{dx}{x^x}$ GR 3.486

5 .2915026221291811810... ≈ $\sqrt{28}=2\sqrt{7}$

.29156090403081878014... ≈ $\frac{G}{\pi}$

.29162205063154922281... ≈ $\frac{1}{4}-\frac{\pi^2}{16}+\frac{\pi^2\log 2}{4}-\frac{7\zeta(3)}{8}=\int_0^1\frac{x^3\arccos^2 x}{1-x^2}dx$

.291666666666666666$\underline{6}$ = $\frac{7}{24}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k(k/2+2)}=\int_0^1\frac{dx}{(x+1)^4}$

.29171761150604061565... ≈ $-\sum_{k=1}^{\infty}\frac{\mu(2k)}{3^k}$

.29183340144492820645... ≈ $\frac{635}{1107}-\frac{\pi 3^{1/4}}{4}\left(\cot\pi 3^{1/4}+\coth\pi 3^{1/4}\right)$

= $\sum_{k=1}^{\infty}3^k(\zeta(4k)-1)=\sum_{k=2}^{\infty}\frac{3}{k^4-3}$

.2918559274883350395... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)}{k^3}=\sum_{k=1}^{\infty}\left(Li_3\left(\frac{1}{k}\right)-\frac{1}{k}\right)$

.2919265817264288065... ≈ $\cos^2 1=1-\sum_{k=1}^{\infty}(-1)^{k+1}\frac{2^{2k-1}}{(2k)!}$ GR 1.412.2

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}4^k k^2}{(2k)!(k+1)}$

1 .29192819501249250731... ≈ $\frac{\pi^3}{24}=\int_1^{\infty}\frac{\arctan^2 x\,dx}{1+x^2}$

.291935813178557193... ≈ $\sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)-1}{k!}=\sum_{k=2}^{\infty}\left(e^{-1/k}-1-\frac{1}{k}\right)$

.29201720291139049247 3... ≈ $\frac{3}{4}-\frac{G}{2}=-\int_0^1 x\,\text{arccot}\,x\,\log x\,dx$

.29215558535053869628... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k}{k!(\zeta(k)-1)}$

.29215635618824943767... $\approx \frac{1}{2\pi^2}+\frac{\pi}{6}-\frac{\coth \pi^{3/2}}{2\sqrt{\pi}} = \sum_{k=1}^{\infty}\frac{1}{k^2(k^2+\pi)}$

.29226465556771149906... $\approx \sum_{k=1}^{\infty}\zeta(2k+1)\big(\zeta(2k+1)-1\big)$

.292453634344560827916... $\approx \frac{1}{2}\big(3-\gamma-\log 2\pi\big) = \sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k+1}$

$= -\sum_{k=2}^{\infty}\left(k\log\left(1-\frac{1}{k}\right)+1+\frac{1}{2k}\right)$

.2924930746750417626... $\approx \frac{\pi}{2\sqrt{5}}\coth\frac{\pi}{\sqrt{5}}-\frac{1}{2} = \sum_{k=1}^{\infty}\frac{1}{5k^2+1}$

.29251457658160771495... $\approx \frac{1}{8}\big(e-e^{\cos 4}\cos(\sin 4)\big) = \sum_{k=1}^{\infty}\frac{\sin^2 k\ \cos^2 k}{k!}$

.29265193313390600105... $\approx \sum_{k=1}^{\infty}\frac{H^{(3)}{}_k}{4^k k}$

.2928932188134524756... $\approx 1-\frac{1}{\sqrt{2}}$

$= \sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{4^k}\binom{2k}{k} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{(2k)!}{(k!)^2 4^k}$

.2928968253$\underline{968253}$ $= \frac{7381}{25200} = \frac{H_{10}}{10} = \sum_{k=1}^{\infty}\frac{1}{k^2+10k} = \sum_{k=6}^{\infty}\frac{1}{k^2-25}$

2 .292998145057285615802... $\approx 2\,PFQ\big[\{1,1,1\},\{2,2,2\},1\big] = 2\sum_{k=1}^{\infty}\frac{1}{k!k^2} = \int_0^1 e^x\log^2 x\,dx$

3 .293143919512913722... $\approx \frac{1}{2}\Phi\left(\frac{1}{2},-\frac{3}{2},1\right) = \sum_{k=1}^{\infty}\frac{k^{3/2}}{2^k}$

3020 .2932277767920675142... $\approx \pi^7$

.29336724568719276586... $\approx \int_0^1\frac{\log(1+x)}{1+x^3}dx$

.29380029841095036422... $\approx \frac{\sinh 1}{4} = \int_1^{\infty}\cosh\left(\frac{1}{x^4}\right)\frac{dx}{x^5}$

1 .2939358818836499924... $\approx \sum_{k=2}^{\infty}\phi(k)\log\zeta(k)$

.293947018268435532717... $\approx \frac{1}{192}\Big(48\pi(G+1)+\pi^3+96\log 2+63\zeta(3)-6\pi^2\big(2+7\log 2\big)\Big)$

$= \int_1^{\infty}\frac{\arctan^3 x}{x^4}\,dx$

.2941176470588235 = $\frac{5}{17}$

.2941592653589935936... ≈ $\frac{\pi}{10}\coth 5\pi - \frac{1}{50} \quad = \quad \sum_{k=1}^{\infty}\frac{1}{k^2+25}$ J124

.2942335427593188655 8... ≈ $\frac{i}{2}\left(\psi^{(1)}(2+i)-\psi^{(1)}(2-i)\right)=\sum_{k=2}^{\infty}\frac{2}{k^3(1-k^{-2})^2}$

= $6\sum_{k=1}^{\infty}(-1)^{k+1}k\left(\zeta(2k+1)-1\right)$

= $\int_0^{\infty}\frac{x\sin x}{e^x\left(e^x-1\right)}$

.2942368609229461505 7... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k\left(\zeta(k)-1\right)}{k^2-k} \; = \; \sum_{k=2}^{\infty}\left(\frac{k+1}{k}\log\left(1+\frac{1}{k}\right)-\frac{1}{k}\right)$

.2943007444634760791 5... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^k}{2^k+1}$

.2943594734442134266... ≈ $I_2(\sqrt{2}) = \sum_{k=0}^{\infty}\frac{k}{k!(k+1)!2^k}$

.2943720976723057236... ≈ $\sum_{k=2}^{\infty}\frac{k^6+k^3}{(k^3-1)^3} \; = \; \sum_{k=1}^{\infty}k^2(\zeta(3k)-1)$

.2945243112740431161 1... ≈ $\frac{3\pi}{32} \; = \; \prod_{k=1}^{\infty}\frac{k(k+3)}{(k+\frac{3}{2})^2}$ J1061

= $\int_0^{\infty}\frac{(\sin x - x\cos x)^3}{x^5}dx$ Prud. 2.5.29.24

.2945800187144293609... ≈ $4-\pi+\frac{\pi^2}{12}-2\log 2 \; = \; \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2k^3+k^2}$

.2947438714045368963 7... ≈ $\sum_{k=2}^{\infty}\left(\frac{1}{4}-\frac{\zeta(k+2)-1}{\zeta(k)-1}\right)$

.2947888598946322357 1... ≈ $\sum_{k=2}^{\infty}\frac{(\zeta(k)-1)^2}{\zeta(k)} \; = \; 1-\sum_{k=2}^{\infty}\left(1-\frac{1}{\zeta(k)}\right)$

= $1+\sum_{k=2}^{\infty}\frac{\mu(k)}{k(k-1)}$

2 .2948565916733137942... ≈ $\prod_{k=2}^{\infty}\zeta(k)$

2 .2949710033282972322 58... ≈ $2e-\pi$

4 .2950786268784303709 7... ≈ $\sum_{k=1}^{\infty}\frac{k^3}{k^k}$

	.29522056775540668012...	≈	$\frac{\pi}{2}\left(I_0(1)+L_0(1)\right)-\frac{\pi}{2e} = \int_0^1 e^{-x}\arcsin x\,dx$	
1	.295370965952553590595...	≈	$G\sqrt{2}$	
	.2954089751509193379...	≈	$\frac{\sqrt{\pi}}{6}$	
	.29543145370663020628...	≈	$\frac{2\log 2}{3}-1 = \sum_{k=1}^{\infty}\frac{1}{2k^2+5k+2}$	
		=	$\int_1^{\infty}\frac{\log(1+x)}{x^4}dx$	
		=	$\int_0^1 x^2\log\left(1+\frac{1}{x}\right)dx$	
3	.295497493360578095...	≈	$e+\gamma$	
	.2955013479145809011...	≈	$\frac{1}{\pi^2}\left(\gamma+\psi(1+\pi^2)\right) = \sum_{k=1}^{\infty}\frac{1}{k(k+\pi^2)} = \frac{H_{\pi^2}}{\pi^2}$	
	.2955231941257974048...	≈	$\psi(1+\pi) = -\gamma+\sum_{k=1}^{\infty}\frac{\pi}{k(k+\pi)}$	
57	.2955779130823208768...	≈	$\frac{180}{\pi}$, the number of degrees in one radian	
	.29559205186903602932...	≈	$\sum_{k=1}^{\infty}\frac{\mu(k)(-1)^k}{4^k}$	
7	.29571465265948923842...	≈	$-\pi\sqrt{3}\csc\pi\sqrt{3} = \prod_{k=1}^{\infty}\frac{k^2+2k}{k^2+2k-2}$	
	.295775076062376274178...	≈	$\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{4k^2+4k-5}$	
	.29583686600432907419...	≈	$3\log 3-3 = \int_0^{\infty}\left(3e^{-x}-\frac{1-e^{-3x}}{x}\right)\frac{dx}{x}$	GR 3.437
3	.29583686600432907419...	≈	$3\log 3 = \int_0^1\frac{x^{n-1}+x^{n-2/3}+x^{n-1/3}-3x^{3n-1}}{1-x}dx$	GR 3.272.2
	.29588784522204717291...	≈	$\frac{\gamma\log^2 2}{2}-\frac{\pi^2\gamma}{12}-\frac{\pi^2\log 2}{12}+\zeta(3) = \sum_{k=1}^{\infty}\frac{\psi(k+1)}{2^k k^2}$	
	.29593742160765668...	≈	$10-14\log 2 = \sum_{k=1}^{\infty}\frac{k^2}{2^k(k+1)(k+2)}$	
5	.2959766377607603139...	≈	$\frac{\cosh\pi-1}{2} = \sinh^2\frac{\pi}{2} = \frac{e^{\pi}+e^{-\pi}-2}{4} = \sum_{k=0}^{\infty}\frac{\pi^{4k}}{(4k)!}$	
	.2962960000000000000$\underline{0}$	=	$\frac{37037}{125000}$, mil/mile	

.2965501589414455374... ≈ $\frac{9-\pi\sqrt{3}}{12} = \int_0^1 x^3 \log\left(1+\frac{1}{x^3}\right)dx$

.29667513474359103467... ≈ $-\frac{1}{\pi}\cos\frac{\pi\sqrt{5}}{2} = \Gamma^{-1}\left(\frac{3-\sqrt{5}}{2}\right)\Gamma^{-1}\left(\frac{\sqrt{5}-2}{2}\right)$

= $\prod_{k=1}^{\infty}\left(1-\frac{1}{k^2+k}\right)$

.29697406928362356143... ≈ $7\zeta(3) - \frac{\pi^4}{12} = \sum_{k=1}^{\infty}\frac{k}{(k+\frac{1}{2})^4}$

.29699707514508096216... ≈ $\frac{1}{2}-\frac{3}{2e^2} = \sum_{k=0}^{\infty}(-1)^{k+1}\frac{2^k k}{(k+1)!}$

.297015540604756039... ≈ $\frac{1}{2}+\frac{\pi\sqrt{3}}{12}\tan\frac{\pi\sqrt{3}}{2} = \sum_{k=1}^{\infty}\frac{1}{4k^2+4k-2}$

.2970968449824711858... ≈ $2\cdot{}_2F_1\left(2,2,\frac{3}{2},-1\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(2k)!!k}{(2k-1)!!}$

1 .2974425414002562937... ≈ $2\sqrt{e}-2 = \sum_{k=0}^{\infty}\frac{1}{(k+1)!2^k}$

3 .2974425414002562937... ≈ $2\sqrt{e} = \sum_{k=0}^{\infty}\frac{2k+1}{k!2^k} = \sum_{k=0}^{\infty}\frac{pf(k)}{2^k}$

.29756636056624138494... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^{k+1}}{12k-6} = \frac{1}{6}+\frac{\pi}{24}$

.2975868359824348472... ≈ $\frac{\pi^2}{6}-3\log^2\left(\frac{1+\sqrt{5}}{2}\right) = \sum_{k=0}^{\infty}\frac{(-1)^k(k!)^2}{(2k)!(2k+1)^2}$ Berndt 32.3

.29765983718974973707... ≈ $\frac{1}{3}\Gamma\left(\frac{4}{3}\right) = \int_0^{\infty}\frac{x^3\,dx}{e^{x^3}}$

1 .29777654593982225680... ≈ $\prod_{k=1}^{\infty}\left(1-\frac{(-1)^{k+1}}{\binom{2k}{k}}\right)$

.2978447548942876738... ≈ $\sum_{k=1}^{\infty}\frac{H_k^{\ 3}}{6^k}$

.29790266899808726126... ≈ $\frac{1}{4}\left(\pi-\pi\sqrt{2}+\sqrt{2}\log(3+2\sqrt{2})\right) = \int_0^1 \arctan x^2\,dx$

.2979053513880541819... ≈ $\sum_{k=1}^{\infty}\frac{H^{(2)}{}_k}{4^k k}$

.29817368116159703717... ≈ $-\frac{e}{2}Ei(-1) = \int_0^{\infty}\frac{xe^{-x^2}dx}{x^2+1}$

4 .29826879918695200481... $\approx \sum_{k=2}^{\infty}\left(\frac{\zeta^3(k)}{\zeta(2k)}-1\right)$

.29827840441917252967... $\approx \sum_{k=2}^{\infty}\frac{1}{k^2}\log\frac{k+2}{k}$

.29843087812308785 65... $\approx \frac{1}{\pi}\zeta'(2) = \frac{1}{\pi}\sum_{k=1}^{\infty}\frac{\log k}{k^2}$

3 .298462247044795629253... $\approx \frac{\pi}{\sqrt{2}}\log(3+\sqrt{2}) = \int_0^1 \frac{\log(x^2+9)}{x^2+2}dx$

.29849353784930260079... $\approx \sum_{k=2}^{\infty}\frac{(-1)^k}{k^k-1} = \sum_{k=2}^{\infty}\sum_{j=1}^{\infty}\frac{(-1)^k}{k^{jk}}$

3 .2985089027387068694... $\approx \frac{32\pi^4}{945}$, volume of the unit sphere in R^9

.2986265782046758335... $\approx \frac{\log 6}{6}$

.29863201236633127840... $\approx \frac{e^2}{8}-\frac{5}{8} = \sum_{k=0}^{\infty}\frac{2^k}{(k+3)!}$

.2986798531646551388... $\approx \frac{\pi}{3\sqrt{3}}\left(\log 3-\frac{\pi}{3\sqrt{3}}\right) = \int_0^{\infty}\frac{x\,dx}{\sqrt[3]{(e^{3x}-1)^2}}$ GR 3.456.2

$= -\int_0^1 \frac{x\log x\,dx}{\sqrt[3]{(1-x^3)^2}}$ GR 4.244.3

.29868312621868806528... $\approx \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)-1}{k+1}$

.2987954712201816344... $\approx \frac{15}{16}-24\pi\sqrt{3}-\frac{3\log 3}{8} = \sum_{k=1}^{\infty}\frac{1}{k(3k+4)}$

1 .29895306805743878110... $\approx \frac{8}{7}+\frac{8}{7\sqrt{7}}\arcsin\frac{1}{2\sqrt{2}} = \sum_{k=0}^{\infty}\frac{1}{\binom{2k}{k}2^k}$

.29933228862677768482... $\approx \frac{\pi^2}{4}+2G-4 = \sum_{k=1}^{\infty}\frac{(-1)^k k\zeta(k+1)}{4^k}$

1 .2993615699382227606... $\approx \sum_{k=2}^{\infty}\frac{\zeta(k)}{k^2-2}$

.29942803519306324920... $\approx \zeta(2)-\frac{1}{2}+\frac{\pi}{2\sqrt{2}}\cot\frac{\pi}{\sqrt{2}} = \sum_{k=1}^{\infty}\frac{k^2-1}{2k^4-k^2}$

$= \sum_{k=1}^{\infty}\frac{\zeta(2k)-\zeta(2k+2)}{2^k}$

.29944356942685078... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k}{k^3-5}$

.2994647090064681986... ≈ $\frac{6}{e^3-e^{-3}}$ J132

8 .29946505124451516171... ≈ $\frac{1}{2}\left(e^e+e^{1/e}\right)=e^{\cosh 1}\cosh(\sinh 1)=\sum_{k=0}^{\infty}\frac{\cosh k}{k!}$ GR 1.471.2

2 .2998054391128603133... ≈ $\pi\left(\sqrt{3}-1\right) = \int_0^{\infty}\log\left(1+\frac{2}{x^2+1}\right)dx$

.299875433839200776952... ≈ $\frac{\pi^2}{48}+\frac{G}{3}+\frac{1}{3}-\frac{\pi\log 2}{4} = \int_1^{\infty}\frac{\arctan^2 x}{x^4}dx$

5 .29991625085634987194... ≈ $\beta(\frac{1}{3},\frac{1}{3}) = \Gamma\left(\frac{1}{3}\right)^2\Gamma^{-1}\left(\frac{2}{3}\right) = \frac{\sqrt{3}}{2\pi}\Gamma\left(\frac{1}{3}\right)^3$

.3000000000000000000$\underline{0}$ = $\frac{3}{10} = \frac{1}{2\cosh\log 3} = \sum_{k=0}^{\infty}(-1)^k e^{-(2k+1)\log 3}$ J943

= $\int_0^{\infty}\frac{\sin^3 x}{e^x}\,dx = \int_0^{\infty}\frac{\cos x}{e^{3x}}\,dx$

1 .30031286185924454169... ≈ $\frac{1}{\pi^2}\cos\frac{\pi\sqrt{1-4i}}{2}\cos\frac{\pi\sqrt{1+4i}}{2} = \prod_{k=1}^{\infty}\left(1+\frac{1}{k^2(k+1)^2}\right)$

.30042833175354624312 9... ≈ $-\sum_{k=1}^{\infty}\frac{\mu(k)\sigma_0(k)}{2^k+1}$

.30046260628866577442 7... ≈ $\frac{\sqrt{13}}{12}$

.30051422578989857135... ≈ $\frac{\zeta(3)}{4} = 2\zeta(3) - \frac{2}{3}\log^3 2 - 2Li_2\left(\frac{1}{2}\right)\log 2 - 2Li_3\left(\frac{1}{2}\right)$

[Ramanujan] Berndt Ch. 9

= $\int_1^{\infty}\frac{\log^2 x}{x^3-x}dx = \int_1^{\infty}\frac{\log^2 x}{(x+1)x(x-1)}dx$

= $\int_1^{2}\frac{\log^2 x}{x-1}dx$ [Ramanujan] Berndt Ch. 9

= $\int_0^{1}\frac{x\log^2 x}{1-x^2}dx$

= $\int_0^{\infty}\frac{x^2\,dx}{e^{2x}-1}$

= $\int_0^{1}\frac{\log^2(1+x)}{x}dx$

= $\int_0^{1}\frac{\log^2(1+x)}{x}dx = \int_0^{1}\frac{\log(1-x^2)\log x}{x}dx$

= $\int_0^{\pi/2}(\log\sin x)^2\tan x\,dx$

2 .30067452872527048713 7... ≈ $2^{\zeta(3)}$

.30082768734653640721 6... ≈ $\sum_{k=1}^{\infty}\frac{H_k}{2^k(2k+1)}$

1 .3010141145324885742... ≈ $\zeta(3)\zeta(4) = \frac{\pi^4\zeta(3)}{90} = \sum_{k=1}^{\infty}\frac{\sigma_1(k)}{k^4} = \sum_{k=1}^{\infty}\frac{\sigma_{-1}(k)}{k^3}$ HW Thm. 290

.30102999566398119521 4... ≈ $\log_{10} 2$

42 .3010475037335080668 6... ≈ $\sum_{k=1}^{\infty}\frac{\sigma_0(k)}{k!}$

.3010931726... ≈ $\sum_{p_k \ prime} \frac{1}{p_k p_{k+1}}$

.30116867893975678925... ≈ $\sin 1 - \cos 1$

= $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k-1)!(2k+1)} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(2k+1)}{(2k)!}$

= $\int_0^1 x \sin x \, dx = \int_1^{\infty} \sin\left(\frac{1}{x}\right) \frac{dx}{x^3}$

= $\int_1^e \frac{\log x \sin \log x}{x} dx$

1 .301225428913118242223... ≈ $\frac{90}{\pi^{10}}\left(10800(\zeta'(4))^2 + \pi^6 \zeta''(2) - 60\pi^4 \zeta''(4) - 360\pi^2 \zeta'(2)\zeta'(4)\right)$

= $\sum_{k=1}^{\infty} \frac{|\mu(k)| \log^2 k}{k^2}$

1 .30129028456857300855... ≈ $\pi\left(\sqrt{2}-1\right) = \int_0^{\infty} \log \frac{1+x^{-4}}{1+x^{-2}} dx = \int_0^{\infty} \log\left(1+\frac{1}{x^2+1}\right) dx$

= $\int_0^{\infty} \frac{dx}{(x^2+1/2)(x^2+1)}$

2 .3012989023072948735... ≈ $\sinh \frac{\pi}{2} = \operatorname{Re}\{\sin(\log i)\} = i \cos\left(\frac{\pi}{2}(1+i)\right)$

= $\prod_{k=0}^{\infty}\left(1+\frac{1}{(2k+1)^2}\right)$

.301376553376076650855... ≈ $\frac{\pi}{6} - \frac{2}{9} = \int_0^1 x^2 \arcsin x \, dx$ GR 4.523.1

.301544001363391640157... ≈ $1 - \frac{\sin\sqrt{2}}{\sqrt{2}} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{2^k k^2}{(2k)!(k+1)}$

.301733853597972457948... ≈ $\sum_{k=1}^{\infty} \frac{1}{5^k - 1} = \sum_{k=1}^{\infty} \frac{\sigma_0(k)}{5^k}$

.301737240203145494461... ≈ $\frac{\log^2 3}{4} = \int_0^1 \frac{\log(1+2x)}{1+2x} dx$

1 .301760336046015099876... ≈ $2 \arctan e^2 - \frac{\pi}{2} = \arcsin \tanh 2 = gd\, 2$

1 .3018463986037126778... ≈ $\log \frac{e^{\pi} - e^{-\pi}}{2\pi} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\zeta(2k)}{k} = \sum_{k=1}^{\infty} \log\left(1+\frac{1}{k^2}\right)$

= $\log\left(\frac{\sinh \pi}{\pi}\right) = -\log \Gamma(1-i) - \log \Gamma(1+i)$

11 .30192195213633049636... ≈ $I_0(4) \;=\; \sum_{k=0}^{\infty} \frac{4^k}{(k!)^2}$

.301996410804989806545... ≈ $8 - \frac{40}{3\sqrt{3}} \;=\; \sum_{k=1}^{\infty} \binom{2k+1}{k} \frac{k}{16^k}$

.302291222970795777178... ≈ $\frac{1}{2} \log\cot\frac{1}{2} \;=\; \sum_{k=1}^{\infty} \frac{\cos(2k+1)}{2k+1}$ GR 1.442.2

.30229989403903630843... ≈ $\frac{\pi}{6\sqrt{3}} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(3k+1)(3k+3)}$

= $\int_0^{\infty} \frac{dx}{x^3+8} \;=\; \int_1^{\infty} \frac{x\,dx}{x^4+x^2+1} \;=\; \int_{-\infty}^{\infty} \frac{dx}{(x^2+3)^2}$

= $\int_0^{\infty} \frac{x^3\,dx}{1+x^{12}} = \int_0^{\infty} \frac{x^7\,dx}{1+x^{12}}$

.30240035386897389123... ≈ $\sum_{k=2}^{\infty} \frac{H_k\left(\zeta(k)-1\right)}{2^k}$

2 .302585092994045684018... ≈ $\log 10$

5 .30263321633763963143... ≈ $56\zeta(3) - 2\pi^3 \;=\; -\psi^{(2)}\left(\frac{3}{4}\right) \;=\; 2\sum_{k=0}^{\infty} \frac{1}{(k+{}^3\!/\!_4)^3}$

= $\sum_{k=3}^{\infty} \frac{(k-1)(k-2)\zeta(k)}{4^{k-3}}$

.30276089444130022385... ≈ $\int_1^{\infty} \left(\zeta(2x)-1\right) dx$

1 .30292004734231464290... ≈ $\frac{\pi^2}{12} + \log^2 2 \;=\; -\int_0^1 \frac{\log(x/2)}{1+x}\,dx$

.303150275147523568676... ≈ $\log\Gamma\left(\frac{2}{3}\right)$

.303265329856316711802... ≈ $\frac{1}{2\sqrt{e}} \;=\; \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!\,2^k}$

3 .30326599919412410519... ≈ $\frac{\pi}{2}\csc\frac{\pi}{5}\sec\frac{\pi}{5} \;=\; \Gamma\left(\frac{2}{5}\right)\Gamma\left(\frac{3}{5}\right)$

.303301072125612324873... ≈ $1 \,+\, \cos\left(\frac{\sin 1}{2}\right)\left(\sinh\left(\frac{\cos 1}{2}\right) - \cosh\left(\frac{\cos 1}{2}\right)\right)$

= $1 - \frac{\cos\left(\frac{\sin 1}{2}\right)}{e^{(\cos 1)/2}} \;=\; \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\cos k}{k!\,2^k}$

1 .303324170670084184712... ≈ $\sum_{k=2}^{\infty} H_k^{(2)}\left(\zeta(k)-1\right)$

.303394881754352755635... ≈ $\frac{\pi}{4}(\log 4-1) = \int_0^{\infty} \log\left(\frac{1+x^2}{x^2}\right)\frac{x^2\,dx}{(1+x^2)^2}$ GR 4.298.19

.30342106428050762292... ≈ $\sum_{k=2}^{\infty} \frac{1}{(k-1)k(k+1)\log k}$

1 .303497944016070234172... ≈ $\sum_{k=1}^{\infty} \log\left(1+\frac{1}{k!}\right)$

.30378945806558801568... ≈ $\frac{8}{9}-\frac{\pi}{3}+\frac{2\log 2}{3} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k(k+3/2)}$

.303826933784633913104... ≈ $\frac{\pi}{2\sqrt{2}}\coth\frac{\pi}{\sqrt{2}}-\frac{5}{6} = \sum_{k=1}^{\infty} (-1)^{k+1}\frac{\zeta(2k)-1}{2^k}$

= $\sum_{k=2}^{\infty} \frac{1}{2k^2+1}$

.30389379330748584659... ≈ $2\left(Li_3\left(-\frac{1}{3}\right) - Li_3\left(-\frac{1}{2}\right)\right) = \int_0^1 \frac{\log^2 x}{(x+2)(x+3)}dx$

.3039485149055946998... ≈ $\frac{\pi^2}{12}-\frac{14}{27} = \int_0^1 x^2 \arcsin^2 x\,dx$

.30396355092701331433... ≈ $\frac{3}{\pi^2}$

.30410500345454707706... ≈ $\left(3-2\sqrt{2}\right)\sqrt{\pi} = \sum_{k=1}^{\infty} \frac{(k-\frac{1}{2})!}{(k+1)!\,2^k}$

.304186489039561045624... ≈ $\frac{\pi}{24}+\frac{\log 2}{4} = \sum_{k=1}^{\infty} \frac{1}{16k^2-12k}$

.304349609021883684177... ≈ $\frac{1}{2}\log\frac{\sinh\pi}{2\pi} = -\frac{1}{2}\log\left(\Gamma(2+i)\Gamma(2-i)\right)$

= $\sum_{k=1}^{\infty} (-1)^{k+1}\frac{\zeta(2k)-1}{2k}$

2 .304632878711566539649... ≈ $-\frac{\pi}{\sqrt{2}}\csc\pi\sqrt{2} = \prod_{k=1}^{\infty}\left(1+\frac{1}{k^2+2k-1}\right)$

2 .304988258242580902703... ≈ $\frac{\pi^3}{8}-\frac{\pi}{2} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}k^2}{(k^2-1/4)^3}$

.305232894324563360615... ≈ $\log\sqrt{2\pi} + 2\log 2 - 2 = \int_2^3 \log\Gamma(x)\,dx$ GR 6.441.1

.3053218647257396717... ≈ $\frac{G}{3}$

.305490036930133642027... ≈ $5040 - 1854e \;=\; \sum_{k=1}^{\infty} \frac{k}{k!(k+7)}$

.305514069416909279097... ≈ $2 + \log^2 2 - \frac{2\log^3 2}{3} + \frac{\pi^2}{6}\left(2\log 2 - 1\right) - 2\log 2 - \zeta(3)$

= $\sum_{k=1}^{\infty} \frac{H^{(2)}{}_k}{2^k (k+2)}$

.305548232301482856123... ≈ $\sum_{k=3}^{\infty} \frac{1}{k!-2}$

.30562962705065479621... ≈ $\frac{3\sqrt{2}}{2}\arcsin\frac{1}{\sqrt{3}} - 1 \;=\; \sum_{k=1}^{\infty} \frac{(2k)!!}{(2k-1)!!3^k(2k+1)}$

.305808077190268473430... ≈ $\sum_{k=2}^{\infty} \frac{1}{k^3 \log^2 k}$

.305986696230598506245... ≈ $\frac{138}{451} \;=\; \sum_{k=1}^{\infty} \frac{F_k{}^3}{6^k}$

.306018059984358556256... ≈ $6 - \frac{\pi^2}{3} - 2\zeta(3) \;=\; -\int_0^1 \log(1-x)\log^2 x\,dx$

.306119389040098524366... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(k+2)-1}{(k-1)!} \;=\; \sum_{k=2}^{\infty} \frac{e^{1/k}}{k^3}$

.30613305077076348915... ≈ $\frac{4\pi\sqrt{3}}{27} - \frac{1}{2} \;=\; \int_0^{\infty} \frac{dx}{(x^2+x+1)^3}$

1 .30619332493997485204... ≈ $\frac{3\sin 2}{4} - \frac{3\cos 2}{2} \;=\; \frac{3\sqrt{\pi}}{2} J_{3/2}(2) \;=\; \sum_{k=1}^{\infty} \frac{(-1)^k 4^k k^3}{(2k)!}$

13 .30625662804500320717... ≈ $\frac{128}{9} - G \;=\; \sum_{k=1}^{\infty} \frac{(-1)^k (3^k - 1)(k+1)}{4^k} \zeta(k+2)$

.306354175616294411282... ≈ $\frac{1}{4}\Gamma\left(\frac{3}{4}\right) \;=\; \int_0^{\infty} x^2 e^{-x^4}\,dx$

1 .306562964876376527857... ≈ $\sqrt{\frac{2+\sqrt{2}}{2}} \;=\; \cos\frac{\pi}{8} + \sin\frac{\pi}{8} \;=\; \sqrt{2}\sin\frac{5\pi}{8}$

= $\prod_{k=0}^{\infty} \left(1 + \frac{(-1)^k}{4k+2}\right)$

.306563211820816809902... ≈ $1 - J_0\left(\frac{2}{\sqrt{3}}\right) \;=\; \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(k!)^2 3^k}$

8 .3066238629180748526... ≈ $\sqrt{69}$

.306634521423027913862... ≈ $\frac{1}{1215}\left(79\pi^4 - 15\psi^{(3)}\left(\frac{1}{3}\right)\right)$

= $\int_1^\infty \frac{\log^3 x}{x^3+x^2+x}dx$

.30683697542290869392... ≈ $\frac{\pi}{4}\coth\pi + \frac{\pi^2}{4}\operatorname{csch}^2\pi - \frac{1}{2} = \sum_{k=1}^\infty \frac{1}{(k^2+1)^2}$

.306852819440054690583... ≈ $1 - \log 2 = \sum_{k=2}^\infty \frac{(-1)^k}{k} = \sum_{k=2}^\infty (-1)^k \frac{\zeta(k)}{2^k}$

= $\sum_{k=2}^\infty \frac{k-1}{2^k k}$ J145

= $\sum_{k=1}^\infty \frac{1}{2^k k(k+1)}$ J149

= $\sum_{k=1}^\infty \frac{1}{k(4k+2)} = 2\sum_{k=1}^\infty \frac{(-1)^{k+1}}{(2k)^3 - 2k}$

= $\sum_{k=1}^\infty \frac{H_k}{(2k+1)(2k+3)}$

= $\int_1^\infty \frac{dx}{x^3+x^2} = \sum_{k=1}^\infty \frac{1}{k^2+k} - \int_1^\infty \frac{dx}{x^2+x}$

= $\int_0^\infty \frac{dx}{e^x\left(e^x+1\right)}$

= $\int_0^\infty \left(e^{-x} - e^{-2x} - \frac{e^{-2x}}{x}\right)\frac{dx}{x}$ GR 3.438.3

= $\int_0^\infty \left((x+1)e^{-x} - e^{-x/2}\right)\frac{dx}{x}$ GR 3.435

= $\int_0^1 \left(x - \frac{x-1}{\log x}\right)\frac{dx}{\log x}$ GR 4.283.1

= $\int_0^\infty \frac{xe^{-x}}{\sqrt{e^{2x}-1}}dx$ GR 3.452.4

= $\int_1^\infty \frac{\log x}{x^2\sqrt{x^2-1}}dx$ GR 4.241.8

= $-\int_0^{\pi/2} \log(\sin x)\sin x\, dx$ GR 4.384.5

1 .306852819440054690583... ≈ $2 - \log 2 = -\int_0^1 \arccos x \log x\, dx$ GR 4.591.2

.30747679967138350672... ≈ $\frac{3\operatorname{arcsinh}1 - \sqrt{2}}{4} = \sum_{k=0}^\infty \frac{(-1)^k}{4^k(2k+1)(2k+3)}\binom{2k}{k}$

$= \int_0^1 x \operatorname{arcsinh} x \; dx$

.307654580328819552466... $\approx \frac{3\zeta(3)}{8} - \frac{\pi^2}{48} + \frac{1}{16} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^3(k+2)}$

$.307692307692\underline{307692307692} = \frac{4}{13}$

.307799372444653646135... $\approx 1 - e^{-1/e} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!e^k}$

.30781323519300713107... $\approx \frac{-\log(\cos 1)}{2} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\sin^2 k}{k}$ J523

.308169071115984935787... $\approx \operatorname{arccot} \pi = \sum_{k=1}^{\infty} \frac{(-1)^k}{\pi^{2k+1}(2k+1)}$

27 .308232836016486629202... $\approx \cosh 4 = \frac{e^4 + e^{-4}}{2} = \sum_{k=0}^{\infty} \frac{16^k}{(2k)!}$ AS 4.5.63

.308337097266942899991... $\approx \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\mu(k)}{3^k + 1}$

.308425137534042456839... $\approx \frac{\pi^2}{32} = \sum_{k=0}^{\infty} \frac{1}{(4k+2)^2} = \sum_{k=2}^{\infty} \frac{(k-1)\zeta(k)}{2^{k+2}}$

$= \int_0^1 \frac{x \log x}{x^4 - 1} dx = \int_1^{\infty} \frac{x \log x}{x^4 - 1} = \int_0^1 \frac{\arctan x}{1 + x^2}$

.30850832255367103953... $\approx \frac{I_0(2)}{e^2} = \sum_{k=0}^{\infty} \frac{(-1)^1 (2k)!}{(k!)^3} = \; = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \binom{2k}{k}$

1 .3085180169126677982... $\approx 3 + \frac{\pi^2}{4} - 6 \log 2 = \int_0^1 \int_0^1 \int_0^1 \frac{x + y + z}{1 + xyz} \, dx \, dy \, dz$

.30860900855623185640... $\approx \sum_{k=0}^{\infty} \frac{(-1)^k}{2^{2^k}}$

2 .308862659606131442426... $\approx 4\gamma$

1 .308996938995747182693... $\approx \frac{5\pi}{12} = \sum_{k=1}^{\infty} \frac{1}{48k^2 - 48k + 9}$

$= \sum_{k=0}^{\infty} \frac{(-1)^{\lfloor k/3 \rfloor}}{2k + 1}$ Prud. 5.1.4.5

.309033126487808472317... $\approx -Li_2\left(-\frac{1}{3}\right)$

= $\frac{1}{2}\left(\log^2 3 - 2\log 3 \log 4 + 4\log^2 2 + 2Li_2\left(\frac{1}{4}\right)\right)$

= $\frac{1}{6}\left(\pi^2 + 3\log^2 3 - 3\log^2 4 - 6Li_2\left(\frac{3}{4}\right)\right)$

= $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{3^k k^2} = \sum_{k=1}^{\infty} \frac{H_k}{4^k k}$

= $-\int_0^1 \frac{\log x}{x+3} dx$

.30938841220604682364... ≈ $\frac{\pi\sqrt{3}}{2} + \frac{\pi^2}{6} + \frac{9\log 3}{2} - 9 = \sum_{k=1}^{\infty} \frac{1}{3k^3 + k^2}$

= $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{\zeta(k+2)}{3^k}$

4 .30940107675850305803 7... ≈ $2 + \frac{4}{\sqrt{3}}$ CFG A10

2 .30967037950216633419... ≈ $\gamma + \frac{1}{\gamma}$

2 .30967883660676806428... ≈ $\frac{3\pi\log 2}{2\sqrt{2}} = \int_0^{\infty} \frac{\log(x^2+2)}{x^2+2}$

16 .30969097075427141216 2... ≈ $6e$

.30985933910111243018 4... ≈ $\frac{1}{2} - \frac{\pi}{2\sqrt{6}} \cot \frac{\pi}{\sqrt{6}} = \sum_{k=1}^{\infty} \frac{1}{6k^2 - 1} = \sum_{k=1}^{\infty} \frac{\zeta(2k)}{6^k}$

.30989965660362137025... ≈ $-\Gamma(i) - \Gamma(-i)$

.30997059737575027469 3... ≈ $\frac{\log 2}{2} + \frac{1}{8}\left(\psi\left(\frac{1+i}{2}\right) + \psi\left(\frac{1-i}{2}\right) - \psi(i) - \psi(-i)\right)$

= $\int_0^{\infty} \frac{\sin^2 x}{e^x + 1} dx$

.31001609182507930307 3... ≈ $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{\zeta(2k) - 1}{k(k+1)} = \sum_{k=2}^{\infty} \left((k^2+1)\log\left(1 + \frac{1}{k^2}\right) - 1\right)$

.31034129822300065748... ≈ $\frac{1}{2}\left((2 + \log(2 + 2\cos 1))\sin\frac{1}{2} - \cos\frac{1}{2}\right) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k+1} \sin\frac{2k+1}{2}$

1 .31048533548919165733 7... ≈ $\sum_{k=1}^{\infty} \frac{k}{2^k H_k}$

1 .31050969912521588230 2... ≈ $\frac{\pi^2}{3} + \frac{\pi}{8}\coth\pi - \frac{3\pi^2}{8}\operatorname{csch}^2\pi + \frac{\pi^3}{4}\coth\pi \operatorname{csch}^2\pi - 2\zeta(3)$

= $\sum_{k=2}^{\infty} (-1)^k k^2 \left(\zeta(k) - \zeta(2k)\right)$

3 .310914970542980894360... ≈ $\frac{9}{e} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^6}{k!} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^7}{k!}$

.311001742407005668121... ≈ $-\frac{159}{1820} - \frac{\pi}{2\sqrt{14}}\cot\pi\sqrt{14} = \sum_{k=1}^{\infty}\frac{1}{k^2+8k+2}$

1 .3110287771460599052... ≈ $\frac{1}{4\sqrt{2\pi}}\Gamma^2\left(\frac{1}{4}\right)$, the lemniscate constant

= $\frac{1}{2}\sqrt{\frac{\pi^3}{2}}\Gamma^{-2}\left(\frac{3}{4}\right) = \frac{\sqrt{2}}{2}K\left(\frac{1}{\sqrt{2}}\right) = \int_0^{\pi/2}\sqrt{1+\sin^2 x}\,dx$

= $\sqrt{\pi}\Gamma\left(\frac{5}{4}\right)\Gamma^{-1}\left(\frac{3}{4}\right) = \int_0^1\frac{dx}{\sqrt{1-x^4}}$

2 .311350336852182164229... ≈ $\sum_{k=0}^{\infty}\frac{1}{k!k!!}$

2 .31145469958184343582... ≈ $\frac{2\pi}{e} = -\int_0^{\infty}\log x\log\left(1+\frac{1}{e^2x^2}\right)dx$

.311612620070115256697... ≈ $\frac{1}{2\sqrt{2}}\log\left(1+\sqrt{2}\right) = \frac{\sqrt{2}}{4}\operatorname{arcsinh}1$

.311696880108669610301... ≈ $\left(\frac{1}{16}\operatorname{csch}^2\frac{\pi}{2}\right)\left(\pi\sinh\pi - \pi^2\right) = \frac{\pi(\sinh\pi-\pi)}{8(\cosh\pi-1)}$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)}{4^k} = \sum_{k=1}^{\infty}\frac{4k^2}{(4k^2+1)^2}$

.31174142278816155776... ≈ $\sin\left(\frac{\sin 1}{2}\right)\left(\cosh\left(\frac{\cos 1}{2}\right) - \sinh\left(\frac{\cos 1}{2}\right)\right) = \frac{1}{e^{(\cos 1)/2}}\sin\left(\frac{\sin 1}{2}\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}\sin k}{k!2^k}$

.311770492301365488... ≈ $\frac{1}{e}\left(\gamma(1+e) - Ei(1) - e\,Ei(-1)\right) = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{H_k}{(k+1)!}$

.311821131864326983238... ≈ $\frac{2i}{\sqrt{3}-3i}Li_2\left(\frac{1-i\sqrt{3}}{2}\right) - \frac{\sqrt{3}-i}{\sqrt{3}-3i}Li_2\left(\frac{1+i\sqrt{3}}{2}\right)$

= $\frac{5\pi^2}{36} - \frac{1}{6}\psi^{(1)}\left(\frac{1}{3}\right) = -\int_0^1\frac{x\log x}{1-x+x^2}dx$ GR 4.233.4

= $\int_0^{\infty}\frac{x}{e^x(e^x+e^{-x}-1)}dx$ GR 3.418.2

.31182733772005388216... ≈ $\frac{\pi}{\sqrt{7}}\tanh\frac{\pi\sqrt{7}}{2} - \frac{7}{8} = \sum_{k=1}^{\infty}\frac{1}{k^2+5k+8}$

8 .31187288206608164149... ≈ $\pi\sqrt{7}$

.311993314369948002... ≈ $1-\frac{\gamma}{2}-\frac{3\log 2}{2}+\gamma\log 2+\frac{\log^2 2}{2}$

= $\sum_{k=1}^{\infty}\frac{(-1)^k k\,\psi(k+1)}{k+1}$

1 $.\underline{312371838687628161}$ = $\frac{1920}{1463}=\left(1+\frac{1}{7}\right)\left(1+\frac{1}{11}\right)\left(1+\frac{1}{19}\right)$

= $\sqrt{2\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{7^2}\right)\left(1-\frac{1}{11^2}\right)\left(1-\frac{1}{19^2}\right)}$

[Ramanujan] Berndt Ch. 22

.312382639940836992172... ≈ $\frac{3\zeta(3)}{4}-\frac{\pi^2}{6}+2\pi+4\log 2-8=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^3(2k+1)}$

2 .3124329444966854611... ≈ $2HypPFQ\left[\{1,1,1,1\},\{2,2,2,2,\},2\right]=\sum_{k=1}^{\infty}\frac{2^k}{k!k^3}$

$.3125000000000000000\underline{0}$ = $\frac{5}{16}=\sum_{k=1}^{\infty}\frac{k}{5^k}=\sum_{k=2}^{\infty}\frac{1}{k^3-2k+k^{-1}}=\sum_{k=2}^{\infty}\frac{1}{k^3(1-k^{-2})^2}$

= $\sum_{k=1}^{\infty}k\left(\zeta(2k+1)-1\right)$

2 .3126789504275163185... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)}{k^2}=\sum_{k=1}^{\infty}Li_2\left(\frac{1}{k^2}\right)$

.312769582219941460697... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!\zeta(k+1)}$

.312821376456508281708... ≈ $\frac{2\pi}{e^3}=\int_{-\infty}^{\infty}\frac{\cos 3x}{(1+x^2)^2}dx$

.312841184986458512 49... ≈ $-\log\Gamma\left(2+\frac{i}{\sqrt{2}}\right)\Gamma\left(2-\frac{i}{\sqrt{2}}\right)=\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)-1}{2^k k}$

.312941524372491884969... ≈ $\frac{\pi}{12}+\frac{1}{6}-\frac{\log 2}{6}=\int_{1}^{\infty}\frac{\arctan x}{x^4}dx$

.313035285499331303636... ≈ $(\coth 1)-1=\frac{2}{e^2-1}=\sum_{k=1}^{\infty}\frac{2}{e^{2k}}=\sum_{k=0}^{\infty}\frac{B_k 2^k}{k!}$

1 .313035285499331303636... ≈ $\coth 1=\frac{e^2+1}{e^2-1}=i\cot i=\sum_{k=0}^{\infty}\frac{4^k B_{2k}}{(2k)!}$ AS 4.5.67

2 .313035285499331303636... ≈ $\frac{2e^2}{e^2-1}=\sum_{k=0}^{\infty}(-1)^{k+1}\frac{B_k 2^k}{k!}$

.313165880450868375872... ≈ $\operatorname{arccsch}\pi$

.31321412527066138178... $\approx$ $\log 3 - \frac{\pi}{4} = \sum_{k=1}^{\infty} \arctan \frac{10k}{(3k^2+2)(9k^2-1)}$

[Ramanujan] Berndt Ch. 2

.31323210397311561467 0... $\approx$ $\sum_{k=1}^{\infty} \frac{1}{k^3+5}$

.31324831465660205311 8... $\approx$ $\frac{1}{e}\left(\gamma - \psi\left(1-\frac{1}{e}\right)\right) = \sum_{k=1}^{\infty} \frac{1}{k^2e^2 - ke} = \sum_{k=1}^{\infty} \frac{\zeta(k)}{e^k}$

.31326168751822283404 9... $\approx$ $\log\left(1+\frac{1}{e}\right) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{e^k k}$

$= \int_1^{\infty} \frac{dx}{e^x+1}$

1 .31326168751822283404 9... $\approx$ $\log(1+e)$

.31329854251154749690 8... $\approx$ $\frac{\zeta(3)-1}{\zeta(2)-1}$

.31330685966024952260... $\approx$ $\frac{1}{2}{}_1\mathrm{F}_1\left(\frac{1}{2},3,-4\right) = \frac{2}{3e^2} I_0(2) + \frac{1}{2e^2} I_1(2)$

$= \sum_{k=0}^{\infty} (-1)^k \frac{1}{(k+2)!}\binom{2k}{k}$

.3133285343288750628... $\approx$ $\sqrt{\frac{\pi}{32}} = \frac{1}{4}\sqrt{\frac{\pi}{2}} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(k-\frac{1}{2})!}{(k-1)!}$

$= \int_0^{\pi/2} x e^{-2\tan^2 x} \frac{2-\cos^2 x}{\cos^4 x \cot x} dx$ GR 3.964.2

.3134363430819095293... $\approx$ $\frac{23}{4} - 2e = \sum_{k=1}^{\infty} \frac{1}{k!(k+4)} = \sum_{k=1}^{\infty} \frac{1}{(k+1)!+3k!}$

.31351374777072838003 6... $\approx$ $2\log 2 + \frac{3}{2}\log 3 - \frac{\pi\sqrt{3}}{2} = \sum_{k=1}^{\infty} \frac{1}{6k^2-k}$

$= -\int_0^1 \frac{\log(1-x^6)}{x^2} dx$

.31353553294958987628 5... $\approx$ $\gamma + \frac{1}{2}\left(\psi\left(1-\frac{i}{\sqrt{3}}\right) + \psi\left(1+\frac{i}{\sqrt{3}}\right)\right) = \sum_{k=1}^{\infty} \frac{1}{3k^3+k}$

$= \gamma - \frac{1}{2}\left(\psi\left(\frac{i}{\sqrt{3}}\right) + \psi\left(-\frac{i}{\sqrt{3}}\right)\right) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\zeta(2k+1)}{3^k}$

.31366612707925392596 9... $\approx$ $\log \frac{\zeta(2)}{\zeta(3)}$

.313707199711785966821... ≈ $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!2^k \zeta(2k+1)}$ Titchmarsh 14.32.3

30 .3137990863619991972... ≈ $\frac{1}{2}\Phi\left(-16,3,\frac{1}{4}\right) = \int_0^1 \frac{\log^2 x}{x^4+1/16}dx$

.313799364234217850594... ≈ $\frac{\pi}{\sqrt{3}} - \frac{3}{2} = hg\left(\frac{2}{3}\right) - hg\left(\frac{1}{3}\right)$

2 .31381006997532558873... ≈ $\frac{\pi^4}{72} + 2\log 2 = \sum_{k=1}^{\infty} \frac{H_k(k+1)}{2k+1}\left(\frac{1}{k^2}+\frac{1}{k^3}\right)$

.3139018813067524238... ≈ $\frac{\zeta''(2)}{\zeta^2(2)} - \frac{2\zeta'(2)}{\zeta^3(2)} = \sum_{k=1}^{\infty} \frac{\mu(k)\log^2 k}{k^2}$

1 .31413511119186663685... ≈ $\frac{3\pi^2}{4} - \frac{\pi^4}{16} = \int_0^{\infty} \frac{x^3}{\sinh^3 x}dx$

.314159265358979323846... ≈ $\frac{\pi}{10}$

.314329805996472663139... ≈ $\frac{7129}{22680} = \frac{H_9}{9} = \sum_{k=1}^{\infty} \frac{1}{k^2+9k}$

1 .3145763107479915715... ≈ $\frac{1036}{225} - \frac{\pi^2}{3} = H^{(2)}{}_{5/2}$

.31461326649400975195... ≈ $\sum_{k=1}^{\infty} \frac{H^{(3)}{}_k}{2^k k(k+1)}$

2 .31462919078223930970... ≈ $e - e\,Ei(-1) - 1 = \sum_{k=1}^{\infty} \frac{\psi(k+1)}{(k-1)!}$

.31506522977259791641... ≈ $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} H^{(3)}{}_k}{2^k}$

.315121018556805747334... ≈ $\frac{1}{\sqrt{3}}\sin\frac{1}{\sqrt{3}} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{3^k(2k-1)!}$

2 .31515737339411700043... ≈ $\frac{\sqrt{\pi}}{2}\zeta\left(\frac{3}{2}\right) = i\psi^{(1/2)}(1) = \int_0^{\infty} \frac{\sqrt{x}\,dx}{e^x-1}$

1 .31521355573534521931... ≈ $\prod_{k=1}^{\infty} \frac{1}{(1-5^{-k})}$

.315236751687193398061... ≈ $e^{-2\gamma}$

2 .315239207248862830396... ≈ $\frac{\pi^3}{16} + \frac{\pi}{4}\log^2 2 = \int_0^{\infty} \frac{\log^2 x}{x^2+4}dx$

.315275214363904697922... ≈ $\sum_{k=1}^{\infty} (-1)^{k+1} k \frac{\zeta(3k)}{2^k} = \sum_{k=1}^{\infty} \frac{2k^3}{(2k^3+1)^2}$

.31546487678572871855... ≈ $\log_9 2 \;=\; \int\limits_1^\infty \frac{dx}{3^x - 3^{-x}}$

.315496761830453469039... ≈ $\frac{\sqrt{3}+3i}{6\cdot 3^{5/6}}\,\psi\!\left(\frac{6-3^{7/6}i-3^{2/3}}{6}\right)+\frac{\sqrt{3}-3i}{6\cdot 3^{5/6}}\,\psi\!\left(\frac{6+3^{7/6}i-3^{2/3}}{6}\right)$

$-\frac{\sqrt{3}}{3\cdot 3^{5/6}}\,\psi\!\left(1+\frac{1}{3^{1/3}}\right)$

= $\sum_{k=1}^\infty \frac{1}{3k^3+1} = \sum_{k=1}^\infty (-1)^{k+1}\frac{\zeta(3k)}{3^k}$

.31559503344046037327... ≈ $\frac{9e}{8}-\frac{15\sqrt{\pi}}{16}\,erfi\,1 \;=\; \sum_{k=1}^\infty \frac{k}{k!(2k+5)}$

.31571845205389007685... ≈ $-\sum_{k=2}^\infty \frac{\mu(k)}{k}\log(\zeta(k)) \;=\; -\sum_{p\ prime}\left(\log\left(1-\frac{1}{p}\right)+\frac{1}{p}\right)$

= $\sum_{k=2}^\infty \frac{\zeta_p(k)}{k}$

.<u>315789473684210526</u> = $\frac{6}{19}$

.3158473598363041129... ≈ $\int\limits_0^1 \frac{\tan x\,dx}{e^x}$

.315888571364487820746... ≈ $\frac{1}{\sqrt{6}}\csc\pi\sqrt{2}\sin\pi\sqrt{3} \;=\; \prod_{k=3}^\infty\left(1-\frac{1}{k^2-2k-1}\right)$

.3160073586544089317... ≈ $\int\limits_0^1 \log(1+\log(1+x))\,dx$

.316060279414278839202... ≈ $\frac{1}{2}-\frac{1}{2e} \;=\; \int\limits_0^1 x\,e^{-x^2}\,dx$

= $\int\limits_1^\infty \cosh\left(\frac{1}{x^2}\right)\frac{dx}{x^5} = \frac{1}{2}\int\limits_1^\infty \cosh\left(\frac{1}{x}\right)\frac{dx}{x^3}$

1 .316074012952492460819... ≈ $3^{1/4}$

.3162277660168379332... ≈ $\frac{\sqrt{10}}{10} \;=\sin\arctan\frac{1}{3}$

.3162417889176087212... ≈ $\log 2\,(\log 3+\frac{\log 2}{2}-\gamma)+\log 3(\gamma+\frac{\log 3}{2})$

= $\sum_{k=1}^\infty (-1)^k\,\frac{\psi(k)}{2^k\,k}$

.316281418603112960885... ≈ $-\int\limits_0^1 \log\left(\frac{3-x}{2}\right)\frac{dx}{\log x} \;=\; \sum_{k=1}^\infty \frac{\log(k+1)}{3^k\,k}$ GR 4.221.3

.316302575782329983254... ≈ $3\zeta(3)-2\zeta(2) = \sum_{k=1}^{\infty}\frac{H_k}{k^2(k+1)^2}$

.31642150902189314370... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^{2^k}} = -\sum_{k=1}^{\infty}\frac{\mu(2k)}{4^k-1} = \sum_{k=1}^{\infty}\frac{\mu(4k-2)}{4^{4k-2}-1}$

.3165164694380020839... ≈ $\frac{1}{4}I_0(1) = \sum_{k=1}^{\infty}\frac{k^2}{(k!)^2 4^k}$

6 .31656383902767884872... ≈ $Ei(e) - Ei(1) = \int_0^1 e^{e^x}\,dx$

3 .316624790355399849115... ≈ $\sqrt{11}$

.316694367640749877787... ≈ $2\left(2\log 2-\log(2+\sqrt{2})\right) = \sum_{k=1}^{\infty}\binom{2k}{k}\frac{1}{8^k k}$

= $\sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!\,2^k k}$

.316737643877378685674... ≈ $\frac{1}{3}-\frac{1}{3e^3} = \int_1^e \frac{dx}{x^4}$

.316792763484165509320... ≈ $\frac{\cosh 1+3\sinh 1}{16} = \frac{e}{8}-\frac{1}{16e} = \sum_{k=1}^{\infty}\frac{k^4}{(2k+1)!}$

1 .316957896924816708625... ≈ $\operatorname{arccosh} 2 = 2\operatorname{arcsinh}\frac{1}{\sqrt{2}} = -\log\left(2-\sqrt{3}\right)$

= $\log\frac{\sqrt{3}+1}{\sqrt{3}-1}$

.3171338092998413134... ≈ $1-\gamma+2(\gamma-1)\log 2+\log^2 2 = \sum_{k=1}^{\infty}\frac{(-1)^k\,\psi(k)}{k(k+1)}$

5 .317361552716548081895... ≈ $3\sqrt{\pi}$

.317418282448647644165... ≈ $13\zeta(3)-\frac{2\pi^3}{3\sqrt{3}}-\frac{28}{8} = \sum_{k=1}^{\infty}\frac{1}{(k+2/3)^3}$

.317430549669142228460... ≈ $1-\frac{\pi}{2\sinh\pi/2} = 2\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{4k^2+1}$

1 .31745176555867314275... ≈ $\frac{13}{48}+\frac{\pi^2}{24}+\frac{11}{12}\log 2 = \int_0^1\frac{\log^2 x}{(x+1)^5}dx$

111 .317778489856226026841... ≈ $e^{3\pi/2} = \cosh\frac{3\pi}{2}+\sinh\frac{3\pi}{2} = i^{-3i}$

.317803023016524494541... ≈ $6-\frac{7\pi^4}{120} = \int_1^{\infty}\frac{\log^3 x}{x^3+x^2}dx = \int_0^{\infty}\frac{x^3}{e^x(e^x+1)}dx$

.31783724519578224472... ≈ $\sqrt{3}-\sqrt{2}$

1 .3179021514544038949... ≈ $eEi(1)-\gamma = \sum_{k=1}^{\infty}\frac{1}{k!k} = \sum_{k=1}^{\infty}\frac{k H_k}{(k+1)!}$

= $\sum_{k=1}^{\infty}\frac{1}{(k+1)!-k!} = -\int_1^e \log\log x\,dx = -\int_0^1 e^x \log x\,dx$

1 .318057480653625305231... ≈ $\frac{1}{2}\left({}_2F_1\left(-i,1,1-i,\frac{1}{2}\right)+{}_2F_1\left(i,1,1+i,\frac{1}{2}\right)\right) = \sum_{k=0}^{\infty}\frac{1}{2^k(k^2+1)}$

1 .318234415786588472402... ≈ $-\psi\left(\frac{2}{3}\right) = \gamma + \frac{3\log 3}{2} - \frac{\pi}{2\sqrt{3}}$ GR 8.366.7

.318309886183790671538... ≈ $\frac{1}{\pi} = \frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty}\frac{(4k)!(26390k+1103)}{(k!)^4 396^{4k}}$ Ramanujan, Borwein-Devlin p. 74

= $12\sum_{k=0}^{\infty}\frac{(-1)^k(6k)!(545140134k+13591409)}{(3k)!(k!)^3 640530^{(3k+3)/2}}$ Borwein-Devlin p. 74

= $\prod_{k=1}^{\infty}\frac{k(k+\pi)}{(k+\pi-1)(k+1)}$ J1061

= $\int_0^1 x\sin\pi x\,dx$

.318335218955105656695... ≈ $\frac{1}{8}\left(\pi\coth\pi - 3\pi^2\operatorname{csch}^2\pi + 2\pi^3\coth\pi\operatorname{csch}^2\pi\right)$

$+\frac{1}{8}\left(i\psi^{(1)}(1-i) - i\psi^{(1)}(i+i) + \psi^{(2)}(1-i) + \psi^{(2)}(1+i)\right)$

= $\sum_{k=2}^{\infty}\frac{(k^2-k)(k^2-1)}{(k^2+1)^3}$

= $\sum_{k=1}^{\infty}(-1)^{k+1}k^2\left(\zeta(2k)-\zeta(2k+1)\right)$

2 .318390655104304125550... ≈ $\frac{\cosh\pi}{5} = \prod_{k=1}^{\infty}\left(1+\frac{4}{(2k+1)^2}\right)$

5 .318400000000000000000$\underline{0}$ = $\frac{3324}{625} = \sum_{k=1}^{\infty}\frac{F_k^{\,2}k^2}{4^k}$

1 .318513092349658917 18... ≈ $\psi^{(1)}\left(\frac{7}{6}\right)$

37 .318513092349658917 18... ≈ $\psi^{(1)}\left(\frac{1}{6}\right)$

.318876248690727246329... ≈ $\frac{3\pi}{4e^2} = \int_0^{\pi/2}\cos(2\tan x)\cos^2 x\,dx$ GR 3.716.5

.31890411718460284010 5... ≈ $\frac{2}{5\sqrt{5}}\left(\operatorname{arccsch}\left(4-2\log 5\right)+Li_2\left(\frac{-3+\sqrt{5}}{2}\right)-Li_2\left(\frac{-3-\sqrt{5}}{2}\right)\right)$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{H_k}{\binom{2k}{k}}$

.31906088542089813244 3... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)-1}{(2k)!} = \sum_{k=2}^{\infty}\left(1-\cos\frac{1}{k}\right)$

.31913525445517744048 6... ≈ $4\sqrt{2}\operatorname{arcsinh}1 - \frac{14}{3} = \sum_{k=0}^{\infty}\frac{1}{2^k(2k+5)}$

1 .31950791077289425937 4... ≈ $4^{1/5}$

.31973780748486194351 4... ≈ $\gamma\log 2-\log^2 2=-\sum_{k=1}^{\infty}\frac{\psi(k+1)}{2^k(k+1)} = 2\sum_{k=2}^{\infty}(-1)^k\frac{\log k}{k}$

.3198729104347680681 1... ≈ $\operatorname{arccot}\left(\cot 1+2\csc 1\right) = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\sin k}{2^k k}$

.3201986326285258763 0... ≈ $\sum_{k=2}^{\infty}\frac{\log k}{k^3-k^2}$

.3203020092788009497 8... ≈ $\frac{2\log^2 2}{3} = \int_0^1\frac{\log(1+3x)}{1+3x}dx$

.32034114251279383627 3... ≈ $\sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)}{k^2} = \sum_{k=1}^{\infty}\left(Li_2\left(-\frac{1}{k}\right)+\frac{1}{k}\right)$

.3204226940738809280 4... ≈ $\sum_{n=1}^{\infty}\frac{1}{(n-1)!}\sum_{k=2}^{\infty}(-1)^{k+1}\frac{\log^n k}{k!}$

9 .32045171228187040668 9... ≈ $\sum_{k=1}^{\infty}\frac{k}{F_k}$

.32065094800515395132 2... ≈ $-Li_3\left(-\frac{1}{3}\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{3^k k^3}$

.32075647616256643140 8... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)-1}{(2k-1)!2^{2k-1}} = \sum_{k=2}^{\infty}\frac{1}{k}\sin\frac{1}{2k}$

1 .32079632679489661923 1... ≈ $\frac{\pi}{2}-\frac{1}{4} = \frac{i}{2}\log\left(-e^{i/2}\right) = \sum_{k=1}^{\infty}\frac{\sin k/2}{k}$

= $\frac{i}{2}\log\frac{1-e^{i/2}}{1-e^{-i/2}} = \sum_{k=1}^{\infty}\frac{\sin k}{\sqrt{k}}$

1 .32080728264223022838 6... ≈ $\frac{\pi}{2}\coth 2\pi - \frac{1}{4} = \int_0^{\infty}\frac{\sin 2x}{e^x-1}dx$

.3209394514234179322 6... ≈ $\sum_{k=1}^{\infty}\left(\zeta(2k)-\zeta(2k+2)\right)^2$

$.320987654\underline{320987654}$ = $\frac{26}{81} = \int_0^2 \frac{dx}{(1+x)^4}$

.321006354171935371018... ≈ $\frac{e^{1/4}}{4} = \sum_{k=1}^{\infty} \frac{k}{k!4^k}$

.321027287319422150883... ≈ $\frac{200}{623} = \sum_{k=1}^{\infty} \frac{(F_k)^4}{8^k}$

.32104630796716535150... ≈ $\frac{\cot 1}{2}$

.32111172497345558138... ≈ $\frac{7\pi^2}{48} - \frac{\pi}{4} - \log 2 + \frac{3\log^2 2}{4} = \int_1^{\infty} \frac{\log(1+x^2)}{x(1+x)^2}dx$

1 .32130639967764964207... ≈ $\frac{4\pi}{5}\sqrt{\frac{2}{5+\sqrt{5}}} = \frac{2\pi}{5}\csc\frac{3\pi}{5} = \int_0^{\infty} \frac{dx}{1+x^{5/2}}$

2 .321358334513407482394... ≈ $\frac{4}{3} - \frac{2\pi}{3\sqrt{3}} + 2\log 3 = \sum_{k=2}^{\infty}\left(\frac{4}{3}\right)^k \left(\zeta(k)-1\right) = \sum_{k=2}^{\infty} \frac{16}{9k^2-12k}$

.32138852111168611157... ≈ $\frac{\sqrt{\pi}}{2} erf\, \frac{1}{3} = \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{k!3^{2k+1}(2k+1)}$

.321492381818579142365... ≈ $\frac{16}{63} + \frac{\pi}{\sqrt{37}}\tan\frac{\pi\sqrt{37}}{2} = \sum_{k=1}^{\infty} \frac{1}{k^2+7k+3}$

.32166162685421011197... ≈ $\sum_{k=1}^{\infty} \frac{|\mu(k)|}{k}\log\zeta(k)$

.3217505543966421934... ≈ $\arctan\frac{1}{3} = \arctan 2 - \frac{\pi}{4} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{3^{2k+1}(2k+1)}$

= $\sum_{k=1}^{\infty} (-1)^{k+1}\arctan\left(\frac{2}{(k+1)^2}\right)$ [Ramanujan] Berndt Ch. 2, Eq. 7.5

= $\sum_{k=1}^{\infty} \arctan\left(\frac{1}{2(k+1)^2}\right)$ [Ramanujan] Berndt Ch. 2, Eq. 7.6

= $\int_1^2 \frac{dx}{x^2+1}$

1 .32177644108013950981... ≈ $HypPFQ\left[\{1,1,1\},\left\{\frac{1}{2},2\right\},\frac{1}{4}\right] = \sum_{k=0}^{\infty} \frac{1}{\binom{2k}{k}(k+1)}$

1 .321779532040728089443... ≈ $\frac{\pi\sin 1}{2} = \int_0^1 \left(\cos x - \cos\frac{1}{x}\right)\frac{dx}{1-x^2}$

1 .321790572608050379284... ≈ $2\zeta(3) - \zeta(4)$

2 .32185317771930238873... ≈ $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{1}{2^{ijk}}$

.32188758248682007492... ≈ $\frac{\log 5}{5} = \sum_{k=1}^{\infty}\frac{F_k^{\ 2}}{4^k k}$

2 .32192809488736234787... ≈ $\log_2 5$

3 .32192809488736234787... ≈ $\log_2 10$

.32195680437221399043... ≈ $\frac{\pi}{\sqrt{3}}\tanh\frac{\pi\sqrt{3}}{2} - \frac{31}{21} = \sum_{k=1}^{\infty}\frac{1}{k^2+5k+7}$

1 .32199784270916889147... ≈ $\frac{1}{\sqrt{2}}\left(\text{csch}^2\pi\sin\left(\pi\sqrt{-1-i}\right)\sin\left(\pi\sqrt{-1+i}\right)\right) = \prod_{k=1}^{\infty}\frac{k^4+2k^2+2}{k^4+2k^2+1}$

1 .32237166979136267848... ≈

$$-\frac{2}{5\left(-5+\sqrt{5}\right)\left(5+2\sqrt{5}\right)^{5/2}}\left(5\pi\left(47+21\sqrt{5}\right)+2\sqrt{5+2\sqrt{5}}\left(75+33\sqrt{5}+2\left(11+5\sqrt{5}\right)arc\operatorname{csc}h2\right)\right)$$

$= \sum_{k=1}^{\infty}\frac{F_k^{\ 2}}{\binom{2k}{k}}$

.32246703342411321823... ≈ $\frac{\pi^2}{12} - \frac{1}{2} = \sum_{k=2}^{\infty}\frac{1}{2k^2}$

$= \frac{\pi^2}{24} - \frac{1}{2}\left(Li_2(e^{2i}) + Li_2(e^{-2i})\right)$

$= \sum_{k=1}^{\infty}\frac{H_k}{k(k+1)(k+2)}$

$= \sum_{k=2}^{\infty}(-1)^k\left(\frac{\zeta(k)+\zeta(k+1)}{2} - 1\right)$

$= \sum(-1)^{k+1}\frac{\cos^2 k}{k^2}$

.322634... ≈ $\sum_{k=1}^{\infty}\frac{\sigma_0(k)\mu(k)}{k^2}$

2 .32272613946042708519... ≈ $\sqrt{2}\,\frac{e^{\sqrt{2}}+1}{e^{\sqrt{2}}-1}$ Borwein-Devlin p. 90

1 .32287565553229529525... ≈ $\frac{\sqrt{7}}{2}$

14 .32305687810051332422... ≈ $2\pi I_0(2) = \int_0^{2\pi} e^{2\cos x} dx$

.32314082274133397351... ≈ $2Li_3\left(\frac{1}{2}\right) - \frac{5\zeta(3)}{8} = \sum_{k=1}^{\infty} \frac{H^{(2)}{}_k}{2^k k(k+1)}$

.323143494240176057189... ≈ $\zeta(2) - 2\zeta(3) + \zeta(4) = \sum_{k=1}^{\infty} \frac{k^2}{(k+1)^4}$

.323283066580692942094... ≈ $\sum_{k=2}^{\infty} \frac{(k-2)\zeta(k)}{k!}$

3 .323350970447842551184... ≈ $\frac{15\sqrt{\pi}}{8} = \Gamma\left(\frac{7}{2}\right)$

2 .323379732200195668706... ≈ $\sum_{k=1}^{\infty} \frac{H_k \log k}{k(k+1)}$

1 .32338804773499197945... ≈ $\frac{9\sinh\pi}{25\pi} = \prod_{k=1}^{\infty} \left(1 + \frac{1}{(k+3)^2}\right)$

.323409591142274671172... ≈ $\sum_{k=0}^{\infty} \frac{(-1)^k}{3^k + 1}$

.3234778813138516209... ≈ $\frac{1}{2}\left(3\log 2 + \log\pi - \gamma - 2\right) = \sum_{k=2}^{\infty} (-1)^k \frac{k}{k+1}\left(\zeta(k) - 1\right)$

.323712439072071081029... ≈ $\sinh\frac{1}{\pi} = \sum_{k=0}^{\infty} \frac{1}{(2k+1)!\pi^{2k+1}}$ AS 4.5.62

6 .323732825290032637102... ≈ $\sin\left(\pi\sqrt{-1-i}\right)\sin\left(\pi\sqrt{-1+i}\right)\csc\left(\pi\sqrt{-1+\sqrt{3}}\right)\operatorname{csch}\left(\pi\sqrt{1+\sqrt{3}}\right)$

= $\prod_{k=1}^{\infty} \frac{k^4 + 2k^2 + 2}{k^4 + 2k^2 - 2}$

.32378629924752936815... ≈ $\frac{\log^2 5}{8} = \int_0^1 \frac{\log(1+4x)}{1+4x} dx$

3 .3239444327545729800... ≈ $\prod_{k=1}^{\infty} \left(1 + \frac{H_k}{2^k}\right)$

.323946106931980719981... ≈ $\operatorname{arccsc}\pi$

.324027136831942699788... ≈ $\frac{1}{2\cosh 1} = \sum_{k=0}^{\infty} (-1)^k e^{-(2k+1)}$ J943

6 .32403324164743037432... ≈ $\frac{11\zeta(3)}{3}+\frac{17\pi^4}{360}-\frac{49\pi^2}{216}-\frac{4}{9} = \sum_{k=1}^{\infty}\frac{H_k H_{k+3}}{k^2}$

.32413774005332981724 1... ≈ $\frac{\pi^2}{6}-\frac{\pi}{2}+\frac{1}{4} = \frac{1}{2}\left(Li_2(e^i)+Li_2(e^{-i})\right) = \sum_{k=1}^{\infty}\frac{\cos k}{k^2}$ GR 1.443.4

.3241866235411497560 9... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{(2k-1)!\,2^{2k-1}} = \sum_{k=2}^{\infty}\frac{1}{k}\sinh\frac{1}{2k}$

.32420742957670372545... ≈ $\frac{\pi\sqrt{15}}{30}\cot\pi\sqrt{15}-\frac{491}{770} = \sum_{k=1}^{\infty}\frac{1}{k^2+8k+1}$

4 .32442695660979614349 7... ≈ $\pi^2-8\log 2 = \int_0^1\frac{\log(1-x)\log x}{x^{3/2}}dx$

6 .324555320336758664... ≈ $\sqrt{40}=2\sqrt{10}$

1 .3246052049335865... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k}{(k-1)(2k-3)\log k}$

1 .32460908925200584666 3... ≈ $\cosh\frac{\pi}{4} = \prod_{k=0}^{\infty}\left(1+\frac{1}{4(2k+1)^2}\right)$ J1079

3 .32465956914051318189 8... ≈ $\sqrt{\frac{3}{2}}\sinh\pi\sqrt{3}\ \operatorname{csc}h\ \pi\sqrt{2} = \prod_{k=0}^{\infty}\frac{k^2+3}{k^2+2}$

1 .32466679189998915649 6... ≈ $\frac{\pi(\pi+1)}{(\pi-1)^3} = \sum_{k=1}^{\infty}\frac{k^2}{\pi^k}$

.32468633830500738525 5... ≈ $\frac{6}{\pi^6}\left(12\gamma\pi^2\zeta'(2)-144\left(\zeta'(2)\right)^2+12\pi^2\zeta''(2)-\pi^4 StieltjesGamma(1)\right)$

= $\sum_{k=1}^{\infty}\frac{|\mu(k)|\log k}{k}$

1 .32471795724474602596 1... ≈ $\frac{1}{3}\left(\frac{27}{2}-\frac{3\sqrt{69}}{2}\right)^{1/3}+\frac{1}{3^{2/3}}\left(\frac{9}{2}+\frac{\sqrt{69}}{2}\right)^{1/3}$

= the plastic constant, unique real root of $x^3-x-1=0$

2 .3247847728404790612 4... ≈ $\sum_{k=1}^{\infty}\frac{t_3(k)}{2^k} = \sum_{k=1}^{\infty}\frac{\sigma_0(k)}{2^k-1} = \sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\frac{1}{2^{jk}-1} = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{1}{2^{ijk}}$

.32525134178898070857... ≈ $\frac{3-\sqrt{6}}{3}\sqrt{\pi} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(k-\frac{1}{2})!}{k!\,2^{k}}$

.325322571142143252138... ≈ $\frac{\pi}{4}\left(\sqrt{2}-1\right) = \int_{0}^{\infty}\frac{x(x-1)\,dx}{1+x^{4}}$

.325440084011503774969... ≈ $\frac{\arctan e^{2}}{2}-\frac{\pi}{8} = \int_{0}^{1}\frac{dx}{e^{2x}+e^{-2x}}$

8 .32552896478319506789... ≈ $\sum_{k=1}^{\infty}\frac{\sigma_{2}(k)}{2^{k}-1}$

.325735007935279947724... ≈ $\frac{1}{\sqrt{3\pi}}$

.325921357147665220333... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{(2k)!} = \sum_{k=2}^{\infty}\left(\cosh\frac{1}{k}-1\right)$

.326149892512184321542... ≈ $\frac{4}{7\sqrt{7}}\left(2\left(2-\log\frac{7}{2}\right)\operatorname{arccsc}2\sqrt{2}+i\left(Li_{2}\left(\frac{3-i\sqrt{7}}{4}\right)-Li_{2}\left(\frac{-3+i\sqrt{7}}{4}\right)\right)\right)$

= $\sum_{k=1}^{\infty}\frac{H_{k}}{\binom{2k}{k}2^{k}}$

.326190726563202248839... ≈ $\sum_{k=1}^{\infty}\frac{\mu(k)}{k!}$

1 .326324405266653433549... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}2^{2k-1}\frac{\zeta(2k)}{(2k)!} = \sum_{k=1}^{\infty}\sin^{2}\left(\frac{1}{k}\right)$

.3264209744985675... ≈ $H^{(2)}{}_{1/6}$

1 .32646029847617125005... ≈ $\frac{224}{\sqrt{3}}-128 = \sum_{k=0}^{\infty}\frac{1}{16^{k}}\binom{2k+2}{k}$

.32654323173422703585... ≈ $\cos^{2}1-\frac{\pi}{2}+si(2) = \int_{1}^{\infty}\frac{\cos^{2}x}{x^{2}}dx$

.32655812671825666123... ≈ $4+\gamma+\frac{3}{2}\log 2\pi-6\zeta'(1) = \sum_{k=1}^{\infty}\frac{k}{k+3}\left(\zeta(k+1)-1\right)$

1 .32693107195602131840... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(4k-2)}{2^{k}} = \sum_{k=1}^{\infty}\frac{k^{2}}{2k^{4}-1}$

3 .326953110002499790192... ≈ $e^{\zeta(3)}$

43 .327073280914999519496... ≈ *imaginary part of zero of* $\zeta(z)$

9 .3273790530888150456... $\approx \sqrt{87}$

.32752671473888716055... $\approx \dfrac{17}{4} + 2\zeta(2) - 6\zeta(3) = \displaystyle\int_1^\infty \frac{\log^2 x}{x^3(x-1)^2}dx$

.3275602576698990809... $\approx \dfrac{1}{3}(\gamma - \log 2 + \log 3) = \displaystyle\sum_{k=1}^\infty (-1)^k \frac{\psi(k)}{2^k}$

3 .32762949032282987473 3... $\approx \dfrac{4}{\zeta(3)}$

3 .32772475341775212043... $\approx 8G - 4 = \displaystyle\sum_{k=1}^\infty \left(\frac{(-1)^{k+1}}{(k-1/2)^2} - \frac{(-1)^{k+1}}{(k+1/2)^2} \right)$

7 .32772475341775212043... $\approx 8G = \displaystyle\sum_{k=1}^\infty \frac{k}{2^k} \zeta\left(k+1, \frac{3}{4}\right)$ Adamchik (28)

129 .32773993753692033334... $\approx 2\pi^3 + 56\zeta(3) = -\psi^{(2)}\left(\dfrac{1}{4}\right) = 2\displaystyle\sum_{k=0}^\infty \frac{1}{(k+\frac{1}{4})^3}$

.32798221428478223143 3... $\approx \displaystyle\sum_{k=2}^\infty \frac{1}{k^2-1} \log \frac{k}{k-1}$

.32813401447599016212... $\approx 1 - \gamma - \dfrac{1}{2}\big(\psi(1+i) + \psi(1-i)\big)$

$= \displaystyle\sum_{k=2}^\infty (-1)^k \big(\zeta(k) - \zeta(2k-1)\big)$

.32819182748668491245... $\approx {}_1\mathrm{F}_1\left(\dfrac{1}{2}, 2, 1\right) - 1 = \sqrt{e}\left(I_0\left(\dfrac{1}{2}\right) - I_1\left(\dfrac{1}{2}\right)\right) - 1 = \displaystyle\sum_{k=1}^\infty \frac{1}{(k+1)!4^k}\binom{2k}{k}$

$= \displaystyle\sum_{k=1}^\infty \frac{(2k-1)!!}{(2k)!!(k+1)!}$

2 .32820945323001176896 2... $\approx \dfrac{256}{35\pi} = \dbinom{4}{1/2}$

1 .32848684293666443478... $\approx \dfrac{3\pi \log 2}{2} - \dfrac{\pi^3}{16} = \displaystyle\int_0^1 \frac{\arcsin^3 x}{x^3}dx$

$= \displaystyle\int_0^{\pi/2} \frac{x^3 \cos x}{\sin^3 x}dx$

.32862117940693902398 9... $\approx \displaystyle\sum_{k=1}^\infty \frac{\mu(k)}{2^k k}$

13 .32864881447509874105... $\approx 3\pi\sqrt{2}$

.32896210586005002361 1... $\approx \dfrac{\pi^2}{3} - 2\log^2 2 - 2 = \displaystyle\sum_{k=0}^\infty \frac{1}{2^k (k+2)^2}$

$= \displaystyle\int_0^\infty \frac{x dx}{e^x\big(e^x - 1/2\big)}$

.328986813369645287295... ≈ $\frac{\pi^2}{30}$

.32923616284981706824... ≈ $\frac{1}{16}\left(2\pi^2 \log 2 - 7\zeta(3)\right) = -\int_0^{\pi/2} x \log\sin x \, dx$

= $\int_0^1\int_0^1 \frac{\log(1+xy)}{1-x^2y^2}\,dx\,dy$

1 .329340388179137020474... ≈ $\frac{3\sqrt{\pi}}{4} = \Gamma\left(\frac{5}{2}\right) = \int_0^\infty e^{-x^2}(1+x^2)dx$

1 .329396468534378841726... ≈ $\sum_{k=1}^\infty \frac{\log k}{(k-2)(2k-3)}$

.32940794999406386902... ≈ $\sum_{k=1}^\infty \frac{|\mu(k)|}{4^k}$

.3294846162243771650... ≈ $\frac{1}{9}(1+2\gamma-2\log 2+2\log 3) = \sum_{k=1}^\infty (-1)^k \frac{\psi(k)k}{2^k}$

3 .329626035009534959936... ≈ $\left(\gamma+3\log 2\right)\sqrt{\frac{\pi}{2}} = -\int_0^1 \frac{x}{\sqrt{-\log x}} \log\log\frac{1}{x}\,dx$ GR 4.325.11

3 .329736267392905745890... ≈ $\frac{2\pi^2}{3} - \frac{39}{12} = \int_0^1 \frac{(1+x)^2 \log x}{x-1}dx$

.329765314956699107618... ≈ $\operatorname{arctanh}\frac{1}{\pi} = \sum_{k=0}^\infty \frac{1}{\pi^{2k+1}(2k+1)}$

.329809295380395661449... ≈ $\sum_{k=1}^\infty \frac{k}{2^k(2^k+1)} = 2 - \sum_{k=1}^\infty \frac{k}{2^k+1}$

.3302299... ≈ $2\sum_{k=1}^\infty \frac{\pi(k)}{k(k^2+3k+2)} = \sum_{p\ prime} \frac{1}{p(p+1)}$

.33031019944919003837... ≈ $\frac{\pi}{16}\left(\sqrt{2}\coth\frac{\pi}{\sqrt{2}} - 3\pi\operatorname{csch}^2\frac{\pi}{\sqrt{2}} + \pi^2\sqrt{2}\coth\frac{\pi}{\sqrt{2}}\operatorname{csch}^2\frac{\pi}{\sqrt{2}}\right)$

= $\sum_{k=1}^\infty (-1)^k k^2 \frac{\zeta(2k)}{2^k} = \sum_{k=1}^\infty \frac{2k^2(2k^2-1)}{(2k^2+1)^3}$

.330357756100234864973... ≈ $\frac{\pi^2}{2} - \frac{1036}{225} = \psi^{(1)}\left(\frac{7}{2}\right)$

3 .330764430653872718842... ≈ $2e\left(\gamma - Ei(-1)\right) - 1 = \sum_{k=1}^\infty \frac{h(k)}{k!}$ where $h(k) = \sum_{i=1}^k H_i$

1 .33080124357305868479... ≈ $\prod_{k=1}^\infty \left(1 + \frac{1}{(k+1)(k+6)}\right)$

23 .330874490725823342456... ≈ $\frac{45\zeta(5)}{2} = \int_0^\infty \frac{x^4\,dx}{e^x+1} = \int_1^\infty \frac{\log^4 x}{x^2+x}dx$

.330893268204054533566... ≈ $-\log(e-2)$

20 .33104603466475394519... ≈ $\frac{1}{2}\left(\cosh(1+e)+\sinh(1+e)-\cosh(1-\cosh 1+\sinh 1)\right)$

$+\frac{1}{2}\left(\sinh(1-\cosh 1+\sinh 1)\right)$

= $\frac{e^{2+e}-e^{1/e}}{2e} = \sum_{k=1}^\infty \frac{\sinh k}{(k-1)!}$

1 .331265897019480833836... ≈ $\frac{27\log 3}{2} - \frac{27}{2} = \sum_{k=1}^\infty \frac{1}{k(k^2-1/9)}$ Prud. 5.1.25.20

1 .331335363800389712798... ≈ $\pi^{1/4}$

31 .331335637532090171911... ≈ $\psi\left(3,\frac{2}{3}\right) = \sum_{k=4}^\infty \frac{(k-1)(k-2)(k-3)\zeta(k)}{3^{k-4}}$

.331746833315620593286... ≈ $\frac{1}{2}\left(\cosh 1\sin 1-\cos 1\sinh 1\right) = \sum_{k=1}^\infty \frac{(-1)^{k+1}2^{2k+1}k}{(4k)!} = \int_0^1 \sin x\sinh x\,dx$

.33194832233889384697... ≈ $\frac{\pi}{4} - \frac{\pi}{4\sqrt{3}} = \int_0^\infty \frac{dx}{(x^2+1)(x^2+3)}$

6 .33212750537491479243... ≈ $\gamma + 2\log 2 + \frac{3\log 3}{2} + \frac{\pi\sqrt{3}}{2} = -\psi\left(\frac{1}{6}\right)$ Berndt 8.6.1

1 .33224156980746598006... ≈ $\sum_{k=1}^\infty \frac{\zeta(3k-1)}{2^k} = \sum_{k=1}^\infty \frac{k}{2k^3-1}$

.33233509704478425512... ≈ $\frac{3\sqrt{\pi}}{16} = \sum_{k=1}^\infty \frac{(-1)^{k+1}(k-\frac{1}{2})!\,3^k}{(k-1)!}$

.33235580126764226213... ≈ $\sum_{k=1}^\infty \frac{\mu(4k-3)}{4^{4k-3}-1}$

1 .33271169523087469727... ≈ $4\gamma^2$

.33302465198892947972... ≈ $\log^3 2$

.333177923807718674318... ≈ γ^2

.3333333333333333333$\underline{3}$ = $\frac{1}{3}$

= $\sum_{k=1}^\infty \frac{1}{k(3k+3)} = \sum_{k=0}^\infty \frac{1}{(3k+1)(3k+4)} = \sum_{k=0}^\infty \frac{1}{(2k+1)(2k+5)}$

$$= \sum_{k=1}^{\infty}\frac{1}{k^2+5k+6} = \sum_{k=1}^{\infty}\frac{1}{4k^2+4k-3}$$

$$= \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^k} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{2^k k}{\binom{2k}{k}}$$

$$= \sum_{k=1}^{\infty}\frac{\mu(k)}{3^k-1} = \sum_{k=1}^{\infty}\frac{\mu(2k)3^k}{9^k-1}$$

$$= \sum_{k=0}^{\infty}\frac{2^k}{k!(k+2)(k+5)} = \sum_{k=1}^{\infty}\frac{k-1}{\binom{2k}{k}}$$

$$= \prod_{k=1}^{\infty}\frac{k(k+3)}{(k+1)(k+2)}$$ J1061

$$= \prod_{k=2}^{\infty}\left(1-\frac{2}{k(k+1)}\right) = \prod_{k=1}^{\infty}\left(1-\frac{4}{(2k+1)^2}\right)$$ J1061, Marsden p. 358

$$= \int_0^{\infty}\frac{dx}{(1+x)^4} = \int_0^{\infty}\frac{x^2}{(1+x)^4}dx$$

$$= \int_0^{\infty}\frac{x^2\,dx}{e^{x^3}} = \int_0^{\infty}\frac{x^5\,dx}{e^{x^3}}$$

$$= \int_0^{\infty}\frac{1}{e^{\pi\sqrt{x}}-1} = \int_0^{\infty}\frac{\sinh x}{e^{2x}}dx$$

1 .33333333333333333333$\underline{3}$ $$= \frac{4}{3} = \beta\left(2,\frac{1}{2}\right) = \sum_{k=0}^{\infty}\frac{1}{4^k(k+2)}\binom{2k}{k}$$

$$= \prod_{k=1}^{\infty}\left(1+\frac{1}{2^{2^k}}\right)$$

.333550707609087859998... $$\approx \log\left(\pi\csc\frac{\pi}{\sqrt{2}}\right) - \frac{3\log 2}{2} = \sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{2^k k}$$

.33357849190965680633... $$\approx \sum_{\mu(k)=1}\frac{1}{4^k-1}$$

.33373020883590495023... $$\approx \sinh 1 - \sin 1$$

.333850657748648636047... $$\approx \frac{\pi}{6} - \frac{\gamma}{\pi^2} - \frac{\psi(1+\pi)}{\pi^2}$$

.33391221206618026333... $$\approx \frac{\pi}{12\sqrt{3}} + \frac{1}{8} + \frac{\log 2}{12} = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}}{12k-8}$$

2 .333938385220254754686... $\approx \sum_{k=1}^{\infty}\frac{\zeta(k+1)}{2^k-1}$

1 .334074248618567496498... $\approx \frac{3\pi\log 2}{4}+\frac{\pi^2}{16}-G = \int_1^{\infty}\frac{\arctan^2 x}{x^2}dx$

.334159265358993593595... $\approx \frac{\pi}{10}\coth 5\pi+\frac{1}{50} = \sum_{k=0}^{\infty}\frac{1}{k^2+25}$

.33432423314852208404... $\approx \frac{\pi}{8}\left(4-4\sqrt{2}+\pi+4\operatorname{arcsinh}1-6\log 2\right)$

$= \int_0^1 \arctan x \arcsin x \, dx$

44 .33433659358390136338... $\approx 6e^2 = \sum_{k=1}^{\infty}\frac{2^k k^2}{k!}$

2 .33443894740312597477... $\approx \sum_{k=1}^{\infty}\frac{\log(k+2)}{k^2}$

1 .334568251529384309456... $\approx e^{\gamma/2}$

3 .334577261949681056069... $\approx \gamma^2+\gamma^{-2}$

.33491542633111789724... $\approx \prod_{k=1}^{\infty}\left(1-\frac{1}{2k!}\right)$

.335158712773266366954... $\approx \frac{\pi}{\sqrt{41}}\tan\frac{\pi\sqrt{41}}{2}-\frac{1}{40} = \sum_{k=1}^{\infty}\frac{1}{k^2+7k+2}$

.33516839399781105400... $\approx \frac{e}{e-1}+e\log(e-1)-e = \sum_{k=0}^{\infty}\frac{k}{e^k(k+1)}$

1 .335262768854589495875... $\approx \frac{\pi^6}{720} =$ volume of unit sphere in R^{12}

.335348484711510163377... $\approx -\sum_{k=1}^{\infty}\frac{\mu(2k)}{2^k}$

1 .335382914379218353317... $\approx \zeta\left(\frac{1}{2},\frac{1}{4}\right)-\zeta\left(\frac{1}{2},\frac{3}{4}\right)$

10 .335425560099940058492... $\approx \frac{\pi^3}{3}$

.335651189631042415916... $\approx \log\pi-2\log 2+\gamma = \sum_{k=2}^{\infty}(-1)^{k+1}\frac{\zeta(k)}{2^{k-1}k}$

$= \sum_{k=1}^{\infty}\left(\frac{1}{k}-2\log\left(1+\frac{1}{2k}\right)\right)$

1 .335705707054747522239... $\approx \frac{\pi^2}{e^2}$

.33600000000000000000$\underline{0}$ = $\frac{42}{125} = \sum_{k=1}^{\infty}\frac{k^2}{6^k}$

.336074461109355209566... ≈ $\frac{46}{75} - \frac{2\log 2}{5} = \sum_{k=1}^{\infty}\frac{1}{k(2k+5)}$

.33613762329112393978... ≈ $4 - 4G = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k+1/2)^2}$

2 .33613762329112393978... ≈ $6 - 4G = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(k^2+1/4)^3}$

2 .33631317605893329200... ≈ $\sum_{k=2}^{\infty}\frac{\log^2 k}{k(k-1)} = \sum_{k=2}^{\infty}\zeta''(k)$

.336409322543576932... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(k+1)}{2^k k^2} = \sum_{k=1}^{\infty}\frac{1}{k} Li_2\left(\frac{1}{2k}\right)$

.336472236621212930505... ≈ $Li_1\left(\frac{2}{7}\right)$

3 .33656672793314242409... ≈ $-2Li_3(-2) = \frac{\pi^2\log 2}{3} + \frac{\log^3 2}{3} - 2Li_3\left(-\frac{1}{2}\right) = \int_0^1 \frac{\log^2 x}{(x+\frac{1}{2})}dx$

1 .3366190702415150752... ≈ $\frac{\pi e}{e^2-1}$

.33689914015761215217... ≈ $1 + \pi - \frac{4\pi}{3\sqrt{3}} - 2\log 2 = \int_0^{\pi/2}\frac{(1-\cos x)^2}{2-\sin x}dx$

.336945609670253572675... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{(k+1)!} = \sum_{k=2}^{\infty}\left(k^2 e^{1/k^2} - k^2 - 1\right)$

.336971553884269995675... ≈ $Li_5\left(\frac{1}{3}\right) = \sum_{k=1}^{\infty}\frac{1}{3^k k^5}$

.3370475079987656871... ≈ $\frac{\pi}{\sqrt{3}} - 2 - 4\log 2 + 3\log 3 = \sum_{k=1}^{\infty}\frac{(k-\frac{1}{2})!}{(k+\frac{1}{2})!(3k+1)}$

2 .337072437550439251351... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)}{(2k-2)!!} = \sum_{k=1}^{\infty}\frac{e^{1/2k^2}}{k^2}$

.3371172055926770144... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{\binom{2k}{k}}$

.337162848489433839615... ≈ $\frac{4}{8+\sqrt{6}+\sqrt{2}}$ CFG D1

.337187715838920906649... ≈ $\sum_{k=2}^{\infty}\frac{k-1}{k^k}$

.33719110708995486687... $\approx \frac{6\pi}{25\sqrt{5}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k(k-\frac{1}{2})!(k+\frac{1}{2})!}{(2k-1)!}$

$= \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(k-\frac{1}{2})!(k+\frac{1}{2})!}{2(2k-1)!}$

.33726479934440691573 1... $\approx -\frac{2}{\pi}\sqrt{\frac{2}{3}}\sin\pi\sqrt{\frac{3}{2}} = \prod_{k=2}^{\infty}\left(1-\frac{3}{2k^2}\right)$

.33740392290096813466 2... $\approx ci(1) = \gamma + \sum_{k=1}^{\infty}\frac{(-1)^k}{(2k)!(2k)} = \operatorname{Re}\{Ei(1)\}$ AS 5.2.16

1 .33758851203344926888 3... $\approx \frac{\pi^2}{48}+\frac{\pi}{4}+\frac{\log 2}{2} = -\int_0^1 \operatorname{arccot} x \log x\, dx$

.33761025731533638615 2... $\approx \frac{4\sqrt{\pi}}{21} = \int_0^{\infty}\frac{(\sin x - x\cos x)^2}{x^{9/2}}dx$

.3376952469966115444... $\approx 2\log\left(1+\frac{1}{2e}\right) = \int_1^{\infty}\frac{dx}{e^x+1/2}$

.33787706640934548356... $\approx -\frac{3}{2} + \log 2\pi = \sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{k(k+1)}$

1 .33787706640934548356... $\approx \sum_{k=1}^{\infty}\frac{\zeta(2k)}{k(k+1)} = 1+\sum_{k=2}^{\infty}\left(1+(k^2-1)\log\frac{k^2-1}{k^2}\right)$

$= -\frac{1}{2}+\log 2\pi$

.33808124740817174589... $\approx \frac{\pi^2}{6} + \log 2 - 2 = \sum_{k=1}^{\infty}\frac{1}{k^3+k^2} - \int_1^{\infty}\frac{dx}{x^3+x^2}$

1 .33810806274358800517 8... $\approx \sum_{k=2}^{\infty}\frac{ppf\,(k)}{2^k}$

.33868264216941619188... $\approx 6-\frac{8}{e}-e = \int_1^{\infty}\cosh\left(\frac{1}{x}\right)\frac{dx}{x^5}$

.33869688733846589456 0... $\approx \frac{1}{(e-1)^2} = -\sum_{k=0}^{\infty}\frac{B_k k}{k!}$

.33876648328794339088... $\approx \frac{18-\pi^2}{24} = -\int_0^1 x\,\log\left(1+\frac{1}{x}\right)\log x\, dx$

$= \int_1^{\infty}\frac{\log(x+1)\log x}{x^3}\,dx$

8 .33891600686386788021 7... $\approx 1+\frac{\pi^2}{2}+2\zeta(3) = \sum_{k=2}^{\infty}k^2\big(\zeta(k)-1\big) = \sum_{k=2}^{\infty}\frac{4k^2-3k+1}{k(k-1)^3}$

.3390469475765505037... $\approx \frac{\sqrt{\pi}\left(2-\sqrt{2}\right)}{8}\zeta\left(\frac{3}{2}\right) = \int_0^\infty \frac{x^2dx}{e^{x^2}+1}$

.339369340124745379664... $\approx \sum_{k=2}^\infty \left(\frac{\zeta(k)\zeta(k+2)}{\zeta^2(k+1)}-1\right)$

.339395839527289266398... $\approx \sum_{k=1}^\infty \frac{\zeta(2k+1)}{4^k k} = -\sum_{j=1}^\infty \frac{1}{j}\log\left(1-\frac{1}{4j^2}\right)$

.339530545262710049640... $\approx \frac{16}{15\pi} = \binom{2}{-1/2}$

.339732142857142857143... $\approx \frac{761}{2240} = \sum_{k=5}^\infty \frac{1}{k^2-16}$

.339785228557380654420... $\approx \frac{e}{8} = \sum_{k=1}^\infty \frac{k^2+23k}{2k!}$

1 .339836732801696602532... $\approx \frac{2^{2/3}}{3}\left((-1)^{1/3}\psi\left(2+\frac{1+i\sqrt{3}}{2^{1/3}}\right)-(-1)^{2/3}\psi\left(2+\frac{1-i\sqrt{3}}{2^{1/3}}\right)\right)$

$-\frac{2^{2/3}}{3}\psi\left(2-2^{2/3}\right)$

$= \sum_{k=1}^\infty 4^k\left(\zeta(3k)-1\right) = \sum_{k=2}^\infty \frac{4}{k^3+4}$

.339836909454121937096... $\approx \operatorname{arccsc} 3$

.339889822998532907346... $\approx \sum_{k=2}^\infty \frac{\zeta(k)-1}{k!(k-1)!} = \sum_{k=2}^\infty \left(\frac{1}{\sqrt{k}}I_1\left(2\sqrt{\frac{1}{k}}\right)-\frac{1}{k}\right)$

6 .340096668892171638830... $\approx \sum_{k=1}^\infty \frac{\sigma_2(k)}{k!}$

.340211994871313585963... $\approx \frac{36}{\pi^6}\left(\pi^2\zeta''(2)-12(\zeta'(2))^2\right) = -\sum_{k=1}^\infty \frac{\mu(k)\log^2 k}{k^2}$

.340331186244460642029... $\approx \frac{\pi^2}{29}$

.340430601039857489999... $\approx \frac{1}{9}\psi^{(1)}\left(\frac{2}{3}\right) = \frac{4\pi^2}{27}-1-\frac{1}{9}\psi^{(1)}\left(\frac{4}{3}\right) = \frac{4\zeta(2)}{9}-\frac{g_2}{2} = \sum_{k=1}^\infty \frac{1}{(3k-1)^2}$

$= \sum_{k=2}^\infty \frac{(k-1)\zeta(k)}{3^k} = \int_1^\infty \frac{\log x}{x^3-1}dx$

.340505841530123945275... $\approx \frac{\pi}{4}\left(\sqrt{2+\sqrt{2}}-\sqrt{2}\right) = \int_0^\infty \frac{x^4(1-x)^2}{x^8+1}dx$

.340530528544255268456... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k!(k-1)}$

.340791130856250752478... ≈ $Li_4\left(\frac{1}{3}\right) = \sum_{k=1}^{\infty}\frac{1}{3^k k^4}$

.34084505690810466423... ≈ $\frac{\pi-1}{2\pi} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(k-\frac{1}{2})!\,(\pi-1)^k}{(k-1)!}$

1 .340916071192457653485... ≈ $\sum_{k=1}^{\infty}\frac{1}{F_k k^2}$

1 .340999646796787694775... ≈ $\frac{1}{3}+\frac{5\pi\sqrt{3}}{27} = \sum_{k=1}^{\infty}\frac{1}{\binom{2k}{k-1}}$

.34110890264882949616... ≈ $\sqrt{\frac{\pi}{27}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(k-\frac{1}{2})!\,2^k}{(k-1)!}$

.341173496088356300777... ≈ $\frac{9}{16\sqrt{e}} - \sum_{k=0}^{\infty}\frac{(-1)^k k^4}{k!2^k}$

.34121287131515428207... ≈ $\sum_{k=1}^{\infty}\left(\zeta(k+1)-1\right)\log k$

1 .341487257250917179757... ≈ $\zeta\left(\frac{5}{2}\right) = \sum_{k=1}^{\infty}\frac{1}{k^{5/2}}$

.34153172745006491290... ≈ $\sum_{k=1}^{\infty}\frac{1}{2k^6+k^3} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}\zeta(3k+3)}{2^k}$

4 .341607527349605956178... ≈ $\sqrt{6\pi}$

1 .341640786499873817846... ≈ $\frac{3}{\sqrt{5}}$

.34176364520545358529... ≈ $\frac{1}{6\sqrt{3}}\left((-1)^{1/3}\left(2^{2/3}(3i+\sqrt{3})\right)\psi\left(\frac{2+(-2)^{2/3}}{2}\right)-2^{4/3}\sqrt{3}\,\psi\left(1-\left(-\frac{1}{2}\right)^{1/3}\right)\right)$

$$+\frac{\sqrt{3}-3i}{6\sqrt{3}}\left(2+3\zeta(3)+2^{2/3}\psi\left(\frac{1}{2^{1/3}}\right)\right)$$

= $\sum_{k=1}^{\infty}\frac{1}{2k^6+k^2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}\zeta(4k+2)}{2^k}$

.34201401950591179357... ≈ $\frac{\pi^2}{12}-\log^2 2 = \sum_{k=1}^{\infty}\frac{H_k}{2^k k(k+1)} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}H_k}{k(k+1)}$

80 .34214769275067096371... ≈ $4e^3 = \sum_{k=0}^{\infty}\frac{3^k(k+1)}{k!}$ GR 1.212

.342192383062178516199... ≈ $\sum_{k=2}^{\infty}\frac{\log\zeta(k)}{k}$

1 .342464048323863395382... ≈ $\sum_{k=0}^{\infty}J_k(1)$

2 .34253190155708315506... ≈ $e^{\gamma}+e^{-\gamma}$

2 .342593808906333507124... ≈ $\sum_{k=1}^{\infty}\frac{k\zeta(3k-1)}{2^k} = \sum_{k=1}^{\infty}\frac{2k^4}{(2k^3-1)^2}$

.3426927443728117484... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(4k+1)}{4^k} = \sum_{k=1}^{\infty}\frac{1}{4k^5-k}$

= $-\gamma-\frac{1}{4}\left(\psi\left(1-\frac{1}{\sqrt{2}}\right)+\psi\left(1-\frac{i}{\sqrt{2}}\right)+\psi\left(1+\frac{1}{\sqrt{2}}\right)+\psi\left(1+\frac{i}{\sqrt{2}}\right)\right)$

52 .34277778455352018115... ≈ $\frac{945\sqrt{\pi}}{32} = \Gamma\left(\frac{11}{2}\right)$

.34294744981683147519... ≈ $\frac{3\pi G}{4}-\frac{\pi^3}{64}+\frac{3\pi^2\log 2}{32}-\frac{105\zeta(3)}{64} = \int_0^1\frac{\arctan^3 x}{x^2}dx$

= $\int_0^{\pi/4}\frac{x^3}{(\sin x)^2}\,dx$

.343120541319968189938... ≈ $8\log 2-\zeta(3)-4 = \sum_{k=1}^{\infty}\frac{1}{4k^5-k^3}$

2 .343145750507619804793... ≈ $8-4\sqrt{2}$

1 .34327803741968934237... ≈ $\frac{3152}{27}-96\zeta(3) = \int_0^1\frac{\log^2 x}{1+x^{1/4}}\,dx$

.34337796155642703283... ≈ $\int_0^{\infty}\frac{\sin x}{(x+1)^2}\,dx$

.3433876819728560451... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^{k+1}}{12k-4} = \frac{\pi}{12\sqrt{3}}+\frac{1}{4}-\frac{\log 2}{12}$

.343476316712196629069... ≈ $12-6\cos 1-10\sin 1=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!(k+2)} = 2\int_1^e\frac{\log^3 x\cos\log x}{x}\,dx$

.343603799420667640923... ≈ $\frac{1-e^{-e}}{e} = \sum_{k=0}^{\infty}\frac{(-1)^k e^k}{(k+1)!}$

1 .3436734331817690185... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^{k+1}-3}$

24 .3440214084656728122... ≈ $\frac{\pi^2}{2}+\frac{7\pi^4}{120}+12\log 2+\frac{9\zeta(3)}{2}$

= $-\int_0^1 \log\left(1+\frac{1}{x}\right)\log^3 x\,dx$

16 .344223744208892043616... ≈ $\sum_{k=2}^{\infty}\frac{k^3\zeta(k)}{k!}$

.34460765191237177512... ≈ $\frac{\pi^2}{18}-\frac{11}{54} = \sum_{k=1}^{\infty}\frac{1}{k^3+3k^2}$

.34461519439543107372... ≈ $\sum_{k=1}^{\infty}\frac{H^{(3)}{}_k}{4^k}$

2 .34468400875238710966... ≈ $\frac{1}{4}\left(\sqrt{e\pi}\,erfi\frac{1}{\sqrt{e}}+\sqrt{\frac{\pi}{e}}\,erfi\sqrt{e}\right) = \sum_{k=0}^{\infty}\frac{\cosh k}{k!(2k+1)}$

.344684541646987370476... ≈ $265e-720 = \sum_{k=1}^{\infty}\frac{k}{k!(k+6)}$

.344740160430584717432... ≈ $\frac{2\pi^2}{\sqrt{\pi^2-1}}-2\pi = -\int_0^{2\pi}\frac{\cos x}{\pi+\cos x}dx$

5 .3447966605779755671... ≈ $\pi\csc\frac{\pi}{5} = \Gamma\left(\frac{1}{5}\right)\Gamma\left(\frac{4}{5}\right)$

= $\int_0^{\infty}\log\left(1+x^{-5}\right)dx$

.34509711176078574369... ≈ $\frac{\sqrt{\pi}}{4e^{1/4}} = \int_0^{\infty}x\,e^{-x^2}\sin x\,dx$

.3451605044614433513... ≈ $\frac{9\sinh 4\pi}{1190000\pi}=\prod_{k=5}^{\infty}\left(1-\frac{256}{k^4}\right)$

.34544332266900905601... ≈ $\frac{\sin 1+\cos 1}{4}=\sum_{k=1}^{\infty}(-1)^k\frac{k^2}{(2k)!}$

1 .345452103219624168597... ≈ $\frac{1}{2}\left(e^{e/2}-e^{1/2e}\right) = \sum_{k=0}^{\infty}\frac{\sinh k}{k!2^k}$

.345471517734513754249... ≈ $\sum_{k=2}^{\infty}(\zeta(k)-1)^2\log k$

.345506031655163187271... ≈ $-\frac{1}{2}-\frac{\pi}{2\sqrt{2}}\cot\frac{\pi}{\sqrt{2}} = \sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{2^k} = \sum_{k=2}^{\infty}\frac{1}{2k^2-1}$

1 .345506031655163187271... ≈ $\frac{1}{2}-\frac{\pi}{2\sqrt{2}}\cot\frac{\pi}{\sqrt{2}} = \sum_{k=1}^{\infty}\frac{\zeta(2k)}{2^k} = \sum_{k=1}^{\infty}\frac{1}{2k^2-1}$

1 .3455732635381379259... ≈ $\int_1^{\infty}x(x+1)\left(\zeta(2x)-1\right)dx$

.345654901949164100392... ≈ $\pi\log 2 - 2G = \int_0^{\pi/2} \frac{x\cos x}{1+\sin x}dx = \int_0^1 \frac{\arcsin x}{1+x}dx$

$= \int_0^1 \left(K(x) - \frac{\pi}{2}\right)\frac{dx}{x}$ GR 6.142

.345729840840857670674... ≈ $\sum_{k=1}^{\infty} \frac{1}{3^{k^2}}$

.3457458387231644802... ≈ $\frac{1}{2} - \frac{I_0(-2)}{2e^2} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)!}\binom{2k+1}{k}$

.345814317225052656510... ≈ $\frac{1}{\sqrt{e}}\left(\log 2 + Ei\left(\frac{1}{2}\right) - \gamma\right) = \sum_{k=1}^{\infty} (-1)^{k+1}\frac{H_k}{k!2^k}$

.346101661375621190852... ≈ $\frac{\sqrt{\pi}}{2} erfi \frac{1}{3} = \sum_{k=0}^{\infty} \frac{1}{k!3^{2k+1}(2k+1)}$

1 .34610229327379490401... ≈ $\int_0^1 \binom{2x}{x} dx$

1 .346217984368230168984... ≈ $-(-1)^{3/4}\pi\sin\left((-1)^{1/4}\pi\right) - (-1)^{1/4}\pi\sin\left((-1)^{3/4}\pi\right)$

.346250624110635744578... ≈ $\frac{\pi}{\sqrt{5}}\tan\frac{\pi\sqrt{5}}{2} - \frac{1}{5} = \sum_{k=1}^{\infty} \frac{1}{k^2+5k+5}$

.346494734701802213346... ≈ $\frac{\zeta'(2)}{\zeta^2(2)} = -\sum_{k=2}^{\infty} \frac{\mu(k)\log k}{k^2}$

.34654697521433430672... ≈ $\frac{1}{\sqrt{\pi}} - \frac{3}{2e} erfi 1 = \frac{4}{3\sqrt{\pi}}\ {}_1F_1\left(2, \frac{5}{2}, -1\right) = \sum_{k=0}^{\infty} \frac{(-1)^k k}{(k+\frac{1}{2})!}$

.34657359027997265471... ≈ $\frac{\log 2}{2} = \operatorname{Re}\left\{\log(1+i)\right\} = -\operatorname{Re}\left\{Li_1(i)\right\}$

$= \operatorname{arctanh}\frac{1}{3} = \operatorname{arcsinh}\frac{1}{2\sqrt{2}} = \sum_{k=0}^{\infty} \frac{1}{3^{2k+1}(2k+1)}$ AS 4.5.64, J941

$= \sum_{k=1}^{\infty} \frac{H^E{}_k}{2^{k+1}} = \sum_{k=1}^{\infty} \left(Li_k\left(\frac{1}{2}\right) - \frac{1}{2}\right)$

$= \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+2} = -\sum_{k=1}^{\infty} \frac{\cos k\pi/2}{k}$

$= \sum_{k=1}^{\infty} \frac{\zeta(2k)-1}{2k} = \sum_{k=1}^{\infty} \left(1 - \beta(k)\right) = \frac{1}{2}\sum_{k=2}^{\infty} \log\left(1 - \frac{1}{k^2}\right)$

$= \int_1^{\infty} \frac{dx}{(x+1)(x+3)} = \int_1^{\infty} \frac{dx}{x^3+x}$

$= \int_0^1 \frac{x-1}{(x+1)(x^2+1)\log x}dx$ GR 4.267.4

$= \int_0^\infty \frac{dx}{e^{2x}+1} = \int_0^\infty \frac{x\,dx}{e^{x^2}+1}$

$= \int_0^{\pi/4} \tan x\,dx = \int_0^{\pi/4} \frac{\cos x-\sin x}{\cos x+\sin x}\,dx$

$= \int_0^\infty (1-e^{-x})\frac{\cos x}{x}\,dx$

$= -\int_0^1 \psi(x)\sin\pi x\sin 3\pi x\,dx$ GR 6.496.2

$= \int_0^\infty \frac{x^2\sinh x}{\cosh^2 x}\,dx$ GR 3.527.14

$= \int_0^\infty \frac{3-4\sin^2 x}{x}\sin^2 x\,dx$ Prud. 2.5.29.25

.346645900013298547510... ≈ $\frac{4\pi}{7}\left(\cos\frac{\pi}{14}-\cos\frac{3\pi}{14}\right) = \int_0^\infty \frac{x^3(1-x)^2}{1+x^7}\,dx$

.34700906595089537284... ≈ $\sum_{k=1}^\infty (-1)^{k+1}\frac{k}{k^3+1}$

.34720038895629346817... ≈ $2\log 2 + 2\log^2 2 - 2 = \sum_{k=1}^\infty \frac{H_k}{2^k(k+2)}$

1 .3472537527357506922... ≈ $Li_{-1/2}\left(\frac{1}{2}\right) = \sum_{k=1}^\infty \frac{\sqrt{k}}{2^k}$

.347296355333860697703... ≈ $2\sin\frac{\pi}{18} = \sqrt{2-\sqrt{2+\sqrt{2-\sqrt{2+...}}}}$ [Ramanujan] Berndt Ch. 22

240 .34729839382610925755... ≈ $\frac{\pi^6}{4} = \int_0^\infty \frac{\log^5 x}{(x+1)(x-1)}\,dx$ GR 4.264.3

.347312547755034762174... ≈ $24\zeta(5) + 12\zeta(3) - \frac{2\pi^4}{5} = \int_0^\infty \frac{x^4}{(e^x-1)^3}$

.347376444584916568018... ≈ $32\log 2 - \frac{131}{6} = \sum_{k=0}^\infty \frac{1}{2^k(k+5)}$

.347403595868883758064... ≈ $erf\,\frac{1}{\pi}$

.347413148261512801365... ≈ $\sum_{k=2}^\infty \frac{\zeta(k)-1}{k}\log k$

2 .347560059129698730937... ≈ $\sum_{k=1}^\infty \frac{2^k}{k}\left(\zeta(k+1)-1\right)$

.34760282551739384823... ≈ $\frac{4\pi}{3\sqrt{3}} - \frac{\pi}{2} - \frac{1}{2} = \int_0^{\pi/2} \frac{\cos^2 x}{(2-\sin x)^2}\,dx$

.347654672476249243683... ≈ $-\frac{2^{1/3}}{3}\left(\psi\left(2+2^{1/3}\right) - (-1)^{1//3}\psi\left(2 - \frac{1}{2^{2/3}} - \frac{i\sqrt{3}}{2^{2/3}}\right)\right)$

$-\frac{2^{1/3}}{3}(-1)^{2//3}\psi\left(2 - \frac{1}{2^{2/3}} + \frac{i\sqrt{3}}{2^{2/3}}\right)$

= $\sum_{k=1}^{\infty}(-1)^{k+1}2^k\left(\zeta(3k)-1\right) = \sum_{k=2}^{\infty}\frac{2}{k^3+2}$

.3477110463168825593... ≈ $\sum_{k=1}^{\infty}\mu(k)\left(\zeta(k+1)-1\right)$

1 .348098986099992181542... ≈ $\frac{\pi^3}{23}$

.348300583743980317282... ≈ $\frac{2\pi^2}{3} + \frac{\pi^4}{90} - 2\zeta(3) + 16\log 2 - 16 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}\zeta(k+4)}{2^k}$

= $\sum_{k=1}^{\infty}\frac{1}{2k^5+k^4}$

1 .348383106634907167508... ≈ $2\pi - \frac{\pi^2}{2} = i\log i^{\pi}$

7 .3484692283495342946... ≈ $\sqrt{54} = 3\sqrt{6}$

7 .348585884767866802762... ≈ $\frac{1}{3} - \cot 3 = \sum_{k=1}^{\infty}\frac{1}{2^k}\tan\frac{3}{2^k}$ Berndt ch. 31

.348827861154840084214... ≈ $Li_3\left(\frac{1}{3}\right) = \sum_{k=1}^{\infty}\frac{1}{3^k k^3}$

.34906585039886591538... ≈ $\frac{\pi}{9} = \int_0^{\infty}\frac{x^{7/2}dx}{1+x^9}$

.3497621315252674525... ≈ $4 - \frac{\pi}{2} - 3\log 2 = \sum_{k=1}^{\infty}\frac{1}{4k^2+k} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}\zeta(k+1)}{4^k}$

= $hg\left(\frac{1}{4}\right) = \gamma + \psi\left(\frac{5}{4}\right)$

= $-\int_0^1 \log(1-x^4)\,dx$

.349785835986326773311... ≈ $\sum_{p\ prime}\frac{1}{2^p+1}$

1 .3498073850089500695... ≈ $\pi\coth 2\pi - \gamma - \frac{1}{2}\left(1 + \psi(1+2i) + \psi(1-2i)\right)$

$$= \sum_{k=1}^{\infty}\frac{4(k-1)}{k^3+4k} = \sum_{k=1}^{\infty}(-1)^{k+1}4^k\left(\zeta(2k)-\zeta(2k+1)\right)$$

$$.349896163880747238828... \approx \frac{1}{2}\Gamma\left(\frac{5}{3}\right) - \frac{1}{3}\Gamma\left(\frac{2}{3},1\right) = \frac{1}{3}\Gamma\left(\frac{2}{3},0,1\right) = \int_0^1 e^{-x^3}dx$$

$$.3499468758478687547 5... \approx \sum_{k=2}^{\infty}\frac{k}{k^2-1}\left(\zeta(k)-1\right)$$

.35000000000000000000$\underline{0}$ = $\frac{7}{20}$

.350183865439569608867... ≈ $\prod_{k=0}^{\infty}\left(1-\frac{1}{2^{2^k}}\right)$

.350188287713896710 88... ≈ $\frac{\pi}{4}-\frac{1}{\sqrt{2}}\arctan\frac{1}{\sqrt{2}} = \int_0^{\pi/4}\frac{\cos^2 x}{1+\cos^2 x}dx$

.350245159507873421 30... ≈ $\sum_{k=1}^{\infty}\frac{|\mu(2k)|}{4^k-1}$

.350246906114333129 67... ≈ $\frac{\pi}{6\sqrt{3}}+\frac{\log 2}{3}-\frac{\log 3}{6}=\int_1^2\frac{x\,dx}{x^3+1}$

.350402387287602913765... ≈ $2\sinh 1-2 = 2\sum_{k=1}^{\infty}\frac{1}{(2k+1)!}$

2 .350402387287602913765... ≈ $e-\frac{1}{e} = 2\sinh 1 = 2\sum_{k=1}^{\infty}\frac{1}{(2k+1)!}$

= $\int_0^{\pi}e^{\cos x}\sin x\,dx$ GR 3.915.1

1 .350643881047675502520... ≈ $E\left(\frac{1}{2}\right)$

141 .350655079870352238735... ≈ $52e = \sum_{k=1}^{\infty}\frac{k^{\infty 5}}{k!} = 4\sum_{k=1}^{\infty}\frac{(2k+1)^2}{k!}$ Berndt 2.9.7

.350836042055995861835... ≈ $\frac{\pi^2}{6}-\frac{2^{1/3}}{6}\left(\psi\left(1+2^{-1/3}\right)+(-1)^{2/3}\psi\left(\frac{4-2^{2/3}-i2^{2/3}\sqrt{3}}{4}\right)\right)$

$+(-1)^{1/3}\frac{2^{1/3}}{6}\psi\left(\frac{4-2^{2/3}+i2^{2/3}\sqrt{3}}{4}\right)$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(3k+2)}{2^k} = \sum_{k=1}^{\infty}\frac{1}{2k^5+k^2}$

1 .3510284515797971427... ≈ $\frac{\zeta(3)}{2}+\frac{3}{4} = \sum_{k=2}^{\infty}k^2\left(\zeta(2k-1)-1\right) = \sum_{k=2}^{\infty}\frac{4k^4-3k^2+1}{k(k^2-1)^3}$

.351176889972281431035... ≈ $\frac{\pi^2}{24}-\frac{\log^2 2}{8}$ Berndt 9.8

15 .351214888072621297967... ≈ $\zeta(2)+6\zeta(4)+6\zeta(3) = \sum_{k=2}^{\infty}(k-1)^3\left(\zeta(k)-1\right)$

= $\sum_{k=2}^{\infty}\frac{k^2+4k+1}{(k-1)^4}$

.351240736552036319658... ≈ $\dfrac{\pi}{4\sqrt{5}} = \int_0^\infty \dfrac{dx}{4x^2+5}$

.351278729299871853151... ≈ $2-\sqrt{e} = \sum_{k=0}^\infty \dfrac{k}{(k+1)!2^k}$

4 .351286269561060410962... ≈ $K^2\left(\dfrac{\sqrt{2}}{2}\right) = \int_0^1 K(k)\dfrac{dk}{k}$ GR 6.143

.35130998899467984127... ≈ $\sum_{k=1}^\infty \dfrac{1}{2k^5+k} = \sum_{k=1}^\infty \dfrac{(-1)^{k+1}\zeta(4k+1)}{2^k}$

.35144720166935875333... ≈ $16+\dfrac{\pi^2-32}{\sqrt{2}} = \int_0^1 \dfrac{\arcsin^2 x}{\sqrt{1+x}}dx$

.351760582118184854439... ≈ $4-2e-3\gamma-e\gamma+eEi(-1)+3Ei(1) = \sum_{k=1}^\infty \dfrac{kH_k}{(k+2)!}$

.351867223826221510414... ≈ $\sum_{k=1}^\infty \dfrac{\zeta(k+1)-1}{2^k k} = -\sum_{k=2}^\infty \dfrac{1}{k}\log\left(1-\dfrac{1}{2k}\right)$

.351945726336114600425... ≈ $-\sum_{k=1}^\infty \dfrac{E_k}{k!}$

1 .351958911849046348014... ≈ $3\pi^2+\dfrac{9\pi\sqrt{3}}{4}-\dfrac{81}{2} = \sum_{k=1}^\infty \dfrac{1}{(k^2-1/9)^2}$

2 .352009605856259578188... ≈ $\dfrac{e^2}{\pi}$

.3520561937363814016... ≈ $\sum_{k=1}^\infty \dfrac{1}{(4^k-1)k^2} = \sum_{k=1}^\infty \dfrac{\sigma_{-2}(k)}{4^k}$

1 .352081566997835463... ≈ $2-\dfrac{3\log 3}{2} = \sum_{k=0}^\infty \dfrac{1}{4^k(2k+1)(2k+3)}$

.352162954741546234686... ≈ $\dfrac{1}{\sqrt{3}}\sinh\dfrac{1}{\sqrt{3}} = \sum_{k=1}^\infty \dfrac{1}{3^k(2k-1)!}$

.35225012976588843278... ≈ $2e^{-\pi(siin1)/2}\cos\left(\dfrac{\pi}{2}\cos 1\right) = i^{ei^i}+(-i)^{e^{-i}}$

24 .352272758500609309110... ≈ $\dfrac{\pi^4}{4}$

1 .352314016054543571075... ≈ $\dfrac{9\zeta(3)}{8} = \int_0^\infty \dfrac{x^3}{\cosh^2 x}dx$

.352416958458576171524... ≈ $\dfrac{1}{3}SinhIntegral(3) = \int_1^\infty \sinh\left(\dfrac{1}{x^2}\right)\dfrac{dx}{x^3}$

.35248587146747709353... ≈ $\dfrac{\pi^2}{28}$

.352513421777618997471... ≈ $\operatorname{arccot} e = \frac{\pi}{2} - \arctan e = \sum_{k=0} \frac{(-1)^{\infty k}}{e^{2k+1}(2k+1)}$

= $\int_1^\infty \frac{dx}{e^x + e^{-x}}$

1 .352592294195119116937... ≈ $\frac{1}{2}\left(e^2 + \gamma - Ei(2) + \log 2 - 1\right) = \sum_{k=0}^\infty \frac{2^k k}{k!(k+1)^2}$

.35263828675117426574... ≈ $1 - \frac{1}{2e} + e\left(Ei(-1) - Ei(-2)\right) = \int_0^1 \frac{dx}{e^x(1+x)^2}$

2 .352645259278140528155... ≈ $54 - 19e = \sum_{k=1}^\infty \frac{k^3}{k!(k+3)}$

2 .352694261688669518514... ≈ $\frac{\pi^2}{2} + 6\pi - 32 = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{(k^2 - 1/4)^3}$

.352809282429438776716... ≈ $\sum_{k=2}^\infty \frac{(-1)^k \zeta(k)}{2^k (k-1)}$

.35283402861563771915... ≈ $J_2(2) = \sum_{k=0}^\infty \frac{(-1)^k}{k!(k+2)!} = \sum_{k=0}^\infty (-1)^k \frac{k^3}{(k!)^2}$ LY 6.117

2 .35289835619154339918... ≈ $\sum_{k=1}^\infty \frac{\zeta(k+1)}{k^2} = \sum_{k=1}^\infty \frac{1}{k} Li_2 \frac{1}{k}$

1 .352904042138922739395... ≈ $\frac{\pi^2}{72} = \frac{5\zeta(4)}{4} = \frac{1}{2}\left(5\zeta(4) - \zeta^2(2)\right) = \sum_{k=1}^\infty \frac{H_k}{k^3}$ Berndt 9.9.5

$.\underline{3529411764705882}$ = $\frac{6}{17}$

.35323671854995984544... ≈ $\prod_{p\ prime}\left(1 - \left(1 - \prod_{k=1}^\infty \left(1 - p^{-k}\right)\right)^2\right)$

probability that two large integer matrices have relatively prime determinants

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.35326483790071991429... ≈ $\frac{\pi}{2}(J_0(1) - \cos 1) = \int_0^1 \sin x \arcsin x \, dx$

.3533316443238137931... ≈ $\sum_{k=1}^\infty \frac{(-1)^{k+1} k^2}{2^k - 1}$

.35349680070142205547... ≈ $\int_0^1 x^{1/x}\, dx$

1 .353540950076501665721... ≈ $\zeta(3)^{\zeta(2)}$

.353546048172969039851... ≈ $\frac{\cos 1}{e} + \frac{\sin 1}{2e} = \int_1^\infty \frac{x \sin x}{e^x} dx$

.35355339059327376220... ≈ $\frac{\sqrt{2}}{4} = \frac{\sqrt{8}}{8} = \sin\frac{3\pi}{8}\sin\frac{\pi}{8}$

.353658182777093571954... ≈ $\frac{\pi}{4} - \frac{1}{2} + \frac{\pi^2}{6} - \frac{\pi}{2}\coth\pi = \sum_{k=1}^{\infty}\frac{1}{k^4+k^2} - \int_1^{\infty}\frac{dx}{x^4+x^2}$

.35382477486069457290... ≈ $\log\frac{\pi}{\sqrt{5}}\csc\frac{\pi}{\sqrt{5}} = \sum_{k=1}^{\infty}\frac{\zeta(2k)}{5^k k}$

.353939237813914641441... ≈ $\log(e-1) - 1 - 2Li_2\left(\frac{1}{e}\right) - 2Li_3\left(\frac{1}{e}\right) + 2\zeta(3) = \sum_{k=0}^{\infty}\frac{B_k}{k!(k+2)}$

= $\int_0^1 \frac{x^2\,dx}{e^x-1}$

1 .354117939426400416945... ≈ $\Gamma\left(\frac{2}{3}\right)$

.35417288226859797042... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(4k)}{4^k} = \sum_{k=1}^{\infty}\frac{1}{4k^4-1}$

= $\frac{1}{2} - \frac{\pi}{4\sqrt{2}}\left(\cot\frac{\pi}{\sqrt{2}} + \coth\frac{\pi}{\sqrt{2}}\right)$

.354208311324197481258... ≈ $\sum_{k=2}^{\infty}(-1)^k\,\frac{\zeta(k)}{k!k}$

.354248688935409409498... ≈ $3 - \sqrt{7}$

.35429350648514965503... ≈ $\frac{1}{12\cdot 2^{1/3}}\left(\left(1+i\sqrt{3}\right)\psi\left(1-(-1)^{1/3}2^{2/3}\right) + \left(1-i\sqrt{3}\right)\psi\left(1+(-2)^{2/3}\right) - 2\psi\left(1+2^{2/3}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{1}{k^3+4}$

.354880888192781965553... ≈ $\frac{2\sqrt{3}}{3}\operatorname{arctanh}\frac{1}{\sqrt{3}} + \log\frac{2}{3} = \sum_{k=1}^{\infty}\frac{1}{3^k k(2k-1)}$

.35496472955084993189... ≈ $1 - \frac{1}{\sqrt{e}} I_0\left(-\frac{1}{2}\right) = \sum_{k=1}^{\infty}\binom{2k}{k}\frac{(-1)^{k+1}}{k!4^k}$

.35497979441174721069... ≈ $\frac{\pi}{2}\left(\sqrt{2} - 1 + \log\frac{2}{1+\sqrt{2}}\right) = \int_0^1 \arctan x \arccos x\,dx$

.355065933151773563528... ≈ $2 - \zeta(2) = Li_2\left(-e^{2i}\right) + Li_2\left(-e^{-2i}\right)$

= $\sum_{k=1}^{\infty}\frac{1}{k(k+1)^2} = \sum_{k=1}^{\infty}\frac{4k+1}{2k^2(2k+1)^2}$

= $\sum_{k=2}^{\infty}\frac{1}{k^3-k^2} = \sum_{k=1}^{\infty}\left(\zeta(k+2)-1\right) = \sum_{k=2}^{\infty}(-1)^k\left(\zeta(k)-\zeta(k+1)\right)$

= $\sum_{k=1}^{\infty}(-1)^{k+1}k\left(\zeta(k+2)\right) = \sum_{k=2}^{\infty}\frac{(-1)^k k\zeta(k+1)}{2^k}$

$= \int_0^1 \log x \log(1-x)\, dx$ GR 4.221.1

.35513731106146721909 1... $\approx \frac{7\pi^4}{1920} = \int_1^\infty \frac{\log^3 x}{x^3+x} dx$

4 .35517218060720426100 1... $\approx 2\pi \log 2$

$= \int_0^\infty \frac{x\, dx}{\sqrt{e^x - 1}}$ GR 3.452.1

$= \int_0^\infty \frac{\log(x^2+9)}{x^2+1} dx$

$= \int_0^\pi \frac{x \sin x\, dx}{1-\cos x}$ GR 3.791.10

$= -\int_0^{2\pi} \log \sin\left(\frac{x}{2}\right) dx$

$= -\int_0^\pi \log^2 (1+\cos x)\, dx$ GR 4.224.12

$= -\int_0^{\pi/2} \log^2 (\cos^2 x)\, dx$ GR 4.226.1

1 .3551733511720076359... $\approx \frac{5\pi\left(\sqrt{3}-1\right)}{6\sqrt{2}} = \int_0^\infty \frac{dx}{1+x^{12/5}}$

.35527011415059982585 7... $\approx \frac{3}{2} - \log \pi = \sum_{k=1}^\infty \frac{\zeta(2k)-1}{k+1} = -\sum_{k=2}^\infty \left(k^2 \log\left(1-\frac{1}{k^2}\right) + 1 \right)$

.3552701168740580298 3... $\approx \zeta(3) - \zeta(2) + \frac{\pi}{\sqrt{3}} \tanh \frac{\pi\sqrt{3}}{2} = \sum_{k=1}^\infty \frac{1}{k^5+k^4+k^3}$

.3553358665526914273... $\approx \sum_{k=1}^\infty \frac{(k!)^2 \zeta(2k)}{(2k)!}$

.35537972237507081128 0... $\approx \frac{1}{3} e^{-2^{2/3}} + \frac{2}{3} e^{2^{-1/3}} \cos \frac{\sqrt{3}}{2^{1/3}} = \sum_{k=0}^\infty \frac{(-1)^k 4^k}{(3k)!}$

.3557217094508993824... $\approx \zeta(3) - \frac{\zeta(6)}{\zeta(3)}$

.3557711587145111366 5... $\approx \sum_{k=2}^\infty \frac{1}{k^4 - 13}$

1 .3559096738634793803 5... $\approx \prod_{k=1}^\infty \left(1 + \frac{1}{4^k}\right)$

.3560959957811571219... ≈ $\sum_{k=1}^{\infty}(\coth k - 1)$

2 .356194490192344928847... ≈ $\frac{3\pi}{4} = \operatorname{arccot}(-1) = \arctan\frac{1}{7} + 2\arctan 2$

= $\sum_{k=1}^{\infty}\arctan\frac{2}{k^2}$ K Ex. 102

[Ramanujan] Berndt Ch. 2

= $\int_0^{\infty}\frac{dx}{x^2-2x+2}$

.356207187108022176514... ≈ $\log_7 2$

.356344287479823877805... ≈ $\frac{\pi}{4}\coth\frac{\pi}{2} - \frac{1}{2} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)}{4^k} = \sum_{k=1}^{\infty}\frac{1}{4k^2+1}$

.356395048612456272216... ≈ $\frac{1}{4}\,{}_2F_1\left(2,2,\frac{3}{2},\frac{1}{8}\right) = \sum_{k=1}^{\infty}\frac{k}{\binom{2k}{k}2^k}$

.356413814402934681864... ≈ $\sum_{k=1}^{\infty}\frac{\phi(k)}{4^k}$

.3565870639063299275... ≈ $\frac{\Gamma(1/2)}{\Gamma(2/3)\Gamma(1/6)} = \prod_{k=1}^{\infty}\left(1+\frac{(-1)^k 2}{3}\right)$ J1028

.356870185443643475892... ≈ $\frac{4}{3}Li_2\left(\frac{1}{4}\right) = \sum_{k=1}^{\infty}\frac{H^{(2)}{}_k}{4^k} = \frac{1}{3}\sum_{k=0}^{\infty}\frac{1}{4^k(k+1)^2}$

.356892371084866755384... ≈ $\int_1^2\frac{dx}{\zeta(x)}$

.3571428571428$\underline{571428}$ = $\frac{5}{14}$

.35738497664673044651... ≈ $\frac{\sqrt{\pi}\left(\sqrt{2}-1\right)}{3}\zeta\left(\frac{1}{2}\right) = \int_0^{\infty}\frac{\sqrt{x}\,dx}{e^{x^3}+1}$

1 .3574072890240502108... ≈ $\sum_{k=1}^{\infty}\frac{k^2}{3^k+1}$

. 35776388450028050232... ≈ $\frac{6G-1}{4\pi} = \sum_{k=0}^{\infty}\binom{2k}{k}^2\frac{1}{16^k(2k+1)(2k+3)}$

6 .3578307026347737355... ≈ $\frac{3\pi\sqrt{3}}{2} - 24\log 2 + \frac{27\log 3}{2}$

= $\sum_{k=0}^{\infty}\frac{1}{(k+1)(k+1/2)(k+1/3)}$

.357907384065669296994... ≈ $1-\cot 1 = \sum_{k=1}^{\infty}\frac{1}{2^k}\tan\frac{1}{2^k}$ Berndt ch. 31

= $\sum_{k=0}^{\infty}\left(\csc\frac{1}{2^k}-2^k\right)$

.358145824525628472970... ≈ $36e-\frac{195}{2} = \sum_{k=0}^{\infty}\frac{k^4}{(k+3)!}$

.358187786013244017743... ≈ $\frac{\sqrt{2}}{\pi}\sin\frac{\pi}{\sqrt{2}} = \prod_{k=1}^{\infty}\left(1-\frac{1}{2k^2}\right)$

1 .358212161001078455012... ≈ $\frac{\pi}{2}\left(1-e^{-2}\right)=\frac{\pi\sinh 1}{e} = \int_0^{\infty}\sin(2\tan x)\frac{dx}{x}$

= $\int_0^{\infty}\sin(2\tan x)\cos x\frac{dx}{x} = \int_0^{\pi/2}\sin(2\tan x)\cot x\,dx$

1439 .358491362377469674739... ≈ $\frac{61\pi^7}{128} = \int_0^{\infty}\frac{x^6}{\cosh x}dx$ GR 3.523.9

4 .358830714150528961342... ≈ $\sum_{k=2}^{\infty}\left(3^{\zeta(k)}-3\right)$

.358832038292189299797... ≈ $\sum_{k=1}^{\infty}\frac{\zeta^2(2k)}{8^k}$

4 .358898943540673552237... ≈ $\sqrt{19}$

.358987373392412508032... ≈ $\frac{1}{12\cdot 2^{1/3}}\left(\left(-1+i\sqrt{3}\right)\psi\left(1-(-1)^{1/3}2^{-2/3}\right)+\left(-1-i\sqrt{3}\right)\psi\left(1+\left(-\frac{1}{2}\right)^{2/3}\right)+2\psi\left(1+2\right.\right.$

= $\sum_{k=1}^{\infty}\frac{k}{4k^3+1}$

5 .35905348274329944827... ≈ $\frac{17\pi^4}{360}-\frac{\pi^2}{6}+2\zeta(3) = \sum_{k=1}^{\infty}\frac{H_k H_{k+1}}{k^2}$

1 .3590982771354826464... ≈ $\int_0^{\infty}\frac{dx}{e^x-x} = \int_0^{\infty}\frac{x\,dx}{e^x-x}$

.359140914229522617680... ≈ $\frac{e}{2}-1 = \sum_{k=1}^{\infty}\frac{k}{k!(2k+4)}$

1 .359140914229522617680... ≈ $\frac{e}{2} = \sum_{k=1}^{\infty}\frac{k}{(2k-1)!} = \sum_{k=1}^{\infty}\frac{k(k-1)}{2k!}$

= $\int_0^{\infty}xe^{1-x^2}dx$

= $-\int_0^{\infty}\frac{1}{e^x(1+x)^3}dx$

= $\displaystyle\int_1^e \frac{\log x}{(1+\log x)^2}\,dx$ GR 4.212.7

.359201008345368281651... ≈ $3\zeta(3) - 3\zeta(4)$

2 .359730492414696887578... ≈ $\pi^{3/4}$

3 .359885666243177553 17... ≈ $\displaystyle\sum_{k=1}^{\infty}\frac{1}{F_k} = \sum_{k=1}^{\infty}\frac{1}{\phi^{-k}-(-\phi)^k}$

.359906901116773247398... ≈ $\displaystyle\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k!} = \sum_{k=2}^{\infty}\left(e^{1/k}-1-\frac{1}{k}\right)$

.359946660398295076602... ≈ $\displaystyle\operatorname{arccot}\big((2-\cos 2)\csc 2\big) = \sum_{k=1}^{\infty}\frac{\sin 2k}{2^k k}$

1 $.3600000000000000000\underline{0}$ = $\displaystyle\frac{34}{25} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{F_k k^3}{2^k}$

.360164879994378648165... ≈ $\displaystyle\frac{\pi}{8}\tanh\frac{\pi}{2} = \sum_{k=1}^{\infty}(-1)^k\frac{\zeta(4k-2)}{4^k} = \sum_{k=1}^{\infty}\frac{k^2}{4k^4+1}$

1 .360349523175663387946... ≈ $\displaystyle\frac{\pi\sqrt{3}}{4} = \sum_{k=1}^{\infty}\frac{1}{(2k-1)^2-1/9}$ GR 1.421

1 .36037328719804788553... ≈ $\displaystyle\frac{51840}{4199\pi}\cos\frac{\pi\sqrt{77}}{2} = \prod_{k=1}^{\infty}\left(1+\frac{1}{k(k+9)}\right)$

.360553929977493959557... ≈ $1+\zeta(2)-\zeta(3)-\zeta(4)$

.360816041724199458377... ≈ $\displaystyle 4-\frac{6}{\sqrt{e}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!2^k(k+2)}$

.361028100573727922821... ≈ $\displaystyle\frac{\pi}{2\sqrt{2}}\coth\pi\sqrt{2}-\frac{3}{4} = \sum_{k=1}^{\infty}\frac{1}{k^2+4k+6}$

1 .361028100573727922821... ≈ $\displaystyle\frac{1}{4}+\frac{\pi}{2\sqrt{2}}\coth\pi\sqrt{2} = \sum_{k=1}^{\infty}\frac{1}{k^2-2k+3}$

$.3611111111111111111\underline{1}$ = $\displaystyle\frac{13}{36} = \sum_{k=1}^{\infty}\frac{1}{k^2+5k+4}$

1 $.3611111111111111111\underline{1}$ = $\displaystyle\frac{49}{36} = H^{(2)}{}_3$

.361328616888222584697... ≈ $\displaystyle -e^2 Ei(-2) = \int_0^{\infty}\frac{dx}{e^x(x+2)}$

.361367123906707805589... ≈ $\displaystyle\arcsin\frac{1}{2\sqrt{2}}$

.361449081708275819112... ≈ $\displaystyle -\frac{1}{32}+\frac{\pi}{8}\coth 4\pi = \sum_{k=1}^{\infty}\frac{1}{k^2+16}$ J124

.361594731322498428488... ≈ $\frac{3}{2}\left(\log\frac{3}{2}-\log^2\frac{3}{2}\right) = \sum_{k=1}^{\infty}\frac{kH_k}{3^k(k+1)}$

5 .3616211867937175442... ≈ $\frac{\gamma^3}{2}+\frac{\gamma\pi^2}{4}+\frac{(6\gamma^2+\pi^2)\log 2}{4}+\frac{3\gamma\log^2 2}{4}+\frac{\log^3 2}{2}+\zeta(3)$

= $-\int_0^\infty \frac{\log^3 x dx}{e^{2x}}$

441 .361656210365328128367... ≈ $162e+1 = \sum_{k=1}^\infty \frac{k^7}{(k+1)!}$

1 .3618358603536914... ≈ $\sum_{k=2}^\infty\sum_{n=2}^\infty \frac{\log k}{k^n-1}$

6 .3618456410625559136... ≈ $\frac{e^3-1}{3} = \sum_{k=0}^\infty \frac{3^k}{(k+1)!}$

.3618811716413163894... ≈ $\frac{1}{12\Gamma\left(-(-2)^{2/3}\right)\Gamma\left(-2^{2/3}\right)\Gamma\left((-1)^{1/3}2^{2/3}\right)} = \prod_{k=2}^\infty \frac{k^3-4}{k^3}$

$.3619047619047\underline{619047}$ = $\frac{38}{105} = \sum_{k=0}^\infty \binom{2k}{k}\frac{1}{4^k(k+1)(k+4)}$

.3620074223642897908... ≈ $\sum_{k=1}^\infty \frac{\sin k}{(2k+1)}$

.36204337091135813481... ≈ $27-\frac{5\pi^3}{6\sqrt{3}}-\frac{39\zeta(3)}{4} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{(k+1/3)^3}$

.362131615407560310248... ≈ $\sum_{k=1}^\infty (-1)^{k+1}\frac{\zeta(2k)}{(2k)^2} = -\frac{1}{4}\sum_{k=1}^\infty Li_2\left(-\frac{1}{k^2}\right)$

.362161639609789904943... ≈ $I_0\left(\frac{2}{\sqrt{3}}\right)-1 = \sum_{k=1}^\infty \frac{1}{(k!)^2 3^k}$

.36219564772174798450... ≈ $\prod_{k=1}^\infty\left(1-\frac{1}{2(2^k-1)}\right)$

.3623062223664980488... ≈ $\sum_{k=1}^\infty \frac{(-1)^{k+1}}{p(k)} = -\sum_{k=1}^\infty\frac{\mu(p(k))}{p(k)}$, $p(k)$ = product of the first k primes

.362360655593222140672... ≈ $\sum_{k=2}^\infty \frac{1}{2^k\phi(k)}$

.362389718306640099275... ≈ $\frac{5\zeta(3)}{4}-\frac{\pi^2\log 2}{6} = \sum_{k=1}^\infty \frac{(-1)^{k+1}H^{(2)}{}_k}{k(k+1)}$

.362532425370295364014... ≈ $\sum_{k=1}^\infty \frac{1}{(3k)^k}$

.3625370065384367141... ≈ $\frac{29}{15}-\frac{\pi}{2} = \sum_{k=2}^\infty(-1)^k\frac{\zeta(k)}{4^{k-1}} - \sum_{k=1}^\infty\frac{\zeta(2k+1)-1}{16^k}$

.362648111766062933408... ≈ $erf\,\frac{1}{3} = \frac{2}{\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!3^{2k+1}(2k+1)}$

.3627598728468435701... ≈ $\frac{\pi}{5\sqrt{3}}$

.3632479734471902766... ≈ $\frac{1}{2}(\log^2 3-\log^2 2) = \int_0^1 \frac{\log(x+2)}{x+2}dx$ GR 4.791.6

2 .363271801207354703064... ≈ $\frac{4\sqrt{\pi}}{3} = \Gamma\left(-\frac{3}{2}\right)$

.363297732806006305... ≈ $-\frac{\sin(\pi\sqrt{3})\sinh(\pi\sqrt{3})}{24\pi^2} = \prod_2^{\infty}\left(1-\frac{9}{k^4}\right)$

.363340818117227199895... ≈ $\sum_{k=1}^{\infty}\frac{\log(k+1)}{e^k k} = \int_0^1 \log\left(\frac{e-1}{e-x}\right)\frac{dx}{\log x}$ GR 4.221.3

.363380227632418656924... ≈ $1-\frac{2}{\pi} = \sum_{k=1}^{\infty}\left(\frac{(2k-1)!!}{(2k)!!}\right)^2\frac{1}{2k-1}$ J385

= $\sum_{k=1}^{\infty}(-1)^{k+1}\left(\frac{(2k-1)!!}{(2k)!!}\right)^3(4k+1)$

.36363636363636363636 = $\frac{4}{11}$

.363715310613311323433... ≈ $\sum_{k=2}^{\infty}\frac{1}{k!k!-1}$

1 .363771665261829837317... ≈ $\sum_{k=1}^{\infty}\frac{1}{F_{k^2}}$

.36380151974304965526... ≈ $\frac{\pi^2-7\zeta(3)}{4} = \sum_{k=2}^{\infty}(-1)^k k(k-1)\frac{\zeta(k)}{2^k} = \sum_{k=1}^{\infty}\frac{4k}{(2k+1)^3}$

.363843197059608668147... ≈ $3-\frac{17e^{1/3}}{9} = \sum_{k=1}^{\infty}\frac{k^3}{(k+1!3^k}$

.363975655751422539510... ≈ $\zeta(3)-\zeta(2)-\log 2+\frac{3}{2} = \sum_{k=2}^{\infty}\frac{1}{k^4-k^3} - \int_2^{\infty}\frac{dx}{x^4-x^3}$

.363985472508933418525... ≈ $\frac{1}{2}-\frac{\pi}{2\sinh\pi} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2+1} = \int_0^{\infty}\frac{\sin x}{e^x+1}dx$

≈ $\int_0^{\infty}\frac{\sin^2 x}{\cosh^2 x}dx$

.364021410879050636089... ≈ $\frac{1}{14}-\frac{\pi}{2\sqrt{7}}\cot\pi\sqrt{7} = \sum_{k=1}^{\infty}\frac{1}{k^2+6k+2}$

1 .3641737363436514341... ≈ $\frac{35\zeta(3)}{16} - \frac{\pi^2 \log 2}{8} + \frac{\pi^2}{4} - \pi G = \sum_{k=0}^{\infty} \frac{2^k}{\binom{2k}{k}(k+1)^2}$

.3642169446891739762 26... ≈ $\sum_{k=0}^{\infty} \frac{H_k}{\binom{2k}{k}(k+1)}$

7 .3643082723672572 56373... ≈ $\prod_{k=1}^{\infty} \left(1 + \frac{1}{(k-1)!}\right)$

2 .3644538928052092 84597... ≈ $\frac{1}{2}\sqrt{\frac{\pi}{2}}\, erfi\sqrt{2} = \sum_{k=0}^{\infty} \frac{2^k}{k!(2k+1)}$

2 .3646081543708727 03765... ≈ $2\left(CoshIntegral(2) - \log 2 - \gamma\right) = \sum_{k=1}^{\infty} \frac{4^k}{(2k)!k}$

.3648185772692609 1499... ≈ $1 - \frac{\log 2}{2} - \frac{\gamma}{2} = \sum_{k=1}^{\infty} (-1)^{k+1} \psi(k+2)$

.3649161225950003 5018... ≈ $\frac{\pi}{3}\left(3\sqrt{6} - 7\right)$ CFG A22

2 .3650621082759657 8895... ≈ $\frac{e\sqrt{\pi}}{4}\left(\gamma + 2\log 2\right) = -\int_0^{\infty} e^{1-x^2} \log x \, dx$

3 .3651389006617155 4308... ≈ $2 + \pi \operatorname{csch}\frac{\pi}{2} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k^2 + 1/4}$

.3651990418090313 606... ≈ $3\sum_{k=1}^{\infty} \frac{1}{(3k+1)^2} = \frac{1}{3}\psi^{(1)}\left(\frac{1}{3}\right) - 3$

= $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k \zeta(k+1)}{3^k}$

1 .3652721186254415 51877... ≈ $\frac{\gamma}{1-\gamma} = \sum_{k=1}^{\infty} \gamma^k$

2 .3652721186254415 51877... ≈ $\frac{1}{1-\gamma}$

.3653814847007192 49363... ≈ $\frac{3\zeta(3)}{\pi^2}$

.3655409037440503 19216... ≈ $\frac{\pi^2}{27} = -\frac{1}{3}\left(Li_2\left(\frac{-1-i\sqrt{3}}{2}\right) + Li_2\left(\frac{-1+i\sqrt{3}}{2}\right)\right)$

26 .3655738204521602 5321... ≈ $\frac{64\pi}{9} + \frac{8\pi^3}{27} - \frac{64\pi}{27}\log 2 = \int_0^{\infty} x^{1/2} \, Li_2(-x)^2 \, dx$

.3656737155547484 36672... ≈

$$\frac{1}{6561}\left(\pi^4 Root[3534848-152832\#1+\#1^3\&,2]+\psi^{(3)}\left(\frac{4}{9}\right)-\psi^{(3)}\left(\frac{7}{9}\right)\right)$$

= $\displaystyle\int_1^\infty \frac{\log^3 x}{x^3+1+x^{-3}}\,dx$

.365746230813041821667... ≈ $\displaystyle\sum_{k=2}^\infty \frac{\zeta(k)-1}{k(k-1)} = \sum_{k=2}^\infty \left(\frac{1}{k}+\left(1-\frac{1}{k}\right)\log\left(1-\frac{1}{k}\right)\right)$

1 .365746230813041821667... ≈ $\displaystyle\sum_{k=2}^\infty \frac{\zeta(k)}{k(k-1)} = \sum_{k=2}^\infty\sum_{j=2}^\infty \frac{\zeta(k)}{k^j} = \gamma + \sum_{k=2}^\infty \frac{\log k}{k(k+1)}$

= $\displaystyle 1+\sum_{k=2}^\infty \left(\frac{1}{k}+\left(1-\frac{1}{k}\right)\log\left(1-\frac{1}{k}\right)\right)$

.36586384076252029575... ≈ $\displaystyle\sum_{k=2}^\infty \frac{1}{k^2-1}\log\frac{k+2}{k}$

1 .365958772478076070095... ≈ $\displaystyle\sum_{k=1}^\infty \frac{(-1)^{k+1}\zeta(2k)}{(k-1)!(2k-1)} = \frac{\sqrt{\pi}}{2}\sum_{k=1}^\infty \frac{1}{k} erf\,\frac{1}{k}$

2 .366025403784438646764... ≈ $\displaystyle\frac{\sqrt{3}}{\sqrt{3}-1}$

= $\displaystyle\cos\frac{\pi}{6} - \sin\frac{\pi}{6}$

= $\displaystyle\prod_{k=1}^\infty \left(1+(-1)^k \frac{2}{6k-3}\right)$ J1029

.36602661434399298363... ≈ $\displaystyle -\sum_{k=1}^\infty \frac{\mu(3k-1)}{2^{3k-1}-1}$

.366059513169529355217... ≈ $\displaystyle\frac{1}{4}\log\frac{1-(-1)^{3/4}}{1+(-1)^{3/4}} - \frac{3G}{2} - \frac{i\pi}{96} - 2iLi_2\left((-1)^{3/4}\right) = \int_0^{\pi/4} x\sec x\,dx$

.366204096222703230465... ≈ $\displaystyle\frac{\log 3}{3} = \sum_{k=1}^\infty \frac{H^O{}_k}{4^k}$

.366210241779537772398... ≈ $\displaystyle\frac{2^{1/3}\pi}{3^{13/6}} = \int_0^\infty \frac{dx}{x^3+6}$

.3662132299770634876... ≈ $\displaystyle Li_2\left(\frac{1}{3}\right) = \zeta(2)+\log 2\log 3-\log^2 3-Li_2\left(\frac{2}{3}\right)$

= $\displaystyle\sum_{k=1}^\infty \frac{1}{3^k k^2} = \sum_{k=1}^\infty \frac{(-1)^{k+1}H_k}{2^k k}$

.36633734902506315584... ≈ $\displaystyle 3\log 2 - G - \frac{\pi}{2} + \frac{\pi^2}{4} - \frac{7}{9} = \sum_{k=2}^\infty \frac{8k-1}{k(4k-1)^2}$

= $\displaystyle\sum_{k=1}^{\infty}\frac{(k+1)\left(\zeta(k+1)-1\right)}{4^k}$

.36651292058166432701... ≈ $-\log\log 2$

.36651914291880921154... ≈ $\displaystyle\frac{7\pi}{60} = \int_0^{\infty}\frac{\left(\sin x-\cos x\right)^4}{x^6}\,dx$

.366563474449934445032... ≈ $\displaystyle 2(-1)^{2/3}\left(Li_4\left(-(-1)^{2/3}\right)-Li_4\left((-1)^{1/3}\right)\right)+2Li_4\left(-(-1)^{2/3}\right)-\frac{371\pi^4}{9720}$

= $\displaystyle\frac{1}{1296}\left(\psi^{(3)}\left(\frac{1}{3}\right)-\psi^{(3)}\left(\frac{5}{6}\right)\right)=\int_1^{\infty}\frac{\log^3 x\,dx}{x^3+1}$

8 .3666002653407554798... ≈ $\sqrt{70}$

138 .36669480800628662109... ≈ $\displaystyle\frac{1160703963}{8388608} = \sum_{k=1}^{\infty}\frac{k^9}{9^k}$

3 .36670337058187066843... ≈ $\zeta(3)+2\zeta(4)$

.36685027506808491368... ≈ $\displaystyle\frac{\pi^2}{16}-\frac{1}{4} = \int_0^1 x\ \arcsin^2 x\,dx$

2 .366904589024876561879... ≈ $\displaystyle\sqrt{\pi}\left(\zeta\left(\frac{1}{2},\frac{1}{4}\right)-\zeta\left(\frac{1}{2},\frac{3}{4}\right)\right) = 2\sqrt{\pi}\sum_{0}^{\infty}\frac{(-1)^k}{\sqrt{2k+1}} = \int_0^{\infty}\frac{dx}{\sqrt{x}\cosh x}$

4 .366976254815624405817... ≈ $\displaystyle\frac{4}{G}$

.367011324983578937117... ≈ $\displaystyle 8-\frac{\pi^2}{3}-4\log 4+\zeta(3) = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(k+3)}{2^k} = \sum_{k=1}^{\infty}\frac{1}{2k^4+k^3}$

.367042715537311626967... ≈ $\displaystyle\sum_{k=1}^{\infty}\frac{\cos^2 k}{k^3} = \frac{\zeta(3)}{2}+\frac{1}{4}\left(Li_3(e^{2i})+Li_3(e^{-2i})\right)$

.367156981956213712584... ≈ $\displaystyle -2\sum\frac{\mu(2k)}{2^k+1} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\mu(k)}{2^k+1}$

.367525688683979145707... ≈ $\displaystyle\sum_{k=2}^{\infty}\frac{k-1}{k^3\log k} = \int_2^3\left(\zeta(s)-1\right)ds$

1 .367630801985022350791... ≈ $\displaystyle\sum_{k=1}^{\infty}\frac{\phi(k)}{2^k}$

.367879441171442321596... ≈ $\displaystyle\frac{1}{e} = i^{2i/\pi} = \Gamma(1,1) = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+2)!}$

= $\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^k k^3}{k!} = \sum_{k=0}^{\infty}\frac{(-1)^k k^4}{k!}$

= $\sum_{k=0}^{\infty}\frac{1}{(2k+1)!(2k+3)}$

= $\sum_{k=1}^{\infty}\frac{\mu(k)}{e^{k}-1}$

= $\sum_{k=1}^{\infty}\frac{1}{StirlingS1(k,1)}$

= $\prod_{k=1}^{\infty}\frac{k(k+e)}{(k+e-1)(k+1)}$ J1061

= $\int_{1}^{\infty}\frac{dx}{e^{x}} = \int_{0}^{\infty}\frac{\log x}{(x+e)^{2}}\,dx$

= $\int_{0}^{1}x\sinh x\,dx = \int_{1}^{\infty}\sinh\left(\frac{1}{x}\right)\frac{dx}{x^{3}}$

1 .368056078023647174276... ≈ $\frac{\pi^{2}}{3}-4\log^{2}2 = \int_{0}^{1}\frac{\log(1-x^{2})}{x^{2}}\,dx$

.368120414067783285973... ≈ $\sum_{k=1}^{\infty}(-1)^{k}\log\zeta(k)$

1 .3682988720085906790... ≈ $\frac{\sinh\sqrt{2}}{\sqrt{2}} = \frac{e^{\sqrt{2}}-e^{-\sqrt{2}}}{2\sqrt{2}} = \sum_{k=0}^{\infty}\frac{2^{k}}{(2k+1)!} = \sum_{k=1}^{\infty}\frac{2^{k}k}{(2k)!}$ GR1.411.2

= $\prod_{k=1}^{\infty}\left(1+\frac{2}{\pi^{2}k^{2}}\right)$

.368325534380705489... ≈ $\frac{\log 2}{3}+\frac{\pi}{3}\operatorname{sech}\frac{\pi\sqrt{3}}{2} = \sum_{k=2}^{\infty}\frac{(-1)^{k}}{k-k^{-2}}$

.$\underline{368421052631578947}$ = $\frac{7}{19}$

1 .368432777620205875737... ≈ $\frac{\zeta(2)}{\zeta(3)} = \sum_{k=1}^{\infty}\frac{\phi(k)}{k^{3}}$ Titchmarsh 1.2.12

= $\prod_{p\ prime}\left(1+\frac{1}{p(p+1)}\right)$

.368552931587935171 74... ≈ $\operatorname{Re}\left\{\psi^{(2)}(i)\right\} = \frac{1}{2}\left(\psi^{(2)}(1+i)+\psi^{(2)}(1-i)\right)$

= $\sum_{k=1}^{\infty}\left(\frac{1}{(k+i)^{3}}+\frac{1}{(k-i)^{3}}\right)$

.368608550929364974778... ≈ $\frac{\pi^{2}}{16}-\frac{\log 2}{2}-\frac{\pi^{2}\log 2}{16}+\frac{7\zeta(3)}{16} = \sum_{k=1}^{\infty}\frac{H_{k}}{(4k-2)^{2}}$

.36873782029464990409... ≈ $\sum_{k=1}^{\infty}\frac{1}{k!\binom{3k}{k}}$

.3688266114776901201... ≈ $\frac{\pi}{2}(1-J_0(1)) = \int_0^1 \sin x \arccos x\, dx$

4 .369280326208524790686... ≈ $\frac{3}{2}+\frac{\pi\sqrt{3}}{2}\coth\frac{\pi}{\sqrt{3}} = \sum_{k=0}^{\infty}\frac{1}{k^2+1/3}$

.369669299246093688523... ≈ $1+\frac{\gamma}{2}-\frac{\log 2\pi}{2} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(k+1)}{k+2}$

= $\sum_{k=1}^{\infty}\left(\frac{1}{2k}+k\log\left(1+\frac{1}{k}\right)-1\right)$

.369878743412848974927... ≈ $\frac{\pi}{3\sqrt{3}}-\frac{4}{3}+\log 3 = \sum_{k=2}^{\infty}\left(\frac{2}{3}\right)^k\left(\zeta(k)-1\right) = \sum_{k=2}^{\infty}\frac{4}{9k^2-6k}$

12 .369885416851899866561... ≈ $\frac{\pi^2}{6}+\frac{\pi^4}{15}+4\zeta(3)-\gamma = \sum_{k=1}^{\infty}\frac{k^4}{k+1}\left(\zeta(k+1)-1\right)$

2 .36998127831969604622... ≈ $\sum_{k=0}^{\infty}\frac{k!}{S_2(2k,k)}$

.37024024484653052058... ≈ $\frac{\pi}{6\sqrt{2}} = \int_0^{\infty}\frac{x^2\,dx}{1+x^{12}} = \int_0^{\infty}\frac{x^8\,dx}{1+x^{12}}$

.370408163265306122449... ≈ $\frac{363}{980} = \frac{H_7}{7} = \sum_{k=1}^{\infty}\frac{1}{k^2+7k}$

.3704211756339267985... ≈ $-\sum_{k=1}^{\infty}\frac{\mu(3k)}{3^k-1} = \sum_{k=1}^{\infty}\frac{1}{(\sqrt[3]{3})^{3^k}} = \sum_{k=0}^{\infty}\frac{1}{3^{3^k}}$

.3705093754425278059... ≈ $\frac{3\log 3}{4}-\frac{\pi}{4\sqrt{3}} = \sum_{k=1}^{\infty}\frac{1}{6k^2-2k} = -\int_0^1\frac{\log(1-x^6)}{x^3}dx$

2 .370579559337007635161... ≈ $\frac{\pi}{4}\left(3+\frac{1}{e^4}\right) = \int_0^{\pi/2}\sin^2(2\tan x)\cot^2 x\,dx$ GR 3.716.11

3 .370580154832343626096... ≈ $\sum_{k=1}^{\infty}\frac{2^k}{k^k}$

1 .37066764204583085723... ≈ $\prod_{k=1}^{\infty}\left(1+\frac{1}{(k+1)(k+5)}\right)$

3 .370690303601868129363... ≈ $-\pi\sec\frac{\pi\sqrt{5}}{2} = \prod_{k=2}^{\infty}\left(1+\frac{1}{k^2-k-1}\right)$

.3708147119305699581... ≈ $\pi\left(\frac{\sqrt{5}}{2}-1\right) = \int_0^{\infty}\log\left(1+\frac{1}{4(x^2+1)}\right)dx$

2 .370879845350002036348... ≈ $\dfrac{\sqrt{3}}{2}\sinh\sqrt{3} \;=\; \displaystyle\sum_{k=1}^{\infty}\frac{3^k k}{(2k)!}$

.37122687271077216295... ≈ $\dfrac{4G}{\pi^2} \;=\; -\displaystyle\int_0^1\left(x-\frac{1}{2}\right)\sec\pi\, x\, dx$

.3713340155290132347... ≈ $\left(1-\dfrac{\sin\pi\sqrt{2}}{\sqrt{2}}\right)\dfrac{\pi^2}{8}\csc^2\dfrac{\pi}{\sqrt{2}} - 2 = \displaystyle\sum_{k=1}^{\infty}\frac{k\big(\zeta(2k)-1\big)}{2^k}$

= $\displaystyle\sum_{k=2}^{\infty}\frac{2k^2}{(2k^2-1)^2}$

2 .3713340155290132347... ≈ $\left(1-\dfrac{\sin\pi\sqrt{2}}{\sqrt{2}}\right)\dfrac{\pi^2}{8}\csc^2\dfrac{\pi}{\sqrt{2}} \;=\; \displaystyle\sum_{k=1}^{\infty}\frac{k\zeta(2k)}{2^k}$

= $\displaystyle\sum_{k=1}^{\infty}\frac{2k^2}{(2k^2-1)^2}$

4 .3715149065915282 2551... ≈ $\dfrac{\pi^2}{4} + 2\zeta(3) - \dfrac{1}{2} \;=\; \displaystyle\sum_{k=1}^{\infty}\frac{H_k H_{k+2}}{k(k+1)}$

.37156907160131848243... ≈ $G - \dfrac{\pi\log 2}{4} \;=\; \displaystyle\sum_{k=0}^{\infty}\frac{(-1)^{k+1}H_{k+1/2}}{2k+1}$ Adamchik (25)

= $\pi\displaystyle\sum_{k=1}^{\infty}\frac{(4^k-1)\zeta(2k)}{4^{2k}(2k+1)}$

= $-\displaystyle\int_0^1\frac{\log(2x)}{1+x^2}dx$ GR 4.295.13

= $-\displaystyle\int_0^1\frac{\log(1-x^2)}{1+x^2}dx$ GR 4.295.13

= $\displaystyle\int_0^{\infty}\frac{\log\cosh(x/2)}{\cosh x}dx$ GR 4.375.1

.3716423345722667425... ≈ $1 + \zeta(2) - \log 2 - \dfrac{15\log^2 2}{6} - \dfrac{2\log^3 2}{3} + \dfrac{\zeta(3)}{2}$

= $\displaystyle\int_0^1\frac{\log^2(1+x)}{x^2(x+1)^2}$

.3719052155237359 72523... ≈ $\dfrac{1}{4}\Big(4+(\cos 2-1)\log(4\sin^2 1)-(\pi-2)\sin 2\Big)$

= $\displaystyle\sum_{k=1}^{\infty}\frac{\cos^2 k}{k(k+1)}$

.37216357638560161556... ≈ $\frac{1}{5}+\frac{4}{5\sqrt{5}}\operatorname{arccsch}2 \;=\; \frac{1}{5}+\frac{4\log\phi}{5\sqrt{5}} \;=\; \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{\binom{2k}{k}}$

.3721849587240681867... ≈ $1-\frac{e}{2}+\frac{\sqrt{\pi}}{4}\,erfi(1) \;=\; \sum_{k=1}^{\infty}\frac{1}{k!\,(4k^2-1)}$

.37234885807547619972 5... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\phi(k)}{2^k+1}$

.37236279102931656488... ≈ $(\pi-2)\sin^2 1-\left(1+\log\csc 1-\log 2\right)\sin 2$

= $-\sum_{k=1}^{\infty}\frac{\sin(2k+2)}{k(k+1)}$ GR 1.444.1

.37252473265828081261... ≈ $\frac{1}{3}\left(H\left(-\left(-\frac{1}{2}\right)^{1/3}\right)+H\left(\frac{1}{2^{1/3}}\right)+H\left(\frac{(-1)^{2/3}}{2^{1/3}}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{1}{2k^4+k}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}\zeta(3k+1)}{2^k}$

1 .37258300203047921917 3... ≈ $24-16\sqrt{2} \;=\; \sum_{k=0}^{\infty}\binom{2k+2}{k}\frac{1}{8^k(k+1)}$

.37272030066601304742 5... ≈ $\frac{1}{4}\left(\pi\coth\pi-\pi^2\operatorname{csch}^2\pi+i\,\psi^{(1)}(1-i)-i\,\psi^{(1)}(1+i)\right)$

= $\sum_{k=1}^{\infty}(-1)^{k+1}k\left(\zeta(2k)-\zeta(2k+1)\right) \;=\; \sum_{k=2}^{\infty}\frac{k(k-1)}{(k^2+1)^2}$

16 .37297619578192516935 7... ≈ $\frac{\pi^3}{3}+4\pi\log^2 2 \;=\; \int_0^{\infty}\frac{x^2\,dx}{\sqrt{e^x-1}}$ GR 3.452.2

.3731854421538599476... ≈ $\sum_{k=1}^{\infty}\frac{1}{(4^k-1)k}=\sum_{k=1}^{\infty}\frac{\sigma_{-1}(k)}{4^k}$

1 .37328090766210649751... ≈ $\gamma^{-\gamma}$

.37350544572552464986... ≈ $\frac{\pi}{2^{3/4}}\left(\frac{1+i}{4}\right)\left(\cot\left(\frac{\pi(1+i)}{2^{3/4}}\right)+\coth\left(\frac{\pi(1+i)}{2^{3/4}}\right)\right)-\frac{1}{2}$

= $\sum_{k=1}^{\infty}\frac{1}{2k^4+1}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}\zeta(4k)}{2^k}$

.37355072789142418039... ≈ $\frac{\pi}{3\sqrt{3}}-\frac{\log 2}{3} \;=\; \sum_{k=1}^{\infty}\frac{1}{6k-4}-\frac{1}{6k-1}$ J80

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{3k+2}$ K ex. 113

= $\int_1^{\infty}\frac{dx}{x^3+1} \;=\; \int_0^1\frac{x\,dx}{x^3+1} \;=\; \int_0^{\infty}\frac{dx}{e^{2x}+e^{-x}}$

.37366311696691568710 2... ≈

$$\frac{1}{186624}\left(448\pi^4 + 3\psi^{(3)}\left(\frac{1}{6}\right) + 6\psi^{(3)}\left(\frac{1}{3}\right) - 3\psi^{(3)}\left(\frac{2}{3}\right) - 6\psi^{(3)}\left(\frac{5}{6}\right)\right)$$

$$= \int_1^\infty \frac{\log^3 x}{x^3 + x^{-3}}\,dx$$

.37382974072903473044... $\approx \frac{20}{9} - \frac{8\log 2}{3} = \sum_{k=0}^\infty \binom{2k}{k}\frac{1}{4^k(k+2)^2}$

.37395581361920228805... $\approx \prod_{p\ prime}\left(1 - \frac{1}{p(p-1)}\right)$, Artin’s constant

.37404368238615126287... $\approx -\zeta^{(3)}(3) = -\sum_{k=1}^\infty \frac{\log^3 k}{k^3}$

3 .37404736674013853601... $\approx 4G - \frac{\pi^2}{4} + \pi\log 2 = \int_0^{\pi/2} \frac{x^2}{1-\cos x}\,dx$ GR 3.791.5

.37411695125501623006... $\approx \frac{4-\pi+2\log 2}{6} = \int_0^1 x^2 \log\left(1 + \frac{1}{x^2}\right)dx$

$$= \int_1^\infty \frac{\log(1+x^2)}{x^4}\,dx$$

.37412529825749809439...$\approx \sum_{k=1}^\infty \frac{(-1)^{k+1}k}{2^k(2^k-1)}$

.37416629939256748277...$\approx 96 - 58\sqrt{e} = \sum_{k=0}^\infty \frac{1}{k!2^k(k+4)}$

.37424376965420048658... $\approx \psi^{(1)}(\pi) = \sum_{k=0}^\infty \frac{1}{(k+\pi)^2}$

1 .37427001649629550601... $\approx \sum_{k=0}^\infty \frac{1}{k^3+k^2+k+1} = \sum_{k=2}^\infty \frac{1}{k^2-k^{-2}} + \sum_{k=2}^\infty \frac{1}{k^3-k^{-1}} + \frac{1}{2}$

$$= \frac{i+1}{4}\big((1-i)\gamma + \psi(-i) - i\psi(i)\big)$$

$$= \frac{1}{2} + \frac{\pi}{4}\coth\pi + \frac{\gamma}{2} + \frac{1}{4}\big(\psi(i) + \psi(-i)\big)$$

.37430862285098974167...$\approx \sum_{k=1}^\infty \frac{\zeta(k+1)-1}{k^2+1}$

.37448702411112149465...$\approx \frac{\pi}{e^2+1}$

.37450901996253823880... $\approx \cos 1 \log 2$

$.37469344944141069360\ldots \approx \log\frac{16}{11} = \sum_{k=1}^{\infty}\frac{L_k}{4^k k}$

$.37475132845083865817\ldots \approx \sum_{k=2}^{\infty}\frac{1}{2^k\zeta(k)} = \sum_{k=1}^{\infty}\frac{\mu(k)}{2k-1} = \frac{1}{2}\sum_{k=1}^{\infty}\frac{\mu(k)}{k(2k-1)}$

$.37480222743935863178\ldots \approx e^{1/\pi} - 1 = \sum_{k=0}^{\infty}\frac{1}{k!\,\pi^k}$

1 $.37480222743935863178\ldots \approx e^{1/\pi}$

2 $.37482082344745189756\ldots \approx \frac{2\pi}{\sqrt{7}} = \sum_{k=1}^{\infty}\frac{1}{k^2-k+44} = 2\int_0^{\infty}\frac{x^2+2}{x^4+3x^2+4}dx$

$= \int_{-\infty}^{\infty}\frac{dx}{x^2+x+2} = \int_{-\infty}^{\infty}\frac{dx}{2x^2+x+1}$

$.37490340686486970379\ldots \approx \gamma - 1 + \frac{1}{2}\left(\psi\left(2+e^i\right)+\psi\left(2+e^{-i}\right)\right)$

$= \sum_{k=1}^{\infty}(-1)^{k+1}\cos(k)\left(\zeta(k+1)-1\right)$

1 $.37492595624331005433\ldots \approx 1 - J_0(2\sqrt{3}) = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{3^k}{(k!)^2}$

.375000000000000000000$\underline{0}$ = $\frac{3}{8} = \sum_{k=0}^{\infty} e^{-(2k+1)\log 3}$

= $\sum_{k=1}^{\infty} \frac{1}{k(2k+4)}$

= $\int_{1}^{\infty} \frac{\log^3 x dx}{x^3}$

1 .3750764222837193865... ≈ $\frac{\gamma^2}{4} + \frac{\pi^2}{24} + \frac{2\gamma\log 2}{2} + \log 2 = \int_{0}^{\infty} \frac{\log^2 x dx}{e^{4x}}$

.3751136755745319894... ≈ $\int_{0}^{1} \frac{dx}{2^{2^x}}$

.37517287391604427859... ≈ $\frac{1}{2}\left(Ei\left(e^{-i}\right) + Ei\left(e^{i}\right)\right) - \gamma$

= $\gamma - \frac{1}{2}\left(\Gamma\left(0,-e^{-i}\right) + \Gamma\left(0,-e^{i}\right)\right) = \sum_{k=1}^{\infty} \frac{\cos k}{k!k}$

.375984929565308899765... ≈ $\frac{4\pi^2}{105} = \prod_{p\ prime} \frac{1-p^{-2}}{1+p^{-2}+p^{-4}}$

.376059253341609274676... ≈ $\sum_{k=2}^{\infty} \frac{1}{k^k - 1} = \sum_{j=1}^{\infty}\sum_{k=2}^{\infty} \frac{1}{k^{jk}}$

.376148261725412056905... ≈ $\frac{1}{5\left(1+\sqrt{5}\right)}\left(\sqrt{5}\log\frac{5+\sqrt{5}}{2} + \log\frac{25+11\sqrt{5}}{2}\right) = \sum_{k=1}^{\infty} \frac{F_k F_{k+1}}{4^k k}$

.37626599344847702381... ≈ $Li_{-1/2}\left(\frac{1}{4}\right) = \sum_{k=1}^{\infty} \frac{\sqrt{k}}{4^k}$

2 .3763277109303385627... ≈ $2G + \frac{\pi\log 2}{4}$

1 .376381920471173538207... ≈ $\sqrt{\frac{5+\sqrt{20}}{5}} = \cot\frac{\pi}{5} = \tan\frac{3\pi}{10}$

.3764528129191954316... ≈ $2\log\left(1+\sqrt{2}\right) - 2\log 2 = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(2k-1)!!}{(2k)!\,k}$

= $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{4^k k}\binom{2k}{k}$

.376674047468581174134... ≈ $\frac{\pi}{2}\coth\pi - \frac{6}{5} = \sum_{k=1}^{\infty} \frac{1}{k^2+4k+5}$

.376700132275158021952... ≈ $9e + \frac{65}{e} - 48 = \sum_{k=0}^{\infty} \frac{1}{(2k+1)!(k+3)}$

.376774759859769486606... ≈ $1-\frac{1}{\sqrt{2}}\operatorname{arctanh}\frac{1}{\sqrt{2}} = 1-\frac{\sqrt{2}}{2}\operatorname{arcsinh}1 = \sum_{k=1}^{\infty}\frac{k}{2^k(2k+1)}$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}(2k)!!}{(2k+1)!!}$

1 .37692133029328224931... ≈ $-\frac{1}{2}\left(G+\log 2\pi\right) = \int_0^1 \psi(x)\sin^2 \pi x\,dx$ GR 6.468

.377294934867740533582... ≈ $\frac{\zeta(3)}{2}-\frac{\pi^2}{24}+\frac{3}{16} = \sum_{k=1}^{\infty}\frac{1}{k^3(k+2)}$

.377540668798145435361... ≈ $\frac{1}{\sqrt{e}+1} = \frac{1}{2}-\frac{1}{2}\tanh\frac{1}{4} = \sum_{k=1}^{\infty}(-1)^{k+1}e^{-k/2}$

= $\frac{1}{2}+\sum_{k=1}^{\infty}\frac{1}{(2k-1)!}B_{2k}\left(\frac{1}{2}\right)$ J134

1 .37767936423331146049... ≈ $\frac{2\pi^2}{3}-\zeta(3)-4 = \sum_{k=2}^{\infty}(-1)^k k^2\left(\zeta(k)-\zeta(k+2)\right)$

= $\sum_{k=2}^{\infty}\frac{4k^3-k^2-2k+1}{k^3(k+1)^2}$

.37788595863278193929... ≈ $\prod_{k=2}^{\infty}\left(1+\frac{\log k}{k^2}\right)$

.377964473009227227217... ≈ $\frac{\sqrt{7}}{7}$

.37802461354736377417... ≈ $\frac{e(\cos 1+\sin 1)-3}{2} = \int_1^e \log^3 x\cos\log x\,dx$

1 .37802461354736377417... ≈ $\frac{e(\cos 1+\sin 1)-1}{2} = \int_1^e \cos\log x\,dx$

.378139567567342472088... ≈ $2+4\log\frac{2}{3} = 4\sum_{k=1}^{\infty}\frac{k-1}{3^k k} = \sum_{k=0}^{\infty}\frac{(-1)^k}{2^k(k+2)}$

.378483910334091972067... ≈ $1+\frac{\pi}{\sqrt{13}}\tanh\frac{\pi\sqrt{13}}{2} = \sum_{k=1}^{\infty}\frac{1}{k^2+5k+3}$

.378530017124161309882... ≈ $2\left(\cos 1+\sin 1+si(1)\right)-\frac{\pi}{4} = \int_1^{\infty}\frac{\sin x}{x^3}dx$

.378550375764186642361... ≈ $1-\frac{\pi\cos 1}{2}-ci(1)\sin 1+si(1)\cos 1 = \int_0^{\infty}\frac{\cos x}{(1+x)^2}dx$

1 .378602612417434327112... ≈ $\frac{1}{5}\left(I_0\left(1-\sqrt{5}\right)+I_0\left(1+\sqrt{5}\right)-2J_0(2)\right) = \sum_{k=1}^{\infty}\frac{F_k F_k}{k!k!}$

.378605489832130816595... ≈ $\sum_{k=2}^{\infty}(-1)^{k+1}\mu(k)\left(\zeta(k)-1\right)$

.378696261703666022771... ≈ $\frac{1295\zeta'(2)\zeta''(2)}{\pi^6} - \frac{7776\left(\zeta'(2)\right)^3}{\pi^8} - \frac{36\zeta^{(3)}(2)}{\pi^4}$

= $-\sum_{k=1}^{\infty}\frac{\mu(k)\log^3 k}{k^2}$

1 .378952026600018143708... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^k}+\sum_{k=1}^{\infty}\frac{1}{2^{2^k}}+\sum_{k=1}^{\infty}\frac{1}{2^{2^{2^k}}}+\dots$

.379094331700329445471... ≈ $\frac{1}{2}\left(e^{-e}+e^{-1/e}\right) = \sum_{k=0}^{\infty}\frac{(-1)^k\cosh k}{k!}$

2 .37919532168081267241... ≈ $2\log 2-\frac{3}{2}+\sqrt{2}\log\frac{\sqrt{2}+1}{\sqrt{2}-1} = \sum_{k=1}^{\infty}\frac{H_{2k+2}}{2^k}$

.379347816325727458823... ≈ $\frac{3}{2}-\frac{\pi}{\sqrt{3}}+\log 2 = \sum_{k=0}^{\infty}\frac{(-1)^k}{k+5/3} = 3\int_1^{\infty}\frac{dx}{x^6+x^3}$

.379349218647907817919... ≈ $\frac{3\zeta(3)}{2}+\log^3 2-6\log^2 2-\frac{\pi^2}{4}\log 2+\frac{\pi^2}{2}+3\pi-24$

= $-\int_0^1 \arcsin x\log^3 x\,dx$

.379600169272667639186... ≈ $\frac{\pi^2}{26}$

.379646336926468163793... ≈ $\sum_{k=1}^{\infty}\frac{1}{k(2k-1)\log k}$.

.379651995080949652998... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k\zeta(k)}{(k!)^2} = \sum_{k=1}^{\infty}\left(J_0\left(2\sqrt{\frac{1}{k}}\right)-1+\frac{1}{k}\right)$

.379879650333370492326... ≈ $\frac{3\pi}{32}+\frac{\sqrt{2}}{40}+\frac{1}{20} = \int_0^{\pi/4}\frac{\cos^6 x}{1+\sin x}dx$

.37988549304172247537... ≈ $\log\frac{2e}{e+1} = \sum_{j=1}^{\infty}\frac{(-1)^k(1-2^k)\zeta(1-k)}{k!}$

= $\int_0^{\infty}\frac{dx}{e^x+1}$

.37989752371128314827... ≈ $\frac{\pi^3}{32}-\frac{3\pi}{16} = \int_0^1 x\arcsin^3 x\,dx$

.380770870193601660684... ≈ $\sum_{k=2}^{\infty}(-1)^k\left(\zeta(k)-1\right)^2$

1 .380770870193601660684... ≈ $\sum_{k=2}^{\infty}(-1)^k\left(\zeta^2(k)-1\right) = \sum_{k=2}^{\infty}\frac{\sigma_0(k)}{k(k+1)}$

.380797077977882444406... ≈ $\frac{e^2-1}{2(e^2+1)} = \sum_{k=1}^{\infty}\frac{2^{2k-1}(2^{2k}-1)B_{2k}}{(2k)!}$

9 .3808315196468591091... ≈ $\sqrt{88} = 2\sqrt{22}$

.38102928179240953333... ≈ $1+\gamma+\frac{\pi}{\sqrt{5}}\tan\frac{\pi\sqrt{5}}{2}+\frac{5+\sqrt{5}}{10}\psi\left(\frac{3-\sqrt{5}}{2}\right)+\frac{5-\sqrt{5}}{10}\psi\left(\frac{3+\sqrt{5}}{2}\right)$

= $\sum_{k=2}^{\infty}(-1)^k F_k\left(\zeta(k)-1\right) = \sum_{k=2}^{\infty}\frac{k-1}{k(k^2+k-1)}$

1 .3810359531144620680... ≈ $\frac{1}{16}\left(2\pi^2\log 2+7\zeta(3)\right) = -\int_0^{\pi/2} x\log\cos x\,dx$

.381294821444817558571... ≈ $-\sum_{k=2}^{\infty}\frac{\nu(k)\mu(k)}{2^k}$

.38171255654242344132... ≈ $\int_0^{\infty}\frac{x\sin x}{1+e^x}dx$

1 .381713265354352115152... ≈ $\sum_{k=1}^{\infty}\frac{1}{(k!!)^2}$

1 .381773290676036224053... ≈ $\cos 1+\sin 1=\sum_{k=1}^{\infty}(-1)^k\frac{(2k)^2}{(2k)!} = \sum_{k=0}^{\infty}\frac{(-1)^{\lfloor k/2\rfloor}}{k!}$

.38189099837355872866... ≈ $-\log\Gamma\left(1+\frac{i}{2}\right)\Gamma\left(1-\frac{i}{2}\right) = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)}{4^k k}$

= $\sum_{k=1}^{\infty}\log\left(1+\frac{1}{4k^2}\right)$

.381966011250105151795... ≈ $2-\varphi = \frac{3-\sqrt{5}}{2} = \sum_{k=1}^{\infty}\frac{(-1)^k(2k)!}{(k!)^2(k+1)}$

2 .381966011250105151795... ≈ $\frac{7-\sqrt{5}}{2} = 4-\varphi = 3-\frac{1}{\varphi}=\sum_{k=0}^{\infty}\frac{1}{F_{2^k}}$ GKP p. 302

.382232359785690548677... ≈ $\frac{5}{16}\left(1+\log\frac{5}{4}\right) = \sum_{k=1}^{\infty}\frac{kH_k}{5^k}$

.382249890174222104596... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k}{2^k-2} = \sum_{k=2}^{\infty}\frac{1}{2^k+2}$

2 .382380444637740041929... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(3k-1)}{k!} = \sum_{k=1}^{\infty} k\left(e^{1/k^3}-1\right)$

.382425345822619425691... ≈ $\frac{1}{2}-\frac{\pi}{2\sqrt{5}}\cot\frac{\pi}{\sqrt{5}} = \sum_{k=1}^{\infty}\frac{\zeta(2k)}{5^k} = \sum_{k=1}^{\infty}\frac{1}{5k^2-1}$

.38259785823210634567... ≈ $\frac{\sqrt{2}+\log(1+\sqrt{2})}{6} = \int_1^{\infty}\frac{\operatorname{arcsinh} x}{x^4}dx$

.382626596031170342250... ≈ $\frac{\zeta(3)}{\pi}$

.38268343236508977172... ≈ $\frac{\sqrt{2-\sqrt{2}}}{4} = \sin\frac{\pi}{8} = \cos\frac{3\pi}{8}$

.382843018043786287416... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\mu(k)}{4^k-1} = -\frac{1}{4} + 2\sum_{k=1}^{\infty}\frac{1}{2^{2^k}}$

.38317658318419503472... ≈ $\frac{\log 2}{2}+\frac{1}{8}\left(-\psi\left(\frac{1-2i}{2}\right)-\psi\left(\frac{1-2i}{2}\right)+\psi(i)+\psi(-i)\right)$

= $\int_0^{\infty}\frac{\cos^2 x}{e^x+1}dx$

.383180102968505756325... ≈ $\log\frac{\pi}{\pi-1} = \sum_{k=1}^{\infty}\frac{1}{k\pi^k}$

.383261210898144452739... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta^2(2k+1)-1\right)$

.383473719274746544956... ≈ $\sum_{n=1}^{\infty}\frac{1}{n!2^n}\sum_{k=2}^{\infty}\frac{\log^n k}{k!}$

.38349878283711983503... ≈ $8G+2\pi+4\log 2-16 = -\int_0^1\frac{\log(1+x)\log x}{\sqrt{x}}$

.38350690717842253363... ≈ $\sum_{k=1}^{\infty}\zeta(2k)\left(\zeta(2k+1)-1\right)$

.383576096602374569919... ≈ $\frac{4}{3}\log\frac{4}{3} = \sum_{k=1}^{\infty}\frac{H_k}{4^k}$

.383838383838383838$\underline{38}$ = $\frac{7}{18}$

.383917497444851630134... ≈ $\frac{61}{112}-\frac{\pi}{4\sqrt{2}}\cot 2\pi\sqrt{2} = \sum_{k=1}^{\infty}\frac{1}{k^2+6k+1}$

.383946831127345788643... ≈ $\frac{2}{\zeta(2)}-\frac{1}{\zeta(3)} = -\sum_{k=1}^{\infty}\frac{\mu(k)}{k}\left(1-\frac{1}{k}\right)^2$

.38396378122983515859... ≈ $\sum_{k=2}^{\infty}\frac{H_k}{k^3-1}$

1 .384161678... ≈ $\int\limits_2^\infty \left(\zeta^2(x)-1\right)dx$

2 .38423102903137172415... ≈ $\prod_{k=1}^\infty \left(1+\frac{1}{2^k}\right) = \sum_{k=1}^\infty \frac{2^{2k-1}}{2^{2k-1}-1} = 1+\sum_{k=1}^\infty \frac{Q(k)}{2^k}$

.384255520734072782182... ≈ $\prod_{k=1}^\infty \left(1-\frac{1}{k^2+2}\right)$

.384373946213432915603... ≈ $\frac{\gamma+\log 2\pi}{2\pi} = \int\limits_0^1 (\sin 2\pi\, x)\log\Gamma(x)\,dx$ GR 6.443.1

.3843772320348969041... ≈ $\frac{1}{8}\left(\psi\left(\frac{i}{\sqrt{2}}\right)+\psi\left(\frac{-i}{\sqrt{2}}\right)-\psi\left(\frac{1}{\sqrt{2}}\right)-\psi\left(\frac{-1}{\sqrt{2}}\right)\right)$

= $\sum_{k=1}^\infty \frac{\zeta(4k-1)}{4^k} = \sum_{k=1}^\infty \frac{k}{4k^4-1}$

.384418702702027416264... ≈ $\frac{21}{4}+12\log\frac{2}{3} = \sum_{k=0}^\infty \frac{1}{3^k(k+1)(k+3)}$

1 .384458393024340358836... ≈ $\frac{\pi}{2}\log(1+\sqrt{2}) = \int\limits_0^1 \frac{\arctan x}{x\sqrt{1-x^2}}dx$ GR 4.531.12

.384488876758268690206... ≈ $\frac{1}{3}-\frac{2G}{3}-\frac{\pi}{6}+\frac{\pi^2}{12}+\frac{\pi}{6}\log 2 = \int\limits_0^{\pi/4} \frac{x^2}{\cos^4 x}dx$

1 .384569170237742574585... ≈ $\frac{\pi^2}{8}+\frac{\log^2 3}{8} = \int\limits_0^\infty \frac{\log x}{(x-3)(x+1)}dx$ GR 4.232.3

.384586577443430977920... ≈ $\sin\frac{1}{2}\cos\frac{1}{2}\left(2-\log(2-2\cos 1)\right)+(1-\pi)\sin^2\frac{1}{2}$

= $\sum_{k=1}^\infty \frac{\sin(k+1)}{k(k+1)}$ GR 1.444.1

$.384615384615\underline{384615}$ = $\frac{5}{13}$

.384654440535487702221... ≈ $\zeta(3)+\log 2\log^2 3-\frac{\log^2 2}{2}\log 3-\frac{\log^3 3}{2}+Li_2\left(\frac{2}{3}\right)\log\frac{2}{3}$

$+Li_3\left(\frac{1}{3}\right)-Li_3\left(\frac{2}{3}\right)$

= $\sum_{k=1}^\infty \frac{H_k}{3^k k^2} = \sum_{k=1}^\infty \frac{(-1)^{k+1}H^{(2)}{}_k}{2^k k}$

7 .38483656489729528833... ≈ $-\psi^{(2)}\left(\frac{2}{3}\right)$

.38494647276779467738... ≈ $\frac{\pi\log 2}{4\sqrt{2}} = \int\limits_0^\infty \frac{\log x}{x^2+2}$

$= \quad -\int_0^{\pi/2} \frac{\arcsin\left((\sin x)/\sqrt{2}\right)\sin x}{\sqrt{4-2\sin^2 x}}\,dx$ Borwein-Devlin, p. 59

.3849484951257119020... $\approx \quad e-\frac{7}{3} = \sum_{k=1}^{\infty}\frac{1}{k!(k+3)!} = \sum_{k=1}^{\infty}\frac{1}{(k+1)!+2k!}$

.384963873744095819376... $\approx \quad (-1)^{2/3}\psi\left(\frac{1}{4}\left(4+2^{1/3}-i2^{1/3}\sqrt{3}\right)\right)-(-1)^{1/3}\psi\left(\frac{1}{4}\left(4+2^{1/3}+i2^{1/3}\sqrt{3}\right)\right)$

$\qquad -\frac{1}{3\cdot 2^{2/3}}\psi\left(1-\frac{1}{2^{2/3}}\right)$

$= \quad \sum_{k=1}^{\infty}\frac{\zeta(3k)}{4^k} = \sum_{k=1}^{\infty}\frac{1}{4k^3-1} = \sum_{k=2}^{\infty}\frac{1+(-1)^k}{k^3-2}$

.38499331087225178096... $\approx \quad \frac{1}{\sqrt{2}}J_1\left(\sqrt{2}\right) = \sum_{k=0}^{\infty}(-1)^{k+1}\frac{1}{2^k k!(k-1)}$

.385061651534227425527... $\approx \quad \frac{\pi}{\sqrt{3}}-\frac{2}{\sqrt{3}}\arctan\frac{5}{\sqrt{2}} = \int_2^{\infty}\frac{1}{x^2+x+1}$

2 .385098206176909244267... $\approx \quad \frac{\pi^3}{13}$

3 .385137501286537721688... $\approx \quad \frac{6}{\sqrt{\pi}}$

.385151911060831655048... $\approx \quad \frac{\gamma}{3}+\frac{1}{6}\left(\psi\left(1+i\sqrt{3}\right)+\psi\left(1-i\sqrt{3}\right)\right) = \sum_{k=1}^{\infty}\frac{1}{k^3+3k}$

5 .38516480713450403125... $\approx \quad \sqrt{29}$

.38533394536693817834... $\approx \quad \log\left(2\sqrt{\cos 1}\right) = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\cos^2 k}{k}$

.38542564304175153356... $\approx \quad \frac{i}{2}\left(\psi\left(2+e^{i}\right)-\psi\left(2+e^{-i}\right)\right) = \sum_{k=1}^{\infty}(-1)^{k+1}\sin(k)\left(\zeta(k+1)-1\right)$

.38594825471984105804... $\approx \quad \gamma^{1/\gamma}$

.385968416452652362535... $\approx \quad \operatorname{arccoth} e = -\frac{1}{2}\log\tanh\frac{1}{2} = \sum_{k=0}^{\infty}\frac{1}{e^{2k+1}(2k+1)}$ J945

$= \quad \int_1^{\infty}\frac{dx}{e^x-e^{-x}}$

1 .386075195716611750758... $\approx \quad \sum_{k=1}^{\infty}\frac{(-1)^{k+1}\zeta(4k-2)}{(k-1)!(2k-1)} = \frac{\sqrt{\pi}}{2}\sum_{k=1}^{\infty}erf\left(\frac{1}{k^2}\right)$

1 .38622692545275801365... $\approx \quad \frac{1+\sqrt{\pi}}{2} = \int_0^{\infty}e^{-x^2}(1+x)\,dx$

.386294361119890618835... $\approx \quad 2\log 2-1 = \sum_{k=1}^{\infty}\frac{1}{2^k(k+1)} = \sum_{k=0}^{\infty}\frac{1}{2^k(2k+4)}$

$$= \sum_{k=1}^{\infty} \frac{1}{(4k^2-1)k}$$

J374, K Ex. 110d

$$= \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k(k+1)}$$

J235, J616

$$= \sum_{k=1}^{\infty} \frac{L_k}{2^k k}$$

$$= \sum_{k=1}^{\infty} \frac{\zeta(k+1)-1}{2^k} = \sum_{k=1}^{\infty} \frac{\zeta(2k+1)}{4^k} = \sum_{k=2}^{\infty} (-1)^k \frac{\Omega(k)}{k}$$

$$= \int_1^{\infty} \log\left(1+\frac{1}{x}\right)\frac{dx}{x^2}$$

$$= \int_0^1 \left(E(k')-1\right)\frac{dk}{k'}$$

GR 6.150.2

$$1 \quad .386294361119890618835\ldots \approx \quad 2\log 2 = \log 4 = Li_1\left(\frac{3}{4}\right) = \sum_{k=1}^{\infty} \frac{3^k}{4^k k}$$

$$= \sum_{k=1}^{\infty} \frac{1}{2k^2-k} = \sum_{k=0}^{\infty} \frac{1}{2^k(k+1)} = \sum_{k=1}^{\infty} \frac{H_k}{2^k}$$

$$= \sum_{k=1}^{\infty} \frac{\zeta(k+1)}{2^k} = \sum_{k=2}^{\infty} \left(Li_k(1)+Li_k(-1)\right)$$

$$= \sum_{k=1}^{\infty} \frac{12k^2-1}{k(4k^2-1)^2}$$

J398

$$= 1+2\sum_{k=1}^{\infty} \frac{1}{8k^3-2k}$$

[Ramanujan] Berndt Ch. 2, 0.1

$$= \sum_{k=0}^{\infty} \frac{1}{(2k+1)\left((2k+1)^2-1/4\right)}$$

Prud. 5.1.26.10

$$= \sum_{k=1}^{\infty} \frac{(2k-1)!!}{(2k)!\ \ k} = \sum_{k=1}^{\infty} \binom{2k}{k} \frac{1}{4^k k}$$

$$= \int_0^1 \frac{x^{n-1}+x^{n-1/2}-2x^{2n-1}}{1-x}\,dx$$

GR 3.272.1

$$= \int_1^{\infty} \frac{\log(1+x)}{x^2}\,dx$$

$$= \int_0^1 \log\left(1+\frac{1}{x}\right)dx$$

$$= \int_0^{\infty} \log\left(1+\frac{1}{x(x+2)}\right)dx$$

$$3\ .386294361119890618835\ldots \approx 2+2\log 2 = \sum_{k=1}^{\infty}\frac{kH_k}{2^k}$$

$$28\ .386294361119890618835\ldots \approx 27+2\log 2 = \sum_{k=1}^{\infty}\frac{k^4}{2^k(k+1)}$$

$$953\ .386294361119890618835\ldots \approx 925+2\log 2 = \sum_{k=1}^{\infty}\frac{k^6}{2^k(k+1)}$$

$$.3863044024175697139\ldots \approx \sum_{k=1}^{\infty}\frac{k}{4^k+1}$$

$$.3863186024133260765 2\ldots \approx \sum_{k=1}^{\infty}\frac{1}{e^{k^2}}$$

$$.38651064687313937401\ldots \approx \frac{\pi}{4}\coth\pi - \frac{\pi^2}{4}\operatorname{csch}^2\pi + \frac{\pi^3}{2}\coth\pi\operatorname{csch}^2\pi - \frac{1}{2}$$

$$= \sum_{k=1}^{\infty}(-1)^{k+1}k^2\big(\zeta(2k)-\zeta(2k+2)\big)$$

$$.38656265602765893614 5\ldots \approx \frac{1}{4}+\frac{\pi}{4\sqrt{5}}\tan\frac{\pi\sqrt{5}}{2} = \sum_{k=1}^{\infty}\frac{1}{4k^2+4k+4}$$

$$.386852807234541586870\ldots \approx \log_6 2$$

$$.386995424210199750135\ldots \approx Li_3\left(\frac{1}{e}\right) = \sum_{k=1}^{\infty}\frac{1}{3^k k^3}$$

$$.38699560053943557616\ldots \approx \frac{\pi G}{2}-\frac{7}{8}\zeta(3) = -2\int_0^{\pi/4}x\log\tan x\ dx = \int_0^1\frac{\arctan^2 x}{x}dx$$

$$.38716135743706271230\ldots \approx \frac{I_0(1)}{4}+\frac{I_1(1)}{8} = \sum_{k=1}^{\infty}\frac{k^3}{(k!)^2 4^k}$$

$$3\ .3877355319520023164\ldots \approx \sum_{k=2}^{\infty}\frac{1}{(k-1)\log^2 k}$$

$$.38779290180460559878\ldots \approx \frac{\sqrt{5}}{4}\log\frac{5+\sqrt{5}}{5-\sqrt{5}}+\frac{5\log 5}{4}-\frac{\pi}{2}\sqrt{1+\frac{2}{\sqrt{5}}} = \sum_{k=1}^{\infty}\frac{1}{5k^2-k}$$

$$= \sum_{k=1}^{\infty}\frac{1}{5k^2-k} = \sum_{k=1}^{\infty}\frac{\zeta(k+1)}{5^k}$$

$$11\ .38796388003128288749\ldots \approx \frac{2\pi^2}{3}+4\zeta(3) = \int_0^{\infty}x^{-2}Li_2(-x)^2\,dx$$

$$.38805555247257460868\ldots \approx \prod_{k=1}^{\infty}\left(1-\frac{k!k!}{(2k)!}\right)$$

$$.38840969994784799075\ldots \approx \frac{1}{2}\log^2\left(1+\sqrt{2}\right) = \frac{\operatorname{arcsinh}^2 1}{2}$$

= $\sum_{k=0}^{\infty}(-1)^k \frac{(2k)!!}{(2k+1)!!}\frac{1}{2k+2}$ J143

= $\int_0^1 \frac{\arcsin x}{1+x^2}dx$

= $\int_0^1 \frac{\operatorname{arcsinh} x}{\sqrt{1+x^2}}dx$

.388730126323020031392... ≈ $\frac{\sqrt{34}}{15}$ CFG D1

7 .389056098930650227230... ≈ $e^2+1 = \sum_{k=0}^{\infty}\frac{2^k}{k!} = \frac{1}{3}\sum_{k=0}^{\infty}\frac{2^k(k+1)}{k!}$ LY 6.40

961 .389193575304437030219... ≈ π^6

.3896467926915307996... ≈ $12\log 2-2\pi-\frac{\pi^2}{6}=\sum_{k=1}^{\infty}\frac{\zeta(k+2)}{4^k} = \sum_{k=1}^{\infty}\frac{1}{4k^3-k^2}$

.39027434400620220856... ≈ $\prod_{k=1}^{\infty}\left(1-\frac{1}{k(3k-1)}\right)$

.39053390439339657395... ≈ $erfi\frac{1}{3} = \frac{2}{\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{1}{k!3^{2k+1}(2k+1)}$

2 .39064... ≈ $\sum_{k=2}^{\infty}\frac{1}{\phi^2(k)}$

.3906934607512251312... ≈ $\sum_{k=0}^{\infty}\frac{S_2(2k,k)}{S_1(2k,k)}$

2 .39074611467649675415... ≈ $8\log 2 - 2\gamma - 2 = \sum_{k=0}^{\infty}\frac{\psi(k+3)}{2^k}$

1 .39088575503593145117... ≈ $\log\left(-\Gamma\left(-\frac{2}{3}\right)\right)$

.3910062461378671068... ≈ $6 - \pi - \frac{\pi^2}{4} = \int_0^1 \arcsin x \arccos^2 x\, dx$

.391031136773827639675... ≈ $\frac{e(e+1)(e^{2i}-1)^2}{4(e-1)(e^{2i}-e)(e^{2i+1}-1)} = \sum_{k=1}^{\infty}\frac{\sin^2 k}{e^k}$

.391235129453969820659... ≈ $\frac{\pi}{8}\tanh\pi = \sum_{k=1}^{\infty}\frac{1}{4k^2-4k+5}$ GR 1.422.2, J949

2 .39137103861534867486... ≈ $\frac{\pi^2\log 2}{3}+\frac{\log^3 2}{3} = \int_0^{\infty}\frac{\log^2 x}{(x+1)(x+2)}dx$

.391490159033032044663... ≈ $Li_e\left(\frac{1}{e}\right) = \sum_{k=1}^{\infty}\frac{1}{e^k k^e}$

.39173373121460048563... ≈ $\frac{\sinh 1}{3} = \int_1^\infty \cosh\left(\frac{1}{x^3}\right)\frac{dx}{x^4}$

.39177... ≈ $\sum_{s=2}^\infty \prod_{k=2}^\infty \frac{1}{1-k^{-s}}$

.39180100022767093557 4... ≈ $\sum_{k=2}^\infty \frac{\left|\mu(k)\right|}{k(k+1)} = \sum_{k=2}^\infty (-1)^k\left(\frac{\zeta(k)}{\zeta(2k)}-1\right)$

.391892330690243774136... ≈ $2+\frac{\pi^2}{6}-4\log 2-\log^2 2 = \sum_{k=1}^\infty \frac{k^2}{2^k(k+1)^2}$

.39190124777049688700... ≈ $\frac{\pi^2}{4}\operatorname{sech}^2\frac{\pi}{2} = \int_0^\infty \frac{x\cos x}{\sinh x}dx = \left(\int_0^\infty \frac{\cos x}{\cosh x}dx\right)^2$

.39207289814597337134... ≈ $1-\frac{6}{\pi^2} = \frac{\zeta(2)-1}{\zeta(2)}$

1 .39208199920792696132 1... ≈ $\frac{\pi^{3/2}}{4}$

1 .39209603026908338957 5...≈ $-\frac{1}{2}\left(H\left(\frac{1+\sqrt{5}}{2}\right)+H\left(\frac{1-\sqrt{5}}{2}\right)\right) = \sum_{k=2}^\infty F_{2k-1}\left(\zeta(2k+1)-1\right)$

5 .392103950584448229216...≈ $\gamma^3+\gamma^{-3}$

.39225871325093557127... ≈ $\frac{e\gamma}{4} = -\int_0^\infty e^{1-x^2}x\log x\,dx$

.3925986596400406773... = $2-\gamma-erfi(1)\sqrt{\pi}+Ei(1) = \sum_{k=1}^\infty \frac{1}{k!k(2k+1)}$

218 .39260013257695631244... ≈ $4e^4 = \sum_{k=1}^\infty \frac{4^k k}{k!}$

.39269908169872415480 8... ≈ $\frac{\pi}{8} = \arctan\left(\sqrt{2}-1\right) = \sum_{k=0}^\infty \frac{(-1)^k}{4k+2}$

= $\sum_{k=0}^\infty \frac{1}{(4k+1)(4k+3)}$

= $\sum_{k=1}^\infty \frac{(-1)^{k+1}}{(2k-1)^2}\sin\frac{2k-1}{2}$ J523

= $\sum_{k=1}^\infty \frac{(4^k-1)\zeta(2k)}{16^k} = \sum_{k=1}^\infty \frac{1}{4k^2-1} - \sum_{k=1}^\infty \frac{1}{16k^2-1}$

= $\sum_{k=0}^\infty \frac{1}{4^k(2k+1)(2k+3)}\binom{2k}{k}$

$$= \sum_{k=1}^{\infty} \frac{1}{2^{\lfloor (k+1)/2 \rfloor} k}\left(\left\lfloor \frac{k+7}{8} \right\rfloor - \left\lfloor \frac{k+6}{8} \right\rfloor + \left\lfloor \frac{k+1}{8} \right\rfloor - \left\lfloor \frac{k+4}{8} \right\rfloor\right)$$ Plouffe

$$= \sum_{k=1}^{\infty} \arctan\left(\frac{1}{(1+k\sqrt{2})^2}\right)$$ [Ramanujan] Berndt Ch. 2

$$= \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} \frac{1}{(4s-2)^{2r}}$$ J1122

$$= \int_0^{\infty} \frac{dx}{x^2+16}$$

$$= \int_0^{\infty} \frac{dx}{x^4+4} = \int_0^1 \frac{xdx}{x^4+4} = \int_1^{\infty} \frac{dx}{x^3+x^{-1}} = \int_0^{\infty} \frac{x\,dx}{(x^4+1)^2}$$

$$= \int_0^{\infty} \frac{dx}{e^{2x}+e^{-2x}}$$

$$= \int_0^{\infty} \frac{\sin^2 x \sin 2x}{x} dx$$

$$= \int_0^{\infty} \frac{\sin x^2 + x^2 \cos x^2}{x^5} dx$$

$$= \int_0^1 x \arcsin x\, dx$$ GR 4.523.2

$$= -\int_0^{\infty} x e^{-x} \log x \cos x\, dx$$

4 .39283843336600648478... ≈ $\frac{\pi^8}{2160} = 4\zeta(4)L(4) = \sum_{k=1}^{\infty} \frac{r(k)}{k^4}$

2 .39292255287307281102... ≈ $8G - \frac{\pi^2}{2} = \int_{-1}^{1} \frac{\arcsin^2 x}{x^2}$

.393096190540936400082... ≈ $-2J_0(2\sqrt{2}) = \sum_{k=1}^{\infty} \frac{(-1)^2 2^k k^2}{(k!)^2}$

.393327464565010863238... ≈ $\frac{7\zeta(3)}{4} - \frac{\pi^2 \log 2}{4} = \sum_{k=1}^{\infty} \frac{H_k}{(2k+1)^2}$

1 .39340203780290253025... ≈ $\frac{32\sqrt{15}}{11\Gamma(4-\sqrt{15})\Gamma(\sqrt{15}-3)} = \prod_{k=1}^{\infty}\left(1+\frac{1}{k(k+8)}\right)$

.393469340287366576396... ≈ $\frac{\sqrt{e}-1}{\sqrt{e}} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!2^k}$

2 .39365368240859606764... ≈ $\frac{6}{\sqrt{2\pi}}$

.39365609958233181993... ≈ $\sum_{k=3}^{\infty}\frac{1}{k!-k}$

1 .39365735212511301102... ≈ $\sum_{k=2}^{\infty}\frac{2^k\left(\zeta(2k)-1\right)}{k^2} = \sum_{k=2}^{\infty} Li_2\left(\frac{2}{j^2}\right)$

1 .393825208413407109555... ≈ $\prod_{k=1}^{\infty}\left(1-\frac{(-1)^k}{k\left(\frac{2k}{k}\right)}\right)$

.393829290521217143801... ≈ $4-3\zeta(3)$

18 .393972058572116079776... ≈ $\frac{50}{e} = \sum_{k=0}^{\infty}\frac{(-1)^k 8^k}{k!}$

.39416851186714529353... ≈ $\frac{\pi}{8}\coth\pi$

4 .39444915467243876558... ≈ $4\log 3$

.39463025213978264327... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}H^{(3)}{}_k}{2^k k}$

.39472988584940017414... ≈ $\log\pi-\frac{3}{4} = \sum_{k=1}^{\infty}\frac{k}{k+1}\left(\zeta(2k)-1\right)$

.394784176043574344753... ≈ $\frac{\pi^2}{25}$

2 .394833099273404716523... ≈ $\frac{1}{\sqrt{2}}I_1\left(2\sqrt{2}\right) = \sum_{k=0}^{\infty}\frac{2^k}{k!(k+1)!}$

.394934066848226436472... ≈ $\frac{\pi^2}{6}-\frac{5}{4} = \psi^{(1)}\left(3\right) = \zeta(2,3) = \sum_{k=3}^{\infty}\frac{1}{k^2}$

= $\frac{1}{2}\left(Li_2(e)+Li_2\left(\frac{1}{e}\right)+i\pi\right)-1$

= $\sum_{k=2}^{\infty}(-1)^k(k-1)(\zeta(k)-1)$

= $\sum_{j=2}^{\infty}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{j^{k+1}}$ Berndt 5.8.4

= $\int_1^{\infty}\frac{\log x}{x^4-x^3}dx$

= $-\int_0^1\frac{x^2\log x}{1-x}dx$

1 .39510551694656703221... ≈ $\prod_{k=3}^{\infty}\zeta(k)$

.395338567367445566052... ≈ $\prod_{k=2}^{\infty}\left(1-\frac{1}{k!}\right)$

.39535201510645914871... ≈ $\frac{\pi(1+e)}{e(1+\pi^2)} = \int_0^{\pi} e^{-x/\pi}\cos x\,dx$

1 .39544221511549034512... ≈ $\psi^{(1)}\left(\frac{5}{8}\right) - \psi^{(1)}\left(\frac{7}{8}\right) = \sum_{k=1}^{\infty}\frac{(3^k-1)(k+1)}{8^k}\zeta(k+2)$

.395599547802009644147... ≈ $120-44e = \sum_{k=1}^{\infty}\frac{k}{k!(k+5)}$

1 .395612425086089528628... ≈ $e^{1/3} = \sum_{k=0}^{\infty}\frac{1}{k!3^k}$

1 .395872562271002... ≈ $\sum_{n=2}^{\infty}\prod_{k=2}^{\infty}\frac{1}{1-k^{-n}}$

.395896037098993835211... ≈ $\frac{3e^2}{8}-\frac{19}{8} = \sum_{k=1}^{\infty}\frac{2^k k^2}{(k+3)!}$

1 .3959158283010590... ≈ $\prod_{k=2}^{\infty}\left(1-1/(2k^2)\right)^{-1}$

1 .396208963809216061296... ≈ $\frac{51664}{11025}-\frac{\pi^2}{3} = H^{(2)}{}_{7/2}$

.39660156742592607639... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\left(\zeta(k+1)-1\right)^2}{k!}$

1 .39689069308728222107... ≈ $\sum_{k=1}^{\infty}\frac{1}{k^2}\log\frac{k+2}{k}$

.39691199727321671968... ≈ $\sqrt{2}\sin\sqrt{2} - 1 = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{2^k k}{(2k)!(k+1)}$

.39710172090497736873... ≈ $\frac{1}{\sqrt{2}}\csc\frac{\pi}{\sqrt{3}}\sin\pi\sqrt{\frac{2}{3}} = \prod_{k=1}^{\infty}\left(1-\frac{1}{3k^2-1}\right)$ J1064

.397116771379659432792... ≈ $\sum_{k=1}^{\infty}\frac{k}{(k^2+1)^2}$ Prud. 5.1.25.29

.397117694981577169693... ≈ $erf\left(\frac{1}{e}\right)$

1 .397149809863847372287... ≈ $1-J_0(4) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}4^k}{(k!)^2}$

.39728466206792444388... ≈ $\frac{3-\sqrt{3}}{4\sqrt{2}}\sqrt{\pi} = \int_0^{\infty}\frac{\sin^3\left(x^2\right)}{x^2}dx$

.397532220692825881258... ≈ $\frac{2\cos 1}{e} = \int_1^{\infty}\frac{x^2\sin x}{e^x}dx$

.397715726853315103139... ≈ $\frac{1}{6}+\frac{\log 2}{3} = \sum_{k=0}^{\infty}\frac{1}{(2k+1)(2k+4)}$

3 .397852285573806544200... ≈ $\frac{5e}{4} = \sum_{k=1}^{\infty}\frac{k^2}{(2k-2)!}$

.3980506524443333859... ≈ $\sum_{k=1}^{\infty}\frac{|\mu(k)|(-1)^{k+1}}{2^k}$

7116 .39826205293050534643... ≈ $\frac{3\pi^8}{4}$

.3983926103133159445... ≈ $\int_0^1 \arctan(\arctan x)\,dx$

.3989068320596832521 4... ≈ $\frac{\pi}{2\sqrt{6}}\coth \pi\sqrt{\frac{2}{3}} - \frac{1}{4} = \sum_{k=1}^{\infty}\frac{1}{3k^2+2}$

.3989422804014326779 4... ≈ $\frac{1}{\sqrt{2\pi}}$

.398971548420202857300... ≈ $1-\frac{\zeta(3)}{2}$

.3990209885941838469... ≈ $\frac{\pi}{2}\cos 2 - \cos 2\; si(2) + \sin(2)\; ci(2) = \int_0^{\infty}\frac{\sin^2 x}{(x+1)^2}$

.39920527550843900193... ≈ $\frac{1}{4}+\frac{\pi}{\sqrt{17}}\tan\frac{\pi\sqrt{17}}{2} = \sum_{k=1}^{\infty}\frac{1}{k^2+5k+2}$

.39931600618316563920... ≈ $\frac{e^2-1}{16} = \sum_{k=0}^{\infty}\frac{2^k}{k!(k+2)(k+4)}$

.39957205218788780748... ≈ $2\,\mathrm{arccsch}^2 2 - \frac{1}{\sqrt{5}}\left(2\log 5(\mathrm{arccsch}\ 2)+Li_2\left(\frac{-3+\sqrt{5}}{2}\right)-Li_2\left(\frac{-3-\sqrt{5}}{2}\right)\right)$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{H_k}{\binom{2k}{k}k}$

.40000000000000000$\underline{0}$ = $\frac{2}{5} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{F_k}{2^k} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{kF_k}{2^k}$

= $\frac{1}{2\cosh\log 2} = \sum_{k=0}^{\infty}(-1)^k e^{(-\log 2)(2k+1)}$ J943

= $\prod_{p\ prime}\frac{1-p^{-2}}{1+p^{-2}}$

= $\int_0^{\infty}\frac{\sin 2x}{e^x}dx = \int_0^{\infty}\frac{\sin^2 x}{e^x}dx = \int_0^{\infty}\frac{\cos x}{e^{2x}}dx$

2 .4000000000000000000$\underline{0}$ = $\frac{12}{5} = \sum_{k=1}^{\infty}\frac{F_{2k}k}{4^k}$

1 .40000426223653766564... ≈ $\sum_{k=1}^{\infty}\left(\zeta(k+1)-1\right)H_{2k-1}$

.40009541070153168409... ≈ $\gamma\log 2$

.400130076223970451846... ≈ e^{-G}

.40037967700464134050... ≈ $e+\gamma-1-Ei(1) = \sum_{k=1}^{\infty}\frac{k}{k!(k+1)^2} = \sum_{k=0}^{\infty}\frac{1}{k!(k+2)^2}$

= $-\int_0^1 e^x x\log x\,dx$

.400450020151755157328... ≈ $\sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)}{2k-1} = \sum_{k=1}^{\infty}\left(\frac{1}{k}-\frac{1}{\sqrt{k}}\arctan\frac{1}{\sqrt{k}}\right)$

.400587476159228733968... ≈ $\sum_{k=1}^{\infty}\mu(k)\log\zeta(2k)$

.400685634386531428467... ≈ $\frac{\zeta(3)}{3} = \sum_{k=1}^{\infty}\frac{\cos(k\pi/3)}{k^3}$

.400939666449397060221... ≈ $\frac{\pi}{4\sqrt{2}}\left(1-\frac{\cos\sqrt{2}+\sin\sqrt{2}}{4\sqrt{2}}\left(\cosh\sqrt{2}-\sinh\sqrt{2}\right)\right)$

= $\int_0^{\infty}\frac{\sin^2 x\,dx}{1+x^4}$

.40130634325064638166... ≈ $\frac{1-\gamma}{6} - 2\zeta'(-1) = \sum_{k=2}^{\infty}\frac{\zeta(k)}{(k+1)(k+2)}$

Adamchick-Srivastava 2.22

1 .40147179615651142485... ≈ $\sum_{k=2}^{\infty}\frac{(k-1)\zeta(k)}{k!} = \sum_{k=1}^{\infty}\left(\frac{e^{1/k}}{k} - e^{1/k} + 1\right)$

.40148051389327864275... ≈ $\log 2 - \frac{7}{24} = \sum_{k=1}^{\infty}\frac{1}{8k^2+44k+48} = \sum_{k=3}^{\infty}\frac{(-1)^{k+1}k}{k^2-4}$

.40162427311452899489... ≈ $9-\frac{12}{e^{1/3}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!3^k(k+2)}$

.40175565098878960367... ≈ $\frac{5\sqrt{2}}{12}-\frac{3}{16} = \int_0^{\pi/4}\frac{\cos^5 x}{1+\sin x}dx$

1 .40176256381563326007... ≈ $\log\left(-\Gamma\left(-\frac{1}{3}\right)\right)$

1 .402182105325454261175... ≈ $\sqrt{\pi}\,\Gamma\left(\frac{4}{3}\right)\Gamma^{-1}\left(\frac{5}{6}\right) = \frac{1}{6\sqrt{\pi}}\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{1}{6}\right)$

= $\int_0^1\frac{dx}{\sqrt{1-x^3}} = \int_0^{\infty}\frac{x\,dx}{\sqrt{1+x^6}}$ Seaborn p. 168

1 .402203300817018964126... ≈ $\frac{3\pi^2}{4}-6 = \int_0^{\pi/2}x^3\sin x\,dx = \int_0^1\arccos^3 x\,dx$

7 .402203300817018964126... ≈ $\frac{3\pi^2}{4}$

.40231643935578326982... ≈ $-\frac{3}{4}-\frac{\pi\sqrt{2}}{4}\csc\pi\sqrt{2} = \sum_{k=2}^{\infty}\frac{(-1)^k}{k^2-2}$

.40268396295210902116... ≈ $Li_3\left(\rho^2\right) = \frac{4\zeta(3)}{5}+\frac{4}{5}\zeta(2)\log\rho-\frac{2}{3}\log^3\rho, \qquad \rho = \frac{\sqrt{5}-1}{2}$

= $\frac{2}{3}\log^2\left(\frac{\sqrt{5}+1}{2}\right) - \frac{2\pi^2}{15}\log\left(\frac{\sqrt{5}+1}{2}\right) + \frac{4\zeta(3)}{5}$

.402892520174484499083... ≈ $\frac{1}{4}\left(\psi\left(\frac{i}{2\sqrt{2}}\right)+\psi\left(-\frac{i}{2\sqrt{2}}\right)-\psi\left(\frac{1}{2}+\frac{i\sqrt{2}}{4}\right)-\psi\left(\frac{1}{2}-\frac{i\sqrt{2}}{4}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{k^2+1/2}$

1 .4029920383519025537... ≈ $\frac{1}{\sqrt{5}}\left(Ei\left(\frac{1+\sqrt{5}}{2}\right)-Ei\left(-\frac{2}{1+\sqrt{5}}\right)-2arc\csc h2\right) = \sum_{k=1}^{\infty}\frac{F_k}{k!k}$

3 .40301920828833358675... ≈ $\sum_{k=1}^{\infty}\frac{k!}{k^{k-1}}$

.403025476056584458847... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k!!}$

.40306652538538174458... ≈ $\frac{2\pi}{9\sqrt{3}} = \sum_{k=1}^{\infty}\frac{1}{15\binom{2k}{k}} = \sum_{k=1}^{\infty}\frac{1}{\frac{27k^2}{2}-\frac{27k}{2}+3} = \int_0^{\infty}\frac{x\,dx}{x^3+27}$

1 .40308551457518902803... ≈ $e^{1/18}\left(1+\sqrt{\frac{\pi}{2}}erf\,\frac{1}{3\sqrt{2}}\right) = \sum_{k=0}^{\infty}\frac{1}{k!!3^k}$

6 .40312423743284868649... ≈ $\sqrt{41}$

1 .40312850681674237042 7... ≈ ${}_0F_1\left(;1;\frac{1}{e}\right) = I_0\left(\frac{2}{\sqrt{e}}\right) = \sum_{k=0}^{\infty}\frac{1}{(k!)^2 e^k}$

1 .40317612235005821879 7... ≈ $\sum_{k=1}^{\infty}\frac{\zeta^2(2k)}{(2k)!}$

2 .40324618157446171142... ≈ $\frac{1}{8}\Gamma^3\left(\frac{1}{3}\right) = -\int_0^1 \frac{\log x}{\sqrt[3]{x(1-x^2)^2}}dx$ GR 4.244.1

.40329218106995884773 7... ≈ $\frac{98}{243} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^6}{2^k}$

.4034184004471487... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^3+k^{-3}}$

.40352052665473937253 5... ≈ $\frac{K_0(2)+\log 2}{2} = \int_0^{\infty}\frac{\cos^2 x}{\sqrt{1+x^2}}dx$

.40365263767680592565 9... ≈ $1+eEi(-1) = \int_0^{\infty}\frac{dx}{e^x(x+1)^2} = \int_0^{\infty}\frac{x\,dx}{e^x(x+1)}$

1 .40365263767680592565 9... ≈ $2 + eEi(-1) = \int_0^{\infty}\frac{x^3dx}{e^x(x+1)}$

.40393682788217832057 6... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^{2^k}-1}$

.40406326728086180804 4... ≈ $\sum_{k=1}^{\infty}\frac{1}{3^k+1} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{3^k-1}$

.4041138063191885708... ≈ $2\zeta(3)-2 = -\psi^{(2)}(2)$

= $\int_1^{\infty}\frac{\log^2 x}{x^3-x^2}dx = \int_0^1\frac{x\log^2 x}{1-x}dx = \int_0^{\infty}\frac{x^2}{e^x(e^x-1)}$ GR 4.26.12

2 .4041138063191885708... ≈ $2\zeta(3) = -\psi^{(2)}(1) = \sum_{k=1}^{\infty}\frac{H_k}{k^2}$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2\binom{2k}{k}k^3}$

= $\sum_{k=1}^{\infty}\frac{\psi^{(1)}(k)}{k}$

$$= \int_0^1 \frac{\log^2(1-x)}{x}dx = \int_0^\infty \frac{x^2\,dx}{e^x-1} = \int_1^\infty \frac{\log^2 x}{x^2-x}dx$$

$$= \int_0^1 \frac{\log^2 x}{1-x}$$

$$= -\int_0^1\int_0^1 \frac{\log(xy)}{1-xy}dx\,dy$$

3 .4041138063191885708... ≈ $2\zeta(3)+1 = \sum_{k=2}^\infty \frac{9k^4-2k^2+1}{k(k^2-1)^3}$

$$= \sum_{k=1}^\infty ((2k+1)^2\left(\zeta(2k)-\zeta(2k+1)\right)$$

15 .4041196075522246832... ≈ $8G+\pi+\frac{\pi^2}{2} = \sum_{k=1}^\infty \frac{(3^k-1)(k+1)}{4^k}\zeta(k+1)$

1 .404129680587576209662... ≈ $\frac{\pi}{2}\coth 3\pi - \frac{1}{6} = \int_0^\infty \frac{\sin 3x}{e^x-1}dx$

.404305930665552284361... ≈

$$\frac{2}{1805}\left(190+\sqrt{190\left(47+77\sqrt{5}\right)}arc\csc\sqrt{2\left(1+\sqrt{5}\right)}+\sqrt{190\left(-47+77\sqrt{5}\right)}arc\csc h\sqrt{2\left(-11+\sqrt{5}\right)}\right)$$

$$= \sum_{k=1}^\infty \frac{(-1)^{k+1}F_k}{\binom{2k}{k}}$$

1 .404362229731386861519... ≈ $\sum_{k=1}^\infty \frac{2^{2k-1}\left(\zeta(2k)-1\right)}{(2k-1)!} = \sum_{k=2}^\infty \frac{1}{k}\sinh\frac{2}{k}$

.4045416129644767448... ≈ $\sum_{k=2}^\infty (-1)^k \frac{\zeta(k)-1}{\zeta(k+1)}$

.40455692812372438043... ≈ $\frac{\sqrt{\pi}\csc\sqrt{\pi}}{2} - \frac{1}{2} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{k^2\pi-1}$

2 .404668471503119219718... ≈ $\sum_{k=1}^\infty \frac{k^2}{k^k}$

.404681828751309385178... ≈ $1-\gamma+Ei\left(\frac{1}{e}\right) = \sum_{k=1}^\infty \frac{1}{k!e^k k}$

.404729481218780527329... ≈ $9\pi-18-\pi^2 = \frac{i}{2}\left(Li_3\left(e^{-6i}\right)-Li_3(e^{6i})\right)$

$= -\sum_{k=1}^\infty \frac{\sin 6k}{k^3}$ GR 1.443.5

.40480806194457133626... ≈ $\frac{\pi}{2}\coth\pi - \gamma - \frac{1}{2}\left(1+\psi(1+i)+\psi(1-i)\right)$

= $\sum_{k=1}^{\infty}\frac{k-1}{k^3+k}=\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(2k)-\zeta(2k+1)\right)$

.404942342299990092492... ≈ $\frac{\cosh\sqrt{2}}{2}-\frac{\sinh\sqrt{2}}{2\sqrt{2}} = \sum_{k=1}^{\infty}\frac{2^k k}{(2k+1)!}$

.40496447288461983657... ≈ $\gamma^{\zeta(2)}$

.405182276330542237261... ≈ $\cos(\sin 2)\left(\cosh(\cos 2)+\sinh(\cos 2)\right) = \sum_{k=0}^{\infty}\frac{\cos 2k}{k!}$

.4051919148243804... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2}{k^5+1}$

.405226729401104788051... ≈ $\frac{3\cos 1}{4} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{(2k-1)!}$

2 .4052386904826758277... ≈ $\frac{e^{\pi/2}}{2}$

.40528473456935108578... ≈ $\frac{4}{\pi^2}$

.405465108108164381978... ≈ $\log\frac{3}{2} = Li_1\left(\frac{1}{3}\right) = \sum_{k=1}^{\infty}\frac{1}{3^k k}$ J102, J117

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^k k} = \sum_{k=0}^{\infty}\frac{(-1)^k}{2^k(2k+2)}$

= $2\operatorname{arctanh}\frac{1}{5} = 2\sum_{k=0}^{\infty}\frac{1}{5^{2k+1}(2k+1)}$ K148

= $\int_1^{\infty}\frac{dx}{(x+1)(x+2)} = \int_0^{\log 3}\frac{dx}{e^x+1} = \int_0^{\infty}\frac{dx}{2e^x+1}$

.405577867597361189695... ≈ $\frac{\pi}{2\sqrt{15}} = \int_0^{\infty}\frac{dx}{3x^2+5}$

.40573159035977926877... ≈ $\sum_{k=1}^{\infty}\frac{H^{(3)}{}_k}{2^k(k+1)}$

2 .40579095178568412446... ≈ $\sum_{k=0}^{\infty}\frac{\zeta(k+2)}{k!2^k} = \sum_{k=1}^{\infty}\frac{e^{1/2k}}{k^2}$

1 .405869298287780911255... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)}{2k-1} = \sum_{k=1}^{\infty}\frac{1}{k}\arctan\frac{1}{k}$

.40587121264167682182... ≈ $\frac{\pi^4}{240} = \int_1^{\infty}\frac{\log^3 x}{x^3-x}dx$

1 .406166544295979230078... ≈ $\sqrt{\zeta(2)\zeta(3)}$

1 .4063601829858151518... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(3k)}{k^3}$

.40638974856794906619... ≈ $2 - \pi + 2\pi G - \frac{7\zeta(3)}{2} = \int_0^1 \frac{x^2 \arccos^2 x}{1-x^2} dx$

2 .40654186013... ≈ sum of row sums in $\sum \frac{1}{k^b - k^a}$

.406612178247656085127... ≈ $\frac{120\zeta(5)}{\pi^5} = \int_0^{\infty} \frac{dx}{e^{\pi x^{1/5}} - 1}$

.40671510196175469192... ≈ $\frac{\sinh 2}{4} - \frac{1}{2} = \int_0^1 \sinh^2 x\, dx$

1 .40671510196175469192... ≈ $\frac{\sinh 2}{4} + \frac{1}{2} = \int_0^1 \cosh^2 x\, dx$

.40690163428942536808... ≈ $\frac{\pi}{5}\sqrt{\frac{1}{2} - \frac{1}{2\sqrt{5}}} + \frac{3\operatorname{arccsch} 2}{10(1+\sqrt{5})} + \frac{\operatorname{arccsch} 2}{2\sqrt{5}(1+\sqrt{5})} + \frac{5+\sqrt{5}}{20(1+\sqrt{5}} \log \frac{5+\sqrt{5}}{5-\sqrt{5}}$

$- \frac{1}{5(1+\sqrt{5})}\left(\sqrt{5}\log 2 + \log(\sqrt{5}-1)\right)$

= $\frac{1}{10}\left(\psi\left(\frac{7}{10}\right) - \psi\left(\frac{1}{5}\right)\right)$

= $\sum_{k=0}^{\infty} \frac{(-1)^k}{5k+2}$

2 .40744655479032851471... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(2k)}{k!} = \sum_{k=1}^{\infty} \left(e^{1/k^2} - 1\right)$

1 .40812520284268745491... ≈ $\frac{1}{6} + \frac{\pi^2}{18} + \log 2 = \int_0^1 \frac{\log^2 x}{(x+1)^4} dx$

.40824829046386301637... ≈ $\frac{\sqrt{6}}{6}$

.40825396283062856150... ≈ $\prod_{k=1}^{\infty} \left(1 - \frac{1}{2^k k}\right)$

$.408333333333333333333\underline{3}$ = $\frac{49}{120} = \frac{H_6}{6} = \sum_{k=1}^{\infty} \frac{1}{k^2 + 6k} = \sum_{k=4}^{\infty} \frac{1}{k^2 - 9}$

.40837751064739508288... ≈ $\frac{1}{16}\left(-4 + \frac{2^{7/4}\pi\left(\sin(2^{3/4}\pi) + \sinh(2^{3/4}\pi)\right)}{\cosh(2^{3/4}\pi) - \cos(2^{3/4}\pi)}\right) = \sum_{k=1}^{\infty} \frac{1}{k^4 + 2}$

.408494655426498515867... ≈ $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k}{k^4 + 1}$

.408590857704773823199... ≈ $14 - 5e$

	.40864369354821208259...	≈	$\frac{\sinh 1}{2} - \frac{\sqrt{\pi}}{8}\left(erfi 1 - erf\,1\right) \ = \ \int\limits_1^\infty \cosh\left(\frac{1}{x^2}\right)\frac{dx}{x^4}$	
	.408754287348896269033...	≈	$Li_2\left(\frac{1}{e}\right) \ = \ \frac{\pi^2}{6} - 1 + \sum_{k=1}^\infty \frac{B_k}{(k+1)!k}$	[Ramanujan] Berndt Ch. 9
		=	$\sum_{k=1}^\infty \frac{1}{e^k k^2}$	
2	.40901454734936102856...	≈	$\frac{e\sqrt{\pi}}{2} \ = \ \int\limits_0^\infty e^{1-x^2}\,dx$	
97	.409091034002437236440...	≈	$\pi^4 \ = \ \psi^{(3)}\left(\frac{1}{2}\right) \ = \ 90\zeta(4)$	
		=	$\sum_{k=4}^\infty \frac{(k-1)(k-2)(k-3)\zeta(k)}{2^{k-4}}$	
193	.409091034002437236440...	≈	$\pi^4 + 96 \ = \ -\psi^{(3)}\left(-\frac{1}{2}\right)$	
5	.4092560642181742843...	≈	$\frac{9\zeta(3)}{2} = \int\limits_0^\infty \frac{dx}{e^{x^{1/3}}+1}$	
	.40933067363147861703...	≈	$\frac{e(\sin 1 - \cos 1)}{2} \ = \ \int\limits_1^e \log^3 x \, \sin\log x\,dx$	
18	.40933386237629256843...	≈	$\pi^2 + e\pi$	Borwein-Devlin p. 35
1	.409376212740900917067...	≈	$\frac{\pi^3}{22}$	
	.409447924890760405753...	≈	$\sum_{k=1}^\infty (-1)^{k+1}\left(\zeta(2k)-1\right)^2$	
1	.409943485869908374119...	≈	$\frac{\pi^2}{7} \ = \ \sum_{k=1}^\infty \frac{a(k)}{k^3}$	Titchmarsh 1.2.13
	.410040836946731018563...	≈	$\sum_{k=2}^\infty \frac{\zeta(k)-1}{\zeta(k)+1}$	
	.41019663926527455488...	≈	$\sum_{k=1}^\infty \frac{H^{(4)}{}_k}{3^k k}$	
1	.410278797207865891794...	≈	$\sum_{k=1}^\infty 2^{-F_k}$	
2	.410491857766297...	≈	$\sum_{k=1}^\infty (-1)^{k+1}\frac{e^{1/k}}{k^2}$	
	.41059246053628130507...	≈	$1 - \frac{1}{\sqrt{2}}\arctan\frac{\tan 1}{\sqrt{2}} \ = \ \int\limits_0^1 \frac{\cos^2 x}{1+\cos^2 x}dx$	

.4106780663853243866... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!!2^k}$

1 .4106861346424479769... ≈ $\sqrt{\frac{e\pi}{2}}erf\frac{1}{\sqrt{2}}=\sum_{k=0}^{\infty}\frac{1}{(2k+1)!!}=\sum_{k=0}^{\infty}\frac{k!2^k}{(2k+1)!}=\sum_{k=1}^{\infty}\frac{2^k}{k!\binom{2k}{k}}$

2 .410686134642447997691... ≈ $1+\sqrt{\frac{e\pi}{2}}erf\left(\frac{1}{\sqrt{2}}\right)=\sum_{k=0}^{\infty}\frac{k!2^k}{(2k)!}=\sum_{k=0}^{\infty}\frac{(2k)!!}{(2k)!}$

.410781290502908695476... ≈ $\sin e$

.410861347933971653047... ≈ $3+9\log\frac{3}{4}=\sum_{k=0}^{\infty}\frac{(-1)^k}{3^k(k+2)}$

.41096126403879067973... ≈ $\sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)\zeta(k+1)-1}{k}$

1 .41100762619286173623... ≈ $\sum_{k=1}^{\infty}\frac{1}{k!!k}$

.411233516712056609118... ≈ $\frac{\pi^2}{24}=\frac{\zeta(2)}{4}=-Li_2(i)-Li_2(-i)=\sum_{k=1}^{\infty}\frac{1}{4k^2}$

= $\int_0^{\infty}\frac{x^3dx}{e^{x^2}+1}$

= $\int_1^{\infty}\frac{\log x}{x^3-x}dx=-\int_0^1\frac{x\log x}{1-x^2}dx$

= $\int_0^{\infty}\log\left(1+e^{-2x}\right)dx=\int_0^{\infty}\log\left(1+\frac{1}{x^2}\right)\frac{dx}{x}$

= $-\int_0^{\pi/2}\log(\sin x)\tan x\,dx$ GR 4.384.12

.41131386544700707263... ≈ $\frac{1}{2}\left(\cos 1\cosh 1+\sin 1\sinh 1-1\right)=\int_0^1\cos x\sinh x\,dx$

2 .411354096688153078889... ≈ $\frac{3}{2}+\frac{\pi\sqrt{3}}{2}\operatorname{csch}\frac{\pi}{\sqrt{3}}=\sum_{k=0}^{\infty}\frac{(-1)^k}{k^2+1/3}$

1 .41135828844117328698... ≈ $\frac{SinhIntegral(2)}{\sqrt{\pi}}=\sum_{k=0}^{\infty}\frac{1}{k!(k+\frac{1}{2})!\,(2k+1)}$

2 .411474127809772838513... ≈ $2\sqrt{3}\cos\frac{\pi}{18}-1=\sqrt{8-\sqrt{8-\sqrt{8+\sqrt{8-...}}}}$ [Ramanujan] Berndt Ch. 22

1 .41164138370300128346... ≈ $\frac{\pi^2-1}{2\pi}=\sinh(\log\pi)=-i\sin(i\log\pi)$

6 .41175915924053310538... ≈ $7G$

.4117647058823529 = $\frac{7}{17}$

.41182942258217115 9052... ≈ $\pi\,\mathrm{sech}\frac{\pi\sqrt{3}}{2} = \Gamma\left(\frac{3-i\sqrt{2}}{2}\right)\Gamma\left(\frac{3+i\sqrt{2}}{2}\right) = \prod_{k=1}^{\infty}\left(1-\frac{1}{k^3+1}\right)$

= $\sum_{k=1}^{\infty}\frac{k^2+k}{k^2+k+1}$

.41197960825054113427... ≈ $\frac{3\log 3}{8} = \int_1^{\infty}\frac{\log(x+2)}{x^4}dx$

1 .41228292743739191461... ≈ $\csc^2 1 = \int_0^{\infty}\frac{\log x\;dx}{x^{\pi}-1}$

1 .41255323171015919824 1... ≈ $\frac{\sqrt{2}}{\pi}\mathrm{csch}\,\pi\sqrt{2}\sinh^2\pi = \prod_{k=1}^{\infty}\frac{k^4+2k^2+1}{k^4+2k^2}$

.4126290491156953842... ≈ $\sum_{k=2}^{\infty}\left(\frac{1}{2}-\frac{\zeta(k+1)-1}{\zeta(k)-1}\right)$

5 .4127967952491337838... ≈ $\sum_{k=1}^{\infty}\frac{k^2}{2^k+1}$

.41291393171447973 4856... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^3\log^3 k}$

.41311451318894539 4253... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)}{((2k)!)^2} = \sum\left(\frac{1}{2}I_0\left(\frac{2}{\sqrt{k}}\right)+J_0\left(\frac{2}{\sqrt{k}}\right)-1\right)$

148 .41315910257660342 1116... ≈ e^5

.4131839356331449 1738... ≈ $-\sum_{k=1}^{\infty}\frac{\mu(4k-1)}{4^k-1}$

.4132180012330178 8416... ≈ $\frac{1}{9}\left(6-\sqrt{3}\log\left(2+\sqrt{3}\right)\right)$

= $\frac{2}{3}-\frac{2\sqrt{3}}{9}\mathrm{arcsinh}\frac{1}{\sqrt{2}}$

= ${}_2F_1\left(1,1,\frac{1}{2},-\frac{1}{2}\right) = \sum_{k=1}^{\infty}\frac{(-1)^k 2^k}{\binom{2k}{k}}$

.41326183385306408 7253... ≈ $\sum_{k=1}^{\infty}\frac{1}{k^3+3}$

.41329211610159433 6627... ≈ $\cos(\log\pi) = \mathrm{Re}\{\pi^i\}$

2 .4134342304287... ≈ $\sum_{n=2}^{\infty}\left(\prod_{k=1}^{\infty}\left(1+\frac{1}{k^n}\right)-2\right)$

.413540437991733040494... ≈ $\frac{2\pi}{3\sqrt{3}\cdot 5^{2/3}} = \int_0^\infty \frac{dx}{x^3+5}$

1 .41365142430970187175... ≈ $\frac{1}{2}\left(\cos\sqrt{\pi} + \cosh\sqrt{\pi}\right) = \sum_{k=0}^\infty \frac{\pi^{2k}}{(4k)!}$

6 .4137673108808189126 9... ≈ $\frac{16\pi^4}{243} = \int_0^\infty \frac{\log^3 x\, dx}{x^3-1}$

2 .414043326710635964496... ≈ $2 - 2e + 2\sqrt{\pi}\mathit{erfi}1 = \sum_{k=0}^\infty \frac{1}{k!(k+1/2)(k+1)}$

2 .414069263277926900573... ≈ $\frac{e^\pi + 1}{10} = -\int_0^\pi e^x \sin^2 x\, \cos x\, dx$

.414151108298000051705... ≈ $\frac{\log 12}{6} = \sum_{k=1}^\infty \frac{H_{2k-1}}{4^k}$

.4142135623730950488... ≈ $\sqrt{2} - 1 = \sum_{k=1}^\infty \frac{(2k-1)!!}{(2k)!\ \ 2^k} = \tan\frac{\pi}{8} = -\cot\frac{5\pi}{8}$]

1 .414213562373095048802... ≈ $\sqrt{2}$

= $\sum_{k=0}^\infty \frac{1}{8^k}\binom{2k}{k}$

= $\prod_{k=1}^\infty \left(1 - \frac{(-1)^k}{2k-1}\right)$ GR 0.261

2 .4142135623730950488... ≈ $1 + \sqrt{2} = \sin\frac{3\pi}{8}\csc\frac{\pi}{8}$

.414301138050208113547... ≈ $\frac{1-\log 2}{3} + \frac{\sqrt{3}+i}{6(\sqrt{3}-i)}\left(\psi\left(\frac{1-i\sqrt{3}}{4}\right) - \psi\left(\frac{3-i\sqrt{3}}{4}\right)\right)$

$+ \frac{2i}{6(\sqrt{3}-i)}\left(\psi\left(\frac{3+i\sqrt{3}}{4}\right) - \psi\left(\frac{1+i\sqrt{3}}{4}\right)\right)$

= $\sum_{k=1}^\infty \frac{(-1)^{k+1}}{k^3+1}$

.414322044321820391 87... ≈ $\sum_{k=1}^\infty \frac{1}{\binom{3k}{k}}$

.414398322117159997798... ≈ $7\zeta(3) - 8 = \zeta\left(3, \frac{3}{2}\right) = \sum_{k=1}^\infty \frac{1}{(k+1/2)^3}$

2 .4143983221171599978... ≈ $7\zeta(3) - 6 = \sum_{k=1}^\infty \frac{k}{(k^2 + \frac{1}{4})^3}$

8 .4143983221171599978... ≈ $7\zeta(3) \; = \; \sum_{k=1}^{\infty}\frac{1}{(k+\frac{1}{2})^{3}}$

.41450227931844820531 1... ≈ $1-\frac{1}{e}+\frac{1}{2}\Big(E(i,1)-\Gamma(1-i)-\Gamma(1+i)+\Gamma(1+i,1)\Big)$
= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!(k^{2}+1)}$

.41461045644810919421 2... ≈ $\frac{7}{6}-\frac{\pi}{2\sqrt{6}}\cot\pi\sqrt{\frac{3}{2}} \; = \; \sum_{k=2}^{\infty}\frac{1}{2k^{2}-3}$

.4146234176385755681... ≈ $\frac{4}{3}-\frac{10\pi}{9\sqrt{3}}+\frac{\pi^{2}}{9}$
= $\sum_{k=1}^{\infty}\frac{k}{\binom{2k}{k}(k+1)}$

.41468250985111166024 8... ≈ $\sum_{p\ prime}\frac{1}{2^{p}} \; = \; \frac{1}{2}\sum\frac{\pi(k)}{2^{k}}$

.41509273131087834417... ≈ $\log 2\pi - 2 + \gamma \; = \; \sum_{k=1}^{\infty}\frac{k}{k+2}\big(\zeta(k+1)-1\big)$
= $2\sum_{k=2}^{\infty}(-1)^{k}\frac{\zeta(k)}{k(k+1)}$

.41510749742059470334 0... ≈ $\frac{2}{e\sqrt{\pi}}$

.41517259238542094625... ≈ $\sum_{k=1}^{\infty}\frac{H^{(3)}{}_{k}}{3^{k}k}$

1 .41530796999421526146 7... ≈ $\sum_{k=2}^{\infty}\sigma_{o}(k-1)\big(\zeta(k)-1\big) \; = \; \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\big(\zeta(jk)+1)-1\big)$

.41549482505898532795 9... ≈ $\frac{11}{6}-\frac{\pi}{2\sqrt{2}}\Big(\coth\pi\sqrt{2}+\cot\pi\sqrt{2}\Big) \; = \; \sum_{k=1}^{\infty}4^{k}\big(\zeta(4k)-1\big)$
= $\sum_{k=2}^{\infty}\frac{4}{k^{4}-4}$

27 .41556778080377394121... ≈ $\frac{25\pi^{2}}{9} \; = \; \int_{0}^{\infty}\frac{\log x\, dx}{x^{6/5}-1}$

.41562626476635340361... ≈ $\sum_{k=1}^{\infty}k^{3}\big(\zeta(3k)-1\big) \; = \; \sum_{k=2}^{\infty}\frac{k^{4}(k^{6}+4k^{3}+1)}{(k^{3}-1)^{4}}$

31 .41592653589793238462 6... ≈ 10π

.415939950581392605845... ≈ $\left(\zeta(2)-1\right)^2 = 1-\frac{\pi^2}{3}+\frac{\pi^4}{36} = \sum_{k=2}^{\infty}\frac{f_2(k)}{k^2} = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{1}{(jk)^2}$

1 .41596559417721901505... ≈ $G+\frac{1}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^3}{(k^2-1/4)^2}$

= $\int_0^1 E(x^2)dx$ Adamchik (17)

.416146836547142386998... ≈ $-\cos 2 = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}4^k}{(2k)!}$

1 .416146836547142386998... ≈ $2\sin^2 1 = 1-\cos 2 = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{4^k}{(2k)!}$ GR 1.412.1

7 .41619848709566294871... ≈ $\sqrt{55}$

7 .416298709205487673735... ≈ $\beta\left(\frac{1}{4},\frac{1}{4}\right) = \frac{1}{\sqrt{\pi}}\Gamma^2\left(\frac{1}{4}\right)$

.41634859079941814194... ≈ $\frac{\arctan 3}{3} = \int_0^{\infty}\frac{dx}{x^2+2x+10}$

.41652027545234683566... ≈ $\frac{3\pi}{16\sqrt{2}} = \int_0^{\infty}\frac{dx}{(2x^2+1)^3}$

.416658739762145975784... ≈ $\zeta(3)-\frac{\pi}{4}$

.41666666666666666666$\underline{6}$ = $\frac{5}{12} = \sum_{k=2}^{\infty}\frac{1}{k^2+2k}$

.41705205719676099877... ≈ $-\sum_{k=1}^{\infty}\frac{\mu(2k+1)}{4^k-1}$

.41715698381931361266... ≈ $\sum_{k=1}^{\infty}\frac{|\mu(k)|}{4^k-1}$

.41716921467172054098... ≈ $\frac{7\zeta(3)}{2}+\frac{\pi^2}{2}-\frac{\pi^3}{8}-4G-\frac{32}{27} = \sum_{k=2}^{\infty}\frac{k(k-1)(\zeta(k)-1)}{4^{k-1}}$

.417418352315624897763... ≈ $\frac{3\sqrt{\pi}}{8}erfi 1-\frac{e}{4} = \sum_{k=0}^{\infty}\frac{k}{k!(2k+3)}$

.417479344942600488698... ≈ $-\operatorname{Im}\left\{\sum_{k=1}^{\infty}\frac{\zeta(k+4)}{(2i)^k}\right\} = \sum_{k=1}^{\infty}\frac{2}{4k^5+k^3}$

.41752278908800767414... ≈ $\zeta(2)+4\log 2-1 = \sum_{k=1}^{\infty}\frac{1}{k^2(2k+1)} = \sum(-1)^{k+1}\frac{\zeta(k+2)}{2^k}$

.417771379105166750403... ≈ $\frac{1}{3}\sqrt{\frac{\pi}{2}} = \int_0^\infty \frac{\sin x^2 + x^2 \cos x^2}{x^4}\,dx$ GR 3.852.5

.41782442413236052667... ≈ $\frac{\pi}{6\sqrt{3}} + \frac{\log 2}{6} = \sum_{k=0}^\infty \frac{(-1)^k}{6k+2} = \int_1^\infty \frac{dx}{x^3 + x^{-3}}$

1 .417845935787357293148... ≈ root of $\zeta(x) = 3$

.418023293130673575615... ≈ $1 + \frac{1}{1-e} = \frac{e-2}{e-1} = \sum_{k=1}^\infty \frac{1}{2^k\left(1+e^{1/2^k}\right)}$

= $-\sum \frac{B_k}{k!} = \sum_{k=1}^\infty \left(\frac{e-1}{e}\right)^k \frac{1}{k(k+1)}$ GR 1.513.5

.418115838076169625908... ≈ $-\frac{\pi^2}{24}\log 2 - \frac{3}{4}\zeta'(2) = \sum_{k=0}^\infty \frac{\log(2k+1)}{(2k+1)^2}$ Prud. 5.5.1.5

.418155449141321676689... ≈ $\operatorname{Im}\{\zeta(i)\} = \operatorname{Im}\{\zeta(-i)\}$

.418168472177128190490... ≈ $\left|\zeta(i)\right|$

.41835932862021769327... ≈ $\frac{1}{32}\left(93\zeta(5) - 7\pi^2\right) = \int_0^1 \frac{\arcsin^2 x \arccos^2 x}{x}\,dx$

2 .418399152312290467459... ≈ $\frac{4\pi}{3\sqrt{3}} = \sum_{k=1}^\infty \frac{k!3^k}{2^k(2k+1)!!}$ J278

= area of unit equilateral triangle CFG G13

= $\sum_{k=1}^\infty \frac{(k-\frac{1}{2})!(k+\frac{1}{2})!}{(2k-1)!}$

= $\int_0^\infty \frac{dx}{x^2 - x + 1} = \int_0^\infty \frac{dx}{1 + x^{3/2}}$

= $\int_{-\infty}^\infty \frac{dx}{e^x + e^{-x} - 1}$

.41868043356886331255... ≈ $\frac{35\pi^3}{2592} = \sum_{k=1}^\infty \frac{\sin(5k\pi/6)}{k^3}$ GR 1.443.5

.41893853320467274178... ≈ $\frac{1}{2}\left(\log 2\pi - 1\right) = \sum_{k=1}^\infty \frac{2^k k}{k+2}\left(\zeta(k+1) - 1\right)$

1 .419290313672675237953... ≈ $\frac{\gamma\pi}{\sqrt{3}}\tanh\frac{\pi\sqrt{3}}{2} + \frac{i}{2\sqrt{3}}\left(\psi\left(\frac{1-i\sqrt{3}}{2}\right)^2 - \psi\left(\frac{1+i\sqrt{3}}{2}\right)^2\right)$

$$+\frac{i}{2\sqrt{3}}\left(\psi^{(1)}\left(\frac{1+i\sqrt{3}}{2}\right)-\psi^{(1)}\left(\frac{1-i\sqrt{2}}{2}\right)\right)$$

$= \quad \sum_{k=1}^{\infty}\frac{H_k}{k^2+k+1}$

97 .41935701625712157163... ≈ $\pi^4+\pi^{-4}$

.41942244179510759771... ≈ $\prod_{k=1}^{\infty}\left(1-\frac{1}{2^k+1}\right)$

2 .419550664714475510996... ≈ $4\log 2\pi+4\gamma-\frac{\pi^2}{6}+2\zeta(3)-8 = \sum_{k=1}^{\infty}\frac{k^3}{k+2}\left(\zeta(k+1)-1\right)$

.41956978951241555130... ≈ $\frac{e\sin 1}{1+e^2-2e\cos 1} = \sum_{k=1}^{\infty}\frac{\sin k}{e^k}$

2 .4195961186788831399... ≈ $\sec(\log\pi) = \frac{2}{\pi^i+\pi^{-i}}$

.4197424917352996473... ≈ $\frac{1}{4}\cos\frac{1}{\sqrt{2}}+\frac{\sqrt{2}}{4}\sin\frac{1}{\sqrt{2}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(2k-1)!\,2^k}$

148 .41989704957568888821... ≈ e^5+e^{-5}

.42026373260709425411... ≈ $7-\frac{2\pi^2}{3} = 4\sum_{k=2}^{\infty}\left(\zeta(2k)-1\right)$

2 .42026373260709425411... ≈ $9-\frac{2\pi^2}{3} = \sum_{k=1}^{\infty}\left(\frac{(-1)^{k+1}}{(k-1/3)^2}+\frac{(-1)^{k+1}}{(k+1/3)^2}\right)$

1 .420308303489193353248... ≈ $\frac{\zeta^2(3)}{\zeta(6)} = \sum_{k=1}^{\infty}\frac{2^{\omega(k)}}{k^3}$ HW Thm. 301

$= \quad \prod_{p\ prime}\left(\frac{1+p^{-3}}{1-p^{-3}}\right)$

3 .42054423192855827242... ≈ $\frac{\pi^2\log 2}{2} = -\int_0^{\pi}x\log\sin x\,dx$ GR 4.322.1

.420558458320164071748... ≈ $\frac{5}{2}-3\log 2 = \sum_{k=0}^{\infty}\frac{1}{2^k(k+1)(k+3)}$

.420571993492055286748... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k!}{k^{k-1}}$

.420659594076960499497... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^3-5}$

17 .420688722428817044006... ≈ $8\pi\log 2 = \int_0^{\infty}\frac{e^x x^2}{\sqrt{\left(e^x+1\right)^3}}dx$ GR 3.455.1

.42073549240394825333... ≈ $\frac{\sin 1}{2} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k}{(2k)!}$

.42102443824070833336... ≈ $K_0(1) = \int_0^{\pi/2} \sin(\tan x) \sin x dx = \int_0^{\infty} \frac{\cos x}{\sqrt{1+x^2}} dx$

.$\underline{421052631578947368}$ = $\frac{8}{19}$

.42109768603342377296... ≈ $\sum_{k=1}^{\infty} \frac{1}{4^k - 1} = \sum_{k=1}^{\infty} \frac{\sigma_0(k)}{4^k} = \sum_{k=1}^{\infty} \frac{4^k + 1}{4^{k^2}\left(4^k - 1\right)}$

6 .42147960099874507241... ≈ $\frac{e}{\pi - e} = \sum_{k=1}^{\infty} \left(\frac{e}{\pi}\right)^k$

1 .4215341675430081940... ≈ $\prod_{k=1}^{\infty} \left(1 - \frac{(-1)^k}{2^k k^2}\right)$

.4215955617044106892... ≈ $\sum_{k=2}^{\infty} \frac{StirlingS2(k,2)}{StirlingS1(k,2)}$

.42164305839584698088 2... ≈ $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{\sigma_0(k)}{2^k - 1}$

.42170495465104728887 5... ≈ $2J_2(\sqrt{2}) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(k+2)!2^k}$

.42175135846410626271 6... ≈ $\frac{3\log 2}{2} - \frac{9\log 3}{16} = \int_0^{\infty} \frac{\sin^6 x}{x^3}$ GR 3.827.13

.42176154823190614057... ≈ $e^{2\cos 2} \sin(2\sin 2) = \sin(2\sin 2)\left(\cosh(2\cos 2) + \sinh(2\cos 2)\right)$

= $-\frac{i}{2}\left(e^{2e^{2i}} - e^{2e^{-2i}}\right) = \sum_{k=1}^{\infty} \frac{2^k \sin 2k}{k!}$

1 .42178051211660675651 3... ≈ $\sum_{k=1}^{\infty} \frac{k^2 H_k}{2^k (2k+1)}$

.42188659581978065544 6... ≈ $\sin^5 1$

.42191... ≈ $\sum_{n=1}^{\infty} \frac{1}{n^2} \sum_{k=1}^{n} \mu(k)$

.4219127175822412287... ≈ $\sum_{k=1}^{\infty} \frac{H_k{}^2}{(k+1)^3}$

1 .4220286795392755555... ≈ $\frac{\log^3 2}{3} - \frac{\pi^2}{6} + \frac{2\pi^2 \log 2}{3} - 4Li_3\left(-\frac{1}{2}\right) - 3\zeta(3)$

= $\int_1^{\infty} \frac{x \log^2 x}{(x+2)(x+1)^2} dx$

.42222013200002956777 2... ≈ $\frac{\zeta(3)}{\zeta(2) + \zeta(3)}$

.422371622722581534146... ≈ $\frac{\pi^2}{12} - \gamma \log 2 \; = \; \sum_{k=1}^{\infty} \frac{\psi(1+k)}{2^k k}$

5 .422630642492632281056... ≈ $\pi + \sqrt{3} \log \frac{\sqrt{3}+1}{\sqrt{3}-1} \; = \; \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1/6)}$

.422645425094160918302... ≈ $\frac{\pi^2}{16} - \frac{1}{4} \log^2 \left(\sqrt{2}-1\right) = \; \chi_2\left(\sqrt{2}-1\right)$ Berndt Ch. 9

.422649730810374235491... ≈ $1 - \frac{1}{\sqrt{3}} \; = \; \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^k} \binom{2k}{2}$

.42278433509846713939... ≈ $1-\gamma \; = \; \psi(2) \; = \; \sum_{k=2}^{\infty} \frac{\zeta(k)-1}{k}$ K Ex. 124g

= $\sum_{k=2}^{\infty} \left(Li_1\left(\frac{1}{k}\right) - \frac{1}{k} \right) \; = \; \sum_{k=2}^{\infty} \left(\log \frac{k}{k-1} - \frac{1}{k} \right)$

= $\int_0^{\infty} \frac{x \log x}{e^x} dx$

= $\int_0^{\infty} \left(\frac{\sin x}{x} - \frac{1}{1+x} \right) \frac{dx}{x}$ GR 3.781.1

= $\int_0^{\infty} \left(\cos x - \frac{1}{1+e\,x} \right) \frac{dx}{x}$

1 .42278433509846713939... ≈ $2-\gamma \; = \; \sum_{k=2}^{\infty} 2^k \frac{\zeta(k)-1}{k}$

1 .42281633036073335575... ≈ $\prod_{k=1}^{\infty} \left(1 + \frac{k}{2^{2^k}} \right)$

.422877820191443979792... ≈ $Li_3\left(\frac{2}{5}\right)$

.422980828774864995699... ≈ $CosIntegral(2) \; = \; \gamma + \log 2 + \sum_{k=1}^{\infty} \frac{(-1)^k 4^k}{(2k)!(2k)}$ AS 5.2.16

.423035525761313159742... ≈ $\sum_{k=1}^{\infty} \left(\zeta(2k)-1\right)^2$

.423057840790268613217... ≈ $\frac{1}{2} {}_2F_1\left(1, \frac{2}{3}, \frac{5}{3}, -\frac{1}{2}\right) \; = \; \sum_{k=0}^{\infty} \frac{(-1)^k}{2^k(3k+2)}$

.4232652661789581641... ≈ $\gamma\zeta(3) - \frac{\pi^4}{360} = -\sum_{k=1}^{\infty} \frac{\psi(k)}{k^3}$

.423310825130748003102... ≈ $\pi - e$

1 .423495485003910360936... ≈ $\frac{\zeta(2)+\zeta(3)}{2}$

.423536677851936327615... ≈ $\log 2 + \frac{1}{4}\left(\psi\left(\frac{1+i}{2}\right)+\psi\left(\frac{1-i}{2}\right)-\psi\left(1+\frac{i}{2}\right)-\psi\left(1-\frac{i}{2}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^3+k}$

.423565396298333635... ≈ $2-\log(e-1)+2Li_2\left(\frac{1}{e}\right)+2Li_3\left(\frac{1}{e}\right)-Li_2(1-e)-2\zeta(3)-\frac{1}{2}$

= $\sum_{k=0}^{\infty}\frac{B_k}{(k+2)!} = -\frac{1}{6}+\sum_{k=0}^{\infty}\frac{(-1)^k B_k}{k!}$

.423580847909971087752... ≈ $\sum_{k=1}^{\infty}\frac{H_k\left(\zeta(k+1)-1\right)}{2^k}$

2 .423641733185364535425... ≈ $\frac{e^2}{3}+\frac{2\cos\sqrt{3}}{3e} = \sum_{k=0}^{\infty}\frac{2^{3k}}{(3k)!}$ J803

.423688222292458284009... ≈ $16\log 2-\frac{32}{3} = \sum_{k=0}^{\infty}\frac{1}{2^k(k+4)}$

.423691008343306587163... ≈ $\frac{1}{2}\left(\gamma-\log 2-CosIntegral\,2\right) = -\int_0^1 \log x\sin 2x\,dx$ GR 4.381.1

.423793303250788679449... ≈ $\frac{\pi}{2\sqrt{7}}\csc\pi\sqrt{7}-\frac{5}{21} = \sum_{k=3}^{\infty}\frac{(-1)^{k+1}}{k^2-7}$

1 .4240406467692369659... ≈ $\prod_{k=1}^{\infty}\left(1+\frac{1}{(k+1)(k+4)}\right)$

.424114777686246519671... ≈ $24-\pi^2-6\zeta(4)-6\zeta(3) = \sum_{k=2}^{\infty}\frac{4}{k^5-k^4}$

= $\int_0^1 \log(1-x)\log^3 x\,dx$

.424235324155305999319... ≈ $\sum_{k=2}^{\infty}\left(\zeta(k)-1\right)^k$

1 .424248316072850947362... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(k+1)}{k^2} = -\sum_{k=1}^{\infty}\frac{1}{k}Li_2\left(\frac{1}{k}\right)$

.424413181578387562050... ≈ $\frac{4}{3\pi} = \binom{1}{-1/2}$

.4244363835020222959... ≈ $\frac{\sqrt{\pi}}{2e^{1/4}}erfi\left(\frac{1}{2}\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!\binom{2k}{k}} = \int_0^{\infty}e^{-x^2}\sin x\,dx$

= $Dawson\left(\frac{1}{2}\right)$

.424563592286456286428... ≈ $\frac{\pi}{\sqrt{21}}\tan\frac{\pi\sqrt{21}}{2} - \frac{7}{15} = \sum_{k=1}^{\infty}\frac{1}{k^2+5k+1}$

.424632181169048350655... ≈ $\sum_{k=2}^{\infty}\left(\zeta(k)^{\zeta(k)-1}-1\right)$

14 .424682837915131424797... ≈ $12\zeta(3) = \int_0^{\infty}\frac{x^2\,dx}{e^{x/2}+1}$

.42468496455270619583... ≈ $\frac{\pi}{2}\left(\gamma+\log 2-1\right) = -\int_0^{\infty}\frac{\log x\sin^2 x}{x^2}dx$

1 .424741778429980889761... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}\zeta(4k-2)}{2k-1} = \sum_{k=1}^{\infty}\arctan\frac{1}{k^2}$

$$= \frac{i}{2}\left(\log\Gamma\left(1-\frac{1-i}{\sqrt{2}}\right)+\log\Gamma\left(1+\frac{1-i}{\sqrt{2}}\right)-\log\Gamma\left(1-\frac{1+i}{\sqrt{2}}\right)-\log\Gamma\left(1+\frac{1+i}{\sqrt{2}}\right)\right)$$

.424755371747951580157... ≈ $\frac{\pi}{32}\operatorname{csc}h^3\pi\left(8\pi^2\cosh\pi+\cosh 3\pi-12\pi\sinh\pi-\cosh\pi\right)$

$$= \sum_{k=1}^{\infty}(-1)^{k+1}k^2\left(\zeta(2k)-1\right) = \sum_{k=2}^{\infty}\frac{k^2(k^2-1)}{(k^2+1)^3}$$

9 .424777960769379715388... ≈ 3π

.42514059588544240897... ≈ $\frac{2}{3}\left(2-2\gamma-\psi\left(\frac{1-i\sqrt{3}}{2}\right)-\psi\left(\frac{1+i\sqrt{3}}{2}\right)\right)$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(3k-1)-\zeta(3k)\right) = \sum_{k=1}^{\infty}\frac{k-1}{k^3+1}$

.425168331587636328439... ≈ $\frac{\pi}{e^2} = \int_{-\infty}^{\infty}\frac{\cos 2x}{1+x^2}dx$

.4252949531584225265... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{H_{2k}}{2^k} = \frac{1}{3}\left(\log\frac{3}{2}+\sqrt{2}\arctan\frac{1}{\sqrt{2}}\right)$

3 .425377149919295511218... ≈ $\pi\coth\frac{\pi}{2}$

.425388333283487769500... ≈ $2\log 2-2\log^2 2 = \sum_{k=1}^{\infty}\frac{H_k}{4k^3-k}$

.4254590641196607726... ≈ $\frac{\operatorname{csch}1}{2} = \frac{e}{e^2-1} = \sum_{k=0}^{\infty}\frac{1}{e^{2k+1}}$ GR1.232.3, J942

.4255110379935434188... ≈ $\sum_{k=0}^{\infty}\frac{B_k}{k!}\binom{2k}{k}$

2 .42562246107371717509... ≈ $\frac{2\pi^2}{3}-\frac{7}{4}-2\zeta(3) = 2\sum_{k=2}^{\infty}\frac{3k^2+3k+1}{k(k+1)^3}$

= $\sum_{k=2}^{\infty}(-1)^k k(k+1)(\zeta(k)-1)$

.425661389276834960423... ≈ $\sum_{k=1}^{\infty}\frac{H^{(2)}{}_k}{3^k k}$

.425803019186545577065... ≈ $\frac{\pi}{12\sqrt{3}}+\frac{\log 3}{4} = \sum_{k=0}^{\infty}\frac{1}{(3k+1)(3k+3)}$

.42606171969698205985... ≈ $\sum_{k=1}^{\infty}(-1)^k\frac{\zeta(2k+1)}{2^k} = \sum_{k=1}^{\infty}\frac{1}{2k^3+k}$

= $\gamma+\frac{1}{2}\left(\psi\left(1-\frac{i}{\sqrt{2}}\right)+\psi\left(1+\frac{i}{\sqrt{2}}\right)\right)$

.42612263885053369442... ≈ $\frac{4}{\sqrt{e}}-2 = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+2)!2^k}$

8 .4261497731763586306... ≈ $\sqrt{71}$

1 .426255120215078990369... ≈ $root\,of\ \psi^{(1)}(x)=1$

2 .42632075116724118774... ≈ $\sum_{k=1}^{\infty}\frac{1}{F_k{}^2}$

.42639620163822527630... ≈ $\sum_{k=1}^{\infty} \frac{\log k}{k(2k+1)}$

.426408806162096182092... ≈ $\frac{\pi^2}{15} - \log^2\left(\frac{\sqrt{5}-1}{2}\right) = Li_2\left(\frac{3-\sqrt{5}}{2}\right)$ Berndt Ch. 9

.426473583054083222216... ≈ $\sum_{k=0}^{\infty} \frac{(-1)^1}{7k+2}$

.426790768515592015944... ≈ $\frac{8}{9} - \frac{2\log 2}{3} = \sum_{k=1}^{\infty} \frac{1}{k(2k+3)}$ J265

1 .426819442923313294753... ≈ $\pi G - \frac{3\zeta(3)}{8} = \int_0^{\infty} \log(1+x)\log\left(1+\frac{1}{x^2}\right)\frac{dx}{x}$

.426847730696115136772... ≈ $\frac{\pi^2}{8} + \log 2 - \frac{3}{2} = \sum_{k=2}^{\infty} \frac{4k-1}{2k(2k-1)^2} = \sum_{k=2}^{\infty} \frac{k(\zeta(k)-1)}{2^k}$

2 .427159054034822045078... ≈ $2\pi(2\log 2 - 1)$

.427199846700991416649... ≈ $\frac{2\cos\sqrt{2} - \sqrt{2}\sin\sqrt{2}}{4} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{2^k k^2}{(2k)!}$

.42721687856314874221... ≈ $\frac{\sqrt{2}-1}{12} + \frac{\pi}{8} = \int_0^{\pi/4} \frac{\cos^4 x}{1+\sin x} dx$

.4275050031143272318... ≈ $\frac{\pi}{\sqrt{3}} - 2\log 2 = \psi\left(\frac{5}{6}\right) - \psi\left(\frac{2}{3}\right)$

= $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{3k^2-k}$

= $-\int_0^{\infty} \log\left(\frac{1-x^2}{1-x+x^2}\right) dx$

= $\int_0^{\infty} \log\left(1+\frac{1}{(x+1)^3}\right) dx$

1 .427532966575886781763... ≈ $\frac{9}{4} - \frac{\pi^2}{12} = \frac{1}{2}\left(Li_2\left(-e^{3i}\right) + Li_2\left(-e^{-3i}\right)\right) = \sum_{k=1}^{\infty} (-1)^k \frac{\cos 3k}{k^2}$
GR 1.443.4

.42772793269397822132... ≈ $\frac{\sqrt{2}-2}{2}\zeta\left(\frac{1}{2}\right) = \sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{2k+2}}$

.42808830136517602165... ≈ $\int_0^1 x \tan x \, dx$

.428144741733412454868... ≈ $\zeta(2)\log 2 - \frac{\log^3 2}{3} - \frac{\zeta(3)}{2} = \sum_{k=1}^{\infty} \frac{H^{(2)}{}_k}{2^k(k+1)}$

2 .428189792098870328736... ≈ $\frac{1}{\pi}\cosh\frac{\pi\sqrt{3}}{2} = \frac{1}{\Gamma\left(-(-1)^{1/3}\right)\Gamma\left((-1)^{2/3}\right)} = \prod_{k=1}^{\infty}\left(1+\frac{1}{k^3}\right)$ Berndt 2.12

= $\prod_{k=1}^{\infty}\left(1+\frac{1}{k^2+k}\right)$

.42821977341382775376... ≈ $-\cos\frac{e\pi}{2} = -\operatorname{Re}\left\{i^e\right\}$

.42828486873452133902... ≈ $\sum_{k=2}^{\infty}\frac{1}{k!}\log\frac{k}{k-1}$

.428504222062849638527... ≈ $\frac{(e+1)(\log(e+1)-1)}{e} = \sum_{k=1}^{\infty}\frac{H_k}{(e+1)^k}$

.428571428571$\underline{428571}$ = $\frac{3}{7}$

.4287201581256108127... ≈ $\gamma - \frac{1}{e} - Ei(-1) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k+1)!k} = \sum_{k=2}^{\infty}\frac{(-1)^k}{k!(k-1)}$

= $\sum_{k=2}^{\infty}\frac{(-1)^k}{(k+1)!-2k!}$

403 .428793492735122608387... ≈ e^6

.42881411674498516061... ≈ $\frac{\pi^2}{12} - \frac{1}{2} + \frac{\log 2}{2} - \frac{\log^2 2}{2} = \int_0^1 \frac{\log(1+x)}{x(x+1)^2}$

1 .42893044794135898678... ≈ $\frac{\sqrt{\pi}}{2}\left(\zeta\left(\frac{3}{2}\right)-1\right) = \int_0^{\infty}\frac{x^{1/2}}{e^x\left(e^x-1\right)}dx$

.42909396011806176818... ≈ $\frac{2e^3-17}{54} = \sum_{k=0}^{\infty}\frac{3^k}{(k+3)!}$

.429113234829972113862... ≈ $\frac{\pi^2}{23}$

1 .42918303074795505331... ≈ $\psi(1+2i) + \psi(1-2i)$

.42920367320510338077... ≈ $2 - \frac{\pi}{2}$

= $\sum_{k=1}^{\infty}\frac{1}{4k^2-1/4} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k+3/2}$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}(k-1/2)!}{(k+1/2)!}$

= $\log 2 + \sum_{k=1}^{\infty}(-1)^k\frac{1-2^{-k}}{2^k}\zeta(k+1)$ GR 8.373

= $\displaystyle\sum_{k=1}^{\infty}\frac{2^k}{\binom{2k}{k}(2k+1)k} = \sum_{k=1}^{\infty}\frac{(2k)!!}{(2k-1)!!2^k k(2k+1)}$

= $\displaystyle -\sum_{k=1}^{\infty}\frac{\sin 4k}{k}$

= $\displaystyle \int_0^1 \arccos x \arcsin x \, dx$

= $\displaystyle \int_0^{\pi/2}\frac{x-\sin x}{1-\cos x}dx$ Prud. 2.5.16.15

= $\displaystyle \int_0^{\infty}\frac{dx}{(1+x^2)\cosh \pi x}$ GR 3.522

.42926856072611099995... ≈ $\displaystyle \frac{\pi}{\sqrt{3}}\log\left(\sqrt{2\pi}\,\Gamma\left(\frac{2}{3}\right)\Gamma^{-1}\left(\frac{1}{3}\right)\right) = \int_0^{\infty}\frac{\log x}{e^x+e^{-x}+1}$ GR 4.332.2

6 .429321984298354638879... ≈ $\displaystyle \frac{\pi}{16}\left(3\pi\csc^2\frac{\pi}{\sqrt{2}} - \sqrt{2}\cot\frac{\pi}{\sqrt{2}} - \pi^2\sqrt{2}\cot\frac{\pi}{\sqrt{2}}\csc^2\frac{\pi}{\sqrt{2}}\right)$

= $\displaystyle \sum_{k=1}^{\infty}k^2\frac{\zeta(2k)}{2^k} = \sum_{k=1}^{\infty}\frac{2k^2(2k^2+1)}{(2k^2-1)^3}$

4 .429468097185688596497... ≈ $\displaystyle \frac{\pi^3}{7}$

.429514620607979544321... ≈ $\displaystyle \frac{35\pi}{256} = \int_0^{\infty}\frac{dx}{(x^2+1)^5}$

.429622300326056192750... ≈ $\displaystyle Li_3\left(-\frac{1}{2}\right) - Li_3\left(\frac{3}{2}\right) + \zeta(3) + \frac{1}{6}\left(3\log 2\log^2 3 - 3\log^2 2\log\frac{9}{2}\right)$

$\displaystyle -\frac{1}{6}\left(3i\pi\left(\log^2 3 - \log 2\log\frac{9}{2}\right) - \log\frac{729}{64}Li_2\left(\frac{3}{2}\right)\right)$

= $\displaystyle \sum_{k=1}^{\infty}\frac{(-1)^{k+1}H_k}{2^k k^2}$

.429623584800571687149... ≈ $\displaystyle \sum_{k=2}^{\infty}\frac{2^k\left(\zeta(k)-1\right)}{k^3} = \sum_{k=2}^{\infty}\left(Li_3\left(\frac{2}{k}\right) - \frac{2}{k}\right)$

1 .429706218737208313187... ≈ $\displaystyle \gamma I_0(2) + K_0(2) = \sum_{k=1}^{\infty}\frac{H_k}{k!k!}$

3 .42981513013245864263... ≈ $\displaystyle \frac{\pi}{G}$

4 .42995056995828085132... ≈ $\displaystyle 2 - \frac{\pi\sqrt{3}}{2}\cot\pi\sqrt{3} = \sum_{k=1}^{\infty}3^k\left(\zeta(2k)-1\right) = \sum_{k=2}^{\infty}\frac{3}{k^2-3}$

1 .429956044565484290982... ≈ $\frac{\pi^2+3}{9} = \int_0^\infty \frac{\log^2 x}{(1+x)^4}\,dx$

.43020360185202489933... ≈ $\sum_{k=1}^\infty \frac{\zeta(2k)}{4^k k^2} = \sum_{k=1}^\infty Li_2\left(\frac{1}{4k^2}\right)$

.43026258785476468625... ≈ $\sum_{k=1}^\infty \frac{1}{2k^3+1} = \sum_{k=1}^\infty \frac{(-1)^{k+1}\zeta(3k)}{2^k}$

.43040894096400403889... ≈ $\frac{2}{\sqrt{5}}\operatorname{arcsinh}\frac{1}{2} = \frac{2}{\sqrt{5}}\log\frac{1+\sqrt{5}}{2} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{\binom{2k}{k}k}$

= $\sum_{k=0}^\infty (-1)^k \frac{(2k)!!}{(2k+1)!!2^{2k+1}}$ J142

= $\sum_{k=1}^\infty \frac{F_k}{2^k k} = \sum_{k=1}^\infty \frac{F_k L_k}{4^k k}$

= $\int_1^\infty \frac{dx}{x^3+x^2-x} = \int_0^\infty \frac{dx}{e^x+e^{-x}+3}$

.43055555555555555555$\underline{5}$ = $\frac{31}{72} = \sum_{k=1}^\infty \frac{k+2}{k(k+3)(k+4)}$

.430676558073393050670... ≈ $\log_5 2$

11 .430937592879539094163... ≈ $48\pi - 64\pi\log 2 = \int_0^\infty x^{-1/2} Li_2(-x) Li_2\left(-\frac{1}{x}\right)dx$

1 .430969081105255501045... ≈ $6^{1/5}$

.431166447015110875858... ≈ $\sum_{k=1}^\infty \frac{(-1)^{k+1}}{2^k(2^k-1)}$

1 .43142306317923008044... ≈ $3G - \frac{3\pi}{4} + \frac{3\log 2}{2} = \int_0^1\int_0^1\int_0^1 \frac{x+y+z}{1+x^2y^2z^2}\,dx\,dy\,dz$

2 .43170840741610651465... ≈ $\frac{24}{\pi^2} = \frac{4}{\zeta(2)}$

.431739980620354737662... ≈ $\frac{1}{2} - \frac{\pi^2}{6} + \frac{\pi}{2}\coth\pi = \sum_{k=2}^\infty (-1)^k\left(\zeta(k)-\zeta(2k)\right)$

= $\sum_{k=2}^\infty \frac{k^3-1}{k^2(k+1)(k^2+1)}$

5 .43175383721778681406... ≈ $\sum_{k=1}^\infty \frac{H_k}{F_k}$

5 .43181977215196310741... ≈ $\int_0^\infty \frac{x\,dx}{\Gamma(x)}$

.43182380450040305811... ≈ $\frac{1}{5}+\frac{1}{2}\arctan\frac{1}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{2^{2k-1}(2k-1)}$

.4323323583816936541... ≈ $\frac{e^2-1}{2e^2} = \sum_{k=0}^{\infty}\frac{(-1)^k 2^k}{(k+1)!} = \int_1^e \frac{dx}{x^3} = \int_0^1 \frac{dx}{e^{2x}}$

.43233438520367637377... ≈ $\frac{i}{4\sqrt{2}}\left(H\left(-\left(-\frac{1}{2}\right)^{1/4}\right)+H\left(\left(-\frac{1}{2}\right)^{1/4}\right)-H\left(-\frac{1-i}{2^{3/4}}\right)-H\left(\frac{1-i}{2^{3/4}}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{1}{2k^3+k^{-1}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}\zeta(4k-1)}{2^k}$

.43251279349014789737 8... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\log\zeta(2k)$

1 .432611111111111111$\underline{1}$ = $\frac{205}{144} = H^{(2)}{}_4$

.43262798971613254360 9... ≈ $\frac{e}{2\pi}$

.43282426490660620365 8... ≈ $\frac{1}{1+\log^2\pi} = \int_0^{\infty}\frac{\sin x}{\pi^x}$

.43288474161982931214 5... ≈ $\arctan e-\frac{\pi}{4} = \frac{\pi}{4}-\arctan\frac{1}{e} = \int_0^1 \frac{dx}{e^x+e^{-x}}$

.43291174867614964545 5... ≈ $\frac{3\gamma}{4} = \int_0^{\infty}\left(e^{-x^4}-e^{-x}\right)\frac{dx}{x}$ GR 3.469.2

33 .4331607081195456292... ≈ $1+7\zeta(2)+6\zeta(4)+12\zeta(3) = \sum_{k=2}^{\infty}k^3\left(\zeta(k)-1\right)$

= $\sum_{k=2}^{\infty}\frac{k(k^2+4k+1)}{(k-1)^4}$

1 .43330457796356855869... ≈ $\frac{81\zeta(3)}{2}-\frac{189}{4} = \int_0^1 \frac{\log^2 x}{1+x^{1/3}}dx$

.433333333333333333$\underline{3}$ = $\frac{13}{30} = \int_1^{\infty}\frac{\arctan x^{1/3}}{x^3}dx$

4 .43346463275973539753... ≈ $\sum_{k=2}^{\infty}\frac{k^2}{k-1}\left(\zeta(k)-1\right) = \sum_{k=2}^{\infty}\frac{1}{k}\left(\frac{3k-2}{(k-1)^2}+\log\frac{k}{k-1}\right)$

.43362938564082704614 9... ≈ $13-4\pi$

.43376342818952433618 4... ≈ $\frac{\pi}{3}\left(\sqrt{2}-1\right) = \int_0^{\infty}\log\left(1+\frac{1}{9x^2+1}\right)dx$

.433780830483027187027... ≈ $\log\frac{e^2+1}{2e} = \log\cosh 1$

= $\log\frac{\Gamma\left(1-\frac{i}{\pi}\right)\Gamma\left(1+\frac{i}{\pi}\right)}{\Gamma\left(1-\frac{2i}{\pi}\right)\Gamma\left(1+\frac{2i}{\pi}\right)}$

= $\sum_{k=1}^{\infty}\frac{2^{2k-1}(2^{2k}-1)B_{2k}}{(2k)!k}$

= $\int_0^1 \tanh x\,dx$

.4339085888754952738714... ≈ $-\frac{\sqrt{2}\sin\pi\sqrt{2}}{\pi} = \prod_{k=1}^{\infty}\left(1-\frac{2}{(k+2)^2}\right)$

= $\prod_{k=1}^{\infty}\left(1-\frac{1}{k(k+2)}\right)$

9 .4339811320566038113... ≈ $\sqrt{89}$

1 .43403667553380108311... ≈ $\prod_{k=1}^{\infty}\left(1+\frac{1}{k(k+7)}\right)$

.434171821180757177936... ≈ $\sum_{k=1}^{\infty}\left(\zeta(k+1)-1\right)\left(\zeta(2k)-1\right)$

3 .43418965754820052243... ≈ $3\log\pi$

1 .434241037204324991831... ≈ $\frac{45}{64}+\frac{79e^{1/4}\sqrt{\pi}}{128}erf\,\frac{1}{2} = \sum_{k=1}^{\infty}\frac{k!k^3}{(2k)!}$

.434294481903251827651... ≈ $\log_{10} e = \frac{1}{\log 10}$

.43435113700621794951... ≈ $\frac{\pi}{2\sqrt{3}}\csc\frac{\pi}{\sqrt{3}}-\frac{1}{2} = \sum_{k=1}^{\infty}\frac{-(1)^{k+1}}{3k^2-1}$

1 .43436420505761030482... ≈ $\sum_{k=1}^{\infty}\frac{1}{\phi\left(2^k k\right)}$

.434591199224467793424... ≈ $\sum_{k=1}^{\infty}\frac{\mu(2k-1)}{2^k-1}$

1 .434901660887806674581... ≈ $\frac{2e^2}{3}I_0(2)-\frac{5e^2}{6}I_1(2) = \sum_{k=0}^{\infty}\binom{2k}{k}\frac{1}{(k+2)!}$

1 .4350443315577618438... ≈ $2^{\sinh 1/2} = \prod_{k=0}^{\infty}2^{1/2^{2k+1}(2k+1)!}$

.4353670915862575299... ≈ $\sum_{k=1}^{\infty}\frac{1}{(k+2)!!+k!!}$

.4353977749799916173... ≈ $\frac{\sin 2}{4}-\frac{\cos 2}{2} = \sum_{k=1}^{\infty}\frac{(-1)^k 4^k k}{(2k+1)!}$

.435943911668455273215... ≈ $\frac{23}{32\sqrt{e}} = \sum_{k=1}^{\infty}\frac{(-1)^k k^5}{k!2^k}$

.435953490803707281686... ≈ $\sqrt{2}\left(arc\coth\sqrt{2}-\operatorname{arctan}h\sqrt{2-\sqrt{3}}\right) = \int\limits_{0}^{\pi/6}\frac{dx}{\sin x+\cos x}$

1 .435991124176917432356... ≈ $\frac{1}{3}+\frac{2\sqrt{3}}{\pi}$, Lebesgue constant

.43617267230422259940... ≈ $1+\frac{\pi^2}{12}-2\log 2 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^3+k^2}$

.436174786372556363834... ≈ $\log(1+e)-\log 2-\frac{1}{2e} = \int\limits_{0}^{1}\frac{\cosh x}{1+e^x}dx$

.43634057925226462320... ≈ $9+\frac{1}{4}\left(\psi^{(1)}\left(\frac{2}{3}\right)-\psi^{(1)}\left(\frac{1}{6}\right)\right)=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k+1/3)^2}$

.43646265045727820454... ≈ $\frac{\pi^2}{25}\csc^2\frac{2\pi}{5} = \sum_{k=1}^{\infty}\left(\frac{1}{(5k-2)^2}+\frac{1}{(5k-3)^2}\right)$

.436563656918090470721... ≈ $2e-5 = \sum_{k=1}^{\infty}\frac{k^2}{(k+2)!}$

5 .436563656918090470721... ≈ $2e = \sum_{k=1}^{\infty}\frac{k^2}{k!} = \sum_{k=0}^{\infty}\frac{k+1}{k!}$

.43657478260848849014... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{H^{(2)}{}_k}{2^k k^2}$

1 .436612586189245788936... ≈ $-2\log\left(\Gamma\left(1-\frac{i}{\sqrt{2}}\right)\Gamma\left(1+\frac{i}{\sqrt{2}}\right)\right) = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)}{2^{k-1}k}$

= $2\sum_{k=1}^{\infty}\log\left(1+\frac{1}{2k^2}\right)$

1 .43674636688368094636... ≈ $-Li_2(-2) = \frac{\pi^2}{6}+\log 2\log 3-\frac{\log^2 3}{2}-Li_2\left(\frac{1}{3}\right)$

= $\int\limits_{0}^{1}\frac{\log^2 x}{(2x+1)^2}dx = \int\limits_{0}^{\infty}\log\left(1+2e^{-x}\right)dx$

= $-\int\limits_{0}^{1}\frac{\log x}{x+1/2}dx$

.436827337720053882161... ≈ $\frac{\pi}{\sqrt{7}}\tanh\frac{\pi\sqrt{7}}{2}-\frac{3}{4} = \sum_{k=2}^{\infty}\frac{1}{k^2+k+2}$

.43722380591824767683... ≈ $\frac{42-\pi^2-18\zeta(3)}{24} = \int_0^1 x\log\left(1+\frac{1}{x}\right)\log^2 x\,dx$

2 .437389166862911755043... ≈ $\zeta(4)-\zeta(2)+3 = \sum_{k=2}^{\infty} k\big(\zeta(k)-\zeta(k+3)\big)$

= $\sum_{k=2}^{\infty}\frac{2k^3+k^2+k-1}{k^4(k-1)}$

.437528726844938330438... ≈ $-\frac{\pi\csc\pi\sqrt{14}}{2\sqrt{14}}-\frac{257}{1820} = \sum_{k=4}^{\infty}\frac{(-1)^k}{k^2-14}$

.43754239163900634264... ≈ $\sum_{k=1}^{\infty}\frac{\big(\zeta(k+1)-1\big)^2}{k!}$

.437795379594438621520... ≈ $e^{-e}\big(Ei(e)-\gamma-1\big) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}e^k H_k}{k!}$

.43787374538331792632... ≈ $\frac{3}{128}\left(16\pi^2-128G-16\zeta\left(2,\frac{5}{4}\right)-\zeta\left(4,\frac{3}{4}\right)-\zeta\left(4,\frac{5}{4}\right)\right)$

= $\int_0^{\infty}\frac{x^3}{\cosh^3 x}dx$

.438747307343219426776... ≈ $-\int_0^1\frac{\log x}{e^x+1}dx$

.43882457311747565491... ≈ $\frac{\pi}{4}-\frac{\log 2}{2} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)(2k+2)} = \sum_{k=1}^{\infty}\frac{1}{8k^2-6k+1}$

= $1+\sum_{k=1}^{\infty}\left(\frac{(-1)^k}{2k}+\frac{(-1)^k}{2k+1}\right)$ J72

= $\sum_{k=1}^{\infty}\frac{(-1)^{\lfloor k/2\rfloor}}{k}$ Prud. 5.1.2.4

= $\int_0^{\pi/4}\frac{x\,dx}{\cos^2 x} = \int_0^1\arctan x\,dx$

.43892130408682951019... ≈ $\frac{\sqrt{\pi}\coth\sqrt{\pi}}{2}-\frac{1}{2} = \sum_{k=1}^{\infty}\frac{1}{k^2\pi+1}$

.439028195096889694210... ≈ $\sum_{k=1}^{\infty}\frac{\big(\zeta(k+1)-1\big)^2}{k}$

.439113833857861850508... ≈ $4+2\zeta(2)-2\pi\coth\frac{\pi}{2} = \sum_{k=1}^{\infty}\frac{2}{4k^4+k^2}$

= $-\operatorname{Im}\left\{4\psi\left(1+\frac{i}{2}\right)-\frac{i\pi^2}{3}\right\} = -\operatorname{Im}\left\{\sum_{k=1}^{\infty}\frac{\zeta(k+3)}{(2i)^k}\right\}$

.439255388906447904336... ≈ $4\log 2 - \frac{7}{3} = \sum_{k=2}^{\infty} \frac{2}{(k+1)(2k+1)} = \sum_{k=2}^{\infty}(-1)^k \frac{2^{k-1}-1}{2^{k-2}}\left(\zeta(k)-1\right)$

2 .439341990042075362979... ≈ $\sum_{k=1}^{\infty} \frac{2^{\omega(k)}}{k!}$, $\omega(k) =$ number of prime factors of *k*

5 .43937804598647351638... ≈ $\frac{7\pi^4}{120}+\frac{\pi^2\log^2 2}{4}+\frac{\log^4 2}{8}+3Li_4\left(-\frac{1}{2}\right)=\int_1^{\infty} \frac{\log^3 x\,dx}{x^2+2x}$

.43943789599870974877... ≈ $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^{k^2}} = \frac{3}{2}\sum_{k=1}^{\infty} (-1)^{k+1}\frac{\lfloor\sqrt{k}\rfloor}{2^k}$

.43944231130091681369... ≈ $\frac{e}{2}-\frac{5}{2e} = \int_1^{\infty} \cosh\left(\frac{1}{x}\right)\frac{dx}{x^4}$

.43947885374310212397... ≈ $4\zeta(7) - \gamma\zeta(6) - \zeta(2)\zeta(5) - \zeta(3)\zeta(4) = \sum_{k=1}^{\infty} \frac{\psi(k+1)}{k^6}$

1 .43961949584759068834... ≈ $\pi^{1/\pi}$

.44001215719551248665... ≈ $\sum_{k=2}^{\infty} \frac{1}{k(k+1)\log k}$

.440018745070821693886... ≈ $\frac{\pi}{5\sqrt{3}}-\frac{1}{5}+\frac{2\log 2}{5} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+2)(3k+1)}$

.44005058574493351596... ≈ $J_1(1) = \frac{1}{2}\sum_{k=0}^{\infty} \frac{(-1)^k}{k!(k+1)!4^k}$

.44014142091505317044... ≈ $\frac{1}{2}\left((\log(2+2\cos 1)-2)\sin\frac{1}{2}+(\pi-1)\cos\frac{1}{2}\right) = \sum_{k=1}^{\infty} \frac{1}{k+1}\sin\frac{2k+1}{2}$

1 .44065951997751459266... ≈ $\frac{\pi}{2}\tanh\frac{\pi}{2} = -\operatorname{Im}\left\{\sum_{k=1}^{\infty} \frac{\zeta(2k+2)}{i^k}\right\}$

$= \sum_{k=1}^{\infty} \frac{1}{2k^2-2k+1}$

$= \int_0^{\infty} \frac{\sin x}{\sinh x}dx$ GR 3.981.1

$= \int_{-\infty}^{\infty} \frac{\sin x}{e^x-e^{-x}}dx$

2 .44068364104948817218... ≈ $\frac{1}{e}\left(Ei(e)-\gamma-1\right) = \sum_{k=0}^{\infty} \frac{e^k}{k!(k+1)^2}$

.440865855581020082512... ≈ $1-J_0(\sqrt{2}) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(k!)^2 2^k}$

.440993278190278937096... ≈ $\frac{3}{4\pi\sqrt{2}}\zeta\left(\frac{3}{2}\right)$ Berndt 2.5.15

3 .44107051809507492847... ≈ $6\zeta(4) - 6\zeta(3) + 12\zeta(2) - 9 \;=\; \sum_{k=1}^{\infty} k^3\left(\zeta(k+3)-1\right)$

= $\sum_{k=2}^{\infty} \frac{k^2+4k+1}{k^2(k-1)^4}$

3 .4412853869452228944... ≈ $-\zeta\left(\frac{3}{4}\right) \;=\; -2^{5/4}\pi^{3/4}\,\Gamma^{-1}\left(\frac{3}{4}\right)\zeta\left(\frac{1}{4}\right)\sin\frac{3\pi}{8}$

.44133249687392728479... ≈ $\sum_{k=2}^{\infty} \frac{1}{k^2(k-1)\log k}$

5 .44139809270265355178... ≈ $\pi\sqrt{3} \;=\; \sum_{k=0}^{\infty} \frac{1}{(k+⅓)(k+⅔)}$

= $\int_0^{\infty} \frac{\log(x^2+3)}{x^2}\,dx$

3 .441523869125335258... ≈ $e\,I_0(1) = \sum_{k=0}^{\infty} \frac{1}{k!2^k}\binom{2k}{k}$

1 .441524242056506473167... ≈ $3\zeta(3) - 2\zeta(4)$

4 .44156786325894319020... ≈ $\pi\sqrt{\frac{5+\sqrt{5}}{10}} + \frac{\sqrt{5}}{4}\operatorname{arccsch} 2 + \log 2 + \frac{\sqrt{5}}{16}\log(161+72\sqrt{5})$

= $\sum_{k=0}^{\infty} \frac{(-1)^k}{k+1/5}$

.44195697563153630635... ≈ $\sum_{k=1}^{\infty} (-1)^k H_k\,(\zeta(k+1)-1) \;=\; \sum_{k=2}^{\infty} \frac{1}{k+1}\log\left(1+\frac{1}{k}\right)$

1 .44224957030740838232... ≈ $3^{1/3}$

5 .44266700406635202647... ≈ $\frac{\pi}{\gamma}$

.442668574102213158178... ≈ $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!k\zeta(k+1)}$

.44269504088896340736... ≈ $\frac{1}{\log 2} - 1 \;=\; \sum_{k=1}^{\infty} \frac{1}{2^k\left(1+2^{1/2^k}\right)} \;=\; \sum_{k=1}^{\infty} \frac{2+2^{1/3^k}}{3^k\left(1+2^{1/3^k}+2^{2/3^k}\right)}$

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1 .44269504088896340736... ≈ $\frac{1}{\log 2}$

.44287716368863215107... ≈ $\zeta(2) - \zeta(3) = \sum_{k=1}^{\infty} \frac{k}{(k+1)^3} \;=\; \sum_{k=2}^{\infty} (-1)^k\left(\zeta(k) - \zeta(k+2)\right)$

= $\sum_{k=1}^{\infty}\frac{H_k}{k(k+1)^2}$

.44288293815836624702... ≈ $\pi\sqrt{2} - 4 = \int_0^1 \frac{\arcsin x}{\sqrt{1+x}}dx$

4 .44288293815836624702... ≈ $\pi\sqrt{2} = \Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$

= $\int_{-\infty}^{\infty}\frac{e^{x/4}}{e^x+1}dx$

= $\int_0^{\infty}\log\left(1+x^{-4}\right)dx = \int_0^{\infty}\log\left(1+2x^{-2}\right)dx$

= $\int_0^{\infty}\frac{\log(x^2+2)}{x^2}dx$

= $\int_{-\pi}^{\pi}\frac{dx}{1+\sin^2 x}$

= $\int_0^{\pi/2}\sqrt{\tan x}\;dx$

.44302272411692258363... ≈ $\log\tan 1 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}2^{2k}(2^{2k-1}-1)B_{2k}}{(2k)!k}$ AS 4.3.73

.44311346272637900682... ≈ $\frac{\sqrt{\pi}}{4} = \int_0^{\infty}x^2e^{-x^2}dx = \int_0^{\infty}xe^{-x^4}dx$ LY 6.30

= $\int_0^{\pi/2}xe^{-\tan^2 x}\frac{1-\cos^2 x}{\cos^4 x\cot x}dx$ GR 3.964.2

.44314718055994530941 7... ≈ $\log 2 - \frac{1}{4} = \sum_{k=2}^{\infty}\frac{(-1)^k k}{k^2-1}$

.44318959229937991744 7... ≈ $\frac{3}{4} - \frac{\pi}{2\sqrt{2}}\cot\pi\sqrt{2} = \sum_{k=1}^{\infty}\frac{1}{k^2+4k+2}$

= $\sum_{k=2}^{\infty}(-1)^k L_{k-2}\left(\zeta(k)-1\right)$

1 .44320777848278572146 5... ≈ $2\zeta(3) - 2\log^2 2 = \sum_{k=1}^{\infty}\frac{(k+1)H_k}{k^2(2k+1)}$

.44325$\underline{9803921568627450}$ = $\frac{3617}{8160} = \zeta(-15)$

.44340944198503695432 9... ≈ $\frac{1}{e^{1/2}+e^{-1/2}} = \frac{1}{2\cosh\left(1/2\right)} = \sum_{k=0}^{\infty}(-1)^k e^{-(2k+1)/2}$ J943

1 .443416518250835438647... ≈ $\sum_{k=2}^{\infty}\pi(k)\left(\zeta(k)-1\right)$

3 .443556031041619830543... ≈ $2HypPFQ\left(\left\{1,1,1,\frac{3}{2}\right\},\{2,2,2,2\},4\right) = \sum_{k=1}^{\infty}\binom{2k}{k}\frac{1}{k!k^2}$

1 .443635475178810342493... ≈ $\operatorname{arcsinh} 2$

.4438420791177483629... ≈ $\gamma-\log 2-Ei(-\frac{1}{2}) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!2^k k}$

.444288293815836624702... ≈ $\frac{\pi}{5\sqrt{2}}$

.44444444444444444444$\underline{4}$ = $\frac{4}{9} = \sum_{k=1}^{\infty}\frac{k}{4^k}$

.44466786100976613366... ≈ $e^{1/e}-1 = -\int_0^{1/e}\frac{1+\log x}{x^x}dx$

1 .44466786100976613366... ≈ $e^{1/e}$, maximum value of $x^{1/x}$

= $\sum_{k=0}^{\infty}\frac{1}{k!e^k}$

5 .4448744564853177341... ≈ $\gamma^3+\frac{\gamma\pi^2}{2}+2\zeta(3) = -\int_0^{\infty}\frac{\log^3 x dx}{e^x}$

1 .44490825889549869114... ≈ $\sum_{k=2}^{\infty}(-1)^k k(k+1)\frac{\zeta(k)}{2^k} = \sum_{k=1}^{\infty}\frac{8k^2}{(2k+1)^3}$

1 .44494079843363423391... ≈ $\zeta^2(3) = \sum_{k=1}^{\infty}\frac{\sigma_0(k)}{k^3}$ Titchmarsh 1.2.1

1 .44501602875629472763... ≈ $\prod_{k=1}^{\infty}\frac{\zeta(2k)}{\zeta(2k+1)}$

3 .44514185336664668616... ≈ $\frac{\pi^3}{9}$

.44518188488072653761... ≈ $3-\frac{\pi}{2\sqrt{3}}-\frac{3\log 3}{2} = \sum_{k=1}^{\infty}\frac{1}{3k^2+k}$

= $\gamma+\psi\left(\frac{4}{3}\right) = hg\left(\frac{1}{3}\right) = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(k+1)}{3^k}$

= $-\int_0^1\log(1-x^3)\,dx$

.445260043082768754407... ≈ $\frac{48}{125}+\frac{42}{125}\log\frac{6}{5} = \sum_{k=1}^{\infty}\frac{k^2H_k}{6^k}$

.445901168918876713517... ≈ $\frac{3}{4}\left(1-\log\frac{3}{2}\right) = \sum_{k=1}^{\infty}\frac{(k-1)H_k}{3^k}$

.446483130925452522454... ≈ $\frac{1}{3}-\frac{2\pi}{3}+2\zeta(2)-\zeta(4) \;=-\frac{1}{2}\left(Li_4(e^{2i})+Li_4(e^{-2i})\right)$

= $-\sum_{k=1}^{\infty}\frac{\cos 2k}{k^4}$ GR 1.443.6

.44648855096786546141... ≈ $2(\cos 1+2\sin 1-2) \;=\; \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!(k+2)}$

.446577031296941135508... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)}{(k!)^2} \;=\; \sum_{k=1}^{\infty}\left(I_0\left(\frac{2}{\sqrt{k}}\right)-1-\frac{1}{k}\right)$

.446817484393661569797... ≈ $\sum_{k=2}^{\infty}\left(1-\frac{\zeta(k+1)}{\zeta(k)}\right)$

2 .4468859331875220877... ≈ $\sum_{k=2}^{\infty}\frac{k-1}{d_k}$

.44714430796140880686... ≈ $2-\cos\sqrt{2}-\sqrt{2}\sin\sqrt{2}$

= $Li_3\left((e^i)+Li_3(e^{-i})\right) \;=\; \sum_{k=1}^{\infty}\frac{(-1)^{k+1}2^k}{(2k)!(k+1)}$

.447213595499957939282... ≈ $\frac{\sqrt{5}}{5}$

.447309717383151082397... ≈ $\frac{128}{27}+\frac{32G}{3}-\frac{4\pi^2}{3}+\frac{24\pi}{27}-\frac{144\log 2}{27}$

= $\int_0^1\frac{\log(1-x)\log x}{x^{1/4}}dx$

.44745157639460216262... ≈ $\frac{\pi}{3\sqrt{3}}\left(2-2^{1/3}\right) \;=\; \int_0^{\infty}\frac{dx}{(x^3+1)(x^3+2)}$

.447467033424113218236... ≈ $\frac{\pi^2}{12}-\frac{3}{8} \;=\; \frac{1}{2}\sum_{k=2}^{\infty}(-1)^k k\left(\zeta(k)-1\right) \;=\; \sum_{k=1}^{\infty}\frac{1}{k^3+2k^2}$

2 .4475807362336582311... ≈ $-\zeta\left(\frac{2}{3}\right) \;=\; -(2\pi)^{2/3}\zeta\left(\frac{1}{3}\right)\Gamma^{-1}\left(\frac{2}{3}\right)$

.44764479114467781038... ≈ $-\tan\frac{\pi\sqrt{3}}{2}$

.44784366244327440446... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^k}{(3^k-1)k} \;=\sum_{k=1}^{\infty}\log\left(1+\frac{1}{3^k}\right)$

.447978320832715134501... ≈ $\frac{\pi}{12}\coth\frac{\pi}{2}-\frac{i\pi}{6}\csc\pi(-1)^{5/6}-\frac{\pi}{6}\operatorname{csch}\frac{\pi-i\pi\sqrt{3}}{2}-\frac{\pi}{12}\tanh\frac{\pi}{2}$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^4+k^{-2}}$

3 .44801942158761786457... ≈ $\zeta(2)+\frac{3\zeta(3)}{2} = \sum_{k=1}^{\infty}\frac{H_k H_{k+1}}{k(k+2)}$

810 .44813687055627345203... ≈ $\frac{e^{e^2}+e}{2} = \frac{1}{2}\left(e+\cosh\left(\cosh 2+\sinh 2\right)+\sinh\left(\cosh 2+\sinh 2\right)\right)$

= $\sum_{k=0}^{\infty}\frac{e^k\cosh k}{k!}$

.44830238673872151773 9... ≈ $\frac{\pi}{4}\left(\frac{\pi}{2}-1\right) = \sum_{k=0}^{\infty}\frac{\cos(2k+1)}{(2k+1)^2}$ GR 1.444.6

.44841420692364620244... ≈ $\frac{\log^2 2}{2}-\log 2\log 3+\frac{\log^2 3}{2}+Li_2\left(\frac{1}{3}\right) = -Li_2\left(-\frac{1}{2}\right)$

= $\frac{1}{6}\left(\pi^2+3\log^2 2-3\log^2 3-6Li_2\left(\frac{2}{3}\right)\right)$

= $\frac{\pi^2}{12}-\frac{\log^2 2}{2}-\frac{1}{2}Li_2\left(\frac{1}{4}\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^k k^2} = \sum_{k=1}^{\infty}\frac{H_k}{3^k k}$

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{2^k(k+1)(2k+2)}$

= $\int_0^1\frac{\log^2 x\,dx}{(x+2)^2} = \int_1^{\infty}\frac{\log^2 x\,dx}{(2x+1)^2} = -\int_0^1\frac{\log x\,dx}{x+2}$

= $\int_0^{\infty}\log\left(1+\frac{e^{-x}}{2}\right)dx$

.44851669227070827912... ≈ $\frac{\pi}{40}\left(2\pi-\sqrt{5}\sin\frac{2\pi}{\sqrt{5}}\right)\csc^2\frac{\pi}{\sqrt{5}} = \sum_{k=1}^{\infty}\frac{k\zeta(2k)}{5^k} = \sum_{k=1}^{\infty}\frac{5k^2}{(5k^2-1)^2}$

1 .44852646163024124099 1... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)}{k^2} = \sum_{k=1}^{\infty}Li_2\left(-\frac{1}{k^2}\right)$

1 .44854567814669101862 8... ≈ $\sum_{k=1}^{\infty}\frac{1}{k!H_k} = \sum_{k=2}^{\infty}\frac{(-1)^k}{S1[k,2]}$

.44857300728001739775... ≈ $\frac{1}{2}\left(Li_3\left(e^i\right)+Li_3\left(e^{-i}\right)\right) = \sum_{k=1}^{\infty}\frac{\cos k}{k^3}$

.44862308908611419722 9... ≈ $\prod_{k=1}^{\infty}\left(1-\frac{1}{\left(\frac{2k}{k}\right)k}\right)$

.44878179516410325383 0... ≈ $\frac{9}{e^{1/3}}-6 = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+2)!3^k}$

	.4487989505128276054 9...	≈	$\frac{\pi}{7} = \int_0^\infty \frac{x^{5/2}dx}{1+x^7}$
6	.4490644061661079132 2...	≈	$\frac{32\pi}{9\sqrt{3}} = \sum_{k=1}^\infty \frac{(k+\frac{1}{2})!(k-\frac{1}{2})!}{(2k-2)!}$
	.4492829744712816644 65...	≈	$Li_2\left(\frac{2}{5}\right)$
	.4493394081788311142 78...	≈	$\frac{1}{2} - \frac{1}{2\pi^2} = \int_1^\pi \frac{dx}{x^3}$
	.4494635869386757726 14...	≈	$\frac{\pi}{18\sqrt{2}} + \frac{2\pi}{9\sqrt{3}} - \frac{\log 2}{9} = \int_0^\infty \frac{dx}{(x^3+1)(x^2+2)}$
2	.4494897427831780981 97...	≈	$\sqrt{6}$
110	.4495549668208546497 72...	≈	$41e - 1 = \sum_{k=1}^\infty \frac{k^6}{(k+1)!}$
	.4495902033541041835 4...	≈	$1 - \frac{\pi}{2\sqrt{2}}\cot\frac{\pi}{2\sqrt{2}} = \sum_{k=1}^\infty \frac{1}{4k^2 - 1/2}$
3	.4499650135236733653...	≈	$9\gamma^2 - 6\gamma - 2\gamma + \frac{3\pi^2}{2} - \gamma\pi^2 - 4\zeta(3) = \int_0^\infty \frac{x^2 \log^3 xdx}{e^{2x}}$

$.4500000000000000000\underline{0}$ = $\frac{9}{20}$

3 .45001913833585334783... ≈ $\frac{\pi^2+\log^2 2}{3} = \int_0^\infty \frac{\log^2(1-x)}{x}\frac{x\log x - x - 2}{(x+2)^2}dx$ GR 4.313.7

.45014414312062407805... ≈ $\log 2 - \frac{\pi\sqrt{3}}{3} + \frac{\pi}{2} = \sum_{k=0}^\infty \frac{(-1)^k}{(2k+1)(3k+2)}$

2 .45038314198839886734... ≈ $\sum_{k=1}^\infty (-1)^1 \frac{2^k}{k!!}$

.450571851280126820771... ≈ $\frac{1}{2} + \frac{\pi}{\sqrt{2}} \frac{\cosh\frac{\pi}{\sqrt{2}}\sin\frac{\pi}{\sqrt{2}} + \sinh\frac{\pi}{\sqrt{2}}\cos\frac{\pi}{\sqrt{2}}}{\cos\pi\sqrt{2} - \cosh\pi\sqrt{2}}$

= $\sum_{k=1}^\infty \frac{(-1)^{k+1}}{k^4+1}$

.45066109396293091334... ≈ $\frac{3-\cos 2}{2\sqrt{\pi}} - \frac{\sin 2}{\sqrt{\pi}} = \sum_{k=0}^\infty \frac{(-1)^k}{(k+2)!(k+\frac{1}{2})}$

. 45071584712271411591... ≈ $\frac{2G+1}{2\pi} = \sum_{k=0}^\infty \binom{2k}{k}^2 \frac{1}{16^k(2k+3)}$

.45077133868484785702... ≈ $\frac{3\zeta(3)}{8} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{2k^3}$

= $\int_0^\infty \frac{x^3\,dx}{e^{2x}+e^{-2x}-2}$

= $-2\left(Li_3(i) + Li_3(-i)\right) = -\int_0^1 Li_2(-x^2)\frac{dx}{x}$

4 .450875896181964986251... ≈ $\frac{\pi^6}{216} = \zeta^3(2)$

.45091498545516623899... ≈ $-\sum_{k=1}^\infty \frac{\mu(2k)}{2^k k}$

.45100662426809780655... ≈ $\frac{\pi^3}{8} - 3\pi + 6 = \int_0^1 \arcsin^3 x\,dx$

.45137264647546680565... ≈ $\frac{1}{3}\Gamma\left(\frac{2}{3}\right) = \int_0^\infty \frac{x\,dx}{e^{x^3}}$

3 .451392295223202661434... ≈ $\pi\log 3 = \int_0^\infty \frac{\log(x^2+4)}{x^2+1}dx$

.45158270528945486473... ≈ $\log\frac{\pi}{2} = \sum_{k=1}^\infty \frac{\zeta(2k)}{4^k k} = -\sum_{k=1}^\infty \log\left(1-\frac{1}{4k^2}\right)$ Wilton

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$= \operatorname{Re}\{\log\log i\}$

$= \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \left(\frac{1}{2k} - \log\frac{2k+1}{2k}\right)$ Prud. 5.5.1.16

$= -\int_0^1 \frac{1-x}{1+x}\frac{dx}{\log x}$ GR 4.267.1

$= \int_0^{\pi/2} \left(\frac{1}{x} - \cot x\right) dx$ GR 3.788

1 .451859777032415201064... $\approx -\frac{i}{4}\left(e^{e^{1+i}} + e^{e^{-1-i}} - e^{e^{1-i}} - e^{e^{-1+i}}\right) = \sum_{k=0}^{\infty} \frac{\sin k \sinh k}{k!}$

.451885886874386023421... $\approx \frac{1}{12}\left(\pi + \sqrt{3}\log\left(2+\sqrt{3}\right)\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{12k+2}$

2 .451991067287107389384... $\approx \frac{\pi^2}{3} + 1 - \log 2\pi = \sum_{k=1}^{\infty}\frac{2^k k^2}{k+2}\left(\zeta(k+1)-1\right)$

$= \sum_{k=2}^{\infty}\left(k\log\frac{k}{k-2} - \frac{2(k(k-3)+1)}{(k-2)^2}\right)$

.4520569031595942854... $\approx \zeta(3) - \frac{3}{4}$

.45224742004106549851... $\approx \sum_{p\ prime} p^{-2} = \sum_{k=1}^{\infty}\frac{\mu(k)}{k}\log\left(\zeta(2k)\right)$ Titchmarsh 1.6.1

$= \sum_{k=2}^{\infty}\frac{(2k+1)\pi(k)}{k^2(k+1)^2}$ Shamos

.45231049760682491399... $\approx \frac{3}{4} - \frac{\pi}{2}\sqrt{\frac{3}{2}}\operatorname{csch}\pi\sqrt{\frac{2}{3}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2+2/3}$

.452404030972285668128... $\approx \frac{1}{4}\left(\pi\coth\pi - 2 - \psi(i) - \psi(-i)\right) = \sum_{k=1}^{\infty}\left(\zeta(4k-2) - \zeta(4k-1)\right)$

$= \sum_{k=2}^{\infty}\frac{1}{k^2+k+1+k^{-1}}$

16 .452627765507230224736... $\approx \frac{\sqrt{\pi}}{2} erfi 2 = \sum_{k=0}^{\infty}\frac{2^{2k+1}}{k!(2k+1)}$

.452628365343598915383... $\approx \frac{1}{\sqrt{6}}\left(\zeta\left(\frac{1}{2},\frac{1}{3}\right) - \zeta\left(\frac{1}{2},\frac{5}{6}\right)\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{\sqrt{3k+2}}$

.452726765998749507286... $\approx \sum_{k=1}^{\infty}\left(\frac{\zeta(2k)}{\zeta(3k)} - 1\right)$

.45281681270310393337... $\approx 1 - \frac{5\pi}{16} + \frac{5\pi}{8\sqrt{2}} - \frac{5\log 2}{2} + \frac{5}{8\sqrt{2}}\log\frac{2+\sqrt{2}}{2-\sqrt{2}}$

= $\sum_{k=2}^{\infty}\frac{(-1)^k 5^k \zeta(k)}{8^k} = \sum_{k=1}^{\infty}\frac{25}{8k(8k+5)}$

.452832425263941397660... ≈ $\log\frac{2+e}{3} = \int_0^1 \frac{e^x}{e^x+2}dx$

.4529232530212652211... ≈ $\frac{1}{2+i} = \frac{1}{2+e^{-\pi/2}}$

1 .452943571383509653781... ≈ $\sum_{k=2}^{\infty}\left(3^{\zeta(k)-1}-1\right)$

.453018350450290206718... ≈ $\frac{2\pi}{\pi^2+4} = \int_0^{\infty}\frac{\sin \pi x/2}{e^x}$

.45304697140984087256... ≈ $\frac{e}{6}$

9 .4530872048294188123... ≈ $\frac{16\sqrt{\pi}}{3} = \int_0^{\infty}\frac{\sin^2(4x^2)}{x^4}$ GR 3.852.3

.453114239502760197533... ≈ $\frac{1}{1+\log^2 3} = \int_0^{\infty}\frac{\sin x}{3^x}$

.453344123097900874903... ≈ $\frac{\pi\gamma}{4} = \int_0^{\infty}\frac{si(x)\cos^2 x}{x}dx$

.453449841058554462649... ≈ $\frac{\pi}{4\sqrt{3}} = \int_0^{\infty}\frac{dx}{3x^2+4} = \int_1^{\infty}\frac{dx}{4x^2+3} = \int_0^{\infty}\frac{x\,dx}{x^4+3}$

.453586433219203359539... ≈ $\frac{4\log 2}{3} - \frac{1}{3} + \frac{\pi}{6}\left(\cot\frac{\pi(3-i\sqrt{3})}{4} - \cot\frac{\pi(3+i\sqrt{3})}{4}\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^4+k} = \sum_{k=1}^{\infty}\frac{2}{8k^2-k}$

.45359237000000000000$\underline{0}$ = pounds/kilogram

.453602813292325043435... ≈ $\pi\cot\frac{7\pi}{8} + 8\log 2 + 2\sqrt{2}\log\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}$

= $\sum_{k=1}^{\infty}\frac{1}{4k^2-k/2} = \sum_{k=2}^{\infty}\frac{\zeta(k)}{2^{3k-4}}$

.45383715497509933263... ≈ $\int_0^{\pi/4}\sqrt{\sin x}\,dx$

.45383998041715262... ≈ $\sum_{n=2}^{\infty}(-1)^n\sum_{k=1}^{\infty}\log\zeta(nk)$

.4538686550208064702... ≈ $\cot(\log\pi) = \frac{\pi^i + \pi^{-i}}{i\pi^{-i} - i\pi^i}$

.45395280311216306689... ≈ $\frac{3\sqrt{\pi}}{4}\left(\zeta\left(\frac{5}{2}\right) - 1\right) = \int_0^\infty \frac{x^{3/2}}{e^x\left(e^x - 1\right)}dx$

.453985269150295583314... ≈ $\frac{5\pi^2}{96} - \frac{\log^2 2}{8} = \frac{1}{2}\left(Li_2\left(\frac{1+i}{2}\right) + Li_2\left(\frac{1-i}{2}\right)\right)$

.454219904863173579921... ≈ $Ei\left(\frac{1}{2}\right) = \gamma - \log 2 + \sum_{k=1}^\infty \frac{1}{k!2^k k}$ AS 5.1.10

4 .45427204221056257189... ≈ $\sqrt{2}\,\cosh\pi\,\sinh\pi\,\operatorname{csch}\pi\sqrt{2}$

= $\prod_{k=1}^\infty \frac{k^2+4}{k^2+2} = \prod_{k=1}^\infty \left(1 + \frac{2}{k^2+2}\right)$

.454313133585223101836... ≈ $\sum_{k=2}^\infty (-1)^k \left(2^{\zeta(k)-1} - 1\right)$

.454407859842733526516... ≈ $\sum_{k=2}^\infty \left(\sqrt{\zeta(k)} - 1\right)$

2 .45443350620295586062... ≈ $\frac{\zeta(3) - 1}{\zeta(4) - 1}$

.454545454545454545$\underline{45}$ = $\frac{5}{11} = \sum_{k=1}^\infty \frac{F_{3k-2}}{6^k}$

1 .4546320952042070463... ≈ $2\zeta(3) - \gamma\zeta(2) = \sum_{k=1}^\infty \frac{\psi(k+1)}{k^2}$

.4546487134128408477... ≈ $\frac{\sin 2}{2} = \sum_{k=0}^\infty (-1)^k \frac{2^{2k}}{(2k+1)!} = \prod_{k=1}^\infty \left(1 - \frac{4}{\pi^2 k^2}\right)$ GR 1.431

= $\sin 1 \cos 1 = \binom{0}{2/\pi}$

.454822555520437524662... ≈ $6 - 8\log 2 = \sum_{k=1}^\infty \frac{k}{2^k(k+2)} = \sum_{k=1}^\infty \frac{1}{B_2(k)}$

3 .454933798924981417101... ≈ $\sum_{k=1}^\infty \frac{\zeta(2k)}{(k-1)!} = \sum_{k=1}^\infty \frac{e^{1/k^2}}{k^2}$

.455119613313418696807... ≈ $\frac{1}{\log 9}$

1 .45520607897219862104... ≈ $\pi^2 - 7\zeta(3) = \int_1^\infty \frac{\log^2 x}{(x-1)^2\sqrt{x}}dx$

.45535620037573410418... ≈ $\frac{i}{4\sqrt{\pi}\sinh\pi}\left(\Gamma\left(\frac{3}{2}+i\right)\Gamma(2-i) - \Gamma\left(\frac{3}{2-i}\right)\Gamma(2+i)\right)$

= $\int_0^{\infty} \frac{e^{-x} \sin x}{\sqrt{e^x - 1}} dx$

.455454848083584498405... ≈ $\int_2^3 \left(e^{\zeta(x)-1} - 1\right) dx$

.455698463397600878... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(k+1)}{4^k k} = \sum_{k=1}^{\infty} \frac{1}{k} \log \frac{4k}{4k-1}$

.45593812776599623677... ≈ $i^{i/2} = e^{-\pi/4}$

.4559418900357336741... ≈ $1 - \frac{2\pi}{\sinh \pi}$

.45594532639051997149 8... ≈ $\frac{9}{2\pi^2} = \sum \frac{1}{q^2}$, q squarefree with an odd number of factors

Berndt 5.30

.45603713201940539132 8... ≈ $\frac{1}{8}\left(i \tanh\left(\frac{1+i}{4}\right) - i \tan\left(\frac{1+i}{4}\right) + 4 \csc\left(\frac{1+i}{2}\right)\right)$

$+ \frac{1}{8}\left(4 \csc h\left(\frac{1+i}{2}\right) - i \cot\left(\frac{1+i}{4}\right) - \coth\left(\frac{1+i}{4}\right)\right)$

= $\int_{-\infty}^{\infty} \frac{e^{-x} \sin x}{1 + e^{-2\pi x}} dx$

979 .45612797347790198907 0... ≈ $\frac{\sinh \pi^2}{\pi^2} = \prod_{k=1}^{\infty} \left(1 + \frac{\pi^2}{k^2}\right)$

.4563620990510890237 8... ≈ $-\frac{1}{2} \cos \frac{\pi\sqrt{3}}{2} = \prod_{k=1}^{\infty} \left(1 - \frac{3}{(2k+1)^2}\right)$

.4564267754028415968 9... ≈ $\frac{\pi}{2\sqrt{3}} \coth \frac{\pi}{\sqrt{3}} - \frac{1}{2} = \sum_{k=1}^{\infty} \frac{1}{3k^2+1} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\zeta(2k)}{3^k}$

1 .456426775402841596895... ≈ $\frac{1}{2} + \frac{\pi}{2\sqrt{3}} \coth \frac{\pi}{\sqrt{3}} = \sum_{k=0}^{\infty} \frac{1}{3k^2+1}$

.4566666666666666666$\underline{6}$ = $\frac{137}{300} = \frac{H_5}{5} = \sum_{k=1}^{\infty} \frac{1}{k^2+5k}$

.4569425624776396611 1... ≈ $-\sum_{k=1}^{\infty} \frac{\mu(2k)}{3^k - 1} = \sum_{k=1}^{\infty} \frac{1}{(\sqrt{3})^{2^k}}$

.45710678118654752440 1... ≈ $\frac{1}{\sqrt{2}} - \frac{1}{4} = \int_0^{\pi/4} \frac{\cos^3 x}{1 + \sin x} dx$

2 .45714277885555220148... ≈ $2\sqrt{\pi}\log 2 \;=\; \sum_{k=1}^{\infty}\frac{(k-\frac{1}{2})!}{k!\,k}$

.45765755436028576375... ≈ $-\cot 2$

.45774694347424732856... ≈ $\frac{\log 2-1}{3}+\frac{1}{12}\left(\psi\left(\frac{3+\sqrt{3}}{2}\right)+\psi\left(\frac{3-\sqrt{3}}{2}\right)-\psi\left(\frac{\sqrt{3}}{2}\right)-\psi\left(-\frac{\sqrt{3}}{2}\right)\right)$

= $\sum_{k=2}^{\infty}\frac{(-1)^k}{k^3-3k}$

.45797009720116330422 7... ≈ $1-\frac{2\pi}{\cosh\pi}$

.45798279708860950752 7... ≈ $\frac{G}{2}$

= $-\int_0^{\pi/4}\log(2\sin x)\,dx \;=\; \int_0^{\pi/4}\log(2\cos x)\,dx$ Adamchik (5), (6)

= $\int_0^{\infty}\frac{x\,dx}{e^{x\sqrt{2}}+e^{x\sqrt{2}}}$

.45848156091517979971 1... ≈ $\frac{\pi^2}{12}+\frac{\pi}{2\sinh\pi}-\frac{1}{2} \;=\; \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^4+k^2}$

.45860108943638148692... ≈ $\frac{\pi^4}{48}-\frac{\pi^3}{4}+\pi^2-\frac{4\pi}{3}+\frac{1}{2}$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\sin^4 k}{k^4}$

.45866551368106356297 7... ≈ $\frac{7\zeta(3)}{4}-\zeta(2) \;=\; \sum_{k=3}^{\infty}(-1)^{k+1}\frac{k(k-1)\zeta(k)}{2^k}$

.45867514538708189102 1... ≈ $1-\log(e-1) \;=\; \sum_{k=1}^{\infty}\frac{1}{e^k k} \;=\; \int_0^{\infty}\frac{dx}{e^x-1}$ J157

= $-\sum_{k=1}^{\infty}\frac{B_k}{k!k}$ [Ramanujan] Berndt Ch. 5

2 .45879503228928277960 6... ≈ $6\zeta(4)-12\zeta(3)+7\zeta(2)-\frac{9}{8} \;=\; \sum_{k=2}^{\infty}(-1)^k k^3\left(\zeta(k)-1\right)$

= $\sum_{k=2}^{\infty}\frac{8k^3+5k^2+4k+1}{k(k+1)^4}$

2 .4587950322892827796 1... ≈ $7\zeta(2)-12\zeta(3)+6\zeta(4)-\frac{9}{8} \;=\; \sum_{k=2}^{\infty}(-1)^k k^3(\zeta(k)-1)$

= $\sum_{k=2}^{\infty}\frac{8k^3+5k^2+4k+1}{k(k+1)^4}$

.45896737372945223436... ≈ $\cos(1+\sin 1)\,(\cosh\cos 1+\sinh\cos 1)$

= $\frac{e^{-i}}{2}\left(e^{2i+e^{i}}+e^{e^{-i}}\right) = \sum_{k=1}^{\infty}\frac{\cos k}{(k-1)!}$

22 .459157718361045473427... ≈ π^{e} Not known to be transcendental

2 .459168619823053134509... ≈ $\frac{1}{8}\left(4K^{2}\left(\frac{\sqrt{2}}{2}\right)+\frac{\pi^{2}}{K^{2}\left(\frac{\sqrt{2}}{2}\right)}\right) = \int_{0}^{1}E(x)\frac{dx}{x'}$ GR 6.151

.459349015616034745... ≈ $\frac{7\zeta(6)}{4} - \gamma\zeta(5) - \frac{\zeta^{2}(3)}{2} = \sum_{k=1}^{\infty}\frac{\psi(k+1)}{k^{5}}$

.45936268493278421889... ≈ $\frac{1}{\sqrt{2}}\sin\frac{1}{\sqrt{2}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k-1)!\,2^{k}}$

.4595386986862860465... ≈ $\prod_{k=1}^{\infty}\left(1-\frac{1}{2^{k}k^{2}}\right)$

.4596976941318602826... ≈ $2\sin^{2}\frac{1}{2} = 1-\cos 1 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k)!}$ GR 1.412.1

= $\int_{0}^{1}\sin x\,dx = \int_{1}^{e}\frac{\sin\log x}{x}dx$

= $\int_{1}^{\infty}\sin\left(\frac{1}{x}\right)\frac{dx}{x^{2}}$

3 .45990253977358391000... ≈ $\pi + \frac{1}{\pi}$

1 .459974052202470135268... ≈ $\sum_{k=2}^{\infty}k\left(\frac{\zeta(k)}{\zeta(k+1)}-1\right)$

.460047975918868477... ≈ $\pi\sqrt{3} - \frac{\pi^{2}}{12} - 6\log 2 = \sum_{k=1}^{\infty}\frac{(-1)^{k}}{3k^{3}-k^{2}}$

.46007559225530505748... ≈ $\left(2-\sqrt{2}\right)\frac{\pi}{4} = \frac{\pi}{2} - \frac{\pi}{2\sqrt{2}}$

= $\int_{0}^{\infty}\frac{dx}{(x^{2}+1)(x^{2}+2)}$

= $\int_{0}^{\pi/2}\frac{\sin^{2}x}{1+\sin^{2}x}dx = \int_{0}^{\pi/2}\frac{\cos^{2}x}{1+\cos^{2}x}dx$

= $\int_{0}^{\infty}\log\left(1+\frac{1}{8x^{2}+1}\right)dx$

.460265777326432934536... ≈ $9 - \pi e$

.460336876902524181992... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)}{k^k}$

.460344282619484866603... ≈ $e^{-1/8}\sqrt{\frac{\pi}{2}}\,erf\,i\left(\frac{1}{2\sqrt{2}}\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!!2^{2k+1}}$

1 .46035450880958681289... ≈ $-\zeta\left(\frac{1}{2}\right)$

1 .4603621167531195477... ≈ $G+\frac{\pi\log 2}{4} = \int_0^{\infty}\frac{\log(x+1)}{x^2+1}dx$

$= \int_1^{\infty}\frac{\log\left(x^2-1\right)}{1+x^2}dx$ GR 4.295.14

$= \int_0^{\infty}\frac{x}{e^x+2e^{-x}-2}dx$

$= -\int_0^{\pi/2}\frac{\cos x-\sin x}{\cos x+\sin x}dx$ GR 3.796.1

$= \int_0^{\pi\infty}\frac{\operatorname{arccot} x}{1+x}dx$ GR 4.531.2

$= \int_0^{1}\frac{\operatorname{arcsinh} x}{\sqrt{1\text{-}\mathrm{x}^2}}dx$

$= \int_0^{\pi/2}\log(1+\tan x)\,dx$ GR 4.227.10

1 .46057828082424381571... ≈ $\frac{1}{\sqrt{7}}\left(\pi+2\arctan\frac{1}{\sqrt{7}}\right) = \int_0^{\infty}\frac{dx}{x^2-x+2}$

.460794988828048829116... ≈ $\sum_{k=1}^{\infty}\frac{\tan k}{k!}$

.46096867284703433665... ≈ $\frac{1}{2}HypPFQ\left[\{1,1\},\left\{\frac{3}{2},2\right\},-\frac{1}{4}\right] = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(k-1)!}{(2k)!}$

.461020092468987023150... ≈ $\frac{1}{3^{5/3}}\left(\gamma-(-1)^{1/3}\left(\gamma+\psi\left(1+\frac{i3^{1/6}-3^{2/3}}{2}\right)\right)+(-1)^{2/3}\left(\gamma+\psi\left(1-\frac{i3^{1/6}+3^{2/3}}{2}\right)\right)\right)$

$= \sum_{k=1}^{\infty}\frac{1}{3k^2+k^{-1}} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(3k-1)}{3^k}$

1 .46103684685569942365... ≈ $\sum_{k=1}^{\infty}\frac{\tanh k}{k!}$

.46119836233472404263... ≈ $\frac{1}{\sqrt{2\cos 1}}\sin\frac{1}{2} = \sum_{k=0}^{\infty}\frac{(-1)^k(2k)!}{(k!)^2 4^k}\sin(2k+1)$ Berndt 9.4.19

.46139068323848162342... ≈ $\prod_{k=1}^{\infty}\left(1-\frac{1}{k^4+1}\right) = \lim_{j\to\infty}\frac{(j!)^4}{\prod_{k=1}^{j}\left(k^4+1\right)}$

.461392167549233569697... ≈ $\frac{3}{4}-\frac{\gamma}{2} = -\int_0^{\infty}\left(\frac{\cos x-1}{x^2}+\frac{1}{2(x+1)}\right)\frac{dx}{x}$ GR 3.783.1

= $\int_0^{\infty} x^5 e^{-x^2}\log x\,dx$

.461455316241865234416... ≈ $-\frac{\sqrt{e}}{2}Ei\left(-\frac{1}{2}\right)$

1 .4615009763662596165... ≈ $\prod_{k=2}^{\infty}\frac{2\zeta(k)}{\zeta(k)+1}$

1 .46168127916026790076... ≈ $\sum_{k=2}^{\infty}\frac{k\log\zeta(k)}{k-1}$

.46178881838605297087... ≈ $\frac{9-2\sqrt{3}+12\log 2}{30} = \int_0^1 x^4\log\left(1+\frac{1}{x^3}\right)dx$

.461939766255643378064... ≈ $\frac{\sqrt{2+\sqrt{2}}}{4} = \cos\frac{\pi}{3}\sin\frac{3\pi}{8}$

.462098120373296872945... ≈ $\frac{2\log 2}{3} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(3k+1)(3k+2)}$

= $\int_0^1 x^2\log\left(1+\frac{1}{x^3}\right)dx$

= $-\int_0^1\frac{\log(1-x^6)}{x^4}dx$

.462117157260009758502... ≈ $\frac{e-1}{e+1} = \tanh\frac{1}{2} = 2\sum_{k=1}^{\infty}\frac{(2^{2k}-1)B_{2k}}{(2k)!}$ AS 4.5.64

36 .462159607207911770991... ≈ π^{π}

1 .462163614976201276864... ≈ $\frac{4\pi^2}{27} = G_2 = \frac{8\zeta(2)}{9}$ J309

= $\sum_{k=1}^{\infty}\left(\frac{1}{(3k-1)^2}+\frac{1}{(3k-2)^2}\right)$

= $\int_0^1\frac{\log x\,dx}{x^3-1}$

= $2-\int_0^1\frac{1+x}{1+x^3}\log x\,dx$ Πρυδ. 2.6.8.5

1 .46243121900695980136... ≈ $\prod_{p\ prime}\left(1+\frac{1}{2^{p}}\right)$

1 .462432881399160603387... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)}{k!!}$

.46260848376583338324... ≈ $\sum_{k=2}^{\infty}\left(\left(\zeta(k)-1\right)^{1/k}-\frac{1}{2}\right)$

1 .4626517459071816088... = $\frac{\sqrt{\pi}}{2}erfi\ 1 \ = \ \sum_{k=0}^{\infty}\frac{1}{k!(2k+1)}$

3 .462746619455063611506... ≈ $\prod_{k=1}^{\infty}\left(1+\frac{1}{2^{k}-1}\right) \ = \ \prod_{k=1}^{\infty}\frac{1}{(1-2^{-k})} \ = \ \sum_{k=1}^{\infty}\frac{p(k)}{2^{k}}$

.46287102628419511533... ≈ $20\log\frac{5}{4}-4 \ = \ \sum_{k=0}^{\infty}\frac{(-1)^{k}}{4^{k}(k+1)(k+2)}$

.4630000966227637863... ≈ $\frac{1}{2}\left(1-\pi^{2}\operatorname{csch}^{2}\pi\right)$

= $\sum_{k=1}^{\infty}(-1)^{k+1}k\left(\zeta(2k)-\zeta(2k+2)\right)$

= $\sum_{k=2}^{\infty}\frac{k(k-1)}{(k^{2}+1)^{2}} \ = \operatorname{Re}\left\{\psi^{(1)}(1+i)\right\}$

= $\int_{0}^{\infty}\frac{x\cos x}{e^{x}-1}dx \ = \ \int_{0}^{\infty}\frac{x\cos x}{e^{x}\left(e^{x}-1\right)}dx$

320 .46306452510147901007... ≈ $\frac{\pi^{6}}{3}$

.463087110750218534897... ≈ $\frac{\pi}{4}\left(-\frac{1}{3}\right)^{3/4}\left(\cot\left(\pi\left(-\frac{1}{3}\right)^{1/4}\right)-i\cot\left(\frac{\pi\left(-1\right)^{3/4}}{3^{1/4}}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{1}{3k^{2}+k^{-2}}$

= $\sum\left(-1\right)^{k+1}\frac{\zeta(4k+2)}{3^{k}}$

.46312964115438878499... ≈ $2\operatorname{arcsinh}^{2}\frac{1}{2} \ = \ 2\log^{2}\frac{1+\sqrt{5}}{2} \ = \ \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{\binom{2k}{k}k^{2}}$

3 .463293989409197163639... ≈ 6γ

.4634219926631045735... ≈ $e\left(Ei(-2)-Ei(-1)\right)=\int_{0}^{1}\frac{dx}{e^{x}(1+x)}$

55 .46345832232543673233... ≈ $\frac{319735800}{5764801} \ = \ \sum_{k=1}^{\infty}\frac{k^{8}}{8^{k}}$

.463518463518$\underline{463518}$ = $\frac{6}{13}$

1 .4636111111111111111$\underline{1}$ = $\frac{5269}{3600} = H^{(3)}{}_5$

.46364710900080611621... ≈ $\arctan\frac{1}{2} = \arcsin\frac{1}{\sqrt{5}}$

= $\operatorname{Im}\left\{\log(2+i)\right\} = \operatorname{Re}\left\{i\log\left(1-\frac{i}{2}\right)\right\}$

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{2k+1}(2k+1)}$

= $\sum_{k=1}^{\infty}\frac{(2k)!!}{(2k-1)!!\,5^k\,k}$

= $\sum_{k=0}^{\infty}\arctan\left(\frac{2}{(2k+3)^2}\right)$ [Ramanujan] Berndt Ch. 2

= $\sum_{k=1}^{\infty}(-1)^{k+1}\arctan\left(\frac{2}{(k+1/\phi)^2}\right)$ [Ramanujan] Berndt Ch. 2, Eq. 7.5

.46388055525527722680... ≈ $\frac{e}{e+\pi} = \sum_{k=1}^{\infty}(-1)^{k+1}\left(\frac{e}{\pi}\right)^k$

3 .46410161513775458705... ≈ $\sqrt{12} = 2\sqrt{3}$

.46416351576125970131... ≈ $\sum_{k=1}^{\infty}\frac{1}{e^k+1} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{e^k-1}$ Berndt 6.14.1

.46418436079435943653 6... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k}{k^3-6}$

1 .46451508680225823041... ≈ $9 - \zeta(2) - \zeta(4) - 4\zeta(3) = \sum_{k=2}^{\infty}(-1)^k k^2\left(\zeta(k)-\zeta(k+3)\right)$

.46453645613140711824... ≈ $9e - 24 = \sum_{k=1}^{\infty}\frac{k}{k!(k+4)}$

24 .46453645613140711824... ≈ $9e$

1 .46459188756152326302... ≈ $\pi^{1/3}$

.46460183660255169040... ≈ $\frac{5-\pi}{4}$ AMM 101,8 p. 732

.46481394722935684757... ≈ $\frac{\pi}{\sqrt{3}}\tanh\frac{\pi\sqrt{3}}{2} - \frac{4}{3} = \sum_{k=2}^{\infty}\frac{1}{k^2+k+1}$

= $\frac{i\sqrt{3}}{3}\left(\psi\left(\frac{5-i\sqrt{3}}{2}\right) - \psi\left(\frac{5+i\sqrt{3}}{2}\right)\right)$

= $\sum_{k=1}^{\infty}\left(\zeta(3k-1)-\zeta(3k)\right)$

.464954643258938815496... ≈ $\sqrt{2}\arctan\frac{1}{\sqrt{2}}-\log\frac{3}{2}=\sum_{k=0}^{\infty}\frac{(-1)^k}{2^k(2k+1)(2k+2)}$

= $\int_0^{\infty}\log\left(1+\frac{1}{2(x+1)^2}\right)dx$

1 .465052383336634877609... ≈ $\frac{2}{\pi}\sinh\frac{\pi}{2}=\binom{0}{i/2}=\prod_{k=1}^{\infty}\left(1+\frac{1}{4k^2}\right)$

.46520414169536317620... ≈ $\frac{e^{1/3}}{3}=\sum_{k=1}^{\infty}\frac{k}{k!3^k}$

.4653001813290246588... ≈ $\sum_{k=2}^{\infty}\left(\zeta(k)-1\right)^2$

1 .4653001813290246588... ≈ $\sum_{k=2}^{\infty}\left(\zeta^2(k)-\zeta(k)\right)$

2 .4653001813290246588... ≈ $\sum_{k=2}^{\infty}\left(\zeta^2(k)-1\right)=\sum_{k=2}^{\infty}\frac{\sigma_0(k)}{k(k-1)}$

.465370005065473114648... ≈ $2\log 2+\frac{3\zeta(3)}{4}-\frac{\pi^2}{12}-1=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^4+k^3}$

.465461510476142649489... ≈ $\frac{\sqrt{3}}{\pi}+\frac{1}{3}-\frac{\log(2+\sqrt{3})}{\pi}$ Discr. Comp. Geom 22:105(1999)

3 .465735902799726547086... ≈ $5\log 2$

.4657596075936404365... ≈ $\frac{I_0(1)}{e}=\frac{I_0(-1)}{e}=\sum_{k=0}^{\infty}\frac{(-1)^k}{k!2^k}\binom{2k}{k}$

.466099528283328703461... ≈ $\gamma-1+\frac{1}{2}\left(\psi(2-i)+\psi(2+i)-i\psi^{(1)}(2-i)+i\psi^{(1)}(2+i)\right)$

= $\sum_{k=1}^{\infty}(-1)^{k+1}(2k+1)(\zeta(2k+1)-1)=\sum_{k=2}^{\infty}\frac{3k^2+1}{k(k^2+1)^2}$

.46618725267275587265... ≈ $\sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)\left(\zeta(k)-1\right)}{k}$

.466942206924259859983... ≈ $\frac{1}{\pi-1}=\sum_{k=1}^{\infty}\frac{1}{\pi^k}$

= $\prod_{k=0}^{\infty}\left(1+\frac{1}{\pi^{2^k}}\right)$ Prud. 6.2.3.1

1 .466942206924259859983... ≈ $\frac{\pi}{\pi-1}=\sum_{k=0}^{\infty}\frac{1}{\pi^k}$

.467058656500326469824... ≈ $Li_2\left(\frac{2}{3}\right)-Li_2\left(\frac{1}{3}\right)$

= $\sum_{k=1}^{\infty}\left(\log\left(1+\frac{1}{k}\right)+\log\Gamma\left(1+\frac{1}{k}\right)+\frac{\gamma-1}{k}\right)$

1 .467078079433975472898... ≈ $\frac{6\log 2}{\pi^2}\left(3\log 2+4\gamma-\frac{24\zeta'(2)}{\pi^2}-2\right)-\frac{1}{2}$, Porter's constant

.46716002464644797643... ≈ $1-\sqrt{2}+\operatorname{arcsinh}1 = \int_0^1 \operatorname{arcsinh} x\,dx$

.467164421397788702939... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)-\zeta(2k+1)}{k!} = \sum_{k=2}^{\infty}\frac{k-1}{k}\left(e^{1/k^2}-1\right)$

.467401100272339654709... ≈ $\frac{\pi^2}{4}-2 = \sum_{k=1}^{\infty}(-1)^{k+1}k\,\frac{\zeta(k+1)}{2^k}$

= $\sum_{k=2}^{\infty}\frac{(k-1)(\zeta(k)-1)}{2^{k-1}}$

= $\sum_{k=1}^{\infty}\frac{(k-\frac{1}{2})!}{(k+\frac{1}{2})!(2k+1)}$

≈ $\int_0^{\pi/2} x^2\cos x\,dx = \int_0^1 \arcsin^2 x\,dx$

1 .46740110027233965471... ≈ $\frac{\pi^2}{4}-1 = \sum_{k=1}^{\infty}\frac{(2k)!!}{(2k-1)!!\,k^2(k+1)}$

2 .46740110027233965471... ≈ $\frac{\pi^2}{4} = -\log^2 i = \arcsin^2 1$

= $\sum_{k=1}^{\infty}\frac{k\zeta(k+1)}{2^k} = \sum_{k=1}^{\infty}\frac{2}{(2k-1)^2} = \sum_{k=1}^{\infty}\frac{k^2}{(k^2-1/4)^2}$

= $\sum_{k=1}^{\infty}(-1)^k\,\frac{k\pi^{2k}}{(2k)!}$

= $\int_0^{\infty}\frac{x\,dx}{\sinh x}$

= $\int_0^{\pi}\frac{x\sin x\,dx}{1+\cos^2 x}$ Borwein-Devlin, p. 50

= $\int_0^{\infty}\frac{\log x}{(x+1)(x-1)}\,dx = \int_0^1 K(k')\,dk$

3 .46740110027233965471... ≈ $\frac{\pi^2}{4}+1 = \sum_{k=1}^{\infty}\frac{(2k)!!}{(2k-1)!!\,k(k+1)}$

1 .46746220933942715546... ≈ $E\left(\frac{1}{4}\right)$

.467613876007544485395... ≈ $\frac{27\log 3}{16} - 2\log 2 = \int_0^\infty \frac{\sin^6 x}{x^5}\,dx$

1 .467813645677547003322... ≈ $\sum_{k=1}^\infty \left(2^{1/k^2} - 1\right)$

.4678273201195659261... ≈ $\prod_{k=1}^\infty \left(1 - \frac{1}{2^{k^2}}\right)$

1 .467891949359644150296... ≈ $\frac{\pi + 2\log\left(\sqrt{2}+1\right)}{\sqrt{2}} - 2 = \int_0^\infty \frac{x\,dx}{(1+x^2)\sin(\pi x/4)}$ GR 3.552.9

3 .467891949359644150296... ≈ $\frac{\pi + 2\log\left(\sqrt{2}+1\right)}{\sqrt{2}} = \sum_{k=0}^\infty \frac{(-1)^k}{k+1/4}$

1 .467961779578029614731... ≈ $8\log 2 - 2\gamma - \frac{7}{2} = \sum_{k=1}^\infty \frac{\psi(k+3)}{2^k}$

.46804322686252540322... ≈ $\frac{\pi}{6}\coth 3\pi - \frac{1}{18} = \sum_{k=1}^\infty \frac{1}{k^2+9}$ J124

3 .468649618760533141431... ≈ $\frac{1}{\sqrt{3}} I_1\left(2\sqrt{3}\right) = \sum_{k=0}^\infty \frac{3^k}{k!(k+1)!}$

.4687500000000000000$\underline{0}$ = $\frac{15}{32} = \sum_{k=1}^\infty \frac{k^2}{5^k}$

.46894666977688414590... ≈ $\Gamma\left(\frac{3}{4}\right)\cos\frac{3\pi}{8} = \int_0^\infty \frac{\sin\left(x^4\right)}{x^2}\,dx$

.46921632103286066895... ≈ $\sum_{k=2}^\infty H_{k-1}(\zeta(k)-1) = \sum_{k=2}^\infty \frac{1}{k(k-1)}\log\frac{k}{k-1}$

= $\sum_{k=1}^\infty \frac{\zeta(2k)-\zeta(2k+1)}{k} = 2\sum_{k=2}^\infty \frac{\log k}{k^3-k}$

= $\sum_{n=1}^\infty \left(-1 + \sum_{k=1}^\infty H^{(n)}{}_k\left(\zeta(k+1)-1\right)\right)$

= $\sum_{k=2}^\infty \left(\frac{1}{k^3+k^2} + \frac{1}{k^3-k^2}\right)\log k$

2 .469506314521047562476... ≈ $\sum_{k=1}^\infty \frac{1}{\sqrt{k!}}$

32 .46969701133414574548... ≈ $\frac{\pi^4}{3}$

.469981161956636124706... ≈ $\frac{\pi^2}{21}$

1 .47043116414999108828... ≈ $\frac{2\sinh\pi}{5\pi} = \prod_{k=1}^{\infty}\left(1+\frac{1}{(k+2)^2}\right)$

.4705882352941176 = $\frac{8}{17}$

.470699046351326775891... ≈ $\frac{\pi\sqrt{3}}{2} - \frac{9}{4} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k(k+2/3)}$

1 .470801229745552347742... ≈ $\frac{\pi\sqrt{3}}{2}\coth\pi\sqrt{3} - \frac{5}{4} = \sum_{k=2}^{\infty}\frac{3}{k^2+3}$

= $\sum_{k=1}^{\infty}(-1)^{k+1}3^k\left(\zeta(2k)-1\right)$

.4709786279302689616... ≈ $\sum_{k=1}^{\infty}\frac{1}{3^k\,\phi(k)}$

.471246045438685502741... ≈ $\frac{\pi^2}{2} + \frac{\pi^4}{8} - \frac{21}{2}\zeta(3) - \frac{31}{8}\zeta(5) = \sum_{k=1}^{\infty}\frac{k^3}{(k+1/2)^5}$

.471392596691172030430... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)}{k(2k+1)} = \sum_{k=1}^{\infty}\left(\log\left(1+\frac{1}{k^2}\right) - 2 + 2k\arctan\frac{1}{k}\right)$

1 .471517764685769286382... ≈ $\frac{4}{e} = \int_0^{\infty} e^{-\sqrt{1+x}}\,dx$

403 .4715872121057150966... ≈ $\frac{1}{4}\left(e^{e^2} + e^{e^{-2}}\right) - \frac{e}{2} = \sum_{k=0}^{\infty}\frac{\sinh^2 k}{k!}$

5 .47168118871888519799... ≈ $\frac{\zeta(3)-1}{\zeta(5)-1}$

1 .471854360049... ≈ $\sum_{k=1}^{\infty}\left(\frac{\zeta(k)\zeta(k+1)}{\zeta(k^2+k)} - 1\right)$

8 .472010016955485001797... ≈ $-\frac{1}{\sqrt{2}}\sinh\pi\ \csc\pi\sqrt{2} = \prod_{k=1}^{\infty}\frac{k^2+2k+2}{k^2+2k-1}$

4 .47213595499957939282... ≈ $\sqrt{20} = 2\sqrt{5}$

.472597844658896874619... ≈ $-Li_3\left(-\frac{1}{2}\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^k k^3}$

3 .47273606009117550064... ≈ $\Gamma\left(\frac{5}{2}\right)\zeta\left(\frac{3}{2}\right) = \frac{3\sqrt{\pi}}{4}\zeta\left(\frac{3}{2}\right) = \int_0^{\infty}\frac{x^{3/2}}{e^x + e^{-x} - 2}$

= $\int_0^{\infty}\frac{dx}{e^{x^{2/3}} - 1}$

.47279971743743015582... ≈ $\frac{4\pi\sqrt{3}}{27} - \frac{1}{3} = \int_0^{\infty}\frac{dx}{(x^2+x+1)^2}$

1 .47279971743743015582... ≈ $\frac{2}{3}+\frac{4\pi}{9\sqrt{3}} = \sum_{k=1}^{\infty}\frac{1}{\binom{2k-1}{k}}$

1 .47282823195618529629... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)}{(2k-1)!} = \sum_{k=1}^{\infty}\frac{1}{k}\sin\frac{1}{k}$

.47290683729585510379... ≈ $6-4\cos 1-4\sin 1 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k-1)!\,k(k+1)}$

.47304153518359150747... ≈ $\frac{si(1)}{2} = \int_{1}^{\infty}\sin\left(\frac{1}{x^2}\right)\frac{dx}{x}$

.473050738552108261... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^5+1}$

1 .47308190605019222027... ≈ $\frac{3\sqrt{e}}{2}-2 = \sum_{k=1}^{\infty}\frac{k-1}{k!\,2^k} = \sum_{k=0}^{\infty}\frac{k^2}{(k+1)!\,2^k}$ GR 1.212

.4730953995155560361013... ≈ $\frac{\log 6}{2}+\gamma-1 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}2^k}{k+1}\left(\zeta(k+1)-1\right)$

.473406103908474284834... ≈ $\frac{\pi}{4\sqrt{6}}\tan\pi\sqrt{\frac{3}{2}}+\frac{1}{5} = \sum_{k=1}^{\infty}\frac{1}{4k^2+4k-5}$

.47351641474862295879... ≈ $\frac{7\pi^4}{1440} = \frac{7\zeta(4)}{16} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2k^4}$

.473684210526315789 = $\frac{9}{19}$

771 .4742498266672251905 36... ≈ $-\psi^{(4)}\left(\frac{1}{2}\right) = 744\zeta(5)$

1 .47443420371745989911... ≈ $\frac{7\zeta(3)}{4}-2\log 2-\frac{\pi^2}{4}\log 2+\frac{\pi^2}{4} = \sum_{k=1}^{\infty}\frac{H_k}{(2k-1)^2}$

.47474085555749076227... ≈ $\frac{\gamma\pi^2}{12}$

.47479960260022965741... ≈ $2-\zeta(2)-\zeta(4)+\zeta(3) = \sum_{k=2}^{\infty}(-1)^k\left(\zeta(k)-\zeta(k+3)\right)$

= $\sum_{k=2}^{\infty}\frac{k^3-1}{k^5+k^4}$

.47480157230323829219... ≈ $2\log\frac{6}{3+\sqrt{3}} = \sum_{k=1}^{\infty}\frac{1}{6^k k}\binom{2k}{k}$

.4749259869231265718... ≈ $\pi-\frac{8}{3} = hg\left(\frac{3}{4}\right)-hg\left(\frac{1}{4}\right) = \int_{0}^{\pi}\frac{\sin^2 x}{(1+\sin x)^2}dx$

.474949379987920650333... ≈ $2^{5/4}\sqrt{\pi}\, e^{\pi/8}\,\Gamma^{-2}\left(\frac{1}{4}\right)$, Weierstrass constant

1 .474990335830026928404... ≈ $\sum_{k=1}^{\infty}\frac{1}{k!\sqrt{k}}$

1 .47514894960971573533... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^k+k-2}$

.47521841920391290915 3... ≈ $\sum_{k=1}^{\infty}\frac{\Phi(k)}{4^k}$

.47522253880723538764 9... ≈ $\frac{e^{-3^{1/3}}}{3^{5/3}}\left(1+e^{3^{4/3}/2}\left(\sqrt{3}\sin\frac{3^{5/6}}{2}-\cos\frac{3^{5/6}}{2}\right)\right) = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{3^k k}{(3k)!}$

1 .4752572671352997213... ≈ $\int_0^{\infty}\frac{x^2\,dx}{e^x+x^2}$

7 .47545308623246530741 6... ≈ $\sum_{k=2}^{\infty}\frac{\zeta^4(k)}{\zeta(2k-1)}$

.47572809161053958879 9... ≈ $Ai\left(-\frac{1}{2}\right)$

1 .47577316159455206927 7... ≈ $7^{1/5}$

.47591234810986255573... ≈ $\int_1^{\infty}x\left(\zeta(2x)-1\right)dx$

.47597817593456487296... ≈ $\sum_{k=1}^{\infty}\frac{\mu(k)\log\zeta(2k)}{k^2}$

.4760284515797971427... ≈ $\frac{\zeta(3)}{2}-\frac{1}{8} = \sum_{k=1}^{\infty}k^2\left(\zeta(2k+1)-1\right) = \sum_{k=2}^{\infty}\frac{k(k^2+1)}{(k^2-1)^3}$

1 .47620871294871749614 7... ≈ $\pi^2+\frac{27}{4}-\frac{3}{2}\psi^{(1)}\left(\frac{1}{3}\right) = \sum_{k=1}^{\infty}\frac{k}{(k^2-1/9)^2}$

.476222388197391301... ≈ $1-\frac{I_0(2)+I_1(2)}{e^2} = 1-{}_1\mathrm{F}_1\left(\frac{1}{2},2,-4\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k+1)!}\binom{2k}{k}$

1 .47625229601045971015... ≈ $\sum_{k=2}^{\infty}\frac{\zeta^3(k)}{2^k}$

1 .47648936572856286549 9... ≈ $\frac{\pi^3}{21}$

.47665018998609361711... ≈ $-\frac{1}{3}-\frac{\pi}{2\sqrt{3}}\cot\pi\sqrt{3} = \sum_{k=1}^{\infty}\frac{1}{k^2+4k+1}$

1 .47665018998609361711... ≈ $\frac{2}{3}-\frac{\pi}{2\sqrt{3}}\cot\pi\sqrt{3} = \sum_{k=2}^{\infty}\frac{1}{k^2-3}$

.4769362762044693977... ≈ $erf^{-1}\left(\frac{1}{2}\right)$

.47712125471966243729 5... ≈ $\log_{10}3$

5 .47722557505166113457... ≈ $\sqrt{30}$

.47746482927568600731... ≈ $\frac{3}{2\pi}$

.47764546494388936239... ≈ $\log\left(\frac{\sinh\sqrt{\pi}}{\sqrt{\pi}}\right)$

= $-\log\left(\Gamma\left(1+\frac{i}{\sqrt{\pi}}\right)\Gamma\left(1-\frac{i}{\sqrt{\pi}}\right)\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}\zeta(2k)}{\pi^k k}$

28 .47765864997501086772 1... ≈ $\frac{2\pi^2}{\log 2}$ [Ramanujan] Berndt Ch. 22

.47794581921291817082 7... ≈ $8-6\log 2-7\log^2 2 = \sum_{k=1}^{\infty}\frac{k^2 H_k}{2^k (k+1)(k+2)}$

.47800599951775938643 4... ≈ $\frac{3\zeta(3)}{4}-\log 2+\frac{1}{4}\left(\psi\left(\frac{2+i}{2}\right)+\psi\left(\frac{2-i}{2}\right)-\psi\left(\frac{1+i}{2}\right)-\psi\left(\frac{1-i}{2}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^5+k^3}$

.47806995958639115713 1... ≈ $\frac{G}{1+G}$

39 .47841760435743447534... ≈ $4\pi^2$

.47842896339334824727 0... ≈ $\frac{3\pi^2}{8}-\log 2-\frac{7\zeta(3)}{4}-\frac{23}{54} = \sum_{k=2}^{\infty}\frac{(-1)^k k^2}{2^k}\left(\zeta(k)-1\right)$

.47854249283227585569... ≈ $\sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)-1}{(k-2)!} = \sum_{k=2}^{\infty}\frac{1}{k^2 e^{1/k}}$

1 .47854534395739361903... ≈ $\frac{\pi}{4}+\log 2 = \sum_{k=0}^{\infty}(-1)^k\left(\frac{1}{k+1}+\frac{1}{2k+1}\right)$

.47893467779382437553... ≈ $4\left(\zeta(3)-\zeta(4)\right)$

.47923495811102680807... ≈ $\frac{\pi}{\sqrt{2}}\csc\frac{\pi}{2\sqrt{2}}-2 = \operatorname{Im}\left\{(-1)^{1/6}\right\} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2k^2-1/4}$

.47942553860420300027 3... ≈ $\sin\frac{1}{2} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!2^{2k+1}}$ AS 4.3.65

= $\frac{\sqrt{\pi}}{4}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!(k+\frac{1}{2})16^k}$

.47952808215101070284 2... ≈ $J_2\left(2\sqrt{2}\right) = \sum_{k=0}^{\infty}(-1)^k\frac{2^{k+1}}{k!(k+2)!}$

.47962348400112945189... ≈ $2\gamma-2ci(1) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k)!k}$

2 .47966052573232990761... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)}{(k-1)!} = \sum_{k=1}^{\infty}\frac{e^{1/k}-1}{k}$

	.47987116980744146966...	≈	$\frac{\pi}{3\sqrt{3}\cdot 2^{1/3}} = \int_0^\infty \frac{dx}{x^3+4}$	
	.4800000000000000000$\underline{0}$	=	$\frac{12}{25} = \sum_{k=1}^\infty \frac{F_k^{\ 2}}{4^k}$	
	.48022960406139837837...	≈	$\sum_{k=2}^\infty \frac{\zeta(k)-1}{\sigma_0(k)}$	
3	.480230906913262026939...	≈	$\left(\gamma + 2\log 2\right)\pi = -\int_0^1 \log\left(\log\frac{1}{x}\right)\frac{dx}{\sqrt{\log\frac{1}{x}}}$	GR 4.229.3
	.480453013918201424667...	≈	$\log^2 2 = \sum_{k=1}^\infty \frac{H_k}{2^k(k+1)} = 2\sum_{k=1}^\infty (-1)^k \frac{H_k}{k+1}$	
		=	$\int_0^\infty \log\left(\frac{(x+1)(x+4)}{(x+2)^2}\right)\frac{dx}{x}$	GR 4.299.1
2	.480453013918201424667...	≈	$2 + \log^2 2 = \sum_{k=1}^\infty \frac{k\ H_k}{2^k(k+1)}$	
1	.48049223762892856680...	≈	$\sum_{k=1}^\infty \frac{\phi(2k-1)}{2^{2k-1}-1}$	
	.48053380079607358092...	≈	$\sum_{k=1}^\infty \frac{(-1)^{k+1}}{k!\zeta(2k+1)}$	Titchmarsh 14.32.3
2	.480548302158708391604...	≈	$\pi^2 - e^2$	
	.48061832131527820986...	≈	$\frac{1}{2}\left(3\zeta(3) - \zeta(2) - 1\right) = \sum_{k=1}^\infty \frac{H_k H_k}{k(k+1)(k+2)}$	
1	.480645673830658464567...	≈	$\sum_{k=1}^\infty \frac{2^k\left(\zeta(2k)-1\right)}{k!} = \sum_{k=2}^\infty \left(e^{2/k^2} - 1\right)$	
1	.480716542675186062751...	≈	$\sum_{k=1}^\infty 2^k\left(\zeta(3k-1)-1\right) = \sum_{k=2}^\infty \frac{2k}{k^3-2}$	
6	.48074069840786023097...	≈	$\sqrt{42}$	
	.48082276126383771416...	≈	$\frac{2\zeta(3)}{5} = \sum_{k=1}^\infty \frac{(-1)^{k+1}k!k!}{(2k)!k^3}$	
	.480898346962987802453...	≈	$\frac{1}{\log 8}$	
	.481008115373192720896...	≈	$\pi - 2\left(\arctan 2 - 2\log 2 + \log 5\right) = 2\arctan\frac{1}{2} - 2\log\frac{5}{4}$	

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{4^k(2k+1)(2k+2)} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^{2k-1}k(2k-1)}$

= $\int_0^{\infty}\log\left(1+\frac{1}{(x+2)^2}\right)dx$

2 .481061019790762697937... ≈ $\sum_{k=1}^{\infty}\frac{\sigma_0(k)}{k!} = \sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\frac{1}{(jk)!}$

.481211825059603447498... ≈ $\log\frac{1+\sqrt{5}}{2} = \log\phi = \operatorname{arccsch}2 = \log\left(2\cos\frac{\pi}{5}\right)$

= $\sum_{k=1}^{\infty}\frac{\cos(\pi k/5)}{k}$

.48164052105807573135... ≈ $2+2\zeta(2)-4\zeta(3) = \int_1^{\infty}\frac{\log^2 x}{x^2(x-1)^2}dx$

4 .481689070338064822602... ≈ $e^{3/2} = \sum_{k=0}^{\infty}\frac{3^k}{k!2^k}$

.48180838242836517607... ≈ $6-\frac{15}{e} = \int_0^1 e^{-x^{1/3}}dx$

.48200328164309555398... ≈ $\frac{\pi(1-\log 2)}{2} = -\int_0^{\pi/2}\left(\log\sin x\right)\tan^2 x\,dx$

.482301750646382872131... ≈ $2\zeta(3)-4\log^2 2 = \sum_{k=1}^{\infty}\frac{H_k}{k^2(2k+1)}$

.483128950540980889382... ≈ $2\log\frac{4}{\pi} = \int_0^{\infty}\frac{\log(1+x^2)}{\cosh\left(\pi x/2\right)}dx$ GR 4.373.2

2 .483197662162246572967... ≈ $\frac{15}{8}+\frac{3}{2}\log\frac{3}{2} = \sum_{k=1}^{\infty}\frac{k^2H_k}{3^k}$

.483204576426538793772... ≈ $\sum_{k=2}^{\infty}\sum_{a=1}^{\infty}\frac{1}{k}\left(\zeta(ak)-1\right)$

7 .4833147735478827712... ≈ $\sqrt{56} = 2\sqrt{14}$

6 .4834162225162414100... ≈ $\sum_{k=2}^{\infty}\frac{\log^3 k}{k(k-1)} = -\sum_{m=2}^{\infty}\zeta^{(3)}(m)$

1 .483522817309552286419... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^k\zeta(4k+2)}{(2k+1)!} = \sum_{k=1}^{\infty}\sin\frac{1}{k^2}$

1 .48355384697159722986... ≈ $-\frac{1}{2}Li_3(-4) = \frac{\pi^2\log 2}{6}+\frac{2\log^3 3}{3}-\frac{1}{2}Li_3\left(-\frac{1}{4}\right)$

$= \quad 8\int_1^\infty \frac{\log^2 x}{x^3+4x} = \int_0^\infty \frac{x^2\,dx}{e^x+4}$

.48415067138157489915 1... $\approx \quad \frac{\gamma}{2} + \frac{1}{4}\left(\psi(1+i\sqrt{2}) + \psi(1-i\sqrt{2})\right)$

.48430448328550912049... $\approx \quad \frac{4-\pi}{\sqrt{\pi}} = \sum_{k=1}^\infty \frac{(k-1)!}{(k+\frac{1}{2})!2^k}$ Dingle p. 70

1 .4844444656045474898... $\approx \quad \frac{\gamma}{3} + \frac{\pi^2}{18} + \frac{2\gamma\log 3}{3} + \frac{\log^2 3}{3} = \int_0^\infty \frac{\log^2 x dx}{e^{3x}}$

.48447307312968469024 2... $\approx \quad \frac{\pi^3}{64}$

.48451065928275476522... $\approx \quad \sum_{k=1}^\infty \left(\zeta^2(3k) - 1\right)$

.48482910699568764631... $\approx \quad \frac{Ei(1)-\gamma}{e} = \sum_{k=1}^\infty (-1)^k \frac{H_k}{k!}$

$= \quad -\int_0^1 \frac{\log(1-x)}{e^x} dx$

8 .4852813742385702928... $\approx \quad \sqrt{72} = 6\sqrt{2}$

.48537125676538642635 9... $\approx \quad \frac{5\pi^2}{48} - \frac{\pi}{2} + \frac{\log^2 2}{4} + Li_2\left(\frac{1+i}{2}\right) + Li_2\left(\frac{1-i}{2}\right)$

$= \quad \int_0^\infty \frac{\log(1+x^2)}{x(1+x)^2}$

1 .48539188203274355242... $\approx \quad \frac{45\sqrt{2}}{7\pi}\sin 2\pi\sqrt{2} = \prod_{k=1}^\infty \left(1+\frac{1}{k(k+6)}\right)$

.48540194215038792366 5... $\approx \quad \frac{\pi}{6\sqrt{3}} + \frac{\log 3}{6} = \sum_{k=0}^\infty \frac{(-1)^k}{2^{3k+1}(3k+1)}$ Berndt 8.14.1

$= \quad \int_2^\infty \frac{dx}{x^2+x^{-1}}$

.4857896527073049465... $\approx \quad \sum_{k=1}^\infty \frac{(-1)^{k+1}}{k^6+1}$

.48600607487864246273... $\approx \quad \frac{2\pi\sqrt{3}}{3} - \pi = \int_0^\infty \log\frac{1+x^{-3}}{1+x^{-2}} dx$

$= \quad \int_0^\infty \log\left(1+\frac{1}{3(x^2+1)}\right) dx$

$= \quad \int_0^\pi \frac{\cos x}{2-\cos x} dx$

1	.486276286405273929718...	≈	$4G - \pi\log 2 = -\int_0^{\pi} \frac{x\cos x}{1+\sin x}\,dx$	GR 3.791.2
		=	$\int_0^{\pi} \log(1+\sin x)\,dx$	GR 4.224.10
4	.48643704003032676232...	≈	$\zeta(4) + 2\zeta(3) + 1 = \sum_{k=1}^{\infty} \binom{k+3}{k}\left(\zeta(k+2)-1\right)$	
9	.486832980505137996...	≈	$\sqrt{90}$	
1	.48685779828884403099...	≈	$\sum_{k=1}^{\infty} \phi(k)\left(\zeta(k)-1\right)$	
5	.48691648995407706207...	≈	$\log 24 + 4\gamma = \sum_{k=1}^{\infty} \left(\frac{4}{k} - \log\frac{k+4}{k}\right)$	Prud. 5.5.1.15
	.487212707001281468487...	≈	$10\sqrt{e} - 16 = \sum_{k=0}^{\infty} \frac{1}{k!\,2^k (k+3)}$	
	.48726356479168678841...	≈	$\frac{\pi}{2\sqrt{2}\,3^{3/4}} = \int_0^{\infty} \frac{dx}{x^4+3}$	
	.48749549439936104836...	≈	$\frac{\pi}{2\sqrt{2}} + \frac{\log\left(3-2\sqrt{2}\right)}{2\sqrt{2}} = \int_0^{\pi/4} \sqrt{\tan x}\,dx$	
1	.487577370170670626342...	≈	$1 + \frac{\pi}{\sqrt{2}}\operatorname{csch}\frac{\pi}{\sqrt{2}} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k^2+1/2}$	
3	.48786197086364633594...	≈	$\sum_{k=2}^{\infty} \frac{\zeta^2(k)}{(k-1)^2}$	
	.48808338775163574003...	≈	$-\frac{15}{364} - \frac{\pi}{4\cdot 14^{3/4}}\left(\csc\left(\pi\sqrt[4]{14}\right) + \operatorname{csch}\left(\pi\sqrt[4]{14}\right)\right)$	
		=	$\sum_{k=2}^{\infty} \frac{(-1)^k}{k^4-14}$	
	.4882207515238431339...	≈	$-\sum_{k=1}^{\infty} \frac{\mu(k)k}{2^k}$	
	.48837592811772608174...	≈	$\frac{3}{4} + 2\log 2 - \frac{3\log 3}{2} = \int_0^1 x\log(x+2)\,dx$	
	.48854802693053907803...	≈	$2\sqrt{2}\arctan\sqrt{2} - \frac{5\pi}{8} - \frac{1}{4} = \int_0^{\pi/4} \frac{\cos^4 x}{1+\sin^2 x}\,dx$	
1	.48858801763883853368...	≈	$\binom{2}{1/3}$	
	.48859459163446827723...	≈	$\sum_{k=2}^{\infty} \frac{H_{k-1}\left(\zeta(k)-1\right)}{k} = \frac{1}{2}\sum_{k=2}^{\infty} \log^2\frac{k}{k-1}$	

.48883153138652018302... ≈ $2 - 4\log 2 + 2\log^2 2 + \frac{\zeta(3)}{4}$

= $\int_0^1 \frac{(1+x)\log^2(1+x)}{x}dx$

.488870533723461898816... ≈ $\sqrt{\pi}\,\Gamma^{-1}\left(\frac{1}{4}\right) = \prod_{k=0}^{\infty}\left(1+\frac{2k}{2k+1}\right)$

8 .4889672125599264671... ≈ $\cosh 2\sqrt{2} = \frac{1}{2}(e^{2\sqrt{2}} + e^{-2\sqrt{2}}) = \sum_{k=0}^{\infty}\frac{8^k}{(2k)!}$

1 .48919224881281710239... ≈ $\Gamma\left(\frac{3}{5}\right)$

1 .489343461075086879741... ≈ $\frac{3}{2} - \log 2 - \gamma - \frac{e^2}{2} + Ei(2) = \sum_{k=1}^{\infty}\frac{2^k}{k!k(k+1)}$

.489461515482517598273... ≈ $3\gamma^2 - \gamma^3 + \frac{\pi^2}{2} - \frac{\pi^2\gamma}{2} - 2\zeta(3) = \int_0^{\infty}\frac{x\log^3 x}{e^x}dx$

2 .48979428564930390972... ≈ $\sum_{k=1}^{\infty}\frac{\pi(k+1)}{k!}$

2 .48985263014562670196... ≈ eG

22 .49008835718767688354... ≈ $\frac{\pi^2}{3} - 1 = \sum_{k=1}^{\infty}k^3\big(\zeta(k+1)+\zeta(k+2)-2\big)$

.490150432910047591725... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{\binom{2k}{k}k^4}$

.49023714757471304409... ≈ $\frac{13}{16} + \frac{\pi^2}{12} - \log\pi = \sum_{k=1}^{\infty}\frac{k^2}{k+1}\big(\zeta(2k)-1\big)$

Adamchick-Srivastava 2.24

.49035775610023486497... ≈ $\frac{\pi^2}{2} - \frac{40}{9} = \psi^{(1)}\left(\frac{5}{2}\right)$

1 .490664735610360715703... ≈ $\zeta(3) + \frac{\gamma}{2}$

.49073850974274782075... ≈ $\sum_{k=2}^{\infty}\frac{1}{k!\zeta(k)}$

.49087385212340519351... ≈ $\frac{5\pi}{32} = \int_0^{\infty}\frac{dx}{(x^2+1)^4}$

= $\int_0^{\infty}\frac{\sin^5 x}{x^3}dx$ GR 3.827.9

= $\int_0^{2\pi} \frac{dx}{(5-\sin x)^2}$

= $\int_0^1 x^3 \arcsin x \, dx$ GR 4.523.3

.49128439615773951371l... ≈ $\sum_{k=2}^{\infty} \frac{\zeta(k)}{k!k} = \sum_{k=1}^{\infty} \left(Ei\left(\frac{1}{k}\right) - \gamma - \log\frac{1}{k} - \frac{1}{k} \right)$

1 .4913888888888888888$\underline{8}$ = $\frac{5369}{3600} = H^{(2)}{}_6$

535 .491655524764736503049... ≈ $e^{2\pi} = i^{-4i}$

.491691443213327803078... ≈ $\frac{1}{3}\left(\frac{1}{e} - 2\sqrt{e}\cos\left(\frac{\sqrt{3}}{2} + \frac{\pi}{3} \right) \right) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(3k+2)!}$

.491714676619541377352... ≈ $\frac{\pi}{e^2-1} = \int_0^{\infty} \frac{x \cot x}{x^2+1} dx = \int_0^{\infty} \frac{\sin(x/\pi)}{e^x - 1} dx$

.49191653769852668982... ≈ $\frac{1}{6\Gamma\left((-3)^{1/3}\right)\Gamma\left(-3^{1/3}\right)\Gamma\left(-(-1)^{2/3}3^{1/3}\right)} = \prod_{k=2}^{\infty} \frac{k^3-3}{k^3}$

5 .491925036856047158345... ≈ $\zeta(3) + 1 + \frac{\pi^2}{3} = \sum_{k=1}^{\infty} \binom{k+2}{k} \left(\zeta(k+1) - 1 \right)$

.492504944583995834017... ≈ $\frac{\pi}{4} - 1 + \frac{1}{\sqrt{2}} = \int_0^{\pi/4} \frac{\cos^2 x}{1+\sin x} dx$

43 .49250925534472376576... ≈ $16e$

1 .492590982680769548804... ≈ $\frac{\pi}{2}\left(1 - e^{-3}\right) = \int_0^{\infty} \frac{\sin(2\tan x)}{x} dx$ GR 3.881.2

.492860388528006732501... ≈ $\frac{\pi}{4\sqrt{2}} \coth 2\pi\sqrt{2} - \frac{1}{16} = \sum_{k=1}^{\infty} \frac{1}{k^2+8}$

.492900960560922053576... ≈ $2\sqrt{2}\,\text{arcsinh}\,1 - 2 = 2\sqrt{2}\log(1+\sqrt{2}) - 2 = \sum_{k=0}^{\infty} \frac{1}{2^k(2k+3)}$

2 .4929299919026930579... ≈ $2\gamma^2 + \frac{\pi^2}{3} + 2 - 6\gamma = \int_0^{\infty} \frac{x^2 \log^2 x dx}{e^x}$

5 .493061443340548456976... ≈ $5\log 3$

.493107418043066689162... ≈ $SinIntegral\left(\frac{1}{2}\right) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!2^{2k+1}(2k+1)}$

.4932828155063024... ≈ root of $Ei(x) = 1 + Ei(-x)$

.493414625918785664426... ≈ $\left(\frac{3}{2}\left(\sqrt[3]{9} - 2\right) \right)^{1/3} = \cos^{1/3}\frac{2\pi}{9} + \cos^{1/3}\frac{4\pi}{9} - \cos^{1/3}\frac{\pi}{9}$

[Ramanujan] Berndt Ch. 22

.493480220054467930942... ≈ $\frac{\pi^2}{20} = \frac{\zeta^2(2)-\zeta(4)}{2\zeta(2)} = T(2) = \sum \frac{1}{n^2}$,

where *n* has an odd number of prime factors
[Ramanujan] Berndt Ch. 5

.493550125985245944499... ≈ $\sum_{k=2}^{\infty} \frac{1}{k^2-2} \log \frac{k+2}{k}$

.4939394022668291491... ≈ $\frac{\pi^4}{15} - 6 = \psi^{(3)}(2) = \int_1^{\infty} \frac{\log^3 x}{x^3-x^2} dx = \int_0^1 \frac{\log x}{x-1} dx$

= $\int_0^{\infty} \frac{x^3}{e^x\left(e^x-1\right)} dx$

6 .4939394022668291491... ≈ $\frac{\pi^4}{15} = 6\zeta(4) = \psi^{(3)}(1)$ AS 6.4.2

= $\int_0^1 \frac{\log^3 x}{x-1} dx$ GR 4.262.2

= $\int_0^{\infty} \frac{x^3}{e^x-1} dx$ Andrews p. 89

.49452203055156817569... ≈ $\zeta(2) - 2\log^2 2 + \frac{\log^3 2}{3} - \frac{\zeta(3)}{4} = \int_0^1 \frac{\log^2(1+x)}{x^2(x+1)}$

.494795105503626129386... ≈ $\sum_{k=1}^{\infty} 2^k \left(\zeta(3k)-1\right) = \sum_{k=2}^{\infty} \frac{2}{k^3-2}$

1 .495348781221220541912... ≈ $5^{1/4}$

.495466871359389861239... ≈ $\frac{\log \pi}{1+\log^2 \pi} = \int_0^{\infty} \frac{\cos x}{\pi^x} dx$

.495488166396944002835... ≈ G^8

.49559995357145358065... ≈ $\frac{1}{16} \Phi\left(-\frac{1}{4}, 3, \frac{1}{2}\right) = \int_0^1 \frac{\log^2 x}{x^2+4}$

= $\frac{i}{2}\left(Li_3\left(-\frac{i}{2}\right) - Li_3\left(\frac{i}{2}\right)\right)$

.49566657469328260843... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(4k-2)}{4^k} = \sum_{k=1}^{\infty} \frac{k^2}{4k^4-1}$

= $\frac{\pi}{8\sqrt{2}}\left(\coth \frac{\pi}{\sqrt{2}} - \cot \frac{\pi}{\sqrt{2}}\right)$

.495691210504695050508... ≈ $\sum_{k=1}^{\infty} \frac{1}{(2^k+1)k}$

2 .495722117742122406049... ≈ $\dfrac{3}{\zeta(3)}$

5 .495793565063314090328... ≈ $6G$

.496100426884797122808... ≈ $\dfrac{2\pi^3}{125} = \displaystyle\sum_{k=1}^{\infty} \frac{\sin 4k\pi/5}{k^3}$ GR 1.443.1

.496460978217119096806... ≈ $\dfrac{7\pi^4}{720} - \dfrac{1}{2} + \dfrac{1}{4}(-1)^{1/4}\pi \csc\left((-1)^{1/4}\pi\right) + i\csc\left((-1)^{3/4}\pi\right)$

= $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^8 + k^4}$

.49646632594971788014... ≈ $\dfrac{\pi(e-1)}{4e} = \displaystyle\int_0^{\infty} \frac{\sin^2(x/2)\,dx}{1+x^2}$

3 .49651490659152822551... ≈ $\dfrac{\pi^2}{4} + 2\zeta(3) - \dfrac{11}{8} = \displaystyle\sum_{k=1}^{\infty} \frac{H_{k+2}}{k^2}$

.496658586741566801990... ≈ $\pi - 1 - \dfrac{\pi^2}{6} = \displaystyle\sum_{k=1}^{\infty} \frac{\cos 2k}{k^2} = -\operatorname{Im}\left\{\sum_{k=1}^{\infty} \frac{\zeta(k+2)}{(2i)^k}\right\}$

.496729413289805061722... ≈ $\dfrac{\pi}{2\sqrt{10}} = \displaystyle\int_0^{\infty} \frac{dx}{2x^2+5}$

.496999822715368986233... ≈ $\dfrac{31\zeta(6)}{32} - \dfrac{7\zeta(4)}{8} + \dfrac{\zeta(2)}{2} - \dfrac{1}{2} + \dfrac{\pi}{4}\left(\coth\dfrac{\pi}{2} - \tanh\dfrac{\pi}{2}\right)$

= $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^8 + k^6}$

.49712077818831410991... ≈ $\xi\left(\dfrac{1}{2}\right) = -\dfrac{1}{2\pi^{1/4}}\Gamma\left(\dfrac{5}{4}\right)\zeta\left(\dfrac{1}{2}\right)$

.497203709819647653589... ≈ $\displaystyle\sum_{k=0}^{\infty} \frac{\sin k}{k!\binom{2k}{k}}$

.497700302470745347474... ≈ $2\log\pi - \log 6 = \log\zeta(2) = \displaystyle\sum_{k=1}^{\infty} \frac{\Lambda(k)}{k^2 \log k}$ Titchmarsh 1.1.9

= $\log\zeta(2) = \displaystyle\sum_{p\ prime} \log\left(\frac{1}{1-p^{-2}}\right)$ HW Sec. 17.7

6 .497848411497744790930... ≈ $\dfrac{7\zeta(3)}{4} + \dfrac{3\pi^2}{8} + \log 2 = \displaystyle\sum_{k=2}^{\infty} \frac{k^2\zeta(k)}{2^k} = \sum_{k=1}^{\infty} \frac{16k^2 - 6k + 1}{2k(2k-1)^3}$

.498015668118356042714... ≈ $\operatorname{Im}\{\Gamma(i)\}$

5 .498075457733667127579... ≈ $\displaystyle\sum_{k=2}^{\infty} (-1)^k\left(\zeta^4(k) - 1\right)$

2 .498215280831227494136... $\approx \sum_{k=0}^{\infty} \frac{\zeta(k+2)}{\binom{2k}{k}}$

.498557794286274894758... $\approx \frac{1}{e^2}\left(Ei(2)-\gamma-\log 2\right) = \sum_{k=1}^{\infty} (-1)^{k+1}\frac{2^k H_k}{k!}$

.498611386672832761564... $\approx \sin\frac{1}{\sqrt{2}}\sinh\frac{1}{\sqrt{2}} = \sum_{k=1}^{\infty} \frac{(-1)^k}{(4k-2)!}$ GR 1.413.1

.49865491180167551221... $\approx \frac{1}{2^{4/3}3^{11/6}}\left(2\sqrt{3}\,\psi\left(1+\left(\frac{3}{2}\right)^{1/3}\right)+(\sqrt{3}+3i)\left(\gamma-H\left((-1)^{2/3}\left(\frac{3}{2}\right)^{1/3}\right)\right)\right)$

$+\frac{\sqrt{3}-3i}{2^{4/3}3^{11/6}}\left(\gamma-H\left(-\left(-\frac{3}{2}\right)^{1/3}\right)\right)$

$= \sum_{k=0}^{\infty} \frac{k}{2k^3+3}$

3 .498861059639376104351... $\approx \sum_{k=2}^{\infty}\left(e^{\zeta(k)}-e\right)$

1 .499005313803354632702... $\approx \sum_{k=2}^{\infty} \frac{1}{2^{k-2}\zeta(k)}$

2 .499187287886491926402... $\approx e^G$

.49932101244307520621... $\approx \sum_{k=2}^{\infty} \frac{k^2-1}{k^4\log k} = \;= \int_2^4 \left(\zeta(s)-1\right)ds$

$$.5000000000000000000\underline{0} = \frac{1}{2}$$

$$= \sum_{k=1}^{\infty} \frac{1}{(k+1)!+k!}$$

$$= \sum_{k=1}^{\infty} \left(\zeta(2k)-\zeta(2k+1)\right)$$

$$= \sum_{k=1}^{\infty} \frac{k\zeta(2k+1)}{4^k} = \sum_{k=1}^{\infty} \frac{4k}{(4k^2-1)^2}$$

$$= \sum_{k=0}^{\infty} \frac{1}{3^k} = \sum_{k=1}^{\infty} \frac{k^2}{3^k}$$

$$= \sum_{k=2}^{\infty} \frac{1}{k^2+k} = \sum_{k=2}^{\infty} \frac{k-1}{k^3-k} = \sum_{k=1}^{\infty} \frac{1}{k(k/2+1)}$$

$$= \sum_{k=1}^{\infty} \frac{1}{4k^2-1}$$

J397, K Ex. 109d

$$= \sum_{k=2}^{\infty} \frac{2k^4+k^3-1}{k^7-k}$$

$$= \sum_{k=1}^{\infty} \frac{k^2+k+1}{(k+2)!}$$

GR 0.142

$$= \sum_{k=0}^{\infty} \frac{1}{k!(k+1)(k+3)} = \sum_{k=1}^{\infty} \frac{1}{k!(k+2)}$$

$$= \sum_{k=1}^{\infty} \frac{2k+1}{(k^2+1)((k+1)^2+1)}$$

K 134

$$= \sum_{k=1}^{\infty} \frac{1}{k^3+k+k^{-1}} = \sum_{k=2}^{\infty} \frac{1}{k^3-3k+k^{-1}}$$

$$= \sum_{k=1}^{\infty} \left(\frac{(2k-1)!!}{(2k)!!}\right)^2 \frac{4k+1}{(2k-1)(2k+2)}$$

J390

$$= \sum_{k=0}^{\infty} \frac{2^k}{k!(k+2)(k+3)}$$

$$= \sum_{k=2}^{\infty} (-1)^k\left(\zeta(k)-1\right) = \sum_{k=1}^{\infty} \left(\zeta(2k)-\zeta(2k+1)\right)$$

$$= \sum_{k=1}^{\infty} \frac{\zeta(2k)}{4^k} = \sum_{k=1}^{\infty} \frac{k\zeta(2k+1)}{4^k}$$

$$= \sum_{k=1}^{\infty} \frac{\mu(k)}{2^k-1}$$

$$= \sum_{k=1}^{\infty} \frac{\mu(2k)2^k}{4^k-1}$$

$$= \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\sin k}{k} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\sin^2 k}{k^2} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\sin^3 k}{k^3}$$

8 .525161361065414300166... ≈ $\log 7!$

.525200397399770342589... ≈ $\zeta(4)-\zeta(3)+\zeta(2)-1 \;=\; \sum_{k=1}^{\infty}\frac{1}{k^5+k^4}$

= $\frac{1}{2}+\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(k+4)-1\right)$

.525223891622454569896... ≈ $\frac{e^{-3^{1/3}/2}}{3^{5/3}}\left(\sqrt{3}\sin\frac{3^{5/6}}{2}+\cos\frac{3^{5/6}}{2}-e^{3^{4/3}/2}\right) \;=\; \sum_{k=1}^{\infty}\frac{3^k k}{(3k)!}$

1 .52569812813416969091... ≈ $\frac{\pi^{3/2}}{2}\coth\pi^{3/2}-\frac{3\pi+1}{2(\pi+1)} \;=\; \sum_{k=1}^{\infty}(-1)^{k+1}\pi^k\left(\zeta(2k)-1\right)$

.52573111211913360602... ≈ $\sqrt{\frac{2}{5+\sqrt{5}}}=\frac{1}{2}\csc\frac{3\pi}{5} \;=\; \sin\left(\frac{\arctan 2}{2}\right)$

.525738554967177620202... ≈ $\frac{1+\gamma}{3}$

3 .525956365299825031078... ≈ $\sum_{k=1}^{\infty}\frac{\phi(k)}{(k-1)!}$

2 .52605639716194462560... ≈ $\frac{\pi^4\log 2}{64}+\frac{3\pi^2\zeta(3)}{16}-93\zeta(5)$

= $-\int_0^{\pi/2}x^3\log\cos x\;dx$

.526207036980384918178... ≈ $\frac{\zeta(3)}{\zeta(3)+\zeta(4)}$

$.\underline{526315789473684210}$ = $\frac{10}{19}$

.52641224653331... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k}{k\log k}$

.52693136530773532050... ≈ $\sum_{k=1}^{\infty}\frac{H_k}{2^k k^4}$

.526949261447891738811... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^k}{k^5+1}$

.526999672990696468620... ≈ $\frac{5}{16}\log\frac{27}{5} \;=\; \int_0^{\infty}\frac{\sin^5 x}{x^2}dx$

3 .527000471852952829762... ≈ $\sum_{k=1}^{\infty}\frac{\sigma_1(k)}{k!}$

.52706165278494586199... ≈ $-\arctan\frac{1}{1-e}$

1 .527525231651946668863... ≈ $\sqrt{\frac{7}{3}} = \sum_{k=0}^{\infty}\binom{2k}{k}\frac{1}{7^k}$

.527600797260494257440... ≈ $\frac{1}{2}\Big((\pi-1)(1-\cos 1)-\log(2-2\cos 1)\sin 1\Big) = \sum_{k=1}^{\infty}\frac{\sin k}{k(k+1)}$

.527694767240394589488... ≈ $\frac{\pi\sqrt{2}}{4}\coth\pi\sqrt{2} - \frac{7}{12} = \sum_{k=2}^{\infty}\frac{1}{k^2+2} = \sum_{k=1}^{\infty}(-1)^{k+1}2^{k-1}\big(\zeta(2k)-1\big)$

= $\sum_{k=1}^{\infty}\frac{1}{k^2+2k+3}$

.527887014709683857297... ≈ $\pi-4+2\log 2 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k(k+1/2)}$

= $\sum_{k=1}^{\infty}\frac{(k-\frac{1}{2})!}{(k+\frac{1}{2})!(2k+\frac{1}{2})}$

1 .52806857261482370575... ≈ $\log\Big(-\pi\sqrt{2}\csc\pi\sqrt{2}\Big) = \sum_{k=1}^{\infty}\frac{2^k\big(\zeta(2k)-1\big)}{k} = -\sum_{k=2}^{\infty}\log\left(1-\frac{2}{k^2}\right)$

.528320833573718727151... ≈ $\log_8 3$

.528407192107340935857... ≈ $\frac{1}{3}\left(\psi\left(\frac{6+3^{7/6}i+3^{2/3}}{6}\right)+\psi\left(\frac{6-3^{7/6}i+3^{2/3}}{6}\right)+\psi\left(1-\frac{1}{3^{1/3}}\right)\right)-\gamma$

= $\sum_{k=1}^{\infty}\frac{\zeta(3k+1)}{3^k} = \sum_{k=1}^{\infty}\frac{1}{k(3k^3-1)}$

.528482235314230713618... ≈ $2-\frac{4}{e} = \int_0^1 e^{-\sqrt{x}}dx$

.52862543768786425729... ≈ $\frac{1}{2}SinhIntegral(1) = \int_1^{\infty}\sinh\left(\frac{1}{x^2}\right)\frac{dx}{x}$

.52870968946993108979... ≈ γG

1 .528999560696888418383... ≈ $\sum_{k=0}^{\infty}\frac{1}{2^k+1/2}$

.529154857165146540082... ≈ $\frac{\pi^4}{40}+\frac{\pi^2\log^2 2}{3}-\frac{2\log^4 2}{3}-4\zeta(3)\log 2 = \sum_{k=1}^{\infty}\binom{2k}{k}\frac{1}{4^k k^4}$

$.\underline{5294117647058823}$ = $\frac{9}{17}$

.529416855888971505585... ≈ $\sum_{k=1}^{\infty}\frac{1}{F_{3^k}}$

2 .52947747207915264818... ≈ $\prod_{k=2}^{\infty}\frac{k!}{k!-1}$

. 52962384375569377748... ≈ $\frac{4G-2}{\pi} = \sum_{k=0}^{\infty}\binom{2k}{k}^2 \frac{1}{16^k(2k+1)(2k+2)}$

5 .529955849000091385901... ≈ $\prod_{k=1}^{\infty}\left(1+\frac{k}{2^k}\right)$

.53001937835233379231... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\left(\zeta(k+1)-1\right)$

.53019091763558444752... ≈ $\zeta(3)-\gamma+\frac{1}{2}-\frac{1}{2}\left(\psi(2+i)+\psi(2-i)\right)$

= $\sum_{k=1}^{\infty}\frac{1}{k^5+k^3} = \frac{1}{2}+\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(2k+3)-1\right)$

.53026303220872523716... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^2}\sqrt{\frac{k-1}{k}}$

.530277182813685398637... ≈ $\sum_{k=1}^{\infty}\frac{1}{(2k)!\sqrt{k}}$

1 .530688394225490738618... ≈ $\frac{\pi^2}{2}-2\zeta(3)-1 = \sum_{k=2}^{\infty}(-1)^k k^2\left(\zeta(k)-1\right) = \sum_{k=2}^{\infty}\frac{4k^2+3k+1}{k(k+1)^3}$

1 .53091503904237871421... ≈ $\sum_{k=1}^{\infty}\frac{1}{k!\zeta(2k+1)}$ Titchmarsh 14.32.3

9488 .531016070574007128576... ≈ π^8

.531249534072353755050... ≈ $\left(\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{6}}\right)\pi = \int_0^{\infty}\log\left(1+\frac{1}{6x^2+1}\right)dx$

.5312500000000000000$\underline{0}$ = $\frac{17}{32} = \sum_{k=1}^{\infty}\frac{k^7}{e^{\pi k}+(-1)^k}$ Prud. 5.3.1.1

.531275083024229991483... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\mu(k)}{k}\log\left(\zeta(2k)\right)$

.531463605386615672817... ≈ $e^{1/e-1} = \sum_{k=1}^{\infty}\frac{k}{k!e^k}$

2 .531895752688993200213... ≈ $\frac{3^{3/4}\pi}{2\sqrt{2}} = \int_0^{\infty}\frac{dx}{x^4+1/3}$

.531972647421808261859... ≈ $8\sqrt{\frac{2}{3}}-6 = \sum_{k=0}^{\infty}\binom{2k}{k}\frac{1}{12^k(k+1)(k+2)}$

.53202116874184233687... ≈ $\frac{\pi^2}{12}-\frac{1}{2}Li_3\left(-\frac{1}{3}\right)-\frac{3}{8}\zeta(3) = \int_0^1\frac{\log^2 x}{(x+1)^2(x+3)}dx$

.532108050640355849704... ≈ $4-\frac{1}{\sqrt{2}}\left(\pi+2\log\left(1+\sqrt{2}\right)\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k+1/4}$

2 .532131755504016671197... ≈ $2I_0(1) = \int\limits_{-1}^{1} e^{-\sin\pi x}\,dx$

.532598899727660345291... ≈ $3 - \frac{\pi^2}{4} = \sum_{k=1}^{\infty} \frac{3}{k(k+2)^2}$

.532839975353552023357... ≈ $\sqrt{2} - \log\left(1+\sqrt{2}\right) = -\int\limits_{0}^{\pi/4} \cos x \log(\sin 2x)\,dx$

46 .532919487437891395550... ≈ $\frac{7e^3-1}{3} = \sum_{k=1}^{\infty} \frac{3^k k^2}{(k+1)!}$

.533290128624160141531... ≈ $\frac{\gamma}{3} - 1 + \frac{\pi^2}{6} + \frac{1}{6}\left((1-i\sqrt{3})\,\psi\left(\frac{3-i\sqrt{3}}{2}\right) + (1+i\sqrt{3})\,\psi\left(\frac{3+i\sqrt{3}}{2}\right)\right)$

= $\frac{1}{2} + \sum_{k=1}^{\infty} (-1)^{k+1}\left(\zeta(3k+2)-1\right) = \sum_{k=1}^{\infty} \frac{1}{k^5+k^2}$

1 .533463799145879760767... ≈ $\sum_{k=2}^{\infty} \left(\pi^{\zeta(k)-1} - 1\right)$

6 .533473364460804592338... ≈ $62e - 162 = \sum_{k=1}^{\infty} \frac{k^4}{k!(k+3)}$

1 .533481373193750995 99... ≈ $-2Li_3(-2) - \frac{3\zeta(3)}{2}$

= $\int\limits_{1}^{\infty} \frac{\log^2 x}{(x+1)(x+2)}\,dx$

1 .533582691060658478950... ≈ $\frac{\pi}{2}\left(I_0(1) + L_0(1) - 1\right) = \int\limits_{0}^{1} e^x \arccos x\,dx$

1 .533626292763742225 15... ≈ $\frac{e}{\sqrt{\pi}}$

.5338450130794810532... ≈ $\prod_{k=2}^{\infty} \frac{2\zeta(k)-1}{\zeta(k)}$

.533876764408056995 00... ≈ $\frac{1}{4\cos 1 - 5}\left(\sin\frac{1}{2} - 2\sin\frac{3}{2}\right) = \sum_{k=1}^{\infty} \frac{1}{2^k} \sin\frac{2k+1}{2}$

1 .533885641474380666 83... ≈ $\sum_{k=1}^{\infty} \frac{|\mu(k)|}{2^k - 1} = \sum_{k=1}^{\infty} \frac{2^{\nu(k)}}{2^k}$

7 .533941598797611904699... ≈ $\Gamma\left(\frac{1}{8}\right)$

.534195237508799281491... ≈ $\prod_{k=1}^{\infty} \left(1 + \frac{(-1)^k}{\binom{2k}{k} k}\right)$

3 .534291735288517393270... ≈ $\frac{9\pi}{8}$

.534304386409498279676... ≈ $\frac{16}{3}\log\frac{4}{3} - 1 = \sum_{k=1}^{\infty}\frac{H_{k+1}}{4^k}$

.534643187572618722264... ≈ $\frac{1}{8}\Phi\left(\frac{1}{2},2,\frac{1}{2}\right) = \sum_{k=1}^{\infty}\frac{1}{2^k(4k^2-4k+1)} = \int_1^{\infty}\frac{\log x}{2x^2-1}dx$

1 .534680913814755890897... ≈ $\frac{1}{2}\left(e^{e/4}+e^{1/4e}\right) = \sum_{k=0}^{\infty}\frac{\cosh k}{k!4^k}$

.534799996739570370524... ≈ $\log\left(1+\frac{1}{\sqrt{2}}\right) = \int_0^{\pi/4}\frac{\cos x}{1+\sin x}dx$

.535034887349803758539... ≈ $\gamma+\frac{1}{4}\left(\psi\left(\frac{1+i}{\sqrt{2}}\right)+\psi\left(\frac{1-i}{\sqrt{2}}\right)+\psi\left(\frac{-1+i}{\sqrt{2}}\right)+\psi\left(\frac{-1-i}{\sqrt{2}}\right)\right)$

= $\frac{1}{2}+\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(4k+1)-1\right) = \sum_{k=1}^{\infty}\frac{1}{k^5+k}$

.535112163829819510520... ≈ $\sin\left(\frac{\sin 1}{2}\right)\left(\cosh\left(\frac{\cos 1}{2}\right)+\sinh\left(\frac{\cos 1}{2}\right)\right) = e^{(\cos 1)/2}\sin\left(\frac{\sin 1}{2}\right)$

= $\sum_{k=1}^{\infty}\frac{\sin k}{k!2^k}$ GR 1.449.1

.5351307235543852976... ≈ $\frac{\sqrt{3}}{\pi}\sin\frac{\pi}{\sqrt{3}} = \prod_{k=1}^{\infty}\left(1-\frac{1}{3k^2}\right)$

.535216927240908236283... ≈ $\sum_{k=2}^{\infty}\frac{1}{k!(k!-1)}$

1 .53537050883625298503... ≈ $\sum_{k=1}^{\infty}\frac{1}{F_{2k}} = \sqrt{5}\sum_{k=1}^{\infty}\frac{\phi^{2k}}{\phi^{4k}-1} = \sqrt{5}\sum_{k=1}^{\infty}\left(\frac{1}{\phi^{2k}-1}-\frac{1}{\phi^{4k}-1}\right)$

1 .535390236492927618733... ≈ $\zeta(3)+\frac{1}{3}$

.5355608832923521188... ≈ $Ai(-1)$

.535898384862245412945... ≈ $2(2-\sqrt{3}) = \left(\sqrt{3}-1\right)^2$ CFG D1

.535962843190222987031... ≈ $\frac{1}{2}+\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(5k)-1\right) = \sum_{k=1}^{\infty}\frac{1}{k^5+1}$

.5360774649700956698... ≈ $\frac{(\sqrt{2}-1)\sqrt{\pi}}{2}\zeta\left(\frac{1}{2}\right) = \int_0^{\infty}\frac{dx}{e^{x^2}+1}$

.536119444744722773227... ≈ $\frac{\pi}{\pi+e}$

1 .53642191914104435977... ≈ $-\frac{\pi\sqrt{2}}{3}\csc\pi\sqrt{2} = \prod_{k=1}^{\infty}\left(1+\frac{1}{k^2+4k+2}\right)$

.536441086350966020557... ≈ $\frac{1}{2}+\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(6k-1)-1\right) = \sum_{k=1}^{\infty}\frac{k}{k^6+1}$

.5365207790738756208... ≈ $\sum_{k=1}^{\infty}\frac{1}{(3^k-1)k^2} = \sum_{k=1}^{\infty}\frac{\sigma_{-2}(k)}{3^k}$

.536526945921177100962... ≈ $\int_2^{\infty}\log\zeta(x)\,dx$

.536683562016736660896... ≈ $\frac{1}{2}+\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(7k-2)-1\right) = \sum_{k=1}^{\infty}\frac{k^2}{k^7+1}$

.536784534109508703918... ≈ $\frac{1}{3}\left((-1+(-1)^{2/3})\psi\left(\frac{1-i\sqrt{3}}{2}\right)-(1+(-1)^{1/3})\psi\left(\frac{1+i\sqrt{3}}{2}\right)\right)-\gamma$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(3k-1)-\zeta(3k+1)\right) = \sum_{k=2}^{\infty}\frac{k-1}{k(k^2-k+1)}$

.53685577365418220547... ≈ $\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\zeta(jk+2)-1$

.536921592069942010965... ≈ $\sum_{k=1}^{\infty}\frac{k}{4^k-1}$

.536999903377236213702... ≈ $\frac{1}{2}+\frac{\pi^2}{2\sinh^2\pi} = -\operatorname{Re}\left\{\psi^{(1)}(i)\right\}$

.537213193608040200941... ≈ $\frac{7\zeta(3)}{8}+\frac{\log^3 2}{6}-\frac{\pi^2\log 2}{12} = Li_3\left(\frac{1}{2}\right) = \sum_{k=1}^{\infty}\frac{1}{2^k k^3}$

= $\sum_{k=1}^{\infty}\frac{H^{(2)}{}_k-1}{k^3}$

= $\int_0^1\frac{\log(1-x/2)\log x}{x}dx$

1 .53739220806790406178... ≈ $\frac{1}{3}e^{\pi^{1/3}}+\frac{2}{3}e^{\pi^{-(\pi^{1/3})/2}}\cos\frac{\pi^{1/3}\sqrt{3}}{2} = \sum_{k=0}^{\infty}\frac{\pi^k}{(3k)!}$

.538011176205005048612... ≈ $\frac{\sqrt{5}}{2}\log\frac{1+\sqrt{5}}{2} = \sum_{k=1}^{\infty}\frac{1}{5^k(8k-4)}$

= $\frac{1}{2}+\sum_{k=1}^{\infty}\frac{(-1)^{k+1}(2k-2)!!}{(2k+1)!!2^{2k+1}}$ J141

.5381869343911341187... ≈ $H^{(3)}{}_{1/4} = 64-27\zeta(3)-\pi^3$

1 .538395665719128167672... ≈ $\sum_{k=1}^{\infty}\sin\frac{1}{k!}$

.538461538461$\underline{538461}$ = $\frac{7}{13}$

1 .538477802727944253157... ≈ $2\cos\log 2 \;=\; 2^{i} + 2^{-i}$

.539114847327011158805... ≈ $\frac{1}{8}\left(i\tanh\left(\frac{1+i}{4}\right) - \tan\left(\frac{1+i}{4}\right) + 4\csc\left(\frac{1+i}{2}\right)\right)$

$+\frac{1}{8}\left(4i\,\mathrm{csch}\left(\frac{1+i}{2}\right) - \cot\left(\frac{1+i}{4}\right) - i\coth\left(\frac{1+i}{4}\right)\right)$

= $\int\limits_{-\infty}^{\infty} \frac{e^{-x}\cos x}{1+e^{-2\pi x}}\,dx$

.53935260118837935667... ≈ $\frac{\Gamma(1/2)}{\Gamma(5/4)\Gamma(3/4)} = \prod_{k=1}^{\infty} 1 + \frac{(-1)^k}{2k}$ J1028

3 .539356459551305228672... ≈ $\frac{1}{3^{2/3}}\left((-1)^{1/3}\psi\left(\frac{4+i2^{1/3}3^{5/6}+6^{1/3}}{2}\right) - (-1)^{2/3}\psi\left(\frac{4-i2^{1/3}3^{5/6}+6^{1/3}}{2}\right)\right)$

$-\frac{2^{1/3}}{3^{2/3}}\psi\left(2-6^{1/3}\right)$

= $\sum_{k=1}^{\infty} 6^k\left(\zeta(3k)-1\right) \;=\; \sum_{k=2}^{\infty} \frac{6k}{k^3-6}$

9 .5393920141694564915... ≈ $\sqrt{91}$

8 .539734222673567065464... ≈ πe Not known to be transcendental

.540235927524947682724... ≈ $-\sqrt{3}\log\left(\sqrt{3}-1\right)$

.540302305868139717401... ≈ $\cos 1 \;=\; \cosh i \;=\; \mathrm{Re}\left\{e^{i}\right\} \;=\; \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}$ AS 4.3.66, LY 6.110

.540345545918092079459... ≈ $\frac{\pi}{4}\cosh\frac{\pi}{4} - \frac{1}{2}$

.540553369576224517937... ≈ $\frac{\pi^2}{8} - \log 2 \;=\; \frac{1}{2}\sum_{k=1}^{\infty} \frac{4k+1}{(2k+1)^2 k} \;=\; \sum_{k=2}^{\infty} \frac{(-1)^k k\zeta(k)}{2^k}$

.540722946704618254025... ≈ $\sin(\cos 1\sin 1)\left(\cosh\left(\frac{\cos 2}{2}\right) - \sinh\left(\frac{\cos 2}{2}\right)\right)$

= $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\sin 2k}{k!2^k}$

6 .540737725975564600708... ≈ $\frac{37}{4\sqrt{2}} \;=\; \sum_{k=0}^{\infty} \binom{2k}{k}\frac{k^3}{8^k}$

5 .540829024068169345604... ≈ $\zeta(\zeta(3))$

.540946264299396859964... ≈ G^7

8 .5409729100234621656 2... ≈ $3\zeta(2) + 3\zeta(3) \;=\; \int\limits_0^1 \frac{\log^3 x}{(x-1)^3}$

45 .54097747674216511051... ≈ $13I_0(2) + 10I_1(2) = \sum_{k=1}^{\infty} \frac{k^6}{(k!)^2}$

.54116161685556909575 8... ≈ $\frac{\pi^4}{180} = -\int_0^1 \frac{\log(1-x)^2 \log x}{x}$

.5411961001461969844... ≈ $\sqrt{\frac{2-\sqrt{2}}{2}} = \cos\frac{\pi}{8} - \sin\frac{\pi}{8} = \prod_{k=1}^{\infty}\left(1+\frac{(-1)^k}{4k-2}\right)$ J1029

.54123573432867053015... ≈ $\int_0^1 \frac{dx}{\Gamma(x)}$

.54132485461291810898... ≈ $\log(e-1) = \sum_{k=1}^{\infty} \frac{(-1)^k B_k}{k!k}$ [Ramanjuan] Berndt Ch. 5

.54160884220466346382 2... ≈ $\sum_{k=0}^{\infty} \frac{B_k}{(k!)^2}$

1 .54169146825401604874 2... ≈ $\sum_{k=0}^{\infty} \frac{1}{\left(2^k\right)!}$

1 .54181051878115749942 1... ≈ $\frac{\log 3}{2} - \frac{\pi}{6\sqrt{3}} - \frac{4\pi^3}{81\sqrt{3}} + \frac{1}{3}\psi^{(1)}\left(\frac{2}{3}\right) + \frac{26\zeta(3)}{27}$

= $\sum_{k=2}^{\infty} \frac{k^2\zeta(k)}{3^k} = \sum_{k=1}^{\infty} \frac{36k^2-9k+1}{3k((3k-1)^3}$

2 .54187964767160649839 8... ≈ $\pi^2 - 8G = \psi^{(1)}\left(\frac{3}{4}\right) = \sum_{k=0}^{\infty} \frac{1}{(k+3/4)^2} = \sum_{k=2}^{\infty} \frac{(k-1)\zeta(k)}{4^{k-2}}$

.54202990279883669577 3... ≈ $\frac{2\pi}{\cosh\pi}$

178 .54212933375474970405 4... ≈ $112 + 96\log 2 = \sum_{k=1}^{\infty} \frac{H_k (k+1(k+2)(k+3)}{2^k}$

.54272082063630350093 5... ≈ $\frac{1}{\sqrt{2}}\sinh\frac{1}{\sqrt{2}} = \sum_{k=1}^{\infty} \frac{1}{(2k-1)!2^k}$

1 .54305583321647144517... ≈ $\sum_{p\ prime} \frac{p}{p!}$

.54308053098328421860 2... ≈ $\frac{\gamma}{2} + \frac{\pi\gamma}{4}\coth\frac{\pi}{2} + \frac{i}{8}\left(\psi\left(-\frac{i}{2}\right)^2 - \left(\psi\left(\frac{i}{2}\right)\right)^2\right)$

$+\frac{i}{8}\left(\psi^{(1)}\left(1+\frac{i}{2}\right) - \psi^{(1)}\left(1-\frac{i}{2}\right)\right)$

= $\sum_{k=1}^{\infty} \frac{H_k}{4k^2+1}$

.543080634815243778478... ≈ $\cosh 1 - 1 = \frac{e+e^{-1}-2}{2} = \sum_{k=1}^{\infty}\frac{1}{(2k)!} = \int_1^{\infty}\sinh\left(\frac{1}{x}\right)\frac{dx}{x^2}$

1 .54308063481524377848... ≈ $\cosh 1 = \cos i = \frac{e+e^{-1}}{2} = \sum_{k=0}^{\infty}\frac{1}{(2k)!}$ GR 0.245.5

= $\prod_{k=0}^{\infty}\left(1+\frac{4}{\pi^2(2k+1)^2}\right)$ J1079

.543195529534402361791... ≈ $\frac{1}{2}\left(\log(1-e^{i}) - e^{2i}\log(1-e^{-i})(\sin 1 + i\cos 1)\right)$

= $\sum_{k=1}^{\infty}\frac{\sin k}{k+1}$

.54323539510879497886... ≈ $\sum_{k=1}^{\infty}\frac{|\mu(k)|\log\zeta(2k)}{k}$

.543258965342976706953... ≈ $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{6}\right)\Gamma\left(\frac{1}{6}\right)$, Landau constant

.543383238748395175193... ≈ $4-\frac{\pi^2}{6}-4\log 2+2\log^2 2 = \sum_{k=1}^{\infty}\frac{H_k}{(k+1)(2k+3)}$

8 .5440037453175311679... ≈ $\sqrt{73}$

.544058109964266325950... ≈ $\frac{2\pi}{\sinh\pi} = \Gamma(2+i)\Gamma(2-i) = \pi\left(\coth\frac{\pi}{2}-\tanh\frac{\pi}{2}\right)$

.54425500107911929426... ≈ $\sum_{k=1}^{\infty}\frac{\log\zeta(2k)}{k}$

.54439652257590053263... ≈ $\frac{\pi\log 2}{4} = -\int_0^{\pi/4}\log\sin(4x)\,dx$

= $-\int_0^{\pi}\frac{\sin x}{\sqrt{1+\sin^2 x}}\log\sin x\,dx$

= $-\int_0^{\pi/4}\left(\left(\frac{\pi}{2}-x\right)\tan x - \cot x\right)\frac{dx}{\cos 2x}$ Prud. 2.5.29.28

.54445873964813266061... ≈ $\sum_{k=1}^{\infty}\mu(k)\left(\zeta(2k)-1\right)$

1 .545177444479562475338... ≈ $8\log 2 - 4 = \sum_{k=1}^{\infty}\frac{1}{k(k^2-1/4)}$

.545454545454545454545... ≈ $\frac{6}{11} = \sum_{k=1}^{\infty}\frac{L_k}{4^k}$

.5456413607650470421... ≈ $\frac{e^{1/4}\sqrt{\pi}}{2}\left(1-erf\,\frac{1}{2}\right) = \int_1^\infty \frac{e^x}{e^{x^2}}dx$

.54588182553060244232... ≈ $\frac{7\pi^4}{60} - 9\zeta(3) = \int_1^\infty \frac{\log^4 x}{(x+1)^3}dx = \int_0^1 \frac{x\log^4 x}{(x+1)^3}dx$

1 .5459306102231431605... ≈ $\frac{\pi + 2\arctan\frac{1}{\sqrt{2}}}{2\sqrt{2}} = \int_0^\infty \frac{dx}{x^2-2x+3}$

.546250624110635744578... ≈ $\frac{\pi}{\sqrt{5}}\tan\frac{\pi\sqrt{5}}{2} = \sum_{k=2}^\infty (-1)^k F_{k-1}\left(\zeta(k)-1\right)$

= $\sum_{k=2}^\infty \frac{2(1+\sqrt{5})}{\left(2k+1+\sqrt{5}\right)\left(k-2+k\sqrt{5}\right)}$

1 .546250624110635744578... ≈ $1+\frac{\pi}{\sqrt{5}}\tan\frac{\pi\sqrt{5}}{2} = \sum_{k=2}^\infty F_{k-1}\left(\zeta(k)-1\right)$

= $\sum_{k=2}^\infty \frac{1}{k^2-k-1} = \sum_{k=1}^\infty \frac{2(1+\sqrt{5})}{\left(2k-1-\sqrt{5}\right)\left(2+k+k\sqrt{5}\right)}$

.54716757473605860234... ≈ $1+\log 3 - \log(e+2) = \int_0^1 \frac{e^x}{e^x + e/2}dx$

.54770133168797308275... ≈ $\frac{\gamma}{2} + \log 2 + \frac{1}{4}\psi\left(\frac{1}{2}+\frac{i}{2}\right) + \frac{1}{4}\psi\left(\frac{1}{2}-\frac{i}{2}\right)$

= $\sum_{k=0}^\infty \frac{1}{(2k+1)\left((2k+1)^2+1\right)}$ Prud. 5.1.26.8

1 .54798240215774230466... ≈ $2\pi G - \frac{7\zeta(3)}{2} = \int_0^1 \frac{\arccos^2 x dx}{1-x^2} = 4\int_0^1 \frac{\arctan^2 x}{x}dx$

= $\int_0^{\pi/2} \frac{x^2}{\sin x}dx$

.54831135561607547882... ≈ $\frac{\pi^2}{18} = \sum_{k=1}^\infty \frac{(k-1)!(k-1)!}{(2k)!}$

= $2\arcsin^2\frac{1}{2}$

= $-\int_0^\infty \frac{\log(1-x^3)}{x}dx$

11 .548739357257748378... ≈ $-i\sin(2\log i)$

.54886543055424064622... ≈ ${}_2F_1\left(2,2,\frac{3}{2},-\frac{1}{4}\right) = \sum_{k=1}^\infty \frac{(-1)^{k+1}(k!)^2}{(2k-1)!}$

.54891491425246963638... ≈ $\sum_{k=1}^\infty \frac{1}{2^k(2^k-1)k}$

	.54926923395857905326...	≈	$\log 2\pi - 1 - \frac{\gamma}{2} = \frac{\zeta'(0)}{\zeta(0)} - 1 - \frac{1}{2}\frac{\Gamma'(1)}{\Gamma(1)}$	Titchmarsh 2.12.8
	.549306144334054845697...	≈	$\frac{\log 3}{2} = \operatorname{arctanh}\frac{1}{2} = \sum_{k=0}^{\infty}\frac{1}{2^{2k+1}(2k+1)}$	AS 4.5.64, J941
		=	$\int_0^{\infty}\frac{dx}{(x+1)(x+3)} = \int_2^{\infty}\frac{dx}{x^2-1} = \int_0^{\infty}\frac{dx}{e^x+2}$	
		=	$\int_1^{\infty}\frac{\log x}{(x+2)^2}$	
	.549428148719873179229...	≈	$\frac{1}{4}\csc\left((-1)^{1/4}\pi\right)\csc\left((-1)^{3/4}\pi\right)\bigl(\cos(\pi\sqrt{2} - \cosh(\pi\sqrt{2}$	
			$+ (-1)^{3/4}\pi\sin\left((-1)^{1/4}\pi\right) + (-1)^{1/4}\pi\sin\left((-1)^{3/4}\pi\right)\bigr)$	
		=	$\sum_{k=0}^{\infty}\frac{(-1)^k}{k^4+1}$	
7	.5498344352707496972...	≈	$\sqrt{57}$	
5060	.549875237639470468574...	≈	$\frac{8\pi^8}{15} = 7!\zeta(8) = \psi^{(1)}(7) = -\int_0^1 \frac{\log^7 x}{1-x}dx$	GR 4.266.2

2 .550166236549955... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{e^{1/k}}{k^3}$

1 .55031383401499100877 4... ≈ $\frac{\pi^3}{20}$

6 .55046438222343660872 5... ≈ $\frac{6}{G}$

.55051025721682190180 3... ≈ $3-\sqrt{6}$

.55052194679956934819 0... ≈ $\sum_{k=2}^{\infty}\pi(k-1)\left(\zeta(k)-1\right)$

1 .55054609673043044028 7... ≈ $\frac{6}{\pi^2-6} = \frac{1}{\zeta(2)-1}$

.55066059449645688395... ≈ $\frac{1}{2\pi^2}+\frac{1}{2}\coth\pi^2 = \sum_{k=0}^{\infty}\frac{1}{(k^2+\pi^2)}$

.55082846128002105106 7... ≈ $\sum_{k=1}^{\infty}\frac{\nu^2(k)}{2^k}$

2 .55141918289255750523 5... ≈ $\frac{\pi^2\gamma^2}{6}-\frac{\pi^2\gamma}{3}+\frac{\pi^4}{36}+2(1-\gamma)\zeta'(2)+\zeta''(2)$

= $\int_0^{\infty}\frac{x\log^s x}{e^x-1}dx$

.55144112954356641551 7... ≈ $\frac{4}{e^2-e^{-2}} = \frac{2}{\sinh 2} = \frac{1}{\sinh 1\cosh 1}$ J132, J713

.55154451810789954008... ≈ $\frac{1}{10}\left(2-\sqrt{5}-2H_{(-3+\sqrt{5})/2}+\pi\left(1+\sqrt{5}\right)\tan\frac{\pi\sqrt{5}}{2}\right)$

= $\sum_{k=2}^{\infty}F_{k-1}F_k\left(\zeta(k)-1\right)$

.5516151617923785687... ≈ $e-\frac{13}{6}=\sum_{k=1}^{\infty}\frac{k^2}{(k+1)!}$

.5516932... ≈ $\sum_{p\ prime}\frac{1}{p^2-1}$

.5523689696799021586... ≈ $\frac{\sqrt{\pi}}{8}\left(erf\,1+\ erfi1\right) = \int_1^{\infty}\cosh\left(\frac{1}{x^4}\right)\frac{dx}{x^3}$

.55242828301478027042... ≈ $\int_1^{\infty}\frac{\zeta(x+1)}{x^3+x^{-3}}dx$

1 .55260145083523513225... ≈ $\frac{8}{\pi}\cos\frac{\pi\sqrt{21}}{2} = \prod_{k=1}^{\infty}\left(1+\frac{1}{k(k+5)}\right)$

.5527568077303040748... ≈ $\frac{\sin\sqrt{\pi}}{\sqrt{\pi}} = \prod_{k=1}^{\infty}\left(1-\frac{1}{\pi k^2}\right)$

.55278640450004206072... ≈ $1-\frac{1}{\sqrt{5}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(2k)!}{(k!)^2}$

.55285569203859119314... ≈ $\sqrt{2}\sin\sqrt{2} + \cos\sqrt{2} - 1 = \sum_{k=0}^{\infty}\frac{(-1)^k 2^k}{(2k)!(k+1)}$

.553303299720515717371... ≈ $\sqrt{2}\,\mathrm{arcsinh}1 - \log 2 = \sum_{k=1}^{\infty}\frac{1}{2^k k(2k-1)}$

2 .553310333104003190087... ≈ $\sum_{k=2}^{\infty}\frac{\sigma_0(k)}{k!-1}$

.5535743588970452515... ≈ $\frac{\arctan 2}{2} = \int_0^{\infty}\frac{dx}{x^2+2x+5}$

.554204472098099291561... ≈ $\frac{i}{8}\left(\psi^{(1)}\left(\frac{2+i}{4}\right)-\psi^{(1)}\left(\frac{2-i}{4}\right)+\psi^{(1)}\left(\frac{4-i}{4}\right)-\psi^{(1)}\left(\frac{4+i}{4}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(k^2+1/4)^2}$

4 .5544893692544693575... ≈ $\sum_{k=1}^{\infty}\binom{2k}{k}\frac{1}{k!!\,2^k}$

1 .55484419730133345731... ≈ $\frac{1}{3}\cosh\frac{\pi}{\sqrt{2}} = \prod_{k=1}^{\infty}\left(1+\frac{2}{(2k+1)^2}\right)$

1 .555290108843124661542... ≈ $\sum_{k=1}^{\infty}\frac{H_k}{F_k k^2}$

.55536036726979578088... ≈ $\frac{\pi}{4\sqrt{2}} = \int_0^{\infty}\frac{dx}{x^2+8} = \int_0^{\pi/2}\sqrt{\cos^3 x\sin x}\,dx$

.5553968826533496289... ≈ $\frac{1}{2}+\frac{\sin 1-\cos 1}{2e} = \int_0^1\frac{\cos x\,dx}{e^x}$

= $\int_1^e\frac{\cos\log x}{x^2}dx$

.55582269181411744686... ≈ $I_0(1)-Struve_0(1) = \int_0^1 e^{-\sin\pi x}\,dx$

.5560460564528891354... ≈ $\frac{\pi\sqrt{2}}{6}+\frac{\log 2}{3}+\frac{\sqrt{2}}{6}\log\frac{2-\sqrt{2}}{2+\sqrt{2}} = \int_0^1 x^2\log\left(1+\frac{1}{x^4}\right)dx$

.55663264611852264179... ≈ $-\frac{1}{2}\left(Li_4(-e^i)+Li_4(-e^{-i})\right)$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\cos k}{k^4}$ GR 1.443.6

.55683841210516333558... ≈ $\frac{1}{5}\left(2-\sqrt{5}-2H_{(-3+\sqrt{5})/2}+\pi\tan\frac{\pi\sqrt{5}}{2}\right)$

= $\sum_{k=2}^{\infty}F_{k-1}F_{k-1}\left(\zeta(k)-1\right)$

.55730495911103659264... ≈ $2-\frac{1}{\log 2} = \sum_{k=1}^{\infty}\frac{1}{2^k\left(1+1/2^{1/2^k}\right)}$

1 .557407724654902230507... ≈ $\tan 1 = \sum_{k=0}^{\infty}\frac{(-1)^k 4^k(1-4^k)B_{2k}}{(2k)}$

6 .55743852430200065234... ≈ $\sqrt{43}$

.5579921445238905154... ≈ $\sum_{k=1}^{\infty}\frac{H_{2k}}{4^k} = \frac{4\log 2-\log 3}{3}$

.558110626551247253717... ≈ $\frac{1}{\log 6} = \log_6 e$ J153

.5582373008332086382... ≈ $\sum_{k=1}^{\infty}\frac{H_k}{2^k k^3}$

1 .558414187408610494023... ≈ $1-\cosh\sqrt{2}+\sqrt{2}\sinh\sqrt{2} = \sum_{k=0}^{\infty}\frac{2^k}{(2k)!(k+1)}$

.558542510602814021154... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(k+1)\log k}{2^k}$

1 .558670436425389042810... ≈ $\sum_{k=2}^{\infty}\frac{\log k}{(k-1)!} = \sum_{n=1}^{\infty}\frac{1}{(n-1)!}\sum_{k=2}^{\infty}\frac{\log^n k}{k!}$

.559134144418979917488... ≈ $J_0(\sqrt{2}) = \sum\frac{(-1)^k}{(k!)^2 2^k}$

162 .559234176474304999069... ≈ $22e^2 = \sum_{k=1}^{\infty}\frac{2^k k^3}{k!}$

.559334724804927427919... ≈ $\sum_{k=2}^{\infty}\left(\frac{\zeta(k)}{\zeta(k+1)}-1\right) = \sum_{k=2}^{\infty}\frac{\phi(k)}{k^2(k-1)}$

.55940740534257614454... ≈ $\sqrt{e}-\frac{5}{2}+\sqrt{\frac{e\pi}{2}}\,erf\left(\frac{1}{\sqrt{2}}\right) = \sum_{k=1}^{\infty}\frac{1}{k!!(k+2)}$

.559615787935422686271... ≈ $\log\frac{7}{4} = Li_1\left(\frac{3}{7}\right) = -Li_1\left(-\frac{3}{4}\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}3^k}{4^k k}$

3 .55989038838791143471... ≈ $27-\frac{39\zeta(3)}{2} = \sum_{k=1}^{\infty}\left(\frac{(-1)^{k+1}}{(k-1/3)^3}+\frac{(-1)^{k+1}}{(k+1/3)^3}\right)$

.5600000000000000000$\underline{0}$ = $\frac{14}{25} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2F_k}{2^k} = \int_0^{\infty}\frac{x\sin^2 x}{e^x}dx$

.560045520492075839181... ≈ $\sum_{k=1}^{\infty}\frac{1}{k!2^k\zeta(2k+1)}$ Titchmarsh 14.32.3

.56012607792794894497... ≈ $\prod_{k=1}^{\infty}\left(1-\frac{1}{3^k}\right)$

= $1+\sum_{k=1}^{\infty}(-1)^k\left(\frac{1}{3^{(3k^2+k)/2}}+\frac{1}{3^{(3k^2-k)/2}}\right)$ Hall Thm. 4.1.3

.560329854485890998637... ≈ $\prod_{k=1}^{\infty}\left(1+\frac{(-1)^k}{\binom{2k}{k}}\right)$

.560511825774805757653... ≈ $9e^{1/3}-12 = \sum_{k=0}^{\infty}\frac{1}{k!3^k(k+1(k+2)}$

.560744611093552095664... ≈ $\frac{10}{3}-4\log 2 = \sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)-1}{2^{k-2}} = \sum_{k=2}^{\infty}\frac{2}{k(2k-1)}$

1 .560910575045920426397... ≈ $\sum_{k=1}^{\infty}\frac{\sigma_0(k)}{(k!)^2}$

.561061199700803776228... ≈ $\frac{2\pi^3}{3\sqrt{3}}+13\zeta(3)-27=-\frac{1}{2}\psi^{(2)}\left(\frac{1}{3}\right)-27=\sum_{k=1}^{\infty}\frac{1}{(k+1/3)^3}$

27 .56106119970080377623... ≈ $-\frac{1}{2}\psi^{(2)}\left(\frac{1}{3}\right) = \sum_{k=0}^{\infty}\frac{1}{(k+⅓)^3}$

1 .561257911504962567051... ≈ $4\zeta(3)-3\zeta(4)$

.56145948356688516982... ≈ $e^{-\gamma} = \sum_{k=0}^{\infty}\frac{(-1)^k\gamma^k}{k!}$ Euler-Mascheroni constant

= $\prod_{k=1}^{\infty}\left(1+\frac{1}{k}\right)e^{-1/k}$ KGP ex. 6.69

= $\underline{\lim}\frac{\phi(n)\log\log n}{n}$ HW Thm. 328

= $\lim_{n\to\infty}\left(\log n\prod_{p\ prime<\ n}\left(1-\frac{1}{p}\right)\right)$ Mertens

23 .5614740840256044961... ≈ $\gamma^4+\gamma^2\pi^2+\frac{3\pi^2}{20}+8\gamma\zeta(3) = \int_0^{\infty}\frac{\log^4 xdx}{e^x}$

.56173205239479373631... ≈ $\frac{1}{2}\left(e+e^{\cos 2}\cos(\sin 2)\right)-1 = \sum_{k=1}^{\infty}\frac{\cos^2 k}{k!}$

.561781717266978021561... ≈ $-\frac{\pi}{6}\sec\frac{\pi\sqrt{5}}{2} = \prod_{k=3}^{\infty}\left(1-\frac{1}{k^2-k-1}\right)$

.56227926848469324079... ≈ $\frac{\pi^3\log 2}{8}-\frac{9\pi\zeta(3)}{16} = \int_0^1\frac{\arcsin^3 x}{x}dx = \int_0^{\pi/2}\frac{x^3}{\tan x}dx$

.56250000000000000000$\underline{0}$ = $\frac{9}{16}$

= $\sum_{k=2}^{\infty}k\left(\zeta(2k-1)-1\right) = \sum_{k=2}^{\infty}\frac{2k^2-1}{k(k^2-1)^2}$

.56256294011622259263... ≈ $\frac{1}{2}\log\left(2+2\cos 1\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}\cos k}{k}$

.562610833137088244956... ≈ $\zeta(2)-\zeta(4)$

.56264005857240015056... ≈ $\frac{1}{\sqrt{2}}\sin\frac{\pi}{\sqrt{2}} = \prod_{k=1}^{\infty}\left(1-\frac{1}{4k^2-1}\right)$ J1064

1 .562796019827922754022... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta^2(2k)-1\right)$

4 .5628229009806368496... ≈ $2+\log 2-\frac{3}{2\sqrt{2}}\log(3-2\sqrt{2}) = \sum_{k=1}^{\infty}\frac{kH_{2k}}{2^k}$

5 .5632758932916066715... ≈ $\sum_{k=1}^{\infty}\frac{pd(k)}{2^k}$

.5634363430819095293... ≈ $6-2e = \sum_{k=0}^{\infty}\frac{1}{k!(k+4)} = \sum_{k=0}^{\infty}\frac{k}{k!(k+3)}$

= $\sum_{k=0}^{\infty}\frac{1}{(k+1)!+3k!}$

8 .5636594207477353768... ≈ $\frac{1}{4}\left(\psi^{(1)}\left(\frac{1}{6}\right)-\psi^{(1)}\left(\frac{2}{3}\right)\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+\frac{1}{3})^2}$

.563812982603190392613... ≈ $\frac{1}{2}\cosh\frac{1}{2}$

1 .56386096544105634313... ≈ $6\zeta(2)+6\zeta(4)-\frac{\zeta(3)}{3}-\frac{3}{8} = \sum_{k=2}^{\infty}\frac{6k^2}{(k+1)^4}$

= $\sum_{k=2}^{\infty}(-1)^k(k-1)k(k+1)(\zeta(k)-1)$

.56418958354775628695... ≈ $\frac{1}{\sqrt{\pi}} = \sum_{k=0}^{\infty}\frac{k}{(k+\frac{1}{2})!2^k}$

10081 .56420457498488257411... ≈ $\frac{17\pi^8}{16} = \int_0^{\infty}\frac{x^7}{\sinh x}dx$ GR 3.523.10

2 .564376988688260377856... ≈ $\pi - \gamma \; = \; 3\psi\left(\frac{1}{2}\right) - 2\psi\left(\frac{1}{4}\right)$ Berndt 8.6.1

.564599706384424320593... ≈ $\sum_{k=2}^{\infty}(-1)^k \frac{\zeta(k)-1}{k-1} = \sum_{k=2}^{\infty}\frac{1}{k}\log\left(1+\frac{1}{k}\right)$

.56469423393426890215... ≈ $\sum_{k=1}^{\infty}\frac{\sin k}{\binom{2k}{k}}$

18 .564802414575552598704... ≈ $erfi\,2 \; = \; \frac{2}{\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{2^{2k+1}}{k!(2k+1)}$

1 .56493401856701153794... ≈ $\prod_{k=1}^{\infty}\left(1+\frac{1}{3^k}\right) \; = \; 1+\sum_{k=1}^{\infty}\frac{Q(k)}{3^k}$

1 .564940517815879282638... ≈ $\frac{\pi}{2}\tanh\pi \; = \; \int_0^{\infty}\frac{\sin 2x}{\sinh x}dx$

1 .565084580073287316585... ≈ $6^{1/4}$

.565098600639974693306... ≈ $\gamma\left(e+\frac{1}{e}\right) - \gamma\cosh 1 - \left(e+\frac{1}{e}\right)\cosh\text{int}\,1 + \left(e-\frac{1}{e}\right)\sinh\text{int}\,1$

$+\frac{1}{2}\left(e+\frac{1}{e}\right)\cosh\text{int}\,2 - \frac{1}{2}\left(e-\frac{1}{e}\right)\sinh\text{int}\,2 - \frac{1}{2}\left(e+\frac{1}{e}\right)\log 2$

= $\sum_{k=1}^{\infty}\frac{H_k}{(2k)!}$

.565159103992485027208... ≈ $I_1(1)$

.565446085479077837537... ≈ $\frac{11\pi^2}{192} \; = \; \frac{1}{2}\left(Li_2\left(\frac{1-i}{\sqrt{2}}\right)+Li_2\left(\frac{1+i}{\sqrt{2}}\right)\right) \; = \; \sum_{k=1}^{\infty}\frac{\cos\pi k/4}{k^2}$

.5655658753012601556... ≈ $\frac{1}{2}\left(Li_3\left(-e^i\right)+Li_3\left(-e^{-i}\right)\right) \; = \; \frac{i}{12}\left(1-\pi^2\right) - Li_3\left(-e^i\right)$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\cos k}{k^3}$

.565623554314631616419... ≈ $\sqrt{2}\arctan\sqrt{2} - \frac{\pi}{4} \; = \; \int_0^{\pi/4}\frac{\cos^2 x}{1+\sin^2 x}dx$

1 .565625835315743374058... ≈ $-\cos 2\,\cosh 2 \; = \; \sum_{k=0}^{\infty}\frac{(-1)^{k+1}64^k}{(4k)!}$

2 .565625835315743374058... ≈ $1 - \cos 2\cosh 2 \; = \; \sum_{k=1}^{\infty}(-1)^{k+1}\frac{2^{6k}}{(4k)!}$ GR 1.413.2

2 .565761892331577699921... ≈ $e - 1 + \gamma(e+1) - eEi(-1) - Ei(1) \; = \; \sum_{k=1}^{\infty}\frac{k^2 H_k}{(k+1)!}$

1 .56586036972271770520... ≈ $-\frac{2}{3}Li_3(-3) = \frac{\pi^2 \log 3}{9} + \frac{\log^3 3}{9} - \frac{2}{3}Li_3\left(-\frac{1}{3}\right)$

= $8\int_1^\infty \frac{\log^2 x}{x^3+3x} = \int_0^\infty \frac{x^2\,dx}{e^x+3}$

. 56588424210451674940... ≈ $\frac{16}{9\pi} = \sum_{k=0}^\infty \binom{2k}{k}^2 \frac{1}{16^k(k+1)(k+2)}$

.565959973761085005603... ≈ $\sqrt{2}\,J_1\left(2\sqrt{2}\right) = \sum_{k=1}^\infty (-1)^{k+1}\frac{2^k k}{(k!)^2}$

.565985876838710482162... ≈ $\frac{\pi}{8} + \frac{\log 2}{4} = \sum_{k=0}^\infty \frac{1}{(4k+1)(4k+2)}$

1 .566082929756350537292... ≈ $I_0(\sqrt{2}) = \sum_{k=0}^\infty \frac{1}{(k!)^2 2^k}$

.566213645893642941755... ≈ $\sum_{k=1}^\infty \frac{\phi(k)}{3^k}$

5 .566316001780235204250... ≈ $\Gamma\left(\frac{1}{6}\right)$

12 .566370614359172953851... ≈ 4π

.5665644010044528364... ≈ $e^2(Ei(-2) - Ei(-1)) - \log 2 = \int_0^1 \frac{\log(1+x)}{e^{x-1}}dx$

9 .5667536626468872669... ≈ $\frac{\sinh \pi\sqrt{2}}{\pi\sqrt{2}} = \prod_{k=1}^\infty \left(1+\frac{2}{k^2}\right)$

2 .566766565764270426298... ≈ $\sum_{k=0}^\infty \frac{k!}{(k!!)^3}$

.566797099699373515726... ≈ $\log\frac{15\sqrt{\pi}}{8} - \frac{3}{2}(1-\gamma) = \sum_{k=2}^\infty (-1)^k \frac{\zeta(k)-1}{k}\left(\frac{3}{2}\right)^k$ Srivastava

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.566911504941009405083... ≈ $\operatorname{arccot}\frac{\pi}{2}$

.567209351351013708132... ≈ $3 + 6\log\frac{2}{3} = \sum_{k=0}^\infty \frac{1}{3^k(k+1)(k+2)}$

.567384114877028322541... ≈ $\sum_{k=1}^\infty \frac{1}{(2k)^k}$

1 .567468255774053074863... ≈ $\Gamma(e)$

.567514220947867313537... ≈ $\log \pi - \gamma = \sum_{k=2}^\infty \frac{\zeta(k)}{2^{k-1}k} = -\sum_{k=1}^\infty \left(\frac{1}{k} + 2\log\left(1-\frac{1}{2k}\right)\right)$

.567667641618306345947... ≈ $\frac{1}{2}+\frac{1}{2e^2} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{2^k}{(k+1)!}$

5 .56776436283002192211... ≈ $\sqrt{31}$

.567783854352055695476... ≈ $2\zeta(3)-\frac{\pi^2\log 2}{3}+\frac{4\log^3 2}{3} = \sum_{k=1}^{\infty}\binom{2k}{k}\frac{1}{4^k k^3}$

.568074442278171... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k\log k}{(k-1)(2k-3)}$

.568260019379645262338... ≈ $\frac{\pi^2}{6}-\frac{\pi}{2}\coth\pi+\frac{1}{2} = \sum_{k=1}^{\infty}\frac{1}{k^4+k^2}$

= $\frac{1}{2}+\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta 2k+2)-1\right) = -\operatorname{Im}\left\{\sum_{k=1}^{\infty}\frac{1}{k^2(k+i)}\right\}$

5 .568327996831707845285... ≈ $\pi^{3/2}$

.568389930078412320021... ≈ $\frac{1}{2}+\sum_{k=2}^{\infty}\frac{\Omega(k)}{2^k} = \sum_{k=2}^{\infty}\sum_{j=1}^{\infty}\frac{1}{2^{k^j}}$

.568417037461477055706... ≈ $\frac{3\sqrt{\pi}}{4}erf\,1-\frac{3}{2e} = \int_0^1 e^{-x^{2/3}}dx$

.568656627048287950986... ≈ $H_0(1) = \frac{2}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k}{\left((2k+1)!!\right)^2} = \frac{4}{\pi}\sum_{k=0}^{\infty}\frac{J_{2k+1}(1)}{2k+1}$

.568685634605209962010... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\mu(k)}{2^k}$

1 .56903485300374228508... ≈ $\gamma\, e$

.569356862306854203563... ≈ $\frac{1}{3^{4/3}}\left((-1)^{1/3}\psi\left(\frac{6+3^{7/6}i+3^{2/3}}{6}\right)-(-1)^{2/3}\psi\left(\frac{6-3^{7/6}i+3^{2/3}}{6}\right)\right)$

$-\frac{1}{3^{4/3}}\psi\left(1-\frac{1}{3^{1/3}}\right)$

= $\sum_{k=1}^{\infty}\frac{\zeta(3k)}{3^k} = \sum_{k=1}^{\infty}\frac{1}{3k^3-1}$

.569451751594934318891... ≈ $4\zeta(2)-5\zeta(3)$

1 .56956274263568157387... ≈ $\sum_{k=1}^{\infty}(\sinh k)\left(\zeta(2k)-1\right)$

.5699609930945328064... ≈ $-\frac{\zeta'(2)}{\zeta(2)} = \frac{1}{\zeta(2)}\sum_{k=2}^{\infty}\frac{\log k}{k^2} = \sum_{p\ prime}\frac{\log p}{p^2-1}$ Berndt Ch. 5

= $\sum_{k=1}^{\infty} \frac{\Lambda(k)}{k^2}$

.57015142052158602873... ≈ $Ei\left(\frac{1}{2}\right) - \gamma + \log 2 = \sum_{k=1}^{\infty} \frac{1}{2^k k! k}$

.57025949722570976065... ≈ $\frac{\sqrt{3}}{2} \operatorname{arctanh} \frac{1}{\sqrt{3}} = \sum_{k=1}^{\infty} \frac{H^o{}_k}{3^k}$

.57049021605145009522 8... ≈ $(\cosh \cos 1 + \sinh \cos 1)(\cos 1 \sin \sin 1 + \sin 1 (\cosh \cos 1 - \sinh \cos 1 - \cos \sin 1))$

= $\sum_{k=1}^{\infty} \frac{\sin k}{(k+1)!}$

2 .57056955072903255045... ≈ $\sum_{k=1}^{\infty} \frac{\phi^2(k)}{2^k}$

.57079632679489661923... ≈ $\frac{\pi}{2} - 1 = \sum_{k=1}^{\infty} \frac{\sin 2k}{k}$

= $\sum_{k=1}^{\infty} \binom{2k}{k} \frac{1}{4^k (2k+1)} = \sum_{k=1}^{\infty} \frac{(2k-1)!!}{(2k)!(2k+1)}$ J388

= $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2k^2 - 1/2}$

= $\int_0^1 \frac{\arccos x}{(1+x)^2} dx$

= $\int_0^{\infty} \frac{\tanh x}{e^x} dx$

= $\int_0^{\infty} \frac{x\,dx}{(1+x^2) \sinh \pi x / 2}$ GR 3.522.7

= $\int_0^{\infty} \frac{x \log(1+x)\,dx}{\sqrt{1-x^2}}$ GR 4.292.2

1 .57079632679489661923... ≈ $\frac{\pi}{2} = -i \log i$

= $\sum_{k=0}^{\infty} \frac{(-1)^k}{k + 1/2}$

= $\sum_{k=0}^{\infty} \frac{1}{(2k+1)^2 - 1/4} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^2}{\left(k^2 - 1/4\right)^2}$

= $\sum_{k=0}^{\infty} \frac{k!}{(2k+1)!!}$ K144

= $\sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k} k} = \sum_{k=0}^{\infty} \frac{(k!)^2 2^k}{(2k+1)!} = \sum_{k=0}^{\infty} \binom{2k}{k} \frac{1}{4^k (2k+1)}$

$$= \sum_{k=1}^{\infty} \frac{(2k)!!}{(2k-1)!!\; k\, 2^k}$$

$$= -\sum_{k=1}^{\infty} \left(Li_k(i) + Li_k(-i) \right)$$

$$= \sum_{k=0}^{\infty} \arctan\left(\frac{2}{(2k+1)^2} \right)$$

[Ramanujan] Berndt Ch. 2

$$= \sum_{k=1}^{\infty} \arctan\left(\frac{1}{(2k-1+\sqrt{5})^2} \right)$$

[Ramanujan] Berndt Ch. 2

$$= \prod_{k=1}^{\infty} \frac{4k^2}{(2k-1)(2k+1)}$$

Wallis's product

$$= \int_0^{\infty} \frac{dx}{x^2+1} \;=\; \int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2} \;=\; \int_0^{\infty} \frac{dx}{e^x+2e^{-x}-2}$$

$$= \int_0^{\infty} \frac{dx}{x^2+x+1/2}$$

$$= \int_0^{\infty} \frac{\sin x}{x}\, dx \;=\; \int_0^{\infty} \frac{\sin^2 x}{x^2}\, dx$$

$$= \int_0^{\infty} \frac{x^2-1}{(1+x^2)^2} \log x\, dx$$

GR 4.234.4

$$= \int_0^{\pi/2} \cos^2 x\, dx$$

GR 3.631.20

$$= \int_0^{\infty} \left(\frac{1}{x} - \cot x \right) \frac{dx}{x}$$

Prud. 2.5.29.10

$$= \int_1^{\infty} \frac{\operatorname{arccosh} x}{x^2}$$

$$= \int_0^{\pi/2} \log(e \tan x)\, dx$$

GR 4.227.3

$$= \int_0^{\infty} \frac{1}{e^x+2e^{-x}-2}$$

$$= \int_0^{\infty} x \log\left(1 + \frac{1}{x^4} \right)\, dx$$

$$= \int_0^{\infty} ci(x) \log x\, dx$$

GR 6.264

2 .570796326794896619231... $\approx$

$$\frac{\pi}{2} + 1 \;=\; \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

3 .570796326794896619231... ≈ $\frac{\pi}{2}+2 = \sum_{k=0}^{\infty} \frac{2^k}{\binom{2k}{k}}$

.571428571428$\underline{571428}$ = $\frac{4}{7} = \prod_{p\ prime} \frac{1-p^{-4}}{1+p^{-2}+p^{-4}}$

1 .57163412158236360955... ≈ $\sum_{k=2}^{\infty} \frac{1}{k^2-k^{3/2}} = \sum_{k=4}^{\infty} \left(\zeta(k/2)-1\right)$

.57171288872391284966... ≈ $\sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{12k-2} = \frac{\pi}{12}+\frac{1}{2}+\log\frac{\sqrt{3}-1}{\sqrt{3}+1}$

.571791629712009706036... ≈ $-\gamma-\frac{1}{2}\left(\psi\left(\frac{1}{\sqrt{3}}\right)+\psi\left(-\frac{1}{\sqrt{3}}\right)\right) = \sum_{k=1}^{\infty} \frac{\zeta(2k+1)}{3^k}$

= $\sum_{k=1}^{\infty} \frac{1}{k(3k^2-1)}$

3 .571815532109087142149... ≈ $\frac{9\zeta(3)}{4}+\frac{111}{128} = \sum_{k=2}^{\infty} k^3\left(\zeta(2k-1)-1\right)$

.572303143311902634417... ≈ $\frac{4}{9}\left(1+\log\frac{4}{3}\right) = \sum_{k=1}^{\infty} \frac{kH_k}{4^k}$

.572364942924700087071... ≈ $\frac{\log\pi}{2} = \log\Gamma\left(\frac{1}{2}\right)$

= $-\int_0^{\infty} \left(\frac{e^{-2x}}{2}-\frac{1}{e^x+1}\right)\frac{dx}{x}$ GR 3.427.5

.572467033424113218236... ≈ $\frac{\pi^2}{12}-\frac{1}{4} = \sum_{k=1}^{\infty} k\left(\zeta(2k)-\zeta(2k+1)\right)$

= $\frac{1}{2}\left(Li_2(-e^{i})+Li_2(-e^{-i})\right)$

= $\sum_{k=2}^{\infty} \frac{k}{k^3+k^2-k-1}$

= $\sum_{k=2}^{\infty} (-1)^k k\left(\frac{\zeta(k)+\zeta(k+1)}{2}-1\right)$

= $\sum_{k=1}^{\infty} (-1)^{k+1}\frac{\cos k}{k^2}$

1 .57246703342411321824... ≈ $\frac{\pi^2}{12}+\frac{3}{4} = \sum_{k=1}^{\infty} \frac{H_k}{k^2-1}$

.57289839112388295085... ≈ $\frac{4-\sqrt{2}}{8}\sqrt{\pi} = \int_0^{\infty} \frac{\sin^4\left(x^2\right)}{x^2}dx$

.57393989404675551338... ≈ $\frac{3\zeta(3)}{2\pi} = \xi(3) = \xi(-2)$

2 .57408909729511984405... ≈ $\prod_{k=1}^{\infty}\left(1+\frac{1}{(k!)^2}\right)$

.574137740053329817241... ≈ $\frac{\pi^2}{6} - \frac{\pi-1}{2} = \sum_{k=1}^{\infty}\frac{\cos^2 k}{k^2}$

.574859404114557591023... ≈ $\frac{1+2\gamma}{3} + \frac{1}{3}\left(\psi\left((-1)^{1/3}\right) + \psi\left(-(-1)^{2/3}\right)\right) = \sum_{k=1}^{\infty}\frac{1}{k^4+k}$

= $\frac{1}{2} + \sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(3k+1)-1\right)$

5 .57494152476088062397... ≈ $e^{e-1} = \prod_{k=1}^{\infty} e^{1/k!}$

.575222039230620284612... ≈ $10 - 3\pi$

.57525421205443264457... ≈ $\displaystyle\int_0^{\pi/2} \frac{x\sin x}{2-\cos^2 x}\,dx$

.57527071983288752368... ≈ $\displaystyle\frac{\pi^2}{4} - 3\log^2 2 - \frac{3\zeta(3)}{8} = \int_0^1 \frac{\log^3(1+x)}{x^3}\,dx$

1 .575317162084868575203... ≈ $\displaystyle 16 - 12\zeta(3) = \int_0^1 \frac{\log^2 x}{1+\sqrt{x}}\,dx$

.575563616497977704066... ≈ $\displaystyle 1 - \frac{\sqrt{\pi}}{2e^{1/4}}\,erfi\frac{1}{2} = \sum_{k=0}^{\infty}\frac{(-1)^k k!}{(2k)!}$

1 .57570459714985838481... ≈ $\displaystyle \log\left(-\Gamma\left(-\frac{3}{4}\right)\right)$

.575951309944557165803... ≈ $\displaystyle \frac{\pi^2}{2} - \gamma - \frac{1}{2}\left(\psi(1+i)+\psi(1-i)\right) + \frac{i}{4}\left(\psi^{(1)}(1-i)+\psi^{(1)}(1+i)\right)$

= $\displaystyle \sum_{k=2}^{\infty}(-1)^k k\left(\zeta(k)-\zeta(2k-1)\right)$

.576247064560645225676... ≈ $\displaystyle \frac{\pi^3-4\pi}{32} = \sum_{k=0}^{\infty}(-1)^k\frac{\cos(2k+1)}{(2k+1)^3}$ Davis 3.37

.576674047468581174134... ≈ $\displaystyle \frac{\pi}{2}\coth\pi - 1 = \sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(2k)-1\right)$

= $\displaystyle \sum_{k=2}^{\infty}\frac{1}{k^2+1} = \sum_{k=1}^{\infty}\left(\zeta(4k-2)-\zeta(4k)\right)$ J124

= $\displaystyle \int_0^{\infty}\frac{\sin x}{e^x\left(e^x-1\right)}$

.576724807756873387202... ≈ $\displaystyle J_1(2) = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!(k+1)!} = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}k}{(k!)^2}$

.57694642787663931446... ≈ $\displaystyle \frac{6\sin\pi\sqrt{5}}{\pi\sqrt{5}} = \prod_{k=1}^{\infty}\left(1-\frac{1}{k(k+4)}\right)$

.57721566490153286061... ≈ γ (Euler's constant, not known to be irrational)

= $\displaystyle \lim_{k\to\infty}\left(H_k - \log k\right)$ Euler

= $\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{2}\log(k(k+1))\right)$ Cesaró

= $\displaystyle 2\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{2k-1} - \log 4n\right)$

= $\displaystyle \lim_{n\to\infty}\left(H_n - \frac{1}{2}\log\left(n^2+n+\frac{1}{3}\right)\right)$

$$= \lim_{n\to\infty}\left(H_n - \frac{1}{4}\log\left(\left(n^2+n+\frac{1}{2}\right)^2 - \frac{1}{45}\right)\right)$$

$$= \lim_{n\to\infty}\left(n - \Gamma\left(\frac{1}{n}\right)\right)$$
Demys

$$= 1 + \sum_{k=2}^{\infty}\left(\frac{1}{k} + \log\left(1 - \frac{1}{k}\right)\right)$$
Euler

$$= \frac{\log 2}{2} + \frac{1}{\log 2}\sum_{k=2}^{\infty}(-1)^k \frac{\log k}{k}$$

$$= 1 - \log\frac{3}{2} - \sum_{k=1}^{\infty}\frac{\zeta(2k+1)-1}{4^k(2k+1)}$$

$$= 1 - \sum_{k=2}^{\infty}\sum_{j=2}^{\infty}\frac{1}{jk^j}$$

$$= \sum_{k=2}^{\infty}\frac{(-1)^k\zeta(k)}{k} = \sum_{k=1}^{\infty}\left(\frac{1}{k} - \log\left(1 + \frac{1}{k}\right)\right)$$

$$= 1 - \sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k}$$
Euler

$$= \sum_{k=2}^{\infty}\frac{k-1}{k}\left(\zeta(k)-1\right) = \sum_{k=2}^{\infty}\left(\log\left(1-\frac{1}{k}\right) + \frac{1}{k-1}\right)$$
Euler (1769)

$$= 1 - \frac{\log 2}{2} - \sum_{k=1}^{\infty}\frac{\zeta(2k+1)-1}{2k+1}$$
Euler

$$= 1 - \log\frac{3}{2} - \sum_{k=1}^{\infty}\frac{\zeta(2k+1)-1}{4^k(2k+1)}$$
Euler-Stieltjes

$$= 1 - \sum_{k=1}^{\infty}\frac{\zeta(2k+1)}{(k+1)(2k+1)}$$
Glaisher

$$= 2 - 2\log 2 - \sum_{k=1}^{\infty}\frac{\zeta(2k+1)-1}{(k+1)(2k+1)}$$
Glaisher

$$= 1 - \log 2 - \sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)-1}{k}$$

$$= \frac{3}{2} - \log 2 - \sum_{k=2}^{\infty}(-1)^k(k-1)\frac{\zeta(k)-1}{k}$$
Flajolet-Vardi

$$= \frac{5}{4} - \log 2 - \frac{1}{2}\sum_{k=3}^{\infty}(-1)^k(k-2)\frac{\zeta(k)-1}{k}$$

$$= \log 8\pi - 3 + 2\sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)-1}{k+1}$$

$$= \log\frac{4}{\pi} + 2\sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)}{2^k k}$$

$$= \quad 1+\log\frac{16}{9\pi}+2\sum_{k=2}^{\infty}(-1)^k\,\frac{\zeta(k)-1}{2^k k}$$

$$= \quad \frac{3}{2}-\log 2-\sum_{k=2}^{\infty}\frac{1}{k}\left(\zeta(k)-1-\frac{1}{2^k}\right)$$

$$= \quad \frac{11}{6}-\log 3-\sum_{k=2}^{\infty}\frac{1}{k}\left(\zeta(k)-1-\frac{1}{2^k}-\frac{1}{3^k}\right)$$

$$= \quad H_s-\log s-\sum_{k=2}^{\infty}\frac{\zeta(k,s)}{k}$$

$$= \quad -\frac{1}{2}\sum_{k=2}^{\infty}\frac{\Lambda(k)-1}{k}\ ,\quad \Lambda(k)=\log p\ \ \textit{if}\ \ k\,\textit{is a power of a prime}\ p$$

$$= \quad \sum_{n=1}^{\infty}\frac{1}{n!}\left(\left(\sum_{k=2}^{\infty}\frac{\log^n k}{k(k-1)}\right)-1\right)$$

Shamos

$$= \quad \sum_{k=1}^{\infty}k\left(\sum_{j=2^k}^{2^{k+1}-1}\frac{(-1)^j}{j}\right)$$

Vacca, Franklin

$$= \quad \sum_{k=1}^{\infty}(-1)^k\,\frac{\lfloor\log_2 k\rfloor}{k}$$

Vacca

$$= \quad -\int_0^{\infty}\frac{\log x}{e^x}\,dx$$

$$= \quad 1-\int_1^{\infty}\frac{x-\lfloor x\rfloor}{x^2}\,dx$$

$$= \quad -\int_0^{1}\log\log\frac{1}{x}\,dx$$

Andrews, p. 87

$$= \quad -\int_0^{2\pi}\log x\ \sin x\ dx$$

GR 4.381.3

$$= \quad \int_0^{\pi/2}\left(1-\sec^2 x\cos(\tan(x))\right)\frac{dx}{\tan x}$$

GR 3.716.12

$$= \quad -\Gamma'(1)\ =\ -\int_0^{\infty}e^{-x}\log x\,dx$$

$$= \quad \int_0^{1}H(x)\,dx$$

$$= \quad -\int_{-\infty}^{\infty}xe^{x-e^x}\,dx$$

Prud. 2.3.18.6

$$= \quad \int_0^{1}\frac{1-e^{-x}-e^{-1/x}}{x}\,dx$$

Barnes

$= \quad -2\log 2 - \frac{4}{\sqrt{\pi}}\int_0^\infty e^{-x^2}\log x\, dx$

$= \quad \frac{1}{2} + 2\int_0^\infty \frac{x\,dx}{(1+x^2)(e^{2\pi x}-1)}$ Andrews, p. 87

$= \quad \int_0^1 (1-e^{-x}-e^{-1/x})\frac{dx}{x}$ Andrews, p. 88

$= \quad \int_0^1 \left(\frac{1}{x}+\frac{1}{\log(1-x)}\right) dx$

$= \quad \int_0^\infty \left(\frac{1}{1-e^x}-\frac{1}{x}\right) e^{-x}\, dx$ GR 3.427.2

$= \quad \int_0^\infty \left(\frac{1}{1+x}-e^{-x}\right)\frac{dx}{x}$ GR 3.435.3

$= \quad \int_0^\infty \left(\frac{1}{1+x}-\cos x\right)\frac{dx}{x}$ GR 3.783.2

$= \quad \int_0^\infty \left(\frac{1}{1+x^2}-\cos x\right)\frac{dx}{x}$ GR 3.783.2

$= \quad \int_0^x \frac{1-\cos x}{x}dx - \int_x^\infty \frac{\cos x}{x}dx - \log x, \qquad x>0$

$= \quad \int_0^x \frac{1-e^{-x}}{x}dx - \int_x^\infty \frac{e^{-x}}{x}dx - \log x, \qquad x>0$

$= \quad \frac{ab}{a-b}\int_0^\infty \frac{e^{-x^a}-e^{-x^b}}{x}dx\,, \qquad a>0, b>0, a\neq b$

$= \quad 1 - \int_0^1 \frac{1}{1+x}\left(\sum_{k=1}^\infty x^{2^k}\right)dx$ Catalan

$= \quad \frac{1}{2} + 2\int_0^\infty \frac{x\,dx}{(e^{2\pi x}-1)(x^2+1)}$ Hermite

$= \quad H_{2n} - \log n + 2\int_0^\infty \frac{x\,dx}{(e^{2\pi x}-1)(x^2+n^2)}$ Hermite

1 .57721566490153286061... $\approx \quad \gamma + 1 = \int_0^\infty si(x)\log x\, dx$ GR 6.264.1

$= \quad \int_0^\infty Ei(-x)\log x\, dx$ GR 6.234

.57735026918962576450... $\approx \quad \frac{\sqrt{3}}{3} = \tan\frac{\pi}{6}$

= $\displaystyle \frac{4}{3\pi}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2-1/9}$ GR 1.421

= $\displaystyle \sum_{k=0}^{\infty}\binom{2k}{k}\frac{1}{6^k(k+1)(k+2)}$

.577779867999970432229... ≈ $\displaystyle \frac{\zeta(2)}{\zeta(2)+\zeta(3)}$

.577863674895460858955... ≈ $\displaystyle \frac{\pi}{2e} = \prod_{k=1}^{\infty}\left(1+\frac{2}{k}\right)^{(-1)^{k+1}k}$

= $\displaystyle \int_0^{\infty}\frac{\cos x}{1+x^2}dx$ AS 4.3.146

= $\displaystyle \int_0^{\infty}\frac{\cos x}{(1+x^2)^2}dx$

= $\displaystyle \int_0^{\infty}\frac{x\sin x}{1+x^2}dx$

= $\displaystyle \int_0^{\pi/2}\cos(\tan x)\cos^2 x\,dx$ GR 3.716.5

= $\displaystyle \int_0^{\infty}\cos(\tan x)\frac{\sin x}{x}dx$ GR 3.881.4

= $\displaystyle \int_0^{1}\left(\cos x+\cos\frac{1}{x}\right)\frac{dx}{1+x^2}$ Prud. 2.5.29.11

= $\displaystyle \int_{-\infty}^{\infty}\frac{x^3\sin x}{(x^2+1)^2}dx$ Marsden p. 259

.5781221858069403989... ≈ $\displaystyle \frac{27\log 3}{6}-\frac{\pi^2}{6}-\frac{\pi\sqrt{3}}{2} = \sum_{k=1}^{\infty}\frac{1}{3k^3-k^2}$

1 .57815429172342929876... ≈ $\displaystyle \frac{\pi^2+3}{3e} = \int_0^{\infty}\frac{\log^2 x}{(x+e)^2}$

.578169737606879079622... ≈ $\displaystyle \sum_{k=1}^{\infty}\frac{\zeta(2k)}{(2k)^2} = \frac{1}{4}\sum_{k=1}^{\infty}Li_2\left(\frac{1}{k^2}\right)$

.578477579667136838318... ≈ $\displaystyle \frac{\pi(\sin\pi\sqrt{2}+\sinh\pi\sqrt{2})}{2\sqrt{2}(\cosh\pi\sqrt{2}-\cos\pi\sqrt{2})}-\frac{1}{2} = \sum_{k=1}^{\infty}\frac{1}{k^4+1}$

= $\displaystyle \frac{1}{2}+\sum_{k=1}^{\infty}(-1)^{k+1}\big(\zeta(4k)-1\big) = \operatorname{Im}\left\{\sum_{k=1}^{\infty}\frac{1}{k^2(k^2+i)}\right\}$

2 .578733971888556748934... ≈ $\displaystyle \sum_{k=2}^{\infty}\frac{\zeta^3(k)}{k!}$

4 .57891954339760069657... ≈ $4\log\pi$

.$\underline{578947368421052631}$ = $\frac{11}{19}$

.57911600484428752980... ≈ $\frac{\pi^2}{2} + \frac{\pi^4}{24} - 7\zeta(3) = \sum_{k=1}^{\infty} \frac{k^2}{(k+½)^4}$

1 .57913670417429737901... ≈ $\frac{4\pi^2}{25}$

= $\sum_{k=1}^{\infty} \left(\frac{1}{(5k-1)^2} + \frac{1}{(5k-2)^2} + \frac{1}{(5k-3)^2} + \frac{1}{(5k-4)^2} \right)$

6 .57925121201010099506... ≈ $\log 6!$

3 .57944154167983592825 2... ≈ $\frac{3}{2}\left(1 + 2\log 2\right) = \sum_{k=2}^{\infty} \left(\frac{3}{2}\right)^2 \left(\zeta(k) - 1\right) = \sum_{k=2}^{\infty} \frac{9}{4k^2 - 6k}$

.57948049437049180131 5... ≈ $\sum_{k=2}^{\infty} \frac{1}{k(k!-1)}$

.5795933814359071842... ≈ $\sum_{k=1}^{\infty} \frac{1}{(3^k - 1)k} = \sum_{k=1}^{\infty} \frac{\sigma_{-1}(k)}{3^k}$

4 .57973626739290574589... ≈ $\frac{2\pi^2}{3} - 2 = \int_0^{\infty} \frac{e^x - e^{-x} + 2}{(e^x - 1)^2} x^2 \, dx$ GR 3.424.5

6 .57973626739290574589... ≈ $\frac{2\pi^2}{3} = \int_0^{\infty} \log^2 x \frac{dx}{(1-x)^2}$ GR 4.261.5

.57975438341923839722... ≈ $-Li_2\left(-\frac{2}{3}\right)$

4 .57982797088609507527 3... ≈ $5G$

2 .58013558949369127019... ≈ $\zeta(\zeta(\zeta(\zeta(2))))$

.58037423860937130785 6... ≈ $\sum_{k=1}^{\infty} \frac{k}{k^5 + 1} = \frac{1}{2} + \sum_{k=1}^{\infty} (-1)^{k+!} \left(\zeta(5k-1) - 1\right)$

5 .58040037250884444363 3... ≈ $\sum_{k=1}^{\infty} \frac{e^k}{k^k}$

.5804376954844022367 3... ≈ $\arctan(\tanh\frac{\pi}{4}) = \sum_{k=0}^{\infty} (-1)^{k+1} \arctan\left(\frac{1}{2k+1}\right)$

[Ramanujan] Berndt Ch. 2

.58055179162194598889 8... ≈ $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{\mu(k)}{3^k - 1}$

6 .58088599101792097085 2... ≈ $2^e = \prod_{k=0}^{\infty} 2^{1/k!}$

.58089172936454176272... ≈ $\frac{3}{4}\zeta(3) + Li_3\left(-\frac{1}{3}\right) = \int_0^1 \frac{\log^2 x}{(x+1)(x+3)} dx$

.58105827123019180794... ≈ $-\frac{2}{\pi}\cos\frac{\pi\sqrt{3}}{2} = \prod_{k=2}^{\infty}\left(1-\frac{1}{2k(k-1)}\right)$

.58106146679532725822... ≈ $\frac{1}{2}\left(3-\log 2\pi\right) = \sum_{k=2}^{\infty}\frac{2^k}{k+1}\left(\zeta(k)-1\right)$

.58132252304236693938... ≈ $\frac{2\pi}{3^{13/6}} = \int_0^{\infty}\frac{dx}{x^3+3}$

.581343706060247836785... ≈ $\frac{i\pi}{6}\left(\cot\pi(-1)^{1/6} - \cot\left(\frac{\pi\sqrt{2-2i\sqrt{3}}}{2}\right) + i\coth\pi\right)$

= $\sum_{k=1}^{\infty}\frac{k^2}{k^6+1} = \frac{1}{2} + \sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(6k-2)-1\right)$

.581824016936632859971... ≈ $\frac{29e^{1/4}}{64} = \sum_{k=1}^{\infty}\frac{k^3}{k!4^k}$

.581832832827806506555... ≈ $\sum_{k=1}^{\infty}\frac{k^3}{k^7+1} = \frac{1}{2} + \sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(7k-3)-1\right)$

.5819767068693264244... ≈ $\frac{1}{e-1} = \sum_{k=1}^{\infty}\frac{1}{e^k} = \sum_{k=1}^{\infty}\frac{1}{2^k\left(1+1/e^{1/2^k}\right)}$

[Ramanujan] Berndt Ch. 31

= $\sum_{k=1}^{\infty}\frac{(-1)^k B_k}{k!}$

= $\prod_{k=0}^{\infty}\left(1+\frac{1}{e^{2^k}}\right)$ Prud. 6.2.3.1

= $\int_1^{\infty}\frac{\log x}{(x+e-1)^2}\,dx$

2 .581931792882360146140... ≈ $\pi\left(\sqrt{5}-\sqrt{2}\right) = \int_0^{\infty}\log\left(1+\frac{3}{x^2+2}\right)dx$

.581954706951074968255... ≈ $\frac{\pi\left(\sqrt{2}-1\right)}{\sqrt{5}} = \int_0^{\infty}\log\left(1+\frac{1}{5x^2+1}\right)dx$

1 .581976706869326424385... ≈ $\frac{e}{e-1} = \sum_{k=0}^{\infty}\frac{1}{e^k} = \sum_{k=0}^{\infty}\frac{(-1)^k B_k}{k!}$

3 .582168726084073272274... ≈ $-2 + 2\sqrt{e}\cosh\frac{\sqrt{5}}{2} = \sum_{k=1}^{\infty}\frac{L_k}{k!}$

.5822405264650125059... ≈ $\frac{\pi^2}{12} - \frac{\log^2 2}{2} = Li_2\left(\frac{1}{2}\right) = \sum_{k=1}^{\infty}\frac{1}{2^k k^2}$ J362

		=	$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{H_k}{k}$	J116
		=	$\sum_{k=1}^{\infty}\frac{H^{(2)}{}_k-1}{k^2}$	MIS
		=	$\int_0^1\frac{\log(1+x)}{x(1+x)}dx = \int_1^2\frac{\log x}{x^2-x}dx$	GR 4.291.12
		=	$\int_2^{\infty}\log\frac{x}{x-1}\cdot\frac{dx}{x}$	
		=	$-\int_0^{1/2}\frac{\log(1-x)}{x}dx$	GR 4.291.3
		=	$-\int_0^1\log\left(1-\frac{x}{2}\right)\frac{dx}{x}$	GR 4.291.4
		=	$-\int_0^1\log\left(\frac{1+x}{2}\right)\frac{dx}{1-x}$	GR 4.291.5
4	.58257569495584000659...	≈	$\sqrt{21}$	
1	.58283662444715458907...	≈	$\sqrt{\frac{3}{2}}\sin\pi\sqrt{2}\,\csc\pi\sqrt{3} = \prod_{k=1}^{\infty}\left(1+\frac{1}{k^2+4k+1}\right)$	
	.58312180806163756027...	≈	$\frac{2G}{\pi}$	
	.58326324064259403627...	≈	$\sin 1\log 2$	
	$.5833333333333333333\underline{3}$	=	$\frac{7}{12}$	
	.58349565395417428579...	≈	$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\sigma_{-1}(k)}{2^k-1}$	
10	.58358414839597534084...	≈	$\frac{3e^2-1}{2} = \sum_{k=0}^{\infty}\frac{2^k k^2}{(k+1)!}$	
	.5838531634528576130...	≈	$2\cos^2 1 = 2-\sum_{k=1}^{\infty}(-1)^{k+1}\frac{2^{2k}}{(2k)!}$	GR 1.412.2
1	.58489319246111348520...	≈	$10^{1/5}$	
1	.58496250072115618145...	≈	$\log_2 3$	
	.5852181544310789058...	≈	$16-\pi^2-8\log 2 = \int_0^1\frac{\log(1-x)\log x}{\sqrt{x}}dx$	
1	.58556188208527645146...	≈	$\sum_{k=2}^{\infty}\frac{1}{2^{\phi(k)}}$	

.585698861949791886453... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^k}{k^3+1}$

.585786437626904951198... ≈ $2-\sqrt{2} = \frac{\sqrt{2}}{1+\sqrt{2}} = \int_0^{\pi/4}\frac{dx}{\sin x+1}$

.58617565462430550835... ≈ $\Gamma\left(\frac{7}{3}\right) - \frac{4}{3}\Gamma\left(\frac{4}{3},1\right) = \int_0^1 e^{-x^{3/4}}dx$

.586186075393236288392... ≈ $\sum_{k=1}^{\infty}\frac{\sigma_0(k)}{(2k)!}$

8 .586241294510575299961... ≈ $\zeta\left(\frac{9}{8}\right)$

4 .586419093909772141909... ≈ $(\pi-1)^2$

.586781998766982115844... ≈ $\frac{1}{3}+\frac{2}{3\sqrt{3}}\operatorname{arccsch}\sqrt{2} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{2^k}{\binom{2k}{k}}$

.586863910482303846601... ≈ $HypPFQ[\{\},\{2,3\},1] = \sum_{k=0}^{\infty}\frac{1}{k!(k+1)!(k+2)!}$

1 .58686891204429363056... ≈ $\frac{G}{\gamma}$

2 .586899392477790854402... ≈ $\frac{512}{63\pi} = \binom{5}{1/2}$

.58698262938250601998... ≈ $\frac{\pi}{2}\tanh\frac{\pi}{8}$

7 .587073064508709895971... ≈ $\frac{1}{3}e^{\pi} + \frac{2}{3}e^{-\pi/2}\cos\frac{\pi\sqrt{3}}{2} = \sum_{k=0}^{\infty}\frac{\pi^{3k}}{(3k)!}$

.587413281200730307975... ≈ $\sum_{k=1}^{\infty}\frac{\log(k+1)}{2^k k} = -\int_0^1\frac{\log(2-x)}{\log x}dx$ GR 422.3

.587436851334876027264... ≈ $\frac{\log 3}{2}+\frac{1}{9}\psi^{(1)}\left(\frac{2}{3}\right)-\frac{\pi}{6\sqrt{3}} = \sum_{k=2}^{\infty}\frac{k\zeta(k)}{3^k} = \sum_{k=1}^{\infty}\frac{6k-1}{3k(3k-1)^2}$

7 .58751808926722988286... ≈ $\zeta\left(\frac{8}{7}\right)$

.58760059682190072844... ≈ $\frac{\sinh 1}{2} = \sum_{k=1}^{\infty}\frac{k}{(2k)!} = \int_1^{\infty}\cosh\left(\frac{1}{x^2}\right)\frac{dx}{x^3}$

.587719754150346292672... ≈ $\frac{\pi}{3\cdot 2^{5/6}} = \int_0^{\infty}\frac{dx}{x^6+2}$

.587785252292473129169... ≈ $\sin\frac{\pi}{5} = \cos\frac{3\pi}{10}$

.58778666490211900819... ≈ $\log\frac{9}{5} = \sum_{k=1}^{\infty}\frac{L_k}{3^k k}$

.5882352941176470 = $\frac{10}{17}$

12 .588457268119895641747... ≈ $9\sqrt{3}-3 = \sum_{k=0}^{\infty}\binom{2k+1}{k}\frac{k^2}{6^k}$

1 .588467085518637731166... ≈ $i\left(\psi^{(1)}(1+i)-\psi^{(1)}(1-i)\right) = i\left(\zeta(2,2+i)-\zeta(2,2-i)\right)$

= $i\sum_{k=1}^{\infty}\left(\frac{1}{(k-i)^2}-\frac{1}{(k+i)^2}\right)$

.58863680070958604544... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}F_k\left(\zeta(2k)-1\right)$

.588645903311846921111... ≈ $\frac{3}{2}-\frac{\pi\sqrt{3}}{2}\operatorname{csch}\frac{\pi}{\sqrt{3}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2+1/3}$

.588718946888426853122... ≈ $2I_2\left(\sqrt{2}\right) = \sum_{k=0}^{\infty}\frac{1}{k!(k+2)!2^k}$

2 .58896621435810891117... ≈ $\frac{\pi^3+\pi}{8\sqrt{e}} = \int_0^{\infty}\frac{\log^2 x}{ex^2+1}dx$

.589048622548086232212... ≈ $\frac{3\pi}{16} = \prod_{k=1}^{\infty}\frac{k(k+2)}{(k+3/2)(k+1/2)}$ J1061

= $\int_0^{\infty}\frac{\sin^5 x}{x}dx = \int_0^{\infty}\frac{\sin^6 x}{x^2}dx = \int_0^{\infty}\frac{dx}{(x^2+1)^3}$ GR 3.827.12

= $\int_0^{\infty}\frac{\cos^3 x\sin x}{x}dx = \int_0^{\infty}\frac{\cos^4 x\sin x}{x}dx = \int_0^{\infty}\frac{\cos^3 x\sin^2 x}{x^2}dx$

= $\int_0^{\infty}\frac{\cos^4 x\sin^2 x}{x^2}dx = \int_0^{\infty}\frac{\cos^4 x\sin^2 x}{x^2}dx$

.5891600374288587219... ≈ $8-2\pi+\zeta(2)-4\log 2 = \sum_{k=1}^{\infty}\frac{(-1)^{k+}}{k^3+k^2/2}$

6 .58921553992655506549... ≈ $\zeta\left(\frac{7}{6}\right)$

5 .589246332394602395131... ≈ $e\left(1+\sqrt{\frac{\pi}{2}}erf1\right) = \sum_{k=0}^{\infty}\frac{(\sqrt{2})^k}{k!!}$

.589255650988789603667... ≈ $\frac{5}{6\sqrt{2}} = \int_0^{\pi/4}\cos^3 x\,dx$

.589296304394759574549... ≈ $\sum_{k=2}^{\infty}\frac{\log^2 k}{2^k}$

1 .58957531255118599032... $\approx$ $\log\left(-\Gamma\left(-\frac{1}{4}\right)\right)$

2 .58980531592437570493... $\approx$ $\frac{\pi\sqrt{e}}{2} = -\int_0^\infty \log x \log\left(1+\frac{e}{x^2}\right) dx$

.58989274325855087144 5... $\approx$ $\frac{1}{6^{5/3}}\left((1+i\sqrt{3})\psi\left(\frac{4+2^{1/3}3^{5/6}i+6^{1/3}}{2}\right)-2\psi(2-6^{1/3})\right)$

$+\frac{(1-i\sqrt{3})}{6^{5/3}}\psi\left(\frac{4-2^{1/3}3^{5/6}i+6^{1/3}}{2}\right)$

$=$ $\sum_{k=2}^\infty \frac{1}{k^3-6}$

.59000116354468792989... $\approx$ $\sum_{k=1}^\infty \frac{S_2(2k,k)}{(2k)!k^2}$

.590159304371654323... $\approx$ $\sum_{k=2}^\infty \frac{(-1)^k\left(\zeta(k)-1\right)}{2k-3} = \sum_{k=2}^\infty\left(\sqrt{\frac{1}{k^3}}\arctan\sqrt{\frac{1}{k}}\right)$

.59023206296500030164... $\approx$ $\sum_{k=0}^\infty \frac{(-1)^k B_k}{(k+2)!} = \frac{1}{6}+\sum_{k=0}^\infty \frac{(-1)^k B_k}{(k+2)!}$

.59032006179560104886 0... $\approx$ $24\sqrt{\frac{3}{5}}-18 = \sum_{k=0}^\infty \binom{2k+2}{k}\frac{(-1)^k}{6^k}$

3 .59048052361289410146... $\approx$ $\frac{9}{\sqrt{2\pi}}$

. 59048927088638507516... $\approx$ $\frac{\pi}{4}-\frac{\pi\sqrt{2}}{2}+\frac{3\sqrt{2}}{2}\arctan\left(\sqrt{2}\right)$

$=$ $\int_0^1 \frac{\arctan\left(\sqrt{x^2+1}\right)}{(x^2+1)\sqrt{x^2+1}}dx$ Borwein-Devlin, p. 53

.59057487283167075234 6... $\approx$ G^6

1 .59063685463732906338 2... $\approx$ $I_1(2) = \sum_{k=0}^\infty \frac{1}{k!(k+1)!} = \sum_{k=1}^\infty \frac{k}{(k!)^2} \sum_{k=1}^\infty \binom{2k}{k}\frac{k}{(2k)!}$ LY 6.113

4 .59084371199880305321... $\approx$ $\Gamma\left(\frac{1}{5}\right)$

4 .59117429878527614807... $\approx$ $2\left(\sqrt{2}+\operatorname{arcsinh}1\right) = \int_0^2 \sqrt{4+x^2}\,dx$

.59133069574507858717 1... $\approx$ $\pi\log\frac{1+\sqrt{2}}{2} = \int_0^1 \log(1+x^2)\frac{dx}{\sqrt{1-x^2}}$ GR 4.295.38

$= \int_0^{\pi/2} (1+\sin^2 x)\,dx$ GR 4.226.2

13 .59140914229522617680... $\approx$ $5e = \sum_{k=1}^{\infty} \frac{k^3}{k!}$ J161

.591418137582957447613... $\approx$ $\frac{1}{5}\left(\frac{\pi}{2} + 2\log 2\right) = -\int_0^1 li\left(\frac{1}{x}\right) \sin(2\log x)\,dx$ GR 6.213.2

14 .591429305318978476834... $\approx$ $-2\pi\sqrt{3}\,\csc\pi\sqrt{3} = \prod_{k=2}^{\infty} \frac{1}{1-3k^{-2}} = \prod_{k=1}^{\infty} \frac{k^2+2k+1}{k^2+2k-2}$

1 .591549430918953357689... $\approx$ $\frac{5}{\pi}$

5 .591582441177750776537... $\approx$ $\zeta\left(\frac{6}{5}\right)$

9 .5916630466254390832... $\approx$ $\sqrt{92}$

6 .59167373200865814837... $\approx$ $6\log 3$

.59172614725621914047... $\approx$ $\int_0^{\infty} \frac{dx}{e^{x^2} + e^{-x^2}}$ Berndt Ch. 28, Eq. 21.4

1 .59178884564103357816... $\approx$ $2e\, erf\left(\frac{1}{\sqrt{2}}\right) = \sum_{k=0}^{\infty} \frac{1}{(k+½)!2^k}$

10 .5919532755215206278... $\approx$ $2\sinh^2 \frac{\pi}{2} = \cosh\pi - 1 = \sum_{k=1}^{\infty} \frac{\pi^{2k}}{(2k)!}$

11 .5919532755215206278... $\approx$ $\cosh\pi = \cos i\pi = \frac{e^{\pi}+e^{-\pi}}{2} = \sum_{k=0}^{\infty} \frac{\pi^{2k}}{(2k)!}$ AS 4.5.63

$= \prod_{k=1}^{\infty}\left(1 + \frac{4}{(2k+1)^2}\right) = \prod_{k=1}^{\infty} \frac{k^2+4}{k^2+1}$ J1079

1 .59214263031556513132... $\approx$ $182\zeta(3) + 4\sqrt{3}\pi^3 - 432 = -\psi^{(2)}\left(\frac{7}{6}\right)$

433 .59214263031556513132... $\approx$ $182\zeta(3) + 4\sqrt{3}\pi^3 = -\psi^{(2)}\left(\frac{1}{6}\right)$

.59229653646932657566... $\approx$ $\frac{\sqrt{\pi}}{2} e^{1/4} erf\,\frac{1}{2} = \sum_{k=1}^{\infty} \frac{k!}{(2k)!} = \int_0^{\infty} e^{-x^2} \sinh x\,dx$

.592394075923426897354... $\approx$ $\sum_{k=2}^{\infty}\left(1 - \frac{\zeta(k+2)}{\zeta(k)}\right)$

.59283762069794257656... $\approx$ $\frac{2\sin 1}{5-4\cos 1} = \frac{\sin 1}{2(1-\cos 1 + 1/4)} = \sum_{k=1}^{\infty} \frac{\sin k}{2^k}$ GR 1.447.1

1 .5930240705604336815... ≈ $\dfrac{\log 2}{2} + \dfrac{\sqrt{2}}{2}\log\dfrac{2+\sqrt{2}}{2-\sqrt{2}} = \dfrac{\log 2}{2} + \dfrac{\log(3+2\sqrt{2})}{\sqrt{2}}$

= $\displaystyle\sum_{k=1}^{\infty}\frac{H_{2k-1}}{2^k}$

.593100317882891074703... ≈ $\dfrac{3}{2} - \dfrac{\pi}{2\sqrt{3}} = \displaystyle\sum_{k=1}^{\infty}\frac{1}{k^2-1/3}$

.59313427658358143675... ≈ $\zeta(2) - \dfrac{7\zeta(3)}{8} = \displaystyle\int_1^{\infty}\frac{\log^2 x}{(x+1)(x-1)^2}dx$

.593350269487182069140... ≈ $-\dfrac{2}{\pi}\cos\dfrac{\pi\sqrt{5}}{2} = \displaystyle\prod_{k=2}^{\infty}\left(1-\frac{1}{k(k+1)}\right)$

.593362978994233334116... ≈ $\displaystyle\sum_{k=1}^{\infty}\frac{H^{(3)}{}_k}{2^k k^2}$

3 .593427941774942960255... ≈ $\zeta(3) + \dfrac{\pi^2\log 2}{3} + \dfrac{\log^3 2}{3} = \displaystyle\sum_{k=1}^{\infty}\frac{H_k{}^3}{2^k}$

.593994150290161924318... ≈ $1-3e^{-2}$

.594197552889060608131... ≈ $\dfrac{1}{2\sin 1} = \dfrac{i}{e^i - e^{-i}}$

.59449608840624638119... ≈ $\pi\operatorname{csch}\pi + \dfrac{\pi^2}{12} - \dfrac{1}{2} = \displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^4+k^2}$

.594534891891835618022... ≈ $1+\log 2-\log 3 = 1+\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^k}{2^k k}$

.594696079786514810321... ≈ $\gamma - 1 + \dfrac{3+i\sqrt{3}}{6}\psi\left(\dfrac{5-i\sqrt{3}}{2}\right) + \dfrac{3-i\sqrt{3}}{6}\psi\left(\dfrac{5+i\sqrt{3}}{2}\right)$

= $\displaystyle\sum_{k=1}^{\infty}\left(\zeta(3k-1)-\zeta(3k+1)\right)$

.594885082800512587395... ≈ $4\sqrt{e} - 6 = \displaystyle\sum_{k=0}^{\infty}\frac{1}{(k+2)!2^k}$

2 .594885082800512587395... ≈ $4\sqrt{e} - 4 = \displaystyle\sum_{k=1}^{\infty}\frac{pf(k+1)}{2^k}$

1 .594963111240958733397... ≈ $\displaystyle\sum_{k=1}^{\infty}\frac{\sigma_0(k)}{k!+1}$

4 .59511182584294338069... ≈ $\zeta\left(\dfrac{5}{4}\right)$

.59530715857726908925... ≈ $\displaystyle\sum_{k=1}^{\infty}|\mu(k)|\log\zeta(2k)$

1 .59576912160573071176... ≈ $\frac{4}{\sqrt{2\pi}}$

.595823236590955745... ≈ $\sin^3 1 \;=\; \frac{1}{4}\sum_{k=1}^{\infty}(-1)^{k+1}\frac{3^{2k+1}-3}{(2k+1)!}$ GR 1.412.3

.595886193680811429201... ≈ $3-2\zeta(3) \;=\; \sum_{k=2}^{\infty}\frac{H_k}{k^2(k-1)}$

.596111293068951500191... ≈ $\frac{1}{\pi}\left(\gamma+\psi(1+\pi)\right) \;=\; \frac{H_\pi}{\pi}$

3 .59627499972915819809... ≈ $\pi\log\pi$

.59634736232319407434... ≈ $-e\,Ei(-1) \;=\; \sum_{k=0}^{\infty}\frac{\psi(k+1)}{k!}$, the Gompertz constant

= $\int_0^{\infty}\frac{dx}{e^x(x+1)} \;=\; \int_0^{\infty}\frac{x^2\,dx}{e^x(x+1)}$

= $\int_1^{\infty}\frac{dx}{x^2(1+\log x)} \;=\; \int_0^{\infty}e^{-x}\log(1+x)\,dx$

4 .59634736232319407434... ≈ $4-e\,Ei(-1) \;=\; \int_0^{\infty}\frac{x^4\,dx}{e^x(x+1)}$

.5963475204797203166... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{2^k-1}$

.596573590279972654709... ≈ $\frac{\log 2}{2}+\frac{1}{4} \;=\; \int_1^{\infty}\frac{x\log x}{(1+x)^3}dx$

1 .59688720677545620285... ≈ $\prod_{k=1}^{\infty}\left(1+\frac{1}{2^{k^2}}\right)$

.596965555578483224579... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^{k-1}}$

.59713559316352851048... ≈ $\frac{80\pi}{243\sqrt{3}} \;=\; \int_0^{\infty}\frac{dx}{(x^3+1)^4}$

.597264024732662556808... ≈ $\frac{e^2-5}{4} \;=\; \sum_{k=1}^{\infty}\frac{2^k}{(k+2)!}$

1 .597264024732662556808... ≈ $\frac{e^2-1}{4} \;=\; \sum_{k=0}^{\infty}\frac{2^k}{k!(k+3)}$

.597446822713573534080... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)-\zeta(2k+2)}{k!} \;=\; \sum_{k=2}^{\infty}\left(\frac{k^2-1}{k^2}\left(e^{1/k^2}-1\right)\right)$

1 .597948797240904373890... ≈ $\sum_{k=2}^{\infty}\sqrt{k}\left(\zeta(k)-1\right) \;=\; \sum_{k=2}^{\infty}\left(Li_{-1/2}\left(\frac{1}{k}\right)-\frac{1}{k}\right)$

.598064144464784678174... ≈ $-\log(2\sin 2) \;=\; \frac{1}{2}\log\left(2-e^{4i}-e^{-4i}\right) \;=\; -\sum_{k=1}^{\infty}\frac{\cos 4k}{k}$

.59814400666130410147... ≈ $\frac{1}{2}\sqrt{\frac{\pi}{2}}\,erf\sqrt{2} \;=\; \sum_{k=0}^{\infty}\frac{(-1)^k 2^k}{k!(2k+1)}$

54 .598150033144239078110... ≈ $e^4 \;=\; \sum_{k=0}^{\infty}\frac{4^k}{k!}$

1 .598313166927325339078... ≈ $\frac{\pi^4}{120}-\frac{\pi^2}{48}+\frac{127}{128} \;=\; \sum_{k=2}^{\infty}k^3\left(\zeta(2k)-1\right) \;=\; \sum_{k=2}^{\infty}\frac{8k^6-5k^4+4k^2-1}{k^2(k^2-1)^4}$

8 .59866457725355536964... ≈ $4G+\frac{\pi^2}{2} \;=\; \sum_{k=1}^{\infty}\frac{k}{2^k}\zeta\left(k+1,\frac{3}{4}\right)$

.599002264993457570659... ≈ $\left(\frac{\pi}{2}-1\right)\cos 1\sin 1+\sin^2 1-\log(2\sin 1)\sin^2 1 \;=\; \sum_{k=1}^{\infty}\frac{\sin^2(k+1)}{k(k+1)}$

1 .59907902990129290008... ≈ $\sum_{k=1}^{\infty}\frac{1}{k!F_k}$

1 .599205922440719577791... ≈ $-\log\left(\zeta(3)-1\right)$

.599314365613468571533... ≈ $1-\frac{\zeta(3)}{3}$

.59939538416978169923... ≈ $\sum_{k=1}^{\infty}\log\zeta(2k)$

.5996203229953586595... ≈ $2-e-\gamma+Ei(1) = \sum_{k=1}^{\infty}\frac{1}{(k+1)!k} = \sum_{k=1}^{\infty}\frac{k^2}{(k+1)!(k+1)}$

= $\sum_{k=2}^{\infty}\frac{1}{(k+1)!-2k!}$

= $\sum_{k=1}^{\infty}\frac{H_k}{k!(k+2)} \;=\; \sum_{k=1}^{\infty}\frac{1}{(k+1)!k}$

1 .599771198411726589334... ≈ $\sum_{k=1}^{\infty}\frac{2^k}{(k+1)^k}$

4 .599873743272337314... ≈ $\frac{17\pi^4}{360} \;=\; \frac{17\zeta(4)}{4} \;=\; \sum_{k=1}^{\infty}\left(\frac{H_k}{k}\right)^2$

Borwein & Borwein, Proc. AMS 123, 4 (1995) 1191-1198

.599971479517856975792... ≈ $\operatorname{arccsch}\frac{\pi}{2}$

$.6000000000000000000\underline{0}$ = $\frac{3}{5} = \sum_{k=1}^{\infty} \frac{F_{2k-1}}{4^k}$

1 $.6000000000000000000\underline{0}$ = $\frac{8}{5} = \sum_{k=1}^{\infty} \frac{L_k L_k}{4^k}$

.600053813264123766513... ≈ $\frac{3\pi}{2} - \frac{5\pi^2}{24} - \frac{\log^2 2}{2} - 2Li_2\left(\frac{1+i}{2}\right) - 2Li_2\left(\frac{1-i}{2}\right)$

= $\int_0^{\infty} \frac{\log(1+x^2)}{x^2(1+x)^2} dx$

.600281176062325408286... ≈ $\pi\sqrt{3} + 6\log 2 - 9 = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k(k+1/3)}$

.6004824307923120424... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(2k) - \zeta(2k+2)}{k} = \sum_{k=2}^{\infty} \frac{k^2-1}{k} \log \frac{k^2-1}{k}$

2 .600894716163993447115... ≈ $\sum_{k=1}^{\infty} \left(e^{1/k!} - 1\right)$

.601028451579797142699... ≈ $\frac{\zeta(3)}{2} = \sum_{k=1}^{\infty} \frac{H_k H_{k+1}}{k(k+1)(k+2)}$

= $\int_0^{\infty} \frac{x^2 dx}{e^x \sinh x} = -\int_0^1 \frac{\log^2 x dx}{\sinh(\log x)} = -\int_{-1}^0 \frac{\log^2 (1+x) dx}{\sinh(\log(1+x))}$

.60178128264873772913... ≈ $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{\zeta(k+1) - 1}{k^2} = -\sum_{k=2}^{\infty} \frac{1}{k} Li_2\left(-\frac{1}{k}\right)$

.601782458374805167143... ≈ $\sum_{k=2}^{\infty} \frac{1}{k+1} \log \frac{k}{k-1}$

.601907230197234574738... ≈ $K_1(1) = \int_0^{\infty} e^{-\sqrt{1+x^2}} dx$

.602056903159594285399... ≈ $\zeta(3) - \frac{3}{5}$

.602059991327962390428... ≈ $\log_{10} 4$

8 .6023252670426267717... ≈ $\sqrt{74}$

2 .60243495809669391311... ≈ $\frac{\sinh \pi\sqrt{2}}{\sqrt{2} \sinh \pi} = \frac{1}{\sqrt{2}} \operatorname{csch} \pi \sinh \pi\sqrt{2}$

= $\prod_{k=1}^{\infty} \frac{k^2+2}{k^2+1} = \prod_{k=1}^{\infty} \left(1 + \frac{1}{k^2+1}\right)$

.602482584806786886836... ≈ $\frac{\pi}{2\sqrt{5}} \coth \pi\sqrt{5} - \frac{1}{10} = \sum_{k=1}^{\infty} \frac{1}{k^2+5}$

175 .602516306812280539986... ≈ $24\zeta 5) + 50\zeta(3) + \frac{2\pi^4}{3} + \frac{5\pi^2}{2} + 1 = \sum_{k=2}^{\infty} k^4\left(\zeta(k)-1\right)$

= $\sum_{k=2}^{\infty} \frac{16k^4 + k^3 + 11k^2 - 5k + 1}{k(k-1)^5}$

4 .602597804614589746926... ≈ $2\sinh\frac{\pi}{2}$ Berndt 7.2.5

1 .60343114675605016067... ≈ $\pi\sqrt{2} - 2\sqrt{2}\arctan\frac{1}{\sqrt{2}} - \log 3$

= $\int_0^{\infty} \log\left(1 + \frac{2}{(x+1)^2}\right) dx$

2 .603451532529716432282... ≈ $-\frac{6\sqrt{2}}{\pi}\sin\pi\sqrt{2} = \prod_{k=1}^{\infty}\left(1 + \frac{2}{k(k+4)}\right)$

2 .603599580529289999450... ≈ $\frac{7\zeta(3)}{4} + \frac{1}{2} = \sum_{k=1}^{\infty} \frac{k^3}{(k^2 - 1/4)^3}$

2 .603611904599514233302... ≈ $\prod_{k=1}^{\infty}\left(1 + \frac{1}{k^k}\right)$

.603782862791487988416... ≈ $\sum_{k=2}^{\infty} \frac{\log k}{k!}$

1 .603912052520052227157... ≈ $\frac{1}{\pi^2}\Gamma\left(1-i\right)\Gamma\left(1+i\right)\cosh^2\frac{\pi\sqrt{3}}{2} = \prod_{k=1}^{\infty}\frac{k^4 + k^2 + 1}{k^4 + k^2}$

.60393978817538095427... ≈ $\frac{1}{2} + \frac{1}{2e^{\pi/2}} = \int_0^{\pi/2} e^{-x}\cos x\, dx$

1 .604303978912866704254... ≈ $\frac{2\pi}{\sqrt{3}}\left(3^{1/3} - 1\right) = \int_0^{\infty} \log\left(1 + \frac{2}{x^3 + 1}\right) dx$

3 .60444734197194674489... ≈ $\sum_{k=1}^{\infty} \frac{\sigma_1(k)}{2^k - 1}$

.604599788078072616865... ≈ $\frac{\pi}{3\sqrt{3}} = \sum_{k=1}^{\infty}\left(\frac{1}{3k-2} - \frac{1}{3k-1}\right)$

= $\frac{\pi}{3\sqrt{3}} = \sum_{k=1}^{\infty} \frac{(2k)!!}{(2k-1)!!\, 4^k\, k}$

= $\sum_{k=0}^{\infty} \frac{1}{3k+1)(3k+2)}$

= $\sum_{k=1}^{\infty} \frac{1}{\binom{2k}{k} k}$ CFG F17

= $\int_0^{\infty} \frac{dx}{2x^2 + 2x + 2} = \int_0^{\infty} \frac{dx}{4x^2 + 2x + 1} = \int_0^{\infty} \frac{x\, dx}{x^4 + x^2 + 1}$

$= \int_0^\infty \frac{dx}{x^2+2x+4} = \int_0^\infty \frac{dx}{x^2+3x+3}$

$= \int_1^\infty \frac{dx}{x^3-x^2+x} = \int_0^\infty \frac{x\,dx}{1+x^6} = \int_0^\infty \frac{x^3\,dx}{1+x^6} = \int_0^\infty \frac{x\,dx}{x^3+8}$

$= \int_0^\infty \frac{dx}{x^5+x^4+x^3+x^2+x+1}$

$= \int_0^\infty \frac{dx}{e^x+e^{-x}+1}$

10 .604602902745250228417... ≈ $\log 8!$

.604898643421630370247... ≈ $\left(1-\sqrt{2}\right)\zeta\left(\frac{1}{2}\right) = \zeta\left(\frac{1}{2},\frac{1}{2}\right) = \eta\left(\frac{1}{2}\right) = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{\sqrt{k}}$

5 .604991216397928699311... ≈ $\sqrt{10\pi}$

.605008895087302501620... ≈ $\frac{7\pi^2}{48} - \frac{\pi\sqrt{3}}{8}\log 3 + \frac{9\log^2 3}{16} - \frac{1}{4}\psi^{(1)}\left(\frac{2}{3}\right)$

.605065933151773563528... ≈ $\frac{9}{4} - \frac{\pi^2}{2} = \int_0^1 \frac{1-x-x^2}{x-1}\log x\,dx$

.605133652503344581744... ≈ $\frac{1}{2e}\left(\pi\, erfi(1) - Ei(1)\right) = \int_0^\infty \frac{dx}{e^{x^2}(x+1)}$

.605139614580041017937... ≈ $\frac{9}{8} - \frac{3\log 2}{4} = \sum_{k=2}^\infty \frac{k^2}{2^k(k^2-1)}$

1 .605412976802694848577... ≈ $si(2) = \sum_{k=0}^\infty \frac{(-1)^k 2^{2k+1}}{(2k+1)!(2k+1)}$ AS 5.2.14

.605509265700126543353... ≈ $3\zeta(2) - 4\zeta(4)$

.6055217888826004477... ≈ $\sum_{k=2}^\infty \frac{1}{k^2\log k} = \int_2^\infty \left(\zeta(x)-1\right)dx$

3 .605551275463989293119... ≈ $\sqrt{13}$

.605591341412105862802... ≈ $\frac{5\pi^3}{256} = \sum_{k=1}^\infty \frac{\sin(3k\pi/4)}{k^3}$ GR 1.443.5

.6056998670788134288... ≈ $\sin\left(\frac{\pi\sqrt{2}}{2}-1\right) - \cos\frac{\pi}{\sqrt{2}} = \prod_{k=1}^\infty \left(1-\frac{2}{(2k+1)^2}\right)$

2 .605840094684629285 81... ≈ $\frac{\pi^2}{6} + 2\log^2 2 = \int_0^1 \log^2\left(1+\frac{1}{x}\right)dx$

.606125795769702052589... ≈ $\frac{\sqrt{\pi}}{4}\left(1+\frac{1}{e}\right) = \int_0^\infty e^{-x^2}\cos^2 x\,dx$

3 .606170709478782856199... ≈ $3\zeta(3) = \sum_{k=1}^\infty \frac{H_k H_k}{k(k+1)}$

= $-\int_0^1\int_0^1 \frac{\log(x^2 y^2)}{1+xy}\,dx\,dy$

.606501028604649267054... ≈ $\sum (-1)^{k+1}\frac{\zeta(2k)-1}{k!} = \sum_{k=2}^\infty\left(1-e^{-1/k^2}\right)$

.606530659712633423604... ≈ $\frac{1}{\sqrt{e}} = i^{i/\pi} = \sum_{k=0}^\infty \frac{(-1)^k}{k!2^k} = \sum_{k=0}^\infty \frac{(-1)^k}{(2k)!!}$

15 .606641563575290687132... ≈ γ^{-5}

.60669515241529176378... ≈ $\sum_{k=1}^\infty \frac{1}{2^{k+1}-1} = \sum_{k=1}^\infty \frac{1}{2^k(2^k-1)} = \sum_{k=2}^\infty \frac{\Omega(2^k)}{2^k}$

1 .60669515241529176378... ≈ $\sum_{k=1}^\infty \frac{1}{2^k-1} = \sum_{k=1}^\infty \frac{\sigma_0(k)}{2^k} = \sum_{k=1}^\infty \frac{2^k+1}{2^{k^2}(2^k-1)}$

= $\sum_{j=1}^\infty\sum_{k=1}^\infty \frac{1}{2^{jk}} = 1+\sum_{k=2}^\infty \frac{\Omega(2^{k)}}{2^k}$

Shown irrational by Erdös in J. Indian Math, Soc. (N.S.) 12 (1948) 63-66

.606789763508705511269... ≈ $\pi\left(\log 2-\frac{1}{2}\right) = \int_0^\infty \frac{xe^{-x}}{\sqrt{e^x-1}}\,dx$ GR 3.452.3

1 .606984244848816274257... ≈ $\sum_{k=1}^\infty \frac{1}{k!\phi(k)}$

.60715770584139372912... ≈ $\frac{erf(1)}{e} = \sum_{k=0}^\infty \frac{(-1)^k}{(k+\frac{1}{2})!}$

3 .6071833342396650534... ≈ $3\zeta(3)+\frac{\zeta(5)}{1024} = \sum_{k=1}^\infty \frac{1}{a(k)^5}$,

where $a(k)$ is the nearest integer to $\sqrt[3]{k}$. AMM 101, 6, p. 579

5 .607330577557532492158... ≈ $1536-563e = \sum_{k=0}^\infty \frac{k^4}{k!(k+4)}$

1 .607695154586736238835... ≈ $12-6\sqrt{3} = \sum_{k=0}^\infty \binom{2k+2}{k}\frac{1}{6^k(k+1)}$

.607927101854026628663... ≈ $\frac{6}{\pi^2} = \frac{1}{\zeta(2)} = \sum_{k=1}^\infty \frac{\mu(k)}{k^2} = \prod_{p\ prime}\left(1-\frac{1}{p^2}\right)$

.607964336255266831093... ≈ $\frac{4\pi^{12}}{6081075} = \prod_{p\ prime} \frac{1}{1+p^{-2}+p^{-4}+p^{-6}+p^{-8}+p^{-10}+p^{-12}}$

.60798640550036075618... ≈ $\frac{\log 3}{4} + \frac{1}{3} = \frac{1}{2}\,{}_2F_1\left(\frac{1}{2},2,\frac{3}{2},\frac{1}{4}\right) = \sum_{k=1}^{\infty} \frac{k}{2^{2k-1}(2k-1)}$

= $\int_2^{\infty} \frac{dx}{x^2+x^{-2}-2}$

= $\int_0^{\infty} \frac{(\sin x - x\cos x)^3}{x^4} dx$

.608006311704856510141... ≈ $\zeta(2)-\zeta(5)$

.608197662162246572967... ≈ $\frac{3}{2}\log\frac{3}{2} = \sum_{k=1}^{\infty} \frac{H_k}{3^k}$

.608199697591539684584... ≈ $\sum_{k=1}^{\infty} \frac{1}{(k-1)!2^k \zeta(2k)}$ Titchmarsh 14.32.1

.608381717863324722684... ≈ $\sum_{p\ prime} \frac{\log p}{p^2-p+1} = \sum_{p\ prime} \left(\frac{p-2}{(p^2-p+1)(p^2-1)} + \frac{1}{p^2-1}\right)\log p$

= $\sum_{p\ prime} \left(\frac{(p-2)\log p}{(p^2-p+1)(p^2-1)}\right) - \frac{\zeta'(2)}{\zeta(2)}$

.608531731029057175381... ≈ $\frac{2\pi^8}{31185} = \prod_{p\ prime} \frac{1}{1+p^{-2}+p^{-4}+p^{-6}+p^{-8}}$

.608699218043767368353... ≈ $\log\frac{e^{\pi}-e^{-\pi}}{4\pi} = \log\frac{\sinh\pi}{2\pi} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\zeta(2k)-1}{k}$ K Ex. 124f

= $\log\frac{1}{\Gamma(2+i)\Gamma(2-i)} = \sum_{k=2}^{\infty} \log\left(1+\frac{1}{k^2}\right)$

1 .6089918989086720365... ≈ $\sum_{k=1}^{\infty} \binom{2k}{k} \frac{\zeta(2k)-1}{k!}$

.609023896230194382244... ≈ $\sum_{k=2}^{\infty} \frac{\zeta(k)-1}{H_k}$

.609245969064096382192... ≈ $\sum_{k=1}^{\infty} \frac{H_k^{\,3}}{4^k}$

4 .609265757423133079297... ≈ $-\pi\sqrt{2}\csc\pi\sqrt{2} = \prod_{k=2}^{\infty} \frac{1}{1-2k^{-2}}$

1 .609437912434100374600... ≈ $\log 5 = Li_1\left(\frac{4}{5}\right)$

.609475708248730032535... ≈ $\frac{2\sqrt{2}}{3} - \frac{1}{3} = \int_0^{\pi/4} \frac{\sin x}{\cos^4 x} dx$

.6100364084437080... $\approx \sum_{k=2}^{\infty} \frac{(-1)^k}{k(k-1)\log k}$

7 .610125138662288363419... $\approx \cosh e = \frac{1}{2}\left(e^e + e^{-e}\right) = \sum_{k=0}^{\infty} \frac{e^{2k}}{(2k)!}$

.610229474375159889916... $\approx \frac{\pi^2}{2} + \frac{\pi^4}{15} - 9\zeta(3) = \int_0^{\infty} \frac{x^3}{(e^x-1)^3}\,dx$

.610350763456124924754... $\approx \sum_{k=2}^{\infty} \log\left(1 + \frac{1}{k!}\right)$

.6106437294514793434... $\approx \frac{2G}{3}$

.61094390106934977277... $\approx 8 - e^2$

.611111111111111111111$\underline{1}$ $= \frac{11}{18} = \frac{H_3}{3} = \sum_{k=1}^{\infty} \frac{1}{k^2+3k} = \sum_{k=4}^{\infty} \frac{1}{k^2-3k}$

.611643938224066294941... $\approx -\frac{1}{2} - \frac{\gamma}{3} - \frac{1}{6}\left((1-i\sqrt{3})\psi\left(\frac{-1-i\sqrt{3}}{2}\right) + (-1-i\sqrt{3})\psi\left(\frac{-1+i\sqrt{3}}{2}\right)\right)$

$= \sum_{k=2}^{\infty} \frac{k}{k^3+1} = \sum_{k=1}^{\infty} (-1)^{k+1}\left(\zeta(3k-1) - 1\right)$

.612018841644809204554... $\approx -\frac{2}{5}\left(\log\frac{1953125}{1048576} + \sqrt{5}\log\frac{5-\sqrt{5}}{5+\sqrt{5}}\right) = \sum_{k=1}^{\infty} \frac{F_k F_{k+3}}{4^k (k+1)}$

5 .6120463970207165486... $\approx -\frac{14\pi^4}{243} = -\int_0^{\infty} \frac{\log^3 x\,dx}{x^3+1}$

1 .612273774471087972438... $\approx \frac{\sinh\sqrt{\pi}}{\sqrt{\pi}} = \binom{0}{2i} = \prod_{k=1}^{\infty}\left(1 + \frac{1}{\pi k^2}\right)$

2 .61237534868548834335... $\approx \zeta\left(\frac{3}{2}\right) = -4\pi\zeta\left(-\frac{1}{2}\right) = \sum_{k=1}^{\infty} \frac{1}{k^{3/2}}$

$= \prod_{p\ prime} \frac{1+p^{-3/2}}{1-p^{-3}} = \zeta(6) \prod_{p\ prime}\left(1 + p^{-3/2} + p^{-3} + p^{-9/4}\right)$

.61251880056266508858... $\approx 14 - \frac{2\pi^2}{3} - \frac{\pi^4}{45} - 3\zeta(3) - \zeta(5) = \sum_{k=2}^{\infty} \frac{1}{(k-1)^2}\left(\frac{1}{k} - \frac{1}{k^5}\right)$

1 .612910874202944353144... $\approx \sum_{k=2}^{\infty}\left(1 - \frac{1}{\zeta^3(k)}\right)$

42 .612923374243529926954... $\approx \frac{\sinh 2\pi}{2\pi} = \frac{1}{\pi}\cosh\pi\sinh\pi = \prod_{k=1}^{\infty}\left(1 + \frac{4}{k^2}\right)$

	.613095585441758535407...	≈	$\frac{1}{2}\log\tan\left(\frac{\pi}{4}+\frac{1}{2}\right) = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\sin(2k-1)}{2k-1}$	GR 1.442.3
	.61314719276545841313...	≈	$\log_6 3$	
3	.613387333659139458760...	≈	$\sum_{k=0}^{\infty}\frac{\zeta(k+2)}{k!} = \sum_{k=1}^{\infty}\frac{e^{1/k}}{k^2}$	
2	.613514090166737576533...	≈	$\frac{\pi}{\zeta(3)}$	
	.613705638880109381166...	≈	$2-2\log 2 = \gamma + \psi\left(\frac{3}{2}\right) = hg\left(\frac{1}{2}\right)$	
		=	$\sum_{k=1}^{\infty}\frac{1}{2k^2+k}$	GR 0.234.8
		=	$\sum_{k=0}^{\infty}\frac{k}{2^k(k+1)}$	
		=	$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(k+1)}{2^k} = \sum_{k=1}^{\infty}\frac{2^k-1}{2^k}\left(\zeta(k+1)-1\right)$	
		=	$\int_0^1 \frac{\log x}{1-x^2}dx$	
		=	$-\int_0^1 \log(1-x^2)\,dx$	
		=	$\int_0^1 \frac{4\log(1+x)}{(x+1)^2}dx = \int_0^1 \frac{\log(1+x)}{(x/2+1/2)^2}dx$	GR 4.291.14
		=	$-\int_0^{\pi/2}\log(\sin 2x)\sin x\,dx$	GR 4.384.10
4	.61370563888010938117...	≈	$6-2\log 2 = \sum_{k=1}^{\infty}\frac{k^3}{2^k(k+1)}$	
128	.61370563888010938117...	≈	$130-2\log 2 = \sum_{k=1}^{\infty}\frac{k^5}{2^k(k+1)}$	
	.613835953026141526250...	≈	$\frac{\pi}{\sqrt{11}}\tanh\frac{\pi\sqrt{11}}{2}-\frac{1}{3} = \sum_{k=1}^{\infty}\frac{1}{k^2+k+3}$	
	.613955913179956659743...	≈	$\frac{2^{7/10}\pi}{5\sqrt{5-\sqrt{5}}} = \int_0^{\infty}\frac{dx}{x^5+2}$	
	.61397358864975783997...	≈	$\frac{1}{2}\log\left(2+\sqrt{2}\right)$	
2	.61406381540519800021...	≈	$4^{\log 2} = \prod_{k=1}^{\infty}4^{(-1)^{k+1}/k}$	

.61478761371472754153 6... ≈ $\prod_{k=1}^{\infty}\left(1-\frac{1}{k(3k+1)}\right)$

.61495209469651098084.... ≈ $erfi\frac{1}{2} = \frac{2}{\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{1}{k!2^{2k+1}(2k+1)}$

$.\underline{615384615384615384}$ = $\frac{8}{13}$

.61547970867038734107... ≈ $\arctan\frac{1}{\sqrt{2}} = \arcsin\frac{\sqrt{3}}{5} = \int_0^{\pi/4}\frac{\cos x\,dx}{1+\sin^2 x}$

.61562647038601426214 7... ≈ $-\log(\cos 1) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}2^{2k-1}(2^{2k}-1)B_{2k}}{(2k)!k}$ AS 4.3.72

7 .6157731058639082857... ≈ $\sqrt{58}$

.61594497904805006741... ≈ $\frac{\pi}{2}\frac{\left(\sin\frac{1}{2}\right)\sin\left(2\pi\cos\frac{1}{2}\right)-\cos\frac{1}{2}\sinh\left(2\pi\sin\frac{1}{2}\right)}{\cos\left(2\pi\cos\frac{1}{2}\right)-\cosh\left(2\pi\sin\frac{1}{2}\right)}-\frac{1}{2}\cot\frac{1}{2}$

= $-\frac{i}{4}\left(e^{-i/2}\pi\left(\cot\left(e^{-i/2}\pi\right)-e^{i}\cot\left(e^{i/2}\pi\right)\right)-2i\cot\frac{1}{2}\right)$

$\sum_{k=1}^{\infty}\left(\zeta(2k)-1\right)\sin k$ Adamchik-Srivastava 2.28

1 .61611037054142350347... ≈ $\sum_{k=1}^{\infty}\frac{\arctan k}{k!}$

9 .6164552252767542832... ≈ $8\zeta(3)$

54 .61646567203297325840 4... ≈ $2\cosh 4 = e^{4}+e^{-4}$

1 .6168066722416746633... ≈ $2^{\log 2} = \prod_{k=1}^{\infty}2^{(-1)^{k+1}/k} = \prod_{k=1}^{\infty}2^{1/2^{k}k}$

.61685027506808491367 7... ≈ $\frac{\pi^2}{16} = \sum_{k=0}^{\infty}\frac{(-1)^k H_k}{k+1} = \sum_{k=1}^{\infty}\frac{k\zeta(2k)}{4^k} = \sum_{k=1}^{\infty}\frac{4k^2}{(4k^2-1)}$

= $\int_0^{\infty}\frac{x\arctan x}{1+x^4}dx$ GR 4.531.8

5 .61690130799559924468... ≈ $2\pi\sqrt{3}+6\log 2-3\pi = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+\frac{1}{2})(k+\frac{1}{3})}$

1 .61705071484731409581... ≈ $\frac{\pi}{\sqrt{6}}\coth\pi\sqrt{\frac{3}{2}}+\frac{1}{3} = \sum_{k=0}^{\infty}\frac{1}{k^2+3/2}$

.61737084509925288889 5... ≈ $\sin 2-\frac{\cos 2}{2}-\frac{1}{2}$

1 .617535655787233699013... ≈ $5\log\frac{10}{5+\sqrt{5}} = \sum_{k=0}^{\infty}\binom{2k+1}{k}\frac{1}{5^k(k+1)}$

.6176178315135553390411... ≈ $\frac{1}{\sqrt{5}}\left(Li_2\left(\frac{1+\sqrt{5}}{4}\right)-Li_2\left(-\frac{1}{1+\sqrt{5}}\right)\right) = \sum_{k=1}^{\infty}\frac{F_k}{2^k k^2}$

4 .617725319212262884852... ≈ γ^8

.61779030404890075888... ≈ $\sum_{k=2}^{\infty}\frac{k}{k-1}(\zeta(k+1)-1)$

1 .61779030404890075888... ≈ $\sum_{k=2}^{\infty}\frac{k^2}{k-1}(\zeta(k+1)-1) = \sum_{k=2}^{\infty}\left(\frac{3k-2}{k(k-1)^2}+\frac{1}{k}\log\frac{k}{k-1}\right)$

3 .6178575491454110364... ≈ $\frac{\pi^2}{2}-\frac{\pi^2\log 2}{2}+\frac{7\zeta(3)}{4} = \sum_{k=0}^{\infty}\frac{4^k}{\binom{2k}{k}(k+1)^2}$

.617966219979193677004... ≈ $\frac{15}{4}-\gamma-\frac{\pi}{2\sqrt{3}}-\frac{3\log 3}{2} = \psi\left(\frac{7}{3}\right)$

2 .61799387799149436539... ≈ $\frac{5\pi}{6} = \int_0^{\infty}\frac{dx}{1+x^{12/5}} = \int_0^{\infty}\frac{dx}{e^x+e^{-x}-\sqrt{3}}$

.618033988749894848204... ≈ $\varphi - 1 = \frac{1}{\varphi} = \frac{\sqrt{5}-1}{2}$

1 .618033988749894848204... ≈ $\varphi = \frac{\sqrt{5}+1}{2} = 2\cos\frac{\pi}{5}$, the Golden ratio

2 .618033988749894848204... ≈ $\varphi + 1 = \varphi^2$

.618107596152659484271... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^k kH_k}$

1 .618157392436715516456... ≈ $4\log 2 - 2\gamma = \sum_{k=0}^{\infty}\frac{\psi(k+2)}{2^k}$

.61847041926350753801... ≈ $\frac{2\pi^4}{315} = \prod_{p\ prime}\frac{1}{1+p^{-2}+p^{-4}}$

.61851608883629551823... ≈ $1+\frac{3\pi}{8}-\frac{9\log 2}{4} = \sum_{k=2}^{\infty}\frac{(-1)^k 3^k\zeta(k)}{4^k}$

1 .618793194732580219157... ≈ $\frac{2}{3\pi}\cosh\frac{\pi\sqrt{3}}{2} = \prod_{k=1}^{\infty}\left(1+\frac{1}{(k+1)(k+2)}\right)$

.61897071820759046667... ≈ $\frac{1}{10}\left(10 + \left(-5+\sqrt{5}\right)H_{(1-\sqrt{5})/2} - \left(5+\sqrt{5}\right)H_{(1+\sqrt{5})/2}\right)$

$= \quad \sum_{k=2}^{\infty} F_{k-2}\left(\zeta(k)-1\right)$

.61928812542301650399 4... ≈ $\frac{16}{3}-\frac{10\sqrt{2}}{3} = \sum_{k=0}^{\infty}\binom{2k}{k}\frac{1}{8^k(k+2)}$

4 .61970402552121281236 6... ≈ $\sum_{k=1}^{\infty}\binom{2k}{k}\frac{1}{k^k}$

.61971867820222486036 8... ≈ $1-\frac{\pi}{\sqrt{6}}\cot\frac{\pi}{\sqrt{6}} = \sum_{k=1}^{\infty}\frac{1}{3k^2-1/2}$

.62008880718956768915 6... ≈ $\frac{\pi^2}{6}+\pi\sqrt{2}\operatorname{csch}\frac{\pi}{\sqrt{2}}-2 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2(k^2+1/2)}$

.62011450695827752463... ≈ $\log\frac{e+1}{2} = \sum_{k=1}^{\infty}\frac{(1-2^k)\zeta(1-k)}{k!}$

$= \quad \sum_{k=1}^{\infty}\frac{(-1)^k(2^k-1)B_k}{k!k}$ [Ramanujan] Berndt Ch. 5

$= \quad \int_0^1\frac{dx}{1+e^{-x}}$

.62027218892715090161 3... ≈ $\frac{4e^{1/3}}{9} = \sum_{k=1}^{\infty}\frac{k^2}{k!3^k}$

4 .62032422925425692375 0... ≈ $\sum_{k=1}^{\infty}\frac{\sigma_0(k)}{(k-1)!}$

.62047113489033260164... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)-1}{2k-1} = \sum_{k=2}^{\infty}\frac{1}{k}\arctan\frac{1}{k}$

.62073133866424427034... ≈ $-\frac{24}{7}\cos\frac{\pi\sqrt{29}}{2} = \prod_{k=1}^{\infty}\left(1-\frac{1}{k(k+5)}\right)$

8 .62074214840238569958... ≈ $\frac{1}{3^{2/3}}\left((-1)^{1/3}\psi\left(\frac{4-i2^{1/3}3^{5/6}+6^{1/3}}{2}\right)-2^{2/3}\psi\left(2-6^{1/3}\right)\right)$

$-\frac{(-1)^{2/3}}{3^{1/3}}\psi\left(\frac{4+i2^{1/3}3^{5/6}+6^{1/3}}{2}\right)$

$= \quad \sum_{k=1}^{\infty}6^k\left(\zeta(3k-1)-1\right) = \sum_{k=2}^{\infty}\frac{6k}{k^3-6}$

1 .62087390360396657265... ≈ $\Phi\left(\frac{1}{\sqrt{2}},\frac{1}{2},0\right)$

9 .62095476193070331095... ≈ $2e^{\pi/2}$

.62113863160551746463 7... ≈ $\frac{\pi^2}{24}+\frac{\pi}{2}-\log 2+\frac{\log^2 2}{2}-Li_2\left(\frac{1-i}{2}\right)-Li_2\left(\frac{1+i}{2}\right)$

$= \frac{\pi}{2} - \frac{\pi^2}{16} - \log 2 + \frac{3\log^2 2}{4}$

$= \int_0^1 \frac{\log(1+x^2)}{x^2(x+1)}$

.62133493455961181 0707... $\approx \frac{1}{\log 5} = \log_5 e$

2 .62140838307586150 5699... $\approx \frac{1}{4}\left(\sqrt{5} - 2 + \sqrt{13 - 4\sqrt{5}} + \sqrt{50 + 12\sqrt{5} - 2\sqrt{65 - 20\sqrt{5}}}\right)$

$= \sqrt{5 + \sqrt{5 - \sqrt{5 - \sqrt{5 + \sqrt{5 + \sqrt{5 - \sqrt{5 - \ldots}}}}}}}$ [Ramanujan] Berndt Ch. 22

.62144962423581335 7639... $\approx \frac{\pi}{2}\cos 1 + ci(1)\sin 1 - ci(1)\cos 1 = ci(1)\sin 1 + \frac{\cos 1}{2}\left(\pi - 2si(1)\right)$

$= \int_0^\infty \frac{\sin x\,dx}{1+x} = \int_0^\infty \frac{\arctan x\,dx}{e^x}$

$= \int_0^\infty \frac{dx}{e^x\left(x^2+1\right)}$

.62182053074983197934... $\approx \sum_{k=2}^\infty \frac{(-1)^k}{2^k\left(\zeta(k)-1\right)}$

.62200846792814621 5588... $\approx \frac{2+\sqrt{3}}{6}$ CFG G1

2 .62205755429211981046... $\approx \frac{1}{2\sqrt{2\pi}}\Gamma^2\left(\frac{1}{4}\right)$, lemniscate constant

1 .62306783661962438 2082... $\approx \sinh^3 1 = \frac{1}{4}\left(\sinh 3 - 3\sinh 1\right) = \frac{1 - 3e^2 + 3e^4 - e^6}{8e^3}$

$= \sum_{k=1}^\infty \frac{(3^k - 3)(1 - (-1)^k)}{8k!}$ J883

$= \frac{1}{4}\sum_{k=1}^\infty \frac{3^{2k-1} - 3}{(2k-1)!}$

.62322524014023051 3394... $\approx \frac{1}{\sqrt{2}}\log\left(1+\sqrt{2}\right) = \frac{1}{\sqrt{2}}\operatorname{arcsinh}1 = \frac{1}{\sqrt{2}}\operatorname{arctanh}\frac{1}{\sqrt{2}}$

$= 1 + \sum_{k=1}^\infty \left(\frac{(-1)^k}{4k-1} + \frac{(-1)^k}{4k+1}\right)$

$= \sum_{k=1}^\infty \frac{1}{2^k(2k-1)}$ GR 1.513.1

$= \sum_{k=0}^{\infty} \frac{(-1)^k (2k)!!}{(2k+1)!!}$ J129

$= \sum_{k=0}^{\infty} \frac{(-1)^{\lfloor (k+1)/2 \rfloor}}{2k+1}$ Prud. 5.1.4.4

$= \sum_{k=1}^{\infty} \frac{1}{2^k} Li_k\left(\frac{1}{2}\right)$ Adamchik-Srivastava 4.12

$= \int_0^1 \frac{x\,dx}{(1+x^2)\sqrt{1-x^2}}$

$= \int_0^{\infty} \frac{dx}{2x^2+4x+1}$

$= \int_0^{\pi/4} \frac{dx}{\sin x + \cos x}$

$= \int_0^{\pi/2} \frac{\sin x}{2-\cos^2 x} dx$

$= \int_2^{\infty} \frac{dx}{x^2-2}$

.623263879436410617495... $\approx \sum_{k=0}^{\infty} \frac{1}{2k^3+3}$

.623658674932063698052... $\approx \frac{3}{4\pi}\zeta\left(\frac{3}{2}\right) = \int_0^{\infty} \frac{dx}{e^{\pi x^{2/3}}-1}$

.623810716364871399208... $\approx 2\log\frac{1+\sqrt{3}}{2}$

.623887006992315787... $\approx \sum_{k=1}^{\infty} \frac{pr(k)}{2^k-1}$

104 .62400000000000000000$\underline{0}$ $= \frac{13078}{125} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^6 F_k}{2^k}$

.624063589774511570688... $\approx \frac{1}{6} + \frac{\pi}{4\sqrt{3}}\coth\frac{\pi\sqrt{3}}{2} = \sum_{k=0}^{\infty} \frac{1}{4k^2+3}$

.624270016496295506006... $\approx -24\zeta(5) - 50\zeta(3) + \frac{2\pi^4}{3} + \frac{5\pi^2}{2} - 4$

$= \sum_{k=1}^{\infty} \left(\zeta(4k-2) - \zeta(4k+1)\right) = \frac{1}{2} + \sum_{k=1}^{\infty} \left(\zeta(4k-1) - \zeta(4k)\right)$

$= \sum_{k=2}^{\infty} \frac{k^2+k+1}{k^4+k^3+k^2+k}$

3 .624270016496295506006... $\approx -24\zeta(5) - 50\zeta(3) + \frac{2\pi^4}{3} + \frac{5\pi^2}{2} - 1 = \sum_{k=2}^{\infty} (-1)^k k^4 \left(\zeta(k) - 1\right)$

$= \sum_{k=2}^{\infty} \frac{16k^4 - k^3 + 11k^2 + 5k + 1}{k(k+1)^5}$

1 .624838898635177482811... $\approx$ $K_2(1) = K_0(1) + 2K_1(1)$

$= \int_0^{\infty} x^2 e^{-\sqrt{1+x^2}}\, dx$

.624841238258061424793... $\approx$ $\zeta(3) - \gamma$

$.62500000000000000000\underline{0} \quad = \quad \frac{5}{8}$

$.62519689192003630759\ldots \quad \approx \quad \frac{2}{3}+\frac{\pi}{2}-\frac{8\pi\sqrt{3}}{27} \;=\; \int_0^{\pi/2}\frac{\sin^2 x}{(2-\sin x)^2}dx$

$.6252442188407330907\ldots \quad \approx \quad \log\frac{\pi}{\sqrt{3}}\csc\frac{\pi}{\sqrt{3}} \;=\; \sum_{k=1}^{\infty}\frac{\zeta(2k)}{3^k k}$

1 $.62535489744912819254\ldots \quad \approx \quad \prod_{k=1}^{\infty}\left(1+\frac{1}{2^k k^2}\right)$

3 $.62560990822190831193\ldots \approx \quad \Gamma\left(\frac{1}{4}\right)$ AS 6.1.10

1 $.62578957571429774104 8\ldots \approx \quad \sum_{k=2}^{\infty}\frac{2^k\left(\zeta(k)-1\right)}{k!} \;=\; \sum_{k=2}^{\infty}\left(e^{-2k}-1-\frac{2}{k}\right)$

$.62602016562607381154 4\ldots \approx \quad \frac{\pi}{2}\operatorname{sech}\frac{\pi}{2} \;=\; \frac{\pi e^{\pi/2}}{e^{\pi}+1} \;=\; \int_0^{\infty}\frac{\cos x}{\cosh x}dx$ GR 3.981.3

$= \quad \int_{-\infty}^{\infty}\frac{\cos x}{e^x+e^{-x}}dx$

$.62605942820612802275 5\ldots \approx \quad \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)-1}{k^2} = -\sum_{k=2}^{\infty}Li_2\left(-\frac{1}{k}\right)$

$.62640943473787862544\ldots \quad \approx \quad \pi^2+16-21\zeta(3) \;=\; 16\sum_{k=1}^{\infty}\frac{k}{(2k+3)^3} \;=\; \int_1^{\infty}\frac{\log^2 x\,dx}{x^{3/2}(x-1)^2}$

$.62650367783334798380 8\ldots \approx \quad \frac{6\sin 1}{(5-4\cos 1)^2} \;=\; \sum_{k=1}^{\infty}\frac{k\sin k}{2^k}$

$.62653492911185378416 4\ldots \approx \quad -\sum_{k=1}^{\infty}\mu(2k)\left(\zeta(2k)-1\right)$

$.62657547937811691154 2\ldots \approx \quad \sum_{k=1}^{\infty}\frac{S_2(2k,k)2^k}{(2k)^{2k}}$

1 $.62657656169778574321 1\ldots \approx \quad 7^{1/4}$

$.62665706865775012560 4\ldots \approx \quad \frac{1}{2}\sqrt{\frac{\pi}{2}}$

3 $.62686040784701876766 8\ldots \approx \quad \sinh 2 \;=\; \frac{e^2-e^{-2}}{2} \;=\; \sum_{k=0}^{\infty}\frac{2^{2k+1}}{(2k+1)!} \;=\; \sum_{k=0}^{\infty}\frac{4^k k}{(2k)!}$ AS 4.5.62

$= \quad \int_0^{\pi}e^{2\cos x}\sin x\,dx$ GR 3.915.1

$.6269531324505805969\ldots \quad \approx \quad -\sum_{k=1}^{\infty}\frac{\mu(3k)}{2^k-1} = \sum_{k=1}^{\infty}\frac{1}{(\sqrt[3]{2})^{3^k}}$

.62702749554145676072... ≈ $3\log 3 + 4\log 2 - \pi\sqrt{3} = \sum_{k=1}^{\infty} \frac{1}{3k^2 - k/2}$

.627591004863777296758... ≈ $\zeta(2) - \zeta 6)$

3 .627598728468435701188... ≈ $\frac{2\pi}{\sqrt{3}} = \Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right) = \beta\left(\frac{1}{3}, \frac{2}{3}\right) = \sum_{k=1}^{\infty} \frac{3^k}{\binom{2k}{k} k}$

$= \int_{-\infty}^{\infty} \frac{dx}{x^2 + x - 1} = \int_0^{\infty} \frac{\log(1 + x^3)}{x^3} dx$

$= \int_0^{2\pi} \frac{dx}{1 + \cos x}$

$= \int_{-\infty}^{\infty} \frac{e^{x/3}}{e^x + 1} dx$

$= \int_0^{\infty} \log\left(1 + x^{-3}\right) dx = \int_0^{\infty} \log\left(1 + \frac{4}{x(x+2)}\right) dx$

.627716860485152532642... ≈ $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{\zeta(3k-1) - 1}{k} = \sum_{k=2}^{\infty} k \log\left(1 + \frac{1}{k^3}\right)$

.627815041275931813278... ≈ $\frac{1}{4}\left(2e - \sqrt{\pi}\, erfi 1\right) = \sum_{k=0}^{\infty} \frac{1}{k!(2k+1)} = \int_1^{\infty} \left(\sinh\left(\frac{1}{x^2}\right) + \cosh\left(\frac{1}{x^2}\right)\right) \frac{dx}{x^4}$

.6278364236143983844... ≈ $\frac{4}{5} - \frac{4\sqrt{5}}{25} \operatorname{arcsinh} \frac{1}{2}$

$= {}_2F_1\left(1, 1, \frac{1}{2}, -\frac{1}{4}\right) = \sum_{k=0}^{\infty} \frac{(-1)^k}{\binom{2k}{k}}$

.628094784476264027477... ≈ $\left(\frac{\sin 1}{2}\right)\left((\pi - 2)\cos 1 + \log(4\sin^2 1)\sin 1\right)$

$= \sum_{k=1}^{\infty} \frac{\sin^2 k}{k(k+1)}$

4 .62815959554093703767... ≈ $2\zeta(3) + \frac{11\pi^2}{36} - \frac{19}{24} = \sum_{k=1}^{\infty} \frac{H_k H_{k+3}}{k(k+1)}$

.628318530717958647693... ≈ $\frac{\pi}{5} = \int_0^{\infty} \frac{x^{3/2} dx}{1 + x^5}$

7 .628383212379538395441... ≈ $\sin\left(\pi(-1)^{1/3}\right) = \sin\left(\frac{\pi}{2} + \frac{i\pi\sqrt{3}}{2}\right)$

= $\cosh\frac{\pi\sqrt{3}}{2} = \prod_{k=0}^{\infty}\left(1+\frac{3}{(2k+1)^2}\right)$

1 .628473712901584447056... ≈ $\sum_{k=1}^{\infty}\frac{1}{k^{k-1}}$

.628507443651989153506... ≈ $Li_3(\gamma) = \sum_{k=1}^{\infty}\frac{\gamma^k}{k^3}$

.628527924724310085412... ≈ $(-1)^{1/4}\frac{\pi}{4}\left(\cot\left((-1)^{3/4}\pi\right)+i\cot\left((-1)^{1/4}\pi\right)\right)-\frac{1}{2}$

= $\frac{(-1)^{1/4}}{4}\left(\psi\left(2-(-1)^{3/4}\right)-\psi\left(2+(-1)^{3/4}\right)\right)$

$+\frac{(-1)^{3/4}}{4}\left(\psi\left(2-(-1)^{1/4}\right)-\psi\left(2+(-1)^{1/4}\right)\right)$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(4k-2)-1\right) = \sum_{k=2}^{\infty}\frac{k^2}{k^4+1} = -\operatorname{Im}\left\{\sum_{k=1}^{\infty}\frac{\zeta(2k)}{i^k}\right\}$

14 .628784140560320753162... ≈ $2\zeta(3)+\frac{5\pi^2}{6}+4 = \sum_{k=2}^{\infty}(k+1)^2\left(\zeta(k)-1\right) = \sum_{k=2}^{\infty}\frac{9k^2-11k}{k(k-1)^3}$

2 .62880133541162176449... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{e^{1/k}}{k^4}$

.628952086030043489235... ≈ $\sum_{k=1}^{\infty}\frac{\sigma_0(k)}{2^{k^2}} = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{1}{2^{k^2 j^2}}$

106 .629190515800789928381... ≈ $24\pi\sqrt{2} = \int_0^{\infty}\frac{dx}{(x^4+1/16)^2}$

5 .629258381564478619403... ≈ $2e-1-2eEi(-1) = \sum_{k=1}^{\infty}\frac{k^2\psi(k+1)}{k!}$

1 .6293779129886049518... ≈ $\frac{\gamma}{2}+\frac{\pi^2}{12}+\gamma\log 2+\frac{\log^2 2}{2} = \int_0^{\infty}\frac{\log^2 x\,dx}{e^{2x}}$

= $\frac{\pi^2}{12}+\frac{(\gamma+\log 2)^2}{12} = \int_0^{\infty}e^{-2x}\log^2 x\,dx$

1 .629629629629629629$\underline{629}$ = $\frac{44}{27} = \sum_{k=1}^{\infty}\frac{k^3}{4^k}$

.630171775633151608 55... ≈ $\frac{\gamma}{G}$

.630330700753906311477... ≈ $\frac{1}{2}\left(\log 2\pi-\gamma\right) = \sum_{k=2}^{\infty}\frac{\zeta(k)}{k(k+1)}$

= $\sum_{k=1}^{\infty}\left((2k-2)\log\left(1-\frac{1}{k}\right)+1-\frac{1}{2k}\right)$

	.63038440599136615460...	≈	$\prod_{p\ prime}\left(1-\frac{1}{2^p}\right)$	
	.6304246388313639332...	≈	$1-\frac{1}{\zeta(2)^2}=1-\frac{36}{\pi^4}$	
	.63062833173102285682 8...	≈	$\zeta(3)-\frac{4}{7}$	
3	.63082455165596093149 8...	≈	$\frac{\pi^2}{e}$	
	.6309297535714574371...	≈	$\log_3 2$	
	.63101092488848778826 8...	≈	$\sum_{k=2}^{\infty}\frac{\log\zeta(k)}{k-1}$	
	.63110323860592122695 8...	≈	$\frac{3\sin 1}{4} = \sum_{k=0}^{\infty}\frac{\sin^3\left(3^k\right)}{3^k}$	Berndt ch. 31
	.63152689898170515210 8...	≈	$\sum_{k=1}^{\infty}\frac{1}{2^k\sigma_1(k)}$	
	$.\underline{631578947368421052}$	=	$\frac{12}{19}$	
11	.63172839656744892914 4...	≈	$\frac{105\sqrt{\pi}}{16} = \Gamma\left(\frac{9}{2}\right)$	
1	.6318696084180513481...	≈	$\int_0^1 e^{\sin x}dx$	
	.6319660112501051518...	≈	$\frac{9}{4}-\varphi=\sum_{k=1}^{\infty}\frac{1}{F_{3\cdot 2^k}}$	GKP p. 302
	.63196619783816790666 2...	≈	$\zeta(3)-\frac{\pi^2}{12}\log 2 = \sum_{k=1}^{\infty}\frac{H_k}{2^k k^2}$	Berndt 9.12.1
	.63212055882855767840 5...	≈	$1-\frac{1}{e} = \sum\frac{(-1)^{k+1}}{k!} = \int_1^e\frac{1}{x^2} = \int_0^1(x-1)e^{-x}\log x\,dx$	GR 4.351.1
		=	$\int_0^1 x\cosh x dx = \int_1^{\infty}\cosh\left(\frac{1}{x}\right)\frac{dx}{x^3}$	
1	.63252691943815284477 3...	≈	$\sqrt{2}^{\sqrt{2}}$	
	.6328153188403884421...	≈	$\sum_{k=1}^{\infty}\frac{\zeta(k+1)}{3^k k} = -\sum_{k=1}^{\infty}\frac{1}{k}\log\left(1-\frac{1}{3k}\right)$	
	.63284301804378628741 6...	≈	$\sum_{k=2}^{\infty}\frac{\lfloor\log_2 k\rfloor}{2^k} = 2\sum_{k=1}^{\infty}\frac{1}{2^{2^k}}$	
1096	.63315842845859926372 0...	≈	e^7	
6	.63324958071079969823...	≈	$\sqrt{44}$	

.633255651314820034552... ≈ $-\cos(e\pi)$

2 .6338893027985365459... ≈ $\pi^2 \log 2 - \frac{7\zeta(3)}{2} = \sum_{k=1}^{\infty} \frac{4^k}{\binom{2k}{k} k^3}$

.633974596215561353236... ≈ $\frac{\sqrt{3}}{1+\sqrt{3}}$

.634028701498111859376... ≈ $\frac{1}{1+\gamma} = \sum_{k=0}^{\infty} (-1)^k \gamma^{-k}$

.634340223870549172483... ≈ $1 - \gamma + \frac{1}{\sqrt{5}} \left(\log\Gamma\left(\frac{3-\sqrt{5}}{2}\right) - \log\Gamma\left(\frac{3+\sqrt{5}}{2}\right) \right)$

= $\sum_{k=2}^{\infty} \frac{F_k}{k} \left(\zeta(k) - 1 \right)$

2 .634415941277498655611... ≈ $-\pi \log\left(\frac{1-e^{-2}}{2}\right) = -\int_0^{\infty} \frac{\log \sin^2 x}{x^2+1} dx$

6 .63446599048208274358... ≈ $Ei(e) - \gamma - 1 = \sum_{k=1}^{\infty} \frac{e^k}{k!k}$

.634861099338284456921... ≈ $2 - \pi \operatorname{csch}\frac{\pi}{2} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2 + 1/4}$

.63486687113357064562... ≈ $\operatorname{arcsec}\sqrt{\frac{15-\sqrt{33}}{6}}$ Associated with a sailing curve. Mel1 p.153

1 .634967033424113218236... ≈ $\frac{\pi^2}{12} + \frac{13}{16} = \sum_{k=1}^{\infty} (k+1)\left(\zeta(2k) - 1\right) = \sum_{k=2}^{\infty} \frac{2k^2 - 1}{(k^2-1)^2}$

.635181422730739085012... ≈ $\frac{\log 2}{2} + \frac{\gamma}{2} \prec \sum_{k=1}^{\infty} (-1)^k \psi(k)$

= $-\int_0^1 x \log\log\left(\frac{1}{x}\right) dx = -\int_0^{\infty} e^{-2x} \log x \, dx$ GR 4.325.8

.635469911917901624599... ≈ $\frac{\pi^2}{4} - 2G = \sum_{k=1}^{\infty} \frac{k\zeta(k+1)}{4^k} = \sum_{k=1}^{\infty} \frac{4}{(4k-1)^2}$

.6356874793986674953... ≈ $\sum_{k=2}^{\infty} \left(1 - \frac{\zeta(2k)}{\zeta(k)} \right)$

.63575432737726430215... ≈ $2\zeta(2) - 2\zeta(3) - \frac{1}{4} = 2\sum_{k=2} \frac{k}{(k+1)^3}$

= $\sum_{k=2}^{\infty} (-1)^k k(k-1)(\zeta(k) - 1)$

.63586172815606855366... ≈ $\frac{I_1\left(\sqrt{2}\right)}{\sqrt{2}} = \sum_{k=1}^{\infty} \frac{k}{(k!)^2 2^k}$

1 .635886323999068092858... ≈ $\frac{1}{4}\left(\psi\left(1+\frac{1}{2\sqrt{2}}\right)+\psi\left(1-\frac{1}{2\sqrt{2}}\right)-\psi\left(\frac{1}{2}+\frac{1}{2\sqrt{2}}\right)-\psi\left(\frac{1}{2}-\frac{1}{2\sqrt{2}}\right)\right)$

= $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k}{k^2-1/2}$

.636014527491066581475... ≈ $\frac{1}{2}+\frac{\pi}{2}\operatorname{csch}\pi = \sum_{k=0}^{\infty} \frac{(-1)^k}{k^2+1}$

= $\int_0^{\infty} \frac{\cos^2 x}{\cosh^2 x}\, dx$

.636363636363636363$\underline{63}$ = $\frac{7}{11} = \sum_{k=1}^{\infty} \frac{F_{3k-1}}{6^k}$

6 .636467172351030914783... ≈ $\frac{2^{\pi}+1}{1+\log^2 2} = \int_0^{\pi} 2^x \sin x\, dx$

.636534159241417807499... ≈ $\frac{\pi^2}{12}-\frac{\pi}{16}+\frac{1}{96} = \sum_{k=1}^{\infty} \frac{\sin k/2}{k^3}$ GR 1.443.5

= $\frac{i}{2}\left(Li_3\left(e^{-i/2}\right)-Li_3\left(e^{i/2}\right)\right)$

.636584789466303609633... ≈ $\zeta(2)-\zeta(7)$

.6366197723675813431... ≈ $\frac{2}{\pi} = \sum_{k=0}^{\infty} \binom{2k}{k}^2 \frac{1}{16^k(2k+2)}$

.6366197723675813431... ≈ $\prod_{k=1}^{\infty}\left(1-\frac{1}{4k^2}\right)$ GR 1.431

= $\prod_{k=1}^{\infty} \cos\frac{\pi}{2^{k+1}}$ GR 1.439.1

= $\binom{0}{1/2}$

= $i \log_i e$

.636823470047540403172... ≈ $\frac{1-\gamma}{5}+\frac{1}{5}\left((-1)^{4/5}\psi\left(\frac{1}{4}\left(3-\sqrt{5}-2i\sqrt{\frac{5-\sqrt{5}}{2}}\right)\right)\right)$

$-\frac{1}{5}\left((-1)^{1/5}\psi\left(\frac{1}{4}\left(3-\sqrt{5}+2i\sqrt{\frac{5-\sqrt{5}}{2}}\right)\right)-(-1)^{2/5}\psi\left(\frac{1}{4}\left(3+\sqrt{5}-i\sqrt{2(5+\sqrt{5})}\right)\right)\right)$

$$- \frac{(-1)^{1/5}}{5}\psi\left(\frac{1}{4}\left(3+\sqrt{5}+i\sqrt{2(5+\sqrt{5})}\right)\right)$$

$$= \sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(5k-3)-1\right) = \sum_{k=2}^{\infty}\frac{k^3}{k^5+1}$$

.637160267117967246437... ≈ $\frac{\pi}{2\sqrt{2}}\coth\frac{\pi}{\sqrt{2}} - \frac{1}{2} = \sum_{k=1}^{\infty}\frac{1}{2k^2+1} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)}{2^k}$

.637464900519471816199... ≈ $\frac{6\sin^2 1}{4\cos 2-5} = \sum_{k=1}^{\infty}\frac{\sin^2 k}{2^k}$

.637752497381454492659... ≈ $\frac{3\pi}{2e^2} = \int_{-\infty}^{\infty}\frac{\cos 2x}{(1+x^2)^2}dx$

.637915805122426308543... ≈ $\frac{1}{\pi^2}\cosh^2\frac{\pi}{2} = \sum_{k=0}^{\infty}\frac{\pi^{4k}}{(4k+2)!}$

.638000580642192011016... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)\zeta(k+1)-1}{k}$

1 .6382270745053706475... ≈ $\sum_{k=2}^{\infty}\frac{1}{d_k}$

.638251240331360040453... ≈ $2\pi - \frac{\pi^2}{6} - 4 = -\sum_{k=1}^{\infty}\frac{\cos 4k}{k^2}$ GR 1.443.3

.6387045287798183656... ≈ $\frac{\pi\sqrt{3}+9\log 3}{24} = \sum_{k=1}^{\infty}\frac{1}{6k^2-4k}$

$= -\int_0^1\frac{\log(1-x^6)}{x^5}dx$

.638961276313634801150... ≈ $\sin\log 2 = \operatorname{Im}\left\{2^i\right\}$

.63900012153645849366... ≈ $\frac{1}{3}\Gamma\left(\frac{5}{3}\right)\zeta\left(\frac{5}{3}\right) = \int_0^{\infty}\frac{x^4\,dx}{e^{x^3}-1}$

.639343615032532580146... ≈ $\frac{i}{2}\log\frac{\Gamma\left(2-\frac{1+i}{\sqrt{2}}\right)\Gamma\left(2+\frac{1-i}{\sqrt{2}}\right)}{\Gamma\left(2-\frac{1-i}{\sqrt{2}}\right)\Gamma\left(2+\frac{1+i}{2}\right)} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(4k-2)-1}{2k-1}$

$= \sum_{k=2}^{\infty}\arctan\frac{1}{k^2}$

.639432937474838301769... ≈ $\frac{1}{5-4\cos 1}\left(arc\cot(\cot 1-2\csc 1)(\cos 1-2)-\sin 1\log\left(\frac{5}{4}-\cos 1\right)\right)$

$= \sum_{k=1}^{\infty}\frac{H_k\sin k}{2^k}$

.639446070022506040443... ≈ $\zeta(3)+\zeta(4)-\zeta(2)$

.6397766840608015768... ≈ $\sum_{k=1}^{\infty}\frac{k}{3^k+1}$

.64000000000000000000$\underline{0}$ = $\frac{16}{25} = \sum_{k=1}^{\infty}\frac{F_k F_{k+1}}{4^k}$

.640022929731504395634... ≈ $\frac{3i}{8}\left(Li_3(e^{-i}) - Li_3(e^{i})\right) + \frac{i}{8}\left(Li_4(e^{3i}) - Li_4(e^{-3i})\right)$

= $\sum_{k=1}^{\infty}\frac{\sin^3 k}{k^4}$

.640186152773388023916... ≈ $\frac{4}{3} - \log 2$

1 .640186152773388023916... ≈ $\frac{7}{3} - \log 2$

.64031785796091597380... ≈ $16\zeta(3) - \frac{502}{27} = \int_0^1 \frac{x\log^2 x}{1-\sqrt{x}}dx$

.640402269535703102846... ≈ $\sum_{k=1}^{\infty}\frac{H^{(3)}{}_k}{2^k(2k-1)}$

.64057590922154613385... ≈ $\frac{1}{2\Gamma\left((-2)^{1/3}\right)\Gamma\left(-2^{1/3}\right)\Gamma\left(-(-1)^{2/3}2^{1/3}\right)} = \prod_{k=2}^{\infty}\frac{k^3-2}{k^3}$

.64085908577047738232... ≈ $2 - \frac{e}{2}$

.640995703458790509... ≈ $27 - 12\zeta(3) - \frac{2\pi^3}{3\sqrt{3}} = H^{(3)}{}_{1/3}$

2 .641188148861605230604... ≈ $6G + \frac{3\pi^2 G}{4} - \frac{3\pi^2}{8} + \frac{1}{256}\left(\psi^{(3)}\left(\frac{3}{4}\right) - \psi^{(3)}\left(\frac{1}{4}\right)\right)$

= $\int_0^{\pi/2}\frac{x^3}{\sin^3 x}dx$

.641274915080932047772... ≈ $\frac{\pi}{2\sqrt{6}} = \int_0^{\infty}\frac{dx}{2x^2+3} = \int_0^{\infty}\frac{dx}{3x^2+2}$

.6413018960103079026... ≈ $-2Li_3\left(-\frac{1}{3}\right) = \int_0^1 \frac{\log^2 x}{x+3}dx$

.641398092702653551782... ≈ $\frac{\pi}{\sqrt{3}} - \frac{24}{5} = hg\left(\frac{5}{6}\right) - hg\left(\frac{1}{6}\right)$

1 .641632560655153866294... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^{k(k-1)/2}} = \prod_{k=1}^{\infty}\frac{1-1/2^{2k}}{1-1/2^{2k-1}}$ [Gauss] I Berndt 303

5 .641895835477562869481... ≈ $\frac{10}{\sqrt{\pi}}$

.642024000514891118449... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^k-k}$

.642051832501655779767... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(4k-2)-1}{(2k-1)!} = \sum_{k=2}^{\infty}\sin\frac{1}{k^2}$

.642092615934330703006... ≈ $\cot 1 = 1-2\sum_{k=1}^{\infty}\frac{1}{k^2\pi^2-1}$ GKP eq. 6.88

= $\sum_{k=0}^{\infty}\frac{(-1)^k B_{2k} 4^k}{(2k)!}$

.642092615934330703006... ≈ $\cot 1$

1 .642188435222121136874... ≈ $\frac{1}{5}+\frac{\sqrt{25-2\sqrt{5}}}{\pi}$, Lebesgue constant

7 .642208445927342710179... ≈ $\frac{7}{G}$

.642761268839978879 11... ≈ $-Li_2\left(-\frac{3}{4}\right)$

.642857142857142857 1 = $\frac{9}{14}$

.643107628915133244616... ≈ $\int_1^2\left(1-\frac{1}{\zeta(x)}\right)dx$

1 .64322179613925445451... ≈ $1-\gamma-\frac{1}{3}\left(\psi(2-6^{1/3})+\psi\left(\frac{4-i2^{1/3}3^{5/6}+6^{1/3}}{2}\right)\right)$

$-\frac{1}{3}\psi\left(\frac{4+i2^{1/3}3^{5/6}+6^{1/3}}{2}\right)$

= $\sum_{k=1}^{\infty}6^k\left(\zeta(3k+1)-1\right) = \sum_{k=2}^{\infty}\frac{6}{k(k^3-6)}$

.643318387340724531847... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{\sqrt{k}} = \sum_{k=2}^{\infty}\left(Li_{1/2}\left(\frac{1}{k}\right)-\frac{1}{k}\right)$

.643501108793284386803... ≈ $2\arctan 2-\frac{\pi}{2} = \arcsin\frac{3}{5} = \arctan\frac{3}{4} = gd\log 2$

9 .6436507609929549958... ≈ $\sqrt{93}$

.6436776435894211956... ≈ $\frac{e\sin 1-1}{2} = \int_1^e \log x\,\sin\log x\,dx$

1 .64371804071094635289... ≈ $\sqrt{e}\sin\sqrt{e} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{e^k}{(2k-1)!}$

.643767332889268748742... ≈ $G - \frac{\pi \log 2}{8} = \frac{i}{2}\left(Li_2\left(\frac{1+i}{2}\right) - Li_2\left(\frac{1-i}{2}\right)\right)$

= $-\int_0^{\pi/4} \log(1-\tan x)\, dx$ GR 4.227.11

= $-\int_0^1 \frac{\log(1-x)}{1+x^2} dx$ GR 4.291.10

= $\int_0^1 \frac{\arctan x}{x(1+x)} dx$ GR 4.531.3

.643796887509858981341... ≈ $\frac{\gamma + \psi(1+e)}{e} = H_e$

.643907357064919657769... ≈ $\frac{1}{6} + \frac{\pi}{4\sqrt{2}} \tan \frac{\pi\sqrt{7}}{2} = \sum_{k=1}^{\infty} \frac{1}{4k^2 + 4k - 6}$

.64399516584922827668... ≈ $\frac{1}{\log^3 2 + 4 \log 2} = \int_0^{\infty} \frac{\sin^2 x}{2^x}$

1 .64429465690499542033... ≈ $\sum_{k=2}^{\infty} \left(2^{\zeta(k)} - 2\right)$

.644329968197302957804... ≈ $\sum_{j=2}^{\infty} \sum_{k=1}^{\infty} \frac{\log j}{2^{j^k} - 1}$

.644756611586665798596... ≈ G^5

1 .644801170454997028679... ≈ $-\frac{4\sqrt{3}}{\pi} \sin \pi\sqrt{3} = \prod_{k=1}^{\infty} \left(1 + \frac{1}{k(k+4)}\right)$

.6448805377924315... ≈ $\sum_{k=1}^{\infty} \frac{(-1)^k}{k(2k-3)\log k}$

.644934066848226436472... ≈ $\frac{\pi^2}{6} - 1 = \psi^{(1)}(2) = \zeta'(2) = \sum_{k=2}^{\infty} \frac{1}{k^2}$ J272, J348

= $\sum_{k=1}^{\infty} \frac{1}{k^2 + k + 1} = \sum_{k=1}^{\infty} \frac{1}{k^2 (k+1)}$

= $\sum_{k=2}^{\infty} \left(\zeta(k) - \zeta(k+1)\right) = \sum_{k=1}^{\infty} \left(\zeta(2k) - \zeta(2k+2)\right)$

= $\sum_{k=2}^{\infty} (-1)^k k \left(\zeta(k) - \zeta(k+1)\right) = \sum_{k=1}^{\infty} k \left(\zeta(k+2) - 1\right)$

= $\sum_{k=1}^{\infty} \frac{1}{k^2} - \int_1^{\infty} \frac{dx}{x^2}$

$= \quad -\int_0^1 \frac{x\log x}{1-x}dx$ GR 4.231.3

$= \quad \int_1^\infty \frac{\log x}{x^3-x^2}dx$

$= \quad \int_0^1 \left(\frac{1}{1-x}+\frac{x\log x}{(1-x)^2}\right)dx$ GR 4.236.2

$= \quad \int_0^\infty \frac{x\,dx}{e^x\left(e^x-1\right)}$ GR 3.411.9

1 .644934066848226436472... $\approx \quad \frac{\pi^2}{6} = \zeta(2) = \psi^{(1)}(1) = \sum_{k=1}^\infty \frac{1}{k^2} = \sum_{k=1}^\infty \frac{2+1/k}{(k+1)^2}$

$= \quad \sum_{k=1}^\infty \frac{H_k}{k(k+1)}$

$= \quad 3\sum_{k=1}^\infty \frac{1}{\binom{2k}{k}k^2}$ CFG F17, K156

$= \quad \sum_{k=1}^\infty \frac{21k-8}{\binom{2k}{k}^3 k^3}$

$= \quad \sum_{k=1}^\infty k\left(\zeta(k+1)-1\right) = \sum_{k=2}^\infty k\left(\zeta(k)-\zeta(k+1)\right)$

$= \quad -\int_0^1 \frac{\log x}{1-x}dx$ GR 4.231.2

$= \quad \int_0^\infty \frac{\log(1+x)}{x(1+x)}$

$= \quad \int_0^1 \frac{\log^2 x}{(1+x)^2}dx = \int_1^\infty \frac{\log^2 x}{(1+x)^2} = \int_0^\infty \frac{\log^2 x}{(1+x)^3}dx = \int_1^\infty \frac{\log x}{x(x-1)}dx$

$= \int_0^1 \frac{\log(1-x^2)}{x^3} dx$

$= \int_0^1 \frac{\log(1+2x+x^2)}{x} dx$ GR 4.296.1

$= -\int_0^\infty \log\left(1-e^{-x}\right) dx$ GR 4.223.2

$= \int_0^\infty \frac{x}{e^x-1} dx$

$= \int_0^\infty \frac{dx}{e^{\sqrt{x}}+1}$

2 .644934066848226436472... $\approx \frac{\pi^2}{6}+1 = \sum_{k=2}^\infty k\left(\zeta(k)-1\right)$

3 .644934066848226436472... $\approx \frac{\pi^2}{6}+2 = \sum_{k=2}^\infty k\left(\zeta(k)+\zeta(k+1)-2\right)$

.645275610234835007040... $\approx \frac{\pi\sqrt{3}}{6} + 2\log 2 - \frac{3\log 3}{2}$

$= \psi\left(\frac{1}{2}\right)-\psi\left(\frac{2}{3}\right) = \sum_{k=1}^\infty \frac{1+(-1)^{k+1}}{3k^2+k}$

$= \int_0^\infty \log\left(1+\frac{1}{(x+1)(x+2)}\right) dx$

.645724718266977128910... $\approx -\sum_{k=1}^\infty \frac{\mu(k)}{F_k}$

2 .645751311064590590502... $\approx \sqrt{7}$

1 .645902515225396119354... $\approx \frac{\sqrt{3}}{\pi}\sinh\frac{\pi}{\sqrt{3}} = \prod_{k=1}^\infty \left(1+\frac{1}{3k}\right)$

.645903590137755193632... $\approx \frac{\gamma}{2}+\frac{1}{4}\left(\psi(1-2i)+\psi(1-2i)\right) = \int_0^\infty \frac{\sin^2 x}{e^x-1}$

.645932474229300336240... $\approx \frac{1}{16}\left(5-2\pi\coth\pi+(1-i)\pi\sqrt{2}\left(\coth\frac{(1+i)\pi}{\sqrt{2}}-\cot\frac{(1+i)\pi}{\sqrt{2}}\right)\right)$

$= \frac{\pi}{8}\coth\pi-\frac{1}{16}+\frac{1}{8}\left((-1)^{3/4}\psi\left(2-(-1)^{1/4}\right)-(-1)^{3/4}\psi\left(2+(-1)^{1/4}\right)\right)$

$+\frac{1}{8}\left((-1)^{1/4}\psi\left(2-(-1)^{3/4}\right)-(-1)^{1/4}\psi\left(2+(-1)^{3/4}\right)\right)$

$= \sum_{k=2}^\infty \frac{1}{k^2-k^{-6}} = \sum_{k=1}^\infty \left(\zeta(8k-6)-1\right)$

.646393061176817551567... ≈ $\sum_{k=2}^{\infty} \frac{\zeta(k)}{k} \left(\zeta(k)-1\right)$

.646435324626649699803... ≈ $\frac{\pi}{4\sqrt{e}} \left(\cos\frac{1}{2} + \sin\frac{1}{2} \right) = \int_0^{\infty} \frac{\cos x \, dx}{4x^4+1}$

.6464466094067262378... ≈ $1 - \frac{1}{2\sqrt{2}}$

.646458473588902984359... ≈ $-\frac{1}{3} Li_2(-3) = \frac{\pi^2}{18} + \frac{\log 3 \log 4}{3} - \frac{2\log^2 2}{3} - \frac{1}{3} Li_2\left(\frac{1}{4}\right)$

= $\int_0^{\infty} \frac{x}{e^x+3} dx$

.646596291662721938667... ≈ $\frac{11}{25} + \frac{24}{25\sqrt{5}} \operatorname{arcsinh}\frac{1}{2} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k!(k+1)!}{(2k)!}$

.646612001608765845265... ≈ $\frac{1}{4}\left(5\sin 1 - 3\cos 1\right) = \sum_{k=1}^{\infty} (-1)^k \frac{k^2}{(2k-2)!}$

.6469934025023697224... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(6k-4)-1}{k} = -\sum_{k=2}^{\infty} k^4 \log(1-k^{-6})$

.64704901239611528732... ≈ $\sum_{k=1}^{\infty} \frac{1}{2k^2+k^{-1}} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}\zeta(3k-1)}{2^k}$

1 .64705860880273600822... ≈ $\prod_{k=1}^{\infty} \left(1 + \frac{\sin k}{2^k}\right)$

$.\underline{6470588235294117}$ = $\frac{11}{17}$

1 .647182343401733581306... ≈ $\int_1^{\infty} \frac{\log x}{\Gamma(x)} dx$

3 .647562611124159771980... ≈ $\frac{36}{\pi^2} = \frac{6}{\zeta(2)}$

1 .647620736401059521105... ≈ $\frac{\sqrt{\pi}}{4}(e+1) = \int_0^{\infty} e^{-x^2} \cosh^2 x \, dx$

.647719422669750427618... ≈ $\frac{1}{2}\left(\cosh\frac{\cos 1}{2} + \sinh\frac{\cos 1}{2} \right) \sin\left(1 + \frac{\sin 1}{2}\right) = \sum_{k=1}^{\infty} \frac{k \sin k}{k! 2^k}$

.647793574696319037017... ≈ $\frac{1}{\sqrt{2}} - \frac{1}{2} + \frac{\log\left(1+\sqrt{2}\right)}{2} = \int_1^{\infty} \frac{\operatorname{arcsinh} x}{x^3} dx$

.647859344852456910440... ≈ $\cos\frac{\sqrt{3}}{2}$

1 .647918433002164537093... ≈ $\frac{3\log 3}{2} = \int_1^\infty \frac{\log(x+2)}{x^2}dx$

.648054273663885399575... ≈ $\operatorname{sech} 1 = \frac{2}{e+e^{-1}} = \frac{1}{\cos i} = \sum_{k=0}^\infty \frac{E_{2k}}{(2k)!}$ AS 4.5.66

.648231038121442835697... ≈ $\frac{3\gamma}{5} + \frac{\arctan 2}{5} + \frac{\log 5}{20} = -\int_0^\infty \frac{\log x \cos^2 x}{e^x}dx$

.648275480106659634482... ≈ $\frac{\pi^2}{3} - \pi + \frac{1}{2} = Li_2\left(e^i\right) + Li_2\left(e^{-i}\right) = 2\sum_{k=1}^\infty \frac{\cos k}{k^2}$

32 .648388556215921310693... ≈ $6\pi\sqrt{3} = \int_0^\infty \log\left(1+\frac{999}{x^3+1}\right)dx$

1 .648721270700128146849... ≈ $\sqrt{e} = \sum_{k=0}^\infty \frac{1}{k!2^k} = \sum_{k=0}^\infty \frac{1}{(2k)!!} = i^{-i/\pi}$

.648793417991217423864... ≈ $\frac{\pi^2}{12} - \frac{3}{4}\log^2\left(\frac{\sqrt{5}-1}{2}\right) = \chi_2\left(\frac{\sqrt{5}-1}{2}\right)$ Berndt Ch. 9

.64887655168558007488... ≈ $\prod_{k=1}^\infty \frac{\zeta(4k)}{\zeta(4k+2)}$

.64907364028134509045... ≈ $\frac{\pi}{2\sqrt{3}}\tanh\frac{\pi\sqrt{3}}{2} - \frac{1}{4}$

= $\sum_{k=2}^\infty \frac{k^4}{k^6-1} = \sum_{k=1}^\infty \left(\zeta(6k-4)-1\right)$

.649185972973479437802... ≈ $9\log\frac{3}{2} - 3 = \sum_{k=0}^\infty \frac{1}{3^k(k+2)}$

.64919269265578293912... ≈ $\sum_{k=1}^\infty \frac{\zeta(5k-3)-1}{k} = -\sum_{k=2}^\infty k^3\log(1-k^{-5})$

.649481824343282643015... ≈ $\frac{\pi}{3} - \frac{\log 2}{3} - \frac{1}{6} = \int_0^{\pi/4} \frac{x\,dx}{\cos^4 x}$

.64963693908006244413... ≈ $\sin\frac{1}{\sqrt{2}}$

2 .649970605053701527022... ≈ $\pi G - \frac{\pi^3}{16} + \frac{\pi^2\log 2}{4} = \int_{-1}^1 \frac{\arccos x \arctan^2 x}{x^2}\,dx$

$.6500000000000000000\underline{0}$ = $\frac{13}{20}$

1 .650015797211168549834... ≈ $\log\frac{e^e-1}{e} = \sum_{k=1}^{\infty}(-1)^k\frac{e^k B_k}{k!k}$ Berndt 5.8.5

.65017457857712101250... ≈ $4320-1589e = \sum_{k=1}^{\infty}\frac{k^2}{k!(k+6)}$

3 .65023786847473254748... ≈ $\frac{\pi}{2}+3\log 2 = \sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^k\zeta(k+1) = \sum_{k=1}^{\infty}\frac{3}{k(4k-3)}$

.650311156858960038983... ≈ $\frac{2^{2/3}}{3}\left((-1)^{1/3}\log\left(1+\frac{(-1)^{1/3}}{2^{1/3}}\right)-(-1)^{2/3}\log\left(1-\frac{(-1)^{2/3}}{2^{1/3}}\right)\right)$

$-\frac{2^{2/3}}{3}\log\left(1-\frac{1}{2^{1/3}}\right)$

= $\frac{1}{3}\Phi\left(\frac{1}{2},1,\frac{2}{3}\right) = \frac{1}{2^{1/3}}\ {}_2F_1\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-1\right] = \sum_{k=0}^{\infty}\frac{1}{2^k(3k+2)}$

.650386106929128806... ≈ $\frac{i}{2}\left(Li_2(-e^i)-Li_2(-e^{-i})\right) = \operatorname{Im}\left\{Li_2(-e^{-i})\right\} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\sin k}{k^2}$

1 .650425758797542876025... ≈ $erfi\,1 = \frac{2}{\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{1}{k!(2k+1)}$

.650604594983248538006... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^{kH_k}}$

.65064514228428650428... ≈ $\frac{\pi}{2}\left(\sqrt{2}-1\right) = \arcsin\left(-\cos\frac{\pi}{\sqrt{2}}\right)$

= $4\sum_{k=1}^{\infty}\frac{(-1)^k}{27(2k-1)^3-2(2k-1)}$

= $\int_0^{\pi/2}\frac{\sin^2 x}{1+\cos^2 x}dx = \int_0^{\pi/2}\frac{\cos^2 x}{1+\sin^2 x}dx$

= $\int_0^1\frac{1-x^2}{x^2}\arctan(x^2)dx$ GR 4.538.2

= $\int_0^{\infty}\log\left(1+\frac{1}{4x^2+1}\right)dx$

.650923199301856338885... ≈ $\frac{1}{2}\log\frac{\sinh\pi}{\pi} = \sum_{k=1}^{\infty}\frac{\zeta(4k-2)-1}{(2k-1)} = \sum_{k=2}^{\infty}\operatorname{arctanh}\frac{1}{k^2}$

2 .65131660816881198157... ≈ $-\frac{1}{2}\psi^{(2)}\left(\frac{3}{4}\right) = 28\zeta(3)-\pi^3 = \sum_{k=0}^{\infty}\frac{1}{(k+3/4)^3}$

.651473539678535823016... ≈ $\frac{15}{32}\log\frac{5}{4}+\frac{35}{64} = \sum_{k=1}^{\infty}\frac{k^2H_k}{5^k}$

1 .651496129472318798043... ≈ $\log_2 \pi$

.651585210821386425593... ≈ $\left(\frac{\pi}{2^{9/4}}\right)\frac{\sin\left(2^{1/4}\pi\right)-\sinh\left(2^{1/4}\pi\right)}{\cos\left(2^{1/4}\pi\right)-\cosh\left(2^{1/4}\pi\right)} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(4k-2)}{2^k}$

= $\sum_{k=1}^{\infty}\frac{1}{2k^2+k^{-2}}$

2 .651707423218151760587... ≈ $\frac{1}{48}\left(6+\left(9-8\zeta(3)\right)\pi^2+\pi^4-120\zeta(3)+168\zeta(5)\right)$

= $\sum_{k=1}^{\infty}\frac{H_k H_k}{k^3}$

1 .651942792704498623963... ≈ $\frac{7\zeta(5)}{2}-\frac{\pi^2}{6}\zeta(3)=\sum_{k=1}^{\infty}\frac{H_k H_k}{k^3}$

1 .652163247071347030652... ≈ $\frac{7\pi\zeta(3)}{16} = \int_0^{\pi/2} x^2 \log\tan x \; dx$

1 .653164280286206248139... ≈ $\sum_{k=2}^{\infty}\frac{\zeta^2(k)}{k^2}$

23 .65322619113844249836... ≈ $\frac{31\pi^6}{1260} = \int_0^1 \log(1+x)\frac{\log x}{x}dx$

2 .653283344230149263312... ≈ $\zeta(2)+\zeta(7)$

.653301013632933874628... ≈ $4\pi-\frac{2\pi^2}{3}-\frac{16}{3} = \frac{i}{2}\left(Li_3\left(e^{-4i}\right)-Li_3\left(e^{4i}\right)\right) = -\sum_{k=1}^{\infty}\frac{\sin 4k}{k^3}$

GR 1.443.5

.653344711643348759976... ≈ $\sum_{k=2}^{\infty}\left(\zeta(k^2-2)-1\right)$

40 .65348145876833758993... ≈ $11\sqrt{e}+7+11\sqrt{\frac{e\pi}{2}}erf\frac{1}{\sqrt{2}} = \sum_{k=1}^{\infty}\frac{k^3}{k!!}$

.65353731412265158046... ≈ $\frac{1}{20}\left(4\gamma+\left(1+\sqrt{5}+2i\sqrt{\frac{5-\sqrt{5}}{2}}\right)\psi\left(\frac{9+\sqrt{5}}{4}-\frac{i}{2}\sqrt{\frac{5-\sqrt{5}}{2}}\right)\right)$

$+\frac{1}{20}\left(1+\sqrt{5}-2i\sqrt{\frac{5-\sqrt{5}}{2}}\right)\psi\left(\frac{9+\sqrt{5}}{4}+\frac{i}{2}\sqrt{\frac{5-\sqrt{5}}{2}}\right)$

$+\frac{1}{20}\left(1-\sqrt{5}+2i\sqrt{\frac{5+\sqrt{5}}{2}}\right)\psi\left(\frac{9-\sqrt{5}}{4}-\frac{i}{2}\sqrt{\frac{5+\sqrt{5}}{2}}\right)$

$+\frac{1}{20}\left(-1+\sqrt{5}+2i\sqrt{\frac{5+\sqrt{5}}{2}}\right)\psi\left(\frac{9-\sqrt{5}}{4}+\frac{i}{2}\sqrt{\frac{5+\sqrt{5}}{2}}\right)$

= $\displaystyle\sum_{k=2}^{\infty}\frac{1}{k^2-k^{-3}} = \sum_{k=1}^{\infty}\bigl(\zeta(5k-3)-1\bigr)$

.653643620863611914639... ≈ $\displaystyle -\cos 4 = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}16^k}{(2k)!}$

1 .653643620863611914639... ≈ $\displaystyle 2\sin^2 2 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}16^k}{(2k)!}$

.653743338243228533787... ≈ $\displaystyle \sum_{k=1}^{\infty}(-1)^{k+!}\frac{\zeta(5k-3)}{2^k} = \sum_{k=1}^{\infty}\frac{1}{2k^2+k^{-3}}$

.653953240369287891 13... ≈ $\displaystyle \sum_{k=1}^{\infty}\frac{\zeta(4k-2)-1}{k} = -\sum_{k=2}^{\infty}k^2\log(1-k^{-4})$

1 .654028242523005382213... ≈ $\displaystyle (\cosh\cos 1+\sinh\cos 1)\ \sin(1+\sin 1) = \sum_{k=1}^{\infty}\frac{k\sin k}{k!}$

.654113806319188570 8... ≈ $\displaystyle 2\zeta(3)-\frac{7}{4} = \sum_{k=3}^{\infty}(-1)^{k+1}k(k-1)\bigl(\zeta(k)-1\bigr) = \sum_{k=2}^{\infty}\frac{2(3k^2+3k+1)}{k^2(k+1)^3}$

5 .654517112077461193 82... ≈ $e^{1/\gamma}$

.654652615605328088 92... ≈ $\displaystyle \sum_{k=1}^{\infty}\frac{\zeta(3k-1)-1}{k^2} = \sum_{k=2}^{\infty}k\,Li_2\left(\frac{1}{k^2}\right)$

.654653670707977143798... ≈ $\displaystyle \sqrt{\frac{3}{7}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{3^k}\binom{2k}{k}$

.654966493222733104 66... ≈ $\displaystyle -\frac{4\sqrt{10}}{3\pi}\sin\pi\sqrt{10} = \prod_{k=1}^{\infty}\left(1-\frac{1}{k(k+6)}\right)$

6 .655258980645659749465... ≈ $\displaystyle \frac{8}{\zeta(3)}$

14 .655449506835504240 87... ≈ $\displaystyle 16G = \sum_{k=1}^{\infty}\frac{(3^k-1)(k+1)}{4^k}\zeta(k+2)$ Adamchik (27)

.655831600867491587281... ≈ $\displaystyle \sum_{k=1}^{\infty}(-1)^{k+1}\frac{k!}{k^k}$

.655878071520253881077... ≈ $\displaystyle \frac{\zeta(2)-\gamma}{2} = -c_1$ Patterson Ex. A.4.2

.655939826098979743 77... ≈ $\displaystyle \sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{k^3} = \sum_{k=2}^{\infty}Li_3\left(\frac{1}{k^2}\right)$

.656002513632980683235... ≈ $\displaystyle Li_3\left(\frac{3}{5}\right)$

1 .656484202473378895 65... ≈ $\displaystyle \sum_{k=2}^{\infty}\frac{k^2\bigl(\zeta(k)-1\bigr)}{k!} = \sum_{k=2}^{\infty}\frac{1}{k^2}\bigl((k+1)e^{1/k}-k\bigr)$

.656517642749665651818... ≈ $\frac{\coth 1}{2} = \frac{1}{2} + \sum_{k=1}^{\infty} \frac{1}{k^2 \pi^2}$ J951

.65660443319200020682... ≈ $\frac{4\log 2}{3} + \frac{2\log 3}{3} - 1 = \sum_{k=1}^{\infty} \frac{H_{2k+1}}{4^k}$

9 .65662747460460224761... ≈ $6 \log 5$

.6568108991872036472... ≈ $4 - \pi + 3\log 2 + \frac{\sqrt{3}}{2} \log \frac{2-\sqrt{3}}{2+\sqrt{3}} = \int_0^1 \log \frac{(1+x)^2}{1+x^6} dx$

1 .65685424949238019521... ≈ $4\sqrt{2} - 4 = \sum_{k=0}^{\infty} \binom{2k+1}{k} \frac{1}{8^k}$

5 .65685424949238019521... ≈ $\sqrt{32} = 4\sqrt{2}$

2 .656973685873328869567... ≈ $3\pi \log 2 - \frac{\pi^3}{8} = \int_{-1}^{1} \frac{\arcsin^3 x}{x^3} dx$

.656987523063179381245... ≈ $\sum_{k=2}^{\infty} \frac{\zeta(k) \log k}{2^k}$

.65777271445891870069... ≈ $-\psi\left(1 + \frac{i}{2}\right) - \psi\left(1 - \frac{i}{2}\right)$

.657926395608213776872... ≈ $\frac{55\pi^4}{2592} = \frac{i}{2}\left(Li_3\left(\frac{\sqrt{3}-i}{2}\right) - Li_3\left(\frac{\sqrt{3}+i}{2}\right)\right)$

= $\sum_{k=1}^{\infty} \frac{\sin k\pi/6}{k^3}$ GR 1.443.5

.65797362673929057459... ≈ $\frac{\pi^2}{15} = \frac{\zeta(4)}{\zeta(2)} = \xi(4) = \xi(-3)$

= $\sum_{k=1}^{\infty} \frac{(-1)^{\rho(k)}}{k^2} = \prod_{p\ prime} \frac{p^2}{p^2+1} = \prod_{p\ prime} \sum_{k=0}^{\infty} \frac{(-1)^k}{p^{2k}}$ HW Thm. 299

= $\sum_{k=1}^{\infty} \frac{\lambda(k)}{k^2}$ HW Thm. 300

1 .6583741831710831673... ≈ $\gamma^2 + \frac{\pi^2}{6} - 2\gamma \log 2 + \log^2 2 = \int_0^{\infty} \frac{\log^2(2x) dx}{e^x}$

.65847232569963413649... ≈ $\frac{\pi^2 \log 2}{4} - \frac{7\zeta(3)}{8} = \int_0^1 \frac{x \arccos^2 x}{1-x^2} dx$

= $\int_0^1 \frac{\arcsin^2 x}{x} dx$

8 .658585869689105532128... ≈ $8\zeta(4) = \frac{4\pi^4}{45}$

2 .658680776358274040947... ≈ $\frac{3\sqrt{\pi}}{2} = 2\Gamma\left(\frac{5}{2}\right) = \frac{\sqrt{\pi}}{6} + \Gamma\left(-\frac{3}{2}\right)$

= $-\int_0^{\pi/2} x e^{-\tan^2 x} \frac{\sin 4x}{\cos^2 x} dx$ GR 3.963.3

1. 658730029128491878509... ≈ $\sum_{k=1}^{\infty} \frac{\pi(k+1)}{2^k}$

.658759815490980624984... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(k+1)-1}{k^4} = -\sum_{k=2}^{\infty} \frac{1}{k} Li_4\left(-\frac{1}{k}\right)$

.658951075727201947430... ≈ $\frac{1}{2}\left(Ei(1)-\gamma\right) = \sum_{k=1}^{\infty} \frac{1}{2k!k}$

.65905796753218002746... ≈ $\frac{\pi^4}{72} - \gamma\zeta(3) = \sum_{k=1}^{\infty} \frac{\psi(k+1)}{k^3}$

3 .659370435304991975484... ≈ $(\cosh 2 - \sinh 2)\left(1 - \cosh e + \cosh(1+e) - \sinh e + \sinh(1+e)\right)$

= $\frac{e^e(e-1)+1}{e^2} = \sum_{k=0}^{\infty} \frac{e^k}{k!(k+2)}$

1 .659688960215841425343... ≈ $\prod_{k=1}^{\infty} \left(1 + \frac{1}{\binom{2k}{k} k}\right)$

.659815254349999514864... ≈ $\sum_{k=2}^{\infty} \frac{(-1)^k \zeta(k)}{k!} = \sum_{k=1}^{\infty} \left(e^{-1/k} - 1 + \frac{1}{k}\right)$

.659863237109041309297... ≈ $40\sqrt{\frac{2}{3}} - 32 = \sum \binom{2k+2}{k} \frac{(-1)^k}{8^k}$

.65998309173598797224... ≈ $\sum_{k=2}^{\infty} \log\left(\left(\left(1+\frac{1}{k}\right)^{k+1}\left(1-\frac{1}{k}\right)^{k-1}\right)^{1/k}\right)$

= $\sum_{k=2}^{\infty}\sum_{n=2}^{\infty} \left(Li_k\left(\frac{1}{n}\right) + Li_k\left(-\frac{1}{n}\right)\right)$

.660161815846869573928... ≈ $\prod_{p\ prime \geq 3} \left(1 - \frac{1}{(p-1)^2}\right) = \prod_{p\ prime \geq 3} \frac{p(p-2)}{(p-1)^2} = \prod_{p\ prime \geq 3} \frac{1-2/p}{(1-1/p)^2}$

= Twin primes constant C_2

8 .6602540378443864676... ≈ $\sqrt{75} = 5\sqrt{3}$

.660349361999116217163... ≈ $\prod_{k=1}^{\infty} \left(1 - \frac{1}{3k^2+k+1}\right)$

.660398163397448309616... ≈ $\frac{\pi}{4} - \frac{1}{8} = \frac{\pi^2}{12} - \frac{1}{4}\left(Li_2\left(e^i\right) + Li_2\left(e^{-i}\right)\right) = \sum_{k=1}^{\infty} \frac{\sin^2(k/2)}{k^2}$

.66040364132111511419... ≈ $\frac{\pi}{4}\coth 2\pi - \frac{1}{8} = \sum_{k=1}^{\infty}\frac{1}{k^2+4}$ J124

$= \int_0^{\infty}\frac{\sin x \cos x \, dx}{e^x - 1}$

.660438500147765439828... ≈ $\frac{\pi}{2^{9/4}} = \int_0^{\infty}\frac{dx}{x^4+2}$

.660653199838824821037... ≈ $\frac{2\pi}{5}\sqrt{\frac{2}{5+\sqrt{5}}} = \frac{\pi}{5}\csc\frac{3\pi}{5} = \int_0^{\infty}\frac{x^2}{x^5+1}dx$

1 .660990476320476376040... ≈ $\pi G \gamma$

1 .6611554434943963640... ≈ $\sum_{k=2}^{\infty}\frac{\log k}{k^2-k-1}$

.66130311266153410544... ≈ $1-\frac{1}{(e-1)^2} = \sum_{k=1}^{\infty}\frac{(-1)^k k B_k}{k!}$

809 .661456252670475428540... ≈ $(\cosh\cosh 2 + \sinh\cosh 2)\cosh\sin 2 = \sum_{k=0}^{\infty}\frac{\cosh 2k}{k!}$

.661566129312475701703... ≈ $\log\left(2\cos\frac{1}{4}\right) = \frac{1}{2}\log\left((1+e^{i/2})(1+e^{-i/2})\right)$

$= \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\cos k/2}{k}$

.661707182267176235156... ≈ $\frac{1}{105}\left(4+17\sqrt{2}-6\sqrt{3}+21\log(1+\sqrt{2})+42\log(2+\sqrt{3})-7\pi\right)$

Robbins' constant, the avg. distance between points in the unit cube

.66197960825054113427... ≈ $\frac{1}{4}+\frac{3\log 3}{8} = \int_1^{\infty}\frac{\log(x+2)}{x^3}dx$

2 .662277128832675576187... ≈ $\zeta(2) + \zeta(6)$

2 .662448289304721483984... ≈ $\sum_{k=1}^{\infty}\frac{1}{\phi(k!)}$

.662743419349181580975... ≈ Laplace limit. Root of $\frac{x e^{\sqrt{1+x^2}}}{1+\sqrt{1+x^2}} = 1$

6 .662895311501290705419... ≈ $\sum_{k=1}^{\infty}\left(\frac{3}{k}\right)^k$

.66333702373429058707... ≈ $\frac{\pi}{4}\coth\pi - \frac{1}{8} = \sum_{k=2}^{\infty}\frac{k^2}{k^4-1} = \sum_{k=1}^{\infty}\left(\zeta(4k-2)-1\right)$

.663502138933028197136... ≈ $\sum_{k=1}^{\infty}\frac{1}{F_{3k}}$

.66359071404430856287 2... ≈ $\frac{1}{2}\left(Li_3\left(\frac{1-i}{\sqrt{3}}\right)+Li_3\left(\frac{1+i}{\sqrt{3}}\right)\right) = \sum_{k=1}^{\infty}\frac{\cos k\pi/4}{k^3}$

1 .66381474516141493 7366... ≈ $\frac{2}{\zeta(3)}$

1 .6638623767088760602... ≈ $4G-2 = \sum_{k=1}^{\infty}\frac{(-1)^k k}{(k^2-\frac{1}{4})^2}$

3 .6638623767088760602... ≈ $4G = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+\frac{1}{2})(k+\frac{1}{2})}$

.66386996876846016 6669... ≈ $28\zeta(3)+\pi^3-64 = -\frac{1}{2}\psi^{(2)}\left(\frac{1}{4}\right)-64 = \sum_{k=1}^{\infty}\frac{1}{(k+\frac{1}{4})^3}$

64 .66386996876846016 6669... ≈ $28\zeta(3)+\pi^3 = -\frac{1}{2}\psi^{(2)}\left(\frac{1}{4}\right) = \sum_{k=0}^{\infty}\frac{1}{(k+\frac{1}{4})^3}$

.66428571428571$\underline{428571}$ = $\frac{93}{140} = \sum_{1}^{\infty}\binom{2k}{k}\frac{1}{4^k(k+1)}$

4 .6645325919040003 7192... ≈ $\cosh\frac{\pi}{\sqrt{2}} = \prod_{k=0}^{\infty}\left(1+\frac{2}{(2k+1)^2}\right)$

.66460274033342744 2793... ≈ ${}_0F_1\left(;1;-\frac{1}{e}\right) = J_0\left(\frac{2}{\sqrt{e}}\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k!)^2 e^k}$

.66467019408956851024... ≈ $\frac{3\sqrt{\pi}}{8} = \int_0^{\infty} x^4 e^{-x^2}dx$

= $\int_0^{\pi/2} e^{-\tan^2 x}\frac{1-2\cos^2 x}{\cos^6 x\cot x}dx$ GR 3.964.3

.66488023368106598117... ≈ $\left(\frac{\pi^2}{6}-1\right)(\gamma-1)-\zeta'(2) = -\int_0^{\infty}\frac{x\log x}{e^x\left(e^x-1\right)}dx$

.66489428584555810021... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(3k-1)-1}{k} = -\sum_{k=2}^{\infty}k\log(1-k^{-3})$

2 .66514414269022518865... ≈ $2^{\sqrt{2}}$ Gelfond Schneider constant, known to be transcendental

1 .66535148216552671854... ≈ $\frac{6}{5}+\frac{3}{10}\left(5\log 5+\sqrt{5}\log\frac{5+\sqrt{5}}{5-\sqrt{5}}-2\pi\sqrt{1+\frac{2}{\sqrt{5}}}\right)$

= $\sum_{k=2}^{\infty}\frac{6^k\left(\zeta(k)-1\right)}{5^k} = \sum_{k=2}^{\infty}\frac{36}{5k(5k-6)}$

.66539301160385631761 7... ≈ $\frac{1}{18}\left(\pi^2-2\pi\sqrt{3}\log 3+2\psi^{(1)}\left(\frac{1}{3}\right)-2\psi^{(1)}\left(\frac{2}{3}\right)\right)$

$= \quad \sum_{k=1}^{\infty} \frac{H_k}{\binom{2k}{k} k}$

2 .665423390461749394547... ≈ $8\gamma^2$

.66544663410202726678... ≈ $-\operatorname{arctanh} \frac{1}{1-e}$

.665738986368222001365... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(k^2)}{k^2}$

.665773750028353863591... ≈ $\arctan \frac{\pi}{4}$

1 .665889619038593371592... ≈ $5\gamma^2$

.666003775286042277660... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(2k)-1}{(k!)^2} = \sum_{k=2}^{\infty} \left(\frac{2}{k} I_0\left(\frac{2}{k}\right) - \frac{2}{k} \right)$

1 .666081101809387342631... ≈ $\frac{3\pi}{4\sqrt{2}} = \frac{\pi + 2\arctan(1-\sqrt{2})}{\sqrt{2}} = \int_0^{\infty} \frac{dx}{e^x + e^{-x} - \sqrt{2}}$

1 .666090786005217177154... ≈ $B_2 = \gamma + \sum_{p\ prime} \left(\log\left(1 - \frac{1}{p}\right) - \frac{1}{p} \right) + \sum_{p\ prime} \frac{1}{p(p-1)}$

HW Thm. 430

.666169509211720802743... ≈ $\sum_{k=2}^{\infty} \frac{\zeta^3(k)}{k^3}$

8 .666196488469696472590... ≈ $\sum_{k=0}^{\infty} \frac{2^k \zeta(k+2)}{k!} = \sum_{k=1}^{\infty} \frac{e^{2/k}}{k^2}$

.666355847615437348637... ≈ $2\gamma^2$

.666366745392880526338... ≈ $\cos \sin 1$

.66653061575479140978... ≈ $\sin 1\ si(1) - \cos 1\ ci(1) - \gamma \cos 1$

$= \quad -\int_0^1 \log x \sin(1-x)\, dx$

$.66666666666666666666\underline{6}$ = $\frac{2}{3} = \sum \frac{(-1)^k}{2^k}$

$= \quad \sum_{k=0}^{\infty} \binom{2k}{k} \frac{1}{4^k (k+1)(k+2)} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(k+3/2)(k-1/2)}$

$= \quad \sum_{k=1}^{\infty} \frac{F_{4k-3}}{8^k}$

$= \prod_{k=1}^{\infty}\left(1-\frac{1}{(k+2)^2}\right)$

$= \prod_{k=2}^{\infty}\left(1-\frac{2}{k^3+1}\right)$ J1034, Mardsen p. 358

$= \int_0^{\pi/2} \sin x\sqrt{\cos x}\ dx$

$= \int_0^{\pi/2} \frac{1}{1+\sin^2 x}dx$

$= \int_0^{\infty} \frac{\cosh x}{e^{2x}}dx$

$= \int_1^{\infty} \cosh\ \log x\ \frac{dx}{x^3}$

$= \int_0^{\infty} \frac{x^8\ dx}{e^{x^3}}$

1 .66666666666666666666$\underline{6}$ $= \frac{5}{3}$

$= \sum_{k=1}^{\infty}\frac{F_{4k-1}}{8^k}$

2 .66666666666666666666$\underline{6}$ $= \frac{8}{3}$

$= \sum_{k=1}^{\infty}\frac{F_{4k}}{8^k}$

2 .66669146825473297034 1... $\approx \sum_{k=0}^{\infty}\frac{1}{(k!!)!}$

.66682207619228132568 2... $\approx 1-\gamma^2$

.66699543830463006824 7... $\approx \frac{\cosh\sqrt{3}}{2}-\frac{\sinh\sqrt{3}}{2\sqrt{3}} = \sum_{k=0}^{\infty}\frac{3^k k}{(2k+1)!}$

.66724523794284173732... $\approx 2\sqrt{\pi}\log\left(\frac{1+\sqrt{2}}{2}\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(k-½)!}{k!k}$

.66745721602838377187 2... $\approx \arccos\frac{\pi}{4}$

.66749054338776451533... $\approx \frac{\gamma}{2}+\frac{1}{4}\left(\psi\left(2-e^{2i}\right)+\psi\left(2-e^{-2i}\right)\right)$

$\sum_{k=1}^{\infty}\left(\zeta(k+1)-1\right)\sin^2 k$

.6676914571896091767... ≈ $\frac{1}{2}\left(\zeta\left(\frac{1}{2},\frac{1}{4}\right)-\zeta\left(\frac{1}{2},\frac{3}{4}\right)\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{\sqrt{2k+1}}$

.66774488357928988201... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{k^2} = \sum_{k=2}^{\infty}Li_2\left(\frac{1}{k^2}\right)$

1 .66828336396657121205... ≈ $-Li_3(-2) = \frac{\pi^2\log 2}{6} + \frac{\log^3 2}{6} - Li_3\left(-\frac{1}{2}\right)$

$= \int_0^1\frac{\log^2 x\,dx}{1+2x} = 8\int_1^{\infty}\frac{\log^2 x\,dx}{x^3+2x} = \int_0^{\infty}\frac{x^2\,dx}{e^x+2}$

.66942898781460986875... ≈ $\sum_{k=1}^{\infty}\frac{\sigma_o(k)}{2^k k^2}$

.669583085926878588096... ≈ $\sum_{k=1}^{\infty}\frac{1}{(2^k-1)(k+1)}$

96 .67012655640149635441... ≈ $2\pi^3\log 2 + 8\pi\log^3 2 + 12\pi\zeta(3) = \int_0^{\infty}\frac{x^3\,dx}{\sqrt{e^x-1}}$

1 .6701907046196043386... ≈ $\sum_{k=1}^{\infty}\frac{k}{2^k+1}$

1 .67040681796633972124... ≈ $\sum_{k=1}^{\infty}\frac{1}{e^{\sqrt{k}}}$

15 .67071684818578136541... ≈ $\gamma^5+\gamma^{-5}$

20 .67085112019988011698... ≈ $\frac{2\pi^3}{3}$

.670894551991274910198... ≈ $-e\left(1+e\log\left(1-\frac{1}{e}\right)\right) = \sum_{k=0}^{\infty}\frac{1}{e^k(k+2)}$

.671204652601083... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k}{k^2-k^{3/2}}$

2 .671533636866672060559... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}e^{1/k}}{k^5}$

1 .67158702286436895171... ≈ $\sin\left(\frac{1}{2}+\sin 1\right)\left(\cosh\cos 1+\sinh\cos 1\right) = \sum_{k=0}^{\infty}\frac{1}{k!}\sin\frac{2k+1}{2}$

.671646710823367585219... ≈ $\frac{e\pi(1-erf\,1)}{2} = \int_0^{\infty}\frac{e^{-x^2}}{1+x^2}\,dx$

.671719601885874542354... ≈ $\frac{2\pi}{\sqrt{3}}\left(\log\Gamma\left(\frac{1}{6}\right)-\frac{5}{6}\log 2\pi\right) = -\int_0^1\log\log\left(\frac{1}{x}\right)\frac{dx}{1-x+x^2}$

GR 4.325.6

= $-\int_0^\infty \frac{\log x}{e^x + e^{-1} - 1}$

.67177754230896957429... ≈ $\frac{10\pi}{27\sqrt{3}} = \frac{1}{9}\sum_{k=1}^{\infty} \frac{1}{\binom{2k}{k}} = \int_0^\infty \frac{dx}{(x^3+1)^3}$

.671865985524009837878... ≈ $\gamma + \frac{1}{2}\left(\psi(i) + \psi(-i)\right) = \sum_{k=1}^{\infty} \frac{1}{k^3+k}$

= $\frac{1}{2} + \sum_{k=1}^{\infty} (-1)^{k+1}\left(\zeta(2k+1) - 1\right) = \frac{1}{2} + \sum_{k=1}^{\infty} \left(\zeta(4k-1) - \zeta(4k+1)\right)$

= $\operatorname{Re}\left\{\sum_{k=1}^{\infty} \frac{1}{k^2(k+i)}\right\}$

.672148922218710419133... ≈ $\frac{92}{75} - \frac{4\log 2}{5} = \sum_{k=1}^{\infty} \frac{1}{k(k+5/2)}$

.672247528206935894883... ≈ $\int_{-1}^{1} \frac{\arcsin^2 x \arctan^2 x}{x^2}\, dx$

.672462966936363624928... ≈ $\frac{1}{2}\left(\pi Y_0(2) - 2\gamma J_0(2)\right) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} H_k}{k!k!}$

.67270395832927634325... ≈ $\frac{90}{\pi^6}\left(30\zeta'(4) - \pi^2\zeta'(2)\right) = \sum_{k=1}^{\infty} \frac{|\mu(k)|\log k}{k^2}$

.672753015827581593636... ≈ $\frac{1}{4} - \frac{\pi}{4\sqrt{2}}\cot\frac{\pi}{\sqrt{2}} = \sum_{k=1}^{\infty} \frac{1}{4k^2 - 2}$

.672942823879357247419... ≈ $\frac{\pi}{4\sqrt{3}} + \frac{1}{3}\log\frac{1+\sqrt{3}}{\sqrt{3}} = \sum_{k=0}^{\infty} \frac{(-1)^k(\sqrt{3}-1)^{3k+1}}{3k+1}$

2 .67301790986491692022... ≈ $\sum_{k=2}^{\infty} \left(\zeta^3(k) - 2\zeta(k) + 1\right)$

3 .67301790986491692022... ≈ $\sum_{k=2}^{\infty} \left(\zeta^3(k) - \zeta(k)\right)$

4 .67301790986491692022... ≈ $\sum_{k=2}^{\infty} \left(\zeta^3(k) - 1\right)$

1 .67302442726824453628... ≈ $\frac{\cos 2 + \cosh 2}{2} = \cos(1+i)\cosh(1+i) = \frac{e^2}{4} + \frac{1}{4e^2} + \frac{\cos 2}{2}$

= $\sum_{k=0}^{\infty} \frac{16^k}{(4k)!}$

.6731729316883003802... ≈ $\frac{1}{2^{3/4}}\Gamma\left(\frac{3}{4}\right)\cos\frac{\pi}{8} = \int_0^\infty \frac{\sin^2\left(x^4\right)}{x^2} dx$

2	.67323814048303015041... ≈	$\frac{2\pi}{e-e^{-1}} = \frac{\pi}{\sinh 1}$	
	.67344457931634517503... ≈	$\sum_{k=1}^{\infty} \frac{1}{q(k)}$	
	.673456768265772964153... ≈	$\frac{1}{2} + \frac{\pi}{2} - 2si(2) = \int_1^{\infty} \frac{\sin^2 x}{x^2} dx$	
	.6736020436407296815... ≈	$\sum_{k=1}^{\infty} \frac{k^2}{4^k + 1}$	
	.673917363376357541664... ≈	$\prod_{k=1}^{\infty} \left(1 - \frac{1}{k(k+1)^2}\right)$	
	.673934654451819223753... ≈	$\frac{\pi^2}{20} - \frac{3}{8} \log \frac{\sqrt{5}-1}{2}$	Berndt 9.8
	.674012554268124725048... ≈	$\sum_{k=2}^{\infty} \frac{\zeta(k)-1}{(k-1)^3} = \sum_{k=1}^{\infty} \frac{1}{k} Li_3\left(\frac{1}{k}\right)$	
3	.674225975055874294347... ≈	$\cosh^3 1 = \sum_{k=1}^{\infty} \frac{(3^k - 3)(1 + (-1)^k)}{8k!}$	J844
	.674600261330919653608... ≈	$\frac{7\zeta(3)}{2} + \frac{\pi^2}{4} - 6 = \sum_{k=1}^{\infty} \frac{k^2}{2^k} \left(\zeta(k+1) - 1\right) = \sum_{k=2}^{\infty} \frac{2(2k+1)}{(2k-1)^3}$	
3	.674643966011328778996... ≈	$\sum_{k=1}^{\infty} \frac{p_k}{2^k}$, p_k the kth prime	
1	.674707331811177969904... ≈	$\sum_{k=2}^{\infty} \left(1 - \frac{1}{2^k(\zeta(k)-1)}\right)$	
	.67475715901610705842... ≈	$\sum_{k=1}^{\infty} \frac{\zeta(2k)-1}{F_{2k}}$	
	.67482388341871655835... ≈	$\sum_{k=2}^{\infty} \frac{\log k}{(k+1)^2}$	
	.675198437911114341901... ≈	$\sum_{p\ prime} \frac{1}{p!} = \sum_{k=1}^{\infty} \frac{\pi(k)k}{(k+1)!}$	
5	.675380154319224365966... ≈	$\frac{6}{5}\left(\pi - \log 2 + \sqrt{3}\log(2+\sqrt{3})\right) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)(k+1/6)}$	
	.67546892106584099631... ≈	$\frac{1}{1+\log^2 2} = \int_0^{\infty} \frac{\sin x}{2^x}$	
	.67551085885603996302... ≈	$\frac{\arctan\sqrt{2}}{\sqrt{2}} = \frac{\pi - 2\arctan\frac{1}{\sqrt{2}}}{2\sqrt{2}} = \int_0^{\infty} \frac{dx}{3x^2+2x+1}$	

$= \displaystyle\int_0^\infty \frac{dx}{x^2+2x+3} = \int_0^{\pi/4} \frac{dx}{1+\sin^2 x}$

14 .67591306671469341644... $\approx \displaystyle\sum_{k=1}^\infty \frac{k^2\sigma_0(k)}{2^k} = \sum_{n=1}^\infty\sum_{m=1}^\infty \frac{m^2n^2}{2^{mn}} = \sum_{m=1}^\infty \frac{k^2\left(2^{-k}+4^{-k}\right)}{(1-2^{-m})^3}$

.675947298512314616619... $\approx \displaystyle\frac{1}{2\sqrt{2}}\arctan 2 + \frac{\log 5}{4\sqrt{2}} = \int_0^{\pi/4} \frac{\cos x}{1+\sin^4 x}dx = \int_0^{1/\sqrt{2}} \frac{1}{1+x^4}dx$

.675977245872051077662... $\approx \displaystyle\frac{1}{\sqrt{5}}e^{-1/(1+\sqrt{5})}\left(e^{\sqrt{5}/2}-1\right) = \sum_{k=1}^\infty \frac{F_k}{k!2^k}$

3 .676077910374977720696... $\approx \displaystyle\frac{\sinh \pi}{\pi} = \binom{0}{i} = \frac{e^\pi + e^{-\pi}}{2\pi} = \sum_{k=0}^\infty \frac{\pi^{2k}}{(2k+1)!}$

$= \displaystyle\prod_{k=1}^\infty\left(1+\frac{1}{k^2}\right) = \prod_{k=1}^\infty\left(1+\frac{2}{k(k+2)}\right) = \prod_{k=1}^\infty \frac{k^2+2k+2}{k^2+2k}$

.676565136147966640827... $\approx \displaystyle\sum_{k=1}^\infty \frac{\zeta(2k)-1}{2k-1} = \sum_{k=2}^\infty \frac{1}{k}\operatorname{arctanh}\frac{1}{k}$

1 .67658760238543093264... $\approx \displaystyle\sum_{k=1}^\infty \frac{|\mu(k)|}{k!}$

.676795322990996747892... $\approx \displaystyle\sum_{k=1}^\infty \frac{|\mu(k)|}{2^k k}$

1 .676974972518977020305... $\approx \displaystyle e^{1/8}\left(1+\frac{\sqrt{\pi}}{2}erf\,\frac{1}{2\sqrt{2}}\right) = \sum_{k=0}^\infty \frac{1}{k!!2^k} \quad \sum_{k=1}^\infty \frac{(k-1)!!}{(2k)!!}$

.67697719835070493892... $\approx \displaystyle\sum_{k=2}^\infty\left(\zeta(k)^{\zeta(k)}-\zeta(k)\right)$

.677365284338832383751... $\approx \displaystyle 12-2e-\frac{16}{e} = 12-18\cosh 1+14\sinh 1 = \sum_{k=0}^\infty \frac{1}{(2k)!(k+2)}$

.67753296657588678176... $\approx \displaystyle\frac{3}{2}-\frac{\pi^2}{12} = \sum_{k=2}^\infty\left(\frac{\zeta(k)+\zeta(k+1)}{2}-1\right)$

.678097245096172464424... $\approx \displaystyle\frac{3\pi}{8}-\frac{1}{2} = \sum_{k=1}^\infty \frac{\sin^3 k}{k^3}$

3 .678581614644620922501... $\approx \displaystyle\sum_{k=2}^\infty \frac{\zeta^2(k)}{(k-1)!}$

17 .67887848183809818913... $\approx \displaystyle e^2\sqrt{2\pi}\,erf\sqrt{2} = \sum_{k=1}^\infty \frac{(2k)!!\,2^k}{(2k-1)!!\,k!}$

2 .67893853470774763366... $\approx \displaystyle\Gamma\left(\frac{1}{3}\right)$

$.678953692032834648496... \approx 1-\frac{\cot 1}{2}$

$.67910608050053922751... \approx \frac{\pi}{4}\left(1-\frac{1}{e^2}\right) = \frac{\pi}{4}-\frac{\sqrt{\pi}}{2}K_{1/2}(2) = \int_0^\infty \frac{\sin^2 x\,dx}{1+x^2}$

$= \int_0^{\pi/2} \sin^2(\tan x)\,dx$ GR 3.716.9

719 $.67924568118873483737... \approx \frac{61\pi^7}{256} = 360i\left(Li_7(-i)-Li_7(i)\right) = \int_0^1 \frac{\log^6 x}{1+x^2}dx$ GR 4.265

$.67927089041591768 1932... \approx \sum_{j=2}^\infty \sum_{k=1}^\infty \frac{1}{2^{j^k}-1}$

2 $.67953572363023946445... \approx \frac{4}{\sqrt{3}}\cosh\pi\,\sinh\pi\,\operatorname{csch}\pi\sqrt{3} = \prod_{k=0}^\infty \frac{k^2+4}{k^2+3}$

$.67954834142940858377... \approx \sum_{k=2}^\infty \gamma^2\left(\zeta^2(k)-1\right)$

$.67957045711476130884... \approx \frac{e}{4} = \sum_{k=1}^\infty \frac{k^2}{(2k)!} = \sum_{k=0}^\infty \frac{1}{4k!} = -\int_0^\infty \frac{dx}{e^x(x+2)^3}$

$.67965885748720616 3727... \approx \sum_{k=2}^\infty \frac{H_k\left(\zeta(k)-1\right)}{k}$

2 $.67984556985870685 2438... \approx \sum_{k=1}^\infty \frac{\log k}{F_k}$

11 $.6800000000000000000\underline{0} = \frac{292}{25} = \sum_{k=1}^\infty \frac{k^2 F_{2k}}{4^k}$

$.680531222042836769403... \approx \sum_{k=2}^\infty \frac{(-1)^k \zeta(k)}{k(k-1)} = \sum_{k=2}^\infty \left(H_k-1\right)\left(\zeta(k)-1\right)$

1 $.680531222042836769403... \approx \sum_{k=2}^\infty H_k\left(\zeta(k)-1\right) = \sum_{k=2}^\infty \frac{k}{k-1}\log\frac{k}{k-1}$

7 $.6811457478686081758... \approx \sqrt{59}$

$.68117691028158186735... \approx \frac{1}{2}-\frac{\sqrt{\pi}\cot\sqrt{\pi}}{2} = \sum_{k=1}^\infty \frac{1}{k^2\pi-1}$

$.681690113816209328462... \approx \frac{\pi-1}{\pi} = \int_1^\pi \frac{dx}{x^2}$

1 $.681792830507429086062... \approx 8^{1/4}$

$.681942519346374694769... \approx \frac{3}{2} - {}_2F_1\left(1,1,1+i,-1\right) - \frac{1-i}{4}{}_2F_1\left(1,1,2-i,-1\right) = \sum_{k=1}^\infty \frac{k^2}{2^k(k^2+1)}$

.68202081730847307059... ≈ $G + \frac{\pi \log 2}{4} - \frac{\pi^2}{16} - \frac{\pi^3}{192} = \int_0^{\pi/4} x^2 \cot^2 x \, dx$

.682153502605238066761... ≈ $\sum_{k=1}^{\infty} \frac{1}{3^k - 1} = \sum_{k=1}^{\infty} \frac{\sigma_0(k)}{3^k} = \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} \frac{1}{\left(3^j\right)^k}$

= $\sum_{k=1}^{\infty} \frac{3^k + 1}{3^{k^2}(3^k - 1)}$ Berndt 6.1.4

3 .6821541361836286282... ≈ $\prod_{k=1}^{\infty} \left(1 + \frac{1}{k!}\right)$

5 .68219697698347550546... ≈ $\frac{7\pi^4}{120} = \frac{21\zeta(4)}{4} = -\int_0^1 \frac{\log^3 x}{1+x^2} dx = \int_1^{\infty} \frac{\log^3 x}{x^2 + x} dx = \int_1^{\infty} \frac{x^3}{1+e^x} dx$

GR 4.262.1

.682245131127166286554... ≈ $1 - \gamma - \frac{1}{2}\left(\psi\left(2+\sqrt{2}\right) + \psi\left(2-\sqrt{2}\right)\right) = \sum_{k=1}^{\infty} 2^k \left(\zeta(2k+1) - 1\right)$

.68256945033085777154... ≈ $\frac{\pi}{2\sinh(\pi/2)}$

.682606194485985295135... ≈ $\log_5 3$

.6826378970268436894... ≈ $\frac{720}{143\pi} \cos\frac{\pi\sqrt{52}}{2} = \prod_{k=1}^{\infty} \left(1 - \frac{1}{k(k+7)}\right)$

.682784063255295681467... ≈ $\pi^{-1/3}$

.682864049225884024308... ≈ $\sum_{k=1}^{\infty} \left(\zeta(k^2+1) - 1\right)$

.68286953823459171815... ≈ $\frac{1}{192}\left(-21 + 4\pi^2 \; 36\pi \coth\pi - 12\pi^2 \operatorname{csc} h^2 \pi\right)$

= $\sum_{k=2}^{\infty} \frac{1}{k^2 \left(1 - 1/k^4\right)^2}$

1 .682933835880662358359... ≈ $\cosh\frac{\pi}{2\sqrt{2}} = \prod_{k=0}^{\infty} \left(1 + \frac{1}{2(2k+1)^2}\right)$

1 .68294196961579301331... ≈ $2\sin 1 = -i(e^i - e^{-i})$

1 .683134192795736841802... ≈ $\left(\cosh\cosh\frac{1}{2} + \sinh\cosh\frac{1}{2}\right) \sinh\sinh\frac{1}{2}$ J712

= $\frac{1}{2}\left(e^{\sqrt{e}} - e^{1/\sqrt{e}}\right) = \sum_{k=0}^{\infty} \frac{\sinh(k/2)}{k!}$

.683150338229591228672... ≈ $\frac{7\pi^2}{24} + \frac{\pi\sqrt{3}}{4}\log 3 + \frac{9}{8}\log^2 3 - \frac{1}{2}\psi^{(1)}\left(\frac{2}{3}\right) = \sum_{k=1}^{\infty} \frac{H_k}{3k^2 + k}$

.683853674822667022202... ≈ $\sum_{k=1}^{\infty} \left(\frac{1}{k} - \log\left(1 + \sin\frac{1}{k}\right)\right)$

3 .683871510540411993356... ≈ $Ei(2) - \log 2 - \gamma = \sum_{k=1}^{\infty} \frac{2^k}{k!k} = -\int_0^1 \frac{1-e^{2x}}{x} dx$

.683899300784123200209... ≈ $\sum_{k=1}^{\infty} \frac{\Omega(k)}{2^k}$

.68390168402925265143... ≈ $\frac{1}{32}\left(192\pi^2 G - 2\pi^4 + \psi^{(3)}\left(\frac{3}{4}\right) - \psi^{(3)}\left(\frac{1}{4}\right)\right)$

= $\int_0^1 \frac{\arcsin^4 x}{x^2} dx$

.6840280390118235871... ≈ $\frac{\pi^2}{6} - 2\log^2 2 = \sum_{k=1}^{\infty} \frac{(2k-1)!!}{(2k)!\,k^2} = \sum_{k=1}^{\infty} \frac{(2k)!}{(k!)^2 4^k k^2}$

= $\sum_{k=1}^{\infty} \frac{H_k}{(k+1)(2k+1)}$

= $\int_0^1 \frac{\log^2(1+x)}{x^2} dx$

$.\underline{684210526315789473}$ = $\frac{13}{19}$

.6842280752259962142... ≈ $\prod_{k=2}^{\infty} \frac{\zeta(k)+1}{2\zeta(k)}$

21 .68464289811076878215... ≈ $\frac{\pi^2}{3} + \frac{7\pi^4}{90} + 9\zeta(3) = \int_0^1 \frac{\log^4 x}{(1+x)^4} dx$

.6847247885631571233... ≈ $\log K_0 \log 2 = \sum_{k=1}^{\infty} \frac{\zeta(2k)-1}{k}\left(\sum_{j=1}^{2k-1}(-1)^{j+1}\frac{1}{j}\right)$,

K_0 = Khintchine's constant

= $\log^2 2 + Li_2\left(-\frac{1}{2}\right) + \frac{1}{2}\sum_{k=2}^{\infty}(-1)^k Li_2\left(\frac{4}{k^2}\right)$

= $\zeta(2) - \frac{1}{2}\log^2 2 + \sum_{k=2}^{\infty} Li_2\left(\frac{-1}{k^2-1}\right)$

= $-\sum_{k=2}^{\infty} \log\left(1-\frac{1}{k}\right)\log\left(1+\frac{1}{k}\right)$

Bailey, Borwein, Crandall, Math. Comp. 66, 217 (1997) 417-431

.68497723153155817271... ≈ $\frac{\pi}{(\pi-1)^2} = \sum_{k=1}^{\infty} \frac{k}{\pi^k} = \sum_{k=1}^{\infty} \frac{\phi(k)}{\pi^k - 1}$

1 .685290077416171813048... ≈ $\sum_{k=1}^{\infty} \frac{2^{k-1}}{k^k} = \int_0^1 x^{-2x} dx$ Prud. 2.3.18.1

1 .68534515180093406468... ≈ $-\frac{\pi}{2}\sec\frac{\pi\sqrt{5}}{2} = \prod_{k=1}^{\infty}\left(1+\frac{1}{k^2+3k+1}\right)$

2 .68545200106530644531... ≈ $\prod\left(1+\frac{1}{k(k+2)}\right)^{\log_2 k}$, Khintchine's constant

1 .685750354812596042871... ≈ $K\left(\frac{1}{4}\right)$

3 .685757453295764112754... ≈ $\frac{5}{4}+\frac{\pi^2}{8}+\zeta(3) = \sum_{k=1}^{\infty}\frac{H_k H_{k+2}}{k(k+2)}$

.686320834104453573562... ≈ $\sum_{k=2}^{\infty}\left(\zeta(k)-1\right)^{k-1}$

.68650334233862388596... ≈ $\frac{1}{6}\left(2\gamma+\left(1+i\sqrt{3}\right)\psi\left(\frac{5-i\sqrt{3}}{2}\right)+\left(1-i\sqrt{3}\right)\psi\left(\frac{5+i\sqrt{3}}{2}\right)\right)$

$= \sum_{k=1}^{\infty}\frac{1}{k^3+1} = \frac{1}{2}+\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(3k)-1\right)$

$= \sum_{k=2}^{\infty}\frac{1}{k^2+k^{-1}} = \sum_{k=1}^{\infty}\left(\zeta(3k-1)-1\right)$

.686827337720053882161... ≈ $\frac{\pi}{\sqrt{7}}\tanh\frac{\pi\sqrt{7}}{2}-\frac{1}{2} = \sum_{k=1}^{\infty}\frac{1}{k^2+k+2}$

.686889658948339591657... ≈ $\frac{4\zeta(3)}{7}$

.6869741956329142613... ≈ $\frac{3G}{4}$

1 .687023683370069268005... ≈ $2G-\frac{\pi^2}{8}+\frac{\pi\log 2}{2} = \int_{-1}^{1}\frac{\arctan^2 x}{x^2}dx$

.687223392972767270914... ≈ $\frac{7\pi}{32} = \int_0^{\infty}\frac{\cos^2 x\sin^3 x}{x^3}dx$

.687438441825156070767... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k^2-3}$

1 .687832499411264734869... ≈ $3\left(\zeta(2)-\zeta(4)\right)$

.687858934422874767598... ≈ $\frac{\pi}{8}\left(3\tanh\frac{\pi}{2}-\tanh\frac{3\pi}{2}\right) = \int_{-\infty}^{\infty}\frac{\sin^3 x}{e^x-e^{-x}}dx$

.688498165926576719332... ≈ $\frac{\sqrt{\pi(2+\sqrt{2})}}{2^{9/4}} = \int_0^{\infty}e^{-x^2}\cos(x^2)dx$

.6885375371203397155... ≈ $\prod_{k=1}^{\infty}\left(1-\frac{1}{4^k}\right)$

1 .688761186655448144358... ≈ $\left(1-2^{-3/2}\right)\zeta\left(\frac{3}{2}\right) = \sum_{k=0}^{\infty}\frac{1}{(2k+1)^{3/2}}$

.688948447698738204055... ≈ $I_2(2) = \sum_{k=0}^{\infty}\frac{1}{k!(k+2)!}$ LY 6.114

.689164726331283279349... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{k!} = \sum_{k=2}^{\infty}\left(e^{1/k^2}-1\right)$

3 .689357624731389088942... ≈ $\pi(\log 2+\operatorname{arccsch} 2) = \int_0^{\infty}\frac{\log(x^2+5)}{x^2+1}dx$

.689723263693965085215... ≈ $\frac{\pi^3}{6}+2\pi\log^2 2-2\pi\log 2-\pi = \int_0^{\infty}\frac{x^2e^{-x}}{\sqrt{e^x-1}}$

.68976567855631552097... ≈ $1+\frac{2\pi}{5}\sqrt{1+\frac{2}{\sqrt{5}}}-\log 5+\frac{1}{\sqrt{5}}\log\frac{3-\sqrt{5}}{2} = \sum_{k=2}^{\infty}\frac{(-1)^k 4^k\zeta(k)}{5^k}$

.690107091374539952004... ≈ $\operatorname{arccsc}\frac{\pi}{2}$

.69019422352157148739... ≈ $\frac{\sqrt{\pi}}{2e^{1/4}} = \int_0^{\infty}e^{-x^2}\cos x\,dx$

7 .690286020676767839767... ≈ $7\log 3$

4 .69041575982342955457... ≈ $\sqrt{22}$

.69054892277090786489... ≈ $\frac{\cosh^2 1}{2}-\frac{1}{2} = \int_0^1 \sinh x\cosh x\,dx$

.690598923241496941963... ≈ $3-\frac{4}{\sqrt{3}} = \sum_{k=0}^{\infty}\binom{2k}{k}\frac{1}{6^k(k+2)}$

.69063902436834890724... ≈ $\frac{1}{2}\log(2-2\cos 3) = -\sum_{k=1}^{\infty}\frac{\cos 3k}{k}$

.690759038686907307796... ≈ $-\frac{3}{4}\left(Li_2\left(-\frac{1}{2}\right)+Li_3\left(-\frac{1}{2}\right)\right) = -\int_0^1\frac{\log^3 x\,dx}{(x+2)^3}$

.69088664533801811203... ≈ $\frac{1}{2}(\cos 1+\sin 1) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{(2k-1)!}$

2 .69101206331032637454... ≈ $1-\frac{\pi}{\sqrt{2}}\cot\frac{\pi}{\sqrt{2}} = \sum_{k=1}^{\infty}\frac{1}{k^2-1/2} = \sum_{k=0}^{\infty}\frac{1}{k^2+2k+1/2}$

= $\sum_{k=1}^{\infty}\frac{\zeta(2k)}{2^{k-1}}$

.691229843692084262883... ≈ $\frac{\pi}{2}J_1(1) = \int_0^{\pi/2}\cos(\sin x)\cos^2 x\,dx = \int_0^{\pi/2}\cos(\sin x)\sin^2 x\,dx$

$= \int_0^{\pi/2} \sin(\sin x)\sin x\, dx$

4 .69135802469135802469... ≈ $\frac{380}{81} = \sum_{k=1}^{\infty} \frac{k^4}{4^k}$

.69136733903629335053... ≈ $\frac{1}{\sqrt{\pi}}\,\Gamma\left(\frac{3}{4}\right) = \prod_{k=0}^{\infty}\left(1+\frac{4k+1}{4k+3}\right)$

.69149167744632896047... ≈ $1-\frac{I_0(2)}{e^2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(2k)!}{(k!)^3}$

.691849430120897679... ≈ $-\frac{\sin(\pi\sqrt{2})\sinh(\pi\sqrt{2})}{6\pi^2} = \prod_{2}^{\infty}\left(1-\frac{4}{k^4}\right)$

.691893583207303682497... ≈ $4-2\log 2-4\log^2 2 = \sum_{k=1}^{\infty}\frac{kH_k}{2^k(k+2)}$

.69203384606099223896... ≈ $\prod_{k=1}^{\infty}\frac{\zeta(2k+1)}{\zeta(2k)}$

1 .692077137409748268193... ≈ $\frac{3\zeta(3)}{2}-\frac{\log^3 2}{3} = \sum_{k=1}^{\infty}\frac{H_k{}^{(2)}H}{2^k}$

.692200627555346353865... ≈ $e^{-1/e} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!e^k}$

.692307692307$\underline{692307}$ = $\frac{9}{13}$

.6926058146742493275... ≈ $\sum_{k=2}^{\infty}\frac{1}{k^2\log^2 k}$

.6929843466740731589... ≈ $\frac{1}{5}\left(4Li_2\left(\frac{1}{4}\right)-\left(2+\sqrt{5}\right)Li_2\left(\frac{-3-\sqrt{5}}{8}\right)+\left(-2+\sqrt{5}\right)Li_2\left(\frac{-3+\sqrt{5}}{8}\right)\right)$

$= \sum_{k=1}^{\infty}\frac{(-1)^{k+1}F_kF_{+3}}{4^k k^2}$

1 .6929924684136012447... ≈ $\frac{3\pi^2 G}{2}+\frac{1}{128}\left(\psi^{(3)}\left(\frac{3}{4}\right)-\psi^{(3)}\left(\frac{1}{4}\right)\right) = \int_0^{\pi/2}\frac{x^3dx}{\sin x}$

.69314718055994530941... ≈ $\log 2 = \eta(1) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}$ J71

$= \sum_{k=1}^{\infty}\frac{1}{2^k k} = Li_1\left(\frac{1}{2}\right)$ J117

$$= 2\operatorname{arcsinh}\frac{1}{2\sqrt{2}} = 2\operatorname{arctanh}\frac{1}{3} = 2\sum_{k=0}^{\infty}\frac{1}{3^{2k+1}(2k+1)}$$
K148

$$= \sum_{k=2}^{\infty}\frac{\zeta(k)}{2^k}$$

$$= \sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{k} = -\sum_{k=2}^{\infty}\log(1-k^{-2})$$
K Ex. 124(d)

$$= \sum_{k=1}^{\infty}\frac{(2k+1)\zeta(2k+1)}{2^{2k+1}} = \sum_{k=1}^{\infty}\frac{(4^k-1)\zeta(2k)}{2\cdot 4^{2k-1}k}$$

$$= \sum_{k=0}^{\infty}\frac{1}{(2k+1)(2k+2)}$$
GR 8.373.1

$$= \sum_{k=1}^{\infty}\frac{(2k-2)!}{(2k)!} = \sum_{k=1}^{\infty}\frac{(k-1)!}{(2k)!!} = \sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!!(2k)}$$
J628, K Ex. 108b

$$= \frac{1}{2}+\sum_{k=1}^{\infty}\frac{1}{8k^3-2k}$$
[Ramanjuan] Berndt Ch. 2

$$= 1+2\sum_{k=1}^{\infty}\frac{(-1)^k}{8k^3-2k}$$
[Ramanujan] Berndt Ch. 2

$$= \frac{3}{4}+\frac{3}{2}\sum_{k=1}^{\infty}\frac{(-1)^k}{27k^3-3k}$$
[Ramanujan] Berndt Ch. 2

$$= \sum_{k=1}^{\infty}\frac{H_k}{4k^2-1}$$

$$= \sum_{n=1}^{\infty}\left(-1+\sum_{k=1}^{\infty}\frac{H^{(n)}{}_k}{2^k}\right)$$

$$= -\sum_{k=1}^{\infty}\frac{\cos k\pi}{k}$$
GR 1.448.2

$$= \lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n+k}$$
[Ramanjuan] Berndt Ch. 2

$$= \sum_{r=2}^{\infty}\sum_{s=1}^{\infty}\frac{1}{(2s)^r}$$
J1123

$$= \int_2^{\infty}\frac{dx}{x^2-x} = \int_1^{\infty}\frac{dx}{x^3+x}$$

$$= \int_0^{\infty}\frac{dx}{e^x+1}$$

$$= \int_{-\infty}^{\infty}\frac{e^{-x}\,dx}{e^{e^{-x}}+1}$$

$$= \int_0^{\infty}\frac{dx}{2x^2+3x+1} = \int_0^{\infty}\frac{dx}{x^2+3x+1}$$

$= \quad -\int_0^1 \frac{\log x}{(x+1)^2}dx = \int_1^\infty \frac{\log x}{(x+1)^2}dx$ GR 4.231.6

$= \quad \int_1^\infty \frac{\log x}{(1+x^2)^2}dx$ GR 4.234.1

$= \quad -\int_0^1 \frac{\log(1-x^4)}{x^3}dx$

$= \quad \int_0^{\pi/2} \frac{x}{1+\sin x}dx = \int_0^{\pi/4} \frac{dx}{(\cos x+\sin x)\cos x}$

$= \quad \int_0^\infty \frac{\sin^4 x}{x^3}dx$

$= \quad \int_0^\infty \frac{x}{\cosh^2 x}\,dx$

$= \quad \int_0^{\pi/2} \sin(x)\log\tan x\,dx = -\int_0^{\pi/2} \cos(x)\log\tan x\,dx$ GR 4.393.1

$= \quad -\int_0^1 \log(\sin\pi\,x)\cos 2\pi\,x\,dx$ GR 4.384.3

$= \quad \int_0^\infty \frac{dx}{(1+x^2)\cosh(\pi\,x/2)}$

$= \quad \int_0^\infty \frac{\tanh(x/2)}{x\cosh x}dx$ GR 3.527.15

$= \quad -\int_0^1 li(x)\,dx$ GR 6.211

2 .69314718055994530941... $\approx \quad 2+\log 2 = \sum_{k=1}^\infty \frac{2k+1}{2^k k}$

$= \quad \int_1^\infty \frac{\log(2x)\log x}{x^2}dx$

.69343678817918319009 8... $\approx \quad J_0\left(\frac{2}{\sqrt{3}}\right) = \sum_{k=0}^\infty \frac{(-1)^k}{(k!)^2 3^k}$

14 .69353284726928223312... $\approx \quad \sum_{k=1}^\infty \frac{\sigma_3(k)}{k!}$

.69384607460674271193... $\approx \quad \gamma\,\zeta(3)$

5 .69398194001564144374... $\approx \quad \frac{\pi^2}{3}+2\zeta(3) = \sum_{k=2}^\infty k(k-1)\big(\zeta(k)-1\big) = \sum_{k=2}^\infty \frac{2k}{(k-1)^3}$

= $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}(i+j)\left(\zeta(i+j)-1\right)$

.694173022150715234759... ≈ $\frac{i}{4}\left(\psi\left(1-(-1)^{1/4}\right)+\psi\left(1+(-1)^{1/4}\right)-\psi\left(1-(-1)^{3/4}\right)-\psi\left(1+(-1)^{3/4}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{k}{k^4+1} = \frac{1}{2}+\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(4k-1)-1\right)$

.694222402351994931745... ≈ $\frac{1}{4}+\frac{3e^{1/4}\sqrt{\pi}}{4}erf\,\frac{1}{2} = \sum_{k=1}^{\infty}\frac{k!k}{(2k)!}$

.694627992246826153124... ≈ $\pi^{-1/\pi}$

1 .69476403337322634865... ≈ $2\left(\log 2-ci(2)+\gamma\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}4^k}{(2k)!k}$

.69515651214315706410... ≈ $\sum_{k=2}^{\infty}\frac{\log k}{k^2+2k}$

7 .69529898097118457326... ≈ $\pi\sqrt{6}$

9 .695359714832658028149... ≈ $\sqrt{94}$

8 .69558909395047469286... ≈ $8-6\gamma+6\log 2 = \sum_{k=1}^{\infty}\frac{\psi(k+1)k^2}{2^k}$

.69568423498636049904... ≈ $\frac{i}{2}\left(\psi\left(2-e^{i}\right)-\psi\left(2-e^{-i}\right)\right)$

$\sum_{k=1}^{\infty}\left(\zeta(k+1)-1\right)\sin k$

1 .696310705168963032458... ≈ $2\sinh^2\left(\frac{\sqrt{e}}{2}\right) = \sum_{k=1}^{\infty}\frac{e^k}{(2k)!}$

2 .696310705168963032458... ≈ $\cosh\sqrt{e} = \sum_{k=0}^{\infty}\frac{e^k}{(2k)!}$

1 .696711837754173003159... ≈ $\sum_{k=1}^{\infty}\frac{u(k)}{2^k-1}$

.69717488323506606877... ≈ $\frac{Ei(1)}{e} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}\left(\psi(k+1)+e\,k\,\gamma\right)}{k!}$

.697276598391672325965... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^k+k}$

1 .697652726313550248201... ≈ $\frac{16}{3\pi} = \binom{2}{1/2}$

.6976557223096801684... $\approx$ $2\,Li_3\left(\frac{1}{3}\right) = \int_1^\infty \frac{\log^2 x}{3x^2 - x}dx$

.69771208414402908015 7... $\approx$ $\frac{\pi}{2}\left(1 - I_0(1) + L_0(1)\right) = \int_0^1 e^{-x} \arccos x\, dx$

.69777465796400798203... $\approx$ $\frac{I_1(2)}{I_0(2)}$ Borwein-Devlin, p. 35

has continued fraction {0, 1, 2, 3, 4, 5, …}

.69778275792395754630... $\approx$ $\frac{\pi^2}{18} - \frac{1}{3}Li_2\left(-\frac{1}{2}\right) = \int_0^1 \frac{\log x}{(x+2)(x-1)}dx$

.69795791415052950187... $\approx$ $\frac{\pi}{4\sqrt{2}}\csc\frac{\pi}{\sqrt{2}} = \prod_{k=2}^\infty \left(1 - \frac{1}{2k^2-1}\right)$

.69803584421223213182 5... $\approx$

$$\frac{1}{5\left(3+\sqrt{5}\right)}\left(\log\frac{625}{2} + 3\sqrt{5}\log\left(5+\sqrt{5}\right) + \log\left(25+11\sqrt{5}\right) - 3\sqrt{5}\log 2\right)$$

$$= \sum_{k=1}^\infty \frac{F_k F_{k+2}}{4^k k}$$

.698098558623443139815... $\approx$ $\sum_{k=1}^\infty \frac{1}{k^5+1} = \frac{1}{2} + \sum_{k=1}^\infty (-1)^{k+1}\left(\zeta(5k-2) - 1\right)$

7 .69834350925012958345... $\approx$ $\frac{\sinh 4}{2\sqrt{\pi}} = \sum_{k=0}^\infty \frac{4^k}{k!(k+½)}$

.6984559986366083598... $\approx$ $\frac{\sin\sqrt{2}}{\sqrt{2}} = \sum_{k=0}^\infty \frac{(-1)^k 2^k}{(2k+1)!}$ GR 1.411.1

$$= \sum_{k=0}^\infty \frac{(-1)^{k+1} 2^k k}{(2k)!}$$

.699524263050534905034... $\approx$ $8 - \pi - 6\log 2 = \sum_{k=1}^\infty \frac{1}{2k^2 + k/2}$

.69957167383575344149 9... $\approx$ $\sum_{k=1}^\infty \frac{|\mu(k)|}{2^k + 1}$

.7000000000000000000$\underline{0}$ = $\frac{7}{10} = \prod_{p\ prime} \frac{1+p^{-2}+p^{-4}}{(1+p^{-2})^2}$

.700078628972919596975... ≈ $\frac{1}{6}\left(\psi(1-i)+\psi(1+i)-(1+(-1)^{2/3}\psi\left(-\frac{1}{2}\sqrt{4-4(-1)^{1/3}}\right)\right)$
$+\frac{1}{6}\left(\left(-1+(-1)^{1/3}\right)\psi\left(-\sqrt{1+(-1)^{2/3}}\right)-\left(1+(-1)^{21/3}\right)\psi\left(\frac{1}{2}\sqrt{4-4(-1)^{1/3}}\right)\right)$
$+\frac{1}{6}\left(\left(-1+(-1)^{1/3}\right)\psi\left(\sqrt{1+(-1)^{2/3}}\right)\right)$
= $\sum_{k=1}^{\infty} \frac{1}{k^3+k^{-3}} = \frac{1}{2}+\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(6k-3)-1\right)$

.700170370211818344953... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(2k+1)}{(k+1)^2} = \sum_{k=1}^{\infty}\left(k\,Li_2\left(\frac{1}{k^2}\right)-\frac{1}{k}\right)$

2 .700217494355001975743... ≈ $\sum_{k=2}^{\infty} k^3\left(\zeta(k)-1\right)^3$

.700367730879139217733... ≈ $\prod_{k=1}^{\infty}\left(1-\frac{1}{2^{2^k}}\right)$

.700946934031977688857... ≈ $\sum_{k=1}^{\infty} \frac{S2(2k,k)}{(2k)!k}$

1 .70143108440857635328... ≈ $\int_0^{\infty} \frac{x^2\,dx}{e^x+x}$

.7020569031595942854... ≈ $\zeta(3)-\frac{1}{2} = \sum_{k=1}^{\infty}\frac{1}{k^3}-\int_1^{\infty}\frac{dx}{x^3}$

.70240403097228566812... ≈ $\frac{1-\gamma}{2}-\left(\frac{1+i}{4}\right)\psi(1+i)-\left(\frac{1-i}{4}\right)\psi(1-i)$
= $\sum_{k=1}^{\infty}\frac{1}{k^2+k+1+k^{-1}}$

.70248147310407263932... ≈ $\frac{\pi}{2\sqrt{5}} = \int_0^{\infty}\frac{dx}{x^2+5}$

.702557458599743706303... ≈ $4-2\sqrt{e} = \sum_{k=0}^{\infty}\frac{1}{k!2^k(k+2)}$

.703156640645243187226... ≈ $\psi\left(\frac{5}{2}\right) = \frac{8}{3}-\gamma-2\log 2$

1 .70321207674618230826... ≈ $\frac{\pi}{3\sqrt{3}}+\log 3 = \sum_{k=2}^{\infty}\left(\frac{2}{3}\right)^k\zeta(k) = \sum_{k=1}^{\infty}\frac{4}{9k^2-6k}$

.703414556873647626384... ≈ $\operatorname{arctanh}\frac{1}{\sqrt{e}} = -\frac{1}{2}\log\tanh\frac{1}{4} = \sum_{k=0}^{\infty}\frac{1}{e^{(2k+1)/2}(2k+1)}$ J945

.703585635137844663429... ≈ $\frac{1}{2}\log\frac{1}{2(1-\cos(1/2))} = -\log\left(2\sin\frac{1}{4}\right)$ GR 1.441.2

= $-\frac{1}{2}\log\left((1-e^{i/2})(1-e^{-i/2})\right) = \sum_{k=1}^{\infty}\frac{\cos(k/2)}{k}$

2 .703588859282703332127... ≈ $2HypPFQ[\{1,1,1\},\{2,2,2\},2] = \sum_{k=1}^{\infty}\frac{2^k}{k!k^2}$

.703734447652443020827... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{2^{k-2}(k-1)} = -\sum_{k=2}^{\infty}\frac{2}{k}\log\left(1-\frac{1}{2^k}\right)$

.703909203233587508431... ≈ G^4

.7041699604374744600... ≈ $\int_1^{\infty}\frac{dx}{x^x}$

.7044422... ≈ $\prod_{p\ prime}\left(1-\frac{1}{p(p+1)}\right)$

48 .70454551700121861822... ≈ $\frac{\pi^4}{2}$

.70521114010536776429... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{\zeta(k)} = \sum_{k=2}^{\infty}\left(1-\frac{1}{\zeta(k)}\right) = -\sum_{k=2}^{\infty}\frac{\mu(k)}{k(k-1)}$

= $\sum_{k=1}^{\infty}\frac{\mu(k)}{k}\psi\left(2-\frac{1}{k}\right)$

= $\sum_{m=1}^{\infty}\sum_{k=2}^{\infty}\frac{1}{m^k\zeta(k)}$

1 .70521114010536776429... ≈ $-\sum_{k=2}^{\infty}\frac{\mu(k)}{k-1}$

.7052301717918009651... ≈ $\sum_{k=1}^{\infty}\frac{1}{p(k)}$, $p(k)$ = product of the first k primes

.70535942170114943096... ≈ $\int_2^{\infty}\left(\exp(\zeta(x)-1)-1\right)dx$

.70556922540701800526... ≈ $\frac{315}{13\pi\sqrt{17}}\sin\pi\sqrt{17} = \prod_{k=1}^{\infty}\left(1-\frac{1}{k(k+8)}\right)$

.70561856485887755343... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(k+1)}{2^k k} = \sum_{k=1}^{\infty}\frac{1}{k}\log\left(1+\frac{1}{2k}\right)$

.70566805723127543830... ≈ $2J_2(2) = 2\sum_{k=0}^{\infty}\frac{(-1)^k}{k!(k+2)!}$

2 .70580808427784547879... ≈ $\frac{\pi^4}{36} = \zeta^2(2) = \sum_{k=1}^{\infty}\frac{\sigma_0(k)}{k^2}$ Titchmarsh 1.2.2

= $\sum_{k=1}^{\infty}\frac{\psi^{(2)}(k)}{k^2}$

.7058823529411764 = $\frac{12}{17}$

.7060568368957990482... ≈ $\pi\left(\sqrt{\frac{3}{2}}-1\right) = \int_0^{\infty}\log\left(1+\frac{1}{2(x^2+1)}\right)dx$

1 .70611766843180047273... ≈ $\frac{137}{60}-\gamma = \psi(6)$

.70615978322619246687... ≈ $\sum_{k=2}^{\infty}2^k\left(\zeta(k)-1\right)^4$

.70673879819458419449... ≈ $-\sum_{k=2}^{\infty}\mu(k+1)\left(\zeta(k)-1\right)$

.7071067811865475244... ≈ $\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}=\cos\frac{\pi}{4}=\cos\frac{\pi}{12}-\sin\frac{\pi}{12}$

= $\sum_{k=0}^{\infty}(-1)^k\frac{1}{4^k}\binom{2k}{k} = \sum_{k=0}^{\infty}\frac{1}{8^k}\binom{2k}{k}$

= $\sum_{k=1}^{\infty}\frac{(2k-1)!!k}{(2k)!!2^k}$

= $\prod_{k=1}^{\infty}\left(1+\frac{(-1)^k}{6k-3}\right) = \prod_{k=0}^{\infty}\left(1-\frac{(-1)^k}{6k+3}\right)$ J1029

= $\prod_{k=1}^{\infty}\left(1+\frac{(-1)^k}{2k+1}\right)$

1 .707291400824076944916... ≈ $\frac{\zeta^3(3)}{\zeta(6)} = \sum_{k=1}^{\infty}\frac{\sigma_0(k^2)}{k^3} = \sum_{k=1}^{\infty}\frac{lc(k)}{k^3}$ Titchmarsh 1.2.9

.707355302630645936751... ≈ $\frac{20}{9\pi} = \sum_{k=2}^{\infty}\binom{2k}{k}^2\frac{1}{16^k(k+2)}$

.70754636565543917208... ≈ $\frac{1}{2}\left(\gamma-1+\log 2\pi\right) = \sum_{k=2}^{\infty}\frac{k}{k+1}\left(\zeta(k)-1\right)$

15 .707963267948966192313... ≈ 5π

.70796428934331696271... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(k+1)-1}{k^2} = \sum_{k=2}^{\infty}\frac{1}{k}Li_2\left(\frac{1}{k}\right)$

.70796587673887025351... ≈ $\frac{7}{2}\left(1-\sqrt{\frac{7}{11}}\right) = \sum_{k=0}^{\infty}\binom{2k}{k}\frac{(-1)^k}{7^k}$

.7080734182735711935... ≈ $\sin^2 1=\frac{1-\cos 1}{2}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}2^{2k-1}}{(2k)!} = H^{(2)}{}_{1/2}$ GR 1.412.1

.70807785056045689679... ≈ $4\log 2 + \psi\left(\frac{2+i}{4}\right) + \psi\left(\frac{2-i}{4}\right) - \psi\left(\frac{4+i}{4}\right) - \psi\left(\frac{4-i}{4}\right)$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k(k^2-1/4)}$

6 .70820393249936908923... ≈ $\sqrt{45} = 3\sqrt{5}$

.70828861415331154324... ≈ $\frac{1}{\sqrt{6}}\left(\zeta\left(\frac{1}{2},\frac{1}{6}\right)-\zeta\left(\frac{1}{2},\frac{2}{3}\right)\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{\sqrt{3k+1}}$

.70849832150917037332... ≈ $\frac{1}{2}\left((\cos\frac{1}{2} + \log(2+2\cos 1)\sin\frac{1}{2}\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\sin\frac{2k+1}{2}$

.70865675970942402651... ≈ $-\sum_{k=2}^{\infty}\mu(k)\log\zeta(k)$

.70869912400316624812... ≈ $\frac{231\pi}{1024} = \int_0^1 \frac{x^{11/2}}{\sqrt{1-x}}dx$ GR 3.226.2

.70871400293733645959... ≈ $1-\sum_{k=2}^{\infty}\frac{1}{k^k}$

.70875960579054778797... ≈ $\frac{\pi^2}{8} - 4G - 2\pi + \frac{3(\pi+8)}{2}\log 2 - \frac{9}{2}\log^2 2$

= $\sum_{k=1}^{\infty}\frac{H_k}{4k^2-k}$

4 .7093001693271033307... ≈ $\frac{e}{\gamma}$

1 .70953870969937540242... ≈ $\frac{2}{\sqrt{\pi}} + \frac{e}{\sqrt{\pi}}\Gamma\left(\frac{3}{2},0,1\right) = \sum_{k=0}^{\infty}\frac{k}{(k+½)!}$

.70978106809019450153... ≈ $\frac{\pi}{8\sqrt{2}}\left((1+i)e^{(-1-i)\sqrt{2}} + (1-i)e^{(-1+i)\sqrt{2}} + 2\right) = \int_0^{\infty}\frac{\cos^2 x}{1+x^4}dx$

1 .709975946676696989353... ≈ $5^{1/3}$

.710131866303547127055... ≈ $4 - 2\zeta(2) = \sum_{k=1}^{\infty}\frac{2}{k(k+1)^2}$

= $\int_0^1 \log(x^2)\log(1-x)dx$

.7102153895707254988... ≈ $\frac{\sinh 2\pi}{120\pi} = \prod_{k=3}^{\infty}\left(1-\frac{16}{k^4}\right)$

1 .710249833905893368351... ≈ $\sum_{k=1}^{\infty}\frac{1}{k\sqrt{k^2-1/9}}$

8 .71034436121440852200... $\approx$ $4\pi\log 2 = \int\limits_0^{\pi} \frac{x^2}{1-\cos x}\,dx$ GR 3.791.6

.71121190491339757872... $\approx$ $\sum_{k=1}^{\infty}(-1)^{k+1}\left(\frac{1}{2^{(3k^2+k)/2}}+\frac{1}{2^{(3k^2-k)/2}}\right)$ Hall Thm. 4.1.3

$= 1-\prod_{k=1}^{\infty}\left(1-\frac{1}{2^k}\right)$

.711392167549233569697... $\approx$ $1-\frac{\gamma}{2} = \sum_{k=1}^{\infty}\frac{2^k}{k+1}\left(\zeta(k+1)-1\right)$

3 .711471966428861955824... $\approx$ $1-\gamma+\frac{1}{5+3\sqrt{5}}\left(-(7+3\sqrt{5})\psi\left(\frac{3-\sqrt{5}}{2}\right)+2\psi\left(2+\frac{2}{1+\sqrt{5}}\right)\right)$

$= \sum_{k=2}^{\infty}F_{k+1}\left(\zeta(k)-1\right)$

.711566197550572432097... $\approx$ $MHS(2,1,2) = \frac{9\zeta(5)}{2}-2\zeta(2)\zeta(3)$

.711662976265709412933... $\approx$ $\frac{3}{2}-\frac{\pi}{4}\coth\pi$

.71181724366742530540... $\approx$ $\frac{4}{3}+\frac{\pi}{\sqrt{13}}\tan\frac{\pi\sqrt{13}}{2}$

1 .71206833444346651034... $\approx$ $-\sum_{k=2}^{\infty}\sigma_0(k)\mu(k)\left(\zeta(k)-1\right)$

1 .71219946584900048341... $\approx$ $\sum_{k=1}^{\infty}\left(\frac{\zeta^2(2k)}{\zeta(4k)}-1\right) = \sum_{s=1}^{\infty}\sum_{k=2}^{\infty}\frac{2^{\omega(k)}}{k^{2s}} = \sum_{k=2}^{\infty}\frac{2^{\omega(k)}}{k^2-1}$

1 .71231792754821907256... $\approx$ $1+\log 3-\log 4 = 1+\sum_{k=1}^{\infty}\frac{(-1)^k}{3^k k}$

.712319821093014476375... $\approx$ $\sum_{k=2}^{\infty}(-1)^k H_k\left(\zeta(k)-1\right) = \sum_{k=2}^{\infty}\left(\frac{1}{k}-\frac{k}{k+1}\log\frac{k+1}{k}\right)$

.71238898038468985769... $\approx$ $\frac{3\pi}{2}-4 = \sum_{k=1}^{\infty}\frac{(k-\frac{1}{2})!\,(k+\frac{1}{2})!}{(k+1)!\,(k+1)!}$

4 .71238898038468985769... $\approx$ $\frac{3\pi}{2} = \sum_{k=0}^{\infty}\frac{2^k(k+1)}{\binom{2k}{k}} = \int\limits_0^{\infty}\frac{\sin^3 2x}{x^3}$

.712414374216044353028... $\approx$ $\log_7 4$

.71250706440044539642... $\approx$ $\sum_{k=0}^{\infty}\frac{(-1)^k}{2k^3+1}$

.71268857495964775609... ≈ $\frac{\pi}{2}\coth\frac{\pi}{2}-1 = \frac{\pi}{e^{\pi}-1}+\frac{\pi}{2}-1 = \sum_{k=0}^{\infty}\frac{B_{2k}\pi^{2k}}{(2k)!}$ J948

= $\int_0^{\infty}\frac{\sin x/2}{e^x-1}dx$

1 .71268857495964775561... ≈ $\frac{\pi}{2}\coth\frac{\pi}{2} = 1+\sum_{k=1}^{\infty}\frac{2}{4k^2+1}$ J948

3 .7128321092365460381... ≈ $3\gamma\log 3+\frac{3\log^2 3}{2} = l\left(-\frac{1}{3}\right)+l\left(-\frac{2}{3}\right)$ Berndt 8.17.8

.71296548717191086579... ≈ $\cos 1\; si(1) - \sin 1\; ci(1) + \gamma \sin 1$

= $-\int_0^1 \log x\, \cos(1-x)\; dx$

1 .713009166242679... ≈ $\int_1^{\infty}\left(1-\frac{1}{\zeta^2(x)}\right)dx$

.71317412781265985501... ≈ $\frac{\Gamma(1/2)}{\Gamma(5/6)\Gamma(1/3)} = \prod_{k=1}^{\infty}\left(1+\frac{(-1)^k}{3k}\right)$ J1028

.71342386534805756839... ≈ $\frac{\pi^2}{16}+\frac{\log 2}{2}-\frac{1}{4} = \int_0^1 x\, \arctan^2 x\, dx$

1 .71359871118296149878... ≈ $\prod_{k=1}^{\infty}\zeta(3k-1)$

1 .71426411262545581 0924... ≈ $\sum_{k=2}^{\infty}\frac{1}{(k-1)(2k-3)\log k}$

$.714285714285\underline{714285}$ = $\frac{5}{7}$

1 .71490756506622932 1317... ≈ $\frac{\pi}{2G}$

.71524955143300686 8051... ≈ $\sum_{k=1}^{\infty}\frac{H_k{}^2}{2^k k^2}$

.71563660203067256 8737... ≈ $-\cos\frac{\sqrt{\pi(\pi+4)}}{2} = \prod_{k=2}^{\infty}\left(1-\frac{1}{\pi\, k(k-1)}\right)$

.71588476228661694 9300... ≈ $\frac{\pi}{2}\log\frac{1+\sqrt{3}}{\sqrt{3}} = \int_0^1\frac{\arcsin x}{x(1+2x^2)}dx$

.71616617919084682703... ≈ $\frac{3}{4}-\frac{1}{4e^2} = \int_0^1\frac{\cosh x}{e^x}dx = 1-\int_0^1\frac{\sinh x}{e^x}dx$

.71637557202648803549... ≈ $\frac{2\sqrt{2}}{\pi}\sin\frac{\pi}{\sqrt{2}} = \prod_{k=2}^{\infty}\left(1-\frac{1}{2k^2}\right)$

.71653131057378925043... ≈ $e^{-1/3} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!3^k}$

.716586630228190877618... ≈ $\log 5 - \arctan 2 - 2 = \int_0^1 \log(1+4x^2)\,dx$

.716814692820413523075... ≈ $7 - 2\pi$

1 .716814692820413523075... ≈ $8 - 2\pi = \sum_{k=1}^{\infty}\frac{(-1)^k}{(k^2-1/4)^2}$

.716863707177261351536... ≈ $\log\frac{256}{125} = \sum_{k=1}^{\infty}\frac{L_k{}^2}{4^k k}$

.716890415241513593513... ≈ $\frac{1}{2}\log\frac{e^2+1}{2} = \int_0^1 \frac{e^x}{e^x+e^{-x}}\,dx$

.717740886799541002535... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k^{k-2}}$

8 .7177978870813471045... ≈ $\sqrt{76} = 2\sqrt{19}$

.718233512793083843006... ≈ $\frac{2+\sqrt{3}}{3\sqrt{3}}$ CFG G1

.7182818284590452354... ≈ $e-2 = \sum_{k=2}^{\infty}\frac{1}{k!} = \sum_{k=0}^{\infty}\frac{1}{k!(k+3)} = \sum_{k=0}^{\infty}\frac{1}{k!(k+1)(k+2)}$

= $\sum_{k=0}^{\infty}\frac{1}{(k+1)!+2k!}$

1 .7182818284590452354... ≈ $e-1 = \sum_{k=0}^{\infty}\frac{k^2}{(k+1)!} = \sum_{k=1}^{\infty}\frac{k}{k!(k+2)} = \sum_{k=2}^{\infty}\frac{H_k}{k(k-2)!}$

2 .7182818284590452354... ≈ $e = i^{-2i/\pi} = \sum_{k=0}^{\infty}\frac{1}{k!}$

= $-\int_0^{\infty}\frac{\log x}{(x+1/e)^2}\,dx$

3 .7182818284590452354... ≈ $e+1 = \sum_{k=0}^{\infty}\frac{k^3}{(k+1)!}$

.718306293094622894468... ≈ $-\log\left(\frac{\pi}{\sqrt{2}}\operatorname{csch}\frac{\pi}{\sqrt{2}}\right) = -\log\Gamma\left(1-\frac{i}{\sqrt{2}}\right)\Gamma\left(1+\frac{i}{\sqrt{2}}\right)$

= $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(2k)}{2^k k} = \sum_{k=1}^{\infty}\log\left(1+\frac{1}{2k^2}\right)$

.71837318344184047318... ≈ $\frac{\pi^2}{12}+\frac{\log 2\log 3}{2}-\frac{\log^2 3}{4}-\frac{1}{2}Li_2\left(\frac{1}{3}\right) = -\frac{1}{2}Li_2(-2)$

= $\int_0^\infty \frac{x}{e^x+2}$

.718402016690736563302... ≈ $6(\zeta(3)-\zeta(4)) = \int_0^\infty \frac{x^3}{(e^x-1)^2}dx$

.719238483455317260038... ≈ $\sum_{k=1}^\infty \frac{\zeta(3k-1)}{3^k} = \sum_{k=1}^\infty \frac{k}{3k^3-1}$

.71967099502103768149... ≈ $\sum_{k=2}^\infty \frac{\nu(k)}{k!}$

.71994831644766095048... ≈ $\frac{11\pi}{48} = -\int_0^\infty \frac{\log x}{(x^2+1)^5}dx = \int_0^\infty \frac{\sin^5 x\cos x}{x^5}dx$

.72032975998875729633... ≈ $\frac{\pi}{4}\tanh\frac{\pi}{2} = \sum_{k=1}^\infty \frac{1}{4k^2-4k+2}$ J950, GR 1.422.2

= $\int_0^\infty \frac{\sin 2x}{\sinh 2x}dx$ GR 3.921.1

= $\int_0^\infty \frac{\sin x}{e^x-e^{-x}}$

.7203448568537890207... ≈ $\sum_{k=1}^\infty \frac{H^{(3)}{}_k}{2^k k}$

.720383533753944655666... ≈ $-\frac{1}{\pi^2}\cos\frac{\pi\sqrt{5}}{2}\cosh\frac{\pi\sqrt{3}}{2} = \prod_{k=1}^\infty\left(1-\frac{1}{k^2(k+1)^2}\right)$

2 .720416382101518554054... ≈ $5\zeta(3) - 2\zeta(2)$

2 .720699046351326775891... ≈ $\frac{\pi\sqrt{3}}{2} = \sum_{k=0}^\infty \frac{(-1)^k}{(k+1)(k+1/3)}$

.72072915625988586463... ≈ $\frac{\pi}{\sqrt{19}}\tanh\frac{\pi\sqrt{19}}{2} = \sum_{k=0}^\infty \frac{1}{k^2+k+5}$

.721225488726779794821... ≈ $\operatorname{arcsinh}\frac{\pi}{4}$

.72123414189575657124... ≈ $\frac{3\zeta(3)}{5}$

.72134752044448170368... ≈ $\frac{1}{\log 4} = \log_4 e$ J153

8 .72146291064405601592... ≈ $\prod_{k=2}^\infty 2^k\zeta(k)$

.721631677641841900027... ≈ $\frac{3}{7}+\frac{4}{7}\sqrt{\frac{3}{7}}\operatorname{arccsch}\frac{2}{\sqrt{3}} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{3^k}{\binom{2k}{k}}$

1 .72194555075093303475... ≈ $\gamma+\log\pi$

.722066688901... ≈ $\prod_{k=1}^{\infty}\left(1+\frac{(-1)^k}{k^2+k+1}\right)$

.72222222222222222222$\underline{2}$ = $\frac{13}{18} = \int_0^{\infty}\log(1+x)\frac{1+x^2}{(1+x)^4}dx$

6 .72253360550709723390... ≈ $\frac{2e^{3/2}}{2} = \sum_{k=1}^{\infty}\left(\frac{3}{2}\right)^k\frac{1}{(k-1)!}$

.72256362741482793148... ≈ $\frac{21\zeta(3)}{16}-\frac{\pi^2\log 2}{8} = \int_0^1\frac{\log(x)\log(1-x)}{1-x^2}dx$

1 .72257092668332334308... ≈ $\frac{\pi^3}{18}$

.72305625265516683539.... ≈ $\frac{9\log 3-\pi\sqrt{3}-3}{2} = \sum_{k=2}^{\infty}\frac{\zeta(k)-1}{3^{k-2}} = \sum_{k=2}^{\infty}\frac{3}{k(3k-1)}$

.72319350127750277100... ≈ $\pi-\frac{4\pi}{3\sqrt{3}} = \int_0^{\pi}\frac{\sin x}{2+\sin x}dx$

.72330131634698230862... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{\phi^2(k)}$

13 .72369128212511182729... ≈ $\frac{2e^3+1}{3} = \sum_{k=1}^{\infty}\frac{3^k k}{(k+1)!}$

1 .724009710793808932286... ≈ $\frac{\pi^2}{12}+\frac{3\zeta(3)}{4} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{k+1}{k^3}$

.72475312199827158952... ≈ $\frac{\pi(3-2\gamma)}{8} = -\int_0^{\infty}\frac{\log x\sin x}{x^3}dx$

1 .724756270009501831744... ≈ $K\left(\frac{1}{\pi}\right)$

4 .7247659703314011696... ≈ $\frac{16\pi^3}{105}$, volume of the unit sphere in R^7

.724778459007076331818... ≈ $\sqrt{\frac{\pi}{2e}}\,erfi\frac{1}{\sqrt{2}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k+1)!!} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{2^k}{k!\binom{2k}{k}}$

.72482218262595674557... ≈ $\frac{1}{2}-\frac{2}{\pi\sqrt{3}}\cot\frac{\pi}{\sqrt{3}} = \sum_{k=1}^{\infty}\frac{\zeta(2k)}{3^k} = \sum_{k=1}^{\infty}\frac{1}{3k^2-1}$

1 .72500956916792667392... ≈ $\frac{\pi^2+\log^2 2}{6} = \int_0^\infty \frac{\log x}{(x+2)(x-1)}\,dx$ GR 4.237.3

.72520648300641149763... ≈ $\frac{\pi}{40}+\frac{97}{150}$, probability that three points in the unit square form an obtuse triangle

.725613396022265329004... ≈ $\frac{e-e\log(e-1)}{e-1} = \sum_{k=1}^\infty \frac{H_k}{e^k}$

1 .72569614761160133072... ≈ $\frac{\pi\log 3}{2} = \int_0^{\pi/2} \log(3\tan x)\,dx$ GR 4.217.3

.72589193317292292137... ≈ $\frac{\pi}{2}\tanh\frac{1}{2} = \int_0^\infty \sin\frac{2x}{\pi}\cdot\frac{dx}{\sinh x}$

.726032415032057488141... ≈ $\frac{\pi}{8}+\frac{1}{3}$

.7264245534... ≈ $\prod_{p_k\ prime}\left(1-\frac{1}{p_k p_{k+1}}\right)$

.72649594689438055318... ≈ $\log^2 3-\log^2 2$

.726760455264837313849... ≈ $2-\frac{4}{\pi} = \int_0^\infty \log(1+x^2)\frac{\sinh\frac{\pi x}{2}}{\cosh^2\frac{\pi x}{2}}\,dx$ GR 4.373.5

.7268191868... ≈ $\prod_{p\ prime}\left(1-\frac{1}{p^2(p-1)^2}\right)$

18 .726879886895150767705... ≈ $7\zeta(2)+6\zeta(3) = \sum_{k=2}^\infty k^3\big(\zeta(k)-\zeta(k+1)\big)$

= $\sum_{k=2}^\infty \frac{8k^3-5k^2+4k-1}{k^2(k-1)^3}$

.72714605086327924743... ≈ $\frac{i}{2}\left(Li_2\left(e^{-2i}\right)-Li_2\left(e^{2i}\right)\right) = \sum_{k=1}^\infty \frac{\sin 2k}{k^2}$

2 .727257300559364627988... ≈ $\zeta(2)+\zeta(4)$

$.727272727272727272\underline{72}$ = $\frac{8}{11}$

.72732435670642042385... ≈ $\frac{1}{2}+\frac{\sin 2}{4} = \int_0^{\pi/2} \cos^2(\cos x)\sin x\,dx$

.727377349295216469724... ≈ $e^{-1/\pi} = \sum_{k=0}^\infty \frac{(-1)^k}{k!\pi^k}$

.727586307716333389514... ≈ $Li_2\left(\frac{3}{5}\right)$

.72760303948609931052... $\approx \frac{\pi^2}{2} - \frac{7}{2}\zeta(3) = \sum_{k=1}^{\infty} \frac{k}{(k+1/2)^3}$

1 .72763245694004473929... $\approx \prod_{k=1}^{\infty} \left(1 + \frac{1}{k!2^k}\right)$

.72784877051247490076... $\approx \prod_{k=2}^{\infty} \left(1 - \frac{1}{k!k!}\right)$

.72797094501786683705... $\approx 1 - \frac{\sinh \pi}{\pi}$

6 .72801174749956538037... $\approx \pi^2 - \pi$

.728102913225581885497... $\approx \frac{\pi}{4\sqrt{3}} + \frac{\log 3}{4} = \int_1^{\infty} \frac{x^2}{x^4 + x^2 + 1} dx$

.728183137492543041149... $\approx \gamma^{\gamma}$

.72856981255742977004... $\approx \log 2 - \frac{\pi^2}{12} + 2 Li_3\left(-\frac{1}{2}\right) + \frac{3}{2}\zeta(3)$

$= \int_0^1 \frac{\log^2 x}{(x+1)^3 (x+2)} dx$

1 .728647238998183618135... $\approx$ *root of* $\zeta(x) = 2$

22 .72878790793390202184... $\approx \frac{7\pi^4}{30} = \int_1^{\infty} \frac{\log^4 x}{(x+1)^2} dx$

.7289104820416069624... $\approx \frac{21\pi}{64\sqrt{2}} = \int_0^{\infty} \frac{dx}{(x^4+1)^3}$

.72898504860058165211... $\approx 1 - \frac{\pi}{\cosh \pi}$

.72908982783518197945... $\approx \frac{45\zeta(5)}{64} = \int_1^{\infty} \frac{\log^4 x}{x^3 + x} dx$

.729329433526774616212... $\approx 1 - 2e^2$

.729439101865540537222... $\approx \frac{4}{3} + \frac{\log 2}{3}\left(2\log 2 - 4\right) = \sum_{k=1}^{\infty} \frac{H_k}{k(2k+3)}$

807 .729855042097228216666... $\approx \frac{1}{2}\left(\cosh(\cosh 2 + \sinh 2) + \sinh(\cosh 2 + \sinh 2) - e\right)$

$= \frac{1}{2}\left(e^{e^2} - e\right) = \sum_{k=0}^{\infty} \frac{e^k \sinh k}{k!}$

1 .7299512698867919550... $\approx \frac{\pi^2 + 1}{2\pi} = \cosh(\log \pi) = \cos(i \log \pi)$

.73018105837655977384... $\approx \frac{\pi \log 2}{8} + \frac{G}{2} = \sum_{k=0}^{\infty} \binom{2k}{k} \left(\frac{\sqrt{2}}{2} \right)^{2k+1} \frac{1}{2^{2k}(2k+1)^2}$ Berndt 9.32.4

$= \int_0^{\pi/4} x \tan x \, dx = \int_0^{\pi/4} x \cot x \, dx$ Borwein-Devlin, p. 138

$= \int_0^{\pi/4} \frac{\pi/4 - x \tan x}{\cos 2x} \, dx$ GR 3.797.3

$= - \int_0^{\pi/4} \log(\cos x - \sin x) \, dx$ GR 4.225.1

$= - \int_0^{\pi/4} \log(\sqrt{\tan x} + \sqrt{\cot x}) \, dx$ GR 4.228.7

$= \int_0^1 \frac{\arctan x}{x(1+x^2)} \, dx$ GR 4.531.7

1 .730234433703700193420... $\approx \frac{e^{1/4}\sqrt{\pi}}{2} \left(1 + erf \, \frac{1}{2} \right) = \int_0^{\infty} \frac{e^x}{e^{x^2}} dx$

.730499243103159179079... $\approx \binom{1/3}{2/3}$

.731058578630004879251... $\approx \frac{e}{e+1} = \frac{1+\tanh 1/2}{2} = \sum_{k=0}^{\infty} (-1)^k e^{-k}$ J944

$= \sum_{k=2}^{\infty} \left(1 - 2^{k+1}\right) \frac{\zeta(-k)}{k!}$

.73108180748810063843... $\approx \frac{2\pi^2}{27} = -\int_0^{\infty} \frac{\log x}{x^3+1} dx = \int_0^{\infty} \frac{x \log x}{x^3+1} dx = -\int_0^1 \frac{1+x}{1-x^3} \log x \, dx$

$= \int_0^{\infty} \frac{x(1-e^{-x})}{e^x(1+e^{-3x})} dx$ GR 3.411.26

1 .73137330972753180577... $\approx \prod_{k=2}^{\infty} \frac{1}{1-2^{-k}}$

1 .73164699470459858182... $\approx 3\gamma$

.731771876673200892827... $\approx \sqrt{e} \left(\gamma - Ei\left(-\frac{1}{2} \right) + \log 2 \right) = \sum_{k=1}^{\infty} \frac{H_k}{k! 2^k}$

.731796870029137225611... $\approx \sum_{k=1}^{\infty} 3k \left(\zeta(3k) - 1 \right) = \sum_{k=2}^{\infty} \frac{3}{k^3 (1-k^{-3})^2}$

2 .731901246276940125002... $\approx \sum_{k=7}^{\infty} \frac{\zeta(k)}{(k-7)!} = \sum_{k=1}^{\infty} \frac{e^{1/k}}{k^7}$

.732050807568877293527... ≈ $\sqrt{3}-1 = \sum_{k=0}^{\infty}\frac{(-1)^k}{2^k(k+1)}\binom{2k}{k}$

1 .732050807568877293527... ≈ $\sqrt{3} = \tan\frac{\pi}{3} = \sum_{k=0}^{\infty}\frac{1}{6^k}\binom{2k}{k} = \sum_{k=0}^{\infty}\frac{k}{6^k}\binom{2k}{k}$

3 .732050807568877293527... ≈ $2+\sqrt{3} = \tan\frac{5\pi}{12}$ AS 4.3.36

21 .732261539348840427114... ≈ $\frac{2\pi^2}{3}+\frac{7\pi^4}{45} = \int_0^{\infty}\frac{\log^4 x}{(1+x)^4}dx$

.732454714600633473583... ≈ $\frac{1-\gamma}{\gamma}$

1 .732454714600633473583... ≈ $\frac{1}{\gamma}$

1 .732560378041069813992... ≈ $\frac{3\pi\log 2}{8}+G = \frac{1}{2}\left(\pi\log 2 + iLi_2\left(\frac{1-i}{2}\right) - i\frac{Li_2(1+i)}{2}\right)$

= $\int_1^{\infty}\log\left(\frac{x^2+1}{x-1}\right)\frac{dx}{x^2+1}$ GR 4.298.14

.732869201123059587436... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)}{k^2+1}$

.7333333333333333333$\underline{3}$ = $\frac{11}{15}$

= $\sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!\,(k+3)} = \sum_{k=1}^{\infty}\binom{2k}{k}\frac{1}{4^k(k+3)}$

.733358251205666769682... ≈ $\sum_{k=1}^{\infty}k\left(\zeta(3k-1)-1\right) = \sum_{k=2}^{\infty}\frac{1}{k^2(1-k^{-3})^2}$

3 .73345333387461085916... ≈ $\pi\csc 1 = \beta\left(1-\frac{1}{\pi},\frac{1}{\pi}\right)$

.73402021044145968963... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^k\sigma_0(k)}$

.7340226499671578742... ≈ $16-16\log 2-\frac{2\pi^2}{3}+2\zeta(3) = \sum_{k=1}^{\infty}\frac{(k-\frac{1}{2})!}{k^3(k+\frac{1}{2})!}$

= $\sum_{k=1}^{\infty}\frac{2}{2k^4+k^3} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{\zeta(k+3)}{2^{k-1}}$

.73417442372548447512... ≈ $\left(\sqrt{2}-1\right)\sqrt{\pi} = \sum_{k=1}^{\infty}\frac{(k-\frac{1}{2})!}{k!\,2^k}$

.7343469699579425786... ≈ $\frac{e\cos 1}{2} = \int_1^{e}\log x\,\cos\log x\,dx$

.73462758513149342362... ≈ $\frac{2}{\pi^2}\sin\frac{\pi}{\sqrt{2}}\sinh\frac{\pi}{\sqrt{2}} = \prod_{k=1}^{\infty}\left(1-\frac{1}{4k^4}\right)$

1 .734956638731129275406... ≈ $\frac{\sqrt{2}}{3}\sinh\pi\sqrt{2}\ \csc h\ \pi = \prod_{k=1}^{\infty}\left(1+\frac{1}{k^2+2k+2}\right)$

.735105193895722732682... ≈ $\frac{4}{\pi\sqrt{3}}$ CFG G1

1 .735143209673105397603... ≈ $3000-1103e = \sum_{k=1}^{\infty}\frac{k^3}{k!(k+5)}$

2 .73526160397004470158... ≈ $\sum_{k=1}^{\infty}\frac{\Phi(k)}{2^k}$

.735689548825915723523... ≈ $\frac{\pi^2}{2}\left(1-\operatorname{csch}1\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2+1/\pi^2}$

.73575888234288464319... ≈ $\frac{2}{e} = \Gamma(2,1) = \sum_{k=1}^{\infty}(-1)^k\frac{k^3+k^2}{(k-1)!} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^5}{k!}$

= $\int_0^{\infty} x e^{-\sqrt{1+x^2}}\,dx$

3 .7360043360892608938... ≈ $\pi 2^{1/4} = \int_0^{\infty}\log\left(1+\frac{1}{2x^4}\right)dx$

1 .736214725290954627894... ≈ $-\psi\left(\frac{1}{2}+\frac{i}{2}\right) - \psi\left(\frac{1}{2}-\frac{i}{2}\right)$

1 .73623672732835559651... ≈ $\sum_{k=1}^{\infty}H_k{}^2\left(\zeta(k+1)-1\right)$

.73639985871871507791... ≈ $\frac{2\pi\sqrt{3}}{27}+\frac{1}{3} = \sum_{k=1}^{\infty}\frac{(2k)!!}{(2k-1)!!4^k} = \sum_{k=1}^{\infty}\frac{1}{\binom{2k}{k}}$ CFG F17

= $\int_0^{\pi/2}\frac{\cos x}{(2-\cos x)^2}dx$

.73655009813423404267... ≈ $\sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)\zeta(k+1)-1}{k}$

.736576852723235053198... ≈ $\int_1^{\infty}\frac{\Gamma(x)}{x^x}dx$

.736651970465936181741... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{(k-1)!} = \sum_{k=2}^{\infty}\frac{e^{1/k^2}}{k^2}$

$.\underline{736842105263157894}$ = $\frac{14}{19}$

1 .736901061414085912424... ≈ $\zeta^3(3)$

.73710586317587034347... ≈ $\psi^{(2)}(i) + \psi^{(2)}(-i)$

.73726611959445520569... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{2k-3} = \sum_{k=2}^{\infty} k^{-3/2}\ \operatorname{arctanh}\sqrt{\frac{1}{k}}$

.73735096638620963334... ≈ $\sum_{k=1}^{\infty}\frac{1}{F_k k\,(k+1)}$

3 .73795615464616759679... ≈ $4\sqrt{2} - \frac{16}{\pi} + \frac{8\sqrt{2}}{\pi}\log\left(\sqrt{2}+1\right) = \int_0^{\infty}\log(1+x^2)\frac{\cosh \pi x/4}{\sinh^2 \pi x/4}$ GR 4.373.6

1 .7380168094946939249... ≈ $\sum_{k=1}^{\infty}\frac{k^2}{3^k-1} = \sum_{k=1}^{\infty}\frac{\sigma_2(k)}{3^k}$

.73806064483085791066... ≈ $Li_3\left(\frac{2}{3}\right)$

4 .73863870268621999272... ≈ $\sum_{k=1}^{\infty}\frac{p_k}{k!}$

.73873854352439301664... ≈ $1 + \frac{5\pi}{4\sqrt{3}} - \frac{5\log 3}{4} - \frac{5\log 2}{3} = \sum_{k=2}^{\infty}\frac{(-1)^k 5^k \zeta(k)}{6^k}$

.73879844144149771531... ≈ $\frac{1}{3} + \log\frac{3}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(k+1)}{2^k k}$

.7388167388167$\underline{388167}$ = $\frac{512}{693} = \beta\left(6,\frac{1}{2}\right)$

.739085133215160641655... ≈ $root\,of\ \arccos x = x$

.7391337000907798849... ≈ $\frac{\pi^2-1}{12} = \frac{i}{2}\left(Li_3(-e^i) + Li_3(-e^{-i})\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}\sin k}{k^3}$ Davis 3.34

19 .739208802178717237669... ≈ $2\pi^2$

.739947943495465512256... ≈ $\sum_{k=1}^{\infty}\frac{1}{k^k+1} = \sum_{k=2}^{\infty}\sum_{j=1}^{\infty}\frac{(-1)^{j+1}}{k^{jk}}$

.740267076581850782580... ≈ $\frac{\pi\sqrt{3}}{6}\coth \pi\sqrt{3} - \frac{1}{6} = \sum_{k=1}^{\infty}\frac{1}{k^2+3}$ J124

7 .740444313946792661639... ≈ $2I_0(2) + 2I_1(2) = I_0(2) + 3I_1(2) + I_2(2) = \sum_{k=1}^{\infty}\frac{k^4}{k!k!}$

.740480489693061041169... ≈ $\frac{\pi}{3\sqrt{2}}$ CFG D10

.740520017895247015105... ≈ $\frac{e}{e^2-e-1} = \sum_{k=1}^{\infty}\frac{F_k}{e^k}$

.740579177528952263276... ≈ $\frac{7\zeta(3)}{4}+\log 2-\frac{5\pi^2}{24} = \sum_{k=3}^{\infty}(-1)^{k+1}\frac{k^2\zeta(k)}{2^k}$

= $\sum_{k=1}^{\infty}\frac{18k^2+11k+2}{2k^2(2k+1)^3}$

.740726784269006256431... ≈ $\sum_{K=2}^{\infty}\frac{1}{S1(k,2)}$

.740740740740740740$\underline{740}$ = $\frac{20}{27} = \sum_{k=1}^{\infty}\frac{k^2}{4^k}$

1 .740802061377891359869... ≈ $\sqrt{3}\log\left(1+\sqrt{3}\right)$

.7410187508850556118... ≈ $\frac{3\log 3}{2}-\frac{\pi\sqrt{3}}{6} = \sum_{k=1}^{\infty}\frac{1}{3k^2-k} = \sum_{k=1}^{\infty}\frac{\zeta(k+1)}{3^k} = -hg\left(-\frac{1}{3}\right)$

= $-\int_0^1\frac{\log(1-x^3)}{x^2}dx$

.741027921523577355842... ≈ $\frac{2}{\sqrt{5e}}\sinh\frac{\sqrt{5}}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}F_k}{k!} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{k!}$

.741089701006868123721... ≈ $\int_0^1\frac{\sin x}{e^x-1}dx$

.741227065224583887196... ≈ $2+\frac{\pi^2}{3}-\pi\sqrt{2}\coth\frac{\pi}{\sqrt{2}} = \sum_{k=1}^{\infty}\frac{1}{k^2(k^2+1/2)}$

1 .741433975699931636772... ≈ $\frac{7}{8}+\frac{5\log 2}{4} = \sum_{k=2}^{\infty}\frac{k^3}{2^k(k^2-1)}$

3 .741657386773941385584... ≈ $\sqrt{14}$

.741982753502604212262... ≈ $\sum_{k=2}^{\infty}(-1)^k\left(e^{\zeta(k)-1}-1\right)$

11 .743029813183056451016... ≈ $\sum_{k=2}^{\infty}k^2\left(\zeta(k)+\zeta(k+1)-2\right)$

.7431381432026369649... ≈ $2G-\frac{\pi\log 2}{2} = \int_0^1\frac{\arcsin x}{x(1+x)}dx$

= $\int_0^1\frac{\arccos x}{1+x}dx$ GR 4.521.2

= $\int_0^{\pi/2} \frac{x\sin x}{1+\cos x}dx$ GR 3.791.12

= $\int_0^1 \frac{\log(1+x)}{\sqrt{1-x^2}}dx$ GR 4.292.1

.74314714161123874... ≈ $\prod_{k=0}^{\infty} \frac{pf(k)}{e}$

.74343322147602010957... ≈ $\sum_{k=2}^{\infty} \frac{1}{(k-1)(k+1)\log k}$

7 .743684425536057169182... ≈ $\frac{\pi^4}{90}\left(11-6\gamma\right)+6\zeta'(4) = \int_0^{\infty} \frac{x^3\log x}{e^x-1}\,dx$

.74391718786976797494... ≈ $\zeta(2)\sum_{k=1}^{\infty}\frac{\mu(k)}{k}\log\zeta(2k) = \sum_{k=1}^{\infty}\frac{\nu(k)}{k^2}$ Titchmarsh 1.6.2

.744033888759488360480... ≈ $\sum_{k=1}^{\infty}\frac{k}{2^k(2^k-1)}$

2 .744033888759488360480... ≈ $\sum_{k=1}^{\infty}\frac{k}{2^k-1} = \sum_{k=1}^{\infty}\frac{1}{2^k-2+2^{-k}} = \sum_{k=1}^{\infty}\frac{\sigma_1(k)}{2^k}$

= $\sum_{k=1}^{\infty}\frac{k}{2^k(2^k-1)}$

.744074106070984777872... ≈ $\zeta(3)-\frac{G}{2}$

.744150640327195684211... ≈ $\frac{3\pi^3}{125} = \sum_{k=1}^{\infty}\frac{\sin k\pi/5}{k^3}$ GR 1.443.5

.74430307976049287481... ≈ $\int_0^{\pi/4}\sqrt{\cos x}\,dx$

.74432715277120323111... ≈ $\frac{2}{5}+\frac{8}{5\sqrt{5}}\operatorname{arcsinh}\frac{1}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{\binom{2k-1}{k}}$

2 .744396466297114626366... ≈ $\frac{\pi}{\log\pi}$

5 .74456264653802865985... ≈ $\sqrt{33}$

1 .74471604990971988354... = $\frac{\pi^2}{4\sqrt{2}} = \int_0^{\infty}\frac{\log x\,dx}{2x^2-1}$

1 .74491107293333335623... ≈ $\frac{1}{4}(3\cosh 1+2\sinh 1) = \frac{5e}{8}+\frac{1}{8e}$

= $\sum_{k=1}^{\infty}\frac{k^2}{(2k-1)!}$

.74521882569020979229... ≈ $\sum_{k=2}^{\infty}|\mu(k)|\log\zeta(k)$

.7453559924999298988... ≈ $\frac{\sqrt{5}}{3} = \sum_{k=0}^{\infty}\frac{(-1)^k}{5^k}\binom{2k}{k}$

.74562414166555788889... ≈ $\sin\sin 1$

1 .745804395794629191549... ≈ $16(G+\log 2)-24 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k(k^2-1/4)^2}$

.74590348439289584727... ≈ $\sum_{k=1}^{\infty}|\mu(k)|\left(\zeta(2k)-1\right)$

7 .7459666924148337704... ≈ $\sqrt{60} = 2\sqrt{15}$

21 .74625462767236188288... ≈ $8e$

2 .746393504673284119151... ≈ $\sum_{k=6}^{\infty}\frac{\zeta(k)}{(k-6)!} = \sum_{k=1}^{\infty}\frac{e^{-1/k}}{k^6}$

9 .7467943448089639068... ≈ $\sqrt{95}$

.7468241328114270254... ≈ $\frac{\sqrt{\pi}}{2}\, erf\, 1 = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!(2k+1)}$ J973

= $\int_0^1 e^{-x^2}dx$

5 .74698546753472378545... ≈ $\frac{2\pi^2}{3}+2\log^2 2+4Li_2\left(-\frac{1}{2}\right) = \int_0^1\frac{\log^2 x}{(x+½)^2}dx$

.747119662029117550247... ≈ $-\sum_{k=2}^{\infty}\frac{\mu(k)\left(\zeta(k)-1\right)}{\phi(k)}$

2 .747238274932304333057... ≈ $\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)$

= $\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5-\ldots}}}}}}}$ [Ramanujan] Berndt Ch. 22

17 .7476761359958500014... ≈ $\frac{59535\zeta(7)-62\pi^6}{24} = -\int_0^1\frac{x\log^7 x}{(x+1)^3}$

.747727141203212136438... ≈ $\sum_{k=2}^{\infty}\left(\frac{\zeta(k)\zeta(k+2)}{\zeta(k+1)}-1\right)$

2 .747896782531657045164... ≈ $3G$

9 .748110948000252951799... ≈ $e^2\left(\gamma + \log 2 - Ei(-2)\right) = \sum_{k=1}^{\infty} \frac{2^k H_k}{k!}$

1 .748493952693942358428... ≈ $\frac{11\zeta(5)}{2} - \frac{\pi^2 \zeta(3)}{3} = \sum_{k=1}^{\infty} \frac{H^{(3)}{}_k}{k^2}$

.748540032992591012012... ≈ $\gamma - 2 + \frac{\pi}{2}\coth\pi + \frac{1}{2}\left(\psi(2-i) + \psi(2+i)\right)$

= $\gamma + \left(\frac{1+i}{2}\right)\psi(1-i) - i\,\psi(1+i)$

= $\sum_{k=2}^{\infty} \frac{k+1}{k^3+k} = \sum_{k=1}^{\infty} (-1)^{k+1}\left(\zeta(2k) + \zeta(2k+1) - 2\right)$

.74867010071672042744... ≈ $\pi + \frac{\pi^2}{2} - 8G = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(3^k - 1)(k+1)}{4^k}$

2 .748893571891069083655... ≈ $\frac{7\pi}{8}$

.748989900377804858183... ≈ $\zeta(4) - \frac{1}{3} = \sum_{k=1}^{\infty} \frac{1}{k^4} - \int_1^{\infty} \frac{dx}{x^4}$

.74912714295902753882... ≈ $\frac{3-\sqrt{3}}{3}\sqrt{\pi} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(k-\frac{1}{2})!\,2^k}{k!}$

.749306001288449023606... ≈ $e^{-\gamma/2}$

.749502656901677316351... ≈ $\sum_{k=1}^{\infty} \frac{1}{2^k \zeta(k+1)}$

.7500000000000000000$\underline{0}$ = $\frac{3}{4} = -i\sin(i\log 2) = \sum_{k=1}^{\infty}\frac{k}{3^k}$

= $\sum_{k=1}^{\infty}\left(\zeta(2k)-1\right) = \sum_{k=2}^{\infty}\frac{1}{k^2-1}$ J369, J605

= $\sum_{k=1}^{\infty}\frac{1}{k^2+2k}$ K133

= $\sum_{k=3}^{\infty}\frac{1}{k^2-2k} = \sum_{k=1}^{\infty}k\left(\zeta(2k)-\zeta(2k+2)\right)$

= $\prod_{k=1}^{\infty}\left(1-\frac{1}{(k+3)^2}\right)$

= $\int_1^{\infty}\frac{\log^4 x dx}{x^3}$

1 .7500000000000000000$\underline{0}$ = $\frac{7}{4}$

2 .7500000000000000000$\underline{0}$ = $\frac{11}{4} = \prod_{p\ prime}\frac{1+p^{-2}+p^{-4}+p^{-6}+p^{-8}}{1-p^{-2}-p^{-6}+p^{-8}}$

372 .7500000000000000000$\underline{0}$ = $\sum_{k=1}^{\infty}\frac{k^6}{3^k}$

.75000426807018511438... ≈ $\frac{4}{\sqrt{3}}\arctan\left(\sqrt{3}\tan\frac{1}{2}\right)-1 = \int_0^1\frac{\cos\theta}{2-\cos\theta}d\theta$

1 .750021380536541733573... ≈ $\gamma+\psi(1+e) = hg(e) = \sum_{k=1}^{\infty}\frac{e}{k(k+e)}$

1 .75013834593848950211... ≈ $\frac{2}{\sin 2} = \prod_{k=0}^{\infty}2^k\tan\frac{1}{2^k}$

.7507511041240729095... ≈ $\zeta^{(iv)}(3) = \sum_{k=2}^{\infty}\frac{\log^4 k}{k^3}$

.751125544464942482859... ≈ $\pi^{-1/4}$

.75128556447474642837... ≈ $\frac{5\zeta(3)}{8} = \sum_{k=1}^{\infty}\frac{H^{(2)}{}_k}{2^k k} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{H_k}{k^2}$

= $-\int_0^1\frac{\log(1-x)\log(1+x)}{x}dx$

.751300296089510340494... ≈ $\frac{\pi}{\sqrt{3}}\left(\sqrt{2}-1\right) = \int_0^{\infty}\log\left(1+\frac{1}{3x^2+1}\right)dx$

7 .75156917007495504387... ≈ $\frac{\pi^3}{4} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k-1/2)^3} = \int_0^{\pi}\log^2\left(\tan\frac{t}{4}\right)dt$

Borwein & Borwein, Proc. AMS 123, 4 (1995) 1191-1198

.75159571455123294364... ≈ $\gamma - \frac{1}{2} + \log 2 - \frac{1}{2e^2} - Ei(-2) = \sum_{k=1}^{\infty} (-1)^k \frac{2^k}{k!k(k+1)}$

.75173418271380822855... ≈ $\frac{1}{2}\left(I_0\left(2(-1)^{1/4}\right) + J_0\left(2(-1)^{1/4}\right)\right) = \frac{1}{2}\left({}_0F_1(1,i) + {}_0F_1(1,-i)\right) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!(2k)!}$

1 .751861371518082937122... ≈ $\sum_{k=0}^{\infty} \frac{\zeta(k+4)}{k!2^k} = \sum_{k=1}^{\infty} \frac{e^{1/2k}}{k^4}$

1 .751938393884108661204... ≈ $\frac{2}{\pi-2} = \sum_{k=1}^{\infty} \left(\frac{2}{\pi}\right)^k$

1 .75217601958756461842... ≈ $\frac{1}{36}\left(\frac{4\sqrt{3}\pi^3}{27} - \psi^{(2)}\left(\frac{1}{3}\right)\right) = \frac{1}{108}\left(\zeta\left(3,\frac{1}{6}\right) - \zeta\left(3,\frac{1}{3}\right)\right)$

= $\frac{\zeta(3)}{2} + \frac{1}{3}\left(Li_3\left(\frac{1+i\sqrt{3}}{2}\right) + Li_3\left(\frac{1-i\sqrt{3}}{2}\right)\right)$

$+ \frac{i\sqrt{3}}{3}\left(Li_3\left(\frac{-1-i\sqrt{3}}{2}\right) + Li_3\left(\frac{-1+i\sqrt{3}}{2}\right)\right)$

= $\int_0^1 \frac{(1-x)\log^2 x}{1-x^6}$ GR 4.261.8

.752252778063675049264... ≈ $\frac{4}{3\sqrt{\pi}}$

.75267323992255463004... ≈ $\frac{23\pi}{96} = -\int_0^{\infty} \frac{\log x}{(x^2+1)^4} dx = \int_0^{\infty} \frac{\cos x \sin^4 x}{x^4} dx$

.753030298866585425452... ≈ $4 - 3\zeta(4)$

11 .753304951941822444427... ≈ $e^2 I_1(2) = \sum_{k=0}^{\infty} \binom{2k+2}{k} \frac{1}{(k+1)!} = \sum_{k=0}^{\infty} \binom{2k}{k} \frac{k}{(k+1)!}$

1 .75331496320289736814... ≈ $\prod_{k=1}^{\infty} \left(1 + \frac{1}{2k^2+k}\right)$

.75338765437709039572... ≈ $\sqrt{e}\, I_0\left(\frac{1}{2}\right) - 1 = \sum_{k=1}^{\infty} \frac{(2k-1)!!}{(2k)!\,k!}$

1. 75338765437709039572... ≈ $\sqrt{e}\, I_0\left(\frac{1}{2}\right) = \int_0^1 e^{\cos^2 \pi x} dx$

.75375485838430604734... ≈ $\frac{\pi}{\sqrt{2}} \coth \pi\sqrt{2} - \gamma - \frac{1}{2}\left(1 + \psi\left(1+i\sqrt{2}\right) + \psi\left(1-i\sqrt{2}\right)\right)$

= $\sum_{k=1}^{\infty} \frac{2(k-1)}{k^3+2k} = \sum_{k=1}^{\infty} (-1)^{k+1} 2^k \left(\zeta(2k) - \zeta(2k+1)\right)$

. 75405876648941912567... ≈ $\sum_{k=2}^{\infty}(-1)^k \frac{\zeta(k+2)}{\zeta(k)}$

9 .75426251387257056568... ≈ $\frac{15}{2}\zeta(5) + \zeta(2)\zeta(3) = \sum_{k=1}^{\infty}\frac{H_k^{\ 3}}{(k+1)^2}$ (conj.) 18 MI 4, p. 15

.754610025770972168663... ≈ $\frac{\pi}{2}\left(H_0(1) - Y_0(1)\right) = \int_0^{\infty}\frac{dx}{e^x\sqrt{1+x^2}} = \int_0^{\infty} e^{-x}\,\text{arcsinh x}\; dx$

.754637885420670280961... ≈ $1 - \frac{e+1}{e^e} = \int_0^e xe^{-x}\,dx$

.75464783578494730955... ≈ $\frac{\pi}{6} + \frac{\log 2}{3} = \int_0^1 x^2 \log\left(1+\frac{1}{x^6}\right)dx$

4 .75488750216346854436... ≈ $3\log_2 3$

1 .75529829246990261963... ≈ $\pi - 2\log 2 = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+1)(k+½)}$

.75539561953174146939... ≈ $\frac{\pi^2}{10} - \log^2\left(\frac{\sqrt{5}-1}{2}\right) = Li_2\left(\frac{\sqrt{5}-1}{2}\right)$ Berndt Ch. 9

= $\frac{\pi^2}{30} + Li_2\left(\frac{3-\sqrt{5}}{2}\right)$ MIS

1 .75571200920378862263... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(5k)}{k!} = \sum_{k=1}^{\infty}\left(e^{1/k^5} - 1\right)$

3 .75642633323976072885... ≈ $\frac{\pi}{2\sqrt{2}}\csc\frac{\pi}{\sqrt{2}} - \frac{\pi^2}{4}\cot\frac{\pi}{\sqrt{2}}\csc\frac{\pi}{\sqrt{2}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{(k^2-1/2)^2}$

6 .756774401573431365862... ≈ $\frac{\sinh^2\pi}{2\pi^2} = \prod_{k=1}^{\infty}\left(1+\frac{4}{k^4}\right)$

.756943617774572695997... ≈ $\sum_{k=0}^{\infty}\frac{B_k}{(2k)!}$

.757342086122175953454... ≈ $-2Ei(-\log 2) = \int_0^{\infty}\frac{dx}{2^x(x+1)}$

.75735931288071485359... ≈ $5 - 3\sqrt{2}$

.7574464462934689241... ≈ $2\zeta(5) - \zeta(2)\zeta(3) + \zeta(3) - \frac{\zeta(4)}{2} = \sum_{k=1}^{\infty}\frac{k^2 H_k}{(k+1)^4}$

.75748739771880741669... ≈ $\sum_{k=0}^{\infty}\frac{1}{e^{2^k}-1}$

.757612458715149840955... ≈ $\zeta(3) - \frac{4}{9}$

.757672098556372684654... ≈ $\frac{\pi^2}{8} - \frac{\zeta(3)}{2} + \frac{1}{8} = \sum_{k=1}^{\infty} k^2\left(\zeta(2k) - \zeta(2k+1)\right)$

= $\sum_{k=2}^{\infty} \frac{k^2+1}{(k+1)^3(k-1)^2}$

1 .757795988957853130277... ≈ $\frac{\pi^3}{32} + \frac{3\zeta(3)}{4}$

= $\frac{3\zeta(3)}{4} + \frac{1}{4}\left((2+2i)Li_3(-i) + (2-2i)Li_3(i)\right)$

= $\int_0^1 \frac{1-x}{1-x^4}\log^2 x\,dx = \int_0^1 \frac{\log^2 x}{1+x+x^2+x^3}\,dx$

.757872156141312106043... ≈ $e\sqrt{\pi}\,\left(1 - erf 1\right) = \int_0^{\infty} \frac{dx}{e^x\sqrt{1+x}}$

1 .758268564325381820406... ≈ $-\frac{\pi}{72}\left(3\sqrt{3}\cot\frac{\pi}{\sqrt{3}} + \pi\left(2\pi\sqrt{3}\cot\frac{\pi}{\sqrt{3}} - 9\right)\csc^2\frac{\pi}{\sqrt{3}}\right)$

= $\sum_{k=1}^{\infty} \frac{k^2}{3^k}\zeta(2k)$

.758546992994776145344... ≈ $\frac{\pi}{\pi+1} = \sum_{k=0}^{\infty} \frac{(-1)^k}{\pi^k}$

.7587833022804503907... ≈ $\frac{1}{e}\left(Ei(1) - Ei(2)\right) + e\log 2 = \int_0^1 e^x \log(1+x)\,dx$

.758981249114944388204... ≈ $\frac{3}{2} + \frac{\pi\sqrt{3}}{6} - \frac{3\log 3}{2} = hg\left(\frac{2}{3}\right)$

.75917973947096213433... ≈ $2\zeta(3) - \zeta(2)$

1 .75917973947096213433... ≈ $2\zeta(3) - \zeta(2) + 1 = \sum_{k=1}^{\infty} k^2\left(\zeta(k+2) - 1\right)$

= $\sum_{k=1}^{\infty} \frac{H_k}{k^2(k+1)}$

.7596695583288265870... ≈ $\int_1^{\infty} \frac{x\,dx}{e^x - e^{-x}}$

.759747105885591946298... ≈ $\sqrt{\gamma}$

.759835685651592547331... ≈ $3^{-1/4}$

.759967033424113218236... ≈ $\frac{\pi^2}{12} - \frac{1}{16} = -\frac{1}{2}\left(Li_2\left(-e^{i/2}\right) + Li_2\left(-e^{-i/2}\right)\right)$

= $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2} \cos \frac{k}{2}$

.75997605244503147746... ≈ $e - 2 + \sum_{k=2}^{\infty} \frac{\Omega(k)}{k!} = \sum_{k=1}^{\infty} \sum_{j=1}^{\infty} \frac{1}{(k^j)!}$

1 .7600000000000000000$\underline{0}$ = $\frac{44}{25} = \sum_{k=1}^{\infty} \frac{F_k F_{k+3}}{4^k}$

.7602445970756301513... ≈ $\cos \frac{1}{\sqrt{2}} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!2^k}$ AS 4.3.66, GR 1.411.3

.7603327958712324201... ≈ $\prod_{k=1}^{\infty} \left(1 - \frac{1}{5^k}\right)$

.760339706025444015735... ≈ $\sum_{k=1}^{\infty} \frac{S1(2k,k)}{(2k)!}$

.760345996300946347531... ≈ $\frac{1}{\sqrt{3}} \log(2+\sqrt{3}) = 1 + \sum_{k=1}^{\infty} \left(\frac{(-1)^k}{6k-1} + \frac{(-1)^k}{6k+1} \right)$ J83

= $\sum_{k=1}^{\infty} \frac{(-1)^k (2k)!!}{2^k (2k+1)!!} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{2^k}{\binom{2k}{k} k}$ J267

= $\sum_{k=0}^{\infty} (-1)^k \frac{k!}{(2k+1)!!}$ J86

= $\int_2^{\infty} \frac{dx}{x^2 - 3}$

1 .76040595997292398625... ≈ $16 \log 2 - \frac{32G}{\pi} = \sum_{k=0}^{\infty} \frac{(2k+1)!^2}{k!^4 16^k (k+1)^3}$

17 .760473930229652590715... ≈ $5I_0(2) + 4I_1(2) = \sum_{k=1}^{\infty} \frac{k^5}{(k!)^2}$

.761130385915375187862... ≈ $\sum_{k=2}^{\infty} (-1)^k \left(\zeta(k)\zeta(k+1) - 1\right) = \sum_{k=2}^{\infty} \frac{\sigma_1(k)}{k^2(k+1)}$

.76122287570839003256... ≈ $-\cot \frac{\pi}{\sqrt{2}}$

.7612801797083721299... ≈ $\sum_{k=1}^{\infty} \frac{1}{k^3} \log \frac{k+1}{k}$

.761310204001103486389... ≈ $1 - \frac{1}{2\sqrt{3}} \left(i \left(\psi\left(\frac{3-i\sqrt{3}}{4}\right) - \psi\left(\frac{5-i\sqrt{3}}{4}\right) \right) - \psi\left(\frac{3+i\sqrt{3}}{4}\right) + \psi\left(5\frac{3+i\sqrt{3}}{4}\right) \right)$

= $\sum_{k=0}^{\infty} \frac{(-1)^k}{k^2+k+1}$

1 .761365221094699778983... ≈ $\sum_{k=1}^{\infty} \frac{u(k)}{k!}$

.761378697273284672250... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(k+1)-1}{k!} = \sum_{k=2}^{\infty} \frac{e^{1/k}-1}{k}$

.761390022682049161058... ≈ $\sum_{k=2}^{\infty} \frac{e(k)}{k!}$

.761393810948301507456... ≈ $\frac{\sqrt{\pi}}{4}(e-1) = \int_0^{\infty} e^{-x^2} \sinh^2 x\, dx$

.761549782880894417818... ≈ $\log(\pi-1)$

.76159415595576488812... ≈ $\tanh 1 = \frac{e^2-1}{e^2+1} = -i \tan i$ J148

= $1 + 2\sum_{k=1}^{\infty} (-1)^k e^{-2k}$

= $\sum_{k=1}^{\infty} \frac{4^k(4^k-1)B_{2k}}{(2k)!}$ AS 4.5.64

= $\int_0^1 \frac{dx}{\cosh^2 x}$

1 .761634557784733194248... ≈ $\frac{97\pi^6}{22680} - \frac{\pi^4}{72} + 2\zeta(3) - 2\zeta^2(3) - \frac{\pi^2}{6}(1+\zeta(3)) + 3\zeta(5)$

= $\sum_{k=1}^{\infty} \frac{H_k H_{k+1}}{k^4}$

.761747999761543071113... ≈ $\frac{\pi 2^{1/3}}{3\sqrt{3}} = \int_0^{\infty} \frac{dx}{x^3+2}$

.761759981416289230432... ≈ $\sin \frac{\sqrt{3}}{2}$

.761964863942319850842... ≈ $1 + \sqrt{\frac{e\pi}{2}} erf \frac{1}{\sqrt{2}} - \sqrt{e} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!!}$

.7619973727342293746... ≈ $\psi^{(1)}\left(\frac{1}{3}\right) + \frac{1}{54}\psi^{(3)}\left(\frac{1}{3}\right) - \frac{4\pi^3}{9\sqrt{3}} - \frac{26}{3}\zeta(3) = \sum_{k=1}^{\infty} \frac{k^2}{(k+1/3)^4}$

.762054692886954765011... ≈ $\frac{\sqrt{e}}{2} K_0\left(\frac{1}{2}\right) = \int_0^{\infty} \frac{dx}{e^{x^2}\sqrt{1+x^2}}$

$$2\quad .76207190622892413594\ldots \approx \frac{\pi^2 \log 2}{3} - \frac{\log^3 2}{6} + Li_3\left(\frac{1}{2}\right) = \int_0^1 \frac{\log(1-2x)\log x}{x}\,dx$$

$$3\quad .7621956910836314596\ldots \approx \cosh 2 = \frac{e^2+e^{-2}}{2} = \sum_{k=0}^{\infty} \frac{4^k}{(2k)!}$$ AS 4.5.3, GR 1.411.2

$$= \prod_{k=0}^{\infty}\left(1+\frac{16}{\pi^2(2k+1)^2}\right)$$ J1079

$$2\quad .76255761415988504657 0\ldots \approx \frac{583}{128\sqrt{e}} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^7}{k!2^k}$$

$$.76258964475273501375\ldots \approx -\sum_{k=2}^{\infty}\frac{\sigma_0(k)\mu(k)}{2^k}$$

$$1\quad .76274717403908605046 5\ldots \approx 2\log\left(1+\sqrt{2}\right) = 2\operatorname{arcsinh}1 = \int_0^{\infty}\frac{dx}{\sqrt{1+e^x}} = \int_1^{\infty}\frac{\operatorname{arcsinh}x}{x^2}\,dx$$

$$19\quad .76331253485059976025 4\ldots \approx \psi^{(3)}\left(\frac{3}{4}\right) = \sum_{k=4}^{\infty}\frac{(k-1)(k-2)(k-3)\zeta(k)}{4^{k-4}}$$

$$.76345904697490388986 4\ldots \approx 2-\frac{3\sqrt{e}}{4} = \sum_{k=1}^{\infty}\frac{k^3}{(k+1)!2^k}$$

$$.76354658135207244811\ldots \approx 2\left(\sin 1+\cos 1-1\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!(k+1)}$$

$$5\quad .7637435616366000772 1\ldots \approx 6\gamma\log 2+7\log^2 2 = l\left(-\frac{1}{4}\right)+l\left(-\frac{3}{4}\right)$$ Berndt 8.17.8

$$.76378480769468103146\ldots \approx \sum_{k=1}^{\infty}\frac{H_k}{2^k H_{k+1}}$$

$$.76393202250021030359\ldots \approx 3-\sqrt{5}$$

$$.76403822629272751139\ldots \approx \sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)}{\sqrt{k}}$$

$$.76410186989382872062\ldots \approx \pi^2-8G-\frac{16}{9} = \sum_{k=1}^{\infty}\frac{1}{(k+3/4)^2}$$

$$.76414465139077619286 2\ldots \approx \sum_{k=2}^{\infty}\left(\zeta(k)-1\right)^{\zeta(k)}$$

$$.76440318231829535698 5\ldots \approx \frac{\left(\pi-1\right)^2}{6} = \sum_{k=1}^{\infty}\frac{\sin^s k}{k^4}$$

$$.76449978034844420919 1\ldots \approx \sum_{k=1}^{\infty}\frac{1}{2^k+1} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2^k-1} = \sum(-1)^{k+1}\frac{(4^k+1)}{2^{k^2}(4^k-1)}$$ Berndt 4.6

$$= 1-\sum_{k=1}^{\infty}\frac{1}{2^k(2^k+1)}$$

= $\sum_{k=2}^{\infty}\frac{(-1)^k}{S2(k,2)}$

6 .76452021069461369697 5... ≈ $\frac{5\pi^4}{72} = \frac{\zeta^2(2)}{\zeta(4)} = \sum_{k=1}^{\infty}\frac{\left(\sigma_0(k)\right)^2}{k^2}$ Titchmarsh 1.2.10

1 .76466773629668253035 6... ≈ $\zeta(2)+\zeta(3)-\zeta(4)$

$.\underline{7647058823529411}$ = $\frac{13}{17}$

.7649865348150167057... ≈ $-\frac{\sin\pi 3^{1/4}\sinh\pi 3^{1/4}}{2\pi^2\sqrt{3}} = \prod_{2}^{\infty}\left(1-\frac{3}{k^4}\right)$

.76519768655796655145... ≈ $J_0(1) = I_0(i) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k!)^2 4^k} = \sum_{k=0}^{\infty}\frac{(-1)^k}{((2k)!!)^2}$

.76558707852592148581 4... ≈ $\frac{2\pi^3}{81} = \sum_{k=1}^{\infty}\frac{\sin 2\pi k/3}{k^3}$ GR 1.443.5

.76586968364661172132... ≈ $\prod_{k=1}^{\infty}\left(1-\frac{1}{k(k+1)(k+2)}\right)$

4 .7659502374326661364... ≈ $\frac{8\pi^6}{63} - 360\zeta(5) + \frac{13\zeta(4)}{2} - 180\zeta(3) + \frac{31\pi^2}{6} - \frac{3}{4}$

= $\sum(-1)^k k^5\left(\zeta(k)-1\right)$

.7662384356489860248... ≈ $\frac{3}{2}\log\frac{5}{3} = \int_0^{\infty}\frac{dx}{e^x+2/3}$

.76624516885374710152 3... ≈ $1-\frac{e}{2}-\frac{1}{e}\left(Ei(1)+Ei(2)\right) = \int_0^1\frac{e^x}{(1+x)^2}dx$

.76665285034506621240 3... ≈ $1-\pi^3\coth\pi\,\text{csch}^2\pi = i\left(\zeta(3,1+i)-\zeta(3,1-i)\right)$

= $i\sum_{k=1}^{\infty}\left(\frac{1}{(k+i)^3}-\frac{1}{(k-i)^3}\right)$

= $\int_0^{\infty}\frac{x^2\sin x}{e^x-1}dx = \frac{1}{2}+\int_0^{\infty}\frac{x^2\sin x}{e^x\left(e^x-1\right)}dx$

8 .76732808757196318956 3... ≈ $2\left(1-\gamma-Ei(-1)\right)-1 = \sum_{k=1}^{\infty}\frac{kH_k}{(k-1)!}$

.76742029124922385396 8... ≈ $\sum_{k=1}^{\infty}(\log 2)^{2^k}$

1 .76776695296636881100 2... ≈ $\frac{5}{2\sqrt{2}} = \sum_{k=1}^{\infty}\frac{(2k-1)!!k^2}{(2k)!!2^k} = \sum_{k=0}^{\infty}\binom{2k}{k}\frac{k^2}{8^k}$

1 .76804762350015971349... ≈ $\frac{8\pi^3}{81\sqrt{3}} = \frac{2i}{\sqrt{3}}\left(Li_3\left(-(-1)^{1/3}\right) - Li_3\left((-1)^{2/3}\right)\right)$

= $\int_0^1 \frac{\log^2 x\,dx}{x^2+x+1} = -\int_0^\infty \frac{\log^2 x\,dx}{x^3-1}$ GR 4.261.3

5 .7681401910013092414... ≈ $\frac{1}{\sqrt{3}}\csc h\pi \sinh \pi\sqrt{3} = \prod_{k=1}^\infty \left(1+\frac{2}{k^2+1}\right)$

.76816262143574939203... ≈ $6G\pi - \left(\frac{1+3i}{8}\right)\pi^3 + 6Li_3(i) - 6Li_3(-i) - \frac{21\zeta(3)}{2}$

= $6G\pi - \frac{\pi^3}{8} - \frac{21\zeta(3)}{2}$

= $\int_0^1 \frac{\arcsin^3 x}{x^2}dx$

.7684029568860641506... ≈ $\frac{13}{8} - \zeta(2) + \frac{\pi}{4}\coth\pi = \sum_{k=2}^\infty \frac{\Omega(k^2)}{k^2}$

= $\sum_{k=1}^\infty \left(\zeta(2k)-1\right) + \sum_{k=2}^\infty \frac{1}{k^6-k^2}$

1 .76842744993939215689... ≈ $2^{\pi^2/12} = \prod_{k=1}^\infty 2^{(-1)^{k+1}/k}$

.76844644669358040493... ≈ $\frac{\left(\sqrt{3}+1\right)}{6\cdot 2^{2/3}}\Gamma\left(\frac{1}{3}\right) = \int_0^\infty e^{-x^3}\cos x^3\,dx$

4 .7684620580627434483... = $\prod_{k=1}^\infty \left(1+\frac{1}{2^{k-1}}\right)$

.768488694016815547546... ≈ G^3

.7687004249195908632... = $\sum_{k=0}^\infty \frac{1}{(k+2)!!+k!!}$

3 .768802042217039515528... ≈ $\cos\left(2(-1)^{1/4}\right)\cosh\left(2(-1)^{1/4}\right) = \sum_{k=0}^\infty \frac{2^{6k}}{(4k)!}$

2 .768916786048680717672... ≈ $\pi\,\text{arcsinh}1 = \pi\log\left(1+\sqrt{2}\right) = \int_0^\infty \frac{\log(x^2+2)}{x^2+1}dx$

6 .769191382058228939651... ≈ $\pi\left(1+\frac{2}{\sqrt{3}}\right) = \int_0^{2\pi} \frac{\cos x}{4+2\cos x}dx$

.769238901363972126578... ≈ $\cos\log 2 = \text{Re}\left\{2^i\right\}$

1 .769461593584860406643... ≈ $2\pi - \pi\log 2 + 4G - 6 = -\int_0^1 \arccos^2 x \log x\,dx$

31 .7694766216224657296700... ≈ $\frac{411}{61}+15\log\frac{3}{2} = \sum_{k=1}^{\infty}\frac{k^4 H_k}{3^k}$

.7698370720456724802l6... ≈ $\frac{1}{4}\left(\pi\coth\pi-\pi^2\operatorname{csch}^2\pi\right) = \frac{\pi}{8}\operatorname{csch}^2\pi\sinh 2\pi-\frac{\pi^2}{4}\operatorname{csch}^2\pi$

= $\sum_{k=2}^{\infty}(-1)^k k\left(\zeta(k)-\zeta(2k)\right) = \frac{1}{4}+\sum_{k=1}^{\infty}(-1)^{k+1}k\left(\zeta(2k)-1\right)$

= $\sum_{k=1}^{\infty}\frac{k^2}{(k^2+1)^2}$

.769934066848226436472... ≈ $\zeta(2)-\frac{7}{8}$

.76998662174450356193... ≈ $\sqrt{2}J_1\left(\sqrt{2}\right) = \sum_{k=0}^{\infty}(-1)^{k+1}\frac{1}{k!(k+1)!2^k}$

.770151152934069858700... ≈ $\cos^2\frac{1}{2} = \frac{1}{2}+\frac{1}{2}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}$ J777

.770747041268399142066... ≈ $\frac{1}{2(\sqrt{e}-1)} = \sum_{k=0}^{\infty}\frac{B_k}{k!2^k}$

.77101309425278560330... ≈ $\frac{\pi}{4}\left(1-\frac{1}{e^4}\right) = \frac{\pi}{4}-\frac{\sqrt{2\pi}}{2}K_{1/2}(2) = \int_0^{\infty}\frac{\sin^2 2x\,dx}{1+x^2}$

= $\int_0^{\pi/2}\sin^2(2\tan x)\,dx$ GR 3.716.9

.771079989624934599824... ≈ $\sum_{k=1}^{\infty}\frac{\zeta^2(2k)}{4^k}$

1 .7711691814070372309... ≈ $\pi G+\frac{1}{16}\left(2\pi^2\log 2-35\zeta(3)\right) = \sum_{k=1}^{\infty}\frac{3^k}{\binom{2k}{k}k^3}$

1 .771508654754528604291... ≈ $4\zeta(2)-4\zeta(3)$

.771540317407621889239... ≈ $\frac{\cosh 1}{2} = \sum_{k=1}^{\infty}\frac{\left\lfloor\frac{k}{2}\right\rfloor}{k!} = e\sum_{k=1}^{\infty}\left(1-\frac{\Gamma(2k,1)}{\Gamma(2k)}\right)$

1 .7720172747445211300... ≈ $\sum_{k=1}^{\infty}\frac{\sqrt{k}}{k^2-1}$

.772029054982133162950... ≈ $\frac{1}{2}+\frac{\pi}{\sinh\pi}$

.77213800480906783028... ≈ $\frac{\pi}{2\sqrt{2}}e^{-1/\sqrt{2}}\left(\cos\frac{1}{\sqrt{2}}+\sin\frac{1}{\sqrt{2}}\right) = \int_0^{\infty}\frac{\cos x}{1+x^4}dx$

1 .772147608481020664137... ≈ $\zeta(3)+\frac{\pi^2}{12}\log 2$

1 .7724538509055160273... ≈ $\sqrt{\pi} = \Gamma\left(\frac{1}{2}\right)$

$= \sum_{k=1}^{\infty} \frac{(k-\tfrac{1}{2})!}{(k+1)!}$

.77258872223978123767... ≈ $4\log 2 - 2 = \sum_{k=1}^{\infty} \frac{(2k-1)!!k}{(2k)!(k+1)^2}$

$= \sum_{k=0}^{\infty} \frac{1}{2^k(k+2)} = \sum_{k=2}^{\infty} \frac{\zeta(k)-1}{2^{k-2}}$

$= \sum_{k=0}^{\infty} \frac{1}{(k+\tfrac{1}{2})(k+1)}$

$= \int_0^{\infty} \frac{dx}{e^x\left(e^x-1/2\right)}$

$= \int_0^1\int_0^1 \frac{x+y}{1+xy}\,dx\,dy$

1 .77258872223978123767... ≈ $4\log 2 - 1 = \sum_{k=1}^{\infty} \frac{H_{k+1}}{2^k}$

2 .77258872223978123767... ≈ $4\log 2 = \sum_{k=1}^{\infty} \frac{(k+1)H_k}{2^k} = \sum_{k=0}^{\infty} \binom{2k+1}{k} \frac{1}{4^k(k+1)}$

$= \sum_{k=0}^{\infty} \frac{1}{(k+1)(k+1/2)}$

$= \sum_{k=2}^{\infty} \frac{\zeta(k)}{2^{k-2}}$

$= \int_0^{\infty} \frac{\sin^4 2x}{x^3}\,dx$

.773126317094363179778... ≈ $\frac{63\pi}{256} = \int_0^1 \frac{x^{9/2}}{\sqrt{1-x}}\,dx$ GR 3.226.2

.773156669049795127864... ≈ $\sum_{k=2}^{\infty} \sum_{p\,prime} \frac{1}{p^k} = \sum_{k=2}^{\infty} \zeta_p(k) = \sum_{p\,prime} \frac{1}{p(p-1)}$

$= 2\sum_{k=2}^{\infty} \frac{\pi(k)}{k^3-k}$

$= \sum_{s=2}^{\infty} \sum_{k=1}^{\infty} \frac{\mu(k)}{k} \log\zeta(sk)$

1 .773241214308580771497... ≈ $\frac{1}{4}\left(2\cos\sqrt{2} + \sqrt{2}\sinh\sqrt{2}\right) = \sum_{k=1}^{\infty} \frac{2^k k^2}{(2k)!}$

.773485474588165713971... ≈ $\zeta(3) - \frac{3}{7}$

.773705614469083173741... ≈ $\log_6 4$

1 .773877583285132343802... ≈ $\sum_{k=1}^{\infty} \frac{1}{F_k F_{k+1}}$

.773942685266708278258... ≈ $\sin(e\pi)$

40 .774227426885678530404... ≈ $15e = \sum_{k=0}^{\infty} \frac{k^4}{k!}$ GR 0.249

1 .774245499958948344427... ≈ $\frac{3\pi^3}{\sqrt{2}} - 64 = \sum_{k=1}^{\infty} \left(\frac{(-1)^{k+1}}{(k-1/4)^3} - \frac{(-1)^{k+1}}{(k+1/4)^3} \right)$

5 .77436967862887418899... ≈ $\frac{1}{2}\sinh\pi = \int_0^{\pi} \sin x \sinh x \, dx$

.7745966692414833770... ≈ $\sqrt{\frac{3}{5}} = \sum_{k=0}^{\infty} \frac{(-1)^k}{6^k} \binom{2k}{k}$

1 .774603255583817852727... ≈ $64 - 44\sqrt{2} = \sum_{k=1}^{\infty} \binom{2k+2}{2} \frac{k}{8^k}$

.774787660168805549421... ≈ $\sum_{k=0}^{\infty} \frac{(-1)^k}{2^k + k}$

2 .77489768853690375126... ≈ $\frac{\pi^2}{2} - \psi^{(1)}\left(\frac{5}{6}\right) = \sum_{k=1}^{\infty} \frac{(3^k - 1)(k+1)}{6^k} \zeta(k+2)$

8 .7749643873921220604... ≈ $\sqrt{77}$

.776109220858764331948... ≈ $1 - J_0(2) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!k!}$

.776194758601534772662... ≈ $\sum_{k=2}^{\infty} \frac{k-1}{k^{k-1}}$

3 .776373136163078927203... ≈ $\pi\zeta(3)$

.77699006965153986787... ≈ $\int_0^1 \frac{dx}{\binom{2x}{x}}$

2 .77749955322549135074... ≈ $\sum_{k=1}^{\infty} \frac{e^{1/k}}{k^5} = \sum_{k=2}^{\infty} \frac{\zeta(k)}{(k-5)!}$

.777504634112248276418... ≈ $\frac{\pi^2}{6} - 1 - \log(e-1) - Li_2\left(\frac{1}{e}\right) = -Li_2(1-e) - \frac{1}{2}$

= $Li_2\left(1 - \frac{1}{e}\right) = \sum_{k=0}^{\infty} \frac{B_k}{(k+1)!}$

= $\int_0^1 \frac{x\,dx}{e^x - 1}$

.7777777777777777777$\underline{7}$ = $\frac{7}{9}$

17 .7777777777777777777$\underline{7}$ = mil/degree

14 .77811219786130045461... ≈ $2e^2 = \sum_{k=0}^{\infty} \frac{2^{k+1}}{k!}$

1 .77827941003892280122 5... ≈ $10^{1/4}$

1 .77844719779253552618... ≈ $\sum_{k=0}^{\infty} \frac{pf(k)}{e\,k!}$

.77870386232745111525 4... ≈ $\frac{\tan 1}{2} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} 2^{2k-1} (2^{2k} - 1) B_{2k}}{(2k)!}$

.77875857955275841504 2... ≈ $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{\zeta(2k)}{(2k)!} = \sum_{k=1}^{\infty} \left(1 - \cos \frac{1}{k}\right)$

.77880078307140486824 5... ≈ $e^{-1/4} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!4^k}$

1 .77882998327623558570 7... ≈ $\pi \log \frac{2}{1 + e^{-2}} = -\int_0^{\infty} \frac{\log \cos^2 x}{1 + x^2}$ GR 4.322.3

1 .77927256806112714600 6... ≈ $\zeta(3) + \gamma$

.77941262495788297845... ≈ $\frac{\pi}{3\sqrt{3}} - \frac{2}{\sqrt{3}} \arctan \frac{e - 2}{e\sqrt{3}} = \int_0^1 \frac{dx}{e^x + e^{-x} - 1}$

1 .78005080846154620742... ≈ $-\frac{6}{\pi} \cos \frac{\pi\sqrt{5}}{2} = \prod_{k=1}^{\infty} \left(1 + \frac{1}{k(k+3)}\right)$

.78071443535926771285 6... ≈ $\tanh \frac{\pi}{3}$

.7808004195655149546... ≈ $\sum_{k=1}^{\infty} \frac{k}{e^k + 1}$

.78083313985336003825 4... ≈ $\sum_{k=1}^{\infty} \frac{\phi(k)}{2^k k}$

1 .78107241799019798523 7... ≈ $e^{\gamma} = \sum_{k=0}^{\infty} \frac{\gamma^k}{k!}$

= $\overline{\lim} \frac{\sigma_1(n)}{n \log \log n}$ HW Thm. 323

= $\lim_{n \to \infty} \frac{1}{\log n} \prod_{p < n} \left(1 - p^{-1}\right)^{-1}$

.781212821300288716547... ≈ $-Y_1(1)$

.781302412896486296867... ≈ $g_2 = \frac{1}{9}\left(\psi^{(1)}\left(\frac{1}{3}\right)-\psi^{(1)}\left(\frac{2}{3}\right)\right) = \sum_{k=1}^{\infty}\left(\frac{1}{(3k-2)^2}-\frac{1}{(3k-1)^2}\right)$ J310

= $-\int_0^1 \frac{\log x}{1+x+x^2}dx$ GR 4.233.1

.78149414807558060039... ≈ $\sum_{k=1}^{\infty}\frac{2^k}{2^{2^k}}$

.781765584213411527598... ≈ $\frac{21}{10}-\gamma+\frac{\pi}{2\sqrt{3}}-\frac{3\log 3}{2} = \psi\left(\frac{8}{3}\right)$

6 .781793170552500031289... ≈ $\zeta(3)+\frac{\pi^2}{3}-1 = \sum k^2\left(\zeta(k)-\zeta(k+2)\right)$

= $\sum_{k=2}^{\infty}\frac{4k^3+k^2-2k+1}{k^3(k-1)^2}$

1 .78179743628067860948... ≈ $2^{5/6} = \prod_{k=1}^{\infty}\left(1+\frac{(-1)^{k+1}}{6k-5}\right)$

.781838631839335317274... ≈ $\sum_{k=1}^{\infty}\left(2^{1/2^k}-1\right)$

1538 .782144009188396022791... ≈ $\psi^{(3)}\left(\frac{1}{4}\right)$

113 .782299427444799775042... ≈ $\frac{1}{2}\left(e^{2e}-e^{e/2}\right) = \sum_{k=1}^{\infty}\frac{2^k \sinh k}{k!}$

6 .78232998312526813906... ≈ $\sqrt{46}$

1 .7824255962226047521... ≈ $2\,HypPFQ[\{1,1,1\},\{2,2,2\},-1] = \int_0^1 \frac{\log^2 x\,dx}{e^x}$

.78245210647762896657426...≈ $\sum_{k=2}^{\infty}F_k\left(\zeta(2k)-1\right)$

.78279749383106249726... ≈ $\frac{1}{6}+\frac{8\log 2}{9} = \int_1^{\infty}\frac{\log(x+3)}{x^3}dx$

.783430510712134407059 3...≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^k} = \int_0^1 x^x dx$ Prud. 2.3.18.1

.783878618887227378020... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{F_k}$

1 .783922996312878767846... ≈ $\frac{1}{20}\left(\sqrt{5}-1\right)e^{(1-\sqrt{5})/2}\left(5+3\sqrt{5}+2\sqrt{5}\,e^{\sqrt{5}}\right) = \sum_{k=2}^{\infty}\frac{F_k}{(k-2)!k}$

$$4\ \ .78400000000000000000\underline{0} \quad = \quad \frac{598}{125} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{F_k k^4}{2^k}$$

$$2\ \ .78416399841585392264 2\ldots \approx \quad \frac{\pi^{3/2}}{2}$$

$$.7853981633974483096\ldots \quad \approx \quad \frac{\pi}{4} = \beta(1) = \arcsin\frac{1}{\sqrt{2}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{2k+1}$$ AS 23.2.21, K135

$$= \quad -i\,\text{arctanh}\, i = \text{Im}\{\log(1+i\} = \text{Im}\{Li_1(i)\}$$

$$= \quad 4\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right)$$ Borwein-Devlin p. 73

$$= \quad 4\arctan\left(\frac{1}{5}\right) - \arctan\left(\frac{1}{239}\right)$$ Borwein-Devlin p. 73

$$= \quad 3\arctan\left(\frac{1}{4}\right) + \arctan\left(\frac{1}{20}\right) + \arctan\left(\frac{1}{1985}\right)$$ Borwein-Devlin p. 73

$$= \quad 1 - 2\sum_{k=1}^{\infty}\frac{1}{(4k-1)(4k+1)}$$ GR 0.232.1

$$= \quad \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k-1)^2}\sin(2k-1)$$ J523

$$= \quad \sum_{k=1}^{\infty}\frac{\sin(2k-1)}{2k-1}$$ GR 1.442.1

$$= \quad \sum_{k=1}^{\infty}\frac{\sin k\pi/2}{k}$$

$$= \quad \sum_{k=1}^{\infty}\frac{\sin^3 k}{k}$$ GR 3.828.3

$$= \quad \sum_{k=0}^{\infty}\frac{(-1)^k\cos(\theta(2k+1))}{2k+1} \quad , |\theta| < \pi/2$$ Berndt Ch. 4

$$= \quad \sum_{k=0}^{\infty}\arctan\left(\frac{2}{(2k+2)^2}\right)$$ [Ramanujan] Berndt Ch. 2, Eq. 7.3

$$= \quad \sum_{k=1}^{\infty}\arctan\left(\frac{1}{2k^2}\right)$$ [Ramanujan] Berndt Ch. 2, Eq. 7.6

$$= \quad \sum_{k=1}^{\infty}\arctan\frac{1}{k^2+k+1}$$ K Ex. 102, LY 6.8

$$= \quad \sum_{k=1}^{\infty}(-1)^{k+1}\arctan\left(\frac{2}{k^2}\right)$$ [Ramanujan] Berndt Ch. 2

$$= \prod_{k=1}^{\infty}\left(1-\frac{1}{(2k+1)^2}\right) \qquad \text{GR 0.261, J1059}$$

$$= \prod_{k=1}^{\infty}\frac{k(k+1)}{(k+1/2)^2} \qquad \text{J1061}$$

$$= \int_0^{\infty}\frac{dx}{(x^2+1)^2} = \int_0^{\infty}\frac{dx}{x^2+4} = \int_0^{\infty}\frac{dx}{x^2+2x+2} = \int_0^{\infty}\frac{dx}{4x^2+1}$$

$$= \int_0^{\infty}\frac{x\,dx}{1+x^4} \qquad \text{GR 2.145.4}$$

$$= \int_0^{\infty}\frac{dx}{x^3+x^2+x+1} = \int_0^{\infty}\frac{dx}{x^4+2^{2/3}}$$

$$= \int_0^{1}\frac{x\,dx}{\sqrt{1-x^4}}$$

$$= -\int_0^{\infty}\frac{\log x}{(x^2+1)^2}dx = -\int_0^{\infty}\frac{\log x}{(x^2+1)^3}dx$$

$$= \int_0^{\infty}\frac{dx}{e^x+e^{-x}}$$

$$= \int_0^{\pi/2}\frac{\sin x}{2-\sin^2 x}dx$$

$$= \int_0^{\infty}\frac{\sin x\cos x}{x}dx = \int_0^{\infty}\frac{\sin x\cos^2 x}{x}dx = \int_0^{\infty}\frac{\sin^2 x\cos^2 x}{x^2}dx$$

$$= \int_0^{\infty}\frac{\sin^2 x\cos x}{x^2}dx = \int_0^{\infty}\frac{\sin^3 x\cos x}{x^3}dx$$

$$= \int_0^{\infty}\frac{\sin^4 x}{x^2}dx$$

$$= \int_0^{\infty}\frac{\sin x - x\cos x}{x^3}$$

$$= \int_0^{\pi/2}\frac{x\,dx}{(\sin x+\cos x)^2}$$

$$= \int_0^{\pi/2}\frac{1-x\cot x}{\sin^2 x}dx$$

$$= \int_0^{\infty} si(x)\sin x\,dx$$

$$= \int_0^{\infty}\left(1-e^{-x}\right)\frac{\sin x}{x}dx$$

= $\int_0^{\pi/2} (\sin^2 x)\log\tan x\,dx$

= $\int_0^1 \frac{x(1+x)}{\log x}\sin\log x\,dx$ GR 4.429

= $\int_0^\infty (1-x\,\mathrm{arccot}\,x)\,dx$ GR 4.533.1

= $-\int_0^1 \psi(x)\sin\pi x\cos\pi x\,dx$ GR 6/469/1

1 .7853981633974483096... ≈ $\frac{\pi}{4}+1 = \sum_{k=0}^{\infty}\frac{2^k}{\binom{2k+3}{k}}$

2 .78556209745682930 8562... ≈ $\sum_{k=1}^{\infty}\frac{p(k)}{k!}$, $p(k)$ = number of partitions of k

.78569495838710218128... ≈ $\cos\frac{\pi}{16}-\sin\frac{\pi}{16} = \prod_{k=1}^{\infty}\left(1+\frac{(-1)^k}{8k-4}\right)$ J1029

.7857142857142$\underline{857142}$ = $\frac{11}{14}$

.78620041769482291712... ≈ $\frac{8}{9}-\frac{8}{27}\operatorname{arcsinh}\frac{1}{2\sqrt{2}}$

= ${}_2F_1\left(1,1,\frac{1}{2},-\frac{1}{8}\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{\binom{2k}{k}2^k}$

1 .78628364173958499949... ≈ $\sum_{k=2}^{\infty}\frac{k\log k}{2^k} = \sum_{n=1}^{\infty}\frac{1}{(n-1)!}\sum_{k=2}^{\infty}\frac{\log^n k}{2^k}$

6 .78653338973407709094... ≈ $\sum_{k=1}^{\infty}\left(\zeta^4(2k)-1\right)$

1 .78657645936592246346... ≈ $\sum_{k=1}^{\infty}\frac{1}{\phi(k)\sigma_1(k)} = \prod_{p\ prime}\left(1+\sum_{k=1}^{\infty}\frac{1}{p^{2k}-p^{k-1}}\right)$

Silverman constant

1 .78657645936592246346... ≈ $\sum_{k=1}^{\infty}\frac{1}{\phi(k)\sigma_1(k)} = \prod_{p\ prime}\left(1+\sum_{k=1}^{\infty}\frac{1}{p^{2k}-p^{k-1}}\right)$

.78668061788579802349... ≈ $\frac{1}{2}\left(e-\cosh(\cosh 2-\sinh 2)-\sinh(\cosh 2-\sinh 2)\right)$

= $\frac{1}{2}\left(e-e^{1/e^2}\right) = \sum_{k=0}^{\infty}\frac{\sinh k}{k!e^k}$

.786927755143369926331... ≈ $\zeta(5) - \frac{1}{4} = \sum_{k=1}^{\infty} \frac{1}{k^5} - \int_1^{\infty} \frac{dx}{x^5}$

.78693868057473315279... ≈ $2 - \frac{2}{\sqrt{e}} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)!2^k}$

.78704440142662875761... ≈ $\zeta(2) - 2Li_3\left(-\frac{1}{2}\right) - \frac{3}{2}\zeta(3) = \int_0^1 \frac{\log^2 x}{(x+1)^2(x+2)} dx$

2 .787252017813457378711... ≈ $\sum_{k=1}^{\infty} \frac{1}{e^{k/2} - 1}$ Berndt 6.14.4

1 .787281880354185304548... ≈ $\frac{e^3 - 4}{9} = \sum_{k=0}^{\infty} \frac{3^k}{(k+2)!}$

4 .78749174278204599425... ≈ $\log 5!$

.787522150360869746141... ≈ $\frac{1}{2} - \frac{\pi}{4}\coth\frac{\pi}{2} + \frac{\log 2}{4} + \frac{\pi^2}{2}\operatorname{csch}\pi$

$+ \frac{1}{4}\left(\psi\left(\frac{1-i}{2}\right) + \psi\left(\frac{1+i}{2}\right) - \psi\left(1 - \frac{i}{2}\right) - \psi\left(1 + \frac{i}{2}\right)\right)$

= $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+1}{k(k^2+1)}$

.78765858104243429... ≈ $8 - 6\zeta(3) = H^{(3)}{}_{1/2}$

.78778045617246654606... ≈ $\sum_{k=2}^{\infty} \frac{1}{lfac(k)}$, $lfac(k) = LCM(1,...,k)$

3 .78792362044502928224... ≈ $\sum_{k=1}^{\infty} \left(\zeta^3(2k) - 1\right)$

.78797060627038829197... ≈ $\frac{15\zeta(5)}{2\pi^2} = \xi(5) = \xi(-4)$

.78800770172316171118... ≈ $\frac{1}{4}\left(\pi \log 2 + i\left(Li_2\left(-\frac{i}{2}\right) - Li_2\left(\frac{i}{2}\right)\right)\right) = \int_1^{\infty} \frac{\log x}{x^2 + 4}\, dx$

1 .78820673932582791427... ≈ $\sum_{k=2}^{\infty} \sigma_0(k) \log\zeta(k) = \sum_{k=2}^{\infty} \log\zeta(k) + \sum_{n=3}^{\infty}\sum_{k=1}^{\infty} \log\zeta(n-k)$

.788230216655105940660... ≈ $\log 2 - \log\sin 2 = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} B_{2k} 4^{2k}}{(2k)!(2k)}$ [Ramanujan] Berndt Ch. 5

= $\int_0^2 \frac{1 - x\cos x}{x} dx$ [Ramanujan] Berndt Ch. 5

.788337023734290587067... ≈ $\frac{\pi}{4}\coth\pi$

	.7885284515797971427...	≈	$\frac{\zeta(3)}{2}+\frac{3}{16} = \sum_{k=2}^{\infty} k(k-1)\left(\zeta(2k-1)-1\right) = \sum_{k=2}^{\infty}\frac{2k^3}{(k^2-1)^3}$
	.78853056591150896106...	≈	$\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k-1} = -\sum_{k=2}^{\infty}\left(\frac{1}{k}\log\left(1-\frac{1}{k}\right)\right) = \sum_{k=2}^{\infty}\frac{\log k}{k^2+k}$
		=	$\sum_{n=1}^{\infty}\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{k^n} = \sum_{n=1}^{\infty}\sum_{k=2}^{\infty}\left(Li_n\left(\frac{1}{k}\right)-\frac{1}{k}\right)$
1	.78853056591150896106...	≈	$\sum_{k=2}^{\infty}\frac{k}{k-1}\left(\zeta(k)-1\right)$
1	.78885438199983175713...	≈	$\frac{4}{\sqrt{5}}$
8	.78889830934487753116...	≈	$8\log 3$
	$.\underline{789473684210526315}$	=	$\frac{15}{19}$
4	.78966619854680943305...	=	$\sqrt{2}I_1\left(2\sqrt{2}\right) = \sum_{k=1}^{\infty}\frac{2\ k}{k!k!}$
	.78969027051749582771...	≈	$\sum_{k=2}^{\infty}(-1)^k\frac{\zeta(k)-1}{\log k}$
2	.789802883501667684222...	≈	$\frac{188}{81}-44\log\frac{4}{3} = \sum_{k=1}^{\infty}\frac{k^2H_k}{4^k}$
	.790701923562543475698...	≈	$\sum_{k=1}^{\infty}\frac{1}{2^k H_k}$
	.7907069804229852813...	≈	$\sum_{k=1}^{\infty}\frac{1}{k!!(k+1)} = \sum_{k=0}^{\infty}\frac{k!!}{(k+2)!} = \sum_{k=1}^{\infty}\frac{1}{(k+2)!!-k!!}$
	.7916115315243421172...	≈	$\sum_{k=1}^{\infty}\frac{H_k{}^3}{(k+1)^3}$
1	.791759469228055000813...	≈	$\log 6 = Li_1\left(\frac{5}{6}\right) = \sum_{k=1}^{\infty}\frac{5^k}{6^k k}$
1	.791831656602118007467...	≈	$\frac{\pi}{\sqrt{2}}\csc\frac{\pi}{\sqrt{2}}-1 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2-1/2}$
1	.792363426894272110844...	≈	$\frac{\gamma}{2}+\frac{\pi\gamma}{2}\coth\pi+\frac{i}{4}\left(\psi(-i)^2-\psi(i)^2+\psi^{(1)}(1+i)-\psi^{(1)}(1-i)\right)$
		=	$\sum_{k=1}^{\infty}\frac{H_k}{k^2+1}$
	.792481250360578090727...	≈	$\log_4 3 = \frac{\log 3}{2\log 2} = \int_1^{\infty}\frac{dx}{2^x-2^{-x}}$
2	.79259596281002874558...	≈	$\frac{7}{\sqrt{2\pi}}$

	.792902785140609359763...	≈	$\sum_{k=1}^{\infty}\frac{1}{e^k+k-2}$	
1	.793209546954886070955...	≈	$\frac{\pi^2}{2}-\pi$	
	.7939888543152131425...	≈	$\frac{\Gamma(1/2)}{\Gamma(9/8)\Gamma(3/8)} = \prod_{k=1}^{\infty}\left(1+\frac{(-1)^k}{4k}\right)$	J1028
	.794023616383282894684...	≈	$\sqrt{2}\,e^{-\gamma}$	Berndt 8.17.17
	.79423354275931886558...	≈	$-\operatorname{Im}\left\{\psi^{(1)}(i)\right\} = \frac{i}{2}\left(\psi^{(1)}(1+i)-\psi^{(1)}(1-i)\right) = \int_0^{\infty}\frac{x\sin x}{e^x-1}dx$	
1	.79426954519229214617...	≈	$\sum_{k=1}^{\infty}\left(\zeta(k+1)-1\right)H_{2k}$	
9	.79428970264856009459...	≈	$\frac{1}{4}\left(I_0(2e)-I_0\left(\frac{2}{e}\right)\right) = \sum_{k=1}^{\infty}\frac{\sinh k\cosh k}{(k!)^2}$	
25	.794350166618684018559...	≈	$\frac{\pi^3}{\zeta(3)}$	
	.794415416798359282517...	≈	$30\log 2-20 = \sum_{k=0}^{\infty}\frac{k^3}{2^k(k+1)(k+2)}$	
	.795371500563939690401...	≈	$\pi^{-1/5}$	
	.7953764815472385856...	≈	$HypPFQ[\{1,1,1\},\{\frac{1}{2},2\},-\frac{1}{4}] = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+1)\binom{2k}{k}}$	
	.79569320156748087193...	≈	$\sin\frac{\pi}{\sqrt{2}}$	
	.795749711559096019168...	≈	$Li_2\left(\frac{1}{e}\right)+Li_3\left(\frac{1}{e}\right)$	
	.7958135686991373424...	≈	$\tan 1-\tanh 1$	
4	.7958315233127195416...	≈	$\sqrt{23}$	
	.795922775805343375904...	≈	$\sum_{k=1}^{\infty}\frac{H_k^{\ 2}}{3^k}$	
	.79659959929705313428...	≈	$\gamma-Ei(-1) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!k} = \int_0^1\frac{1-e^{-x}}{x}dx$	J289
		=	$HypPFQ[\{1,1\},\{2,2\},-1]$	

= $\int_{-\infty}^{0} xe^{x-e^{x}}\,dx$

= $-\int_{0}^{1} \frac{\log x\,dx}{e^{x}}$

= $\sum_{k=0}^{\infty} \frac{(-1)^{k}}{k!(k+1)^{2}} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(k+1)!-k!}$

8 .7965995992970531343... ≈ $8+\gamma-Ei(-1) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{(2k+1)^{4}}{k!k}$ Berndt 2.9.8

.796718392705881508606... ≈ $\sum_{k=1}^{\infty} \left(\zeta^{3}(3k)-1\right)$

.79690219140466371781... ≈ $\sum_{k=2}^{\infty} \left(\frac{\zeta(k)}{\zeta(k+2)}-1\right)$

.797123679538449558285... ≈ $\frac{\log 2}{9}\left(\pi^{2}+\log^{2} 2\right) = \int_{0}^{\infty} \frac{\log^{2} x}{(x-1)(x+2)}dx$ GR 4.261.4

.797267445945917809011... ≈ $1+\frac{1}{2}\log\frac{2}{3} = 1+\sum_{k=1}^{\infty} \frac{(-1)^{k}}{2^{k+1}k}$

1 .7977443276183680508... ≈ $\sum_{k=2}^{\infty} \frac{\log^{2} k}{k(k+1)}$

.79788456080286535588... ≈ $\frac{2}{\sqrt{2\pi}}$

.797943096840405714600... ≈ $2-\zeta(3) = \sum_{k=2}^{\infty} \frac{k-1}{k^{3}} = \sum_{k=2}^{\infty} (-1)^{k}\left(\zeta(k)-\zeta(k+2)\right)$ ∴

9 .7979589711327123928... ≈ $\sqrt{96}$

.7981472805626901809... ≈ $\frac{\pi}{\sqrt{3}}\tanh\frac{\pi}{2\sqrt{3}} - 1 = \frac{\pi}{\sqrt{3}} - \frac{2\pi\sqrt{3}}{3+3e^{\pi\sqrt{3}}} - 1$

= $\sum_{k=0}^{\infty} \zeta(6k+2)+\zeta(6k+3)-\zeta(6k+5)-\zeta(6k+6)$

1 .7981472805626901809... ≈ $\frac{\pi}{\sqrt{3}}\tanh\frac{\pi}{2\sqrt{3}} = \frac{\pi}{\sqrt{3}} - \frac{2\pi\sqrt{3}}{3+3e^{\pi\sqrt{3}}}$

= $1+\frac{i\sqrt{3}}{3}\left(\psi\left(\frac{3-i\sqrt{3}}{2}\right) - \psi\left(\frac{3+i\sqrt{3}}{2}\right)\right)$

= $=\sum_{k=0}^{\infty} \frac{1}{k^{2}+k+1}$

3 .79824572977119450443... ≈ $e^{\left(1-\sqrt{5}\right)/2}\ \dfrac{2+(3+\sqrt{5})e^{\sqrt{5}}}{5+\sqrt{5}} = \displaystyle\sum_{k=1}^{\infty}\frac{F_k}{(k-1)!}$

.79851213284450825918 7... ≈ $\displaystyle\sum_{k=3}^{\infty}(-1)^{k+1}\frac{k(k-1)}{k!}\zeta(k)$

.79863201236633127840 3... ≈ $\dfrac{e^2-1}{8} = \displaystyle\sum_{k=0}^{\infty}\frac{2^k}{k!(k+1)(k+3)}$

.7987656017062165664 2... ≈ $\dfrac{\gamma-1-\psi(2-\gamma)}{\gamma} = \dfrac{2\gamma-\gamma^2+(1-\gamma)\psi(1-\gamma}{\gamma(\gamma-1)}$

= $\displaystyle\sum_{k=2}^{\infty}\frac{1}{k(k-\gamma)} = \sum_{k=2}^{\infty}\gamma^{k-2}\left(\zeta(k)-1\right)$

.79877599144788949810 3... ≈ $\dfrac{3}{2}-\dfrac{\cos 2}{2}-\sin 2 = \displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}4^k}{(2k)!(k+1)}$

.7989752939540047561... ≈ $\dfrac{2\sin^2 1}{\sqrt{\pi}} = \displaystyle\sum_{k=0}^{\infty}\frac{(-1)^k}{(k+1)!(k+\frac{1}{2})}$

.7991743158073551498 1... ≈ $\displaystyle\sum_{k=1}^{\infty}\frac{\zeta(2k+1)}{2^k k} = -\sum_{k=1}^{\infty}\frac{1}{k}\log\left(1-\frac{1}{2k^2}\right)$

1 .79931736044827240636 5... ≈ $\dfrac{3}{2}+\displaystyle\sum_{k=2}^{\infty}\frac{\Omega(k)k}{2^k} = \sum_{k=2}^{\infty}\sum_{j=1}^{\infty}\frac{k^j}{2^{k^j}}$

.79938610537951043634 7... ≈ $\dfrac{1}{4}+\dfrac{\pi}{4\sinh\pi}+\dfrac{\log 2}{2}$

$+\dfrac{1}{8}\left(\psi\left(1-\dfrac{i}{2}\right)+\psi\left(1+\dfrac{i}{2}\right)-\psi\left(\dfrac{1-i}{2}\right)-\psi\left(\dfrac{1+i}{2}\right)\right)$

= $\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^k}{k^3+k^2+k+1}$

.7997832325421110159 2... ≈ $\dfrac{\pi}{4}\left(1+e^{-4}\right) = \displaystyle\int_0^{\pi/2}\cos(2\tan x)dx$ GR 3.716.10

.8000000000000000000$\underline{0}$ = $\frac{4}{5} = \sum_{k=1}^{\infty} \frac{(-1)^k}{4^k}$ J352

= $\sum_{k=1}^{\infty} \frac{F_{2k}}{4^k}$

.80018201476676943168 1... ≈ $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\zeta(2k)}{(2k-1)!2^{2k-1}} = \sum_{k=1}^{\infty} \frac{1}{k}\sin\frac{1}{2k}$

1 .8007550560052829915... ≈ $\sum_{k=1}^{\infty} \frac{\log(k+1)}{k^2}$

= $\int_1^{\infty} \frac{H(x)}{x^2}\,dx$

6 .8008734907968819647 8... ≈ $\sum_{k=1}^{\infty} \frac{\psi^{(3)}(k)}{k}$

.80137126877306285693... ≈ $\frac{2\zeta(3)}{3}$

.80179989026464499972 5... ≈ $\frac{7\zeta(3)}{8} - \frac{1}{4} = \sum_{k=1}^{\infty} \frac{k^2\zeta(2k+1)}{4^k}$

12 .80182748008146961121... ≈ $\log 9!$

.8020569031595942854... ≈ $\zeta(3) - \frac{2}{5}$

.8025725173496056544 5... ≈ $\frac{1}{5} + \frac{14}{5\sqrt{5}}\operatorname{arccsch} 2 = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\binom{2k}{k-1}}$

1 .80268539948873336735 6... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(4k)}{k!} = \sum\left(e^{\frac{1}{k^4}} - 1\right)$

.80269608533157794645... ≈ $\frac{3\sqrt{3}}{2\pi}\sin\frac{\pi}{\sqrt{3}} = \prod_{k=2}^{\infty}\left(1 - \frac{1}{3k^2}\right)$

.8027064884013474243... ≈ $\frac{si(2)}{2} = \sum_{k=0}^{\infty} \frac{(-1)^k 4^k}{(2k+1)!(2k+1)}$

1 .8030853547393914281... ≈ $\frac{3\zeta(3)}{2} = 2\eta(3) = 2\sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)^3}$

= $\int_0^1 \frac{\log^2 x}{1+x}\,dx$ GR 4.261.11

= $\displaystyle\int_0^1 \frac{x\log^2 x}{(1+x)^2}dx$ GR 4.261.19

= $\displaystyle\int_1^\infty \frac{\log^2 x}{x^2+x}dx = \int_0^\infty \frac{x^2}{e^x+1}dx$

= $\displaystyle -\int_0^1\int_0^1 \frac{\log(xy)}{1+xy}dx\,dy$

.80325103994524191678... ≈ $\displaystyle 4G - 1 - \frac{\pi}{2} + \pi\log 2 - \frac{\pi^2}{4} = \int_0^{\pi/2} \frac{x^2-\sin^2 x}{1-\cos x}dx$

.8033475762076458819... ≈ $\displaystyle\sum_{k=1}^\infty \frac{1}{2^{k+1}-2}$

1 .8037920349100624... ≈ $\displaystyle\sum_{k=1}^\infty (-1)^{k+1}e^{1/k}$

.80384757729336811942... ≈ $6-3\sqrt{3}$

2 .80440660163403792825... ≈ $\displaystyle\frac{3\pi^3}{2} - 12$

.80471895621705018730... ≈ $\displaystyle\frac{\log 5}{2} = \mathrm{Re}\{\log 2 + i\}$

1 .804721402853249668... ≈ $\displaystyle\prod_{k=1}^\infty \left(1+\frac{1}{2^k k}\right)$

2 .80480804893839018764... ≈ $\displaystyle\sum_{k=1}^\infty \frac{e^{1/k^4}}{k^4}$

1 .8049507064891153143... ≈ $\displaystyle\sum_{k=1}^\infty \frac{\zeta(3k+1)}{k!} = \sum_{k=1}^\infty \frac{\left(e^{1/k^3}-1\right)}{k}$

.8053154534982627922... ≈ $\displaystyle\frac{1}{4} + \frac{\gamma^2}{2} + \pi^2\left(\frac{1}{4} + \frac{\operatorname{csch}^2\pi}{2} - \frac{\csc h^2(\pi(1+i))}{4}\right) +$

$\displaystyle + \frac{\gamma}{2}\psi(-i) + \frac{\psi^2(-i)}{4} + \frac{\psi(i)}{2} + \frac{\psi^2(i)}{4}$

= $\displaystyle\sum_{k=1}^\infty \frac{H_k}{k(k^2+1)}$

.80568772816216494092... ≈ $\displaystyle\prod_{k=1}^\infty \left(1-\frac{1}{6^k}\right)$

.80612672304285226132... ≈ $\displaystyle L_{1/2}\left(\frac{1}{2}\right) = \sum_{k=1}^\infty \frac{1}{2^k\sqrt{k}}$

.8061330507707634892... ≈ $\displaystyle\frac{4\pi}{9\sqrt{3}} = \sum_{k=1}^\infty \frac{k+1}{\binom{2k}{k}}$

= $\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{5}{3}\right) = \prod_{k=1}^{\infty}\frac{k(k+2)}{(k+1/3)(k+2/3)}$ J1061

= $\int_0^{\infty}\frac{dx}{(x^3+1)^2}$

1 .8061330507707634892... ≈ $1+\frac{4\pi}{9\sqrt{3}} = \sum_{k=1}^{\infty}\frac{k+1}{\binom{2k}{k}}$

= $\int_0^{\infty}\frac{dx}{(x^3+1)^2}$

.8063709187317308602... ≈ $-2-2\log 2+\log^2 2+3\log 3-2\log 2\log 3+\log^2 3+2Li_2\left(\frac{1}{3}\right)$

= $\int_0^1 \log x\,\log(x+2)\,dx$

.80639561620732622518... ≈ $\int_0^{\infty}\frac{dx}{e^x+x} = \int_0^{\infty}\frac{x\,dx}{e^x+x}$

1 .806457594638064753844... ≈ $\psi\left(\frac{9}{10}\right)-\psi\left(\frac{2}{5}\right)$

1 .80649706588366077562... ≈ $\sum_{k=0}^{\infty}\frac{k}{k!+1}$

.806700467600632558527... ≈ $\frac{2\sqrt{2}}{\pi}\sin\frac{\pi}{2\sqrt{2}} = \prod_{k=1}^{\infty}\left(1-\frac{1}{2(2k)^2}\right)$

.80687748637583628319... ≈ $128-32G-4\pi^2-8\pi-48\log 2$

= $\int_0^1 \frac{\log(1-x)\log x}{x^{3/4}}dx$

2 .80699370501978937177... ≈ $\sum_{k=1}^{\infty}\frac{2^{1/k}}{k^2}$

.8070991138260185935... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k^2-2)}{k^2}$

.80714942020070612009... ≈ $\prod_{k=2}^{\infty}\left(1-\frac{(-1)^k}{2^k}\right)$

2 .80735492205760410744... ≈ $\log_2 7$

.807511182139671452858... ≈ $\Gamma\left(\frac{4}{3}\right)-\frac{1}{3}\Gamma\left(\frac{1}{3},1\right) = \int_0^1 e^{-x^3}dx$

.80768448317881541034... ≈ $1-\gamma^3$

2 .80777024202851936522... ≈ $\int_0^\infty \frac{dx}{\Gamma(x)}$, Fransén-Robinson constant

.80816039364547495593... ≈ $\sum_{k=2}^\infty \frac{\zeta(k)-1}{\phi(k)}$

2 .8082276126383771416... ≈ $4\zeta(3) - 2$

= $\int_0^1 \frac{(1+x)\log^2 x}{1-x} dx$

4 .8082276126383771416... ≈ $4\zeta(3)$

= $\int_0^1 \frac{\log(1-x^{1/2})\log x}{x} dx$

= $-\int_0^1\int_0^1 \frac{\log(x^2y^2)}{1-xy} dx\,dy$

5 .8082276126383771416... ≈ $4\zeta(3) + 1 \ = \ \sum_{k=1}^\infty k^2\left(\zeta(k+1)+\zeta(k+2)-2\right)$

.80901699437494742410... ≈ $\frac{\varphi}{2} = \frac{1+\sqrt{5}}{4}$

.80907869621835775823... ≈ $2\log 2 - \gamma \ = \ \sum_{k=1}^\infty \frac{\psi(k+1)}{2^k}$

.8093149189514646786... ≈ $\zeta\left(\frac{1}{2}, -\frac{1}{2}\right)$

.80939659736629010958... ≈ $\frac{1}{3\pi}\cosh\left(\frac{\pi\sqrt{3}}{2}\right) \ = \ \frac{1}{3\Gamma\left((-1)^{1/3}\right)\Gamma\left(-(-1)^{2/3}\right)}$

= $\frac{1}{\Gamma\left(\frac{5-i\sqrt{3}}{2}\right)\Gamma\left(\frac{5+i\sqrt{3}}{2}\right)}$

= $\prod_{k=2}^\infty \left(1-\frac{1}{k^3}\right) \ = \ \exp\left(-\sum_{k=1}^\infty \frac{\zeta(3k)-1}{k}\right)$

1 .8098752090298617066... ≈ $\sum_{k=1}^\infty \frac{H_k\zeta(k+1)}{2^k}$

.81019298730410784673... ≈ $120 - 4\pi^2 - 24(\zeta(4)+\zeta(5)+\zeta(6)) \ = \ -\int_0^1 \log(1-x)\log^4 x\,dx$

7 .8102496759066543941... ≈ $\sqrt{61}$

.81040139440963627981... ≈ $\frac{1}{2}+\sqrt{2}\,\mathrm{csch}\frac{\pi}{\sqrt{2}} \ = \ \sum_{k=0}^\infty \frac{(-1)^k}{2k^2+1}$

11	.81044097243059865654...	≈	$\sum_{k=1}^{\infty}\left(\frac{4}{k}\right)^k$
4	.81047738096535165547...	≈	$e^{\pi/2} = e^{-i\log i}$
	.81056946913870217...	≈	$\frac{8}{\pi^2} = \frac{4}{3\zeta(2)} = \sum_{k=1}^{\infty}\frac{\mu(2k)}{k^2} = \sum_{k=1}^{\infty}\frac{\mu(2k-1)}{(2k-1)^2}$
1	.81068745341566177051...	≈	$36-2\pi^2\sqrt{3} = \sum_{k=1}^{\infty}\left(\frac{(-1)^{k+1}}{(k-1/6)^2}+\frac{(-1)^{k+1}}{(k+1/6)^2}\right)$
	.81086229142430104 23...	≈	$\frac{\sqrt{\pi}\sin\sqrt{\pi}}{\pi-1} = \prod_{k=2}^{\infty}\left(1-\frac{1}{\pi k^2}\right)$
	.81093021621632876 4...	≈	$2(\log 3-\log 2) = \Phi\left(-\frac{1}{2},1,1\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{2^k(k+1)}$
		=	$\int_0^{\infty}\frac{dx}{e^x+1/2}$
	.81114730330175217177...	≈	$\frac{\pi}{\sqrt{15}}\tanh\frac{\pi\sqrt{15}}{2} = \sum_{k=0}^{\infty}\frac{1}{k^2+k+4}$
551	.8112111771861827781...	≈	$203e = \sum_{k=0}^{\infty}\frac{k^6}{k!}$
	.81139857387224308909...	≈	$\sum_{k=2}^{\infty}\frac{\log(k/(k-1))}{(k^2-3)}$
1	.81152627246085310702...	≈	$\operatorname{arccosh}\pi$
	.8116126200701152567...	≈	$\frac{1}{2}+\frac{\sqrt{2}}{4}\operatorname{arcsinh}1 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(2k)!!}{(2k-1)!!}$
		=	$\sum_{k=1}^{\infty}\frac{(-1)^{k+1}4^k}{\binom{2k}{k}}$
	.8116826944033783883...	≈	$4\log 2-2\log^2 2-1 = \sum_{k=2}^{\infty}\frac{H_k}{2k^2-k}$
1	.8116826944033783883...	≈	$4\log 2-2\log^2 2 = \sum_{k=1}^{\infty}\frac{H_k}{2k^2-k}$
		=	$\log 2+\sum_{k=2}^{\infty}H_{k-1}\left(\zeta(2k)-1\right)$
1	.8117424252833536436...	≈	$MHS(2,2) = \frac{3\zeta(4)}{4} = \sum_{k>j\geq 1}\frac{1}{k^2j^2}$
	.8117977862339265120...	≈	$\sum_{k=1}^{\infty}H_k(\zeta(2k)-1) = -\sum_{k=2}^{\infty}\frac{\log\left(1-k^{-2}\right)}{1-k^{-2}}$

143 .81196393273824608939... ≈ $\displaystyle\sum_{k=1}^{\infty}\frac{\sigma_5(k)}{k!}$

.81201169941967615136... ≈ $\displaystyle\frac{6}{e^2} = \sum(-1)^{k+1}\frac{2^k k^4}{k!}$

1 .81218788563936349024... ≈ $\displaystyle\frac{2e}{3}$

.81251525878906250000... ≈ $\displaystyle\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^{2^2}}+\frac{1}{2^{2^{2^2}}}+\ldots$

$.8126984126984\underline{126984}$ = $\displaystyle\frac{256}{315}=\beta\left(5,\frac{1}{2}\right)$

.81320407336421530767... ≈ $\displaystyle\sqrt{\frac{2}{3}}\sin\frac{\pi\sqrt{2}}{3}=\prod_{k=1}^{\infty}1-\frac{1}{9k^2-1}$ J1064

.81327840526189165652... ≈ $\displaystyle-\zeta\left(\frac{1}{4}\right) = -\frac{2^{1/4}}{\pi^{3/4}}\zeta\left(\frac{3}{4}\right)\Gamma\left(\frac{3}{4}\right)\sin\frac{\pi}{8}$

1 .81337649239160349961... ≈ $\pi\gamma$

1 .8134302039235093838... ≈ $\displaystyle\frac{\sinh 2}{2}=\sum_{k=0}^{\infty}\frac{4^k}{(2k+1)!}$ GR 1.411.2

= $\displaystyle\prod_{k=1}^{\infty}\left(1+\frac{4}{\pi^2k^2}\right)$ GR 1.431.2

.81344319810303241352... ≈ $\displaystyle\frac{11}{4}-6\log 2+4\log^2 2+\frac{\zeta(3)}{4}$

= $\displaystyle\int_0^1\frac{(1+x)^2\log(1+x)}{x}dx$

.81354387406375956482... ≈ $\displaystyle\frac{4}{9}+\frac{\log 768}{18} = \sum_{k=1}^{\infty}\frac{kH_{2k}}{4^k}$

1 .81379936423421785059... ≈ $\displaystyle\frac{\pi}{\sqrt{3}} = \psi\left(\frac{1}{2}\right)-\psi\left(\frac{1}{2}\right) = hg\left(-\frac{1}{3}\right)-hg\left(-\frac{2}{3}\right)$

= $\displaystyle\frac{2\pi}{3}\sin\frac{2\pi}{3}$

= $\displaystyle\int_0^{\pi}\frac{d\theta}{2+\cos 2\theta}$

= $\displaystyle\int_0^{\infty}\log\left(1+\frac{1}{x(x+1)}\right)dx = \int_0^{\infty}\frac{\log(1+x^3)}{x^3}dx$

= $\displaystyle\int_0^{\pi/2}\frac{\sin x}{1-(\sin^2 x)/4}E\left(\frac{\sin x}{2}\right)dx$ GR 6.154

.814748604226469748138... ≈ $4-2\gamma+\frac{\pi^2}{6}-2\log 2\pi = \sum_{k=1}^{\infty}\frac{k^2}{k+2}\left(\zeta(k+1)-1\right)$

.8149421467733263011... ≈ $\frac{\pi^2}{3}-\pi+\frac{2}{3} = \sum_{k=1}^{\infty}\frac{\sin 2k}{k^3}$ GR 1.443.5

.8152842096899305458... ≈ $\pi\frac{3\sqrt{2}-2\sqrt{3}}{3} = \int_0^{\infty}\log\frac{1+x^{-4}}{1+x^{-3}}dx$

47 .81557574770022707230... ≈ $\frac{5\pi^5}{32} = \int_0^{\infty}\frac{\log^2 x\,dx}{x^2+1}$

= $\frac{5\pi^5}{16}+24i\left(Li_5(i)-Li_5(-i)\right) = \int_{-\infty}^{\infty}\frac{x^4}{e^x+e^{-x}}$

= $\int_0^{\infty}\frac{x^4\,dx}{\cosh x}$ GR 3.523.7

.815690849684834190314... ≈ $\sum_{k=1}^{\infty}\frac{1}{E_{2k}}$

.81595464948157421514... ≈ $\frac{\pi^3}{38}$

.8160006632992495351... ≈ $\frac{\pi^2-1}{\pi^2+1} = \tanh(\log\pi)$

.816048939098262981077... ≈ $\Gamma^{-1}\left(\frac{3}{4}\right) = \prod_{k=0}^{\infty}\left(1+\frac{2k+1}{2k+2}\right)$

2 .816190603250374170303... ≈ $\zeta(3)+\frac{\pi^2}{12}+\frac{19}{24} = \sum_{k=1}^{\infty}\frac{H_k H_k}{k(k+3)}$

3 .816262076667919561... ≈ $\frac{1}{2}\left(I_0\left(\frac{2}{\sqrt{e}}\right)+I_0\left(2\sqrt{e}\right)\right) = \sum_{k=0}^{\infty}\frac{\cosh k}{(k!)^2}$

2 .81637833042278439185... ≈ $\frac{6}{12-\pi^2} = \frac{1}{2-\zeta(2)} = \sum_{k=1}^{\infty}\frac{f(k)}{k^2}$ Titchmarsh 1.2.15

.81638547489578... ≈ $\sum_{k=1}^{\infty}\left(\zeta(3k-1)-\zeta(3k+1)+\zeta(3k)-1\right)$

.8164215090218931437... ≈ $-\sum_{k=1}^{\infty}\frac{\mu(2k)}{2^k-1} = \sum_{k=1}^{\infty}\frac{\mu(2k-1)}{2^{2k-1}-1}$

= $\sum_{k=1}^{\infty}\frac{1}{(\sqrt{2})^{2^k}} = \sum_{k=2}^{\infty}\frac{\lceil\log_2 k\rceil}{2^k} = \sum_{k=0}^{\infty}\frac{1}{2^{2^k}}$

.81649658092772603273... ≈ $\sqrt{\frac{2}{3}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{8^k}\binom{2k}{k}$ J168

.81659478386385079894... ≈ $\frac{3\pi\log 2}{8} = \int \log\left(\frac{1+x^2}{1+x}\right)\frac{dx}{1+x}$ GR 4.298.12

1 .81704584026913895975... ≈ $\sum_{k=1}^{\infty}\frac{H^{(3)}{}_k}{k!}$

1 .81712059283213965889... ≈ $\sqrt[3]{6}$

5 .8171817154095476464... ≈ $33-10e = \sum_{k=1}^{\infty}\frac{k^5}{(k+2)!}$

.817222646239917754960... ≈ $\frac{\pi}{e\sqrt{2}} = \sqrt{\pi}\,K_{1/2}(1) = \int_0^{\infty}\frac{\cos x-\sin x}{\sqrt{x}\,(x+1)}dx$

= $\int_0^{\infty}\frac{\cos x+\sin x}{x^2+1}\sqrt{x}\,dx$ Prud. 2.5.29.23

.81734306198444913971... ≈ $\zeta(6)-\frac{1}{5} = \sum_{k=1}^{\infty}\frac{1}{k^6}-\int_1^{\infty}\frac{dx}{x^6}$

.81745491353646342122... ≈ $\frac{3\pi}{2}-\frac{9}{4}-\frac{\pi^2}{6} = -\sum_{k=1}^{\infty}\frac{\cos 3k}{k^2}$ GR 1.443.2

= $\frac{1}{2}\left(Li_2\left(e^{3i}\right)+Li_2\left(e^{-3i}\right)\right)$

4 .81802909469872205712... ≈ $e\sqrt{\pi}$

.81818181818181818181 = $\frac{9}{11}$

1 .81844645923206682348... ≈ $\operatorname{arccsch}\frac{1}{3}$

4 .818486542041838256132... ≈ $\sum_{k=0}^{\infty}\frac{1}{K_k(1)}$

1 .81859485365136339079... ≈ $2\sin 2 = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{4^k}{(2k-1)!}$

2 .81875242548180183413... ≈ $\frac{\pi^3}{11}$

.8187801401720233053... ≈ $-\gamma-2\log 2+\log\pi = -\int_0^{\infty}\frac{\log x}{\cosh^2 x}dx$ GR 4.371.3

1 .81895849954020784402... ≈ $\zeta(2)^{\zeta(3)}$

3 .819290222081750098649... ≈ $26-32\log 2 = \sum_{k=1}^{\infty}\frac{k^3}{2^k(k+2)}$

26 .819615475040601592527... ≈ $-\sqrt{3}\sinh\pi\csc\pi\sqrt{3} = \prod_{k=1}^{\infty}\frac{k^2+2k+2}{k^2+2k-2}$

1 .81963925367740924975... ≈ $\frac{1}{4}\Phi\left(-4,3,\frac{1}{2}\right)=\int_1^{\infty}\frac{\log^2 x}{x^2+4}dx$

.82025951154241682326... ≈ $\sum_{k=1}^{\infty}\frac{1}{e^k-1} = \sum_{k=1}^{\infty}\frac{\sigma(k)}{e^k}$ Berndt 6.14.4

1 .82030105482706493402... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(8k-6)}{(2k-1)!} = \sum_{k=1}^{\infty}k^2\sinh\frac{1}{k^4}$

1 .82101745149929239041... ≈ $\prod_{k=1}^{\infty}\zeta(2k)$

.8221473284524977102... ≈ $\sum_{k=2}^{\infty}\left(2^{\zeta(k)-1}-1\right)$

1 .82222607449402372567... ≈ $3\zeta(3) - 2\gamma\,\zeta(3) + 2\zeta'(3) = \int_0^{\infty}\frac{x^2\log x}{e^x-1}dx$

1 .82234431433954947433... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(k+3)}{k!} = \sum_{k=1}^{\infty}\frac{e^{1/k}-1}{k^3}$

.82246703342411321824... ≈ $\frac{\pi^2}{12}$

= $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2}$ J306

= $\sum_{k=1}^{\infty}\frac{H_k}{2^k k}$

= $\sum_{k=1}^{\infty}\frac{S1(k,1)}{k!k}$

= $\sum_{k=2}^{\infty}k\left(\frac{\zeta(k)+\zeta(+1)}{2}-1\right)$

= $\int_0^{\infty}\frac{x}{1+e^x}dx = \int_1^{\infty}\frac{\log x}{x^2+x}dx = -\int_0^1\frac{\log(1-x^2)}{x}dx$

= $\int_0^{\log 2}\frac{x}{1-e^{-x}}dx$ GR 3.411.5

= $\int_0^1\frac{\log(1+x)}{x}dx = \int_0^{\infty}\log\left(1+e^{-x}\right)dx$ Andrews p.89, GR 4.291.1

= $-\int_0^1\frac{\log(1-x^2)}{x}dx$ GR 4.295.11

= $\int_1^2\frac{\log x}{x-1}dx$ [Ramanujan] Berndt Ch. 9

$= \quad -\int_0^1 \frac{\log x}{x+1} dx$ GR 4.231.1

$= \quad -\int_0^{\infty 1} \log\left(1+e^{-x}\right) dx$

$= \quad \int_0^\infty \frac{\log(1+x^2)}{x(1+x^2)} dx$ GR 4.295.18

$= \quad \int_0^\infty \log\left(\frac{1+x^2}{x^2}\right) \frac{x}{1+x^2} dx$ GR 4.295.16

$= \quad \int_1^\infty \log\left(1+\frac{1}{x}\right) \frac{dx}{x}$

$= \quad \int_0^\infty \frac{\log(x^2+1)}{x(x^2+1)} dx$

$= \quad \int_0^\infty \frac{x^3}{e^{x^2}-1} dx$

$= \quad \int_{-\infty 0}^\infty \frac{e^{-2x}}{e^{e^{-x}}+1} dx$ GR 3.333.2

$= \quad \int_0^\infty \frac{xe^{-x}}{\sin x} dx$

$= \quad \int_0^\infty \frac{x^2}{\cosh^2 x} dx$

1 .82303407111176773008... $\approx \quad \sum_{k=1}^\infty \frac{\zeta(4k-2)}{(2k-1)!} = \sum_{k=1}^\infty \sinh\left(\frac{1}{k^2}\right)$

.82329568993650930401... $\approx \quad \sum_{k=1}^\infty \frac{\mu(k)}{2^{2k-1}-1}$

.<u>8235294117647058</u> $= \quad \frac{14}{17}$

.8236806608528793896... $\approx \quad \gamma^2 + \frac{\pi^2}{6} - 2\gamma = \int_0^\infty \frac{x \log^2 x dx}{e^x}$

1 .82378130556207988599... $\approx \quad \frac{18}{\pi^2} = \frac{3}{\zeta(2)}$

1 .82389862825293059856... $\approx \quad \frac{\pi^3}{17}$

.82395921650108226855... $\approx \quad \frac{3\log 3}{4} = \sum_{k=1}^\infty \frac{\sin^3 x}{x^2}$

.82413881935225377438... ≈ $\frac{1}{2}\left(\gamma - ci(\pi) + \log\pi\right) = \int_0^{\pi/2} \frac{\sin^2 x}{x}$

.82436063535006407342... ≈ $\frac{\sqrt{e}}{2} = \sum_{k=0}^{\infty} \frac{1}{k!2^{k+1}} = \sum_{k=0}^{\infty} \frac{k}{k!2^k} = \sum_{k=0}^{\infty} \frac{k}{(2k)!!}$

.82437753892628282491... ≈ $5-\frac{2\pi^2}{3}+2\zeta(3) = \sum_{k=1}^{\infty} k^2\left(\zeta(k+3)-1\right)$

= $\sum_{k=2}^{\infty} \frac{k+1}{k^2(k+1)^3}$

.82447370907780915443... ≈ $\frac{1}{4}\sin 2 \sinh 2 = \prod_{k=1}^{\infty}\left(1-\frac{16}{\pi^4 k^4}\right)$

1 .824515157406924568142... ≈ $\frac{\sqrt{5}}{4} EllipticTheta^2\left[2,0,\frac{3-\sqrt{5}}{2}\right]$

= $\sum_{k=0}^{\infty} \frac{1}{F_{2k+1}} = \sqrt{5}\sum \frac{\phi^{2k+1}}{\phi^{4k+2}+1}$

.82468811844839402431... ≈ $\frac{4}{3}+\frac{\pi}{2}-3\log 2 = hg\left(\frac{3}{4}\right) = \sum_{k=1}^{\infty}(-1)^{k+1}\left(\frac{3}{4}\right)^k \zeta(k+1)$

= $\sum_{k=1}^{\infty} \frac{3}{k(4k+3)}$

.8248015620896503745... ≈ $-\sum_{k=1}^{\infty} \frac{\mu(2k)}{k!}$

8 .8249778270762876239... ≈ 2^{π}

.825041972433790848534... ≈ $\frac{i}{2}\left(\psi^{(1)}\left(\frac{2+i}{2}\right)-\psi^{(1)}\left(\frac{2-i}{2}\right)\right) = \sum_{k=1}^{\infty} \frac{k}{(k^2+1/4)^2}$

1 .82512195293745377856... ≈ $J_3(2)-6J_2(2)+7J_1(2)-J_0(2) = \sum_{k=0}^{\infty} \frac{(-1)^k k^5}{(k!)^2}$

13 .82561975584874069795... ≈ $(1+e)^2 = \sum_{k=0}^{\infty} \frac{2^k+2}{k!}$ LY 6.41

9 .82569657333515491309... ≈ $\frac{9}{G}$

2 .82589021611626168568... ≈ $\sum_{k=2}^{\infty} \frac{k}{k!-1}$

.825951986065842738464... ≈ $\sum_{k=0}^{\infty} \frac{(-1)^k p(k)}{2^k} = \prod_{k=1}^{\infty}\left(\frac{1}{1-(-1/2)^k}\right)$

.82606351573817904... ≈ $\sum_{k=2}^{\infty} \frac{(1)^k}{k^4+k^3+k^2+k+1}$

	.82659663289696621390...	≈	$\sum_{k=2}^{\infty}\frac{k\zeta(2k+1)}{3^k} = \sum_{k=1}^{\infty}\frac{3k}{(3k^2-1)^2}$	
	.8268218104318059573...	≈	$\sin^2 2 = \frac{1-\cos 2}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}2^{4k-1}}{(2k)!}$	GR 1.412.1
3	.82691348635397815802...	≈	$\frac{\pi\gamma^2}{2} + \frac{\pi^3}{24} + \pi\gamma\log 2 + \frac{\pi\log^2 2}{2}$	
		=	$\int_0^{\infty}\log^2 x\sin 2x\frac{dx}{x}$	GR 4.424.1
	.8269933431326880743...	≈	$\frac{3\sqrt{3}}{2\pi} = \prod_{k=1}^{\infty}\left(1-\frac{1}{9k^2}\right)$	GR 1.431
		=	$\prod_{k=1}^{\infty}\cos\frac{\pi}{3\cdot 2^k}$	
		=	$\binom{0}{1/3}$	
	.8270569031595942854...	≈	$\zeta(3)-\frac{3}{8}$	
	.82789710131633621783...	≈	$\frac{1}{1+e^{-\pi/2}} = \frac{1}{1+i^i}$	
4	.82831373730230112380...	≈	$3\log 5$	
	.828427124746190097603...	≈	$2\sqrt{2}-2 = \sum_{k=0}^{\infty}\frac{(-1)^k}{4^k(k+1)}\binom{2k}{k}$	
1	.828427124746190097603...	≈	$2\sqrt{2}-1 = \sum_{k=1}^{\infty}\frac{(2k+1)!!}{(2k)!!2^k}$	
	.82853544969022304438...	≈	$\frac{1}{\log^2 3} = \int_0^{\infty}\frac{x\,dx}{3^x}$	
5	.82857445396727733488...	≈	$-12Li_4\left(-\frac{1}{2}\right) = \int_0^{\infty}\frac{x^3\,dx}{e^x+1/2}$	
	.82864712767187850804...	≈	$\sum_{k=2}^{\infty}\frac{\log k!}{k!}$	
	.8287966442343199956...	≈	$14\zeta(3)-16 = -\psi^{(2)}\left(-\frac{1}{2}\right) = 2\sum_{k=2}^{\infty}\frac{1}{(k-\frac{1}{2})^3}$	
		=	$\int_0^{\infty}\frac{x^2}{e^{x/2}\left(e^x-1\right)}dx$	

16	.8287966442343199956...	≈	$14\zeta(3) \;=\; -\psi^{(2)}\left(\frac{1}{2}\right) \;=\; \sum_{k=0}^{\infty}\frac{1}{(k+\frac{1}{2})^3}$	
	.8293650197022233205...	≈	$\sum_{k=1}^{\infty}\frac{\pi(k)}{2^k} \;=\; 2\sum_{p\ prime}\frac{1}{2^p}$	
	.82945430337211926235...	≈	$\sum_{k=21}^{\infty}\frac{1}{k!\log k!}$	
	.82979085694614562146...	≈	$\zeta(3) - \gamma\zeta(2) + \gamma \;=\; \sum_{k=1}^{\infty}\frac{\psi(k+1)}{(k+1)^2}$	
1	.83048772171245191927...	≈	$\frac{1+\cos 1}{\sin 1} \;=\; \frac{i\left(e^i+1\right)}{e^i-1} = \frac{i\left(1+\cos 1+i\sin 1\right)}{(\cos + i\sin i - 1)}$	
	.83067035427178011249...	≈	$\sum_{k=2}^{\infty}\log\zeta(k)$	
5	.83095189484530047087...	≈	$\sqrt{34}$	
	.83105865748950903...	≈	$H^{(2)}{}_{2/3}$	
2	.83106601246967972516...	≈	$\frac{\pi}{\sqrt{3}}\log 7 - \frac{2\pi}{3}\operatorname{arccot}\frac{5}{\sqrt{3}} \;=\; \int_0^{\infty}\frac{\log(x^2+8)}{x^2+1}dx$	
	.8313214416001597873...	≈	$\frac{1}{2} - \frac{\pi}{2\sqrt{e}}\cot\frac{\pi}{\sqrt{e}} = \sum_{k=2}^{\infty}\frac{\zeta(2k)}{e^k} \;=\; \sum_{k=1}^{\infty}\frac{1}{ek^2-1}$	
8	.8317608663278468548...	≈	$\sqrt{78}$	
	.83190737258070746868...	≈	$\frac{1}{\zeta(3)} \;=\; \sum_{k=1}^{\infty}\frac{\mu(k)}{k^3} \;=\; \prod_{p}\left(1-\frac{1}{p^3}\right)$	
	.83193118835443803011...	≈	$2G-1 \;=\; \int_0^{\infty}\frac{x\tanh x}{e^x}dx$	
1	.83193118835443803011...	≈	$2G \;=\; i\left(Li_2(-i) - Li_2(i)\right)$	
		=	$\sum_{k=0}^{\infty}\frac{(k!)^2 4^k}{(2k)!(2k+1)^2}$	Berndt 9
		=	$\int_0^{\pi/2}\frac{x}{\sin x}dx$	Adamchik (2)
		=	$\int_0^{\infty}\frac{x}{\cosh x}dx$	
		=	$\int_0^{1}\frac{\arcsin x}{x^2-1}dx$	

= $\int_0^1 \frac{\operatorname{arcsin} h\, x}{x^2+1}\,dx$

= $\int_0^\infty \frac{\arctan x}{x\sqrt{1+x^2}}\,dx$ GR 4.531.11

= $\int_0^1 K'(x)\,dx$ GR 6.141.1

= $\int_0^1 K(x^2)\,dx$ Adamchik (16)

.83205029433784368302... ≈ $\frac{3}{\sqrt{13}} = \sum_{k=0}^\infty \frac{(-1)^k}{9^k}\binom{2k}{k}$

3 .83222717267891019358... ≈ $\frac{1}{4}\left(9\log 3+\pi\sqrt{3}\right) = \sum_{k=0}^\infty \frac{1}{(k+1)(k+1/3)}$

.83240650875238373999... ≈ $\frac{1}{4}\left(\gamma+3\log 2\right)\sqrt{\frac{\pi}{2}} = -\int_0^\infty e^{-2x^2}\log x dx$ GR 4.383.1

1 .83259571459404605577... ≈ $\frac{7\pi}{12}$

.83261846163379315125... ≈ $\gamma^{1/3}$

5 .8327186226814558356... ≈ $\frac{45\zeta(5)}{8} = -\int_0^1 \log(1+x)\frac{\log^3 x}{x}dx$

.83274617727686715065... ≈ $\frac{\gamma}{\log 2}$

.8330405509046936713... ≈ $\frac{3\pi}{8\sqrt{2}} = \int_0^\infty \frac{dx}{(x^4+1)^2}$

= $\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right) = \prod_{k=1}^\infty \frac{k(k+1)}{(k+1/4)(k+3/4)}$ J1061

.83327188647738995744... ≈ $Li_2\left(\frac{2}{3}\right)$

.833333333333333333$\underline{3}$ = $\frac{5}{6}$

= $\sum_{k=0}^\infty \frac{(-1)^k}{5^k}$

= $\sum_{k=1}^\infty \frac{1}{(k+1)(k+2)(k+3)}$

= $\sum_{k=2}^\infty \frac{k^5+k^4-k^3+k^2+k+1}{k^7-k}$

= $\displaystyle\sum_{k=1}^{\infty}\frac{1}{4^k(k+2)}\binom{2k}{k} = \sum_{k=1}^{\infty}\frac{(2k-1)!!}{(2k)!\,(k+2)}$

.8333953224586338242... ≈ $\displaystyle\sum_{k=1}^{\infty}\frac{|\mu(k)|(-1)^k}{2^k-1} = \sum_{k=1}^{\infty}\frac{|\mu(2k)|2^k}{4^k-1}$

.83373002513114904888... ≈ $\displaystyle\cos 1\cosh 1 = \sum_{k=0}^{\infty}(-1)^k\frac{4^k}{(4k)!}$

1 .83377265168027139625... ≈ root of $\zeta(x) = x$

1 .83400113670662422217... ≈ $\displaystyle\sum_{k=1}^{\infty}\frac{\zeta(2k)}{(2k-1)!} = \sum_{k=1}^{\infty}\frac{1}{k}\sinh\frac{1}{k}$

7 .8341682873053609132... ≈ $\displaystyle\frac{15\pi^2-90}{\pi^4-90} = \frac{\zeta(2)-1}{\zeta(4)-1}$

.83471946857721096222... ≈ $\displaystyle\frac{1}{3}\left(\frac{1}{e}+2\sqrt{e}\cos\frac{\sqrt{3}}{2}\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(3k)!}$

3 .83482070063335874733... ≈ $\displaystyle\sum_{k=0}^{\infty}\frac{pf(k)}{k!}$

.83501418762228265843... ≈ $\displaystyle\zeta(3)-\frac{1}{4}\left(Li_3(e^{2i})+Li\left(e^{-2i}\right)\right) = \sum_{k=1}^{\infty}\frac{\sin^2 k}{k^3}$

.83504557817601534828... ≈ $\displaystyle\frac{\pi^2}{3}+8\log 2-8 = \sum_{k=1}^{\infty}\frac{1}{k^3+k^2/2}$

= $\displaystyle 8\sum_{k=3}^{\infty}\frac{(-1)^{k+1}\zeta(k)}{2^k}$

= $\displaystyle\sum_{k=1}^{\infty}\frac{(k-\frac{1}{2})!}{k^2(k+\frac{1}{2})!}$

.835107636... ≈ $\displaystyle\sum_{k=1}^{\infty}\frac{h(k)}{k^2}$ 100 AMM 296 (Mar 1993)

.8352422189577681... ≈ $\displaystyle\sum_{k=1}^{\infty}\frac{1}{F_k k}$

4 .8352755061892873386... ≈ $\displaystyle\frac{31\pi^6-28350\zeta(5)}{84} = \int_0^1\frac{x\log^6 x\,dx}{(x+1)^3}$

1 .835299163476728902487... ≈ $\displaystyle\frac{5\pi^2}{24}-\frac{\pi}{2}(2-\log 2)+2G = \int_0^{\infty}\log(1+x)\log\left(1+\frac{1}{x^2}\right)dx$

.83535353257772361697... ≈ $\displaystyle\frac{135-\pi^4}{45} = 3-2\zeta(4)$

.83554275821033350805... ≈ $\displaystyle\frac{\sqrt{2\pi}}{3} = \int_0^{\infty}\frac{\sin x-x\cos x}{x^{5/2}}dx$ Prud. 2.5.29.24

.83564884826472105334... ≈ $\frac{\pi}{3\sqrt{3}} + \frac{\log 2}{3} = \sum_{k=1}^{\infty}\left(\frac{1}{6k-5} - \frac{1}{6k-2}\right)$ J79, K135

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{3k+1}$ K Ex. 113

= $\int_1^{\infty}\frac{dx}{x^2+x^{-1}} = \int_0^1\frac{dx}{x^3+1} = \int_0^{\infty}\frac{dx}{e^x+e^{-2x}}$

2 .83580364656519516508... ≈ $\frac{5\pi^3}{3\sqrt{3}} - 27 = \sum_{k=1}^{\infty}\left(\frac{(-1)^{k+1}}{(k-1/3)^3} - \frac{(-1)^{k+1}}{(k+1/3)^3}\right)$

1 .83593799333382666716... ≈ $\sum_{k=1}^{\infty}\frac{1}{(k!!)!!}$

.83599833270096432297... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)}{k^2} = \sum_{k=1}^{\infty}\left(Li_2\left(\frac{1}{k}\right) - \frac{1}{k}\right)$

.8362895669572625675... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k}{k!-1}$

.8366854409273997774... ≈ $e - e^{1-1/e} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+1)!e^k}$

6 .8367983046245809349... ≈ $2 + \frac{8\pi}{3\sqrt{3}} = \sum_{k=0}^{\infty}\frac{3^k}{\binom{2k+1}{k}}$

.83772233983162066800... ≈ $4 - \sqrt{10}$

4 .83780174825215167555... ≈ $4\csc^2 2 = \int_0^{\infty}\frac{\log x\ dx}{x^{\pi/2}-1}$

.837866940980208240895... ≈ $\operatorname{Coshint} 1$

.837877066409345483561... ≈ $\log(2\pi) - 1 = \sum_{k=1}^{\infty}\frac{\zeta(2k)}{k(2k+1)}$ Wilton
Messenger Math. 52 (1922-1923) 90-93

1 .837877066409345483561... ≈ $\log 2\pi$

.83791182769499313441... ≈ $\cos\frac{1}{\sqrt{3}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!3^k}$ AS 4.3.66, LY 6.110

.83800747784594108582... ≈ $\frac{\pi^3}{37}$

1 .838038955187488860348... ≈ $\frac{\sinh\pi}{2\pi} = \frac{1}{\Gamma(2+i)\Gamma(2-i)} = \sum_{k=0}^{\infty}\frac{\pi^{4k}}{(4k-1)!}$

= $\prod_{k=2}^{\infty}\left(1+\frac{1}{k^2}\right) = \prod_{k=1}^{\infty}\frac{k^2+2k+2}{k^2+2k+1}$

.83856063842880436639... ≈ $\log\frac{2e^2}{e^2-1}$ J157

= $-\sum_{k=1}^{\infty}\frac{B_k 2^k}{k!k}$ [Ramanujan] Berndt Ch. 5

.83861630300787491228... ≈ $\frac{125}{124\zeta(3)} = -\sum_{k=1}^{\infty}\frac{\mu(5k)}{k^3}$

.83899296971642587682... ≈ G^2

95 .83932260689807153299... ≈ $e^{\pi}(\pi+1) = \sum_{k=0}^{\infty}\frac{\pi^k(k+1)}{k!}$ GR 1.212

.83939720585721160798... ≈ $\frac{5}{e}-1 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^6}{(k+1)!}$

1 .83939720585721160798... ≈ $\frac{5}{e} = \Gamma(3,1)$

2 .83945179140231608634... ≈ $\log 3 + 2\sqrt{2}\arctan\frac{1}{\sqrt{2}} = \int_0^1 \log\left(1+\frac{2}{x^2}\right)dx$

.84006187044843982621... ≈ $\frac{\pi}{2}\log\frac{1+\sqrt{2}}{\sqrt{2}} = \int_0^1 \frac{\arcsin x}{x(1+x^2)}dx$ GR 4.521.6

1 .8403023690212202299... ≈ $\pi\left(2-\sqrt{2}\right) = \int_0^{2\pi}\frac{\sin^2 x}{1+\sin^2 x}dx = \int_0^{\infty}\log\left(1+\frac{2}{x^2+2}\right)dx$

.84042408656431894883... ≈ $\frac{7e^2-45}{8} = \sum_{k=0}^{\infty}\frac{2^k}{k!(k+7)}$

.84062506302375727160... ≈ $\pi\sqrt{2}\left(\sqrt[4]{2}-1\right) = \int_0^{\infty}\log\left(1+\frac{1}{x^4+1}\right)dx$

.840695833076274062... ≈ $-\frac{\sin\pi 2^{1/4}\sinh\pi 2^{1/4}}{\pi^2\sqrt{2}} = \prod_{2}^{\infty}\left(1-\frac{2}{k^4}\right)$

.84075154011728336936... ≈ $2-\frac{\zeta(3)}{\zeta(5)}$

.84084882503400147649... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{3^k-2^k}$

.84100146843744567255... ≈ $\frac{\pi^2-\pi}{8}$

1 .84107706809181431409... ≈ $\prod_{k=2}^{\infty}\left(1+\frac{1}{k!}\right)$

6 .84108846385711654485... ≈ $\pi^2\log 2 = \int_0^{\pi/2}\frac{4x^2\cos x+(\pi-x)x}{\sin x}dx$ GR 3.789

2 .84109848849173775273... ≈ $\frac{7\pi^4}{240} = \int_0^\infty \frac{x^4}{\cosh^2 x}dx$

= $-\int_0^1\int_0^1\int_0^1 \frac{\log xyz}{1+xyz}dx\,dy\,dz$

3 .84112255850390000063... ≈ $2\zeta(3) + \frac{111\pi^2}{36} - \frac{341}{216} = \sum_{k=1}^\infty \frac{H_{k+3}}{k^2}$

5 .84144846701247819072... ≈ $\pi\log 2 + 4G = \int_0^\pi \frac{\left(\frac{\pi}{2} - x\right)\cos x}{1-\sin x}dx$ GR 3.791.4

= $-\int_0^\pi \log(1-\sin x)\,dx$ GR 4.4334.10

.8414709848078965067... ≈ $\sin 1 = \operatorname{Im}\{e^i\} = \operatorname{Im}\{i^{2/3}\} = \sum_{k=0}^\infty \frac{(-1)^k}{(2k+1)!}$ AS 4.3.65

= $\binom{0}{1/\pi}$

= $\sqrt{\frac{\pi}{2}}J_1\left(\frac{1}{2},1\right)$

= $\prod_{k=1}^\infty \left(1 - \frac{1}{\pi^2 k^2}\right)$

= $\prod_{k=1}^\infty \cos\frac{1}{2^k}$ GR 1.439.1

= $\prod_{k=1}^\infty \left(1 - \frac{4}{3}\sin^2\frac{1}{3^k}\right)$ GR 1.439.2

= $\int_0^1 \cos x\,dx = \int_1^e \frac{\cos\log x}{x}dx$

.84168261071525616017... ≈ $\zeta(7) - \frac{1}{6} = \sum_{k=1}^\infty \frac{1}{k^7} - \int_1^\infty \frac{dx}{x^7}$

.841722868303139881174... ≈ $\frac{1}{2}(1 - 2\gamma + \log 2\pi) = \sum_{k=2}^\infty \frac{2^k}{k(k+1)}(\zeta(k) - 1)$

.84178721447693292514... ≈ $\pi\left(2 - \sqrt{3}\right) = \int_0^\infty \log\left(1 + \frac{1}{x^2+3}\right)dx$

1 .84193575527020599668... ≈ $\frac{1}{2}\left(Ei(2) - \log 2 - \gamma\right) = \sum_{k=0}^\infty \frac{2^k}{k!(k+1)^2}$

.84204887648141666667 1... ≈ $2K_0(1) = \int\limits_0^\infty \frac{\cos x}{\sqrt{1+x^2}}dx$

.<u>842105263157894736</u> = $\frac{16}{19}$

119 .842226666212576253... ≈ $\frac{1}{4096}\left(\psi^{(5)}\left(\frac{1}{4}\right)-\psi^{(5)}\left(\frac{3}{4}\right)\right) = -\int\limits_0^1 \frac{\log^5 x}{(x^2+1)}dx$

.84233963038644474542... ≈ $\sqrt{3}J_1\left(\frac{2}{\sqrt{3}}\right) = {}_0F_1\left(;2;-\frac{1}{3}\right) = \sum_{k=0}^\infty \frac{(-1)^k}{k!(k+1)!3^k}$

.84240657224478804816... ≈ $\sum_{k=1}^\infty \frac{1}{2^k F_k}$

.8425754910523763415... ≈ $\frac{1}{3\sqrt[3]{e}-3} = \sum_{k=1}^\infty \frac{B_k}{3^k k}$

.8427007929497148693 4... ≈ $erf\,(1) = \frac{2}{\sqrt{\pi}}\sum_{k=0}^\infty \frac{(-1)^k}{k!(2k+1)}$ AS 7.1.5

= $\frac{2}{\sqrt{\pi}}\int\limits_0^1 e^{-x^2}dx$

6 .84311396670851712778... ≈ $3\sqrt{3}\log\frac{\sqrt{3}+1}{\sqrt{3}-1} = \sum_{k=0}^\infty \frac{(-1)^k}{(k+1/6)(k+5/6)}$

10 .84311984239192954237... ≈ $3\gamma\log 3 + 4\gamma\log 2 + \frac{3}{2}\log^2 12 - \log^2 4$

= $l\left(-\frac{1}{6}\right)+l\left(-\frac{5}{6}\right)$ Berndt 8.17.8

.8432441918208866651... ≈ $\sum_{k=1}^\infty \frac{k^2}{4^k-1}$

.8435118416850346340... ≈ $G - \frac{\pi^2}{16} + \frac{\pi\log 2}{4} = \int\limits_0^1 \frac{\arctan^2 x\,dx}{x^2}$

= $\int\limits_0^{\pi/4} \frac{x^2 dx}{\sin^2 x}$ GR 3.837.2

.8437932862030881776... ≈ $\cos^3\frac{1}{3} = \frac{1}{4}\sum_{k=0}^\infty (-1)^k\,\frac{(1+3^{1-2k})}{(2k)!}$ GR 1.412.4

16 .84398368125898806741... ≈ $e^2 I_0(2) = \sum_{k=0}^\infty \binom{2k}{k}\frac{1}{k!}$

.8440563052346255265... ≈ $2\sin^2\frac{1}{\sqrt{2}} = \sum_{k=1}^\infty \frac{(-1)^{k+1}2^k}{(2k)!} = \sum_{k=0}^\infty \frac{(-1)^k 2^k}{(2k+1)!(k+1)}$

.84416242155207814332... ≈ $\frac{483\log 2}{2} + \frac{261\log^2 2}{2} - \frac{917}{4}$

= $\sum_{k=1}^{\infty} \frac{k^4 H_k}{2^k (k+1)(k+2)(k+3)}$

1185 .84441907811295583887... ≈ $120\zeta(6) + 360\zeta(5) + 39\zeta(4) + 180\zeta(3) + 31\zeta(2) + 1$

= $\sum_{k=2}^{\infty} k^5 \left(\zeta(k) - 1\right)$

.84442580886220444850... ≈ $Li_3\left(\frac{3}{4}\right)$

9666 .84456304416576336457... ≈ $\cosh \pi^2 \ = \ \sum_{k=0}^{\infty} \frac{\pi^{4k}}{(2k)!}$

.844657620095592823802... ≈ $\frac{4}{11} + \left(\frac{2}{11} \sqrt{\frac{2}{55}(151 + 75\sqrt{5})}\, arc\csc \sqrt{2\left(\sqrt{5} - 1\right)} \right)$

$+ \ \frac{2(5+\sqrt{5})}{5\left(3 + 2\sqrt{5}\right)^{3/2}}\, arc\csc h \sqrt{2\left(1 + \sqrt{5}\right)}$

= $\sum_{k=1}^{\infty} \frac{F_k}{\binom{2k}{k}}$

350 .8446720000000000000$\underline{0}$ = $\frac{5481948}{15625} \ = \ \sum_{k=1}^{\infty} \frac{k^4 F_k^{\ 2}}{4^k}$

.84483859475710240076... ≈ $\frac{1}{4}\Gamma\left(\frac{1}{4}, 0, 1\right) \ = \ \Gamma\left(\frac{5}{4}\right) - \frac{1}{4}\Gamma\left(\frac{1}{4}, 1\right) \ = \ \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(4k+1)}$

.84502223280969297766... ≈ $\cos \frac{1}{\sqrt{\pi}} \ = \ \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!\pi^k}$ AS 4.3.66, LY 6.110

.8451542547285165775... ≈ $\sqrt{\frac{5}{7}} \ = \ \sum_{k=0}^{\infty} \frac{(-1)^k}{10^k} \binom{2k}{k}$

.84515451462286429392... ≈ $9 - 3e \ = \ \sum_{k=1}^{\infty} \frac{k^3}{(k+2)!}$

.84528192903489711771... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(2k)}{(2k-1)!2^{2k-1}} \ = \ \sum_{k=1}^{\infty} \frac{1}{k} \sinh \frac{1}{2k}$

.84529085018832183660... ≈ $\frac{\pi^2}{8} - \frac{1}{2}\log^2\left(1 + \sqrt{2}\right) \ = \ \sum_{k=0}^{\infty} \frac{(-1)^k (k!)^2 4^k}{(2k)!(2k+1)^2}$ Berndt Ch. 9, 32

= $\int_0^{\pi/2} \frac{x \sin x}{2 - \sin^2 x} dx$

1 .84556867019693427879... ≈ $3 - 2\gamma \ = \int_0^{\infty} \frac{x^2 \log x}{e^x} dx$

.846153846153$\underline{846153}$ = $\frac{11}{13}$

.84633519370869490299... ≈ $\frac{\zeta(6)}{\zeta(3)} = \sum_{k=1}^{\infty}\frac{\lambda(k)}{k^3}$

1 .84633831078181334102... ≈ $\frac{7}{6}\left(\sqrt{21}-3\right) = \sum_{k=0}^{\infty}\binom{2k}{k}\frac{1}{7^k}$

.84642193400371817729... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k\zeta(k)}{(k-2)!} = \sum_{k=1}^{\infty}\frac{1}{k^2 e^{1/k}}$

.84657359027997265471... ≈ $\frac{\log 2}{2}+\frac{1}{2} = \int_2^{\infty}\frac{\log x}{x^2} = \int_1^{\infty}\cosh(\log(1+x))\frac{dx}{x^3}$

.84662165026149600620... ≈ $2(\pi-e)$

.84699097000782072187... ≈ $\zeta(2)+\zeta(3)-2 = \sum_{k=2}^{\infty}\left(\zeta(k)-\zeta(k+2)\right)$

= $\sum_{k=1}^{\infty}\frac{2k+1}{k^5+2k^4+k^3}$

2 .84699097000782072187... ≈ $\zeta(2)+\zeta(3)$

.8470090659508953726... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^k}{k^5+k^4+k^3+k^2+k+1}$

.8472130847939790866... ≈ $\frac{1}{\sqrt{\pi}}\Gamma^2\left(\frac{3}{4}\right) = \int_0^{\pi/2}\sqrt{\sin x\cos x}\,dx$

.847406572244788... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^k F_k}$

.8474800638725324646... ≈ $\gamma(1+e)-Ei(1)-eEi(-1) = \sum_{k=1}^{\infty}\frac{H_k}{(k+1)!}$

= $\sum_{k=1}^{\infty}\frac{\gamma+\psi(1+k)}{(k+1)!}$

= $\sum_{k=1}^{\infty}\sum_{m=1}^{k}\frac{1}{m!(k+1)!} = \sum_{k=1}^{\infty}\sum_{m=1}^{k}\frac{(-1)^{k+1}e}{m!(k+1)!}$ AMM 101,7 p.682

3 .8474804859040022123... ≈ $\sum_{k=1}^{\infty}\frac{1}{(k!)^3}\binom{2k}{k}$

1 .84757851036301103063... ≈ $\log 2+2\gamma = \sum_{k=1}^{\infty}\left(\frac{2}{k}-\log\frac{k+2}{k}\right)$ Prud. 5.5.1.15

= $\int_0^2 H(x)\,dx$

.84760282551739384823... ≈ $\pi\left(\frac{4\sqrt{3}}{9}-\frac{1}{2}\right) = \sum_{k=1}^{\infty}\frac{(k-\frac{1}{2})!(k+\frac{1}{2})!}{(2k)!}$

2 .84778400685138213164... ≈ $\sum_{k=0}^{\infty}\frac{\zeta(k+4)}{k!} = \sum_{k=1}^{\infty}\frac{e^{1/k}}{k^4}$

.84782787797694820792... ≈ $\frac{7\pi^3}{256} = \sum_{k=1}^{\infty}\frac{1}{k^3}\sin\frac{k\pi}{4}$ GR 1.443.5

= $\left(\frac{1}{4}+\frac{i}{4}\right)(-1)^{1/4}\sqrt{2}\left(Li_3\left(e^{-\pi i/4}\right)-Li_3\left(e^{i\pi/4}\right)\right)$

.84830241699385145616... ≈ $\sum_{k=1}^{\infty}3^k\left(\zeta(3k)-1\right) = \sum_{k=2}^{\infty}\frac{3}{k^3-3}$

3 .84831187770367927099 3... ≈ $-\frac{i}{2}\left(Li_2(e^{i/2})-Li_2(e^{-i/2})\right) = \sum_{k=1}^{\infty}\frac{1}{k^2}\sin\frac{k}{2}$

1 .84839248149318749178... ≈ $\frac{8\log 2}{3} = \int_1^{\infty}\frac{\log(x+3)}{x^2}dx$

.84863386796488363268... ≈ $\sum_{k=2}^{\infty}\left(\frac{\zeta(k)}{\zeta(2k)}-1\right)$ = ?? = $\sum_{k=2}^{\infty}\frac{|\mu(k)|}{k(k-1)}$

= sum of inverse non-trivial powers of products of distinct primes

5 .84865445990480510746... ≈ $\frac{16\pi^2}{27} = \int_0^{\infty}\frac{\log x\ dx}{x^{3/2}-1}$

.8488263631567751241... ≈ $\frac{8}{3\pi} = \prod_{k=2}^{\infty}\left(1-\frac{1}{4k^2}\right)$

9 .8488578017961047217... ≈ $\sqrt{97}$

.84887276700404459187... ≈ $\frac{\sqrt{\pi}}{e^{1/4}}erfi\frac{1}{2} = \sum_{k=0}^{\infty}\frac{(-1)^k k!}{(2k+1)!} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!!2^k}$

.84919444474737692368... ≈ $-\frac{1}{54\zeta(3)}\psi^{(2)}\left(\frac{1}{3}\right) = \frac{1}{\zeta(3)}\sum_{k=1}^{\infty}\frac{1}{(3k-2)^3}$

.84932046884046441263 3... ≈ $\sum_{k=1}^{\infty}\frac{\Phi(k)}{3^k}$

18 .84955592153875943078... ≈ 6π

.84958150985033544395 5... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k!!)^2}$

.849732991384718766... ≈ $\sum_{k=1}^{\infty}\frac{\sigma_0(k)\mu(k)}{k^4}$

.84985193128795296818... ≈ $\sum_{k=1}^{\infty}H_{2k-1}\left(\zeta(2k)-1\right)$

= $\sum_{k=2}^{\infty}\frac{1}{2(k^2-1)}\left(k\log\left(1+\frac{1}{k}\right)-k\log\left(1-\frac{1}{k}\right)-\log\left(1-\frac{1}{k^2}\right)\right)$

= $\frac{\log 2}{4}+\sum_{k=1}^{\infty}\frac{\zeta(2k)-1}{2k-1} = \frac{\log 2}{4}+\sum_{k=2}^{\infty}\left(\frac{1}{k}\operatorname{arctan}h\frac{1}{k}\right)$

.8504060682786322487... ≈ $3-\frac{3}{\sqrt[3]{e}}=\sum_{k=0}^{\infty}\frac{(-1)^k}{(k+1)!3^k}$

.8506818787873336234... ≈ $\frac{\pi}{8}+\frac{G}{2}=\int_0^1\frac{\log x}{(x^2+1)^2}dx$

.85069754062437546892... ≈ $2\gamma+4\log 2+\psi\left(\frac{1}{2}+\frac{i}{4}\right)+\psi\left(\frac{1}{2}-\frac{i}{4}\right)$

1 .85081571768092561791... ≈ $\sec 1=\sum(-1)^k\frac{E_{2k}}{(2k)!}$

.8509181282393215451... ≈ $\operatorname{csch} 1=\frac{i}{\sin i}=\frac{2}{e-e^{-1}}$ J132

= $2\sum_{k=0}^{\infty}\frac{1}{e^{2k+1}}$ GR 1.232.3

= $1-\sum_{k=1}^{\infty}\frac{2(2^{2k-1})B_{2k}}{(2k)!}$ AS 4.5.65

.851441200981879906279... ≈

$$\frac{1}{3\cdot 2^{2/3}}\left((1+i\sqrt{3})\psi\left(\frac{2-(-2)^{1/3}}{4}\right)+(1-i\sqrt{3})\psi\left(\frac{1}{2}+\frac{1}{2}\left(-\frac{1}{2}\right)^{2/3}\right)-2\psi\left(\frac{2+2^{1/3}}{4}\right)\right)$$

= $\sum_{k=0}^{\infty}\frac{1}{(2k+1)\left((2k+1)^2+1/4\right)}$

.85149720152646256755... ≈ $-H_{-1/e}=-\gamma-\psi\left(1-\frac{1}{e}\right)=\sum_{k=1}^{\infty}\frac{1}{k(ek-1)}$

= $\sum_{k=2}^{\infty}\frac{\zeta(k)}{e^{k-1}}$

.85153355273320083373... ≈ $e\log(1+e)-e=\sum_{k=0}^{\infty}\frac{(-1)^k}{e^k(k+1)}$

.85168225614364627497... ≈ $\frac{i}{4}\left(\psi\left(\frac{1-i}{4}\right)-\psi\left(\frac{1+i}{4}\right)-\psi\left(\frac{3-i}{4}\right)+\psi\left(\frac{3+i}{4}\right)\right)$

= $\int_0^{\infty}\frac{\sin x}{\cosh x}dx$

1 .851937051982466170361... ≈ $si(\pi)=\int_0^{\pi}\frac{\sin x}{x}dx$, Wilbraham-Gibbs constant

4 .85203026391961716592... ≈ $7\log 2$

.85212343939396411969... ≈ $2\gamma+\psi\left(\frac{i}{\sqrt{2}}\right)+\psi\left(-\frac{i}{\sqrt{2}}\right)=\sum_{k=1}^{\infty}\frac{1}{k(k^2+1/2)}$

.85225550915074454010... ≈ $2^{2^{\frac{1}{2}\log 2-\gamma}}=\prod_{k=1}^{\infty}\frac{(2k-1)^{1/(2k-1)}}{(2k)^{1/2k}}$ Berndt 8.17.8

1 .85236428911566370893... ≈ $\frac{1}{4}\Phi\left(-3,3,\frac{1}{2}\right) = \int_1^\infty \frac{\log^2 x}{x^2+3}dx$

.85247896683354111790... ≈ $3\zeta(2)-\zeta(4)-3 = \sum_{k=2}^\infty (-1)^k k\big(\zeta(k)-\zeta(k+3)\big)$

= $\sum_{k=2}^\infty \frac{2k^4+k^3-2k-1}{k^4(k+1)^2}$

.85306006230743608251...≈ $\frac{4}{\sqrt{\pi}\,\Gamma\left(\frac{(-1)^{1/3}}{2}\right)\Gamma\left(\frac{-(-1)^{2/3}}{2}\right)} = \prod_{k=1}^\infty\left(1-\frac{1}{8k^3}\right)$

.85358153703118403189... ≈ $\frac{4}{3}\left(\frac{4}{3}-\log 2\right) = \sum_{k=1}^\infty \frac{1}{k(k+3/2)}$

= $\int_0^\infty xe^{-x}\sqrt{1-e^{-x}}\,dx$ GR 3.451.1

.85369546139223027354... ≈ $2\log 2+\frac{\pi^2}{4}-3 = \sum_{k=2}^\infty \frac{k(\zeta(k)-1)}{2^{k-1}}$

= $\sum_{k=2}^\infty \frac{4k-1}{(2k-1)^2 k}$

1 .85386291731316255056... ≈ $\sum_{k=1}^\infty \frac{1}{2^k-k}$

7 .85398163397448309616... ≈ $\frac{5\pi}{2}$

1 .85407467730137191843... ≈ $\frac{1}{4\sqrt{\pi}}\Gamma^2\left(\frac{1}{4}\right) = 2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-1\right) = \int_0^\infty \frac{dx}{\sqrt{x^4+1}}$

= $K\left(\frac{1}{2}\right)$

.85410196624968454461... ≈ $\frac{3\sqrt{5}-5}{2} = \sum_{k=0}^\infty \frac{(-1)^k}{5^k(k+1)}\binom{2k}{k}$

4 .854318108069644090155...≈ $4\pi\big(2\log 2-1\big) = \int_0^\infty \log x\log\left(1+\frac{16}{x^2}\right)dx$ GR 4.222.3

1 .85450481290441694628... ≈ $\sum_{k=2}^\infty \frac{\zeta(k)}{(2k-3)!} = \sum_{k=1}^\infty k^{-3/2}\sinh\sqrt{\frac{1}{k}}$

.85459370928947691247... ≈ $\frac{\gamma}{2}+\frac{\pi}{8}+\frac{\log 2}{4} = -\int_0^\infty \frac{\log x\cos x}{e^x}dx$

6 .85479719023474902805... ≈ $\frac{e^e-e^{1/e}}{2} = e^{\cosh 1}\sinh(\sinh 1) = \sum_{k=1}^\infty \frac{\sinh k}{k!}$ GR 1.471.1

.85562439189214880317... ≈ $\sqrt{\frac{\pi}{2}}\, erf\, \frac{1}{\sqrt{2}} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!2^k(2k+1)}$

2 .85564270285481672333... ≈ $\sum_{k=0}^{\infty} \frac{bp(k)}{2^k} = \prod_{k=0}^{\infty} \frac{1}{1-2^{2^k}}$

6 .85565460040104412494... ≈ $\sqrt{47}$

1 .85622511836152235066... ≈ $\sum_{k=2}^{\infty} \frac{(-1)^k k^3}{k!} \left(\zeta(k)-1\right)$

= $\sum_{k=2}^{\infty} \frac{k^2 e^{1/k} - k^2 + 3k - 1}{k^3 e^{1/k}}$

8 .856277053111707602292...≈ $\frac{2e^{\pi}-2}{5} = \int_0^{\pi} e^x \sin^2 x\, dx$

.85646931665632420671... ≈ $\pi\left(1-e^{-1/\pi}\right) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\pi^k (k+)1)}$

.85659704311393584941... ≈ $\zeta(2) - \frac{\pi}{4} \coth \pi$

$.857142857142\underline{857142}$ = $\frac{6}{7} = \sum_{k=0}^{\infty} \frac{(-1)^k}{6^k}$

.85745378253311466066... ≈ $\frac{\pi}{4G}$

.85755321584639341574... ≈ $\cos\cos 1$

.85758012275100904702... ≈ $\sum_{k=2}^{\infty} \frac{\zeta(k)}{2^k-1}$

.857842886877026832381...≈ $4\zeta(5) - \frac{\pi^2}{3} = -\int_0^{\infty} x^{-1} Li_2(-x)^2\, dx$

.85788966542159767886... ≈ $\frac{3}{2}\zeta(3) + 2Li_3\left(-\frac{1}{2}\right) = \int_0^1 \frac{\log^2 x}{(x+1)(x+2)} dx$

.858007616357088565688...≈ $\sum_{k=3}^{\infty} \frac{S1(k,3)}{2^k}$

.85840734641020676154... ≈ $4 - \pi = \int_0^{\pi} \frac{\cos^2 x}{(1+\sin x)^2}$

= $\int_0^1 \frac{\arccos x}{\sqrt{1+x}} dx$

= $\int_0^\infty \log(1+x^2)\frac{\cosh\pi x + \pi x\sinh\pi x}{\cosh^2\pi x}\frac{dx}{x^2}$ GR 4.376.11

.85902924121595908864... ≈ $\frac{35\pi}{128} = \int_0^1 \frac{x^{7/2}dx}{\sqrt{1-x}}$

.85930736722073099427... ≈ $24 - e^\pi$

7 .85959984818146412704... ≈ $\sum_{k=0}^\infty \frac{2^k\zeta(k+3)}{k!} = \sum_{k=1}^\infty \frac{e^{2k}}{k^3}$

.859672000699493639687... ≈ $\sum_{k=1}^\infty \sigma_0(k)\left(\zeta(2k)-1\right)$

5 .85987448204883847382... ≈ $\pi + e$ Not known to be transcendental

.86009548753481444032... ≈ $\sum_{k=1}^\infty \frac{\zeta^2(2k)-1}{(2k)!}$

.86013600393341676239... ≈ $96 - 35e = \sum_{k=1}^\infty \frac{k^2}{k!(k+4)}$

.860525175709529372495... ≈

$$\frac{1}{3\cdot 2^{1/3}}\left(\left(1-i\sqrt{3}\right)\psi\left(\frac{2+(-2)^{2/3}}{2}\right)+\left(1+i\sqrt{3}\right)\psi\left(1-\left(-\frac{1}{2}\right)^{1/3}\right)-2\psi\left(1+\frac{1}{2^{1/3}}\right)\right)$$

$$= \sum_{k=1}^\infty \frac{1}{k^3+1/2} = \sum_{k=1}^\infty \frac{(-1)^{k+1}\zeta(3k)}{2^{k-1}}$$

1 .8608165667814527048... ≈ $\frac{4}{3}e^{1/3} = \sum_{k=0}^\infty \frac{k+1}{k!3^k}$

.8608178819280080778... ≈ $\frac{4}{\sqrt{5}}\operatorname{arcsinh}\frac{1}{2} = \frac{4}{\sqrt{5}}\log\left(\frac{1+\sqrt{5}}{2}\right) = \sum_{k=1}^\infty \frac{F_k}{2^k k}$

$$= \sum_{k=0}^\infty \frac{(-1)^k}{(2k+1)\binom{2k}{k}} = \int_2^\infty \frac{dx}{x^2-x-1}$$

.8610281005737279228... ≈ $\frac{\pi}{2\sqrt{2}}\coth\pi\sqrt{2} - \frac{1}{4} = \sum_{k=1}^\infty \frac{1}{k^2+2}$ J124

.861183557332564183... ≈ $\frac{1}{4}\Phi(2,2,\frac{1}{2}) = \int_1^\infty \frac{\log x dx}{x^2+2}$

.8612202133408014822... ≈ $\zeta(8) - \frac{1}{7} = \sum_{k=1}^\infty \frac{1}{k^8} - \int_1^\infty \frac{dx}{x^8}$

.8612854633416616715... ≈ $\frac{\pi^3}{36}$

1 .86148002026189245859... ≈ $8\log 2 + 3\zeta(3) - \frac{\pi^2}{3} - 4 = \int_0^1\int_0^1\int_0^1\int_0^1 \frac{w+x+y+z}{1+wxyz}\,dw\,dx\,dy\,dz$

.86152770679629637239... ≈ $\sqrt{\pi}\,erf 1 - 1 + \frac{1}{e} = -1 - \frac{1}{2}\Gamma\left(-\frac{1}{2},0,1\right) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!(2k-1)}$

.86213147853485134492... ≈ $\frac{\pi^2}{8} + \frac{\pi\log 2}{4} - G = \int_0^{\pi/2}\frac{x^2\,dx}{(\sin x + \cos x)^2}$

.86236065559322214... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^k\,\phi(k)}$

20 .8625096400513696180... ≈ $\frac{1}{2e}\left(e^{1/e} + e^{2+e}\right) = \sum_{k=0}^{\infty}\frac{k\cosh k}{k!}$

.8630462173553427823... ≈ $3\log\frac{4}{3} = \sum_{k=0}^{\infty}\frac{(-1)^k}{3^k(k+1)} = \int_0^{\infty}\frac{1}{e^x + 1/3}$

.8632068016894392378... ≈ $\frac{\pi^2}{6} + \sum_{k=1}^{\infty} kH_k\left(\zeta(k+1) - 1\right) = \sum_{k=1}^{\infty}\frac{1}{k^2}\log\frac{k+1}{k}$

4 .8634168148322130293... ≈ $\frac{48}{\pi^2} = \frac{8}{\zeta(2)}$

3 .86370330515627314699... ≈ $\sqrt{2}\left(1+\sqrt{3}\right) = \csc\frac{\pi}{12}$ AS 4.3.46

.86390380998765775594... ≈ $\frac{27}{26\zeta(3)} = -\sum_{k=1}^{\infty}\frac{\mu(3k)}{k^3}$

.86393797973719314058... ≈ $\frac{11\pi}{40} = \int_0^{\infty}\frac{\sin^6 x}{x^6}dx$ GR 3.827.15

.86403519948264378410... ≈ $\frac{1}{8}\left(3e - 4e^{\cos 2}\cos\sin 2 + e^{\cos 4}\cos\sin 4\right) = \sum_{k=1}^{\infty}\frac{\sin^4 k}{k!}$

1 .864387735234080589739... ≈ $\left(6\left(\sqrt[3]{9} - 1\right)\right)^{1/3} = \sec^{1/3}\frac{2\pi}{9} + \sec^{1/3}\frac{4\pi}{9} - \sec^{1/3}\frac{\pi}{9}$

[Ramanujan] Berndt Ch. 22

.8645026534612020404... ≈ $\frac{1}{8}\left(\zeta\left(\frac{3}{2},\frac{1}{4}\right) - \zeta\left(\frac{3}{2},\frac{3}{4}\right)\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)^{3/2}}$

.8646647167633873081... ≈ $1 - e^{-2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}2^k}{k!}$

2 .86478897565411604384... ≈ $\frac{9}{\pi}$

.864988826349078353... ≈ $\sum_{k=2}^{\infty}\frac{1}{3^k(\zeta(k) - 1)}$

.86525597943226508721... ≈ $\frac{e}{\pi}$

1 .8653930155985462163... ≈ $7\zeta(2) + 12\zeta(4) - 18\zeta(3) - 1$

= $\sum_{k=2}^{\infty} (-1)^k k^3 \big(\zeta(k) - \zeta(k+1)\big)$

= $\sum_{k=2}^{\infty} \frac{8k^4 - k^3 - k^2 - 3k - 1}{k^2 (k+1)^4}$

.86558911417184854991... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(8k-6)}{(2k)!} = \sum_{k=1}^{\infty} k^6 \left(\cosh\left(\frac{1}{k^4}\right) - 1 \right)$

.86562170856366484625... ≈ $\frac{\gamma}{1-\gamma^2}$

.86576948323965862429... ≈ $2 \arctan e - \frac{\pi}{2} = gd1$

= $\arctan(\sinh 1) = \sum_{k=-\infty}^{\infty} (-1)^{k+1} \arctan\left(\frac{1}{(k+1/2)\pi} \right)$

[Ramanujan] Berndt Ch. 2, Eq. 11.4

= $\arcsin(\tanh 1)$

.86602540378443864676... = $\frac{\sqrt{3}}{2} = \sin\frac{\pi}{3} = \cos\frac{\pi}{6}$

= $\sum_{k=0}^{\infty} \frac{(-1)^k}{12^k} \binom{2k}{k}$

.86683109649187783728... ≈ $\frac{1}{2} + \frac{\pi\sqrt{5}}{10} \operatorname{csch} \frac{\pi}{\sqrt{5}} = \sum_{k=0}^{\infty} \frac{(-1)^k}{5k^2+1}$

.86697298733991103757... ≈ $\frac{1}{4\sqrt{2}} \left(\pi + 2\log\left(1+\sqrt{2}\right) \right) = \sum_{k=0}^{\infty} \frac{(-1)^k}{4k+1}$ K135

= $\sum_{k=1}^{\infty} \left(\frac{1}{8k-7} - \frac{1}{8k-3} \right)$ J82, K ex. 113

= $\frac{\pi}{16} - \frac{1}{4\sqrt{2}} \left(2\arctan\left(2+\sqrt{2}\right) - 2\arctan\left(2-\sqrt{2}\right) + \log\frac{2-\sqrt{2}}{2+\sqrt{2}} \right)$

= $\frac{\pi}{16} \left(4\sqrt{2} + \cot\frac{7\pi}{8} - \cot\frac{3\pi}{8} \right) - 4\sqrt{2} \left(\log\sin\frac{\pi}{8} - \log\sin\frac{3\pi}{8} \right)$

= $\int_1^{\infty} \frac{dx}{x^2 + x^{-2}} = \int_0^1 \frac{dx}{1+x^4}$

4 .86698382463297672998... ≈ $\frac{2}{3}(\pi + 6\log 2) = \sum_{k=0}^{\infty} \frac{1}{(k+1)(k+1/4)}$

.86718905113631807520... ≈ $2\operatorname{arcsinh}\frac{1}{\sqrt{2}} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}2^{k}}{k^{2}\binom{2k}{k}}$

5 .8673346208932640333... ≈ $\frac{1}{128}\left(\psi^{(3)}\left(\frac{1}{4}\right)-\psi^{(3)}\left(\frac{3}{4}\right)\right)-6 = \int_{0}^{\infty}\frac{x^{3}\tanh x}{e^{x}}dx$

11 .8673346208932640333... ≈ $\frac{1}{128}\left(\psi^{(3)}\left(\frac{1}{4}\right)-\psi^{(3)}\left(\frac{3}{4}\right)\right) = \int_{0}^{\infty}\frac{x^{3}\,dx}{\cosh x}$

1 .86739232503269523903... ≈ $\frac{1}{\pi}\sin\left(\pi\sqrt{-1+i}\right)\sin\left(\pi\sqrt{-1-i}\right)\,\operatorname{csc}h\,\pi\sqrt{2} = \prod_{k=1}^{\infty}\frac{k^{4}+2k^{2}+2}{k^{4}+2k^{2}}$

5 .86768692676152924896... ≈ $\sum_{k=1}^{\infty}\frac{\phi(k)}{F_{k}}$

.86768885868496331882... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k}}{S_{2}(2k,k)}$

1 .86800216804463044688... ≈ $\frac{\pi}{2^{3/4}} = \int_{0}^{\infty}\frac{dx}{x^{4}+1/2}$

.86815220783748476168... ≈ $\frac{13\zeta(3)}{18} = -\frac{13\zeta(3)}{108}-\frac{1}{432}\left(\psi^{(2)}\left(\frac{1}{6}\right)+\psi^{(2)}\left(\frac{5}{6}\right)\right)$

= $\nu_{3} = \sum_{k=0}^{\infty}\left(\frac{(-1)^{k}}{(3k+1)^{3}}-\frac{(-1)^{k}}{(3k+2)^{3}}\right)$ J312

.86872356982626095207... ≈ $\zeta(3)-\frac{1}{3}$

1 .86883374092715470879... ≈ $\zeta^{2}(3)+\frac{\pi^{6}}{2268} = MHS(4,2)$

.86887665265855499815... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k}}{(2^{k}-1)k} = \sum_{k=1}^{\infty}\log\left(1+\frac{1}{2^{k}}\right)$

.86900199196290899881... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)}{(2k)!} = \sum_{k=1}^{\infty}\left(\cosh\left(\frac{1}{k}\right)-1\right)$

114 .86936465607933273489... ≈ $\frac{1}{2}\left(e^{2e}+e^{2/e}-2\right) = \sum_{k=1}^{\infty}\frac{2^{k}\cos k}{k!}$

3 .8695192413979994957... ≈ $\prod_{k=2}^{\infty}\left(\frac{k+1}{k-1}\right)^{1/k}$

.869602181868503186151... ≈ $\frac{\sqrt{2}}{16}\left(\psi\left(\frac{1}{\sqrt{2}}\right)^{2}-\psi\left(-\frac{1}{\sqrt{2}}\right)^{2}+\psi^{(1)}\left(-\frac{1}{\sqrt{2}}\right)-\psi^{(1)}\left(\frac{1}{\sqrt{2}}\right)\right)$

$-\frac{\gamma}{4}-\frac{2\pi\gamma\sqrt{2}}{16}\cot\frac{\pi}{\sqrt{2}}$

= $\sum_{k=1}^{\infty} \frac{H_k}{4k^2 - 2}$

1 .86960440108935861883... ≈ $\pi^2 - 8 = \sum_{k=1}^{\infty} \frac{1}{\left(k^2 - 1/4\right)^2}$

9 .86960440108935861883... ≈ $\pi^2 = \int_0^{\infty} \log^2 \left(\frac{1 + \sin x}{1 - \sin x} \right) \frac{dx}{x}$ GR 4.324.1

7 .8697017727613086918... ≈ $\sum_{k=1}^{\infty} \frac{2^k \zeta(k)}{k!} = \sum_{k=1}^{\infty} \left(e^{2/k^3} - 1 \right)$

.87005772672831550673... ≈ $\sqrt{\pi} \left(\frac{\gamma}{4} + \frac{\log 2}{2} \right) = -\int_0^{\infty} e^{-x^2} \log x \, dx$ GR 4.333

3 .87022215697339633082... ≈ $I_0(2) + I_1(2) = \sum_{k=1}^{\infty} \frac{k^3}{(k!)^2} = \sum_{k=0}^{\infty} \frac{k^3}{(2k)!} \binom{2k}{k}$

.87028205599121212145966... ≈ $-\frac{1}{\pi^3} \cos\left(\frac{\pi \sqrt{-1 - 2i\sqrt{3}}}{2} \right) \cos\left(\frac{\pi \sqrt{-1 + 2i\sqrt{3}}}{2} \right) \cos \frac{\pi \sqrt{5}}{2}$

= $\sum_{k=1}^{\infty} \left(1 - \frac{1}{k^3 (k+1)^3} \right)$

.87041975136710319747... ≈ $\sqrt{2} \arcsin \frac{1}{\sqrt{3}} = \sqrt{2} \arctan \frac{1}{\sqrt{2}}$

= $\sum_{k=0}^{\infty} \frac{(-1)^k}{2^k (2k+1)} = \sum_{k=1}^{\infty} \frac{(2k)!!}{(2k-1)!! 3^k k}$

.87060229149253986726... ≈ $\frac{\pi^4}{72} + 4 \log^2 2 - 2\zeta(3) = \sum_{k=1}^{\infty} \frac{H_k (k+1)}{k^3 (2k+1)}$

.87163473191790146006... ≈ $\gamma^{1/4}$

1 .87166017819754922915 6... ≈ $3e - 2\pi$

.87235802495485994177... ≈ $\frac{\pi^2}{8\sqrt{2}} = 1 + \sum_{k=1}^{\infty} \frac{(-1)^k}{(4k-1)^2} + \sum \frac{(-1)^k}{(4k+1)^2}$ J327

= $\frac{1}{64} \left(\psi\left(\frac{9}{8}\right) - \psi\left(\frac{7}{8}\right) + \psi\left(\frac{5}{8}\right) - \psi\left(\frac{3}{8}\right) \right)$

= $\sum_{k=0}^{\infty} \frac{(-1)^{\lfloor (k+1)/2 \rfloor}}{(2k+1)^2}$ Prud. 5.1.4.4

= $-\int_0^{\infty} \frac{\log x \, dx}{x^4 + 1}$

.8724555364666678115... ≈ $\frac{5e}{16}+\frac{1}{16e} = \sum_{k=1}^{\infty}\frac{k^3}{(2k)!}$

.87247296676432182087... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{\zeta(k+1)}$

.87278433509846713939... ≈ $\frac{29}{20}-\gamma = \psi(7)-1$

1 .8729310037130083205... ≈ $\int_1^{\infty}\frac{x^2\,dx}{e^x-e^{-x}}$

.87298334620741688518... ≈ $\sqrt{15}-3 = \sum_{k=0}^{\infty}\frac{(-1)^k}{6^k(k+1)}\binom{2k}{k}$

3 .87298334620741688518... ≈ $\sqrt{15}$

.87300668467724777193... ≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(3k-1)+\zeta(3k)+\zeta(3k+1)-3\right)$

= $-\frac{1}{2}+\sum_{k=1}^{\infty}\frac{2}{k^3+1}$

2 .87312731383618094144... ≈ $4e-8 = \sum_{k=1}^{\infty}\frac{k^3}{k!(k+2)}$

9 .87312731383618094144... ≈ $4e-1 = \sum_{k=1}^{\infty}\frac{k^4}{(k+1)!}$

10 .87312731383618094144... ≈ $4e$

1 .8732028500772299332... ≈ $\prod_{k=2}^{\infty}\left(1+\frac{\zeta(k)-1}{\zeta(k)}\right) = \prod_{k=2}^{\infty}\frac{2\zeta(k)-1}{\zeta(k)}$

.87356852683023186835... ≈ $\frac{1}{\log\pi}$ J153

1 .87391454266861876559... ≈ $\prod_{k=1}^{\infty}\left(1+\frac{1}{\binom{2k}{k}}\right)$

2 .87400584363103583797... ≈ $\frac{\pi^3+\pi\log^2 2}{8\sqrt{2}} = \int_0^{\infty}\frac{\log^2 x}{2x^2+1}dx = \int_0^{\infty}\frac{\log^2 x}{x^2+2}dx$

7 .8740078740118110197... ≈ $\sqrt{62}$

.87401918476403993682... ≈ $\frac{1}{6\sqrt{2\pi}}\Gamma^2\left(\frac{1}{4}\right) = \int_0^{\pi/2}\sin^{3/2}x\,dx = \int_0^{\pi/2}\cos^{3/2}x\,dx$ GR 3.621.2

$.87408196435930900580\ldots \approx \displaystyle\sum_{k=2}^{\infty}\frac{\zeta(k)}{(2k-2)!} = \sum_{k=1}^{\infty}\left(\frac{1}{k}\cosh\sqrt{\frac{1}{k}}-\frac{1}{k}\right)$

$.87427001649629550601\ldots \approx \displaystyle\frac{\gamma}{2}+\left(\frac{1+i}{4}\right)\psi(2-i)+\left(\frac{1-i}{4}\right)\psi(2+i)$

$.87435832830683304713\ldots \approx \displaystyle\frac{1}{4}\left(3\log\frac{3}{2}+\sqrt{3}\log\frac{3+\sqrt{3}}{3-\sqrt{3}}\right) = \sum_{k=1}^{\infty}\frac{H_{2k}}{3^k}$

$.87446436840494486669\ldots \approx \displaystyle -\sum_{k=2}^{\infty}\mu(k)\left(\zeta(k)-1\right) = -\sum_{k=2}^{\infty}\sum_{n=2}^{\infty}\frac{\mu(k)}{n^k} = \sum_{\substack{\omega\text{ a nontrivial}\\ \text{integer power}}}\frac{1}{\omega}$

$.87468271209245634042\ldots \approx \displaystyle\frac{3G}{\pi}$

$.8750000000000000000\underline{0} = \displaystyle\frac{7}{8} = \sum_{k=0}^{\infty}\frac{(-1)^k}{7^k} = \sum_{k=1}^{\infty}(2k+1)\left(\zeta(2k+1)-1\right)$

$\displaystyle = \sum_{k=1}^{\infty}\frac{H_k}{(k+1)(k+3)}$

$1\quad .8750000000000000000\underline{0} = \displaystyle\frac{15}{8} = \prod_{k=1}^{\infty}\left(1+\frac{1}{k(k+5/2)}\right)$

$3\quad .8750000000000000000\underline{0} = \displaystyle\frac{31}{8} = \sum_{k=1}^{\infty}\frac{k^9}{e^{\pi k}-(-1)^k}$ Prud. 5.3.1.1

$1\quad .87578458503747752193\ldots \approx \displaystyle\frac{\pi^3}{8}-2 = \int_0^{\infty}\frac{x^2\tanh x}{e^x}dx$

$3\quad .87578458503747752193\ldots \approx \displaystyle\frac{\pi^3}{8} = \int_0^{\infty}\frac{\log^2 x}{x^2+1}dx = \int_0^{\infty}\frac{x^2}{\cosh x}dx$ GR 3.523.5

$\displaystyle = \frac{\pi^3}{4}+2i\left(Li_3(i)-Li_3(-i)\right) = \int_{-\infty}^{\infty}\frac{x^2}{e^x+e^{-x}}$

$.87758256189037271612\ldots \approx \displaystyle\cos\frac{1}{2} = \operatorname{Re}\left\{(-1)^{1/6}\right\}$

$\displaystyle = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!4^k}$ AS 4.3.66, LY 6.110

$2\quad .87759078160816115178\ldots \approx \pi G$

$.87764914623495130981\ldots \approx \displaystyle\frac{\pi}{2}-\log 2 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2k^2-k}$

$\displaystyle = -\sum_{k=2}^{\infty}\left(Li_k(i)+Li_k(-i)\right)$

$\displaystyle = \int_0^1\log\left(1+x^2\right)\frac{dx}{x}$ GR 4.295.2

$= \int_0^\infty \log\left(1+\frac{1}{(x+1)^2}\right)dx$

$= \int_0^1 \log\left(\frac{1+x^2}{1-x^2}\right)dx$

$= \int_0^{\pi/2} \frac{x\,dx}{1+\cos x}$ Prud. 2.5.16.18

$= \int_0^1\int_0^1 \frac{x+y}{1+x^2y^2}\,dx\,dy$

1 .87784260817365902037... $\approx \sqrt{\pi}\coth\sqrt{\pi}$

1 .8780463498072068965... $\approx \frac{109e^{1/3}}{81} = \sum_{k=1}^\infty \frac{k^4}{k!3^k}$

5 .8782379806992663077... $\approx \frac{3}{8}\Phi\left(-2,4,\frac{1}{2}\right) = \int_1^\infty \frac{\log^3 x}{x^2+2}dx$

.878429128033657480583... $\approx 2\sqrt{2}\sin\frac{1}{\sqrt{2}} + 4\cos\frac{1}{\sqrt{2}} - 4 = \sum_{k=0}^\infty \frac{(-1)^k}{(2k)!2^k(k+1)}$

.879103174146665828812... $\approx \sum_{k=1}^\infty (-1)^{k+1}\frac{\zeta(2k+1)}{k} = \sum_{k=1}^\infty \frac{1}{k}\log\left(1+\frac{1}{k^2}\right)$

.87914690810034325118... $\approx \sum_{k=1}^\infty \frac{\log k}{k^2+1}$

.87917903628698039171... $\approx \sum_{k=1}^\infty \frac{1}{k!k\,\zeta(k+1)}$ Related to Gram’s series

.87919980032218190636... $\approx \prod_{k=0}^\infty \frac{2^k}{2^k+(-1)^k}$

.87985386217448944091... $\approx \sum_{k=1}^\infty \frac{k!}{k^k}$

.8799519802963893219... $\approx \sum_{k=1}^\infty \frac{\sigma_0(k)}{3^k-1} = \sum_{k=1}^\infty\sum_{j=1}^\infty \frac{1}{\left(3^j\right)^k-1}$

$= \sum_{i=1}^\infty\sum_{j=1}^\infty\sum_{k=1}^\infty \frac{1}{3^{ijk}}$

.88010117148986703192... $\approx 2J_1(1) = \sum_{k=0}^\infty \frac{(-1)^k}{k!(k+1)!4^k}$

1 .880770870194... $\approx \sum_{k=1}^\infty \frac{\sigma_0(k)}{k(k+1)} = \sum_{k=1}^\infty \frac{1}{k}\left(\gamma + \psi\left(1+\frac{1}{k}\right)\right)$

$= \sum_{j=1}^\infty\sum_{k=1}^\infty \frac{1}{jk(jk+1)}$

4 .88079258586502408561... ≈ $I_0(3)$

.88079707797788244406... ≈ $\frac{e}{e^2+1} = \frac{1+\tanh 1}{2} = \sum_{k=0}^{\infty}\frac{(-1)^k}{e^{2k}}$ J994

5 .88097711846511480802... ≈ ${}_2F_1\left(2, 2, \frac{1}{2}, \frac{1}{4}\right) = \sum_{k=1}^{\infty}\frac{(k!)^2}{(2k-2)!}$

.88107945069109218989... ≈ ${}_0F_1\left(; 4, -\frac{1}{2}\right) = 6\sum_{k=0}^{\infty}\frac{(-1)^{k2}}{k!(k+3)!2^k}$

1 .88108248662437028022... ≈ $9\log 3 + 12\log 2 - 3\pi\sqrt{3} = \sum_{k=0}^{\infty}\frac{1}{(k+1)(k+5/6)}$

= $\sum_{k=2}^{\infty}\frac{\zeta(k)}{6^{k-2}}$

1 .88109784554181572978... ≈ $\frac{\cosh 2}{2}$

2 .88131903995502918532... ≈ $\pi\tanh\frac{\pi}{2} = \sum_{k=0}^{\infty}\frac{1}{k^2+k+1/2}$

= $i\left(\psi\left(\frac{1-i}{2}\right) - \psi\left(\frac{1+i}{2}\right)\right)$

.881373587019543025232... ≈ $\log(1+\sqrt{2}) = \operatorname{arcsinh} 1 = \sum_{k=0}^{\infty}\frac{(-1)^k}{4^k(2k+1)}\binom{2k}{k}$ J85

= $\log\tan\frac{\pi}{8} = \log\sin\frac{3\pi}{8} - \log\sin\frac{\pi}{8}$

= $\operatorname{arctanh}\frac{1}{\sqrt{2}} = -i\arcsin i$

.88195553680044057157... ≈ $-\sum_{k=1}^{\infty}\frac{\psi\left(k-\frac{1}{2}\right)}{2(2k-1)^2}$

1 .88203974588235075456... ≈ $\sum_{k=1}^{\infty}\frac{1}{k^2\log(k+1)}$

.88208139076242168... ≈ $\frac{\sqrt{\pi}}{2} erf\, 2 = \sum_{k=0}^{\infty}\frac{(-1)^k 2^{2k+1}}{k!(2k+1)}$

$.\underline{8823529411764705}$ = $\frac{15}{17}$

.88261288770494821175... ≈ $\sum_{k=2}^{\infty}\left(e^{1/k!}-1\right)$

.88261910877658153974... ≈ $-\frac{1}{3} - \frac{\pi\sqrt{3}}{6}\csc\pi\sqrt{3} = \sum_{k=2}^{\infty}\frac{(-1)^k}{k^2-3}$

10 .88279618540530710356... ≈ $2\pi\sqrt{3} = \int_0^{\infty}\log\left(1+\frac{63}{x^2+1}\right)dx$

.883093003564743525207... ≈ $\sum_{k=1}^{\infty}\frac{2^k}{4^k+1}$

3 .88322207745093315469... ≈ $\pi\left(\sqrt{5}-1\right) = \int_0^{\infty}\log\left(1+\frac{4}{x^2+1}\right)dx$

.88350690717842253363... ≈ $\sum_{k=1}^{\infty}\zeta(2k+1)\left(\zeta(2k)-1\right)$

3 .8836640437859815948... ≈ $e\left(\gamma+1-Ei(-1)\right) = \sum_{k=1}^{\infty}\frac{H_k}{(k-1)!}$

.88383880441620186129... ≈ $HypPFQ\left[\{1,1\},\{2,2,2\},-1\right] = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k!)^2 k}$

.88402381175007985674... ≈ $\frac{4\pi^3}{81\sqrt{3}} = g_3$ J310

1 .88410387938990024135... ≈ $\frac{64\pi^5}{10395}$, volume of the unit sphere in R^5

1 .88416938536372010990... ≈ $e\log 2$

1 .8841765026477007091... ≈ $\frac{11}{4}-\frac{3\gamma}{2} = \int_0^{\infty}x^7\,e^{-x^2}\log x\,dx$

.88418748809595071501... ≈ $\sum_{k=1}^{\infty}\frac{H_k k!k!}{(2k)!}$

$= \frac{2}{27}\left(\psi^{(1)}\left(\frac{1}{3}\right)-\psi^{(1)}\left(\frac{2}{3}\right)+\pi\sqrt{3}\,(2-\log 3)\right)$

$= \frac{2}{3\sqrt{3}}\left(\frac{2\pi}{3}-\frac{\pi\log 3}{3}+iLi_2\left(-\frac{1+i\sqrt{3}}{2}\right)-iLi_2\left(\frac{i\sqrt{3}-1}{2}\right)\right)$

.88468759253939275201... ≈ $\frac{16}{17}-\frac{16}{17\sqrt{17}}\operatorname{arcsinh}\frac{1}{4} = \sum_{k=0}^{\infty}(-1)^k\frac{k!k!}{(2k)!4^k}$

3 .88473079679243020332... ≈ $2G+\pi-\frac{\pi\log 2}{2} = \sum_{k=1}^{\infty}\frac{H_k 2^k (k!)^2}{(2k)!}$

1 .88478816701605060798... ≈ $\frac{2}{3}\left(1-\cosh\sqrt{3}+\sqrt{3}\sinh\sqrt{3}\right) = \sum_{k=0}^{\infty}\frac{3^k}{2^k(k+1)}$

1 .88491174043658839554... ≈ $G+\frac{\pi^2}{32} = \int_0^1\frac{\log^2 x dx}{(1+x^2)^2}$

1 .8849555921538759431... ≈ $\frac{3\pi}{5}$

.88496703342411321824... ≈ $\frac{\pi^2}{12}+\frac{1}{16} = \sum_{k=2}^{\infty}\frac{k^2}{(k^2-1)^2}$

$= \quad \sum_{k=1}^{\infty} k\left(\zeta(2k)-1\right)$

1 .885160573807327148806... $\approx \quad \frac{\pi^2}{6}+\frac{\log^2 2}{2} = \int_0^\infty \frac{\log^2 x\,dx}{(x+2)^2}$

25 .88527799493115613083... $\approx \quad e^2\left(1+\gamma-2Ei(-2)+2\log 2\right)-1 = \sum_{k=1}^{\infty}\frac{2^k kH_k}{k!}$

1 .88538890738563... $\approx \quad \int_2^3 \zeta^2(x)\,dx$

2 .885397258254306968762... $\approx \quad 6-\frac{\pi^2}{12}-4\log 2+\log^2 2 = \int_0^1 \arccos x \log^2 x\,dx$

.88575432737726430215... $\approx \quad 2\left(\zeta(2)-\zeta(3)\right)$

$= \quad \sum_{k=1}^{\infty}\frac{2H_k}{k(k+1)^2}$

$= \quad \int_1^\infty \frac{\log^2 x}{x(x-1)^2}dx = \int_0^1 \frac{x\log^2 x}{(x-1)^2}dx$

$= \quad \int_0^\infty \frac{x^2 dx}{\left(e^x-1\right)^2} = \int_0^\infty \frac{x^2 dx}{e^x\left(e^x+e^{-x}-2\right)}$

8 .88576587631673249403... $\approx \quad \pi\sqrt{8}$

1 .88580651860617413628... $\approx \quad \frac{5}{4}+\frac{15\log 2}{4}-\frac{5\pi}{8} = \sum_{k=2}^{\infty}\frac{5^k\left(\zeta(k)-1\right)}{4^k}$

.8858936194371377193... $\approx \quad \frac{\pi^3}{35}$

5 .8860710587430771455... $\approx \quad \frac{16}{e} = \Gamma(4,1)$

2 .88607832450766430303... $\approx \quad 5\gamma$

.88620734825952123389... $\approx \quad \frac{\sqrt{\pi}}{2} erf\, 3 = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}3^{2k+1}}{k!(2k+1)}$

.88622692545275801365... $\approx \quad \frac{\sqrt{\pi}}{2}$ AS 6.4.2

$= \quad \Gamma\left(\frac{3}{2}\right)$ AS 6.1.9

$= \quad \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(k-\frac{1}{2})!\,3^k}{k!}$

$= \quad \prod_{k=0}^{\infty}\left(1+\frac{k+1/2}{k+3/2}\right)$

$= \int_0^\infty \frac{\sin^2(x^2)}{x^2}dx$

$= \int_0^\infty e^{-x^2}dx$ LY 6.29

$= \int_0^\infty e^{-\tan^2 x}\frac{\sin x}{x\cos^2 x}dx$ GR 3.963.1

.88626612344087823195... $\approx 24(\zeta(5)-1) = \int_0^\infty \frac{x^4}{e^x(e^x-1)}dx$

24 .88626612344087823195... $\approx 24\zeta(5) = -\psi^{(4)}(1) = \int_0^1 \frac{\log^4 x}{1-x}dx$

.88629436111989061883... $\approx 2\log 2 - \frac{1}{2}$

1 .886379184598759834894... $\approx \frac{5}{2} - \frac{\pi\sqrt{2}}{2}\cot \pi\sqrt{2} = \sum_{k=2}^\infty \frac{2}{k^2-2} = \sum_{k=1}^\infty 2^k(\zeta(2k)-1)$

.88642971056031277996... $\approx \sum_{k=1}^\infty \frac{(-1)^k}{k^{k+1}}$

2 .88675134594812882255... $\approx \frac{5}{\sqrt{3}}$

.88679069171991648737... $\approx \frac{1}{2} + \frac{2}{\sqrt{3}}\operatorname{csch}\frac{\pi}{\sqrt{3}} = \sum_{k=0}^\infty \frac{(-1)^k}{3k^2+1}$

.8868188839700739087... $\approx \operatorname{sech}\frac{1}{2} = \frac{2}{e^{1/2}+e^{-1/2}} = \sum_{k=0}^\infty \frac{E_{2k}}{(2k)!4^k}$ AS 4.5.66

.887146038222546250877... $\approx -\log\Gamma\left(\frac{1+i\sqrt{3}}{2}\right) - \log\Gamma\left(\frac{1-i\sqrt{3}}{2}\right)$

$= \sum_{k=1}^\infty \log\left(1+\frac{1}{k^3}\right) = \sum_{k=1}^\infty (-1)^{k+1}\frac{\zeta(3k)}{k}$

24 .8872241141319937809... $\approx \sum_{k=2}^\infty \frac{\log^4 k}{k(k-1)}$

9 .88751059801298722256... $\approx 9\log 3$

.88760180330217531549... $\approx \sum_{k=1}^\infty \frac{(-1)^{k+1}}{3^k-2}$

2 .88762978583276585795... $\approx \sum_{k=1}^\infty \frac{F_k{}^2}{k!}$

.8876841582354967259... ≈ $2\left(\gamma-\log 2-Ei(-\frac{1}{2})\right)=\sum_{k=0}^{\infty}\frac{(-1)^k}{(k+1)!2^k(k+1)}$

38 .88769219039634478432... ≈ $\sum_{k=2}^{\infty}k^5\left(\zeta(k)-1\right)^2$

.88807383397711526216... ≈ $\sqrt{\frac{3+\sqrt{3}}{6}}$ CFG D14

8 .8881944173155888501... ≈ $\sqrt{79}$

.88831357265178863804... ≈ $\frac{1+\sqrt{5}}{80}\left(4\pi\sqrt{2-\frac{2}{\sqrt{5}}}\right)+\log\left(\frac{445+199\sqrt{5}}{32(85-38\sqrt{5})}\right)+\sqrt{5}\log\left(56-24\sqrt{5}\right)$

= $\frac{1}{10}\left(\psi\left(\frac{3}{5}\right)-\psi\left(\frac{1}{10}\right)\right)=\sum_{k=0}^{\infty}\frac{(-1)^k}{5k+1}$

.8888888888888888888$\underline{8}$ = $\frac{8}{9}=\sum_{k=0}^{\infty}\frac{(-1)^k}{8^k}$

1 .88893178779049819496... ≈ $25-\pi^2-\frac{3\pi^2}{\sqrt{5}}=\sum_{k=1}^{\infty}\left(\frac{(-1)^{k+1}}{(k-1/5)^2}+\frac{(-1)^{k+1}}{(k+1/5)^2}\right)$

7 .8892749112949219724... ≈ $\sum_{k=1}^{\infty}\frac{1}{k!\left(\zeta(k+1)-1\right)}$

.8894086363241... ≈ $\prod_{k=1}^{\infty}\left(1-\frac{1}{k(k+2)(k+4)}\right)$

1 .89039137863558749847... ≈ $-4Li_3\left(-\frac{1}{2}\right)=\int_0^{\infty}\frac{x^2\,dx}{e^x+1/2}$

.89045439704411550353... ≈ $\frac{\pi\left(2\pi+3\sqrt{3}\right)}{81}=-\int_0^{\infty}\frac{\log x}{(x^3+1)^2}dx$

5 .89048622548086232212... ≈ $\frac{15\pi}{8}$

.89072941267226124064 3...≈ $\gamma+2\log 2+\frac{3}{2}\log 3=-\psi\left(\frac{5}{6}\right)$

1 .89082115433686314276... ≈ $\frac{1}{4}\Phi\left(-2,3,\frac{1}{2}\right)=\int_1^{\infty}\frac{\log^2 x}{x^2+2}dx$

.89089871814033930474 0...≈ $2^{-1/6}=\prod_{k=1}^{\infty}\left(1+\frac{(-1)^k}{6k+1}\right)$

.8912127981113023761... ≈ $HypPFQ[\{1,1,1\},\{2,2,2\},-1]=\frac{1}{2}\int_0^1\frac{\log^2 x dx}{e^x}$

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{k!(k+1)^3} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!k^2}$

.89164659564565004368... ≈ $\frac{3\sqrt{\pi}}{8}\zeta\left(\frac{5}{2}\right) = \int_0^{\infty}\frac{x^4\,dx}{e^{x^2}-1}$

.89169024629435739173... ≈ $\frac{\pi}{4}\left(1+e^{-2}\right) = \int_0^{\pi/2}\cos^2(\tan)\,dx$ GR 3.716.10

= $\int_0^{\infty}\frac{\cos^2 x}{1+x^2}dx$

.891809... ≈ $\sum_{k=1}^{\infty}\frac{|\mu(k)|}{k(k+1)}$

.89183040111256296686... ≈ $\int_1^{\infty}\frac{dx}{x^2\log(1+x)}$

.89257420525683902307... ≈ $4\log\frac{5}{4} = \sum_{k=0}^{\infty}\frac{(-1)^k}{4^k(k+1)} = 4\sum_{k=1}^{\infty}\frac{1}{5^k k} = \int_0^{\infty}\frac{dx}{e^x+1/4}$

.89265685271018665906... ≈ $\binom{1/3}{1/2}$

1 .89278926071437231113... ≈ $3\log_3 2$

3 .89284757490956280439... ≈ $e^{e/2} = \sum_0^{\infty}\frac{e^k}{k!2^k}$

.89289457145126609045 7... ≈ $\sum_{k=2}^{\infty}\sum_{p\ prime}\frac{1}{k^p} = \sum_{p\ prime}\left(\zeta(p)-1\right)$

.89293411695542293745 8... ≈ $B_1 = \gamma - \sum_{p\ prime}\left(\log\left(1-\frac{1}{p}\right)+\frac{1}{p}\right)$ in Mertens' Theorem HW 22.7

= $\gamma - \sum_{k=2}^{\infty}\mu(k)\frac{\log\zeta(k)}{k}$

.89296626185090504491... ≈ $-Li_4\left(e^{2i}\right) - Li_4\left(e^{-2i}\right)$

.89297951156924921122... ≈ $\Gamma\left(\frac{4}{3}\right) = \frac{1}{3}\Gamma\left(\frac{1}{3}\right) = \int_0^{\infty}e^{-x^3}dx$

.89324374097502616834... ≈ $HypPFQ\left[\{1\},\left\{\frac{3}{2},\frac{3}{2}\right\},-\frac{1}{4}\right] = \sum_{k=0}^{\infty}\frac{(-1)^k}{((2k+1)!!)^2}$

= $\frac{\pi}{2}H_0(1) = \int_0^{\pi/2}\sin(\cos x)\,dx$

2 .89359525517877983691... ≈ $\displaystyle\int_0^\infty \frac{x^2\log(1+x)}{e^x-1}dx$

1 .89406565899449183515... ≈ $\displaystyle\frac{7\pi^4}{360}=\sum_{k=1}^\infty \frac{H^{(2)}{}_k}{k^2}$

= $\displaystyle\int_0^1 \log(1+x)\frac{\log^2 x}{x}dx$

1 .89431799614475676131... ≈ $\displaystyle 2\cos\left(\frac{\pi}{2}e^{-\pi/2}\right)=i^{i^i}+(-i)^{(-i)^{-i}}$

.894374836885259781174... ≈ $\displaystyle 8G+\pi^2-\frac{\pi^3}{2}+\frac{1}{96}\psi^{(3)}\left(\frac{1}{4}\right)-14\zeta(3)=\sum_{k=1}^\infty \frac{k^2}{(k+1/4)^4}$

.8944271909999158786... ≈ $\displaystyle\frac{2}{\sqrt{5}}=\sum_{k=0}^\infty \frac{(-1)^k}{16^k}\binom{2k}{k}$

.<u>894736842105263157</u> = $\displaystyle\frac{17}{19}$

.89493406684822643647... ≈ $\displaystyle\frac{\pi^2}{6}-\frac{3}{4}=\sum_{k=2}^\infty \frac{2k+1}{k(k+1)^2}=\sum_{k=2}^\infty (-1)^k k(\zeta(k)-1)$

= $\displaystyle\sum_{k=2}^\infty \left(\frac{1}{k^2-k}-\frac{1}{k^3-k^2}\right)=\sum_{k=2}^\infty \big(\zeta(k)-\zeta(2k)\big)$

= $\displaystyle\sum_{k=1}^\infty \frac{2}{k^3+2k^3}=\sum_{k=2}^\infty \frac{k^2+k-1}{(k-1)k^2(k+1)}$

1 .89493406684822643647... ≈ $\displaystyle\frac{\pi^2}{6}+\frac{1}{4}=\int_0^1 \frac{(1+x-x^2)\log x}{x-1}dx$

.895105505200094223399... ≈ $\displaystyle\sum_{k=1}^\infty \frac{\zeta(k+1)-1}{(k-1)!}=\sum_{k=2}^\infty \frac{e^{1/k}}{k^2}$

1 .89511781635593675547... ≈ $\displaystyle Ei(1)=li(e)=\gamma+\sum_{k=1}^\infty \frac{1}{k!k}$ AS 5.1.10

.89533362517016905663... ≈ $\displaystyle\frac{si(\sqrt{2})}{\sqrt{2}}=\sum_{k=0}^\infty \frac{(-1)^k 2^k}{(2k+1)!(2k+1)}$

.89580523867937996258... ≈ $\displaystyle\frac{i}{2}\big(Li_4(e^{-i})-Li_4(e^i)\big)=\sum_{k=1}^\infty \frac{\sin k}{k^4}$

.89591582830105900373... ≈ $\displaystyle\frac{\pi\sqrt{2}}{4}\csc\frac{\pi}{\sqrt{2}}-\frac{1}{2}$

= $\displaystyle\sum_{k=1}^\infty \frac{(-1)^{k+1}}{2k^2-1}$

.89601893592680657945... ≈ $\sin\frac{\pi}{2\sqrt{2}} = \prod_{k=1}^{\infty}\left(1-\frac{1}{16k^2-1}\right)$ J1064

.89603955949396541657... ≈ $\frac{\pi}{\sqrt{3}}\log 3 - \frac{\pi^2}{9} = \int_0^{\infty}\frac{\log(x^2+1)}{x^2+1}dx$

.896488781929623341300... ≈ $\int_0^1 x^{x^2}dx$

.89682841384729240489... ≈ $-2Li_2\left(-\frac{1}{2}\right) = \Phi\left(-\frac{1}{2},2,1\right)$

= $\log^2 2 - 2\log 2\log 3 + \log^2 3 + 2Li_2\left(\frac{1}{3}\right)$

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{2^k\,(k+1)^2} = \int_0^{\infty}\frac{x}{e^x+\frac{1}{2}}$

1 .89693992379675385782... ≈ $\sum_{k=0}^{\infty}\frac{\zeta(k+3)}{k!2^k} = \sum_{k=1}^{\infty}\frac{e^{1/2k}}{k^3}$

.89723676373539623808... ≈ $\frac{\pi^2}{11}$

1 .89724269164081069031... ≈ $\frac{7\pi\left(\sqrt{3}-1\right)}{6\sqrt{2}} = \int_0^{\infty}\frac{dx}{1+x^{12/7}}$

.8984375000000000000$\underline{0}$ = $\frac{115}{128} = \Phi\left(\frac{1}{5},-3,0\right) = \sum_{k=1}^{\infty}\frac{k^3}{5^k}$

.89887216380918402542... ≈ $\frac{1}{2}\sum_{k=1}^{\infty}\frac{\log^2 k}{k(k+1)}$

.8989794855663561964... ≈ $2\sqrt{6}-4 = \sum_{k=0}^{\infty}\frac{(-1)^k}{8^k(k+1)}\binom{2k}{k}$

4 .8989794855663561964... ≈ $\sqrt{24} = 2\sqrt{6}$

.89917234483108405356... ≈ $\frac{\sqrt{3\pi}}{2}erf\,\frac{1}{\sqrt{3}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!3^k(2k+1)}$

.89918040670820836708... ≈ $2-\frac{\pi}{\sqrt{2}}\cot\frac{\pi}{2\sqrt{2}} = \sum_{k=1}^{\infty}\frac{1}{2k^2-1/4}$

.89920527550843900193... ≈ $\frac{3}{4}+\frac{\pi}{\sqrt{17}}\tan\frac{\pi\sqrt{17}}{2} = \sum_{k=1}^{\infty}\frac{1}{k^2+3k-2}$

9 .8994949366116653416... ≈ $\sqrt{98}$

5 .899876869190837124551... ≈ $3\zeta(3)+\frac{17\pi^4}{360}-\frac{5\pi^2}{24}-\frac{1}{4} = \sum_{k=1}^{\infty}\frac{H_kH_{k+2}}{k^2}$

11 .89988779249402195865... $\approx \dfrac{1}{1-G}$

$.900000000000000000\underline{0}$ = $\frac{9}{10} = \sum_{k=0}^{\infty}\frac{(-1)^k}{9^k}$

.9003163161571060696... ≈ $\frac{2\sqrt{2}}{\pi} = \prod_{k=1}^{\infty}\left(1-\frac{1}{16k^2}\right)$ GR 1.431

= $\binom{0}{1/4}$

= $\prod_{k=1}^{\infty}\cos\left(\frac{\pi}{2^{k+2}}\right)$ GR 1.439.2

2 .9005626934093224662 79...≈ $\frac{3\zeta(3)}{16} + \frac{\pi^2}{24} + \frac{\pi}{2} + \log 2 = \int_0^1 \operatorname{arccot} x \log^2 x\, dx$

1 .9007149859685018278 7... ≈ $\sum_{k=1}^{\infty}\frac{\zeta^2(2k)}{2^k}$

2 .9010223877769939899 7... ≈ $\sum_{k=1}^{\infty}\frac{1}{k!-1/2}$

30 .9011001130494975889 6... ≈ $11e + 1 = \sum_{k=1}^{\infty}\frac{k^5}{(k+1!}$

.9012862993604472987 3... ≈ $\frac{2}{\pi}K\left(-\frac{1}{2}\right) = \sum_{k=0}^{\infty}\binom{2k}{k}^2\frac{(-1)^k}{32^k}$

.9013532442006737121 4... ≈ $\frac{1}{2} - \frac{si(-2)}{4} = -\int_0^1 \log x \cos^2 x\, dx$

.9015426773696957140 5... ≈ $\frac{3\zeta(3)}{4} = \eta(3) = -\Phi(-1,3,0) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^3}$ AS 23.2.19

= $-\int_0^1 \log(1+x)\frac{\log x}{x}dx$ GR 4.315.1

= $\int_0^{\infty}\frac{x^5dx}{e^{x^2}+1}$

.9016442585275096718 14...≈ ${}_2F_1\left(1,\frac{1}{3},\frac{4}{3},-\frac{1}{2}\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{2^k(3k+1)}$

1 .902160577783278... ≈ Brun's constant, the sum of the reciprocals of the twin primes: $\sum_{\substack{p\, atwin \\ prime}}\frac{1}{p}$

Proven by Brun in 1919 to converge even though it is still unknown whether the number of non-zero terms is finite.

.9023782210458315169 12...≈ $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k\zeta^2(k+1)}{2^k}$

.90268536193307106616... ≈ $-\cos\pi^2$

.9027452929509336113... ≈ $\Gamma\left(\frac{5}{3}\right) = \frac{2}{3}\Gamma\left(\frac{2}{3}\right)$

2 .90282933102514164426... ≈ $\sum_{k=2}^{\infty}(-1)^k\left(\zeta^3(k)-1\right)$

1 .90290989453828734 7105...≈ $\sum_{k=0}^{\infty}\frac{I_k(1)}{k!}$

1 .9036493924... ≈ $\sum_{2}^{\infty}\frac{1+2\log k}{k^2\log^2 k} = \int_2^{\infty}x\left(\zeta(x)-1\right)dx$

.90367462377639553 66... ≈ $-\sin\frac{e\pi}{2} = -\operatorname{Im}\left\{i^e\right\}$

.90372628629864941069... ≈ $\sum_{k=2}^{\infty}\left(\frac{\zeta(k)}{\zeta(k+3)}-1\right)$

.90377177374877204684... ≈ $\frac{\pi}{6}+\frac{\sqrt{3}}{6}\log\left(2+\sqrt{3}\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{6k+1}$

.90406326728086180804... ≈ $\sum_{k=0}^{\infty}\frac{1}{3^k+1}$

1 .90414879267562492957 1...≈ $\frac{2}{3}{}_1\mathrm{F}_1\left(\frac{3}{2},2,\frac{4}{3}\right) = \frac{2}{3}e^{2/3}\left(I_0\left(\frac{2}{3}\right)+I_1\left(\frac{2}{3}\right)\right)$

= $\sum_{k=0}^{\infty}\frac{k}{k!3^k}\binom{2k}{k}$

6 $.9042968750000000000\underline{0}$ = $6\frac{463}{512} = \Phi\left(\frac{1}{5},-5,0\right) = \sum_{k=1}^{\infty}\frac{k^5}{5^k}$

.90435488931927417 31... ≈ $\frac{3\pi^2}{8}-\log 2-\frac{7\zeta(3)}{4} = \sum_{k=}^{\infty}\frac{16k^2+6k+1}{2k(2k+1)^3}$

= $\sum_{k=2}^{\infty}\frac{(-1)^k k^2\zeta(k)}{2^k}$

.90452423790027208 15... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!(4k+1)}$ J974

= $\sqrt{\frac{\pi}{2}}C\left(\sqrt{\frac{2}{\pi}}\right) = \int_1^{\infty}\cos\left(\frac{1}{x^2}\right)\frac{dx}{x^2}$

9 .90506305772752071350 1...≈ $8\zeta(3)+\frac{\gamma}{2} = \sum_{k=1}^{\infty}\frac{2^k}{k^3}\left(\zeta(k+1)-1\right)$

.90514825364486643824... ≈ $\frac{e^3-1}{e^3+1}$ J148

16 .90534979423868312757... ≈ $\frac{4108}{243} = \Phi\left(\frac{1}{4},-5,0\right) = \sum_{k=1}^{\infty}\frac{k^5}{4^k}$

.90542562608908937546... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)-1}{F_{k-1}}$

1 .90547226473017993689... ≈ $\frac{\pi}{\sqrt{e}}$

.90575727880447613109... ≈ $\frac{si(2)}{\sqrt{\pi}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!(k+\frac{1}{2})(2k+1)}$

.90584134720168919417... ≈ $2\log 2-\log^2 2 = \sum_{k=1}^{\infty}\frac{kH_k}{2^k(k+1)} = 1-\sum\frac{(-1)^{k+1}H_k}{(k+1)(k+2)}$

= $\sum_{k=1}^{\infty}\frac{H_k}{4k^2-2k}$

.90587260444153744936... ≈ $\sum_{k=1}^{\infty}\frac{1}{2^k\left(\zeta(2k+1\right)}$

6 .90589420856805831818... ≈ $\frac{3}{5}\left(\pi\sqrt{3}+3\log 3+4\log 2\right) = \sum_{k=0}^{\infty}\frac{1}{(k+1)(k+1/6)}$

.90609394281968174512... ≈ $\frac{e}{3}$

12 .906323358973207156143...≈ $8\zeta(3)+\frac{\pi^2}{3} = \sum_{k=1}^{\infty}2^k k^2\left(\zeta(k+1)-1\right)$

.90640247705547707798... ≈ $\Gamma\left(\frac{5}{4}\right)=\frac{1}{4}\Gamma\left(\frac{1}{4}\right)$

= $\int_0^{\infty}e^{-x^4}dx$

.90642880001713865550... ≈ $\sum_{k=1}^{\infty}\frac{1}{k!+k}$

.9064939198463331845.... ≈ ${}_2F_1\left(\frac{1}{2},\frac{1}{2},2,-1\right)=\sum_{k=0}^{\infty}\frac{(-1)^k}{16^k(k+1)}\binom{2k}{k}^2$

.9066882461958017498... ≈ $-\frac{\pi\gamma}{2} = -\int_0^{\infty}\frac{\log x\sin x}{x}dx$

.9068996821171089253... ≈ $\frac{\pi}{2\sqrt{3}} = \frac{\pi}{3}\sin\frac{\pi}{3} = \sum_{k=0}^{\infty}\left(\frac{1}{6k+1}-\frac{1}{6k-5}\right)$ J84, K ex. 109e

= maximum packing density of disks CFG D10

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{(k+1)(3k+1)}$

$= \sum_{k=0}^{\infty} \frac{(-1)^k}{3^k(2k+1)}$ J273

$= \int_0^{\infty} \frac{dx}{x^2+3} = \int_0^{\infty} \frac{dx}{x^4+x^2+1} = \int_0^{\infty} \frac{dx}{3x^2+1}$

$= \int_0^{\infty} \frac{x^2\,dx}{x^4+x^2+1}$

$= \int_0^1 x^2 \log\left(1+\frac{1}{x^3}\right) dx$

1 .90709580309488459886... $\approx \frac{1}{4}\cosh\frac{\pi\sqrt{3}}{2} = \prod_{k=1}^{\infty}\left(1+\frac{3}{(2k+1)^2}\right)$

8 .90747490429497721368l...$\approx \sqrt{3}\,\mathrm{csc}\,h\frac{\pi}{\sqrt{2}}\sinh\pi\sqrt{\frac{3}{2}} = \prod_{k=0}^{\infty}\frac{k^2+3/2}{k^2+1/2}$

.90749909976283678172... $\approx \sum_{k=1}^{\infty}\frac{b(k)}{k^2} = \zeta(2)\sum_{k=1}^{\infty}\frac{\phi(k)}{k}\log\zeta(2k)$ Titchmarsh 1.6.3

.90751894316228530929... $\approx \sum_{k=2}^{\infty}\left(\zeta(k)-1\right)\log k$

23 .90778787385011353615... $\approx \frac{5\pi^5}{64} = \int_0^{\infty}\frac{x^4}{e^x+e^{-x}}dx$

$= \int_0^{\pi/2}\left(\log\tan x\right)^4 dx$ GR 4.227.3

$= \int_0^1 \frac{\log^4 x}{1+x^2}dx$ GR 4.263.2

.907970538300591166629...$\approx \frac{5\pi^2}{48}-\frac{\log^2 2}{4} = Li_2\left(\frac{1+i}{2}\right)+Li_2\left(\frac{1-i}{2}\right)$

.90812931549667023319... $\approx \frac{\pi^4}{90}-\frac{\pi^2}{48}+\frac{\pi}{96}-\frac{1}{768} = \sum_{k=1}^{\infty}\frac{\cos(k/2)}{k^4}$ GR 1.443.6

$= \frac{1}{2}\left(Li_4\left(e^{i/2}\right)-Li_4\left(e^{-i/2}\right)\right)$

.90819273744789092436... $\approx \frac{1}{3}\left(2\gamma+\psi\left(\frac{5-i\sqrt{3}}{2}\right)+\psi\left(\frac{5+i\sqrt{3}}{2}\right)\right)$

$= \sum_{k=2}^{\infty}\frac{k+1}{k^3-1} = \sum_{k=1}^{\infty}\left(\zeta(3k-1)+\zeta(3k)-2\right)$

$= \frac{5}{6}+\sum_{k=1}^{\infty}\frac{1}{k^4+k}$

.90857672552682615638... $\approx \psi^{(1)}\left(\frac{4}{3}\right)+\frac{1}{6}\psi^{(2)}\left(\frac{4}{3}\right) = \sum_{k=1}^{\infty}\frac{k}{(k+1/3)^3}$

.90861473697907307078... ≈ $\sum_{k=1}^{\infty}\frac{\sigma_0(k)}{2^k k} = \sum_{k=1}^{\infty}\frac{-\log(1-2^{-k})}{k} = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{1}{2^{jk}jk}$

.90862626717044620367... ≈ $\sum_{k=2}^{\infty}(-1)^k\left(2^{\zeta(k)}-2\right)$

.909090909090909090$\underline{90}$ = $\frac{10}{11} = \sum_{k=0}^{\infty}\frac{(-1)^k}{10^k}$

.90917842589178437832... ≈ $\sum_{k=2}^{\infty}\frac{1}{k!\log k}$

.9092974268256816954... ≈ $\sin 2 = \sum_{k=0}^{\infty}\frac{(-1)^k 2^{2k+1}}{(2k+1)!} = \sum_{k=1}^{\infty}\frac{2^k\sin(k\pi/2)}{k!}$ AS 4.3.65

.90933067363147861703... ≈ $\frac{1-e(\cos 1-\sin 1)}{2} = \int_1^e \sin\log x\,dx$

1 .90954250488443845535... ≈ $3\log 3 - 2\log 2 = \int_0^1 \log\left(1+\frac{2}{x}\right)dx$

1 .90983005625052575898... ≈ $\frac{5}{2}\left(3-\sqrt{5}\right) = 1-\cos\frac{\pi}{5} = \sum_{k=0}^{\infty}\frac{1}{5^k(k+1)}\binom{2k+2}{k}$

1 .90985931710274402923... ≈ $\frac{6}{\pi}$

1 .91002687845029058832... ≈ $1-\zeta(2)+\zeta(3)+\frac{5}{4}\zeta(4) = \sum_{k=1}^{\infty}\frac{H_{k+1}}{k^3}$

.91011092585744407479... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)}{2^k k^2} = \sum_{k=1}^{\infty}Li_2\left(\frac{1}{2k^2}\right)$

.91023922662683739361... ≈ $\frac{1}{\log 3} = \int_0^{\infty}\frac{dx}{3^x}$ J153

.91040364132111511419... ≈ $\frac{1}{8}-\frac{\pi}{4}\coth 2\pi = \sum_{k=0}^{\infty}\frac{1}{k^2+4}$

.91059849921261470706... ≈ $\sin(\log\pi) = \mathrm{Im}\left\{\pi^i\right\}$

.91060585540584174737... ≈ $Li_3\left(\frac{4}{5}\right)$

1 .910686134642447997691...≈ $\frac{1}{2}+\sqrt{\frac{e\pi}{2}}\,erf\left(\frac{1}{\sqrt{2}}\right) = \sum_{k=1}^{\infty}\frac{k!2^k k}{(2k)!}$

244 .91078944598913500831... ≈ $\frac{8\pi^3}{3}+96\pi-64\pi\log 2 = -\int_0^{\infty}x^{-3/2}Li_2(-x)^2\,dx$

.911324776360696926457...≈ $\sum_{k=2}^{\infty}\left|\mu(k)\right|\left(\zeta(k)-1\right) = \sum_{k=2}^{\infty}\sum_{n=2}^{\infty}\frac{\left|\mu(k)\right|}{n^k}$

.91189065278103994299... ≈ $\frac{9}{\pi^2}$

.91194931412646529928... ≈ $\frac{\pi^3}{34}$

1 .911952134811735459360...≈ $\frac{2}{3}\cosh\frac{\pi\sqrt{11}}{2}\operatorname{sech}\frac{\pi\sqrt{7}}{2} = \prod_{k=1}^{\infty}\left(1+\frac{1}{k^2+k+2}\right)$

23 .9121636761437509037... ≈ $\frac{65}{e} = \Gamma(5,1)$

.91232270129516167141... ≈ $\frac{1}{112}\left(-19+14\pi\sqrt{2}\csc(2\pi\sqrt{2})\right)$

1 .9129311827723891012... ≈ $\sqrt[3]{7}$

.91339126049357314062... ≈ $-\sum_{k=1}^{\infty}\frac{1}{\begin{bmatrix}2k\\k\end{bmatrix}}$

.9134490707088278551... ≈ $\zeta(3)-\frac{\gamma}{2}$

3 .91421356237309504880... ≈ $\frac{5}{2}+\sqrt{2}$ CGF D4

.9142425426232080819... ≈ $\frac{\pi}{\sqrt{7}}-\frac{2}{\sqrt{7}}\arctan\frac{1}{\sqrt{7}} = \int_0^{\infty}\frac{dx}{x^2+x+2}$

$.9142857142857\underline{142857}$ = $\frac{32}{35} = \beta(4,1/2) = \sum_{k=0}^{\infty}\frac{1}{4^k(k+4)}$

2 .91457744017592816073... ≈ $\cosh\sqrt{3} = \sum_{k=0}^{\infty}\frac{3^k}{(2k)!}$

.91524386085622595963... ≈ $\frac{1}{2}\cot\frac{1}{2}$

= $\frac{i\left(e^i+1\right)}{2\left(e^i-1\right)} = \frac{i}{2}\left(\frac{\cos 1+i\sin 1+1}{\cos 1+i\sin 1-1}\right) = \sum_{k=0}^{\infty}\frac{(-1)^k B_{2k}}{(2k)!}$

= $\lim_{a\to 0}\sum_{k=1}^{\infty}\frac{\sin k}{k^a}$

.91547952683760158139... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^k}{7k+1}$

4 .91569309392969918879... ≈ $120(46e-125) = \sum_{k=1}^{\infty}\frac{k^4}{k!(k+5)}$

3 .91585245904074342359... ≈ $\pi\sqrt{3}\tanh\frac{\pi}{2\sqrt{3}} = \sum_{k=0}^{\infty}\frac{1}{k^2+k+1/3}$

.91596559417721901505... $\approx$ $G = \beta(2) = \operatorname{Im}\left\{Li_2(i)\right\}$, Catalan's constant LY 6.75

$$= \frac{\pi\log 2}{8} + \frac{i}{2}\left(Li_2\left(\frac{1-i}{2}\right) - Li_2\left(\frac{1+i}{2}\right)\right)$$

$$= \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)^2}$$

$$= \sum_{k=1}^{\infty}\left(\frac{1}{(4k-3)^2} - \frac{1}{(4k-1)^2}\right)$$

$$= \frac{1}{2}\sum_{k=0}^{\infty}\frac{(k!)^2 4^k}{(2k)!(2k+1)^2}$$ Adamchik (23)

$$= \frac{1}{16}\sum_{k=1}^{\infty}\frac{(3^k-1)(k+1)}{4^k}\zeta(k+2)$$ Adamchik (27)

$$= \frac{1}{8}\sum_{k=1}^{\infty}\frac{k}{2^k}\zeta\left(k+1,\frac{3}{4}\right)$$ Adamchik (28)

$$= \frac{\pi}{8}\log\left(2+\sqrt{3}\right) + \frac{3}{8}\sum_{k=0}^{\infty}\frac{1}{\binom{2k}{k}(2k+1)^2}$$

$$= \int_0^{\infty}\frac{x}{e^x+e^{-x}}dx$$

$$= -\int_0^1\frac{\log x}{x^2+1}dx = \int_1^{\infty}\frac{\log x}{x^2+1}dx$$

$$= \int_0^1\log\left(\frac{1+x}{1-x}\right)\frac{dx}{1+x^2}$$ GR 4.297.4

$$= -\int_0^1\log\left(\frac{1-x}{\sqrt{x}}\right)\frac{dx}{x^2+1}$$ Adamchik (12)

$$= -\int_0^1\log\left(\frac{1-x^2}{2}\right)\frac{dx}{x^2+1}$$ Adamchik (13)

$$= -\int_0^{\pi/4}\log\tan x\,dx$$ GR 4.227.4

$$= \frac{1}{2}\int_0^{\pi/2}\frac{x}{\sin x}dx$$ Adamchik (2)

$$= \frac{1}{2}\int_0^{\infty}x\operatorname{sech}x\,dx$$ Adamchik (4)

$= \int_0^{\pi/2} \operatorname{arcsinh}(\sin x)\,dx$ Adamchik (18)

$= -2\int_0^{\pi/4} \log(2\sin x)\,dx \;=\; 2\int_0^{\pi/4} \log(2\cos x)\,d$ Adamchik (5), (6)

$= \int_0^1 \frac{\arctan x}{x}dx \;=\; \int_1^\infty \frac{\arctan x}{x}\,dx$ GR 4.531.1

$= -\int_0^\infty \frac{\arctan x}{1-x^2}dx$

$= \frac{1}{2}\int_0^1 K(x^2)\,dx$ Adamchik (16)

$= -\frac{1}{2} + \int_0^1 E(x^2)\,dx$ Adamchik (17)

$= \int_0^1\int_0^1 \frac{1}{(x+y)\sqrt{1-x}\sqrt{1-y}}\,dx\,dy$ Adamchik (18)

.9159981926890739016... $\approx \frac{\pi(8+3\pi-18\log 2)}{12\sqrt{2}} \;=\; \int_0^\infty \frac{\log(1+x^4)}{x^4(1+x^4)}dx$

5 .91607978309961604256... $\approx \sqrt{35}$

.91629073187415506518... $\approx \log\frac{5}{2} \;=\; Li_1\left(\frac{3}{5}\right) \;=\; \sum_{k=1}^\infty \frac{3^k}{5^k k}$

$.916666666666666666\underline{6}$ $= \frac{11}{12} = \sum_{k=0}^\infty \frac{(-1)^k}{11^k}$

$= \sum_{k=2}^\infty \frac{k^5+k^4+k^2+k+1}{(k^4+k)(k^3-1)} \;=\; \sum_{k=2}^\infty \frac{1}{k^7-k} \;+\; \sum_{k=2}^\infty \frac{k+1}{k^3-1}$

.9167658563152321288... $\approx \cos\sqrt[4]{2}\;\cosh\sqrt[4]{2} \;=\; \sum_{k=0}^\infty \frac{(-1)^k 2^k}{(4k)!}$

.91681543444030774529... $\approx HypPFQ\left[\{\;\},\left\{\frac{1}{4},\frac{3}{4},1\right\},-\frac{1}{64}\right] \;=\; \sum_{k=0}^\infty \frac{(-1)^k}{(4k)!}\binom{2k}{k}$

.91767676628886180848... $\approx 2-\zeta(4) \;=\; \sum_{k=2}^\infty \frac{k^3-k+1}{k^5-k^4}$

.91775980474025243402... $\approx \frac{\pi}{9}\left(\pi - 3(\log 3-1)\sqrt{3}\right) \;=\; \int_0^\infty \frac{\log(1+x^3)}{x^3(1+x^3)}dx$

.91816874239976061064... $\approx \Gamma\left(\frac{6}{5}\right)$

.91872536986556843778... ≈ $\sqrt{2}\sin\frac{1}{\sqrt{2}} = \frac{\sqrt{\pi}}{2^{1/4}}J_{1/2}\left(\frac{1}{\sqrt{2}}\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!2^k}$

.91874093588132784272... ≈ $\frac{7}{16}+\frac{13}{32}e^{1/4}\sqrt{\pi}\,erf\left(\frac{1}{2}\right) = \sum_{k=0}^{\infty}\frac{k!k^2}{(2k)!}$

.91893853320467274178... ≈ $\log\sqrt{2\pi} = \lim_{n\to\infty}\frac{n!e^n}{n^{n+1/2}} = -\zeta'(1)$

= Stirling's constant σ, GKP 9.99

the constant in $\log n! = n\log n - n - \frac{\log n}{2} + \sigma + \frac{1}{12n} - ...$

= $\int_0^1 \log\Gamma(x)\,dx$ GR 6.441.2

= $\int_0^1\left(\frac{1}{\log x}+\frac{x}{1-x}-\frac{x}{2}\right)\frac{dx}{x\log x}$ GR 4.283.3

.9190194775937444302... ≈ $\frac{\sinh\pi}{4\pi} = \prod_2^{\infty}\left(1-\frac{1}{k^4}\right) = \exp\left(-\sum_{k=1}^{\infty}\frac{\zeta(4k)-1}{k}\right)$

= $\frac{1}{2\Gamma(2+i)\Gamma(2-i)}$

.9190625268488832338... ≈ $\Gamma\left(\frac{7}{4}\right)$

.91917581667117981829... ≈ $\psi^{(1)}(i)\psi^{(1)}(-i)$

2 .91935543953838864415... ≈ $\sum_{k=1}^{\infty}\frac{H_k^{\ 2}}{k!}$

.9193953882637205652... ≈ $2-2\cos 1 = 4\sin^2\frac{1}{2} = \sum_{k=1}^{\infty}\frac{(-1)^k}{(2k-1)!k}$

.92015118451061011495... ≈ $\pi-\frac{\pi\sqrt{2}}{2} = \int_0^{\pi}\frac{\sin^2 x}{1+\sin^2 x}dx = \int_0^{\infty}\log\left(1+\frac{1}{2x^2+1}\right)dx$

.920673594207792318945...≈ $\frac{e}{(e-1)^2} = \Phi\left(\frac{1}{e},-1,0\right) = \sum_{k=0}^{\infty}\frac{k}{e^k}$

= $\sum_{k=1}^{\infty}\frac{\phi(k)}{e^k-1}$

.92068856523896973321... ≈ $\frac{\sqrt{2\pi}}{e^{1/8}}erfi\frac{1}{2\sqrt{2}} = \sum_{k=0}^{\infty}\frac{(-1)^k k!}{(2k+1)!2^k}$

2 .92072423350623909536... ≈ $\frac{\pi\log 2}{2}+2G = -\int_0^1\frac{\log(1-x)}{\sqrt{1-x^2}}dx$ GR 4.292.1

= $\int_0^1\frac{\arccos x}{1-x}dx$ GR 4.521.2

$= \int_0^{\pi/2} \frac{x\sin x}{1-\cos x}\,dx$ GR 3.791.9

12 .92111120529372434463... $\approx 2 + 8\operatorname{csch}\frac{\pi}{2} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k^2+1/4}$

.92152242033286316168... $\approx 2 - \frac{\pi\left(\sin\pi\sqrt{2}+\sinh\pi\sqrt{2}\right)}{2\sqrt{2}\left(\cosh\pi\sqrt{2}-\cos\pi\sqrt{2}\right)}$

$= 1-\sum_{k=2}^{\infty}\frac{1}{k^4+1} = 1-\sum_{k=1}^{\infty}(-1)^{k+1}\left(\zeta(4k)-1\right)$

1 .92181205567280569867... $\approx 4\log^2 2 = \sum_{k=1}^{\infty}\frac{H_k}{k(k+1/2)}$

7 .9219918406294940337... $\approx \frac{e^2 I_0(2)}{2}-\frac{1}{2} = \sum_{k=0}^{\infty}\frac{1}{(k+1)!}\binom{2k+1}{k}$

.92203590345077812686... $\approx \frac{\pi^2}{6}-\frac{\pi}{4}+\frac{1}{16} = \sum_{k=1}^{\infty}\frac{\cos(k/2)}{k^2}$

$= \frac{1}{2}\left(Li_2\left(e^{i/2}\right)+Li_2\left(e^{-i/2}\right)\right)$

.92256201282558489751... $\approx \sqrt{\pi}\,erf\,\frac{1}{2} = \sum_{k=0}^{\infty}\frac{(-1)^1}{k!4^k(2k+1)}$

.92274595068063060514... $\approx \int_0^1 \Gamma(x+1)\,dx$

.92278433509846713939... $\approx \frac{3}{2}-\gamma = \psi(3)$

1 .92303552576131315974... $\approx \sum_{k=1}^{\infty}\left(\zeta^2(2k)-1\right)$

$.923076923076\underline{923076}$ $= \frac{12}{13} = \sum_{k=0}^{\infty}\frac{(-1)^k}{12^k}$

.92311670684442125248... $\approx \sum_{k=1}^{\infty}\frac{(-1)^{k+1}\zeta(4k)}{(2k-1)!} = \sum_{k=1}^{\infty}\frac{1}{k^2}\sin\frac{1}{k^2}$

.92370103378742659168... $\approx \frac{\pi}{\sqrt{4\pi-1}}\tanh\frac{\pi\sqrt{4\pi-1}}{2} = \sum_{k=0}^{\infty}\frac{1}{k^2+k+\pi}$

.92387953251128675612...$\approx \frac{\sqrt{2+\sqrt{2}}}{2} = \cos\frac{\pi}{8} = \sin\frac{3\pi}{8}$

.92393840292159016702... $\approx \frac{90}{\pi^4} = \frac{1}{\zeta(4)} = \sum_{k=1}^{\infty}\frac{\mu(k)}{k^4}$

.9241388730... $\approx$ point at which Dawson's integral, $e^{-x^2}\int_0^x e^{-t^2}dt$, attains its maximum of 0.5410442246. AS 7.1.17

.92419624074659374589... ≈ $\frac{4\log 2}{3} = 1 + 2\sum_{k=1}^{\infty}\frac{(-1)^k}{27k^3 - 3k}$ Berndt 2.5.4

= $\sum_{k=1}^{\infty}\frac{H_{k/2}}{2^k}$

.924299897222937... ≈ $\sum_{k=2}^{\infty}\frac{(-1)^k}{\log k}$

.924442728488380072202... ≈ $\frac{\pi^2}{25}\cot\frac{\pi}{5}\csc\frac{\pi}{5} = -\int_0^{\infty}\frac{\log x}{x^5+1}dx$

.92465170577553802366... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^k}{8k+1}$

5 .924696858532122437317... ≈ $\frac{5}{3} + \frac{5\log 3}{2} + \frac{5\pi}{6\sqrt{3}} = \sum_{k=2}^{\infty}\frac{5^k\left(\zeta(k)-1\right)}{3^k}$

.92491492293323294696... ≈ $\frac{1}{3\sqrt{2}}\left(\pi + \log(3+2\sqrt{2})\right) - \frac{\log 2}{3}$

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{(k+1)(4k+1)}$ Prud. 5.1.8.9

.92527541260212737052... ≈ $\frac{3\pi^2}{32} = \sum_{k=1}^{\infty}\frac{k^2\zeta(2k)}{4^k} = 4\sum_{k=1}^{\infty}\frac{k^2(4k^2+1)}{(4k^2-1)^3}$

= $\int_1^{\infty}\frac{\arctan x}{1+x^2}dx$

.925303491814363217608... ≈ $\sqrt{\pi}\,erfi(1) - 2 = \sum_{k=1}^{\infty}\frac{(k-\frac{1}{2})!}{k!(k+\frac{1}{2})!}$

.92540785884046280896... ≈ $\frac{\sec 1}{2} = \frac{1}{e^i + e^{-i}}$

.925459064119660772567... ≈ $\frac{1+\operatorname{csch}1}{2} = \frac{e^2+2e-1}{2\left(e^2-1\right)}$

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{k^2\pi^2+1}$

.9260001932455275726... ≈ $1 - \frac{\pi^2}{\sinh^2\pi} = \sum_{k=1}^{\infty}\left(\frac{1}{(k+i)^2} + \frac{1}{(k-i)^2}\right)$

= $\sum_{k=1}^{\infty}\frac{2k^2-2}{k^4+2k^2+1}$

.92614448971326014734... ≈ $3 - 2\zeta(5)$

.92655089611797091088... ≈ $1 - \frac{\pi^2}{\cosh^2\pi}$

1 .92684773069611513677... ≈ $\frac{\pi^2}{8} + \log 2 = \sum_{k=2}^{\infty} \frac{k\zeta(k)}{2^k} = \sum_{k=1}^{\infty} \frac{4k-1}{2k(2k-1)^2}$

8 .92697404116274799947... ≈ $\frac{1}{12}\left(\pi^2 + \log^2 2\right) = \int_0^{\infty} \frac{\log^3 x}{(x+2)(x-1)} dx$ GR 4.262.3

.92703733865068595922... ≈ $\frac{1}{8\sqrt{\pi}} \Gamma^2\left(\frac{1}{4}\right) = {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -1\right) = \int_1^{\infty} \frac{dx}{\sqrt{x^4+1}}$

.927099379463425416951...≈ $\frac{3\log^2 3}{2} - 6\log 2 \log 3 + 6\log^2 2 + 3Li_2\left(\frac{1}{4}\right)$

= $\sum_{k=0}^{\infty} \frac{(-1)^k}{3^k (k+1)^2} = \int_0^{\infty} \frac{x}{e^x + \frac{1}{3}}$

.92727990590304527791... ≈ $\gamma - 1 + \frac{5+3\sqrt{5}}{10} \psi\left(\frac{5+\sqrt{5}}{2}\right) + \frac{5-3\sqrt{5}}{10} \psi\left(\frac{5-\sqrt{5}}{2}\right)$

= $\sum_{k=2}^{\infty} \frac{2k-1}{k(k^2+k-1)} = \sum_{k=2}^{\infty} (-1)^k F_{k+1}\left(\zeta(k) - 1\right)$

.927295218001612232429...≈ $2\arctan\frac{1}{2} = \arcsin\frac{4}{5} = \frac{\pi}{4} + \arctan\frac{1}{7}$

= $i\log\left(1 - \frac{i}{2}\right) - i\log\left(1 + \frac{i}{2}\right) = \sum_{k=0}^{\infty} \frac{(-1)^k}{4^k (2k+1)}$

.92753296657588678176... ≈ $\frac{7}{4} - \frac{\zeta(2)}{2}$

.927695310470230431223...≈ $\frac{1}{2}\left(Li_3\left(e^{i/2}\right) - Li_3\left(e^{-i/2}\right)\right) = \sum_{k=1}^{\infty} \frac{\cos(k/2)}{k^3}$

.92770562889526130701... ≈ $\frac{\pi^4}{105} = \frac{\zeta(8)}{\zeta(4)} = \sum_{k=1}^{\infty} \frac{\lambda(k)}{k^4}$ HW Thm. 300

2 .92795159370697612701... ≈ $e^{2\cos 1} \sin(2\sin 1) = \sin(2\sin 1)\left(\cosh(2\cos 1) + \sinh(2\cos 1)\right)$

= $-\frac{i}{2}\left(e^{2e^{i}} - e^{2e^{-i}}\right) = \sum_{k=1}^{\infty} \frac{2^k \sin k}{k!}$

1 .92801312657238221592... ≈ $2\pi(1 - \log 2) = -\int_0^{\infty} \log\left(1 + \frac{4}{x^2}\right) \log x \, dx$ GR 4.222.3

3 .928082040192025996... ≈ $\sum_{k=1}^{\infty} \frac{pd(k)}{k!}$

.92820323027550917411... ≈ $4\sqrt{3} - 6 = \sum_{k=0}^{\infty} \frac{(-1)^k}{12^k (k+1)} \binom{2k}{k}$

6 .92820323027550917411... ≈ $\sqrt{48} = 4\sqrt{3} = \sum_{k=1}^{\infty} \binom{2k}{k} \frac{k^2}{6^k}$

$.9285714285714\underline{285714}$ = $\frac{13}{14} = \sum_{k=0}^{\infty}\frac{(-1)^k}{13^k}$

1 .92875702258534176183... ≈ $\frac{\pi^2}{3} - \frac{49}{36} = \int_0^1 \frac{(1+x^3)\log x}{x-1}dx$

.92875889011460955439... ≈ $\frac{1}{2}I_2(2\sqrt{2}) = \sum_{k=0}^{\infty}\frac{2^k}{k!(k+2)!}$

.92920367320510338077... ≈ $\frac{5-\pi}{2} = \frac{i}{2}\log\frac{1-e^{-5i}}{1-e^{5i}} = -\sum_{k=1}^{\infty}\frac{\sin 5k}{k}$

4 .92926836742289789153... ≈ $\pi e \gamma$

.92931420371895891339... ≈ $\zeta(2) + \zeta(3) + \zeta(4) - 3 = \sum_{k=2}^{\infty}\frac{k^2+k+1}{k^4}$

3 .92931420371895891339... ≈ $\zeta(2) + \zeta(3) + \zeta(4) = \sum_{k=1}^{\infty}\frac{k^2+k+1}{k^4}$

1 .92935550865482645193... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(3k)}{k!} = \sum_{k=1}^{\infty}\left(e^{1/k^3}-1\right)$

.92939892490022826891 4...≈ $3\pi\left(\log 3 - 1\right) = \int_0^{\infty}\log x \log\left(1+\frac{9}{x^2}\right)dx$ GR 4.222.3

1 .92951861052078948 4999...≈ $\frac{\pi^2}{3} - \frac{\pi\sqrt{3}}{4} = \sum_{k=1}^{\infty}\frac{k^2}{(k^2-1/9)^2}$

.92958455909241192604... ≈ ${}_2F_1\left(1,\frac{1}{3},\frac{4}{3},-\frac{1}{3}\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{3^k(3k+1)}$

.93002785739908278355... ≈ $-\frac{263}{2310} - \frac{\pi}{2\sqrt{15}}\csc \pi\sqrt{15}$

.93019136710263285866 8...≈ $\sqrt{\frac{e}{\pi}}$

.93052667763586197073... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)}{k!(k-1)!} = \sum_{k=1}^{\infty}\left(\frac{1}{\sqrt{k}}I_1\left(2\sqrt{\frac{1}{k}}\right) - \frac{1}{k}\right)$

.93122985945271217726... ≈ $\frac{1}{\sqrt{2}}\log\left(2+\sqrt{3}\right) = \sqrt{2}\,\text{arcsinh}\frac{1}{\sqrt{2}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{8^k(2k+1)}\binom{2k}{k}$

.93137635638313358185... ≈ $\sum_{k=1}^{\infty}\frac{|\mu(k)|}{2^k}$

6 .93147180559945309417... ≈ $10\log 2$

.9314760947318097375... ≈ $\zeta(3) - \frac{\pi^4}{360} = \sum_{k=1}^{\infty}\frac{k\,H_k}{(k+1)^3}$

1	.9316012105732472119...	≈	$\frac{e+e^{e^{-2\infty}}}{2} = \sum_{k=0} \frac{\cosh k}{k!e^k}$	
	.93194870235104284677...	≈	$\frac{\pi(3\pi+4)}{32\sqrt{2}} = -\int_0^\infty \frac{\log x}{(x^4+1)^2}dx$	
	.93203042415025124362...	≈	$\sum_{k=0}^\infty \frac{(-1)^k}{9k+1}$	
3	.932239737431101510706...	≈	$-\zeta'\left(\frac{3}{2}\right) = \sum_{k=1}^\infty \frac{\log k}{k^{3/2}}$	
	.9326813147863510178...	≈	$\frac{4}{\pi^2}\sinh\frac{\pi}{2} = \prod_{k=1}^\infty \left(1-\frac{1}{16k^4}\right)$	
	.932831259045789401392...	≈	$\pi - \frac{\pi^2}{12} - 2\log 2 = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{2k^3-k^2} = -\int_0^1 Li_2(-x^2)\frac{dx}{x^2}$	
1	.932910130264518932837...	≈	$\frac{37}{16} - \zeta(3) + \frac{\pi^2}{12} = \int_0^\infty x\, Li_2(-x)^2\, dx$	
	.93309207559820856354...	≈	$\cos\frac{1}{e} = \sum_{k=0}^\infty \frac{(-1)^k}{(2k)!e^{2k}}$	AS 4.3.66, LY 6.110
2383	.93316355858267141097...	≈	$877e = \sum_{k=1}^\infty \frac{k^7}{k!}$	
	.93333333333333333333$\underline{3}$	=	$\frac{14}{15} = \sum_{k=0}^\infty \frac{(-1)^k}{14^k}$	
5	.9336673104466320167...	≈	$\frac{1}{1256}\left(\psi^{(3)}\left(\frac{1}{4}\right) - \psi^{(3)}\left(\frac{3}{4}\right)\right) = \int_1^\infty \frac{\log^3 x\, dx}{x^2+1}$	
1	.93388441384851971997...	≈	$\frac{\pi+1}{\pi-1}$	
	.93401196415468746292...	≈	$1-e^{-e} = 1+\cosh e+\sinh e = \sum_{k=1}^\infty \frac{(-1)^{k+1}e^k}{k!}$	
		=	$\int_0^e e^{-x}dx$	
151	.93420920380567881895...	≈	$\frac{413}{e} = \sum_{k=0}^\infty \frac{(-1)^k k^{10}}{k!}$	
	.93478807021696950745...	≈	$\frac{2}{\sqrt{\pi}}\Gamma\left(\frac{7}{6}\right)\Gamma\left(\frac{4}{3}\right) = \prod_{k=1}^\infty \frac{k(k+\frac{1}{2})}{(k+\frac{1}{3})(k+\frac{1}{6})}$	J1061
	.93480220054467930941...	≈	$\frac{\pi^2}{2} - 4 = \sum_{k=1}^\infty \frac{(2k)!!}{(2k-1)!!k^2(2k+1)} = \psi^{(1)}\left(\frac{3}{2}\right)$	
		=	$\sum_{k=1}^\infty \frac{1}{(k+1/2)^2} = \sum_{k=2}^\infty \frac{(-1)^k(k-1)\zeta(k)}{2^{k-2}} = \sum_{k=1}^\infty \frac{4}{(2k+1)^2}$	

1 .934802200544679309417... ≈ $\frac{\pi^2}{2} - 3 = \int_0^1\int_0^1\int_0^1 \frac{x+y+z}{1-xyz}\,dx\,dy\,dz$

2 .934802200544679309417... ≈ $\frac{\pi^2}{2} - 2 = \int_0^\pi \frac{x\sin^2 x\,dx}{1-\cos x}$

4 .934802200544679309417... ≈ $\frac{\pi^2}{2} = \psi^{(1)}\left(\frac{1}{2}\right) = -i\,\pi\log i$

= $3\zeta(2) = \sum_{k=0}^{\infty}\frac{1}{(k+\frac{1}{2})^2} = \zeta\left(2,\frac{1}{2}\right)$

= $\sum_{k=1}^{\infty}\frac{(2k)!!}{(2k-1)!!k^2} = \sum_{k=1}^{\infty}\frac{4^k}{\binom{2k}{k}k^2} = \sum_{k=21}^{\infty}\frac{(k-1)!(k-1)!4^k}{(2k)!}$

= $\sum_{k=2}^{\infty}k^2\left(\zeta(k)-\zeta(k+1)\right) = \sum_{k=2}^{\infty}\frac{4k^2-k+1}{k^2(k-1)^2}$

= $2\arcsin^2 1$

= $-\int_0^\infty \frac{\log^3 x}{(x+1)^4}dx$

= $\int_0^\infty \log^2\left(\frac{1+x}{1-x}\right)\frac{dx}{x(1+x^2)}$ GR 4.297.5

= $\int_0^\infty \log^2\left(\frac{1+\tan}{1-\tan x}\right)\frac{dx}{x}$ GR 4.323.3

8 .93480220054467930941... ≈ $\frac{\pi^2}{2} + 4 = \psi^{(1)}\left(-\frac{1}{2}\right)$

.93534106617011942477... ≈ $\frac{7\pi^4}{729} = 1 + \sum_{k=1}^{\infty}\left(\frac{(-1)^k}{(3k-1)^4} + \frac{(-1)^k}{(3k+1)^4}\right) = \nu_4$

.93548928378863903321... ≈ $\frac{5}{2\pi\sqrt{2}}\sqrt{5-\sqrt{5}} = \frac{1}{2\pi}\sin\frac{\pi}{5} = \binom{0}{1/5} = \prod_{k=1}^{\infty}\left(1-\frac{1}{25k^2}\right)$

.93594294416994206293... ≈ $-Li_3\left(e^{2i}\right) - Li_3\left(e^{-2i}\right)$

.93615021799926654078... ≈ $\log 2 + \frac{\pi}{\sqrt{3}} + \frac{\pi}{2} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)(3k+1)}$

.93615800314842911627... ≈ $\sum_{k=2}^{\infty}\left(1 - \frac{1}{\zeta(k)\zeta(k+1)}\right)$

.936342294367604449... ≈ $\frac{\sin 1}{1-\pi^{-2}} = \prod_{k=2}^{\infty}\left(1-\frac{1}{\pi^2 k^2}\right)$

5 .936784413887151464... ≈ $\int_1^\infty \frac{x^3\,dx}{e^x - e^{-x}}$

.93709560427462468743... $\approx$ $\frac{\pi}{8}(1+2\log 2) = -\int_0^{\pi/2}\log(\sin x)\cos^2 x\,dx$ GR 4.384.10

$= -\int_0^1 \sqrt{1-x^2}\log x\,dx$ GR 4.241.9

$= \int_0^\infty xe^{-x}\sqrt{1-e^{-2x}}\,dx$ GR 3.431.2

7 .9372539331937717715... $\approx$ $\sqrt{63} = 3\sqrt{7}$

1 .93738770119278726439... $\approx$ $\frac{3}{2} - \frac{\gamma}{2} + \frac{\pi^2}{6} - \frac{\log 2\pi}{2} = \sum_{k=1}^\infty \frac{k^2}{k+1}\left(\zeta(k)-1\right)$

Adamchick-Srivastava 2.23

.9375000000000000000$\underline{0}$ = $\frac{15}{16} = \sum_{k=0}^\infty \frac{(-1)^k}{15^k}$

.93750000000013113727... $\approx$ $\sum_{k=1}^\infty \frac{(-1)^{k+1}}{k^{k^k}}$

.93754825431584375370... $\approx$ $-\zeta'(2) = \sum_{k=1}^\infty \frac{\log k}{k^2}$ Berndt 8.23.8

$= \sum_{n=1}^\infty \frac{1}{(n-1)!}\sum_{k=1}^\infty \frac{\log^n k}{k^3}$

.93755080503059465056... $\approx$ $\sum_{k=1}^\infty \frac{(-1)^{k+1}}{k^{k^2}}$

1 .93789229251873876097... $\approx$ $\frac{\pi^3}{16} = \int_0^\infty \frac{\log^2 x\,dx}{(x^2+1)^2} = \int_0^\infty \frac{x^2\,dx}{e^x + e^{-2x}}$

$= i\left(Li_3(-i) - Li_3(i)\right)$

$= \int_0^1 \frac{\log^2 x\,dx}{x^2+1} = \int_1^\infty \frac{\log^2 x\,dx}{x^2+1}$ GR 4.261.6

$= -\int_0^\infty \frac{\log^2 x\;dx}{x^4-1}$

$= \int_0^{\pi/4} \left(\log\tan x\right)^2 dx$ GR 4.227.7

$= -\int_0^1\int_0^1 \frac{\log(x^2y^2)}{1+x^2y^2}\,dx\,dy$

.93795458450141751816... $\approx$ $\frac{1}{24}\left(15 + \pi^2 + 3i\left(\psi^2(i) - \psi^2(-i)\right)\right) = \sum_{k=2}^\infty \frac{H_k}{k^4-1}$

.93809428703288482665... $\approx$ $\sum_{k=0}^\infty \frac{(-1)^k}{10k+1}$

.93836264953979373002... ≈ $8\cos\frac{1}{2}+4\sin\frac{1}{2}-8=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!4^k(k+1)}$

.93846980724081290423... ≈ $J_1\left(\frac{1}{2}\right)=\sum_{k=0}^{\infty}\frac{(-1)^k}{(k!)^2 16^k}$

.93852171797024950374... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}\zeta(2k)}{(k-1)!}=\sum_{k=1}^{\infty}\frac{1}{k^2 e^{1/k^2}}$

6 .938535628628181848... ≈ $\sum_{k=1}^{\infty}\frac{k^2 H_k^{(3)}}{2^k}$

2 .93855532107125917158... ≈ $\sum_{k=2}^{\infty}\left(\sigma_1(k)-1\right)\left(\zeta(k)-1\right)$

1 .93872924635600957846... ≈ $\pi\sqrt{2}\csc\frac{\pi}{\sqrt{2}}-\frac{\pi^2}{6}-2=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2(k^2-1/2)}$

.93892130408682951018... ≈ $\frac{\sqrt{\pi}}{2}\coth\sqrt{\pi}$

1 .93896450902302799392... ≈ $\frac{5\sqrt{5}}{4}\log\frac{5+\sqrt{5}}{5-\sqrt{5}}+\frac{25\log 5}{4}-\frac{5\pi}{2}\sqrt{1+\frac{2}{\sqrt{5}}}$

= $\sum_{k=1}^{\infty}\frac{5}{k(5k-1)}=\sum_{k=0}^{\infty}\frac{1}{(k+1)(k+4/5)}=\sum_{k=2}^{\infty}\frac{\zeta(k)}{5^{k-2}}$

4 .939030025676363443... ≈ $3\zeta(2)+\frac{\zeta(4)}{256}=\frac{\pi^2}{2}+\frac{\pi^4}{23040}=\sum_{k=1}^{\infty}\frac{1}{a(k)^4}$,

where $a(k)$ is the nearest integer to $\sqrt[3]{k}$. AMM 101, 6, p. 579

1 .939375420766708953077...≈ $-Li_2(-3)=\int_0^1\frac{\log x}{x+1/3}dx$

.93958414182726727804... ≈ $\frac{\pi^3}{33}$

1 .9395976608404063362... ≈ $\log 2-\frac{1}{\sqrt{2}}\log\frac{2+\sqrt{2}}{2-\sqrt{2}}=\sum_{k=1}^{\infty}\frac{H_{2k}}{2^k}$

4 .9395976608404063362... ≈ $3+\log 2-\frac{1}{\sqrt{2}}\log\frac{2+\sqrt{2}}{2-\sqrt{2}}=\sum_{k=1}^{\infty}\frac{kH_{2k+1}}{2^k}$

.93982139966885912103... ≈ $\sum_{p_k=kth\ \ prime}\phi(k)\left(\zeta(p_k)-1\right)$

.9399623239132722494... ≈ $\frac{2\pi^2}{21}=\frac{\zeta(6)}{\zeta(4)}=\prod_{p\ prime}\frac{1+p^{-2}}{1+p^{-2}+p^{-4}}$

.94002568113... ≈ g_4 J310

.94015085153961392796... ≈ $\sum_{k=2}^{\infty}\frac{\zeta(k)}{(k-1)k!}$

1 .94028213339253981177... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k+1)}{k!} = \sum_{k=1}^{\infty}\frac{e^{1/k^2}-1}{k}$

.94084154990319328756... ≈ $\frac{115\pi}{384} = \int_0^{\infty}\frac{\sin^5 x}{x^5}$ GR 3.827.11

1 .94112549695428117124... ≈ $24\sqrt{2} - 32 = \sum_{k=0}^{\infty}\frac{1}{8^k}\binom{2k+2}{k}$

.$\underline{9411764705882352}$ = $\frac{16}{17} = \sum_{k=0}^{\infty}\frac{(-1)^k}{16^k}$

.94226428525106763631... ≈ $8\log\frac{9}{8} = \sum_{k=0}^{\infty}\frac{(-1)^k}{8^k(k+1)}$

4 .9428090415820633659... ≈ $4+\frac{2\sqrt{2}}{3}$ CFG D4

.94286923678411146019... ≈ $\frac{\pi^2}{6}-\frac{\pi}{4}+\frac{1}{12} = \sum_{k=1}^{\infty}\frac{\sin k}{k^3}$ Davis 3.32

= $\frac{i}{2}\left(Li_3\left(e^{-i}\right) - Li_3\left(e^{i}\right)\right)$

.942989417$\underline{989417}$ = $\frac{7129}{7560} = \sum_{k=1}^{\infty}\frac{1}{k(3+k/3)}$

.94308256800936130684... ≈ $HypPFQ[\{1,1,1,1\},\{2,2,2,2\},-1]$

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{k!(k+1)^4}$

.94318959229937991745... ≈ $\frac{5}{4}-\frac{\pi\sqrt{2}}{4}\cot\pi\sqrt{2} = \sum_{k=2}^{\infty}\frac{1}{k^2-2}$

= $\sum_{k=1}^{\infty}2^{k-1}\left(\zeta(2k)-1\right)$

1 .94359643682075920505 7...≈ $\frac{\zeta(2)\zeta(3)}{\zeta(6)} = \sum_{k=1}^{\infty}\frac{|\mu(k)|}{k\cdot\phi(k)}$

= $\prod_{p\ prime}\left(1+\frac{1}{p(p-1)}\right) = \prod_{p\ prime}\left(\frac{1-p^{-6}}{(1-p^{-2})(1-p^{-3})}\right)$

5 .94366218996591215818... ≈ $3+2\sqrt{6}\,\mathrm{csch}\frac{\pi}{\sqrt{6}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k^2+1/6}$

.94409328404076973180... ≈ $\frac{\pi\zeta(3)}{4} = \int_0^1\frac{\pi\arcsin^2 x - 2\arcsin^3 x}{x}dx$

= $-\int_0^{\pi/2} x^2 \log(2\cos x)\,dx$

Latter result due to Peter J.C. Moses and Seiichi Kirikami (A193712)

.94427190999915878563... ≈ $4\sqrt{5}-8 \;=\; \sum_{k=0}^{\infty} \frac{(-1)^k}{16^k(k+1)}\binom{2k}{k}$

8 .94427190999915878563... ≈ $\sqrt{80} \;=\; 4\sqrt{5}$

.9444444444444444444$\underline{4}$ = $\frac{17}{18} \;=\; \sum_{k=0}^{\infty} \frac{(-1)^k}{17^k}$

.94495694631473766439... ≈ $\cos\frac{1}{3} \;=\; \sum \frac{(-1)^k}{(2k)!9^k}$ AS 4.3.66, LY 6.110

.94519568931779374921... ≈ $-2Li_3\left(-\frac{1}{2}\right) \;=\; \sum_{k=0}^{\infty} \frac{(-1)^k}{2^k(k+1)^3}$

= $\int_1^{\infty} \frac{\log^2 x}{2x^2+x}dx \;=\; \int_0^{\infty} \frac{x^2}{2e^x+1}dx$

= $\int_0^{1} \frac{\log^2 x}{x+2}dx$

.94529965343349825190... ≈ $\sum_{k=1}^{\infty} \frac{\sigma_0(k)}{(k+1)!}$

.94530872048294188123... ≈ $\frac{8\sqrt{\pi}}{15} = -\Gamma\left(-\frac{5}{2}\right)$

1 .94558907762210954515... ≈ $\sum_{k=1}^{\infty} \frac{\zeta(2k)}{(k!)^2} \;=\; \sum_{k=1}^{\infty}\left(I_0\left(\frac{2}{k}\right)-1\right)$

1 .94591014905531330511... ≈ $\log 7 \;=\; Li_1\left(\frac{6}{7}\right)$

.94608307036718301494... ≈ $si(1) \;=\; \int_0^{1} \frac{\sin x}{x}dx$

= $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!(2k+1)}$ AS 5.2.14

= $\int_1^{\infty} \sin\left(\frac{1}{x}\right)\frac{dx}{x}$

= $-\int_0^{1} \log x \cos x\, dx$

4 .94616381210038444055... ≈ $3\sqrt{e} \;=\; \sum_{k=0}^{\infty} \frac{pf(k)k}{2^k}$

.94619079903112072202 6...≈ $\log 6 - 2\gamma - 2 \;=\; \sum_{k=2}^{\infty} (-1)^k \frac{2^k}{k}\left(\zeta(k)-1\right)$

1 .9467218571726023989... $\approx \sum_{k=1}^{\infty} \sin\frac{k}{2^k}$

.946738452432832464562... $\approx \sum_{k=1}^{\infty} \frac{\pi(k)}{k!}$

5 .94678123535278519168... $\approx \pi\sqrt{5}\tan\frac{\pi}{2\sqrt{5}} = \sum_{k=0}^{\infty} \frac{1}{k^2+k+1/5}$

.94684639467237257934... $\approx \frac{1}{4}\left(3e^{\cos 1}\sin(\sin 1) - e^{\cos 3}\sin(\sin 3)\right) = \sum_{k=1}^{\infty} \frac{\sin^3 k}{k!}$

$= \frac{i}{8}\left(3e^{e^{-i}} - 3e^{e^{i}} - e^{e^{-3i}} + e^{e^{3i}}\right)$

.94703282949724591758... $\approx \frac{7\pi^4}{720} = \eta(4) = \frac{7\zeta(4)}{8} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^4}$ J306

.947123885654706166761... $\approx \frac{\pi}{2^{3/4}}\left(\frac{1-i}{4}\right)\left(\cot\pi(-2)^{1/4} - \coth\left(\frac{\pi(1+i)}{2^{1/4}}\right)\right)$

$= \sum_{k=1}^{\infty} \frac{k^2}{k^4+2}$

.94716928635947485958... $\approx \frac{\pi}{\sqrt{11}}\tanh\frac{\pi\sqrt{11}}{2} = \sum_{k=1}^{\infty} \frac{1}{k^2-k+3}$

.947368421052631578947... $\approx \frac{18}{19} = \sum_{k=0}^{\infty} \frac{(-1)^k}{18^k}$

1 .94752218030078159758... $\approx \frac{\sqrt{\pi}}{16}\left(2\gamma^2 + \pi^2 + 8\gamma\log 2 + 8\log^2 2\right)$

$= \int_0^{\infty} e^{-x^2}\log^2 x\,dx$ GR 4.335.2

.947786324594343627... $\approx \sqrt{\pi}\,\sin\frac{1}{\sqrt{\pi}} = \prod_{k=1}^{\infty}\left(1 - \frac{1}{\pi^3 k^2}\right)$

.94805944896851993568... $\approx \frac{\pi}{8}\left(1+\sqrt{2}\right) = \sum_{k=1}^{\infty}\left(\frac{1}{8k-7} - \frac{1}{8k-1}\right)$ J78, J264

1 .94838823193110849310... $\approx \sum_{k=1}^{\infty} \frac{Q(k)}{k!}$

.94899699000260690729... $\approx \frac{\zeta(2)+\zeta(3)}{3}$

4 .949100893673262820934... $\approx \frac{1}{\zeta(3)-1}$

.94939747687277389162... $\approx \arctan(\sinh 1 \csc 1) = \sum_{k=-\infty}^{\infty} (-1)^{k+1}\arctan\left(\frac{1}{k\pi+1}\right)$

[Ramanujan] Berndt Ch. 2, Eq. 11.4

.9494332401401025766... $\approx \sum_{k=1}^{\infty} \frac{k}{3^k - 1} = \sum_{k=1}^{\infty} \frac{\sigma_1(k)}{3^k}$

.9494817111114981524546... $\approx \frac{\pi^2 \gamma}{6}$

.94949832897257497482... $\approx \frac{2 \sin 1}{\sqrt{\pi}} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(k+\frac{1}{2})4^k}$

.9497031262940093953... $\approx \frac{\pi^2}{6\sqrt{3}} = 1 + \sum_{k=1}^{\infty} \left(\frac{(-1)^k}{(6k-1)^2} + \frac{(-1)^k}{(6k+1)^2} \right)$ J329

$= -\int_0^{\infty} \frac{\log x}{x^6 + 1} dx$

9 .9498743710661995473... $\approx \sqrt{99}$

.9500000000000000000$\underline{0}$ = $\dfrac{19}{20} = \sum_{k=0}^{\infty} \dfrac{(-1)^k}{19^k}$

.95023960511664325898... ≈ $\dfrac{\pi^2}{6} - 3\log^2 \dfrac{1+\sqrt{5}}{2} = \sum_{k=0}^{\infty} \dfrac{(-1)^k (k!)^2}{(2k)!(2k+1)^2}$ Berndt Ch. 9

.95038404818064742915... ≈ $\dfrac{\pi}{\sqrt{6}} \coth \pi \sqrt{\dfrac{3}{2}} - \dfrac{1}{3} = \sum_{k=1}^{\infty} \dfrac{1}{k^2 + 3/2}$

.95075128294937996421... ≈ $\dfrac{8}{7\zeta(3)} = -\sum_{k=1}^{\infty} \dfrac{\mu(2k)}{k^3}$

.95105651629515357212... ≈ $\dfrac{1}{4}\sqrt{10+\sqrt{20}} = \sin \dfrac{2\pi}{5} = \cos \dfrac{\pi}{10}$

.9511130618309... ≈ $\sum_{k=1}^{\infty} (-1)^{k+1} \dfrac{|\mu(k)|}{k^4}$

1 .95136312812584743609... ≈ $2\sin^2 \sqrt{2} = \sum_{k=1}^{\infty} \dfrac{(-1)^{k+1} 8^k}{(2k)!}$

.95136798763366872l6... ≈ $\dfrac{15-e^2}{8} = \sum_{k=0}^{\infty} \dfrac{2^k}{k!(k+6)}$

.95151771341641504187... ≈ $\dfrac{1}{36}\left(\psi^{(1)}\left(\dfrac{1}{6}\right) - \psi^{(1)}\left(\dfrac{2}{3}\right)\right) = \sum_{k=0}^{\infty} \dfrac{(-1)^k}{(3k+1)^2}$

= $-\int_0^1 \dfrac{\log x}{1+x^3}\,dx = \int_1^{\infty} \dfrac{x \log x}{1+x^3} = \int_0^{\infty} \dfrac{x\,dx}{e^x + e^{-2x}}$

.95205690315959428854... ≈ $\zeta(3) - \dfrac{1}{4} = \sum_{k=2}^{\infty} \left(\dfrac{1}{k^2 - k} - \dfrac{1}{k^5 - k^3} \right)$

= $\sum_{k=2}^{\infty} \left(\zeta(k) - \zeta(2k+1) \right)$

.95217131705368432233... ≈ $\dfrac{2+\log 2}{2\sqrt{2}} = -\int_0^{\pi/4} \cos x \log(\sin x)\,dx$

.952380952380$\underline{952380}$ = $\dfrac{20}{21} = \sum_{k=0}^{\infty} \dfrac{(-1)^k}{20^k}$

.95247416912953901173... ≈ $\dfrac{1}{4}\Phi\left(-\dfrac{1}{2}, 2, \dfrac{1}{2}\right) = \sum_{k=0}^{\infty} \dfrac{(-1)^k}{2^k (2k+1)^2}$

2 .95249244201255975651... ≈ $(e-1)^2 = \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} \dfrac{1}{j!k!}$

.95257412682243321912... ≈ $\dfrac{e^3}{e^3+1} = \dfrac{1}{2}\left(1 + \tanh \dfrac{3}{2}\right) = \sum (-1)^k e^{3k}$ J944

.95273613236508996884... ≈ $\dfrac{\pi}{2\sqrt{e}} = \int_0^{\infty} \dfrac{1}{1+t^2} \cos \dfrac{t}{2}\,dt$

5 .9527777854112600533338...≈ $\displaystyle \frac{11\pi^4}{180} = \frac{11\zeta(4)}{2} = \frac{1}{\pi}\int_0^\pi x^2 \log^2\left(2\cos\frac{x}{2}\right)dx$

Borwein & Borwein, Proc. AMS 123, 4 (1995) 1191-1198

.9535301255514155563988...≈ $\displaystyle \sum_{k=2}^\infty \left(\frac{\zeta(k)}{\zeta(k+4)}-1\right)$

.9535353256364312286 1... ≈ $\displaystyle \frac{551}{2} - 101e = \sum_{k=0}^\infty \frac{k^5}{(k+3)!}$

.9540853578760062131 45...≈ $\displaystyle \sum_{k=2}^\infty \frac{\phi(k)}{k!} = \sum_{k=1}^\infty \frac{k\Phi(k)}{(k+1)!}$, where $\displaystyle \Phi(n) = \sum_{k=1}^n \phi(k)$

4 .9542343560018901633 8... ≈ $\displaystyle Ei(2) = \gamma + \log 2 + \sum_{k=1}^\infty \frac{2^k}{k!k}$ AS 5.1.10

.9546045214322317348 2... ≈ $1 - \zeta(4) + \zeta(5)$

3 .9546087005945922363 94...≈ $\displaystyle \frac{\pi^2\zeta(3)}{3} = 2\zeta(2)\zeta(3)$

.9547712524422192276 8... ≈ $\displaystyle \frac{3}{2}\log 3 - \log 2 = \int_0^1 \log\left(1+\frac{1}{2x}\right)dx$

7 .9549265210128452745 13...≈ $\displaystyle 2\pi I_0(1) = \int_0^{2\pi} e^{\sin x}dx$

.9549296585513720146 1... ≈ $\displaystyle \frac{3}{\pi} = \prod_{k=1}^\infty \left(1-\frac{1}{36k^2}\right)$ GR 1.431

= $\displaystyle \prod_{k=1}^\infty \cos\left(\frac{\pi}{6\cdot 2^k}\right)$ GR 1.439.1

= $\displaystyle \binom{0}{1/6}$

2 .9555568855073028145 3... ≈ $\displaystyle \sum_{k=2}^\infty \binom{2k}{k}\frac{\zeta(k)-1}{k!}$

.9562301773696957140 5... ≈ $\displaystyle \frac{7}{128} + \frac{3\zeta(3)}{4} = \sum_{k=1}^\infty k^3(\zeta(2k+1)-1)$

= $\displaystyle \sum_{k=2}^\infty \frac{k(k^4+4k^2+1)}{(k^2-1)^4}$

2 .9565757385002539605 6... ≈ $\displaystyle 293e - \frac{1587}{2} = \sum_{k=0}^\infty \frac{k^6}{(k+3)!}$

.9567860817362277502 26...≈ $\displaystyle 1 - \cosh\pi + \sinh\pi = \int_0^\pi e^{-x}dx$

.9569838481574018572 7... ≈ $\displaystyle \frac{5\pi^3}{162} = \sum_{k=1}^\infty \frac{\sin k\pi/3}{k^3}$ GR 1.443.5

.95706091455937067489... ≈ $\sqrt{G}$

6 .9572873234262029614... ≈ $\int\limits_2^{\infty} x^2\left(\zeta(x)-1\right)dx$

.957536385054047800178... ≈ $\sum_{s=2}^{\infty}\sum_{n=1}^{\infty}\log\left(\zeta(sn)\right)$

.95765755436028576375O... ≈ $\frac{1}{2}-\cot 2 = \sum_{k=1}^{\infty}\frac{1}{2^k}\tan\frac{1}{2^{k-1}}$ Berndt ch. 31

1 .95791983595628290328... ≈ $\frac{\pi\,\operatorname{arcsinh}1}{\sqrt{2}} = \frac{\pi}{\sqrt{2}\log\left(1+\sqrt{2}\right)} = \int\limits_0^{\infty}\frac{\log(x^2+1)}{x^2+2}$

2980 .95798704172827474359... ≈ e^8

.958168713149316111695... ≈ $\frac{\pi^2}{2}-\gamma-\frac{1}{2}\left(\psi(1+i)+\psi(1-i)\right)-\frac{1}{8}\left(\psi^{(2)}(1+i)+\psi^{(2)}(1-i)\right)$

$+\frac{5i}{8}\left(\psi^{(1)}(1-i)-\psi^{(1)}(1+i)\right)-2\zeta(3)$

$= \sum_{k=2}^{\infty}(-1)^k k^2\left(\zeta(k)-\zeta(2k-1)\right)$

.95833608906520793715... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k^2)!}$

.95835813283300701621... ≈ $\cos\frac{1}{\sqrt{2}}\cosh\frac{1}{\sqrt{2}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(4k)!}$

.95838045456309456205... ≈ $\frac{1}{1024}\left(\psi^{(2)}\left(\frac{5}{8}\right)+\psi^{(2)}\left(\frac{3}{8}\right)-2\psi^{(2)}\left(\frac{1}{8}\right)-2\pi^3\cot\frac{\pi}{8}\csc^2\frac{\pi}{8}\right)$

$= 1+\sum_{k=1}^{\infty}\left(\frac{(-1)^k}{(4k-1)^3}+\frac{(-1)^k}{(4k+1)^3}\right)$

$= \sum_{k=0}^{\infty}\frac{(-1)^{\lfloor (k+1)/2\rfloor}}{(2k+1)^3}$

.95853147061909644370... ≈ $\frac{1}{3}\left(\cos\left(\frac{\sqrt{3}}{2}-\frac{\pi}{3}\right)-\frac{1}{e}\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(3k+1)!}$

.95857616783363717319... ≈ $\frac{1+\tanh(\pi/2)}{2} = \frac{e^{\pi/2}}{e^{\pi/2}+e^{-\pi/2}} = \sum_{k=0}^{\infty}(-1)^k e^{-\pi k}$ J944

2 .95867511918863889231... ≈ $\Gamma\left(\frac{1}{4}\right)\Gamma^{-1}\left(\frac{3}{4}\right)$

.9588510772084060005... ≈ $2\sin\frac{1}{2} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!4^k} = \prod_{k=1}^{\infty}\left(1-\frac{1}{4\pi^2k^2}\right)$ GR 1.431

$= \binom{0}{1/2\pi}$

.95924696800225890378... $\approx 4\left(\gamma + ci(1)\right) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k-1)!k^2}$

.95951737566747185975... $\approx \frac{1}{2\sinh(1/2)} = \frac{\sqrt{e}}{e-1} = \sum_{k=0}^{\infty} e^{-(k+1/2)}$ J942

.95974233961488293932... $\approx \frac{2^{2/3}\pi}{3\sqrt{3}} = \int_0^{\infty} \frac{x\,dx}{x^3+2}$

.95985740794367609075... $\approx \frac{1}{2}\left((\pi-1)\cos\frac{1}{2} - \log\left(2-2\cos 1\right)\sin\frac{1}{2}\right) = \sum_{k=1}^{\infty} \frac{1}{k}\sin\frac{2k+1}{2}$

.95993649079456585556... $\approx \sum_{k=1}^{\infty} \left|\mu(k)\right|\left(\zeta(k+1)-1\right)$

1 .96029978618557568714... $\approx \sum_{k=2}^{\infty} \frac{k(\zeta^2(k)-1)}{k!}$

.96031092683032321590... $\approx \frac{16}{\sqrt{17}}\operatorname{arcsinh}\frac{1}{4} = \sum_{k=0}^{\infty} \frac{(-1)^{k+1}(k!)^2}{(2k+1)!4^k}$

.96032715793678926 80... $\approx \frac{4}{e^{1/4}} - 4 + 2\sqrt{\pi}\, erf\, \frac{1}{2} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)!4^k(2k+1)}$

.96033740390091314086... $\approx \frac{32\sqrt{2}}{15\pi} = \prod_{k=2}^{\infty}\left(1 - \frac{1}{16k^2}\right)$

1 .96040661651099501 05... $\approx \frac{\pi^2}{48} + \frac{\pi}{8} + \frac{\log 2}{4} + \frac{\log^2 2}{4} = \int_0^{\pi/2} \left(\log\sin x\right)^2 \cos^2 x\,dx$

.96090602783640284933... $\approx 2\log^2 2 = \sum_{k=1}^{\infty} \frac{H_k}{k(2k+1)}$

.960984475257857133608...$\approx \sum_{k=1}^{\infty} (-1)^{k+1}\frac{\phi(k)}{2^k-1}$

.96120393268995345712... $\approx 2\sqrt{2}\arctan\frac{1}{2\sqrt{2}} = \sum_{k=0}^{\infty} \frac{(-1)^k}{8^k(2k+1)}$

1 .96123664263055641973... $\approx 3\zeta(3) - \zeta(2)$

.961533506612144893495...$\approx 8 - 8\pi + \frac{11\pi^2}{6} = Li_2\left(-e^{4i}\right) + Li_2\left(-e^{-4i}\right)$

.96164552252767542832... $\approx \frac{4\zeta(3)}{5}$

2 .96179751544137251057... $\approx \sum_{k=1}^{\infty} \frac{\zeta(2k+1)}{(k-1)!} = \sum_{k=1}^{\infty} \frac{e^{-k^2}}{k^3}$

.96201623074638544179... $\approx 4\left(\log\frac{4}{5} + \arctan\frac{1}{2}\right) = \sum_{k=0}^{\infty} \frac{(-1)^k}{4^k(2k+1)(k+1)}$

.962423650119206895... ≈ $2\operatorname{arcsinh}\frac{1}{2} \;=\; 2\,\log\frac{1+\sqrt{5}}{2} \;=\; \sum_{k=0}^{\infty}\frac{(-1)^k}{16^k(2k+1)}\binom{2k}{k}$

.9625788994443448 2607... ≈ $\log 7 - 2 + \frac{\pi}{2\sqrt{3}} + \frac{1}{\sqrt{3}}\operatorname{arccot} 3\sqrt{3}$

= $1 + \sum_{k=1}^{\infty}\frac{(-1)^k}{3^{3k-1}}\left(\frac{1}{6k-1} + \frac{1}{6k+1}\right)$

1 .962858173209645782869... ≈ $\sum_{k=1}^{\infty}\frac{1}{L_k}$

.96307224485663007367... ≈ $2 - \zeta(5)$

.96325656175755909737... ≈ $\left(5\Gamma\left((-1)^{1/5}\right)\Gamma\left(-(-1)^{2/5}\right)\Gamma\left((-1)^{3/5}\right)\Gamma\left(-(-1)^{4/5}\right)\right)^{-1}$

= $\prod_{k=2}^{\infty}\left(1-\frac{1}{k^5}\right)$

1 .96349540849362077404... ≈ $\frac{5\pi}{8} \;=\; \sum_{k=1}^{\infty}\frac{1}{32k^2-32k+6}$

.963510026021423479441... ≈ $\gamma \;+ 2\log 2 \;-1 \;=\; -\psi\left(\frac{1}{2}\right) \;=\; \frac{\Gamma'(1/2)}{\Gamma(1/2)} - 1$

= $-\psi\left(\frac{3}{2}\right) - 1$

.96400656328619110796... ≈ $\pi(1-\log 2) \;=\; \int_0^{\infty}\frac{\log(1+x^2)}{x^2(1+x^2)}dx$

.9640275800758168839... ≈ $\tanh 2 \;=\; \frac{e^2-e^{-2}}{e^2+e^{-2}}$

3 .9640474536922083937... ≈ $(i-1)\,\psi(-i) - (i+1)\,\psi(i)$

.9641198407218830887... ≈ $2\pi - \frac{\pi^2}{6} - 4\log 2 - \frac{3\zeta(3)}{4} \;=\; \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2k^4-k^3}$

.96438734042926245913... ≈ $\frac{1}{\zeta(5)} \;=\; \sum_{k=1}^{\infty}\frac{\mu(k)}{k^5}$

.96534649609163603621... ≈ $\frac{\zeta(10)}{\zeta(5)} \;=\; \sum_{k=1}^{\infty}\frac{\lambda(k)}{k^5}$ HW Thm. 300

.96558322350755543793... ≈ $\sum_{k=0}^{\infty}\frac{1}{2^k+2}$

.9656623982056352867... ≈ $-\sum_{k=1}^{\infty}\frac{\mu(2k)}{2^{2k-1}-1}$

	.9659258262890682867...	≈	$\frac{\sqrt{2}}{4}\left(1+\sqrt{3}\right) = \frac{1}{\sqrt{6}-\sqrt{2}} = \sin\frac{5\pi}{12}$	AS 4.3.46
	.96595219711110745375...	≈	$\frac{\pi\left(6+5\pi\sqrt{3}\right)}{108} = -\int_0^\infty \frac{\log x}{(x^6+1)^2}dx$	
	.96610514647531072707...	≈	$erf\,\frac{3}{2} = \frac{2}{\sqrt{\pi}}\sum\frac{(-1)^k 3^{2k+1}}{k!2^{2k+1}(2k+1)}$	
4	.96624195886232883972...	≈	$\zeta(2)+\zeta(3)+\zeta(4)+\zeta(5)$	
1	.9663693785883453904...	≈	$\frac{1}{4}\Phi\left(-\frac{1}{2},3,\frac{1}{2}\right) = i\sqrt{2}\left(Li_3\left(-\frac{i}{\sqrt{2}}\right)-Li_3\left(\frac{i}{\sqrt{2}}\right)\right)$	
		=	$\int_1^\infty \frac{\log^2 x}{x^2+1/2}dx$	
	.9667107481003567015...	≈	$\frac{1}{2}\left(\cosh 1\sin 1+\cos 1\sinh 1\right) = \int_0^1 \cos x\cosh x\,dx$	
4	.96672281024822179388...	≈	$\prod_{k=1}^\infty\left(1+\frac{1}{k!!}\right)$	
	.96676638530855214796...	≈	$\frac{7}{\pi}\sin\frac{\pi}{7} = \prod_{k=1}^\infty\left(1-\frac{1}{49k^2}\right)$	GR 1.431
		=	$\binom{0}{1/7}$	
1	.96722050079850408134...	≈	$\pi+\frac{\pi^3}{8}-4G-2\log 2 = \int_0^1\int_0^1\int_0^1\int_0^1 \frac{w+x+y+z}{1+w^2x^2y^2z^2}dw\,dx\,dy\,dz$	
	.96740110027233965471...	≈	$\frac{\pi^2}{4}-\frac{3}{2} = \sum_{k=2}^\infty(-1)^k k^2\left(\frac{\zeta(k)+\zeta(k+1)}{2}-1\right)$	
	.96761269589907614401...	≈	$\sum_{k=1}^\infty\frac{\mu(4k-3)}{2^{4k-3}-1}$	
	.9681192775644117...	≈	v_5	
	.96854609799918165608...	≈	$\frac{1}{64}\left(\psi^{(1)}\left(\frac{1}{8}\right)-\psi^{(1)}\left(\frac{5}{8}\right)\right) = \sum_{k=0}^\infty\frac{(-1)^k}{(4k+1)^2}$	
		=	$-\int_0^1\frac{\log x}{1+x^4}dx = \int_1^\infty\frac{x^2\log x}{1+x^4}dx$	
		=	$\int_1^\infty\frac{x\log x}{x^3+x^{-1}}dx = \int_1^\infty\frac{\log x}{x^2+x^{-2}}dx$	
3	.96874800690391485217...	≈		

$$\frac{2}{5\left(1+\sqrt{5}\right)}\left(2\left(5+2\sqrt{5}\right)\left(\log 2+\operatorname{arcsinh} 2\right)+\left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}\right)-5\left(1+\sqrt{5}\right)\operatorname{arccsch} 2\right)$$

		$=$	$\sum_{k=1}^{\infty}\frac{F_k H_k}{2^k}$	
4	.96888875288688326456...	$\approx$	$\sum_{k=2}^{\infty}\frac{\zeta^2(k)}{(k-2)!}$	
	.96891242171064478414...	$\approx$	$\cos\frac{1}{4}=\operatorname{Re}\left\{(-1)^{1/12}\right\} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!16^k}$	
	.96894614625936938048...	$\approx$	$\frac{\pi^3}{32} = \beta(3) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)^3}$	AS 23.2.21
		$=$	$\operatorname{Im}\left\{Li_3(i)\right\} = -\frac{i}{2}\left(Li_3(i) - Li_3(-i)\right)$	
		$=$	$\sum_{k=1}^{\infty}\frac{\sin k\pi/2}{k^3}$	
		$=$	$\int_0^1\int_0^1\int_0^1\frac{dx\,dy\,dz}{1+x^2y^2z^2}$	
5	.96912563469688101600...	$\approx$	$\sum_{k=1}^{\infty}\frac{2^k}{k!\zeta(2k+1)}$	Titchmarsh 14.32.3
	.96924634455436308315 7...	$\approx$	$\sum_{k=2}^{\infty}\left(\frac{\zeta(k)}{\zeta(4k)}-1\right)$	
	.96936462317800478923...	$\approx$	$\frac{\pi}{2\sqrt{2}}\csc\frac{\pi}{\sqrt{2}} - \frac{\pi^2}{12} - 1$	
		$=$	$\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2k^4-k^2}$	
	.96944058923515320145...	$\approx$	$\frac{4\pi^5}{729\sqrt{3}} = g_5$	J310
2	.9706864235520193362...	$\approx$	$\sqrt{2}^{\pi}$	
9	.9709255847316963903...	$\approx$	$\pi^2+\pi^{-2}$	
	.97116506982589819634...	$\approx$	$G^{1/3}$	
54	.9$\underline{711779448621553884}$	$=$	$\frac{43867}{798} = B_9$	
	.97201214975728492545...	$\approx$	$2\pi\left(\frac{2}{\sqrt{3}}-1\right)=\int_0^{2\pi}\frac{\cos x}{2-\cos x}dx=-\int_0^{2\pi}\frac{\sin x dx}{2+\sin x}$	
3	.97202236169854872739 5...	$\approx$	$\sum_{k=2}^{\infty}k\left(\zeta(k)\zeta(k+1)-1\right)$	

379 .97207708399179641258... ≈ $276 + 150\log 2 = \sum_{k=1}^{\infty} \frac{k^4 H_k}{2^k}$

.97211977044690930594... ≈ $\frac{15\zeta(5)}{16} = \eta(5) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^5} = -\Phi(-1,5,0)$ AS 23.2.19

.97211977044690930594... ≈ $\frac{15\zeta(5)}{16} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)^5}$

.97218069963151458411... ≈ $\frac{\pi\sqrt{2}}{3} - \frac{\pi}{2} + \frac{2\sqrt{2}}{3}\log(1+\sqrt{2}) + \frac{\log 2}{3}$

= $\sum_{k=0}^{\infty} \frac{1}{(k+1)(2k+1)(4k+1)}$

.97245278724951431491... ≈ $\sqrt{6}\sin\frac{1}{\sqrt{6}} = \sum_{k=0}^{\infty} \frac{(-1)^k}{6^k (2k+1)!}$

= $\prod_{k=1}^{\infty}\left(1 - \frac{1}{6\pi^2 k^2}\right)$

.97295507452765665255... ≈ $\frac{\log 7}{2}$

= $1 + \sum_{k=1}^{\infty}\left(\frac{(-1)^k}{3^{3k-1}(6k-1)} + \frac{(-1)^k}{3^{3k}(6k+1)}\right)$ K ex. 108

1 .97298485638739759731... ≈ $\sum_{k=2}^{\infty} H_{k+1}\left(\zeta(k) - 1\right)$

1 .97318197252508458260... ≈ $\frac{5\pi^3}{81\sqrt{3}} + \frac{13\zeta(3)}{8}$

= $\frac{1}{216}\left(\psi^{(2)}\left(\frac{2}{3}\right) - \psi^{(2)}\left(\frac{1}{6}\right)\right) = \int_0^{\infty} \frac{x^2\,dx}{e^x + e^{-2x}}$

= $\frac{\zeta(3)}{2} + \frac{2}{3}\left((-1)^{1/3} Li_3\left(-(-1)^{2/3}\right) - (-1)^{2/3} Li_3\left((-1)^{1/3}\right)\right)$

= $\int_0^1 \frac{\log^2 x}{x^3+1}\,dx = \int_1^{\infty} \frac{\log^2 x}{x^2 + x^{-1}}\,dx = \int_1^{\infty} \frac{x\log^2 x}{x^3+1}\,dx$

.97336024835078271547... ≈ $-\zeta\left(\frac{1}{3}\right) = -\frac{1}{(2\pi)^{2/3}}\Gamma\left(\frac{2}{3}\right)\zeta\left(\frac{2}{3}\right)$

2 .97356555791412288969... ≈ $4\zeta(2) - 3\zeta(3)$

1 .97373351671205660911 8...≈ $\frac{2\pi^2 + 75}{48} = \sum_{k=1}^{\infty} \frac{kH_k H_k}{(k+1)(k+2)(k+3)}$

.97402386878476711062... ≈ $\sum_{k=1}^{\infty} \frac{S_2(2k,k)}{(2k)!}$

.97407698418010668087... ≈ $\log(1+\sqrt{e})$

.974428952452842215934... ≈ $\frac{\pi^2+1}{3e+3} = \int_0^\infty \frac{\log^2 x}{(x-1)(x+e)}dx$

.9744447165890447142... ≈ $i\left(Li_2\left(-\frac{i}{2}\right)-Li_2\left(\frac{i}{2}\right)\right)$

= $\frac{1}{4}\Phi\left(-\frac{1}{4},2,\frac{1}{2}\right) = \sum_{k=0}^\infty \frac{(-1)^k}{4^k(2k+1)(2k+1)}$

.97449535840443264512... ≈ $\frac{8}{\pi}\sin\frac{\pi}{8} = \frac{2}{\pi}\sqrt{8-2\sqrt{2}} = \prod_{k=1}^\infty \cos\left(\frac{\pi}{2^{k+3}}\right)$ GR 1.439.1

.97499098879872209672... ≈ $\frac{1}{\sqrt{2}}\left(\pi - 2\log\left(1+\sqrt{2}\right)\right) = \sum_{k=0}^\infty \frac{(-1)^k}{k+3/4}$

= $\int_0^\infty \frac{dx}{(1+x^2)\cosh \pi x/4}$ GR 3.527.10

1 .97532397706265487214... ≈ $\frac{3\pi^4}{64}+\frac{\pi^2\log 2}{16}+\frac{\pi\log^2 2}{16} = \int_0^\infty \frac{\log^2 x}{x^2+4x^{-2}}dx$

.97536797208363138516... ≈ $\sin\left(-\frac{\pi^2}{2}\right) = \text{Im}\left\{(-i)^\pi\right\}$

1 .975416977098902409461... ≈ $\frac{3\pi^2\log 2}{4}-\frac{21\zeta(3)}{8} = \int_0^{\pi/2} \frac{x^3}{\sin^2 x}dx$

1 .97553189198547105414... ≈ $2\sin\sqrt{2} = \sum_{k=1}^\infty \frac{(-1)^{k+1}16^k}{(2k)!}$

25 .97575760906731659638... ≈ $\frac{4\pi^4}{15} = 24\zeta(4) = \int_0^\infty \frac{x^4}{e^x+e^{-x}-2}$

1 .97630906368989922434... ≈ $I_0(1)+L_0(1)$

= $\int_0^1 \exp(\sin \pi x)\, dx$

2 .9763888927056300267... ≈ $\frac{11\pi^4}{360} = \frac{3}{2}\zeta(4)+\frac{1}{2}\zeta^2(2) = \sum_{k=1}^\infty \frac{H_k^{\ 2}}{(k+1)^2}$ 18 MI 4, p. 15

.97645773647066825248... ≈ $\sum_{k=1}^\infty \sin\frac{1}{2^k} = \sum_{k=1}^\infty \frac{(-1)^{k+1}}{(2k-1)!(2^{2k-1}-1)}$

.976525256762151736913... ≈ $\frac{\pi^2}{\pi^2-1}\cos\frac{1}{2} = \prod_{k=1}^\infty \left(1-\frac{1}{\pi^2(2k+1)^2}\right)$

.976628016120607871084... ≈ $h_2 = \frac{1}{36}\left(\psi^{(1)}\left(\frac{1}{6}\right)-\psi^{(1)}\left(\frac{5}{6}\right)\right)$ J314

1 .9773043502972961182... ≈ $\zeta(2)\zeta(3) = \frac{\pi^2\zeta(3)}{6} = \sum_{k=1}^\infty \frac{\sigma_1(k)}{k^3} = \sum_{k=1}^\infty \frac{\sigma_{-1}(k)}{k^2}$ HW Thm. 290

.97735462865789295940 4... ≈ $\frac{7\zeta(3)}{2} - \gamma\zeta(2) - \frac{\pi^2 \log 2}{3} = \sum_{k=1}^{\infty} \frac{\psi\left(k+\frac{1}{2}\right)}{k^2}$

.97745178 ... ≈ $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{\phi(k)}{k^3}$

3 .97746326050642263726... ≈ $\pi I_0(1) = \int_0^{\pi} e^{\cos x} dx = \int_0^{\pi} e^{\cos 2x} dx$

2 .9775435572485657664... ≈ $\pi\left(1 - \gamma + \frac{\gamma^2}{2} + \frac{\pi^2}{24} - \log 2 + \gamma \log 2 + \frac{\log^2 2}{2}\right)$

= $\int_0^{\infty} \frac{\log^2 x \sin^2 x}{x^2} dx$

.97774106744692379763 2... ≈ $2\sqrt{\pi}\,\Gamma^{-1}\left(\frac{1}{4}\right) = \prod_{k=0}^{\infty}\left(1 + \frac{2k+2}{2k+3}\right)$

4 .9777855916963031499... ≈ $e\left(I_0(1) + I_1(1)\right) = {}_1F_1(\frac{3}{2},2,2) = \sum_{k=0}^{\infty} \frac{k}{k!2^k}\binom{2k}{k}$

109 .9778714378213816731... ≈ $55 + \frac{35\pi}{2} = \sum_{k=0}^{\infty} \frac{2^k k^3}{\binom{2k}{k}}$

1 .97793978981013327634 6... ≈ $\frac{7G}{4} + \frac{3}{8} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^5}{(k^2 - 1/4)^3}$

.97797089479890401141... ≈ $\frac{1}{100}\left(\psi^{(1)}\left(\frac{1}{10}\right) - \psi^{(1)}\left(\frac{3}{5}\right)\right) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(5k+1)^2}$

= $\int_1^{\infty} \frac{x \log x}{x^3 + x^{-2}} dx = \int_0^1 \frac{\log x}{x^5 + 1} dx$

1 .9781119906559451108... ≈ $\gamma^2 + \frac{\pi^2}{6} = \int_0^{\infty} \frac{\log^2 x dx}{e^x}$ GR 4.355.1

.97829490163210534585... ≈ $G^{1/4}$

40400 .97839874763488532782... ≈ $40320\zeta(9) = -\psi^{(8)}(1)$

.97846939293030610374... ≈ $Li_2\left(\frac{3}{4}\right)$

.97895991797814145164... ≈ $\frac{\pi}{4\sqrt{2}}\left(\log(3 + 2\sqrt{2}) + i\left(\pi + 2arc\cot(1 - \sqrt{2} - 2arc\cot(1 + \sqrt{2})\right)\right)$

= $\int_0^{\pi/2} \frac{x\,dx}{\sin x + \cos x}$

.97917286680253558830... ≈ $\cos\frac{1}{2^{3/4}}\cosh\frac{1}{2^{3/4}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(4k)!2^k}$

.97929648814722060913... ≈ $2\sqrt{2}\sin\frac{1}{2\sqrt{2}} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!8^k}$

.97933950487701827107... ≈ $16\sin^2\frac{1}{4} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!4^k(k+1)}$

.97987958188123309882... ≈ $\pi\log\frac{1+\sqrt{3}}{2} = \int_0^1\frac{\log(1+x^2)}{\sqrt{1-x^2}}dx$ GR 4.295.38

.98025814346854719171... ≈ $\sqrt{2}\log 2 = \sum_{k=0}^{\infty}\binom{2k}{k}\frac{(-1)^k}{32^k(2k+1)}$

2 .98132947122072143140 6...≈ $\gamma + 2\zeta(3) = \sum_{k=1}^{\infty}\frac{k^3}{k+1}\left(\zeta(k+1)-1\right)$

.98158409038845673252... ≈ $3\sin\frac{1}{3} = \sum_{k=0}^{\infty}\frac{(-1)^k}{9^k(2k+1)!}$

= $\prod_{k=1}^{\infty}\left(1-\frac{1}{9\pi^2k^2}\right)$

.98174770424681038702... ≈ $\frac{5\pi}{16} = \int\frac{x^{5/2}}{\sqrt{1-x}}dx$ GR 3.226.2

.98185090468537672707... ≈ $\sum_{k=1}^{\infty}\frac{\zeta(2k)}{k!2^k} = \sum_{k=1}^{\infty}\frac{\zeta(2k)}{(2k)!!} = \sum_{k=1}^{\infty}\left(e^{1/2k^2}-1\right)$

.98187215105020335672... ≈ $\frac{i}{2}\left(Li_2\left(-(-1)^{3/4}\right) + Li_2\left((-1)^{1/4}\right)\right)$

= $\sum_{k=1}^{\infty}\frac{\sin\pi k/4}{k^2}$

1 .98195959951647446311... ≈ $\frac{6\pi}{5}\sqrt{\frac{2}{5+\sqrt{5}}} = \frac{3\pi}{5}\csc\frac{3\pi}{5} = \int_0^{\infty}\frac{dx}{1+x^{5/3}}$

.98201379003790844197... ≈ $\frac{1+\tanh 2}{2} = \frac{e^2}{e^2+e^{-2}} = \sum_{k=0}^{\infty}(-1)^k e^{-4k}$ J944

.98209763246798892755 3...≈ $\frac{19e^{1/3}}{27} = \sum_{k=0}^{\infty}\frac{k^3}{k!3^k}$

1 .9823463240711475715... ≈ $\frac{22}{9}-\frac{2\log 2}{3} = -\int_0^{\infty}\left(\frac{2e^{-x}}{3}-\frac{2}{x}-\frac{2}{x^2}-\frac{1-e^{-2x}}{x^3}\right)\frac{dx}{x}$. GR 3.438.2

.98265693801555086029... ≈ $2-\zeta(6)$

.98268427774219251832... ≈ $\frac{1+\cosh\pi\sqrt{3}}{12\pi^2} = \frac{1}{6\pi^2}\sin^2\left(\frac{\pi}{2}+\frac{i\pi\sqrt{3}}{2}\right) = \prod_{k=2}^{\infty}\left(1-\frac{1}{k^6}\right)$

.982686076702760592745... ≈ $\mathit{HypPFQ}\left[\left\{\frac{1}{2},\frac{1}{2},1,1\right\},\left\{\frac{3}{2},\frac{3}{2},\frac{3}{2}\right\},-\frac{1}{4}\right] = \sum_{k=0}^{\infty}\frac{(-1)^k}{\binom{2k}{k}(2k+1)^3}$

.98295259226458041980... ≈ $\frac{945}{\pi^6} = \frac{1}{\zeta(6)} = \sum_{k=1}^{\infty}\frac{\mu(k)}{k^6}$

.983118479820648366... ≈ $\pi\sin\frac{1}{\pi} = \prod_{k=1}^{\infty}\left(1-\frac{1}{\pi^4 k^2}\right)$

.98318468929417269520... ≈ $\frac{1}{8}\Phi\left(-\frac{1}{2},3,\frac{1}{2}\right) = \int_0^1\frac{\log^2 x}{x^2+2}$

= $\frac{i}{\sqrt{2}}\left(Li_3\left(-\frac{i}{\sqrt{2}}\right)-Li_3\left(\frac{i}{\sqrt{2}}\right)\right)$

.983194483680076021738... ≈ $\frac{691\pi^6}{675675} = \prod_{p\ prime}\frac{1}{1+p^{-6}}$

.98358502084677797943... ≈ $\zeta(2)+\zeta(3)+\zeta(4)+\zeta(5)+\zeta(6)$

.98361025039925204067... ≈ $\frac{\pi}{2}+\frac{\pi^3}{16}-2G-\log 2 = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k(2k-1)^3}$

.98372133768990415044... ≈ $\frac{1}{144}\left(\psi^{(1)}\left(\frac{1}{12}\right)-\psi^{(1)}\left(\frac{7}{12}\right)\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(6k+1)^2}$

= $\int_1^{\infty}\frac{x\log x}{x^3+x^{-3}}dx$

.98385007371209431669... ≈ $2+4\zeta(2)+2\zeta(3) = 2\sum_{k=2}^{\infty}\frac{3k^2-3k+1}{k(k-1)^3}$

= $\sum_{k=2}^{\infty}(k^2+k)\left(\zeta(k)-1\right)$

.98419916936149897912... ≈ $\frac{403\pi^6}{393660} = \nu_6$ J312

.9843262438190419103... ≈ $\frac{2\pi^3}{63} = \xi(6) = \xi(-5)$

.9845422648017221585... ≈ $\frac{\pi^2}{6}-2\log^2 2+\frac{\zeta(3)}{4} = \int_0^1\frac{(1+x)\log^2(1+x)}{x^2}dx$

.98481748453515847115... ≈ $\sum_{k=1}^{\infty}\frac{(-1)^k}{k^{k+4}}$

.98517143100941603869... ≈ $2\left(\sqrt{2}-1\right)2^{1/4}$, from elliptic integrals

.98542064692776706919... ≈ $\log\Gamma\left(\frac{1}{3}\right)$

.9855342964496961782... ≈ $\frac{96}{\pi^4} = \frac{16}{15\zeta(4)} = -\sum_{k=1}^{\infty}\frac{\mu(2k)}{k^4}$

.9855510912974351041... ≈ $\frac{31\pi^6}{30240} = \eta(6) = \frac{31\zeta(6)}{32} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^6}$ J306

4 .98580192112184410715... ≈ $4\sqrt{2}\left(\log\sin\frac{3\pi}{8} - \log\sin\frac{\pi}{8}\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k+\frac{1}{4})(k+\frac{3}{4})}$

1 .98610304049995312993... ≈ $\frac{1}{512}\left(\psi^{(2)}\left(\frac{5}{8}\right) - \psi^{(2)}\left(\frac{1}{8}\right)\right) = \int_0^1\frac{\log^2 x}{x^4+1}dx$

= $\int_1^{\infty}\frac{\log^2 x}{x^2+x^{-4}}dx$

.98621483608613337832... ≈ $2si\left(\frac{1}{2}\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!4^k(2k+1)}$

6 .98632012366331278404... ≈ $\frac{5e^2-9}{4} = \sum_{k=0}^{\infty}\frac{2^k k^2}{k!(k+3)}$

.9865428606939705039... ≈ $\frac{11\pi^4}{768\sqrt{2}} = 1 + \sum_{k=1}^{\infty}\left(\frac{(-1)^k}{(4k-1)^4} + \frac{(-1)^k}{(4k+1)^4}\right)$ J327, J344

= $\sum_{k=0}^{\infty}\frac{(-1)^{\lfloor (k+1)/2\rfloor}}{(2k+1)^4}$ Prud. 5.1.4.4

.98659098626254229130... ≈ $\frac{5\pi^3}{162\sqrt{3}} + \frac{13\zeta(3)}{2} = \frac{1}{432}\left(\psi^{(2)}\left(\frac{2}{3}\right) - \psi^{(2)}\left(\frac{1}{6}\right)\right)$

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{(3k+1)^3}$

12 .98673982624599908550... ≈ $\frac{28402}{2187} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^{10}}{2^k}$

.98696044010893586188... ≈ $\frac{\pi^2}{10}$

8 .98747968146102986535... ≈ $\frac{1856}{243} - \frac{380}{81}\log\frac{4}{3} = \sum_{k=1}^{\infty}\frac{k^4 H_k}{4^k}$

.98776594599273552707... ≈ $\sin\sqrt{2}$

.98787218579881572198... ≈ $-\frac{4}{105} - \frac{7\pi\sqrt[4]{15}}{420}\left(\csc\left(\pi\sqrt[4]{15}\right) + \operatorname{csch}\left(\pi\sqrt[4]{15}\right)\right)$

= $\sum_{k=2}^{\infty}\frac{(-1)^k}{k^4-15}$

6 .98787880453365829819... ≈ $\frac{2\pi^4}{15} - 6 = \int_0^1\frac{(x+1)\log^3 x}{1-x}dx$

.98829948773456306974... ≈ $\frac{\pi^2}{18\left(\sqrt{3}-1\right)^3\sqrt{2}} = -\int_0^\infty \frac{\log x}{x^{12}+1}dx$

.98861592946536921938... ≈ $\frac{3\sqrt{2}}{\pi}\left(\sqrt{3}-1\right) = \prod_{k=1}^\infty \left(1-\frac{1}{144k^2}\right)$ GR 1.431

= $\prod_{k=1}^\infty \cos\left(\frac{\pi}{3\cdot 2^{k+2}}\right)$

= $\binom{0}{1/12}$

.98889770576286509638... ≈ $\sin 1 \sinh 1 = \sum_{k=0}^\infty (-1)^k \frac{2^{2k+1}}{(4k+2)!}$ GR 1.413.1

= $\prod_{k=1}^\infty \left(1-\frac{1}{\pi^4 k^4}\right)$

.98894455174110533611... ≈ $\frac{1}{1536}\left(\psi^{(3)}\left(\frac{1}{4}\right) - \psi^{(3)}\left(\frac{3}{4}\right)\right)$

= $-\frac{i}{2}\left(Li_4(i) - Li_4(-i)\right)$

= $\sum_{k=0}^\infty \frac{(-1)^k}{(2k+1)^4} = \sum_{k=1}^\infty \frac{\sin \pi k/2}{k^4}$

1 .98905357643767967767... ≈ $\frac{\pi^3}{9\sqrt{3}} = -\int_0^\infty \frac{\log^2 x\, dx}{x^6-1}$

.98905599532797255554... ≈ $\frac{\gamma^2}{2} + \frac{\pi^2}{12} = \int_0^\infty \frac{\log^2(2x)dx}{e^{2x}}$

1 .98928023429890102342... ≈ $\zeta''(2) = \sum_{k=1}^\infty \frac{\log^2 k}{k^2}$

1 .98930064124495395454 3...≈ $\psi(3+i) + \psi(3-i) = \frac{4}{5}\left(\psi(2+i) + \psi(2-i)\right)$

.98943827421728483862... ≈ $\sum_{k=1}^\infty \frac{\sigma_0(k)}{\binom{2k}{k}}$

.98958488339991993644... ≈ $\cos\frac{1}{2}\cosh\frac{1}{2} = \sum_{k=0}^\infty \frac{(-1)^k}{(4k)!4^k}$

.98961583701809171839... ≈ $4\sin\frac{1}{4} = \sum_{k=0}^\infty \frac{(-1)^k}{(2k+1)!16^k}$

= $\prod_{k=1}^\infty \left(1-\frac{1}{16\pi^2 k^2}\right)$

.98986584618905381178... ≈ $4\operatorname{arcsinh}\frac{1}{4} = \sum_{k=0}^{\infty}\binom{2k}{k}\frac{(-1)^k}{64^k(2k+1)}$

1 .98999249660044545727... ≈ $1-\cos 3 = 2\sin^2\frac{3}{2} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}9^k}{(2k)!}$

.99004001943815994979... ≈ $\frac{1}{3456}\left(\psi^{(2)}\left(\frac{5}{12}\right)+\psi^{(2)}\left(\frac{7}{12}\right)-2\psi^{(2)}\left(\frac{1}{12}\right)\right) - \frac{(7+4\sqrt{3})\pi^3}{432}$

= $1+\sum_{k=1}^{\infty}\left(\frac{(-1)^k}{(6k-1)^3}+\frac{(-1)^k}{(6k+1)^3}\right)$

.990079321507338635294...≈ $\frac{1}{256}\left(4\pi^2\sqrt{20+14\sqrt{2}}+\psi^{(1)}\left(\frac{7}{16}\right)-\psi^{(1)}\left(\frac{15}{16}\right)\right)$

= $\int_1^{\infty}\frac{\log x\ dx}{x^2+x^{-6}}$

4 .990384604362124949925...≈ $\frac{10}{3}+\frac{74\pi}{81\sqrt{3}} = \sum_{k=1}^{\infty}\frac{k^3}{\binom{2k}{k}}$

78 .99044152834577217825... ≈ $1+\frac{212}{e} = \sum_{k=0}^{\infty}\frac{(-1)^k k^{10}}{(k+1)!}$

.99076316985337943267... ≈ $\sum_{k=2}^{\infty}k\left(\zeta(k)-1\right)^2$

.99101186342305209843... ≈ $\frac{1}{4}\left((i-1)\psi(-i)-(i+1)\psi(i)\right)$

21 .99114857512855266924... ≈ 7π

5 .991155403803739739770668...≈ $\frac{1}{4096}\left(176\pi^4\sqrt{2}+\psi^{(3)}\left(\frac{3}{8}\right)-\psi^{(3)}\left(\frac{7}{8}\right)\right)$

= $\int_1^{\infty}\frac{\log^3 x\ dx}{x^2+x^{-2}}$

4 .9914442354842448121... ≈ $\frac{6}{\zeta(3)}$

.99166639109270971002... ≈ $HypPFQ\left[\{\ \},\left\{\frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{4}{5}\right\},\frac{1}{3125}\right] = \sum_{k=0}^{\infty}\frac{1}{(5k)!}$

.99171985583844431043... ≈ $\frac{1}{\zeta(7)} = \sum_{k=1}^{\infty}\frac{\mu(k)}{k^7}$

6	.99193429822870080627...	≈	$\sum_{k=2}^{7}\zeta(k)$	
2	.9919718574637504583...	≈	$\sqrt[3]{2}+\sqrt{3}$	Borwein-Devlin p. 35
	.99213995900836...	≈	ν_7	
1	.992173853105526785479...	≈	$\sum_{k=0}^{\infty} I_k(1)$	
	.99220085376959424562...	≈	$\frac{4\pi^3}{125} = \sum_{k=1}^{\infty}\frac{\sin 2k\pi/5}{k^3}$	GR 1.443.5
	.99223652952251116935...	≈	$\frac{56\pi^7}{94815\sqrt{3}} = g_7$	J310
1	.992294767124987392926...	≈	$\frac{e(e+1)}{(e-1)^3} = \Phi\left(\frac{1}{e},-2,0\right) = \sum_{k=1}^{\infty}\frac{k^2}{e^k}$	
1	.992315656154176128013...	≈	$\frac{\pi^5}{768} = i\left(Li_5(-i)-Li_5(i)\right)$	
	.99259381992283028267...	≈	$\frac{63\zeta(7)}{64} = \eta(7) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^7}$	AS 23.2.19
	.99293265189943576028...	≈	$\frac{\pi}{2}(1-e^{-1}) = \int_0^{\infty}\sin(\tan x)\frac{dx}{x}$	GR 3.881.2
	.99297974679457358056...	≈	$\sum_{k=1}^{\infty}\frac{\mu(3k-2)}{2^{3k-2}-1}$	
	.99305152024997656497...	≈	$\frac{1}{1024}\left(\psi^{(2)}\left(\frac{5}{8}\right)-\psi^{(2)}\left(\frac{1}{8}\right)\right)$	
	.99331717348313360398...	≈	$2\pi-2-\frac{\pi^2}{3} = -Li_2\left(e^{2i}\right)-Li_2\left(e^{-2i}\right)$	
	.993317230688294041051...	≈	$2\left(2\gamma+\psi\left(1+\frac{i}{2}\right)+\psi\left(1-\frac{i}{2}\right)\right) = \sum_{k=1}^{\infty}\frac{1}{k(k^2+1/4)}$	
	.993703033793554457744...	≈	$\frac{17\zeta(4)}{4}-3\zeta(3) = \sum_{k=1}^{\infty}\frac{H_k H_k}{k^2(k+1)}$	
	.9944020304296468447...	≈	$6\log 2+\log^2 2-2-\frac{\pi^2}{6} = \sum_{k=1}^{\infty}\frac{k^3}{2^k(k+1)^2}$	
	.99452678821883983884...	≈	$\frac{\pi^3}{18\sqrt{3}} = h_3$	J314
1	.9947114020071633897...	≈	$\frac{5}{\sqrt{2\pi}}$	
	.99532226501895273416...	≈	$erf\,2 = \frac{2}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k 2^{2k+1}}{k!(2k+1)}$	

1 .9954559575001380004... ≈ $\int_0^\infty \frac{dx}{x^x}$

1 .99548129134760281506... ≈ $\frac{\pi}{2}(\gamma + \log 2) = \int_0^\infty \log x \sin x \frac{dx}{x}$ GR 4.421.1

.99551082651341250945... ≈ $\frac{(91+45\sqrt{5})\pi^4}{18750} = 1 + \sum_{k=1}^\infty \left(\frac{(-1)^k}{(5k-1)^4} + \frac{(-1)^k}{(5k+1)^4} \right)$

10 .99557428756427633462... ≈ $\frac{7\pi}{2}$

21 .99557428756427633462... ≈ $\frac{7\pi}{2} + 11 = \sum_{k=1}^\infty \frac{2^k k^2}{\binom{2k}{k}}$

.99563385631296745634 2...≈ $\frac{1}{786432}\left(\psi^{(4)}\left(\frac{3}{8}\right) + \psi^{(4)}\left(\frac{5}{8}\right) - \psi^{(4)}\left(\frac{1}{8}\right) - \psi^{(4)}\left(\frac{7}{8}\right) \right)$

= $1 + \sum_{k=1}^\infty \left(\frac{(-1)^k}{(4k-1)^5} + \frac{(-1)^k}{(4k+1)^5} \right)$

= $\sum_{k=0}^\infty \frac{(-1)^{\lfloor (k+1)/2 \rfloor}}{(2k+1)^5}$ Prud. 5.1.4.4

.99585422505141623107... ≈ $\sum_{k=2}^\infty \frac{\zeta^2(k)}{2^k}$

.99591638044188511306... ≈ $\sum_{k=1}^\infty \frac{\arctan k}{2^k}$

.99592331507778367120... ≈ $-\frac{\sinh \pi}{8\pi^3} \sin\left(\pi(-1)^{1/4}\right) \sin\left(\pi(-1)^{3/4}\right) = \prod_{k=2}^\infty \left(1 - \frac{1}{k^8}\right)$

= $\frac{\sinh \pi}{16\pi^3}\left(\cosh \pi\sqrt{2} - \cos \pi\sqrt{2}\right)$

7 .99601165442664514565... ≈ $\sum_{k=2}^8 \zeta(k)$

1 .99601964118442342116... ≈ $142e - 384 = \sum_{k=0}^\infty \frac{k^3}{k!(k+4)}$

.99608116021237162307... ≈ $\frac{5207\pi^8}{49601160} = \nu_8$ J312

15 .99610183565115862273 3...≈ $\frac{32}{3} + \frac{238\pi}{81\sqrt{3}} = \sum_{k=1}^\infty \frac{k^4}{\binom{2k}{k}}$

.99610656865... ≈ g_8 J310

.99615782807708806401... ≈ $\frac{5\pi^5}{1536} = \beta(5) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)^5}$ AS 23.2.21, J345

.99623300185264789923... ≈ $\frac{127\pi^8}{1209600} = \eta(8) = \frac{127\zeta(8)}{128} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^8}$ J306

.99624431360434498902... ≈ $2 - \frac{19\pi^5}{4096\sqrt{2}}$

= $1 + \sum_{k=1}^{\infty}\left(\frac{(-1)^k}{(4k-1)^5} - \frac{(-1)^k}{(4k+1)^5}\right)$

.99627207622074994426 5...≈ $\tanh\pi = \frac{\tan\pi i}{i} = \frac{e^{\pi} - e^{-\pi}}{e^{\pi} + e^{-\pi}}$

.99633113536305818711... ≈ $\frac{\pi\log 3}{4\sqrt{3/4}} = \int_0^{1/2}\frac{K(k)k}{(k')^2\sqrt{\frac{1}{2} - k^2}}dk$ GR 6.153

8 .99646829575509185269... ≈ $2\pi^3\log 2 - 9\pi\zeta(3) = \int_0^{\pi}\frac{x^3\sin x\,dx}{1-\cos x}$

.99749486472136872703 2...≈ $\sum_{k=2}^{\infty}(-1)^k\frac{k\zeta^2(k)}{2^k}$

119 .99782676761602472146... ≈ $\frac{e(e^4 + 26^3 + 66e^2 + 26e + 1)}{(e-1)^6} = \Phi\left(\frac{1}{e}, -5, 0\right) = \sum_{k=1}^{\infty}\frac{k^5}{e^k}$

.99784841149774479093... ≈ $\frac{3\pi^2}{8} - \frac{11}{2} + \log 2 + \frac{7\zeta(3)}{4} = \sum_{k=2}^{\infty}\frac{16k^2 - 6k + 1}{2k(2k-1)^3}$

= $\sum_{k=2}^{\infty}\frac{k^2(\zeta(k)-1)}{2^k}$

8 .99802004725272736007... ≈ $\sum_{k=2}^{9}\zeta(k)$

.998043589097895... ≈ v_9 J312

.99805019565707723 72... ≈ $\frac{3236\pi^9}{55801305\sqrt{3}} = g_9$ J310

.99807159983792868 73... ≈ $\frac{23\pi^4}{1296\sqrt{3}} = 1 + \sum_{k=1}^{\infty}\left(\frac{(-1)^k}{(6k-1)^4} + \frac{(-1)^k}{(6k+1)^4}\right)$

.99809429754160533077... ≈ $\eta(8) = \frac{255\zeta(9)}{256} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^9}$ AS 23.2.19

2 .99809593260204657254 7...≈ $\sum_{k=1}^{\infty}\frac{H_k H_k}{k(2k-1)}$

	.99813603811037497213...	≈	$\frac{1+\tanh\pi}{2}=\frac{e^{2\pi}}{1+e^{2\pi}}=\sum(-1)^k e^{-2\pi k}$	J944
	.9983343660981139742...	≈	$\pi^3\left(\frac{3}{128\sqrt{2}}+\frac{1}{64}\right) = \sum_{k=1}^{\infty}\left(\frac{1}{(8k-5)^3}-\frac{1}{(8k-3)^3}\right)$	
17	.99851436155475931539...	≈	$\frac{1016}{81}+\frac{512}{27}\log\frac{4}{3} = \sum_{k=1}^{\infty}\frac{(k+1)(k+2)(k+3)H_k}{4^k}$	
	.99851515454428730477...	≈	$\pi\left(\sqrt{3}-\sqrt{2}\right) = \int_0^{\infty}\log\left(1+\frac{1}{x^2+2}\right)dx$	
	.99857397195353054767...	≈	$\frac{19^2\pi^4}{2^{14}3\cdot 5\sqrt{3}} = 1+\sum_{k=1}^{\infty}\left(\frac{(-1)^k}{(4k-1)^6}+\frac{(-1)^k}{(4k+1)^6}\right)$	J327
2	.9986013142694680689...	≈	$9\gamma^2$	
	.99861111319878665371...	≈	$HypPFQ\left[\{\ \},\left\{\frac{1}{6},\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{5}{6}\right\},-\frac{1}{46656}\right] = \sum_{k=0}^{\infty}\frac{(-1)^k}{(6k)!}$	
	.99868522221843813544...	≈	$\beta(6)=\frac{1}{491520}\left(\psi^{(5)}\left(\frac{1}{4}\right)-\psi^{(5)}\left(\frac{3}{4}\right)\right)=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)^6}$	AS 23.2.21
	.998777285931944...	≈	h_4	J314
33	.99884377071514942382...	≈	$\frac{5e^4-1}{8} = \sum_{k=0}^{\infty}\frac{4^k k}{k!(k+2)}$	
	.999022588776486298...	≈	$\frac{48983\pi^{10}}{4591650240} = \nu_{10}$	
	.99903950759827156564...	≈	$\frac{73\pi^{10}}{6842880} = \eta(10) = \frac{511\zeta(10)}{512} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^{10}}$	J306
1	.99906754284631204591...	≈	$6\gamma^2$	
	.99915501340010950975...	≈	$\frac{\pi^4\sin 1\sinh 1}{\pi^4-1} = \prod_{k=2}^{\infty}\left(1-\frac{4}{\pi^4k^4}\right)$	
1	.99916475865341931390...	≈	$\frac{e^e-e^{2+1/e}+e^2-1}{2e} = \sum_{k=1}^{\infty}\frac{\sinh k}{(k+1)!}$	
1	.99919532501168660629...	≈	$\frac{7\pi}{11}$	
1	.99935982878411178897...	≈	$\frac{\pi^3\left(7+4\sqrt{3}\right)}{216} = -\int_0^{\infty}\frac{\log^2 x\ dx}{x^{12}-1}$	
	.99953377142315602297...	≈	$3\gamma^2$	
	.9995545078905399095...	≈	$\frac{61\pi^7}{184320} = \beta(7) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)^7}$	AS 23.2.21

.99955652539434499642... ≈ $2-\frac{307\pi^7\sqrt{2}}{1310720} = 1+\sum_{k=1}^{\infty}\left(\frac{(-1)^k}{(4k-1)^7}-\frac{(-1)^k}{(4k+1)^7}\right)$

.999735607648751739... ≈ $\frac{11\pi^5}{1944\sqrt{3}} = h_5$ J314

.9997427569925535489... ≈ $2-\frac{305\pi^5}{93312} = 1+\sum_{k=1}^{\infty}\left(\frac{(-1)^k}{(6k-1)^5}+\frac{(-1)^k}{(6k+1)^5}\right)$

.9997539139218932560... ≈ $-\frac{\sinh\pi}{12\pi^5}\cosh^2\left(\frac{\pi\sqrt{3}}{2}\right)\sin\left(\pi(-1)^{1/6}\right)\sin\left(\pi(-1)^{5/6}\right)$

= $\frac{\sinh\pi}{24\pi^5}\cosh^2\left(\frac{\pi\sqrt{3}}{2}\right)\left(\cosh\pi-\cos\pi\sqrt{3}\right)$

= $\prod_{k=2}^{\infty}\left(1-\frac{1}{k^{12}}\right)$

.99980158731305804717... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^k}{(7k)!}$

.99984521547922560046... ≈ $\frac{24611\pi^8}{165150720\sqrt{2}} = 1+\sum_{k=1}^{\infty}\left(\frac{(-1)^k}{(4k-1)^8}+\frac{(-1)^k}{(4k+1)^8}\right)$ J327

.99984999024682965634... ≈ $\beta(8) = \frac{1}{330301440}\left(\psi^{(7)}\left(\frac{1}{4}\right)-\psi^{(7)}\left(\frac{3}{4}\right)\right)$

= $\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)^8}$

.99992821782510442180... ≈ $\frac{1681\pi^6}{933120\sqrt{3}} = 1+\sum_{k=1}^{\infty}\left(\frac{(-1)^k}{(6k-1)^6}+\frac{(-1)^k}{(6k+1)^6}\right)$ J327

.99994968418722008982... ≈ $\frac{6385\pi^9}{41287680} = \beta(9) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)^9}$ AS 23.2.21

.99997519841274620747... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^k}{(8k)!}$

.9999883774094057879... ≈ $\frac{301\pi^7}{524880\sqrt{3}} = h_7$ J314

.99998844843872824784... ≈ $2-\frac{33367\pi^7}{100776960} = 1+\sum_{k=1}^{\infty}\left(\frac{(-1)^k}{(6k-1)^7}-\frac{(-1)^k}{(6k+1)^7}\right)$

.9999972442680777576... ≈ $\sum_{k=0}^{\infty}\frac{(-1)^k}{(9k)!}$

.99999727223893094715... ≈ $\frac{25743\pi^8}{1410877440\sqrt{3}} = 1+\sum_{k=1}^{\infty}\left(\frac{(-1)^k}{(6k-1)^8}+\frac{(-1)^k}{(6k+1)^8}\right)$ J329

$$.9999997244268077605 5\ldots \approx \sum_{k=0}^{\infty}\frac{(-1)^k}{(10k)!}$$