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Low-Interference Topology Control for Wireless Ad Hoc Networks

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Topology control has been well studied in wireless ad hoc networks. However, only a few topology control methods take into account the low interference as a goal of their methods. Some researchers tried to reduce the interference by lowering node energy consumption (i.e. by reducing the transmission power) or by devising low degree topology controls, but none of those protocols can guarantee low interference. Recently, Burkhart *et al.* [3] proposed several methods to construct topologies whose maximum link interference is minimized while the topology is connected or is a spanner for Euclidean length. In this paper we give algorithms to construct a network topology for wireless ad hoc networks such that the maximum (or average) link (or node) interference of the topology is either minimized or approximately minimized.

Keywords: Topology control, interference, wireless ad hoc networks.

1 INTRODUCTION

Wireless networks have become increasingly important with the requirement for enhanced data and multimedia communications in ad hoc environments. While single hop wireless networks, or *infrastructured networks* are common, there are a growing number of applications which require multi-hop wireless infrastructure which does not necessarily depend on any fixed base-station. Wireless ad hoc network needs some special treatments as it intrinsically has its own special characteristics and some unavoidable limitations compared with wired networks. For example, wireless nodes are often powered by batteries only and they often have limited memories. A transmission by a wireless device is often received by many nodes within its vicinity, which possibly causes signal interferences at these neighboring nodes. On the other hand, we can also utilize this property to save the communications needed to send some information. Unlike most traditional static communication devices, the wireless devices often move during the communication. Therefore, it is more challenging to design a network protocol for wireless ad hoc networks. which is suitable for designing an efficient routing scheme to save energy and storage memory consumption, than the traditional wired networks. For simplification, we assume that the wireless nodes are quasi-static for a period of time.

Energy conservation is one of the critical issues in designing wireless ad hoc networks. Many aspects of the networking will affect the energy consumption of the wireless networks, such as the physical electronic design, the medium access control (MAC) protocols, the routing protocols, and so on. Topology control, a layer between MAC and routing protocol, provides another dimension to save the energy consumption of the wireless networks. In the literature, most of the research in the topology control is about adjusting the transmission power, or designing some *sparse* network topologies that can result in more efficient routing methods. However, less attention is paid to minimize the interference caused by this structures when we perform routing on top of them. Notice that, if a topology has a large interference, then either many signals sent by nodes will collide (if no collision avoidance MAC is used), or the network may experience serious delay at delivering data for some nodes, which in turn may cause larger energy consumption.

In wireless ad hoc networks, each wireless device can selectively decide which nodes to communicate either by adjusting its transmission power, or by only maintaining the communication links with some special nodes within its transmission range. Maintaining a small number of communication links will also speed up the routing protocols in addition to possibly alleviate the interferences among simultaneous transmissions,and also to possibly save the energy consumption. The question in topology control we have to deal with is how to design a network such that it ensures attractive network features such as bounded node degree, low-stretch factor (or called spanning ratio), linear number of links, and more importantly, low interference. In recent years, there was a substantial amount of research on topology control for wireless ad hoc networks [6–9, 11, 13, 15]. However, none of these structures proposed in the literature can *theoretically* bound the ratio of the interference of the constructed structure over the interference of the respected optimum structure. A common assumption in the topology control methods is that *low node degree implies small interference*, which is not always true, as shown in [3]. Notice that, in practice, almost all topology control methods will select short links and avoid longer links. However, even selecting “short” links only cannot guarantee that the interference of the resulting topology is within a constant factor of that of the optimum structure. Further, even letting each node only connect to its nearest neighbors[1], the resulting communication graph[2] may still have an interference arbitrarily, up to $O(n)$ factor, larger than the optimum. Recently, Burkhart *et al.* [3] proposed several methods to construct topologies whose maximum link interference is minimized while the topology is connected or is a spanner for Euclidean length.

In this paper we give algorithms to construct a network topology for wireless ad hoc network such that the maximum link (or node), or the average interference of the topology is either minimized or approximately minimized. We also study how to construct topology locally with small interference while it is power efficient for unicast routing.

The remainder of the paper is organized as follows. In Section 2, we specifically discuss what network model is used in this paper, and how we define the interference of a topology. In Section 3, we propose several methods to construct various topologies such that the maximum link interference or the average link interference of the topology is minimized. In Section 4, we propose several methods to construct various topologies such that the maximum node interference or the average node interference of the topology is minimized. We conclude our paper in Section 8 and also point out some future works.

2 PRELIMINARIES

2.1 Network Model

We consider a wireless ad hoc network (or sensor network) with all nodes distributed in a two dimensional plane. It is assumed in our paper that all wireless nodes have distinctive identities and each wireless node u has a maximum transmission range R_u. We only consider undirected (symmetric) communication links meaning that a message sent over a link can be acknowledged by the receiver. In other words, link uv exists if and only if the Euclidean distance between nodes u and v, denoted by $\|uv\|$, is less than R_u and R_v. It is required that the graph is connected if all nodes use their maximum transmission ranges, otherwise devising a topology that preserves connectivity is not possible.

Energy conservation is a critical issue in wireless ad hoc networks. The energy needed to support the communication between from node u to

[1] Here we assume a symmetric communication. In other words, the radius r_u of a node u is set as $\max(ux, uy)$, where node u is the nearest neighbor of node x, and node y is the nearest neighbor of node u.

[2] Two nodes u and v are connected if $uv \leq \min(r_u, r_v)$.

another node v is composed of three parts: (1) the energy used by node u to process the signal, (2) the energy needed to compensate the path loss of the signal from u to v, and (3) the energy needed by node v to process the signal. In the literature, the following path loss model is widely adopted: the signal strength received by a node v is p_1/r^α, where p_1 is the signal strength at one meter, r is the distance of node v from the source node u, and α is the path loss gradient, depending on the transmission environment. Consequently, the least signal needed to support the communication between two nodes u and v separated by distance r is $c_1 + c_2 r^\alpha$, where c_1, and c_2 are some constants depending on the electronic characteristics and the antenna characteristics of the wireless devices. Thus, we define the energy cost $c(uv)$ for each link as $c(uv) = c_1 + c_2 \cdot \|uv\|^\alpha$.

We also assume that each wireless device can adjust its transmission power to any value from 0 to its maximum transmission power or to a given sequence of transmission powers. Furthermore, in the literature it is often assumed that each wireless device u can adjust its transmission power for every transmission depending on the intended receiver v: node u will use the minimum transmission power available to reach node v. Some researchers assume that, given an undirected network topology H, each wireless device will only adjust its transmission power to the minimum power such that it can reach its farthest neighbor in H. In this paper, we will consider all possible power adjustments.

2.2 Topology Control

Due to the limited power and memory, a wireless node prefers to only maintain the information of a subset of neighbors it can communicate, which is called *topology control*. In recent years, there is a substantial amount of research on topology control for wireless ad hoc networks [6, 7, 11, 13, 15]. These algorithms are designed for different objectives: minimizing the maximum link length (or node power) while maintaining the network connectivity [11]; bounding the node degree [15]; bounding the spanning ratio [6, 7]; constructing planar spanner locally [6]. Here a subgraph H of a graph G is a length (or power) spanner of G if, for any two nodes, the length (or power) of the shortest-path connecting them in H is no more than a constant factor of the length of the shortest-path connecting them in the original graph G. Planar structures are used by several localized routing algorithms [2]. In [14], Li *et al.* proposed the first localized algorithm to construct a bounded degree planar spanner. Recently, Li, Hou and Sha [5] proposed a novel local MST-based method for topology control and broadcasting. In [8, 9], Li *et al.* proposed several new structures that approximate the Euclidean minimum spanning tree while the structures can be constructed using local information only and with $O(n)$ total messages.

However, none of these structures proposed in the literature can *theoretically* bound the ratio of the interference of the constructed structure over the interference of the respected optimum structure. Recently, Burkhart *et al.* [3] proposed several methods to construct topologies whose maximum link interference is minimized while the topology is connected or is a spanner for Euclidean length.

2.3 What is interference?

As mentioned earlier, the ultimate goal of the topology control is to conserve the energy consumption of the wireless networks. It has been pointed out that the topology control algorithms should not only consider adjusting the transmission power of nodes, bounding the number of nodes a node has to communicate, or bounding the power spanning ratio of the structure, but also to minimize the inherent interference of the structure so multiple parallel transmissions can happen simultaneously, and the number of retransmissions is decreased. Then a natural question is "*What is the interference of a structure*?". In this subsection, we will discuss different models of defining the interference of a structure.

The interference model proposed in [1] is based on the current network traffic. However, this model requires a priori information about the traffic in a network, which is often not available when designing the network topology due to the fact that the amount of the network traffic is often random and depends on the upper application layer. Thus, when we design a network topology to minimize the "interference", we prefer a static model of interference that depends solely on the distribution of the wireless nodes and, maybe, their transmission ranges.

Notice that, symmetric links are often preferred in wireless communications. In other words, a link uv exists in the communication graph if these two nodes u and v can communicate with each other directly, i.e., $|uv| \leq \min(r_u, r_v)$. Using this observation, Burkhart *et al.* [3] defined the interference of a link uv as the number of nodes covered by two disks centered at u and v with radius $\|uv\|$. Let $D(u, r)$ denote the disk centered at node u with radius r. Specifically, they defined the coverage of a link uv as

$$cov(uv) = \{w \mid w \textit{ is covered by } D(u, |uv|) \textit{ or } D(v, |uv|)\}.$$

Here, $cov(uv)$ represents the set of all nodes that could be affected by node u or by node v when they communicate with each other using exactly the minimum power needed to reach each other. We call this interference model as Interference based on Coverage model, and will use $IC(uv)$ to denote the interference of a link uv under this model. This model is chosen since whenever a link uv is used for a send-receive transaction all nodes whose distance to node u or to node v is less than $\|uv\|$ will be affected.

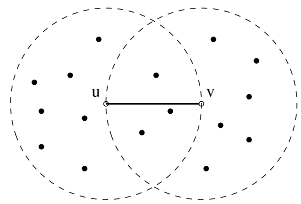


FIGURE 1
The interference of link uv is the number of wireless nodes whose distance to node u or to node v is less than $\|uv\|$.

The network is then represented by a geometric undirected *weighted* graph, $G = (V, E, W)$, with vertices representing wireless nodes, and edges representing communication links. The weight of each link uv is its interference number $IC(uv)$. See Figure 5 for an illustration. After assigning weights to all links, we call the graph the *interference graph*. Then, Burkhart *et al.* [3] proposed centralized methods to select a connected spanning subgraph of this interference graph while the maximum interference of selected links is minimized. They also proposed centralized and localized methods to select subgraphs with additional requirement that the subgraph is an Euclidean length spanner of the original communication graph.

Thus, given a subgraph H of the original communication graph G of n wireless devices, the maximum interference, denoted as $MIC(H)$, of this structure H is defined as $\max_{e \in H} IC(e)$, and the average interference, denoted as $TIC(H)$, of this structure H is defined as $\sum_{e \in H} IC(e)/n$.

Notice that, in practice, the wireless devices often cannot adjust their transmission powers to any number from 0 to their maximum transmission powers. Usually, there are a sequence of discrete power levels that the wireless device can choose from. In this discrete power model, we clearly can extend the interference based on coverage model IC as follows. Given any link uv, let P_{uv} be the minimum power level such that nodes u and v can reach each other using this power, and let r_{uv} be the corresponding transmission range using the power P_{uv}, i.e. $P_{uv} = c_1 + c_2 \cdot r_{uv}^{\alpha}$. Then, the coverage of a link uv is defined as

$$cov(uv) = \{w \mid w \ is \ covered \ by \ D(u, r_{uv}) \ or \ D(v, r_{uv})\}.$$

And the interference $IC(uv)$ of a link uv is then the cardinality of $cov(uv)$. In the remainder of the paper, we will not distinguish this model from the model used in [3]: we always use $IC(uv)$ to denote the interference of a link in both models.

Notice that the interference model used in [3] implicitly assumes that the node u will send message to v and node v will send message to u at the same time. We argue that when u sends data to node v, typically node v only has to send a very short ack message to u. The communication then becomes one way by ignoring this small ack message from v. Clearly, when v is receiving message from node u, the nodes "nearby" node v cannot send any data, otherwise, the signal from u to v will be colliding and thus interference will occur. Theoretically speaking, the transmission by another node w causes the interference with the transmission from node u to node v if the signal to noise ratio (SINR) of the signal received by node v is below the threshold [3] of node v when node w transmits at a given power. To simplify the analysis of SINR, we assume that the transmission of a node w causes such interference if node v is within the transmission range of w. In other words, we say interference occurs when v is within the transmission ranges of both node u and node w, and both node u and node w transmit signal to v. The number of such nodes w is the total number of nodes whose transmission will cause interference with the signal received by node v. Considering such a node w, then the transmission of node w may cause interference to *all* nodes within its transmission range. Thus, to alleviate the interference, we would like to minimize the number of nodes within the transmission range of node w. We call such interference model as Interference based on Transmission model and will use $IT_H(w)$ to denote the interference of a node w under a given network topology H.

Thus, given a subgraph H of the original communication graph G, the transmission range of each node u is defined as $r_u = \max_{uv \in H} \|uv\|$. The interference number $IT_H(u)$ of a node u under Interference based on Transmission model is then defined as the cardinality of the set $\{v \mid \|uv\| \le r_u\}$. The maximum interference of this structure H is defined as $\max_{u \in V} IT_H(u)$, and the average interference of this structure H is defined as $\sum_{u \in V} IT_H(u)/n$.

3 LINK BASED INTERFERENCE

In this section, we design algorithms for topology control that minimize the maximum or the average interference of the resulting topology while preserving some properties of the network topology such as connectivity.

3.1 Minimizing the Maximum Interference

Definition 1 *The* Min-Max *link interference with a property $\mathcal{P}$ problem is to construct a subgraph H of a given communication graph $G = (V, E)$ such that the maximum interference $MIC(H)$ of structure H achieves the minimum among all subgraphs of G that have the given property $\mathcal{P}$.*

[3] The threshold of node v depends on the sensitivity of the antenna of node v, the modulation technique of the signal, and other factors.

Essentially, in [3], Burkhart *et al.* gave a centralized method to construct a connected topology that minimizes the maximum interference. He also introduced centralized and localized methods for the the Min-Max link interference problem with a property *bounded Euclidean spaning ratio*. In their algorithm (called LIFE) edges are sorted by their weights (interference) in ascending order. Starting from the edge with minimum weight, in each iteration of the algorithm an edge uv is processed. If nodes u and v are already connected, the edge uv is just ignored and otherwise it will be added to the topology. The algorithm continues till a connected graph is constructed. Clearly, the time complexity of this approach is $O(m \log m + hn)$, where h is the number of links in the final structure H. If a t-spanner structure is needed, they [3] add a link uv if the shortest path connecting u and v using previously added "short" links has length larger than t times the length of link uv; otherwise, link uv will not be added. Clearly, the time complexity of this approach is $O(m \log m + h(h + n \log n))$.

A graph property $\mathcal{P}$ is called *polynomially verifiable* if we can test whether any given graph H has this property $\mathcal{P}$ in polynomial time in the size of the graph H. For example, the connectivity property is polynomially verifiable, the bounded spanning ratio property is polynomially verifiable, the k-connectivity property is polynomially verifiable. Assume that we are given a *polynomially verifiable* property $\mathcal{P}$, the following binary search based approach is straightforward.

Algorithm 1 *Min-Max Link Interference with a given property* $\mathcal{P}$

1 Sort the weight (i.e., interference number) of all links in ascending order. Let $w_1, w_2, \cdots, w_m$ be the sorted list of link weights. Let $U = m$ and $L = 1$. Repeat the following steps until $U = L$.

2 Let $i = \lfloor \frac{L+U}{2} \rfloor$ and $w = w_i$.

3 Test if the structure H formed by all links with weight $\leq w$ has the property $\mathcal{P}$. If it does, then $U = i$, otherwise, then $L = i$.

Assume that the time complexity to test whether a given structure H (with n vertices and at most m links) has a property $\mathcal{P}$ takes time $\beta_{\mathcal{P}}(m, n)$. It is easy to show that the above binary search based approach has time complexity $O(m \log m + \beta_{\mathcal{P}}(m, n) \cdot \log n)$. For example, to test whether a structure is connected can be done in time $O(n)$, which implies that the min-max interference with connectivity can be done in time $O(m \log m + n \log n) = O(m \log n)$. Testing whether a given structure H is a t-spanner of the original graph G can be done in time $O(n(n \log n + m)) = O(n^2 \log n + mn)$, which implies that the min-max interference with t-*spanner* can be done in time $O(m \log m + n^2 \log^2 n + mn \log n) = O(n \log n(m + n \log n))$ using a binary search based approach described by Algorithm 1.

The following theorem is obvious.

Theorem 1 *For a given property $\mathcal{P}$, Algorithm 1 gives the optimum solution for* Min-Max *link interference problem* .

3.2 Minimizing the Average Interference

The maximum interference of the structure captures the worst link on the structure, however, it does not capture the overall performance of the structure in terms of the interference. In this section, we design algorithms that will minimize the *average* interferences of the structure while preserving some additional property $\mathcal{P}$.

Definition 2 *The* Min-Average *link interference with a property $\mathcal{P}$ problem is to construct a subgraph H of a given communication graph $G = (V, E)$ such that the average interference $TIC(H)$ of structure H achieves the minimum among all subgraphs of G that have the given property $\mathcal{P}$.*

When the given property $\mathcal{P}$ is just merely the connectivity of structure, to solve Min-Average link interference with a property $\mathcal{P}$ problem, it suffices to find the minimum spanning tree of the interference graph. The following lemma proves that the MST gives the optimum answer.

Lemma 2 *MST gives the optimum solution for* Min-Average *link interference with* connectivity.

Proof. Assume the optimum graph, say G', is not MST and preserves connectivity. Since the average link interference in G' is equivalent to summation of link weights in G' it requires that G' has the minimum weight. Since G' is not MST, it must have weight less than MST which is impossible. □

Note that we will construct the minimum spanning tree of the interference graph, which is different from the Euclidean minimum spanning tree. Actually, the Euclidean MST (i.e. where the weight of each edge is the Euclidean length of the edge) can be $\Omega(n)$ times worse than the optimum for link interference Min-Average problem. The example illustrated by Figure 5 in [3] can be used to show that the Euclidean MST can be very bad for both link interference Min-Max and the link interference Min-Average problems. For that example, the maximum interference of the Euclidean MST is $O(n)$ and the average interference of the Euclidean MST is also $O(n)$, while in the optimum structure, the maximum interference is $O(1)$, and the average interference is also $O(1)$. Thus, Euclidean MST is $\Omega(n)$ times worse than the optimum solution for both criteria. Notice that $\Omega(n)$ is the worst possible ratio for any structure for both the Min-Max and the Min-Average link interference problems.

Preserving the connectivity of the final structure H and minimizing the average interference can be optimally solved using the minimum spanning tree, it will be NP-hard to find the optimum structure when the property $\mathcal{P}$ is additive, e.g., being a t-spanner.

4 NODE INTERFERENCE

In this section we define interference for each node instead of defining interference for each link. To study node interference problem we define two models. The first model is based on link interference and the second model is based on the number of nodes that are in the transmission region of a node.

4.1 Node Interference via Link

Given a network topology H, a node u will then only communicate using links in H. If node u communicates with a neighbor v with $uv \in H$, node u may experience the interference from $IC(uv)$ number of nodes. We then would like to know what is the worst interference number experienced by node u, i.e., we are then interested in $IC(u) = \max_{uv \in H} IC(uv)$. In this model the interference of each node u is the maximum link interference of all links incident to it.

Definition 3 Node Interference (Model 1)*: Given a structure H, the interference of a node u is defined as the maximum interference of all links incident on u, i.e.,*

$$IC_H(u) = \max_{uv \in H} IC(uv).$$

Then the maximum node interference of a structure is defined as $MNIC(H) = \max_{u \in V} IC_H(u)$, and the average node interference of a structure is defined as $TNIC(H) = \sum_{u \in V} IC_H(u)/n$.

4.1.1 Minimizing the Maximum Interference

First, we would like to minimize the maximum node interference.

Definition 4 *The* Min-Max *node interference with a property $\mathcal{P}$ problem is to construct a subgraph H of a given communication graph $G = (V, E)$ such that the maximum node interference $MNIC(H)$ of structure H achieves the minimum among all subgraphs of G that have the given property $\mathcal{P}$.*

Notice that the node interference of a node now depends on the final topology, which introduces a level of difficulty compared with the link interference studied in subsection 3.1. We first study how to find a connected topology whose maximum node interference is minimized. Surprisingly enough, we found that the minimum spanning tree based approach still produces the optimum network topology.

Theorem 3 *MST produces an optimum structure for* Min-Max *Node Interference for connectivity problem.*

Proof. Assume the MST is not optimum and *OPT* is an optimum structure. Consider the edge with the highest interference in MST, say e. Then edge e doesn't belong to *OPT* (otherwise structure MST would have been the optimum) and also the interference of all edges in *OPT* is less than the interference of edge e. This means a connected graph can be constructed with using edges whose interference is less than the interference of edge e, and this violates the definition of MST. □

4.1.2 Minimizing the Average Interference

Similarly, we can also minimize the average node interference of the structure.

Definition 5 *The* Min-Average *node interference with a property $\mathcal{P}$ problem is to construct a subgraph H of a given communication graph $G = (V, E)$ such that the average node interference $TNIC(H)$ of structure H achieves the minimum among all subgraphs of G that have the given property $\mathcal{P}$.*

Solving the Min-Average node interference with a property $\mathcal{P}$ is not easy and since the simple form of this problem by requiring the connectivity property is similar to the min-average power symmetric connectivity, which is well-known to be NP-Hard. Thus, instead of trying to solve it optimally, we first give a good approximation algorithm to achieve the connectivity property. The following theorem proves that the MST (of the interference graph G) is a 2-approximation for the Min-Average node interference with connectivity.

Theorem 4 *MST is a 2-approximation for the* Min-Average *node interference with connectivity problem.*

Proof. Consider any spanning tree T and let $I(T)$ denote the average node interference of graph T and let $W(T)$ denote the total weight of the links of graph T. Note that here the weight of each link is the interference of that link. Since the weight of each edge is assigned to at most two nodes, $I(T) \leq 2W(T)$. On the other hand, consider the spanning tree as a tree rooted at some node. For any leaf node u, the interference of the link that connects u to its parent is the interference that is assigned to node u; for any internal node v, the interference assigned to node u is less than or equal to the interference of the link between node v and its parent in the tree; and the interference assigned to root is some value greater than zero. Thus, the total interference of the nodes is greater than the total interference of the links and we have $W(T) < I(T)$. Now let OPT be the optimum structure. Clearly OPT is a spanning tree (i.e., cycles can be removed if there is any without increasing the average interference). We have $I(MST) \leq 2W(MST)$. Since MST is the minimum weight spanning tree, $W(MST) \leq W(OPT)$ and $W(OPT) < I(OPT)$. Consequently, $I(MST) < 2I(OPT)$. This finishes the proof. □

The MST based heuristics also works if the weight of each edge is some quality such as the power needed to support the link, the delay of the link, or the SINR (Signal to Interference and Noise Ratio). Again, the Euclidean MST can be $\Omega(n)$ times worse than the optimum. Since the maximum interference is at most $O(n^2)$, obviously $\Omega(n)$ is the worst possible ratio.

4.2 Transmission based Interference

Notice that, when a topology H is used for routing, each wireless node typically adjusts its transmission power to the minimum that can reach its farthest neighbor in H. Considering this power level, we say that the the interference of each node u is the number of nodes inside its transmission range. Let r_u denote the transmission range of node u then the node interference is defined as follows:

Definition 6 Node Interference (Model 2)*: Given a structure H, the interference of a node u is number of nodes inside its transmission range, i.e.,*

$$IT_H(u) := |\{v \mid \|uv\| \leq r_u\}|.$$

Here $r_u = \max_{uv \in H} \|uv\|$.

Then similarly the maximum node interference of a structure is defined as $MNIT(H) = \max_{u \in V} IT_H(u)$, and the average node interference of a structure is defined as $TNIT(H) = \sum_{u \in V} IT_H(u)/n$.

4.2.1 Minimizing the Maximum Interference

First, we would like to minimize the maximum node interference.

Definition 7 *The* Min-Max *node interference with a property $\mathcal{P}$ problem is to construct a subgraph H of a given communication graph $G = (V, E)$ such that the maximum node interference $MNIT(H)$ of structure H achieves the minimum among all subgraphs of G that have the given property $\mathcal{P}$.*

Consider node u and let $N(u)$ be the number of neighbors of node u when node u adjusts its transmission range to maximum. Node u can adjust its transmission range to have exactly k neighbors ($0 \leq k \leq N(u)$). In other words, each node u can set its interference to any value between 0 and $N(u)$ by using the appropriate transmission range. Having this property, solving the Min-Max node interference with a property $\mathcal{P}$ problem is only a simple binary search.

Algorithm 2 Min-Max *Node Interference with a property $\mathcal{P}$ for model 2.*

1 Let $U = n - 1$ and $L = 1$. Repeat the following steps until $U = L$.
2 Let $i = \lfloor \frac{L+U}{2} \rfloor$ and let H_i be the graph formed by connecting each node u to its first i-shortest links. Notice that, if u has less than i neighbors in the original graph, then u will only connect to all its $N(u)$ neighbors.
3 Test if the structure H_i has the property $\mathcal{P}$. If it does, then $U = i$, otherwise, then $L = i$.

Assume Algorithm 2 gives an interference value i. Since setting the interference of each node to a value less than i cannot preserve the property $\mathcal{P}$. The following theorem is then obvious.

Theorem 5 *Algorithm 2 produces the optimum solution for the* Min-Max *Node Interference with a property $\mathcal{P}$.*

4.2.2 Minimizing the Average Interference

Similarly, we can also minimize the average node interference of the structure.

Definition 8 *The* Min-Average *node interference with a property $\mathcal{P}$ problem is to construct a subgraph H of a given communication graph $G = (V, E)$ such that the average node interference $TNIT(H)$ of structure H achieves the minimum among all subgraphs of G that have the given property $\mathcal{P}$.*

Solving the Min-Average node interference problem for Model 2 is not easy and it seems to be NP-Hard to find the optimum answer. Here we give an efficient heuristics to find a structure that is practically good.

We construct a directed graph $G' = (V', E', W')$ as follows: for each edge uv of G, we introduce two additional vertices $[uv]$ and $[vu]$. Each node u, sorts its neighbors $v_1, v_2, ..., v_k$ in ascending order of distances from u. Then we connect node u to node $[uv_1]$ using directed link $u[uv_1]$ and we assign weight 1 to it; we also define a directed link $[uv_1]u$ and we assign weight 0 to link $[uv_1]u$. We also connect vertices $[uv_i]$ and $[uv_{i+1}]$ using two directed links $[uv_i][uv_{i+1}]$ and $[uv_{i+1}][uv_i]$ $(1 \leq i < k)$ and assign weight 1 to all those links $[uv_i][uv_{i+1}]$ and we assign weight 0 to all links $[uv_{i+1}][uv_i]$ $(1 \leq i < k)$. All pairs $[uv], [vu]$ are connected also. Assume node u is the p^{th} nearest neighbor of node v and node v is the q^{th} nearest neighbor of node u. Then we assign weight p to the edge $[uv][vu]$ and weight q to $[vu][uv]$. See Figure 2 and Figure 3 for

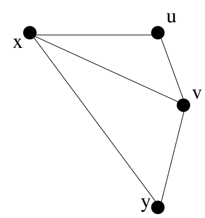


FIGURE 2
The original communication graph.

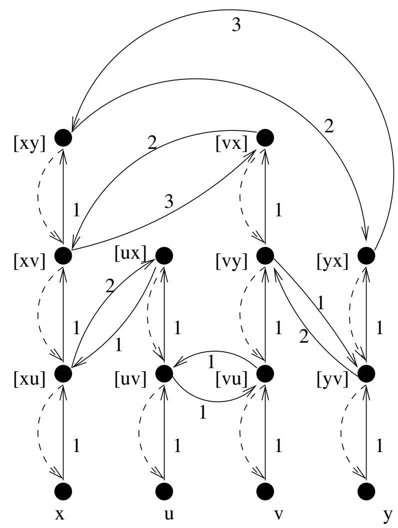


FIGURE 3
The transformed graph.

an illustration. Figure 2 depicts the original graph and Figure 3 shows the transformed graph. All dashed edges have weight 0. Now we start from any node $u \in V$ and we solve the min-cost multicast problem to all other nodes $v \in V$. It is easy to show that the min-cost multicast problem in G' is equal to the min-average node interference graph in G.

We then introduce a greedy based algorithm for this multicast problem in the directed graph G'. The algorithm starts with an empty set of *processed nodes*, denoted by A, and picks a random node u and puts it in the set A. We define the distance between a node v that does not belong to set A and set A as the shortest path starting from a node in set A to v. Then in each iteration the node that is the closest to the set A is added to set A and the distances of nodes to the set A are updated. The algorithm continues till all desired nodes are in A. Let H_u be the final structure constructed when node u is first put to the set A.

To find the best structure possible, we will construct the structures H_{v_i} for all nodes $v_i \in V$ and then find the structure with the minimum average nodal interference.

The approach used in this algorithm is like the Prim's algorithm. The set of nodes V is divided into two sets S and $V - S$, a random node is put in S and in each iteration the node *closest* to the set S is added to it till $S = V$. Now we have to define the distance between a node $v \in V - S$ and the set S. Consider edge uv such that $u \in S$ and $v \in V - S$, if this edge is added then the interference of nodes u and v might increase, we define this incremental interference as the weight of edge uv, and like Prim's algorithm the distance of node v from the set S is the weight of the shortest edge connecting v to S. Whenever an edge uv is added The adjustable transmission range of nodes u and v is updated if necessary.

There is another heuristic to solve this problem. This heuristic in only slightly different. We start from n components and each component has exactly one node. In each iteration two components that are the closest to each other are merged. Edge weights are defined the same way and the distance between two components is defines as the weight of the shortest edge connecting them. The algorithm continues till there is only one component left. Our simulation results show that this simple trick slightly improves the performance.

5 LOCALIZED APPROACHES

In the previous sections, we discussed in detail several centralized methods for topology control to minimize the interference while preserving some property $\mathcal{P}$. Although these centralized methods can find the optimum or near optimum structures for wireless ad hoc networks, but they are too expensive to be implemented in wireless ad hoc networks. In this section, we shift our attention to localized topology control methods to minimize the interference, with an additional requirement such as hop spanner, length spanner or power spanner. Here the desired spanning ratio is given. If the structure is required to be t-length spanner, as shown in [3], for each link uv we only need the information of $(t/2).\|uv\|$ neighborhood (i.e. nodes whose distance to node u or to node v is less than $(t/2).\|uv\|$). Similarly for t-hop spanner it suffices to gather the information of $\lceil k/2 \rceil$ hops of nodes u and v (i.e. nodes which are at most $\lceil k/2 \rceil$ hops away from node u and node v). Here we say that a structure H is a t-spanner for power consumption if for any pair of nodes u and v, the minimum power needed to connect them in H is no more than t times the minimum power of the best path connecting them in the original communication graph. Remember that, the power needed to support a link $e = (x, y)$, denoted by $p(e)$, is $c_1 + c_2 \cdot \|xy\|^{\alpha}$.

The total power of a path Π, denoted by $v_0 v_1 \cdots v_k$, connecting u and v is $p(\Pi) = \sum_{i=0}^{k-1} p(v_i v_{i+1}) = k \cdot c_1 + c_2 \cdot \sum_{i=0}^{k-1} \|v_i v_{i+1}\|^\alpha$. Here u is node v_1 and v is node v_k. Let $u \to_H v$ be the path connecting u and v using links in H with the minimum total power consumption, and its power consumption is then denoted by $p(u \to_H v)$. Formally speaking, a structure H is a t-power-spanner of original graph G if

$$\max_{u,v \in V} \frac{p(u \to_H v)}{p(u \to_G v)} \leq t.$$

In the remainder of the paper, we assume that the maximum transmission range of every node is R_0 (i.e., the maximum transmission power of every node is $c_1 + c_2 R_0^\alpha$).

Lemma 6 *Consider any structure H that is a t-power-spanner. For any link uv in the original graph G, the t-power spanner path $u \to_H v$ has an Euclidean length at most $t \cdot \mathcal{A} \cdot (c_1 + c_2\|uv\|^\alpha)$, where $\mathcal{A} = \frac{c_2^{1/\alpha}(\alpha-1)^{1+1/\alpha}}{\alpha c_1^{1-1/\alpha}}$ is a constant.*

Proof. Remember that the power cost of using a link uv is $c_1 + c_2\|uv\|^\alpha$. We define the *mileage* of this model as $\max_{0<x} \frac{x}{c_1 + c_2 x^\alpha}$. In other words, milage is the maximum distance a message can be sent using unit amount of energy. It is easy to see that $x = \sqrt[\alpha]{\frac{c_1}{(\alpha-1)c_2}}$ achieves the maximum mileage for this energy model. Clearly the maximum mileage is $\frac{c_2^{1/\alpha}(\alpha-1)^{1+1/\alpha}}{\alpha c_1^{1-1/\alpha}}$. Hereafter, we use $\mathcal{A}$ to denote such mileage.

We then show that the least power path $u \to_H v$ has an Euclidean length, say x, within some constant factor of the Euclidean length $\|uv\|$. From the definition of mileage, we know that the total power of the path $u \to_H v$ is at least $\frac{x}{\mathcal{A}}$. Since it is a t-power-spanner path for uv, we have $\mathcal{A} \cdot x \leq t(c_1 + c_2\|uv\|^\alpha)$. In other words, $x \leq t \cdot \mathcal{A} \cdot (c_1 + c_2\|uv\|^\alpha)$. □

This lemma implies that node u can locally decide whether a link uv will be kept in a t-power spanner H by using only the information of nodes within distances $\frac{x}{2} + \|uv\|$ to node u. The above lemma also implies that the minimum power path for any link uv uses only local neighborhood nodes as long as the mileage (the maximum ratio of the length of a link over the power needed to support the direct communication of this link) is bounded from above by a constant.

Then similar to [3], we can construct a network topology H such that the maximum interference is minimized while the structure H is a t-power spanner of the original communication graph. For the completeness of the presentation, we still include the algorithm here. The algorithm is presented from the point view of a node u.

Algorithm 3 *Min-Max Link Interference with a t-power spanner*

1 Each wireless device collects the information of nodes with distance $q \cdot t \cdot R_0$.
2 Sort the interference number in ascending order of all links formed by nodes within distance $q \cdot t \cdot R_0$ from u. Let $w_1, w_2, \cdots, w_m$ be the sorted list of link weights. Let $U = m$ and $L = 1$. Repeat the following steps until $U = L$.
3 Let $i = \lfloor \frac{L+U}{2} \rfloor$ and $w = w_i$.
4 Test if the structure H formed by all links with interference $\leq w$ has a path that is a t-power spanner for each physical link uv. In other words, the path has total power at most $t \cdot (c_1 + c_2 \|uv\|^2$. If it does, then $U = i$, otherwise, then $L = i$.

6 PERFORMANCES ON RANDOM DEPLOYED NODES

In the previous sections, we studied how to design topologies with low maximum or average interferences in the worst case. Worst case performance analysis provides us the insight how bad these methods could behave. However, the worst case does happen rarely in practice. Another important performance analysis is average performances analysis, which gives us insight how a structure will perform generally. In this section, we will show that the most commonly used structures in the literature could have arbitrarily large maximum node interferences, but their average interferences are often bounded by a small constant.

6.1 Theoretical Analysis

For average performance analysis, we consider a set of wireless nodes distributed in a two-dimensional unit square region. The nodes are distributed according to either the uniform random point process or homogeneous Poisson process. A point set process is said to be a *uniform random point process*, denoted by $\mathcal{X}_n$, in a region Ω if it consists of n independent points each of which is uniformly and randomly distributed over Ω. The standard probabilistic model of *homogeneous Poisson process* is characterized by the property that the number of nodes in a region is a random variable depending only on the area (or volume in higher dimensions) of the region. In other words,

- The probability that there are exactly k nodes appearing in any region Ψ of area A is $\frac{(\lambda A)^k}{k!} \cdot e^{-\lambda A}$.
- For any region Ψ, the conditional distribution of nodes in Ψ given that exactly k nodes in the region is *joint uniform*.

Given a set V of wireless nodes, several structures (such as relative neighborhood graph RNG, Gabriel graph GG, Yao structure, etc) have been proposed for topology control in wireless ad hoc networks. The *relative neighborhood graph*, denoted by $RNG(V)$, consists of all edges uv such that the intersection of two circles centered at u and v and with radius $\|uv\|$ do not contain any vertex w from the set V. The *Gabriel graph* [4] $GG(V)$ contains edge uv if and only if the disk using link uv as diameter, denoted by $disk(u, v)$, contains no other nodes of V. We will study the expected maximum node interference and the expected average node interference for structures Euclidean Minimum Spanning Tree (EMST), Gabriel Graph (GG) and the Relative Neighborhood Graph (RNG).

Let d_n be the longest edge of the Euclidean minimum spanning tree of n points placed independently in 2-dimensions according to standard poisson distribution with density n. In [10], they showed that $\lim_{n\to\infty} P_r(n\pi d_n^2 - \log n \leq \alpha) = e^{-e^{-\alpha}}$. Notice that the probability $P_r(n\pi d_n^2 - \log n \leq \log n)$ will be sufficiently close to 1 when n goes to infinity, while the probability $P_r(n\pi d_n^2 - \log n \leq -\log\log n)$ will be sufficiently close to 0 when n goes to infinity. That is to say, with high probability, $n\pi d_n^2$ is in the range of $[\log n - \log\log n, 2\log n]$.

Given a region with area A, let $m(A)$ denote the number of nodes inside this region by a Poisson point process with density δ. According to the definition of Poisson distribution, we have $P_r(m(A) = k) = \frac{e^{-\delta A}(\delta A)^k}{k!}$. Thus, the expected number of nodes lying inside a region with area A is

$$E(m(A)) = \sum k \cdot P_r(m(A) = k) = \sum_{k=1}^{\infty} \frac{e^{-\delta A}(\delta A)^k}{k!} k$$
$$= \delta A \sum_{k=1}^{\infty} \frac{e^{-\delta A}(\delta A)^{k-1}}{(k-1)!} = \delta A.$$

For a Poisson process with density n, let uv be the longest edge of the Euclidean minimum spanning tree, and $d_n = \|uv\|$. Then, the expected number of nodes that fall inside $\mathcal{D}(u, d_n)$ is $E(m(\pi d_n^2)) = n\pi d_n^2$, which is larger than $\log n$ almost surely, when n goes to infinity. That is to say, the expected maximum interference of Euclidean MST is $\Theta(\log n)$ for a set of nodes produced according to a Poisson point process. Consequently, the expected maximum node interference of containing MST is at least $\Omega(\log n)$. Thus, the expected maximum node interference of structure GG, RNG and Yao structures are also at least $\Omega(\log n)$. The above analysis shows that all commonly used structures for topology control in wireless ad hoc networks generally have a large maximum node interference even for *randomly* deployed nodes.

Our following analysis will show that the average interference of all nodes of these structures is small for a randomly deployed network.

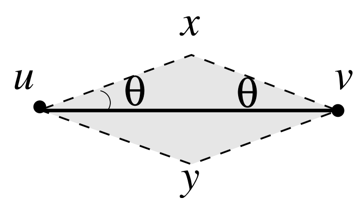


FIGURE 4
The diamond expanded from link uv.

Consider a set V of wireless nodes produced by Poisson point process. Given a structure G, let $I_G(u_i)$ be the node interference caused by a node u_i, i.e., the number of nodes inside the transmission region of node u_i. Here the transmission region of node u_i is a disk centered at u_i whose radius is the length r_i of the longest incident link of G at node u_i. Hence, the expected average node interference is

$$\begin{aligned} E(\frac{\sum_{i=1}^{n} I_G(u_i)}{n}) &= \frac{1}{n} E(\sum_{i=1}^{n} I_G(u_i)) = \frac{1}{n} \sum_{i=1}^{n} E(I_G(u_i)) \\ &= \frac{1}{n} \sum_{i=1}^{n} E(m(\pi r_i^2)) = \frac{1}{n} \sum_{i=1}^{n} (n\pi r_i^2) \\ &= \sum_{i=1}^{n} (\pi r_i^2) \leq 2 \sum_{e_i \in G} (\pi e_i^2). \end{aligned}$$

The last inequality follows from the fact that r_i is the length of some edge in G and each edge in G can be used by at most two nodes to define its radius r_i.

The *open diamond* subtended by a line segment uv, denoted by $D(uv, \theta)$, is the rhombus with sides of length $\|uv\|/(2\cos\theta)$, where $0 \leq \theta \leq \pi/3$ is a parameter. See Figure 4 for an illustration. It was proven in [12] that the diamonds defined with parameter $\theta = \pi/6$ by any two edges of the Euclidean minimum spanning tree do not overlap. In addition they proved that the total area of these diamonds defined by edges of EMST of a set of points inside a unit disk is at most $4\pi/3$. Let e_i, $1 \leq i \leq n-1$ be the length of all edges of the EMST of n points inside a unit disk. Consequently, they showed that $\sum_{e_i \in EMST} e_i^2 \leq 8\pi/\sqrt{3}$. They further improved this to $\sum_{e_i \in EMST} e_i^2 \leq 12$ using a more refined approach.

Thus, the expected average node interference of the structure EMST is

$$E(\frac{\sum_{i=1}^{n} I_{EMST}(u_i)}{n}) \leq 2 \sum_{e_i \in EMST} (\pi e_i^2) \leq 24\pi.$$

For RNG graph, similar to the proof of [12], we can show that $\sum_{e_i \in RNG} e_i^2 \leq 8\pi/\sqrt{3}$. This implies that

$$E(\frac{\sum_{i=1}^{n} I_{RNG}(u_i)}{n}) \leq 2 \sum_{e_i \in RNG} (\pi e_i^2) \leq 16\pi^2/\sqrt{3}.$$

We then summarize the above discussions by the following theorem.

Theorem 7 *For a set of nodes produced by a Poisson point process with density n, the expected maximum node interferences (thus link interferences) of EMST, GG, RNG and Yao structures are at least $\Theta(\log n)$ with high probability; the expected average node interferences (thus link interferences) of EMST and RNG are bounded from above by some constants with high probability.*

7 SIMULATION STUDIES

7.1 Simulation Environment

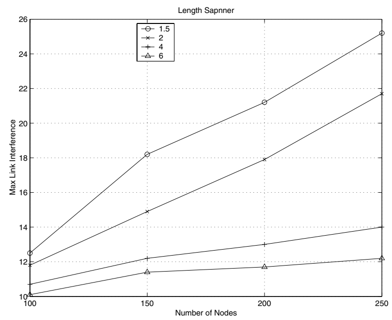
We conducted extensive simulations to study the performance of different models and approaches introduced in this paper. The network is modelled by unit disk graph. We put different number of nodes that are randomly placed in a 7×7 square and the maximum transmission range of each node is set to 1.

7.2 Link Based Interference

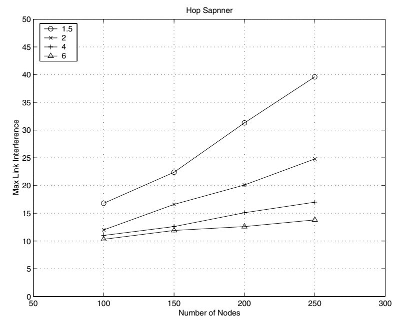
We first study the performance of the optimum structures when different spanning ratio requirement is posted. Our simulation results are plotted in Figure 5. A critical observation is that the maximum interference does increase with the increasing of network density as we showed theoretically.

7.3 Node based Interference

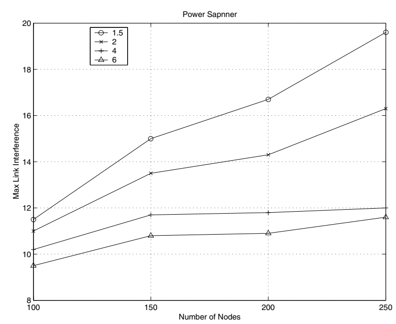
For the first model of the node based interference, minimizing the maximum node interference is equivalent to minimizing the the maximum link interference we studied in the previous subsection. In addition, we know that the minimum spanning tree of the link-weighted interference graph defined for link interference has an average node interference no more than 2 times of the optimum. Thus, we will concentrate our simulation studies in the second model of the node interference.



(a) length spanner



(b) hop spanner



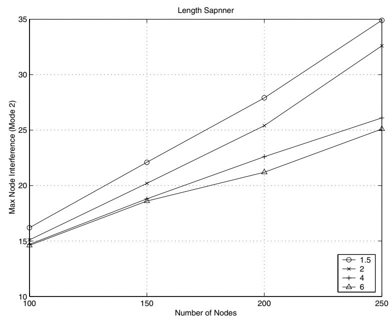
(c) power spanner

FIGURE 5
The maximum interference of the optimum structures with different length spanning ratio requirement (a), hop spanning ratio requirement (b), and power spanning ratio requirement (c).

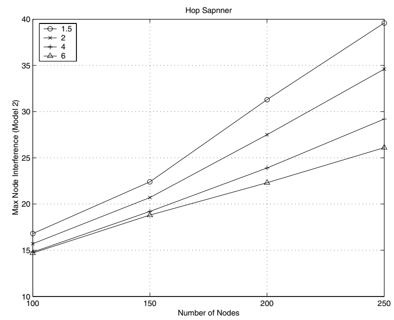
For node interference that only considers the number of nodes within the transmission range of a node, our experimental results are plotted in Figure 6.

7.4 Comparison of Structures

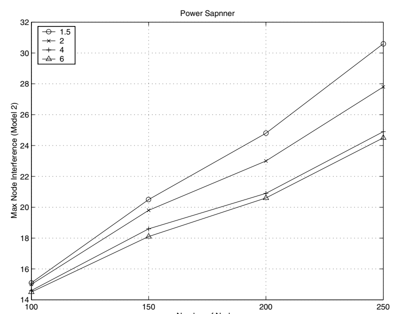
We also compared the performance of our centralized (almost) optimum connected structures that minimize the maximum link interference, or average node interferences with various locally constructed structures such as Gabriel graph, relative neighborhood graph and the local minimum spanning tree. Figure 7 illustrates the performance comparisons of various structures in terms of link interference and node interference. An observation is that although the localized structures are not optimum, their performances are comparable with the optimum solution, especially the local minimum spanning tree.



(a) length spanner



(b) hop spanner

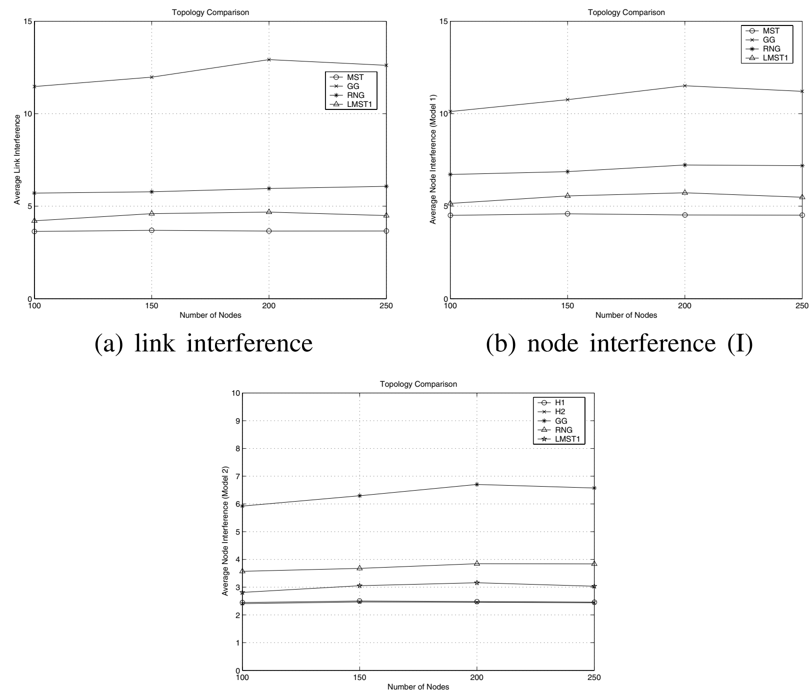


(c) power spanner

FIGURE 6
The maximum interference of the optimum structures with different length spanning ratio requirement (a), hop spanning ratio requirement (b), and power spanning ratio requirement (c).

8 CONCLUSION AND FUTURE WORK

Topology control draw considerable attentions recently in wireless ad hoc networks for energy conservation. In this paper, we studied various problems of topology control when we have to minimize the interference of the constructed structure. We optimally solved some problems, gave approximation algorithms for some NP-hard questions, and also gave some efficient heuristics for some questions that seem to be NP-hard. We conducted extensive simulations to see how these new structures perform for random wireless networks. We also theoretically showed that the most commonly used localized structures in the literature have large maximum interference even for random networks. On the other hand, we show that the local minimum spanning tree and the relative neighborhood graph have constant bounded average interference ratios for randomly deployed networks. As a future work, we would like to know whether our greedy



(a) link interference (b) node interference (I)

(c) node interference (II)

FIGURE 7
Comparison of various topologies in terms of the average link interference (a), the average node interference defined by the first model (b), and the average node interference defined by the second model (c).

heuristics for the min-average node interference gives a constant approximation guarantee.

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