Computational Verb Cellular Networks: Part II–One-Dimensional Computational Verb Local Rules

Tao Yang

***Abstract*— Computational verb cellular networks (CVCNs) are a new kind of cellular computational platform where the local rules are computational verb rules. In a sister paper[60] 2D CVCNs were studied. In this paper, 1D CVCNs with 1D computational verb local rules are studied. The bifurcations of patterns in 1D CVCNs with computational verb local rules consisting of two computational verbs** decrease **and** increase **are studied in details. As two examples, the "Rule 30 1D CVCN" and "Rule 110 1D CVCN" are studied.** *Copyright © 2009 Yang's Scientific Research Institute, LLC. All rights reserved.*

***Index Terms*— Computational verb, cellular network, pattern formation, local rule, computational verb rule.**

I. INTRODUCTION

COMPUTATIONAL verb cellular networks (CVCNs) (verb cellular networks, for short) are cellular networks, of which the local rules are computational verb rules. This paper is the second part of a paper series on CVCNs. In the first part[60] of this series 2D CVCN's were studied. In this paper, 1D CVCNs are explored.

The organization of this paper is as follows. In Section II, the brief history of computational verb theory will be given. In Section III, the architectures of 1D computational verb cellular networks will be presented. In Section IV, the bifurcation of patterns in "Rule 30 1D CVCN" will be studied. In Section V, the bifurcation of patterns in "Rule 110 D CVCN" will be studied. In Section VI, some concluding remarks will be included.

II. A BRIEF HISTORY OF COMPUTATIONAL VERB THEORY

As the first paradigm shift for solving engineering problems by using verbs, the computational verb theory[30] and physical linguistics[33], [50], [24] have undergone a rapid growth since the birth of computational verb in the Department of Electrical Engineering and Computer Sciences, University of California at Berkeley in 1997[15], [16]. The paradigm of implementing verbs in machines was coined as *computational verb theory*[30]. The building blocks of computational theory are *computational verbs*[25], [19], [17], [26], [31]. The relation between verbs and adverbs was mathematically defined in [18]. The logic operations between verb statements were studied in [20]. The applications of verb logic to verb reasoning were addressed in [21] and further studied in [30]. A logic paradox was solved based on verb logic[27]. The mathematical concept of set was generalized into verb set in[23]. Similarly, for measurable attributes, the number systems can be generalized into verb numbers[28]. The applications of computational verbs to predictions were studied in [22]. In [32] fuzzy dynamic systems were used to model a special kind of computational verb that evolves in fuzzy spaces. The relation between computational verb theory and traditional linguistics was studied in [30], [33]. The theoretical basis of developing computational cognition from a unified theory of fuzzy and computational verb theory is the theory of the UNICOGSE that was studied in [33], [38]. The issues of simulating cognition using computational verbs were studied in [34]. In [63] the correlation between computational verbs was studied. A method of implementing feelings in machines was proposed based on grounded computational verbs and computational nouns in [40]. In [47] a theory of how to design stable computational verb controllers was given. In [41] the rule-wise linear computational verb systems and their applications to the design of stable computational verb controllers and chaos in computational verb systems were presented. In [45] the concept of computational verb entropy was used to construct computational verb decision tree for data-mining applications. In [44] the relation between computational verbs and fuzzy sets was studied by using computational verb collapses and computational verb extension principles. In [46] the distances and similarities of saturated computational verbs were defined as normalized measures of the distances and similarities between computational verbs. Based on saturated computational verbs, the verb distances and similarities are related to each other with a simple relation. The distances and similarities between verbs with different life spans can be defined based on saturated computational verbs as well. In [48] the methods of using computational verbs to cluster trajectories and curves were presented. To cluster a bank of trajectories into a few representative computational verbs is to discover knowledge from database of time series. We use cluster centers to represent complex waveforms at symbolic levels. In [13] computational verb controllers were

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used to control a chaotic circuit model known as Chua's circuit. Computational verb controllers were designed based on verb control rules for different dynamics of the region-wise linear model of the control plant. In [12] computational verb controllers were used to synchronize discrete-time chaotic systems known as Hénon maps. Different verb control rules are designed for synchronizing different kinds of dynamics. In [52], how can computational verb theory functions as the most essential building block of cognitive engineering and cognitive industries was addressed. Computational verb theory will play a critical important role in personalizing services in the next fifty years. In [49], [51] computational verb theory was used to design an accurate flame-detecting systems based on CCTV signal. In [55] the learning algorithms were presented for learning computational verb rules from training data. In [53] the structures and learning algorithms of computational verb neural networks were presented. In [61] the ambiguities of the states and dynamics of computational verbs were studied. In [54] the history and milestones in the first ten years of the studies of computational verb theory were given. In [3] a case study of modeling adverbs as modifiers of computational verbs was presented. In [14] computational verb rules were used to improve the training processes of neural networks. In [56] the classifications and interactions between computational verb rule bases were presented. In [57] the simplest verb rules and their verb reasoning were connected to many intuitive applications of verb rules before the invention of computational verbs. In [58] the interactions between computational verbs were used as a powerful tools to understand the merging and splitting effects of verbs. In [62] computational verb rules were trained by using prescribed training samples of functions. In [59] the trend-based computational verb similarity was given as a way to decrease the computational complexities of verb similarities. In [4] computational verb PID controller was used to control linear motors. In [11] computational verb controller was used to control an auto-focusing system.

The theory of computational verb has been taught in some university classrooms since 2005[1]. The latest active applications of computational verb theory are listed as follows.

1) Computational Verb Controllers. The applications of computational verbs to different kinds of control problems were studied on different occassions[29], [30]. For the advanced applications of computational verbs to control problems, a few papers reporting the latest advances had been published[36], [35], [47], [41], [64]. The design of computational verb controllers was also presented in a textbook in 2005[1].
2) Computational Verb Image Processing and Image Understanding. The recent results of image processing by using computational verbs can be found in[37]. The applications of computational verbs to image understanding can be found in [39]. The authors of [2] applied computational verb image processing to design the vision systems of RoboCup small-size robots.
3) Stock Market Modeling and Prediction based on computational verbs. The product of Cognitive Stock Charts[7] was based on the advanced modeling and computing reported in [42]. Computational verb theory was used to study the trends of stock markets known as Russell reconstruction patterns [43].

Computational verb theory has been successfully applied to many industrial and commercial products. Some of these products are listed as follows.

1) Visual Card Counters. The *YangSky-MAGIC* card counter[9], developed by Yang's Scientific Research Institute and Wuxi Xingcard Technology Co. Ltd., was the first visual card counter to use computational verb image processing technology to achieve high accuracy of card and paper board counting based on cheap webcams.
2) CCTV Automatic Driver Qualify Test System. The *DriveQfy* CCTV automatic driver qualify test system[10] was the first vehicle trajectory reconstruction and stop time measuring system using computational verb image processing technology.
3) Visual Flame Detecting System. The *FireEye* visual flame detecting system[5] was the first CCTV or webcam based flame detecting system, which works under color and black & white conditions, for surveillance and security monitoring system.
4) Smart Pornographic Image and Video Detection Systems. The *PornSeer*[8] pornographic image and video detection systems are the first cognitive feature based smart porno detection and removal software.
5) Webcam Barcode Scanner. The *BarSeer*[6] webcam barcode scanner took advantage of the computational verb image processing to make the scan of barcode by using cheap webcam possible.
6) Cognitive Stock Charts. By applying computational verbs to the modeling of trends and cognitive behaviors of stock trading activities, cognitive stock charts can provide the traders with the "feelings" of stock markets by using simple and intuitive indexes.
7) TrafGo ITS SDK. Computational verbs were applied to model vehicle trajectories and dynamics of optical field and many other aspects of dynamics in complex environments for applications in intelligent transportation systems (ITS).

III. ARCHITECTURES OF 1D COMPUTATIONAL VERB CELLULAR NETWORKS

The architectures of general CVCNs were presented in [60]. In a 1D CVCN, the cells are arranged along a line and each cell only has cells immediately to its left and right as its neighbors. Therefore, in a 1D CVCN the neighborhood of a center cell is arranged within a line segment. The verb local rules of 1D

[1] Some computational verb theory related college courses are

- Dr. G. R. Chen, EE 64152 - Introduction to Fuzzy Informatics and Intelligent Systems, Department of Electronic Engineering, City University of Hong Kong.
- Dr. D. H. Guo, Artificial Intelligence, Department of Electronic Engineering, Xiamen University.
- Prof. T. Yang, Computational Methodologies in Intelligent Systems, Department of Electronic Engineering, Xiamen University.
- Dr. Mahir Sabra, EELE 6306: Intelligent Control, Electrical and Computer Engineering Department, The Islamic University of Gaza.

TABLE I
VERB LOCAL RULES OF A 1D CVCN BASED ON A SET OF STANDARD VERBS.

$x_{i-1}(k)$	$x_i(k)$	$x_{i+1}(k)$	$x_i(k+1)$
V_1	V_1	V_1	$\widetilde{\mathsf{V}}_1$
V_2	V_1	V_1	$\widetilde{\mathsf{V}}_2$
V_1	V_2	V_1	$\widetilde{\mathsf{V}}_3$
V_2	V_2	V_1	$\widetilde{\mathsf{V}}_4$
$\vdots$			
V_{n-1}	V_n	V_n	$\widetilde{\mathsf{V}}_{n^3-1}$
V_n	V_n	V_n	$\widetilde{\mathsf{V}}_{n^3}$

CVCN have the outputs of cells as antecedents. The reasoning results of verb local rules determine the state of a center cell.

In this paper, only 1D CVCNs with 1-neighborhood are studied. The verb local rules for 1-neighborhood is given by

$$\begin{aligned}&\text{IF } x_{i-1}(k)\ \mathsf{V}_{p,-1} \text{ AND } x_i(k)\ \mathsf{V}_{p,0} \text{ AND } x_{i+1}(k)\ \mathsf{V}_{p,1},\\ &\text{THEN } x_i(k+1)\ \mathsf{V}_p;\\ &p=1,\cdots,m. \end{aligned} \tag{1}$$

Observe that the dynamics of the 1D CVCNs are determined by a set of m verb rules. If we cluster all dynamics of cells into a few computational verbs[48], which are called *standard verbs* henceforth, then we can build the local rules by exhausting all combinations of standard verbs. Let $S_{\mathsf{V}} = \{\mathsf{V}_1, \ldots, \mathsf{V}_n\}$ be the set of standard verbs, then consider a 1D CVCN of 1-neighborhood, the rule base consists of all combination of standard verbs is given by

$$\begin{aligned}&\text{IF } x_{i-1}(k)\ \mathsf{V}_{\alpha} \text{ AND } x_i(k)\ \mathsf{V}_{\beta} \text{ AND } x_{i+1}(k)\ \mathsf{V}_{\gamma},\\ &\text{THEN } x_i(k+1)\ \widetilde{\mathsf{V}}_p;\\ &\alpha,\beta,\gamma=1,\cdots,n; p=1,\cdots,n^3. \end{aligned} \tag{2}$$

This verb rule base can be represented in Table I. Since $\widetilde{\mathsf{V}}_p \in S_{\mathsf{V}}, p = 1, \cdots, n^3$, there are total n^3 possible patterns for a neighborhood and n^{n^3} possible 1D CVCNs constructed from S_{V}. Since n^{n^3} grows very fast when n increases, one can imagine how complex a social network can be just by considering that when $n = 3$ the number is $n^{n^3} > 7.6256e+012$ and when $n = 4$ the number is $n^{n^3} > 3.4028e + 038$. Therefore, we only study the case when $n = 2$ and we only need to study $n^{n^3} = 256$ 1D CVCNs.

When $n = 3$, we choose the standard verb set as $S_{\mathsf{V}} = \{\mathsf{decrease}, \mathsf{stay}, \mathsf{increase}\}$. When $n = 2$, we choose the standard verb set as $S_{\mathsf{V}} = \{\mathsf{decrease}, \mathsf{increase}\}$. When $n = 2$ the rule base in Eq. (2) is explicitly given by Eq. (3) on the top of the next page.

In this case, the verb local rules are listed in Table II.

The verb reasoning of the eight verb rules in Table II results in

$$x_i(k+1) = \frac{\sum_{p=1}^{8} g_p f(x_{i+\alpha}(k)) \bigwedge_{j=-1}^{1} S(\mathsf{V}_{p,j}, x_{i+j}(k))}{\sum_{p=1}^{8} \bigwedge_{j=-1}^{1} S(\mathsf{V}_{p,j}, x_{i+j}(k))} \tag{4}$$

TABLE II
VERB LOCAL RULES OF THE 1D CVCN BASED ON A SET OF STANDARD VERBS $S_{\mathsf{V}} = \{\mathsf{decrease}, \mathsf{increase}\}$.

Rule	$x_{i-1}(k)\ \mathsf{V}_{p,-1}$	$x_i(k)\ \mathsf{V}_{p,0}$	$x_{i+1}(k)\ \mathsf{V}_{p,1}$	$x_i(k+1)$
1	decrease	decrease	decrease	$\widetilde{\mathsf{V}}_1$
2	increase	decrease	decrease	$\widetilde{\mathsf{V}}_2$
3	decrease	increase	decrease	$\widetilde{\mathsf{V}}_3$
4	increase	increase	decrease	$\widetilde{\mathsf{V}}_4$
5	decrease	decrease	increase	$\widetilde{\mathsf{V}}_5$
6	increase	decrease	increase	$\widetilde{\mathsf{V}}_6$
7	decrease	increase	increase	$\widetilde{\mathsf{V}}_7$
8	increase	increase	increase	$\widetilde{\mathsf{V}}_8$

where $\mathsf{V}_{p,-1}$, $\mathsf{V}_{p,0}$, and $\mathsf{V}_{p,1}$ are the computational verbs listed in the second, third and fourth column of the pth rule in Table II, respectively. $\bigwedge$ is a t-norm. g_p is a parameter to model the consequent verbs $\widetilde{\mathsf{V}}_p$. g_p is given by

$$g_p = \begin{cases} g_I, & \text{if } \widetilde{\mathsf{V}}_p = \mathsf{increase}, \\ g_D, & \text{if } \widetilde{\mathsf{V}}_p = \mathsf{decrease}. \end{cases} \tag{5}$$

$f(\cdot)$ is the *output function*. $\alpha = -1, 0$, and 1 is the directional influential index that defines the output of which neighbor influences $x_i(k + 1)$. When $\alpha = -1, 0$, and 1 we call the 1D CVCN is *left-influential*, *self-influential*, and *right-influential*, respectively.

However, $x_i(k+1)$ is not necessary to be influenced by the output of a single neighbor cell. It can be influenced by the outputs of all neighbor cells at the same time. For example, when a 1D CVCN is *sum-influential*, then Eq. (4) is recast into

$$x_i(k+1) = \frac{\sum_{p=1}^{8} g_p \sum_{\alpha=-1}^{1} f(x_{i+\alpha}(k)) \bigwedge_{j=-1}^{1} S(\mathsf{V}_{p,j}, x_{i+j}(k))}{\sum_{p=1}^{8} \bigwedge_{j=-1}^{1} S(\mathsf{V}_{p,j}, x_{i+j}(k))}. \tag{6}$$

When a 1D CVCN is *co-influential*, then Eq. (4) is recast into

$$x_i(k+1) = \frac{\sum_{p=1}^{8} g_p \bigwedge_{j=-1}^{1} S(\mathsf{V}_{p,j}, x_{i+j}(k)) f(x_{i+j}(k))}{\sum_{p=1}^{8} \bigwedge_{j=-1}^{1} S(\mathsf{V}_{p,j}, x_{i+j}(k))}. \tag{7}$$

Since the richness of linguistic representations of local rules, there are virtually infinitely many kinds of 1D CVCNs with the same neighborhood structure. In this paper, I will only study right-influential 1D CVCN; namely, the case of $\alpha = 1$.

To avoid clutter and follow the convention of the established labeling method widely used in the study of cellular automata(CA), I will use *Wolfram notation* to label 1D CVCNs when $S_{\mathsf{V}} = \{\mathsf{decrease}, \mathsf{increase}\}$. For example, corresponding to "rule 30 CA", the local rules of "rule 30 1D CVCN" is given by Table III. The last column in Table III lists a binary representation $\{b_p\}$ of elements in $S_{\mathsf{V}} = \{\mathsf{decrease}, \mathsf{increase}\}$. Here, when $\widetilde{\mathsf{V}}_p = \mathsf{decrease}$ the corresponding $\{b_p\} = 0$ is denoted and when $\widetilde{\mathsf{V}}_p = \mathsf{increase}$ the corresponding $\{b_p\} = 1$

$$\begin{aligned}
&\text{IF } x_{i-1}(k) \text{ decrease AND } x_i(k) \text{ decrease AND } x_{i+1}(k) \text{ decrease, THEN } x_i(k+1)\ \widetilde{\mathsf{V}}_1;\\
&\text{IF } x_{i-1}(k) \text{ increase AND } x_i(k) \text{ decrease AND } x_{i+1}(k) \text{ decrease, THEN } x_i(k+1)\ \widetilde{\mathsf{V}}_2;\\
&\text{IF } x_{i-1}(k) \text{ decrease AND } x_i(k) \text{ increase AND } x_{i+1}(k) \text{ decrease, THEN } x_i(k+1)\ \widetilde{\mathsf{V}}_3;\\
&\text{IF } x_{i-1}(k) \text{ increase AND } x_i(k) \text{ increase AND } x_{i+1}(k) \text{ decrease, THEN } x_i(k+1)\ \widetilde{\mathsf{V}}_4;\\
&\text{IF } x_{i-1}(k) \text{ decrease AND } x_i(k) \text{ decrease AND } x_{i+1}(k) \text{ increase, THEN } x_i(k+1)\ \widetilde{\mathsf{V}}_5;\\
&\text{IF } x_{i-1}(k) \text{ increase AND } x_i(k) \text{ decrease AND } x_{i+1}(k) \text{ increase, THEN } x_i(k+1)\ \widetilde{\mathsf{V}}_6;\\
&\text{IF } x_{i-1}(k) \text{ decrease AND } x_i(k) \text{ increase AND } x_{i+1}(k) \text{ increase, THEN } x_i(k+1)\ \widetilde{\mathsf{V}}_7;\\
&\text{IF } x_{i-1}(k) \text{ increase AND } x_i(k) \text{ increase AND } x_{i+1}(k) \text{ increase, THEN } x_i(k+1)\ \widetilde{\mathsf{V}}_8. \qquad (3)
\end{aligned}$$

TABLE III

VERB LOCAL RULES OF "RULE 30 1D CVCN" BASED ON A SET OF STANDARD VERBS $S_{\mathsf{V}} = \{\text{decrease}, \text{increase}\}$.

$x_{i-1}(k)$	$x_i(k)$	$x_{i+1}(k)$	$x_i(k+1)$	b_p
decrease	decrease	decrease	$\widetilde{\mathsf{V}}_1$ = decrease	0
increase	decrease	decrease	$\widetilde{\mathsf{V}}_2$ = increase	1
decrease	increase	decrease	$\widetilde{\mathsf{V}}_3$ = increase	1
increase	increase	decrease	$\widetilde{\mathsf{V}}_4$ = increase	1
decrease	decrease	increase	$\widetilde{\mathsf{V}}_5$ = increase	1
increase	decrease	increase	$\widetilde{\mathsf{V}}_6$ = decrease	0
decrease	increase	increase	$\widetilde{\mathsf{V}}_7$ = decrease	0
increase	increase	increase	$\widetilde{\mathsf{V}}_8$ = decrease	0

TABLE IV

VERB LOCAL RULES OF "RULE 110 1D CVCN" BASED ON A SET OF STANDARD VERBS $S_{\mathsf{V}} = \{\text{decrease}, \text{increase}\}$.

$x_{i-1}(k)$	$x_i(k)$	$x_{i+1}(k)$	$x_i(k+1)$	b_p
decrease	decrease	decrease	$\widetilde{\mathsf{V}}_1$ = decrease	0
increase	decrease	decrease	$\widetilde{\mathsf{V}}_2$ = increase	1
decrease	increase	decrease	$\widetilde{\mathsf{V}}_3$ = increase	1
increase	increase	decrease	$\widetilde{\mathsf{V}}_4$ = increase	1
decrease	decrease	increase	$\widetilde{\mathsf{V}}_5$ = decrease	0
increase	decrease	increase	$\widetilde{\mathsf{V}}_6$ = increase	1
decrease	increase	increase	$\widetilde{\mathsf{V}}_7$ = increase	1
increase	increase	increase	$\widetilde{\mathsf{V}}_8$ = decrease	0

is denoted. The binary number $b_8b_7b_6b_5b_4b_3b_2b_1$ is used as a unique identification number for a 1D CVCN. In the case shown in Table III this binary number is $b_8b_7b_6b_5b_4b_3b_2b_1 = 00011110 = 30$. Therefore, the 1D CVCN, of which the verb local rules are listed in Table III, is identified as "rule 30 1D CVCN".

For example, corresponding to "rule 110 CA", the local rules of "rule 110 1D CVCN" is given by Table IV. In this case, the identification number for this 1D CVCN is $b_8b_7b_6b_5b_4b_3b_2b_1 = 01101110 = 110$.

It follows from [57] that the simplest verb similarities are given by

$$\begin{aligned}
S(\text{increase}, x(k)) &= \frac{1}{1+e^{-\Delta_x/\Delta}},\\
S(\text{decrease}, x(k)) &= \frac{1}{1+e^{\Delta_x/\Delta}},\\
S(\text{stay}, x(k)) &= \frac{2}{1+e^{\kappa|\Delta_x|}} \qquad (8)
\end{aligned}$$

where $\Delta > 0$, $\kappa > 0$ and $\Delta_x = x(k) - x(k-1)$. Here we choose the nonlinear output function as

$$f(x) = \frac{1}{1+e^{-x}}. \qquad (9)$$

IV. BIFURCATION OF PATTERNS IN "RULE 30 1D CVCN"

Let us choose the parameters as $g_D = 1/g_I$, $\Delta = 0.5$ and $\kappa = 1$ and g_I to be the bifurcation parameter. We use 101 cells to construct the "Rule 30 1D CVCN", of which the initial conditions are $x_i(-1) = x_i(0) = 0, i = 1, \cdots, 101$ except for $x_{51}(0) = 1$.

A. Product as the t-Norm

We choose the t-norm $\bigwedge$ to be product in Eq. (4) with $\alpha = 1$, then it can be recast into

$$x_i(k+1) = \frac{\sum_{p=1}^{8} g_p f(x_{i+1}(k)) \prod_{j=-1}^{1} S(\mathsf{V}_{p,j}, x_{i+j}(k))}{\sum_{p=1}^{8} \prod_{j=-1}^{1} S(\mathsf{V}_{p,j}, x_{i+j}(k))}. \qquad (10)$$

Figure 1 shows the evolution of the "rule 30 1D CVCN" when the bifurcation parameter is $g_I = -5$. The horizontal direction denotes the location of cells while the vertical direction denotes the direction of evolving iterations. The CVCN evolves 101 iterations. Observe that at the beginning, all cells transit to an equilibrium state coded in the color of light blue except for the cells near the center cell whose initial state is different from others. Observe that the central region of the CVCN generates disturbances, which propagate from both sides into the homogenous light-blue region.

When $g_I = -4.5$ the evolving process of the "rule 30 1D CVCN" is shown in Fig. 2. Comparing with the result shown in Fig. 1, observe that at the beginning all cells transit to an equilibrium state except for the center region as well. However, in this case, the propagating speed of the central region to the homogenous region becomes low. Otherwise, the results when $g_I = -4.5$ and $g_I = -5$ are qualitatively the same though there are more disturbances to the left-hand half of the wave front in Fig. 1.

When we further increase the bifurcation parameter to $g_I = -4$, the evolving process of the "rule 30 1D CVCN" is shown in Fig. 3. The strength of the propagation of the central region becomes much weaker than both cases shown in Figs. 1 and

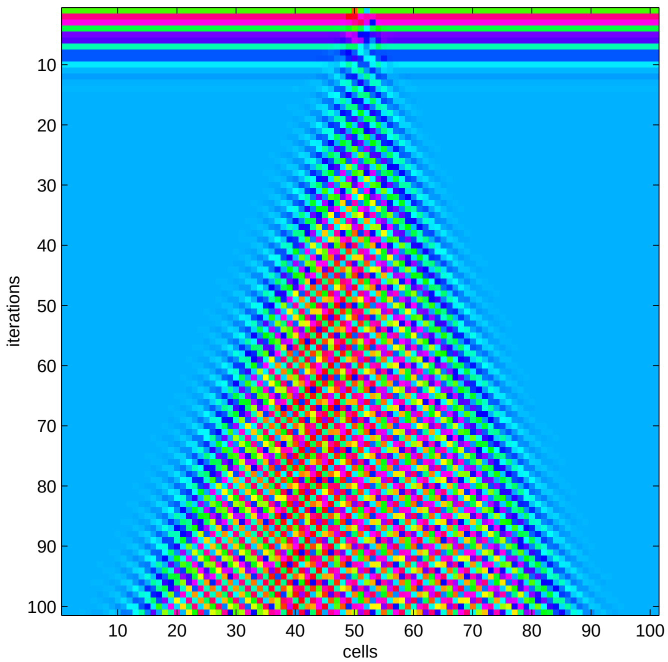

Fig. 1. Evolving process of “rule 30 1D CVCN” of 101 cells. The bifurcation parameter is $g_I = -5$.

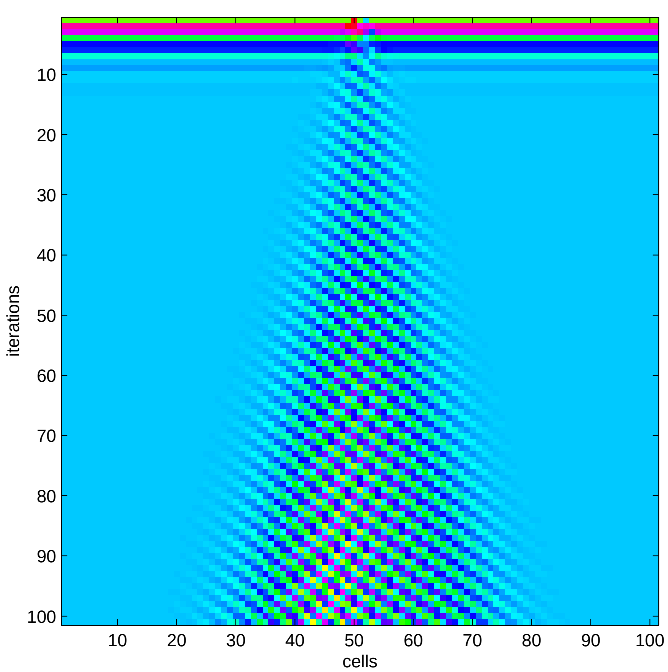

Fig. 2. Evolving process of “rule 30 1D CVCN” of 101 cells. The bifurcation parameter is $g_I = -4.5$.

2. When we keep increasing g_I when it satisfies $g_I < -1$, the propagation becomes too weak to influence the homogenous region and dies out very soon within a few iterations.

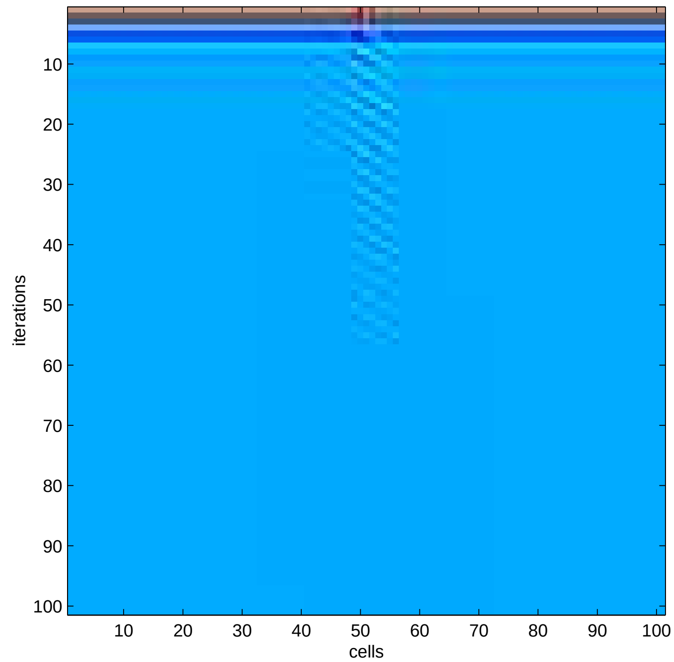

Fig. 3. Evolving process of “rule 30 1D CVCN” of 101 cells. The bifurcation parameter is $g_I = -4$.

When we increase the bifurcation parameter to $g_I \in (-1, 0)$, the absolute value of g_D is bigger than $|g_I|$, the qualitative behaviors of “rule 30 1D CVCN” change. When $g_I = -0.2$ the evolving process of the “rule 30 1D CVCN” is shown in Fig. 4. Observe that the disturbance of the central cell dies out very fast and the CVCN is dominated by a synchronized oscillation among all cells. This kind of synchronized oscillation results in alternatively line patterns.

When g_I becomes positive, the “rule 30 1D CVCN” has qualitatively different patterns comparing with the cases when g_I is negative. When $g_I = 0.1$ the evolving process of the “rule 30 1D CVCN” is shown in Fig. 5. Observe that the disturbance of the central cell results in two slim wave fronts propagating to left and right. All cells between both wavefronts are synchronized while the cells outside both wave fronts oscillated with small amplitudes.

When $g_I = 0.2$ the evolving process of the “rule 30 1D CVCN” is shown in Fig. 6. Observe that the disturbance of the central cell results in two slim wave fronts propagating to left and right in the same way as that shown in Fig. 5 when $g_I = 0.1$. Almost all cells between both wavefronts are synchronized while the cells outside both wave fronts oscillated with medium amplitudes. Within the region of both wavefronts a small groups of oscillated cells propagate from left to right.

When g_I increases the cells between and out both wave fronts become synchronized at two different levels. Between both synchronized regions are two boundary regions where checkerboard patterns are found as shown in Fig. 7 with parameter $g_I = 0.4$.

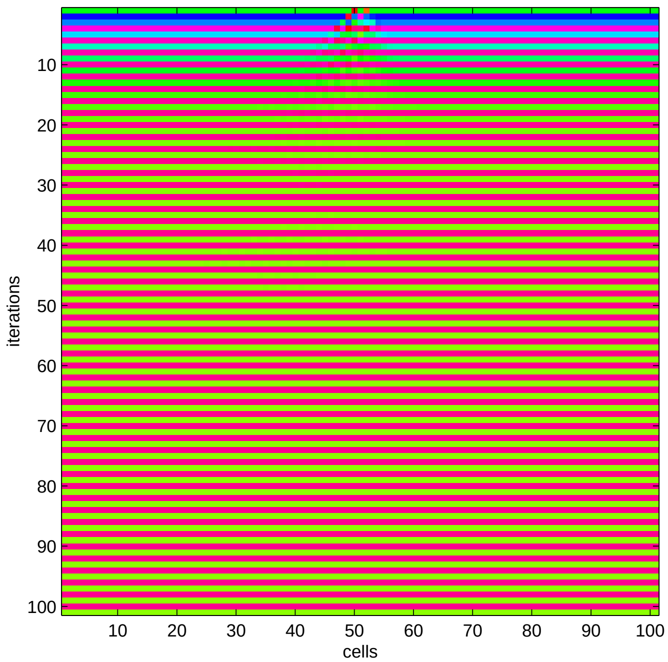

Fig. 4. Evolving process of "rule 30 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = -0.2$.

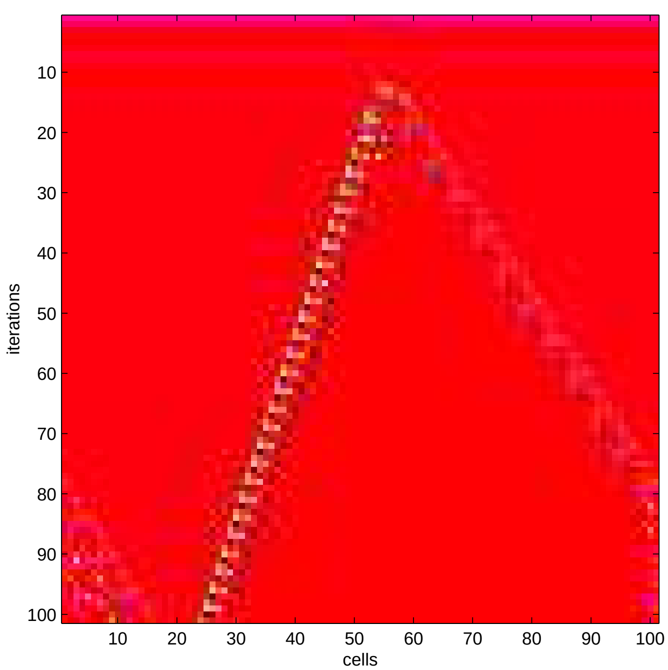

Fig. 5. Evolving process of "rule 30 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = 0.1$.

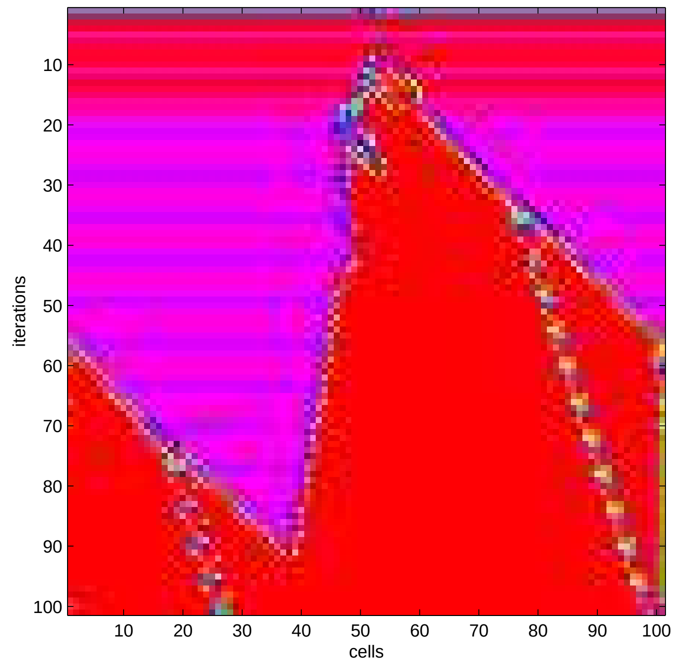

Fig. 6. Evolving process of "rule 30 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = 0.2$.

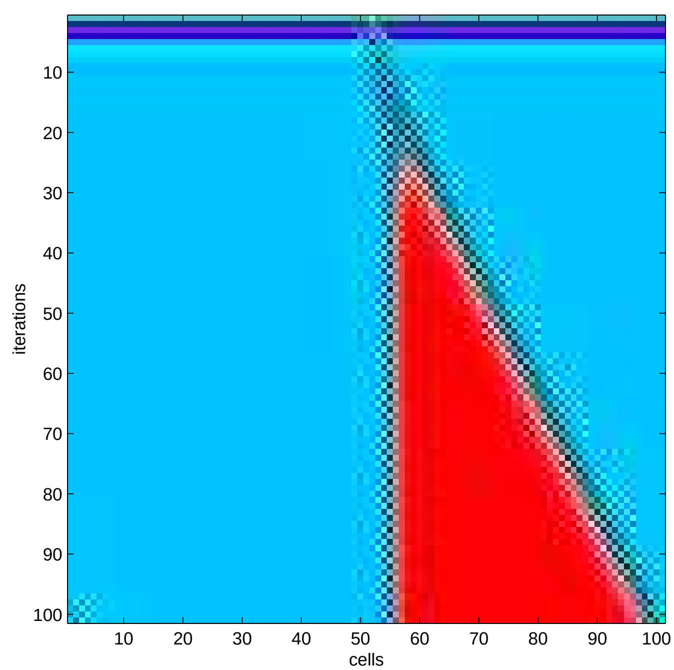

Fig. 7. Evolving process of "rule 30 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = 0.4$.

However, when the parameter further increases to $g_I = 0.5$, the disturbance initiates from the central regions dies out fast and all cells settle at the same value as shown in Fig. 8. This equilibrium point of CVCN is stable.

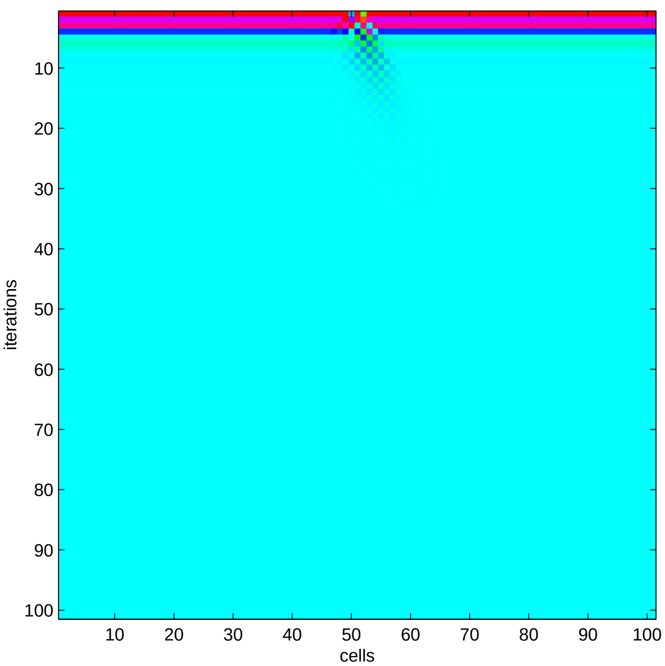

Fig. 8. Evolving process of "rule 30 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = 0.5$.

When g_I increases to bigger than 1 and is not big enough, the CVCN converges to trivial patterns. When $g_I = 4$ the evolving process of the "rule 30 1D CVCN" is shown in Fig. 9. Observe that the disturbance of the central cell results in a complex pattern of propagation. All cells that are outside the central disturbance are synchronized in an oscillation with big amplitude and high-frequencies.

When g_I increases further to $g_I = 5$, the evolving process of the "rule 30 1D CVCN" is shown in Fig. 10. The propagating speed of the central disturbance becomes bigger and more irregular comparing with the case when $g_I = 4$ shown in Fig. 9. The cells outside the disturbance synchronize in an oscillation with big amplitude and lower frequencies than the case shown in Fig. 9.

When g_I increases to a degree that the disturbance becomes too weak comparing with the synchronized oscillation dominating all cells over the CVCN as shown in Fig. 11 when $g_I = 10$.

When g_I becomes bigger than 10, the patterns become more likely to be homogenous.

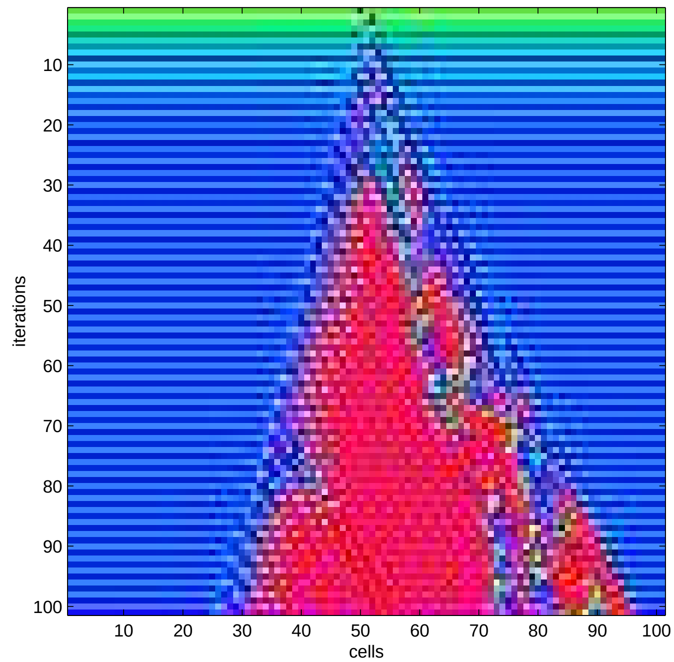

Fig. 9. Evolving process of "rule 30 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = 4$.

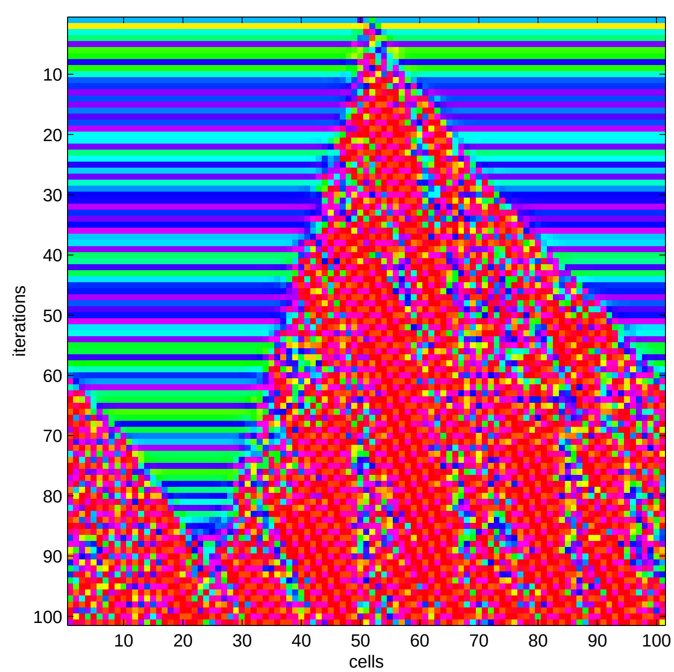

Fig. 10. Evolving process of "rule 30 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = 5$.

B. Minimum as the t-Norm

We choose $\bigwedge$ to be $\min$ operator in Eq. (4) with $\alpha = 1$, then it can be recast into

$$x_i(k+1) = \frac{\sum_{p=1}^{8} g_p f(x_{i+1}(k)) \min_{j=-1}^{1} S(\mathsf{V}_{p,j}, x_{i+j}(k))}{\sum_{p=1}^{8} \min_{j=-1}^{1} S(\mathsf{V}_{p,j}, x_{i+j}(k))}. \quad (11)$$

When $g_I = -10$ the evolving process of the "rule 30 1D CVCN" is shown in Fig. 12. Observe that in this case the strong synchronized oscillation of the entire network dies out quickly and the disturbance of the central region grows into a periodically oscillated region.

When g_I increases to $g_I = -8$, the speed of propagation of the central region becomes lower in this case. The pattern formation mechanism is qualitatively the same to the case of $g_I = -10$. The evolving process of the "rule 30 1D CVCN" is shown in Fig. 13.

When g_I increases to $g_I = -6$, the disturbance of the central region dies out fast and the CVCN settles at an equilibrium point.

When g_I increases to $g_I = 0.2$, the disturbance had two wavefronts to separate the CVCN into two regions as shown in Fig. 15 . Within the disturbance region there is a small region of oscillation propagates from left to right and outside the disturbance region all cells form a homogenous pattern,

When g_I increases to bigger than 1, the behaviors of CVCN change qualitatively from the case when g_I is positive and smaller than 1. If g_I is small, then the CVCN converges to a homogenous pattern as shown in Fig. 16 with $g_I = 5$.

When g_I increases to big enough, then different kinds of patterns are formed in the CVCN. One example is shown in Fig. 17 with $g_I = 7$. Observe that in this case high-frequency random patterns are generated within the disturbance region. The oscillation outside the disturbance region dies out after around 30 iterations.

When g_I increases further to $g_I = 8$, the propagating speed of the disturbance increases and the oscillation outside the disturbance region has big amplitudes and high frequency.

V. Bifurcation of Patterns in "Rule 110 1D CVCN"

The local rules of "Rule 110 1D CVCN" are listed in Table IV. In this section the pattern formations of "Rule 110 1D CVCN" are studied. All the settings are the same as those used in Sec. IV. g_I is chosen as the bifurcation parameter.

A. Product as the t-Norm

In this section we study the bifurcation of pattern formation in "Rule 110 1D CVCN" when we choose $\bigwedge$ in Eq. (4) to be product and choose $\alpha = 1$.

When $g_I = -10$ the evolving process of the "rule 110 1D CVCN" is shown in Fig. 19. Observe that in this case the strong synchronized oscillation of the entire network sets up almost immediately after the network runs and the disturbance from the central cell has no effect to the global dynamics. The

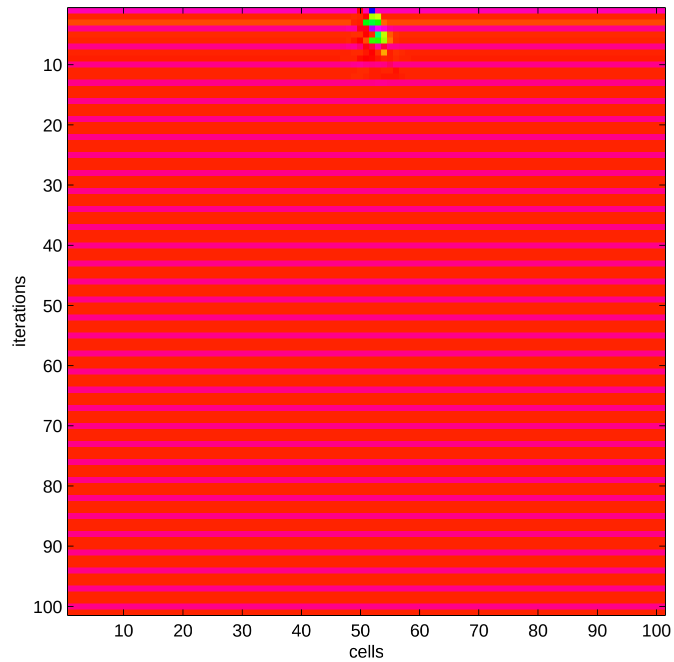

Fig. 11. Evolving process of "rule 30 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = 10$.

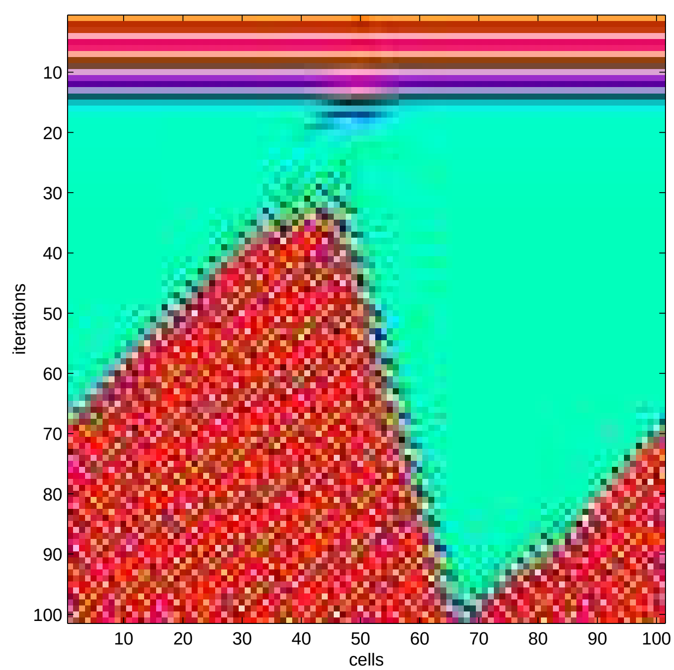

Fig. 12. Evolving process of "rule 30 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = -10$.

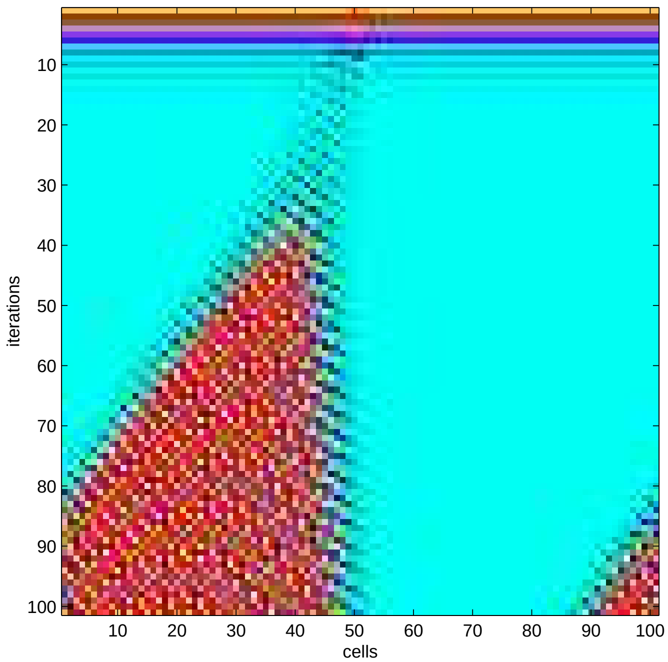

Fig. 13. Evolving process of "rule 30 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = -8$.

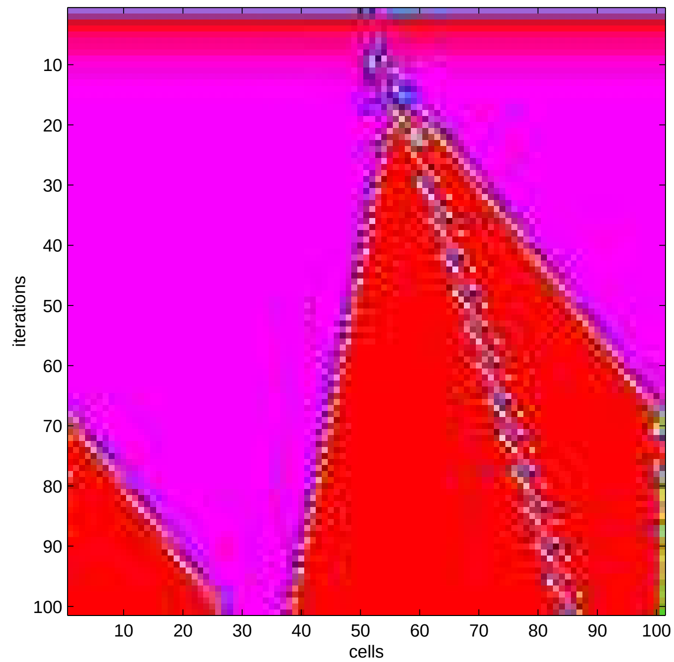

Fig. 15. Evolving process of "rule 30 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = 0.2$.

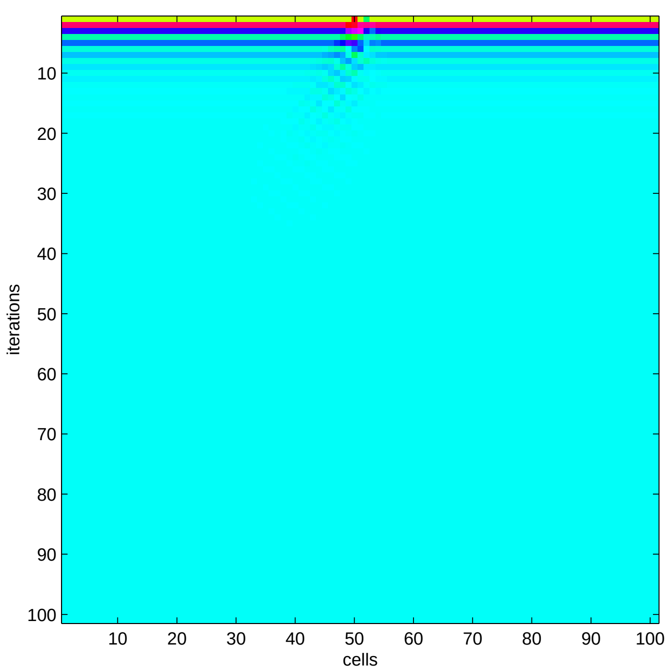

Fig. 14. Evolving process of "rule 30 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = -6$.

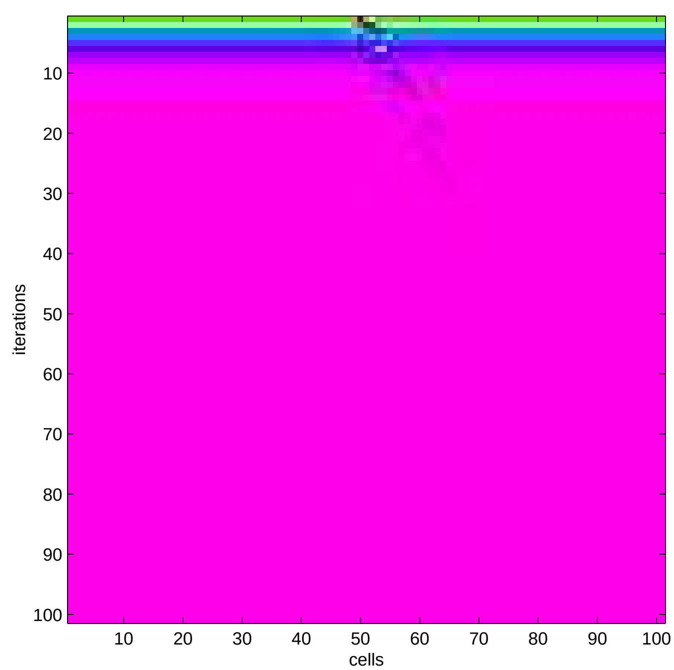

Fig. 16. Evolving process of "rule 30 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = 5$.

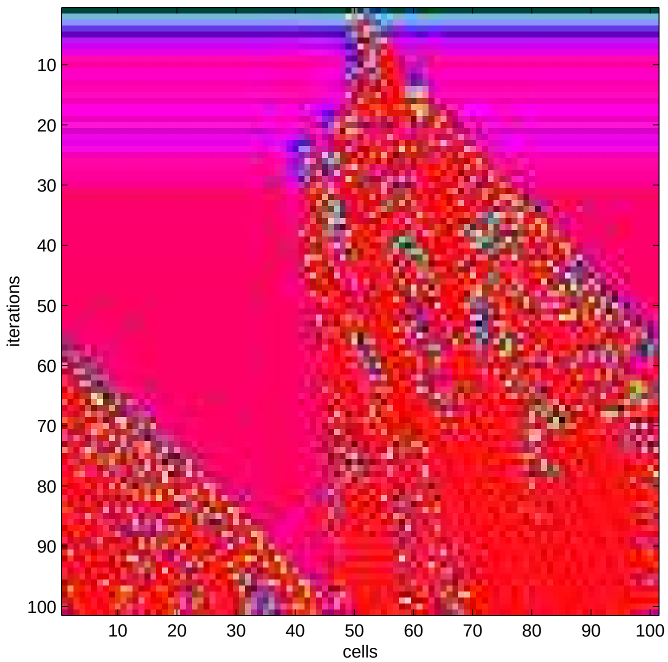

Fig. 17. Evolving process of “rule 30 1D CVCN” of 101 cells. The bifurcation parameter is $g_I = 7$.

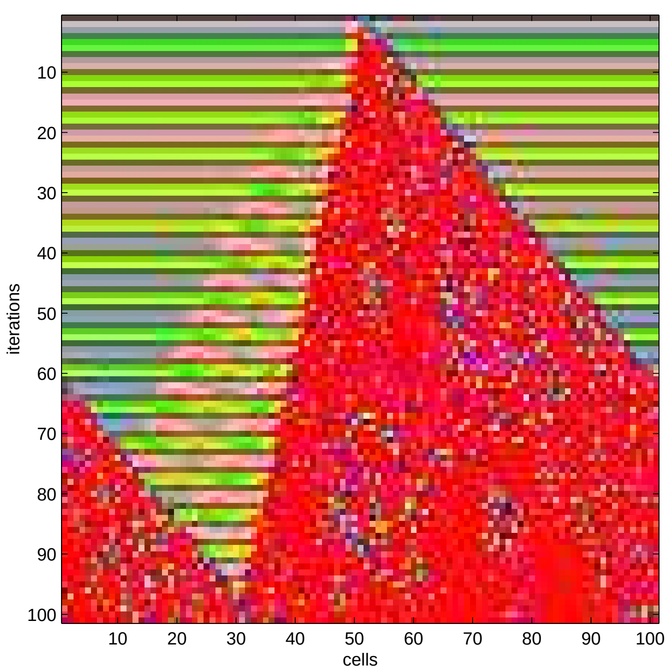

Fig. 18. Evolving process of “rule 30 1D CVCN” of 101 cells. The bifurcation parameter is $g_I = 8$.

amplitudes of the oscillation grows slowly while the frequency of the oscillation increases slowly as well.

When $g_I = -8$ the evolving process of the “rule 110 1D CVCN” is shown in Fig. 20. Observe that in this case the strong synchronized oscillation also set up at the beginning as that shown in Fig. 19. However, it is different from the case of $g_I = -10$, the synchronized oscillation is not strong enough to survive for a long period, the disturbance from the central cell emerges after 60 iterations. Observe that the disturbance appears only after a long-term accumulation of its effect under the dominate synchronized oscillation.

When g_I increases, the synchronized oscillation becomes more unstable and the disturbance grows up faster. When $g_I = -5$ the evolving process of the “rule 110 1D CVCN” is shown in Fig. 21. Comparing with the result shown in Fig. 20 that the iteration when the disturbance begins to show its effects becomes much earlier. However, the patterns formed within the disturbance regions are qualitatively similar in both cases.

However, when g_I increases further, the effect of the disturbance becomes weaker and the cells are dominated by a homogenous pattern. There is no trace of the strong synchronized oscillation as shown in Fig. 19, instead the competition is now between a homogenous pattern and the oscillation within the disturbance. This trend continues when g_I increases until the disturbance disappears very soon and the entire CVCN dominated by a homogenous pattern.

When g_I keeps increasing further, the disturbance dies out soon such that the entire CVCN converges to a homogenous pattern. However, when $|g_I| < 1$ the qualitative behaviors of CVCN change and when g_I is close enough to zero, patterns appear in CVCN. When $g_I = -0.1$ the pattern is shown in Fig. 23. Observe that in this pattern, the disturbance region competes with a homogenous pattern.

However, the window of parameters for this kind of pattern is narrow, outside the window, the CVCN is dominated by homogenous patterns. When $g_I = -0.08$ the pattern is shown in Fig. 24. Observe that in this pattern, the disturbance region become more narrow than that shown in Fig. 23. When g_I increases a little bit, the disturbance becomes so weak that it can only survive for a few iterations before it is entirely taken out by a homogenous pattern.

When $g_I = -0.07$ the disturbance dies out fast and the CVCN converges to a homogenous pattern. When $g_I = 0$ the disturbance dies out fast and the CVCN converges to a homogenous pattern. When $g_I > 0$ the qualitative behaviors of CVCN change. When $g_I = 0.08$ the pattern is shown in Fig. 25. Observe that in this case the homogenous pattern becomes unstable because of the oscillation caused by the disturbance. The oscillation of the disturbance is very weak.

When $g_I = 0.25$ the pattern is shown in Fig. 26. Observe that in this case the disturbance becomes much stronger than that shown in Fig. 25. In this case, the disturbance becomes strong enough to dominate the entire CVCN. Observe that in this case the disturbance consists of at least two different kinds of oscillations.

When g_I increases to near 1, the disturbance dies out fast and the CVCN converges to a homogenous pattern. When g_I is bigger than 1, as g_I increases the disturbance becomes more

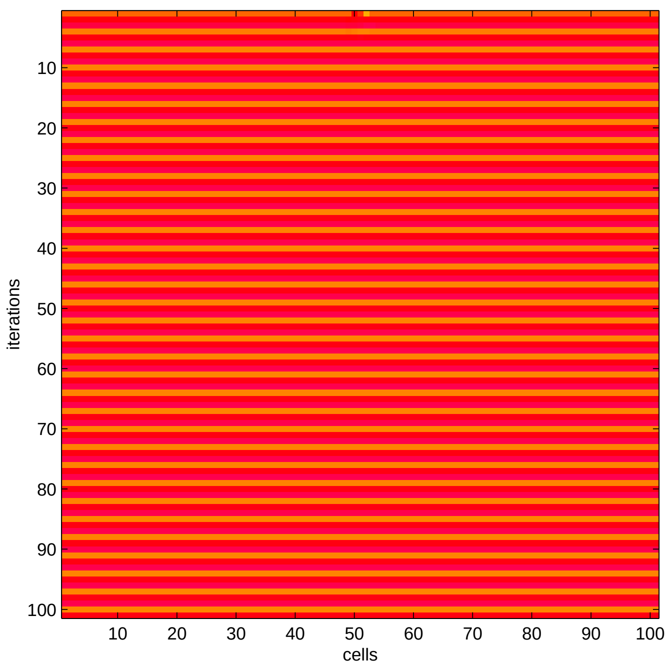

Fig. 19. Evolving process of "rule 110 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = -10$.

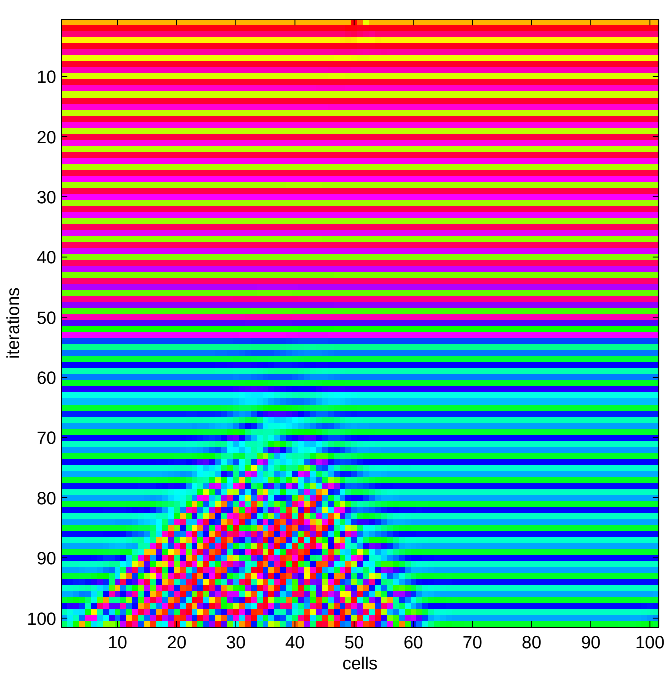

Fig. 20. Evolving process of "rule 110 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = -8$.

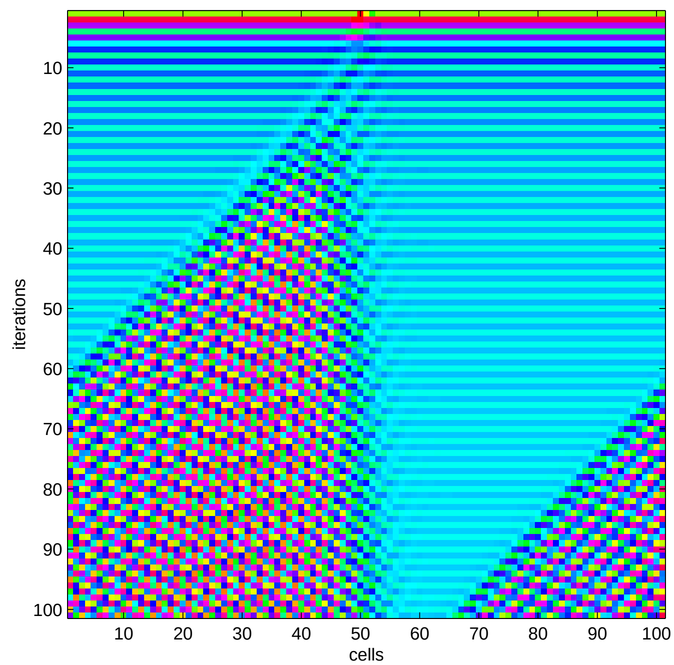

Fig. 21. Evolving process of "rule 110 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = -5$.

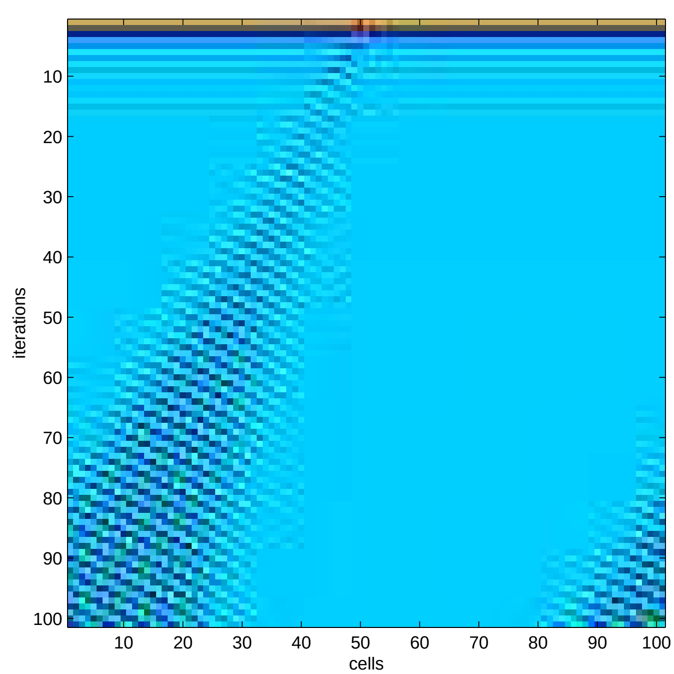

Fig. 22. Evolving process of "rule 110 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = -4$.

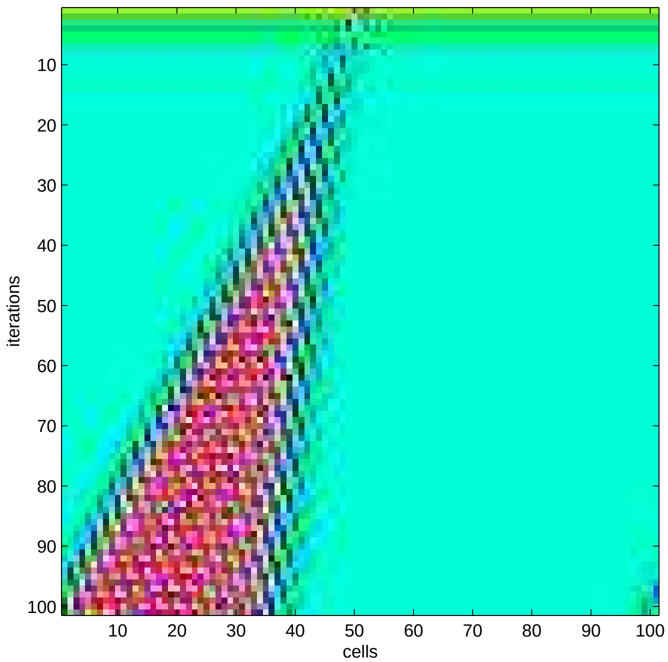

Fig. 23. Evolving process of “rule 110 1D CVCN” of 101 cells. The bifurcation parameter is $g_I = -0.1$.

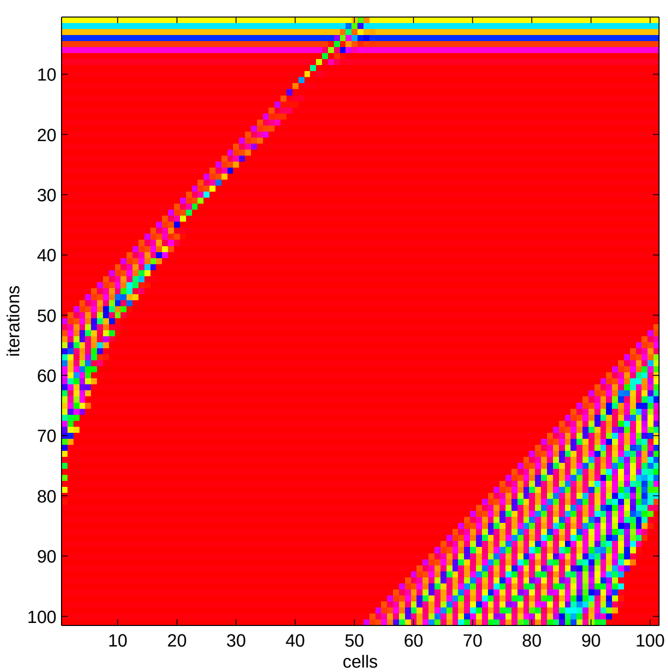

Fig. 24. Evolving process of “rule 110 1D CVCN” of 101 cells. The bifurcation parameter is $g_I = -0.08$.

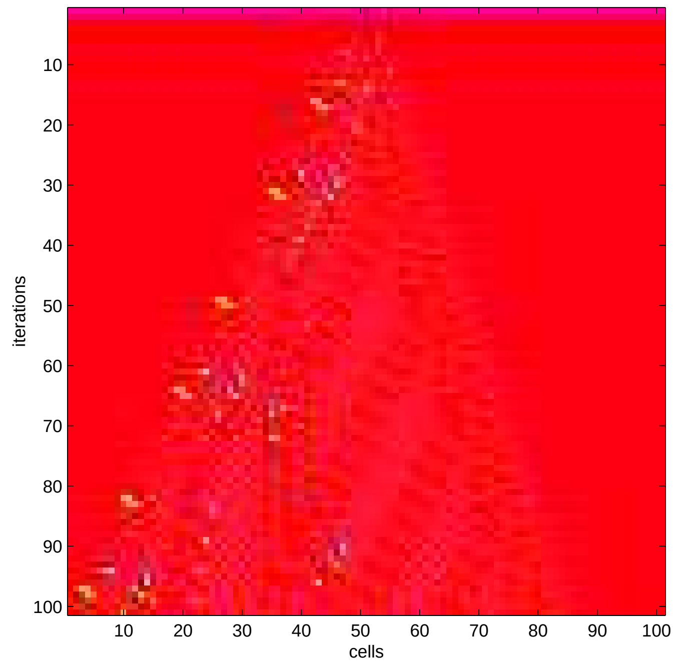

Fig. 25. Evolving process of “rule 110 1D CVCN” of 101 cells. The bifurcation parameter is $g_I = 0.08$.

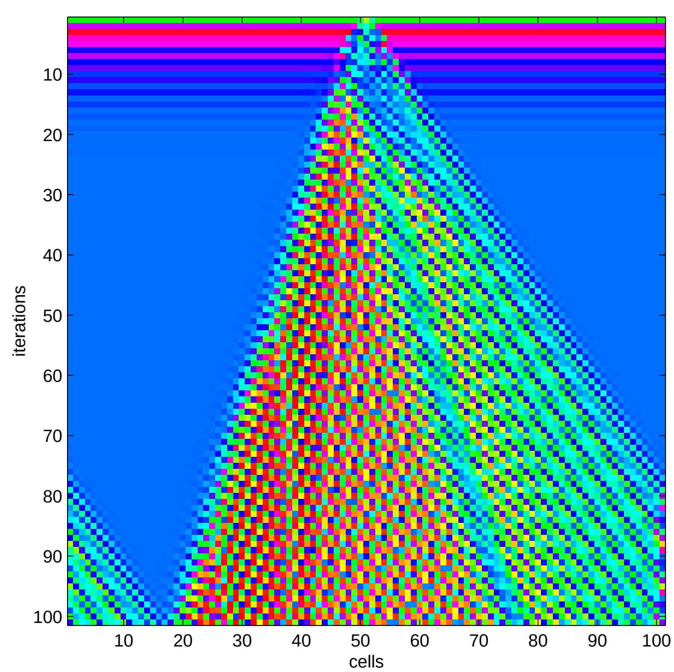

Fig. 26. Evolving process of “rule 110 1D CVCN” of 101 cells. The bifurcation parameter is $g_I = 0.25$.

robust till g_I becomes big enough such that different patterns appear. When $g_I = 2$ the disturbance dies out fast and the CVCN settles down at a homogenous pattern.

When $g_I = 3$ the pattern is shown in Fig. 27. Observe that the pattern within the disturbance becomes robust enough to eat out the homogenous blue region. The diagonal line patterns grow stronger when g_I increases.

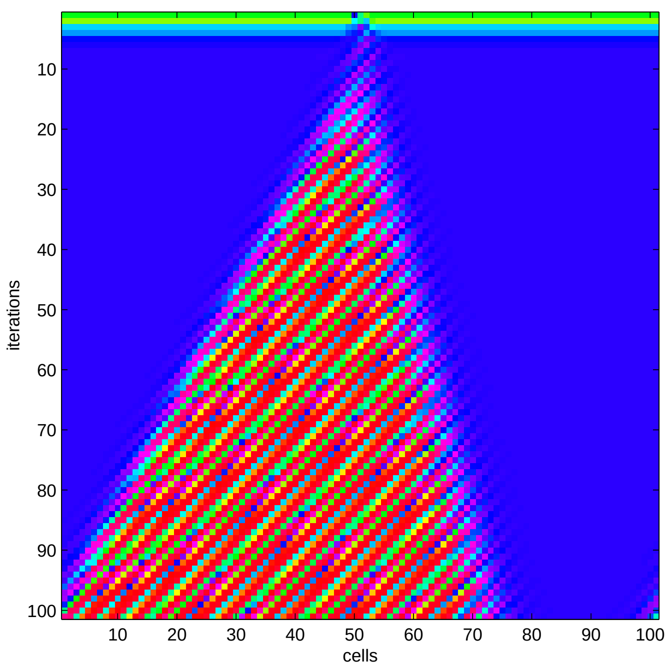

Fig. 27. Evolving process of "rule 110 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = 3$.

When $g_I = 4$ the pattern is shown in Fig. 28. Observe that the propagating speed of the disturbance region becomes much bigger comparing with that shown in Fig. 27.

When $g_I = 5$ the pattern is shown in Fig. 29. The synchronized oscillation outside the disturbance region becomes much stronger and survives for a longer time comparing with the cases of $g_I = 3$ and $g_I = 4$.

When g_I increases, the synchronized oscillation outside the disturbance region becomes dominated and the competition between the disturbance and the synchronized oscillation costs more iterations for the disturbance to dominate the CVCN. When $g_I = 6$ the pattern is shown in Fig. 30. Observe that only at iteration 40 that the disturbance becomes strong enough to compete against the synchronized oscillation.

When g_I increases to big enough, the synchronized oscillation becomes dominating the entire CVCN such that the disturbance dies out very fast as shown in Fig. 31 when $g_I = 7$.

When g_I increases further, all cells synchronize at a strong oscillation and the initial disturbance dies very fast.

B. Minimum as the t-Norm

In this section we study the bifurcation of pattern formation in "Rule 110 1D CVCN" when we choose $\bigwedge$ to be minimum in Eq. (4) and choose $\alpha = 1$.

When $g_I = -10$ the CVCN generates a pattern similar to that shown in Fig. 19; namely, the disturbance dies out

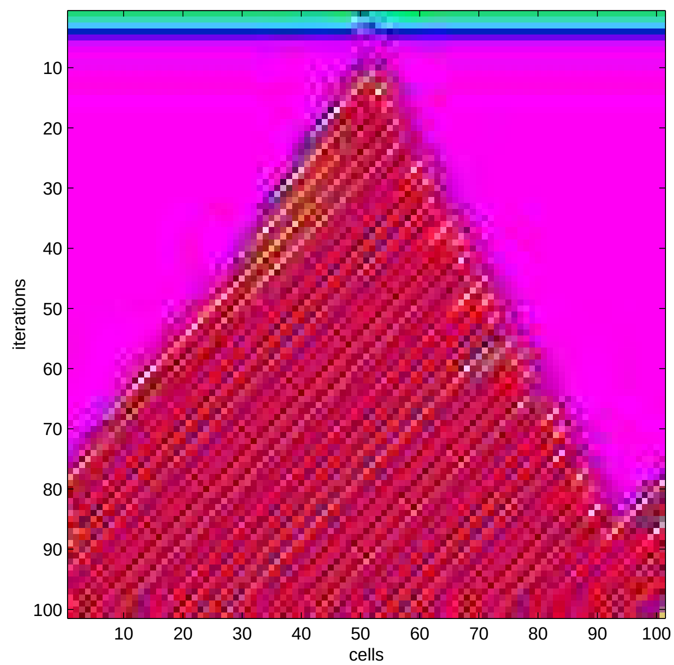

Fig. 28. Evolving process of "rule 110 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = 4$.

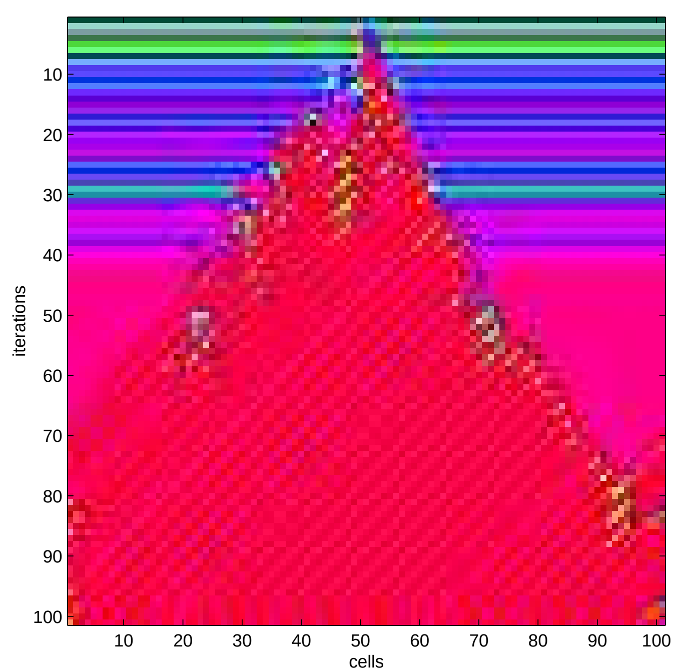

Fig. 29. Evolving process of "rule 110 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = 5$.

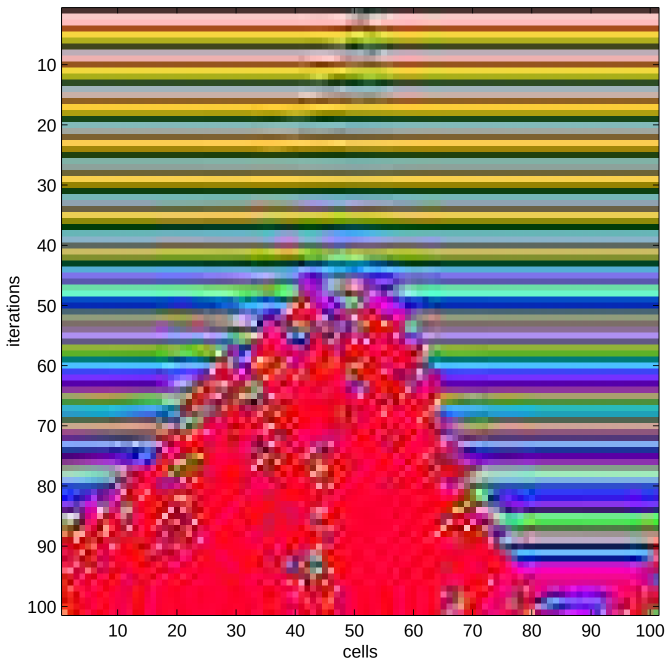

Fig. 30. Evolving process of "rule 110 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = 6$.

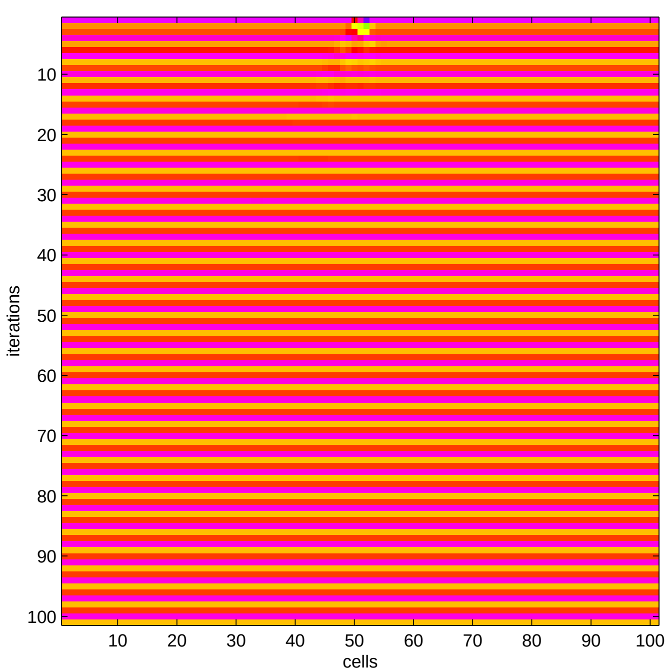

Fig. 31. Evolving process of "rule 110 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = 7$.

very fast and the synchronized oscillation dominates the entire CVCN.

When $g_I = -8$ the pattern is shown in Fig. 32. Observe that only at iteration 20 that the disturbance becomes strong enough to compete over the synchronized oscillation.

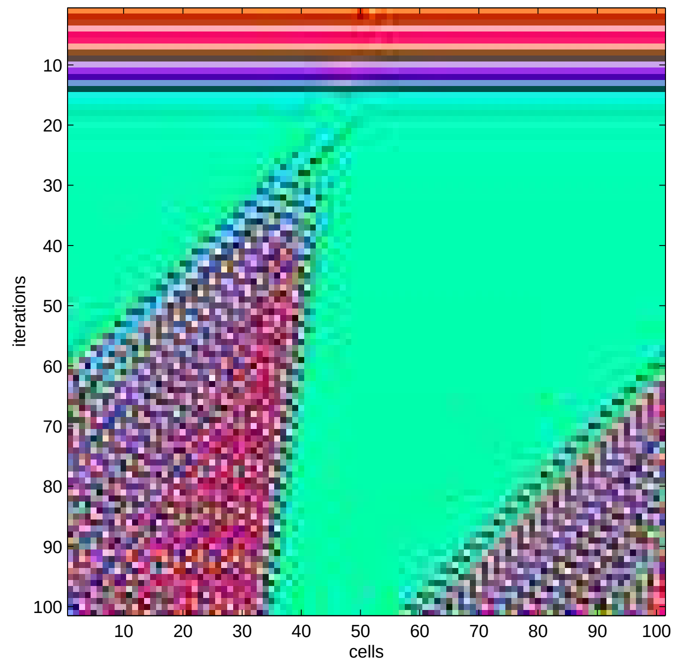

Fig. 32. Evolving process of "rule 110 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = -8$.

When g_I increases, the disturbance dies out at the beginning few iterations and the CVCN is dominated by homogenous pattern. When the absolute value of g_I is too small, a negative g_I results in a homogenous pattern.

However, when $g_I > 0$ the behaviors of CVCN changes qualitatively. When $g_I = 0.2$ the pattern is shown in Fig. 33. Observe that in the case the irregular oscillation in disturbance region competes with the synchronized oscillation outside the disturbance region. However, the parameter window for this pattern formation is very narrow.

As g_I increases to bigger than 1, the patterns only appear when g_I becomes bigger enough. When $g_I = 5$ the pattern is shown in Fig. 34. Observe that the oscillation within the disturbance region is weak while the wavefronts are strong.

When g_I increases to 6, the right-hand side wavefront becomes must more irregular as shown in Fig. 35.

When g_I increases to 7 the dominated pattern becomes a synchronized oscillated pattern. As g_I increases further, the amplitudes of the oscillation become weaker and the patterns become more homogenous.

VI. Concluding Remarks

Comparing to the 2D CVCNs studied in [60], the 1D CVCNs have much simpler dynamical patterns to generate. However, comparing to its counterparts where only Boolean local rules are used, 1D CVCNs are much more complex than 1D CAs and 1D CNN. The lack of a solid mathematical theory to study the pattern formations in 1D CVCNs will be overcome when the behaviors of them will be revealed more and more. The varieties of 1D CVCNs are tremendously huge simply by considering the extensions of the results presented in this paper along the following directions.

1) The standard verb set can be more comprehensive than the set consisting of two verbs used in this paper. However, since local rule bases become much more complex, it will impose a big challenge to the exploration of the local rule bases.
2) Different verb similarities used in the verb reasoning of verb local rules can enrich the pattern formations in 1D CVCN.
3) Different sizes of neighborhood can introduce more spatial filtering effects combining with different choices of wights of interactions.

Since CVCNs are used to model human social behaviors based on the modeling of interactions between human individuals via communication in natural languages, the primary results presented in the first two papers of a series of papers on CVCNs paved a path to a deeper understanding to the organization of different types of human societies. Many more researches will follow this path in the future from my group.

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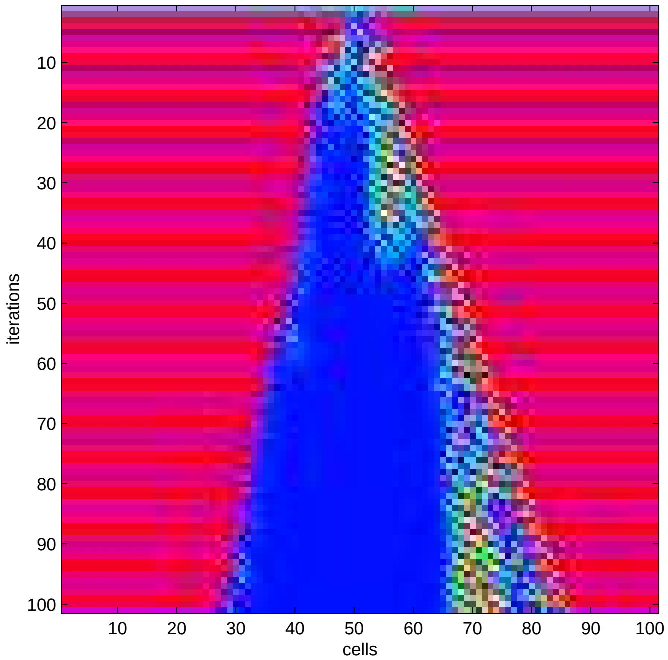

Fig. 33. Evolving process of "rule 110 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = 0.2$.

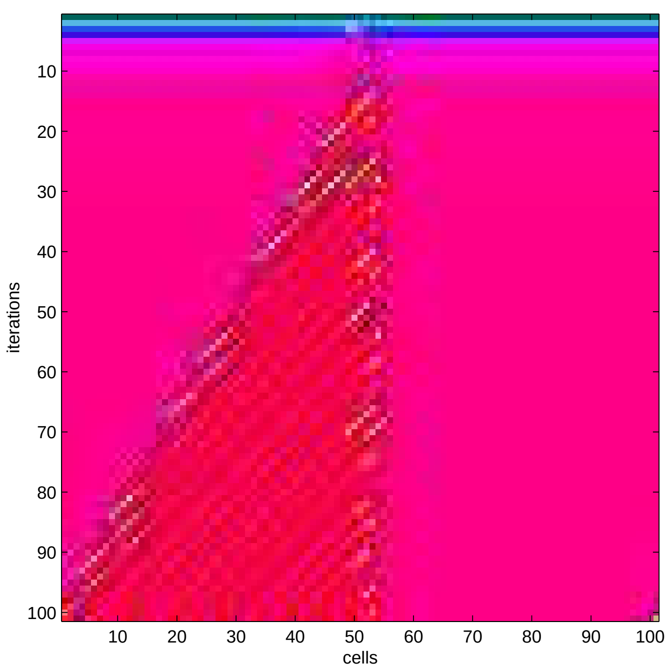

Fig. 34. Evolving process of "rule 110 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = 5$.

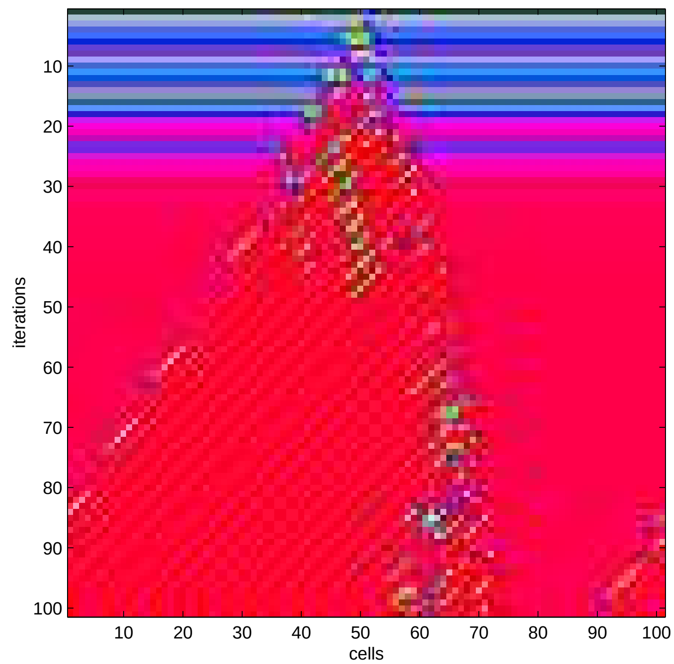

Fig. 35. Evolving process of "rule 110 1D CVCN" of 101 cells. The bifurcation parameter is $g_I = 6$.