# Space-Time User Benefit and Utility Accessibility Measures for Individual Activity Schedules

Olu Ashiru

John W. Polak (corresponding author)

Robert B. Noland

Centre for Transport Studies

Dept of Civil and Environmental Engineering

Imperial College of Science, Technology and Medicine

London, SW7 2BU

United Kingdom

Tel: +44-(0)20-7594-6089

Fax: +44-(0)20-7594-6102

Email: j.polak@ic.ac.uk

Word count: 6379 + 4 Figures

Submitted for *PRESENTATION* and *PUBLICATION* to the 82nd Annual Meeting of the Transportation Research Board.

**Abstract**

Accessibility is a fundamental concept in human existence, which goes to the heart of our notion of society, equity and justice. However despite the importance of the concept the mathematical measures which have historically been used to quantify accessibility levels, have been relatively poorly defined encompassing a limited range of observed forms of travel behavior. In this paper we extend existing space-time locational benefit measures to encapsulate more realistic temporal constraints on activity participation and the associated perceived user benefit. We go on to outline the development of a family of space-time route benefit measures, which despite their apparent theoretical attractiveness have hitherto not been utilized by researchers. We demonstrate how these route benefit measures can be utilized to develop an associated family of disaggregate activity based space-time utility accessibility measures, applicable to individual activity schedules, with the ability to incorporate income constraints. Finally, we sketch how stochastic frontier models in conjunction with existing travel/activity diary datasets can be utilized to operationalize the proposed measure of accessibility.

## INTRODUCTION

The concept of accessibility plays an important role in a number of existing theories of spatial and travel behavior. Moreover, transport and land-use planners have also made extensive use of various accessibility measures to assess alternative land-use-transport system policy measures. A wide variety of definitions of accessibility exist, ranging from those based on notions of reach or separation (e.g., 1-4) to those based on notions of activity participation (e.g., 5-7). Indeed, a number of authors have commented on the apparent lack of consensus on the meaning of the term accessibility (1,2,7,8). In this paper we propose that '…*accessibility is a measure of the overall utility that an individual derives from participating in one or more linked activities within an integrated land-use-transport environment…*'.

Despite the diversity of the interpretations of accessibility that can be found in the literature, there are some important common features that accessibility measures possess. In particular, the majority of the measures proposed to date have considered only a single purpose trip or activity, usually from the home to a non-home based destination, such as to a place of education, employment, shopping or leisure and often analyzed at the aggregate level masking individual variations in accessibility. As such these measures tend to ignore the potential for trip chaining or the sequential linking of activities which often forms part of an individual's daily activity program and which serve to increase an individual's overall level of accessibility. The potential significance of trip chaining in the assessment of accessibility has long been recognized in the literature (9,10,11) but to date little progress has been made towards developing the necessary theoretical or operational treatments. Consideration of trip chaining is particularly important in analyzing the impacts of public transport related policy measures.

The aim of this paper is to propose a more behaviorally realistic series of space-time user benefit measures and a related family of space-time utility accessibility measures. These space-time utility accessibility measures are disaggregate in nature and consequently applicable to individual activity schedules, and implicitly incorporate within the measure the utility of activity participation of all activities present within the activity schedule.

## SPACE-TIME USER BENEFIT MEASURES

In this section we provide a brief review on the definition and use of space-time user benefit measures and then proceed to extend these benefit measures to accommodate considerations of delay and waiting time, associated with the travel and activity participation components of behavior.

### Locational and Route Benefit Measures

Burns (5) utilized a space-time prism, depicted in figure 1, to propose two user benefit measures based upon an individual undertaking a discretionary activity constrained by upstream and downstream mandatory activities.

The locational benefit measure is defined in terms of properties of a particular location, such as the spatial, temporal or cost separation associated with travel between the mandatory-discretionary-mandatory activity locations, the maximum amount of time the individual can spend at the discretionary activity location (the stay time) and the attractiveness or value to the user of the particular discretionary activity location.

The locational benefit function defines the benefit ($BM_k$) to an individual of a location $k$ as a function of the spatial separation ($d_k$), the attractiveness of the opportunity ($a_k$) and the stay time ($T_k$) at the activity location. Typically, a multiplicative functional form has been assumed, so that the locational benefit is expressed as:

$$BM_k = \oplus g(d_k) a_k u(T_k) \qquad 1$$

Where ⊕ represents a binary operation such as addition or maximisation, representing how an individual derives benefit from the choice set as a whole.

The route benefit measure is defined in terms of properties of the spatial route and properties of the locations served by the route, such as the length of the route, the cumulative amount of time the individual can spend at all the locations served by the route (the total stay time) and the attractiveness or value to the user of the particular locations encountered along the route.

The route benefit ($BM_r$) to an individual of a route *r* is defined as a function of the total aggregate spatial separation of all relevant opportunities located along the route ($d_r$), the total aggregate attractiveness of all relevant opportunities located along the route ($a_r$) and the total aggregate stay time ($T_r$) at all relevant opportunities located along the route. This leads to the following functional form:

$$BM_r = \oplus g(d_r) a_r u(T_r) \quad 2$$

Burns' locational and route benefit measures essentially consider the benefit to the individual of being able to reach and stay at a particular discretionary activity location. Implicit in Burns' locational benefit measure and others derived from it (12,13,14), is the presumption that the ability of an individual to reach and remain at a discretionary activity location leads to useful participation in the discretionary activity. However, depending on the actual arrival time relative to the opening/closing time of the opportunity or the earliest/latest start times of the activity, not all the time spent at the destination may be productively spent in the desired activity. A simple example is arriving at a cinema early for a movie then waiting until the movie begins.

Moreover, the locational benefit measures utilized to date do not assess the utility to the individual of actual activity participation, but instead assume that being able to reach and remain at a potential activity location confers an element of utility to the individual. Further, no account is taken of both travel and non-travel related delays, such as time spent waiting for public transport and time spent waiting due to early arrival. These effects due to the scheduling of travel and activities will affect the total utility derived from a particular activity location.

## Incorporating Delay and Waiting Time

Burns' locational and route benefit measures can be modified to overcome these limitations by introducing the notions of route delay, facility wait time and activity wait time. This is done by first defining the following space-time prism, which represents an extension of the definition utilized by Kwan (15) and Kwan and Hong (16):

$$\mathrm{PPS}_{lS_1} = \left\{ (k_1, t_1) \middle| t_i + t_{ik_1} + D_{ik_1} + w_{ik_1} + W_{ik_1} \le t_1 \le t_j - t_{k_1 j} - D_{k_1 j} - w_{k_1 j} - W_{k_1 j} \right\} \quad 3$$

Where:

*PPS* Denotes the potential path space or space-time prism.
*l* Denotes the individual type/person under consideration.
$S_1$ An activity schedule containing only one flexible or discretionary activity.
($k_1$, $t_1$) Denotes all possible activity locations in space-time for undertaking the discretionary activity, situated within the space-time prism and consequently satisfying an individual's coupling constraints.
$t_1$ The start time of the single discretionary activity located at $k_1$ constrained by upstream and downstream coupling activities located at i and j respectively.
$t_i$ The latest end time of the upstream coupling/mandatory activity located at i.
$t_j$ The latest start time of the downstream coupling/mandatory activity located at j.
$t_{ik_1}$ The travel time associated with the minimum time routing between the upstream coupling activity location i and the discretionary activity location $k_1$ under consideration.
$t_{k_1 j}$ The travel time associated with the minimum time routing between the upstream discretionary activity location $k_1$ under consideration and the downstream coupling activity location j.
$D_{ik_1}$ Delay time encountered along the route between the upstream coupling activity location i and the downstream discretionary activity location $k_1$ under consideration,

which may include considerations of wait time penalties, modal interchange time penalties, parking and other non-travel related time spent in transit.

$D_{k_1j}$ Delay time encountered along the route between the upstream discretionary activity location $k_1$ under consideration and the downstream coupling activity location j.

$w_{ik_1}$ Wait time penalty incurred as a consequence of arrival at the discretionary activity location $k_1$ ahead of the scheduled opening times of the opportunity/facility.

$W_{ik_1}$ Wait time penalty incurred as a consequence of arrival at the discretionary activity location $k_1$ ahead of the earliest scheduled start time of the activity as defined within the activity schedule.

$w_{k_1j}$ Wait time penalty incurred as a consequence of arrival at the downstream mandatory activity location j ahead of the scheduled opening times of the opportunity/facility.

$W_{k_1j}$ Wait time penalty incurred as a consequence of arrival at the downstream mandatory activity location j ahead of the earliest scheduled start time of the activity as defined within the activity schedule.

Figure 2 depicts the shape of a space-time prism resulting from participation in a discretionary activity. The change in the shape and structure of the space-time prism as a consequence of the route delay, facility wait time and activity wait time encountered en-route between the constraining upstream and downstream mandatory activities are highlighted. The figure shows that as these variables are increased then the potential path space and the potential path area (the space-time and spatial regions available for discretionary activity participation) decreases. Figure 3 depicts the variation in total utility derived during the course of the day for a constrained discretionary activity. The two figures also depict the relative locations of the travel time, route delay, facility wait time and activity wait time of the discretionary activity and the mandatory activity locations.

If it is assumed that:

- The discretionary activity has an associated minimum activity duration or threshold below which the individual derives no utility.
- Useful activity participation time arises only within the context of one contiguous time block during which the facility/opportunity in question is open and available for use.
- Arrival outside of the formal opening times of an opportunity or outside of the formal activity start times results in the activity location under consideration being ignored with an implicit utility of zero being assigned to the activity location in question.

It thus follows that the space-time prism is defined by:

$$\mathrm{PPS}_{lS_1} = \{(\mathrm{k}_1, \mathrm{t}_1) \mid T_{k_1} \geq T_{1_{\min}}\} \quad 4$$

Where,

$$\mathrm{T}_{\mathrm{k}_1} = t_j - t_i - (t_{ik_1} + t_{k_1j}) - (D_{ik_1} + D_{k_1j}) - (w_{ik_1} + w_{k_1j}) - (W_{ik_1} + W_{k_1j}) \quad 5$$

$T_{1_{min}}$ Minimum discretionary activity duration or threshold required for the individual *l* to derive utility from participating in a single discretionary activity.

$T_{k_1}$ Maximum discretionary activity duration for an individual *l* participating in a single discretionary activity at location $k_1$.

If it is assumed that each discretionary activity has an earliest and latest start time, then it follows that for participation in a discretionary activity to occur, the following condition must be satisfied:

$$t_{E_1} - w_{ik_1} - W_{ik_1} \leq t_{ak_1} \leq t_{L_1} - w_{ik_1} - W_{ik_1} \quad 6$$

Where:

$t_{E_1}$ Earliest start time of the discretionary activity under consideration situated at location $k_1$.

$t_{Ej}$ Earliest start time of the downstream mandatory or coupling activity situated at location $j$.
$t_{L_1}$ Latest start time of the discretionary activity under consideration situated at location $k_1$.
$t_{Lj}$ Latest start time of the downstream mandatory or coupling activity situated at location $j$.
$t_{ak_1}$ Arrival time at the discretionary activity location under consideration situated at location $k_1$.
$t_{aj}$ Arrival time at the downstream mandatory activity location j.

Where:

$$W_{ik_1} = 0 \quad \text{if, } t_{ak_1} + w_{ik_1} \geq t_{E_1} \qquad 7$$

Else given that the late start temporal constraint is satisfied then,

$$W_{ik_1} = t_{E_1} - t_{ak_1} - w_{ik_1} \qquad 8$$

Where:

$$W_{k_1 j} = 0 \text{ if, } t_{aj} + w_{k_1 j} \geq t_{E_j} \qquad 9$$

Else given that the late start temporal constraint is satisfied then,

$$W_{k_1 j} = t_{E_j} - t_{aj} - w_{k_1 j} \qquad 10$$

It is further assumed that participation in an activity can only arise when that activity is open. When arrival is before the opening time of the facility then a wait time penalty is incurred. This wait time penalty effectively extends the opening time of the opportunity activity location by an amount dependent upon the wait time penalty (with no corresponding utility of activity participation, only a disutility of wait time).

If it is assumed that the discretionary activity location has a single opening session, it then follows that in order for activity participation to occur:

$$t_{o_1 k_1} - w_{ik_1} - W_{ik_1} \leq t_{ak_1} \leq t_{c_1 k_1} - w_{ik_1} - W_{ik_1} - T_{1_{min}} \qquad 11$$

Where:

$t_{o_1 k_1}$ The time at which the opportunity/facility situated at location $k_1$ commences or starts its opening session, where $t_{c_1 k_1} > t_{o_1 k_1}$.
$t_{c_1 k_1}$ The time at which the opportunity/facility situated at location $k_1$ ends or closes its opening session, where $t_{c_1 k_1} > t_{o_1 k_1}$.

Where:

$$w_{ik_1} = 0 \quad if,\ t_{ak_1} + W_{ik_1} \geq t_{o_1 k_1} \qquad 12$$

Else:

$$w_{ik_1} = t_{o_1 k_1} - t_{ak_1} - W_{ik_1} \qquad 13$$

Equations 3-13 represent the mathematical formulation of the space-time prism associated with participation in a single discretionary activity with associated route delay, facility wait and activity wait terms introduced.

Utilizing the following definitions:

$$\mathrm{d}_{k_1} = d_{ik_1} + d_{k_1 j} \qquad 14$$
$$\mathrm{D}_{k_1} = D_{ik_1} + D_{k_1 j} \qquad 15$$

$$\mathrm{w}_{\mathrm{k}_1} = w_{ik_1} + w_{k_1 j} \quad (16)$$

$$\mathrm{W}_{\mathrm{k}_1} = W_{ik_1} + W_{k_1 j} \quad (17)$$

$$\mathrm{t}_{\mathrm{k}_1} = t_{ik_1} + t_{k_1 j} \quad (18)$$

Utilizing a multiplicative user benefit, it is thus possible to derive the following generalised user benefit function:

$$BM_{PlS_1} = \sum_{k_1=1}^{m} q\left(t_{k_1}\right) g\left(D_{k_1}\right) h\left(w_{k_1}\right) v\left(W_{k_1}\right) z\left(a_{k_1}\right) u\left(T_{k_1}\right) \quad (19)$$

Where, *q*, *g*, *h*, *v*, *z* and *u* are functions respectively denoting how spatial/temporal separation, route delay, facility wait time, activity wait time, opportunity/activity attractiveness and activity participation time are individually perceived by the individual *l*. There are a range of curvilinear deterrence functions which can be utilized to reflect the disutility associated with increased, spatial/temporal/cost separation, route delay, facility wait time and activity wait time on activity participation and spatial interaction. These include the inverse/negative power function (3,4,15,17,22), the negative exponential function (3,5,8,12,13,14,15,17,18), the negative Gaussian (3,15) and the negative combined function. The effect on the utility of a general activity associated with each of the aforementioned types of deterrence functions is depicted graphically in figure 4.

The negative exponential deterrence type function, of the form used by Burns and others, is utilized in the following analysis to define the functions *q*, g, *h*, and *v*. This is principally due to the behavioral characteristics, the mathematical properties and the popularity of the negative exponential function in behavioral and spatial interaction choice modelling.

A positive power, positive exponential or positive combined function can be utilized for *z* and *u*, reflecting the utility associated with increasing activity participation time and attractiveness at the specific activity location in question. As with the negative exponential function the positive power function is selected principally due to the behavioral characteristics, the mathematical properties and the popularity of the function in behavioral and spatial interaction choice modelling.

It is worth noting that the techniques adopted in the development of the series of space-time user benefit measures and the associated family of space-time utility accessibility measures can equally be applied to negative power, negative Gaussian and negative combined deterrence functions associated with the disutility of spatial/temporal/cost separation, route delay, facility wait time and activity wait time. Equally the positive power function utilized to define the utility of activity participation time and opportunity/activity attractiveness, can in turn be replaced by a positive exponential or positive combined function without any loss of generality in the use of the techniques outlined in the following discussions.

Utilizing the negative exponential and power function model forms, it thus becomes possible to derive the following user benefit function:

$$BM_{PlS_1} = \sum_{k_1=1}^{m} \left(a_{k_1}^{\alpha_{l_1}}\right)\left(T_{k_1}^{\beta_{l_1}}\right) \exp\left[-\left(\lambda_{l_1} t_{k_1} + \mu_{l_1} D_{k_1} + \gamma_{l_1} w_{k_1} + \eta_{l_1} W_{k_1}\right)\right] \quad (20)$$

Where:

$a_{k_1}$ Spatial component of accessibility. A finite non-negative real number representing the relative attractiveness of the activity/opportunity location under consideration.

$t_{k_1}$ Transportation component of accessibility, reflecting the temporal or cost separation associated with travel between respective upstream and downstream coupling activities.

$T_{k_1}$ Temporal component of accessibility, reflecting the amount of time an individual can spend undertaking an activity at the location opportunity in question.

$D_{k_1}$ Route delay component of accessibility, reflecting the amount of non-travel time (interchange time, queuing time, parking time etc) spent in en-route between adjacent upstream and downstream coupling activities.

$w_{k_1}$ Facility wait component of accessibility, reflecting the amount of time spent waiting for an opportunity to open.

$W_{k_1}$ Activity wait component of accessibility, reflecting the amount of time spent waiting to commence an activity.

It can easily be shown that the improved six term locational benefit measure presented in equation 20 has a number of properties which are analogous of those associated with Burns' (5) three term user benefit function.

## AN IMPROVED SPACE-TIME UTILITY ACCESSIBILITY MEASURE

In this section we use the space-time locational benefit measure formulated above to derive a space-time route benefit measure and associated space-time utility accessibility measure. We develop three space-time utility accessibility measures applicable to an activity schedule composed of a single constrained discretionary activity. These three measures correspond to three approaches used for translating user benefit measures into accessibility measures, previously used by Miller (8), who reconciled three hitherto independent approaches to the assessment of accessibility, namely gravity/Hansen measures (4,15,18), user benefit/utility measures (5,9) and constraint based space-time measures (5,19,20,21), to define a series of accessibility measures which were internally consistent with Weibull's (22,23) axiomatic framework. Weibull's axiomatic framework details a robust and rigorous series of rules that an accessibility measure should satisfy in order to be both internally and externally consistent for the behavioral mechanism under consideration.

The user benefit translation mechanisms used to derive the accessibility benefit measures included:

- Consumer welfare or consumer surplus maximisation.
- Consumer welfare aggregation.
- Utility maximizing choice behavior implemented within a random utility framework.

### Consumer Welfare/Consumer Surplus Maximisation

We assume that Wilson's approach to the maximisation of total consumer welfare developed within an aggregate based spatial interaction framework (17) can be applied at an individual level. Under this assumption we can write:

$$a_k^{\alpha} = \exp(\alpha \ln a_k) \qquad 21$$

$$T_k^{\beta} = \exp(\beta \ln T_k) \qquad 22$$

It then follows that equation 20 can be expressed as:

$$BM_{PlS_1} = \sum_{k_1=1}^{m} \exp\left[\left(\alpha_{l_1} \ln a_{k_1} + \beta_{l_1} \ln T_{k_1}\right) - \left(\lambda_{l_1} t_{k_1} + \mu_{l_1} D_{k_1} + \gamma_{l_1} w_{k_1} + \eta_{l_1} W_{k_1}\right)\right] \qquad 23$$

Where:

$\alpha_{l_1}$ A parameter defining the marginal utility of the attractiveness of the opportunity/facility.

$\beta_{l_1}$ A parameter defining the marginal utility of activity participation time $dU(T_k)/dT_k$.
If $\beta_{l_1} < 1$ then the marginal utility of activity participation time, diminishes as $T_k$ increases.
If $\beta_{l_1} > 1$ then the marginal utility of activity participation time, increases as $T_k$ increases.

If $\beta_{l_1}$ = 0 then the marginal utility of activity participation time is zero. Conventional spatial accessibility measures, such as the gravity/Hansen and the cumulative opportunity measures, represent special cases.

$\lambda_{l_1}$ Spatial/temporal based travel impedance parameter for individual *l*, positive in magnitude, used to define the effect of increased spatial or temporal separation on the user benefit measure.

$\mu_{l_1}$ Route delay temporal impedance parameter for individual *l*, positive in magnitude, used to define the effect of increased non-transit delay on the user benefit measure.

$\gamma_{l_1}$ Facility temporal impedance parameter for individual *l*, positive in magnitude, used to define the effect of increased facility wait time on the user benefit measure.

$\eta_{l_1}$ Activity temporal impedance parameter for individual *l*, positive in magnitude, used to define the effect of increased activity wait time on the user benefit measure.

Rearranging and removing binary addition as the binary operation (the binary operation will be reintroduced later) and replacing the benefit measure annotation, BM, present within equation 23 with that of utility U, it can be shown that:

$$U_{PlS_1} = \exp\left[\left(\lambda_{l_1}\mu_{l_1}\gamma_{l_1}\eta_{l_1}\right)\left(\left(\frac{\alpha_{l_1}\ln a_{k_1} + \beta_{l_1}\ln T_{k_1}}{\lambda_{l_1}\mu_{l_1}\gamma_{l_1}\eta_{l_1}}\right) - \left(\frac{t_{k_1}}{\mu_{l_1}\gamma_{l_1}\eta_{l_1}} + \frac{D_{k_1}}{\lambda_{l_1}\gamma_{l_1}\eta_{l_1}} + \frac{w_{k_1}}{\lambda_{l_1}\mu_{l_1}\eta_{l_1}} + \frac{W_{k_1}}{\lambda_{l_1}\mu_{l_1}\gamma_{l_1}}\right)\right)\right] \qquad 24$$

This expression is analogous to Wilson's consumer welfare formulation, in which the term contained within the second closed parenthesis represents the utility or benefit derived from activity participation at the particular destination in question. The term contained within the third of the closed parentheses represents the disutility or disbenefit associated with travel time, delay time, facility wait time and activity wait time. The utility formulation presented in equation 24 together with the user benefit measures presented in equations 19, 20, and 23 are equal to zero if the attractiveness term $a_k$ or activity participation time $T_k$ are zero.

If an individual *l* is assumed to behave according to Wilson's (17) concept of maximisation of net interaction benefits or consumer welfare, it then follows that a space-time utility accessibility measure of the following form can be derived by introducing maximisation as the binary operation:

$$STUAM_{PlS_1} = \max_{k_1}\left[\exp\left[\left(\lambda_{l_1}\mu_{l_1}\gamma_{l_1}\eta_{l_1}\right)\left(\left(\frac{\alpha_{l_1}\ln a_{k_1} + \beta_{l_1}\ln T_{k_1}}{\lambda_{l_1}\mu_{l_1}\gamma_{l_1}\eta_{l_1}}\right) - \left(\frac{t_{k_1}}{\mu_{l_1}\gamma_{l_1}\eta_{l_1}} + \frac{D_{k_1}}{\lambda_{l_1}\gamma_{l_1}\eta_{l_1}} + \frac{w_{k_1}}{\lambda_{l_1}\mu_{l_1}\eta_{l_1}} + \frac{W_{k_1}}{\lambda_{l_1}\mu_{l_1}\gamma_{l_1}}\right)\right)\right]\right] \qquad 25$$

Equation 25 represents the maximum locational benefit that an individual *l* derives from the available choice set located within the space-time prism P, when undertaking an activity schedule $S_1$ composed of a single discretionary activity. This expression can be considered to be a maxitive accessibility measure.

## Consumer Welfare Aggregation

If it is assumed that the individual values a space-time prism according to the range of choice available, then a formulation of the space-time utility accessibility measure which naturally follows on from Wilson's (17) concept of net interaction benefits or consumer welfare is the sum or aggregation of the net locational benefits that are available to the individual within the space-time prism. It then follows that a space-time utility accessibility measure of the following form can be derived by reintroducing the binary operation of addition:

$$STUAM_{PlS_1} = \sum_{k_1=1}^{m}\exp\left[\left(\lambda_{l_1}\mu_{l_1}\gamma_{l_1}\eta_{l_1}\right)\left(\left(\frac{\alpha_{l_1}\ln a_{k_1} + \beta_{l_1}\ln T_{k_1}}{\lambda_{l_1}\mu_{l_1}\gamma_{l_1}\eta_{l_1}}\right) - \left(\frac{t_{k_1}}{\mu_{l_1}\gamma_{l_1}\eta_{l_1}} + \frac{D_{k_1}}{\lambda_{l_1}\gamma_{l_1}\eta_{l_1}} + \frac{w_{k_1}}{\lambda_{l_1}\mu_{l_1}\eta_{l_1}} + \frac{W_{k_1}}{\lambda_{l_1}\mu_{l_1}\gamma_{l_1}}\right)\right)\right]$$

26

This expression can be considered to be an additive standard attraction accessibility measure.

### Utility Maximizing Choice Behavior: Random Utility Framework

It can be assumed that an individual behaves in a manner that values a space-time prism according to the expected maximum utility of the opportunities located within the space-time prism (9). If the individual undertakes a discrete choice according to a random utility maximizing process (in which the random component is IID Gumbel distributed), it accordingly follows that a logsum space-time utility accessibility measure of the following form can be derived:

$$STUAM_{PlS_1} = \frac{1}{\lambda_{l_1}\mu_{l_1}\gamma_{l_1}\eta_{l_1}} \times \ln\sum_{k_1=1}^{m}\exp\left(\exp\left[\left(\lambda_{l_1}\mu_{l_1}\gamma_{l_1}\eta_{l_1}\right)\left(\left(\frac{\alpha_{l_1}\ln a_{k_1}+\beta_{l_1}\ln T_{k_1}}{\lambda_{l_1}\mu_{l_1}\gamma_{l_1}\eta_{l_1}}\right)-\left(\frac{t_{k_1}}{\mu_{l_1}\gamma_{l_1}\eta_{l_1}}+\frac{D_{k_1}}{\lambda_{l_1}\gamma_{l_1}\eta_{l_1}}+\frac{w_{k_1}}{\lambda_{l_1}\mu_{l_1}\eta_{l_1}}+\frac{W_{k_1}}{\lambda_{l_1}\mu_{l_1}\gamma_{l_1}}\right)\right)\right]\right) \quad 27$$

This expression can be considered to be a transform additive standard attraction measure. Equations 25-27 inclusive are analogous to the expressions derived by Miller (8,12) and have been derived from similar principles but the formulation has been extended to incorporate factors of non-travel delay and the actual time available for activity participation reflected in the facility wait time and activity wait time terms.

### Extension To Multi-Activity Based Activity Schedules

In the remainder of this section we demonstrate how a space-time route benefit measure can be developed incorporating one or more constrained discretionary activities and how this can be used to develop a series of space-time utility accessibility measures applicable to activity schedules. The space-time prism and user benefit/utility formulations presented in equations 3-5 and 23-24 are extended to the general case of an activity schedule consisting of two or more discretionary activities framed by upstream and downstream coupling/mandatory activities.

If an activity schedule $S_a$ is considered which includes $n$ discretionary activities then by considering the *a-th* discretionary activity, it can be shown that the associated space-time prism (potential path space) can be defined by the following expressions, in which the associated upstream and downstream activities are treated as temporally constraining activities:

$$\mathrm{PPS}_{lS_a} = \left\{(\mathrm{k_a},\mathrm{t_a})\middle|\,\mathrm{t_{a\text{-}1}} + T_{k_{a-1}} + t_{k_{a-1}k_a} + D_{k_{a-1}k_a} + w_{k_{a-1}k_a} + W_{k_{a-1}k_a} \le t_a \le t_{a+1} - t_{k_ak_{a+1}} - D_{k_ak_{a+1}} - w_{k_ak_{a+1}} - W_{k_ak_{a+1}}\right\} \quad 28$$

$$\mathrm{PPS}_{lS_a} = \left\{(\mathrm{k_a},\mathrm{t_a})\middle|\; T_{k_a} \ge T_{a_{\min}}\right\} \quad 29$$

Where,

$$\mathrm{T_{k_a}} = \left(t_{a+1} - t_{a-1} - \left(t_{k_{a-1}k_a} + t_{k_ak_{a+1}}\right) - \left(D_{k_{a-1}k_a} + D_{k_ak_{a+1}}\right) - \left(w_{k_{a-1}k_a} + w_{k_ak_{a+1}}\right) - \left(W_{k_{a-1}k_a} + W_{k_ak_{a+1}}\right) - \mathrm{T_{k_{a\text{-}1}}}\right) \quad 30$$

A generic macro level route benefit measure, representing the overall benefit to the individual of the route(s) located within the space-time prism(s), which satisfy the individual's entire activity schedule, is presented below.

$$BM_{RlS_{1...m}} = \oplus_1 \left[ f\left(t_{Sk_1}, D_{Sk_1}, w_{Sk_1}, W_{Sk_1}, a_{Sk_1}, T_{Sk_1}\right) \cdots \oplus_2 \cdots f\left(t_{Sk_a}, D_{Sk_a}, w_{Sk_a}, W_{Sk_a}, a_{Sk_a}, T_{Sk_a}\right) \cdots \oplus_2 \cdots f\left(t_{Sk_n}, D_{Sk_n}, w_{Sk_n}, W_{Sk_n}, a_{Sk_n}, T_{Sk_n}\right) \right] \quad 31$$

Where,

$\oplus_1$ The primary binary operation (e.g. addition or maximisation) representing the manner in which the meso level route benefit measures associated with a single complete activity chain, are combined to form the overall macro level route benefit measure.

$\oplus_2$ The secondary binary operation (e.g. addition or multiplication) representing the manner in which the individual micro level route benefit measures associated with a single activity within the activity chain, are combined to form the overall meso level route benefit measure associated with a single complete activity chain.

$k_a$ Denotes the identifier/location of the *a-th* activity present within the activity schedule.

$S$ The subscript denotes the number of different routes ($m$) available within the space-time prism, which satisfy the individual's principle coupling constraints.

Implementing the improved location benefit measure developed in equations 19 and 20 within the context of each individual activity within the activity chain, it then becomes possible to develop the following series of macro level route benefit measures:

$$BM_{RlS_{1...m}} = \oplus_1 \left[ q\left(t_{Sk_1}\right) g\left(D_{Sk_1}\right) h\left(w_{Sk_1}\right) v\left(W_{Sk_1}\right) z\left(a_{Sk_1}\right) u\left(T_{Sk_1}\right) \cdots \oplus_2 \cdots q\left(t_{Sk_a}\right) g\left(D_{Sk_a}\right) h\left(w_{Sk_a}\right) v\left(W_{Sk_a}\right) z\left(a_{Sk_a}\right) u\left(T_{Sk_a}\right) \cdots \oplus_2 \cdots q\left(t_{Sk_n}\right) g\left(D_{Sk_n}\right) h\left(w_{Sk_n}\right) v\left(W_{Sk_n}\right) z\left(a_{Sk_n}\right) u\left(T_{Sk_n}\right) \right] \quad 32$$

$$BM_{RlS_{1...m}} = \oplus_1 \left[ \left(a_{Sk_1}^{\alpha_{l_1}}\right)\left(T_{Sk_1}^{\beta_{l_1}}\right) exp\left[-\left(\lambda_{l_1} t_{Sk_1} + \mu_{l_1} D_{Sk_1} + \gamma_{l_1} w_{Sk_1} + \eta_{l_1} W_{Sk_1}\right)\right] \cdots \oplus_2 \cdots \left(a_{Sk_a}^{\alpha_{l_a}}\right)\left(T_{Sk_a}^{\beta_{l_a}}\right) exp\left[-\left(\lambda_{l_a} t_{Sk_a} + \mu_{l_a} D_{Sk_a} + \gamma_{l_a} w_{Sk_a} + \eta_{l_a} W_{Sk_a}\right)\right] \cdots \oplus_2 \cdots \left(a_{Sk_n}^{\alpha_{l_n}}\right)\left(T_{Sk_n}^{\beta_{l_n}}\right) exp\left[-\left(\lambda_{l_n} t_{Sk_n} + \mu_{l_n} D_{Sk_n} + \gamma_{l_n} w_{Sk_n} + \eta_{l_n} W_{Sk_n}\right)\right] \right] \quad 33$$

Where,

$a_{Sk_a}$ A scalar parameter, denoting the attractiveness of the location/opportunity in question.

$\alpha_{l_a}$ A parameter defining the marginal utility of the attractiveness of the activity undertaken at the opportunity/facility location for individual *l*.

$\beta_{l_a}$ A parameter defining the marginal utility of the activity participation time associated with the activity undertaken at the opportunity/facility location for individual *l*.

$\lambda_{l_a}$ Spatial/temporal based travel impedance parameter for individual *l*, positive in magnitude, used to define the effect of increased spatial or temporal separation on the user benefit measure associated with activity a.

$\mu_{l_a}$ Route delay temporal impedance parameter for individual *l*, positive in magnitude, used to define the effect of increased non-transit delay on the user benefit measure associated with activity a.

$\gamma_{l_a}$ Facility temporal impedance parameter for individual *l*, positive in magnitude, used to define the effect of increased facility wait time on the user benefit measure associated with activity a.

$\eta_{l_a}$ Activity temporal impedance parameter for individual *l*, positive in magnitude, used to define the effect of increased activity wait time on the user benefit measure associated with activity a.

Utilizing multiplication as the secondary binary operation it follows that equation 33 can be rearranged as:

$$BM_{RlS_1} = \oplus_1 \left(a_{Sk_1}^{\alpha_{l_1}} ...x...a_{Sk_a}^{\alpha_{l_a}} ...x...a_{Sk_n}^{\alpha_{l_n}}\right)\left(T_{Sk_1}^{\beta_{l_1}} ...x...T_{Sk_a}^{\beta_{l_a}} ...x...T_{Sk_n}^{\beta_{l_n}}\right)\left(\exp\left(-\left(\lambda_{l_1} t_{Sk_1} ...+...\lambda_{l_a} t_{Sk_a} ...+\lambda_{l_n} t_{Sk_n}\right)\right)\right)$$
$$\times\left(\exp\left(-\left(\mu_{l_1} D_{Sk_1} ...+...\mu_{l_a} D_{Sk_a} ...+...\mu_{l_n} D_{Sk_n}\right)\right)\right)\left(\exp\left(-\left(\gamma_{l_1} w_{Sk_1} ...+...\gamma_{l_a} w_{Sk_a} ...+...\gamma_{l_n} w_{Sk_n}\right)\right)\right) \quad 34$$
$$\times\left(\exp\left(-\left(\eta_{l_1} W_{Sk_1} ...+...\eta_{l_a} W_{Sk_a} ...+...\eta_{l_n} W_{Sk_n}\right)\right)\right)$$

Using the following definitions:

$$\alpha_{l_1} = \alpha_{l_a} = \alpha_{l_n} = \alpha_l \quad 35$$
$$\beta_{l_1} = \beta_{l_a} = \beta_{l_n} = \beta_l \quad 36$$
$$\lambda_{l_1} = \lambda_{l_a} = \lambda_{l_n} = \lambda_l \quad 37$$
$$\mu_{l_1} = \mu_{l_a} = \mu_{l_n} = \mu_l \quad 38$$
$$\gamma_{l_1} = \gamma_{l_a} = \gamma_{l_n} = \gamma_l \quad 39$$
$$\eta_{l_1} = \eta_{l_a} = \eta_{l_n} = \eta_l \quad 40$$

Introducing Wilson's (17) transformation of a power function outlined earlier in equations 21 and 22 it follows that equation 34 can be expressed as:

$$BM_{RlS_{1...m}} = \oplus_1 \left(\exp\left(\alpha_l \ln\left(a_{Sk_1} ...+...a_{Sk_a} ...+...a_{Sk_n}\right)\right)\right)\left(\exp\left(\beta_l \ln\left(T_{Sk_1} ...+...T_{Sk_a} ...+...T_{Sk_n}\right)\right)\right)$$
$$\times\left(\exp\left(-\lambda_l \left(t_{Sk_1} ...+...t_{Sk_a} ...+...t_{Sk_n}\right)\right)\right)\left(\exp\left(-\mu_l \left(D_{Sk_1} ...+...D_{Sk_a} ...+...D_{Sk_n}\right)\right)\right) \quad 41$$
$$\times\left(\exp\left(-\gamma_l \left(w_{Sk_1} ...+...w_{Sk_a} ...+...w_{Sk_n}\right)\right)\right)\left(\exp\left(-\eta_l \left(W_{Sk_1} ...+...W_{Sk_a} ...+...W_{Sk_n}\right)\right)\right)$$

Introducing the following definitions:

$$t_S = \sum_{a=1}^{n}\left(t_{Sk_a}\right) = t_{Sk_1} ...+...t_{Sk_a} ...+...t_{Sk_n} \quad 42$$
$$D_S = \sum_{a=1}^{n}\left(D_{Sk_a}\right) = D_{Sk_1} ...+...D_{Sk_a} ...+...D_{Sk_n} \quad 43$$
$$w_S = \sum_{a=1}^{n}\left(w_{Sk_a}\right) = w_{Sk_1} ...+...w_{Sk_a} ...+...w_{Sk_n} \quad 44$$
$$W_S = \sum_{a=1}^{n}\left(W_{Sk_a}\right) = W_{Sk_1} ...+...W_{Sk_a} ...+...W_{Sk_n} \quad 45$$
$$T_S = \sum_{a=1}^{n}\left(T_{Sk_a}\right) = T_{Sk_1} ...+...T_{Sk_a} ...+...T_{Sk_n} \quad 46$$
$$a_S = \sum_{a=1}^{n}\left(a_{Sk_a}\right) = a_{Sk_1} ...+...a_{Sk_a} ...+...a_{Sk_n} \quad 47$$

Where,

$t_S$ Total travel time encountered along the route S, reflecting the cumulative travel time required to reach all activity locations situated on the route S.

$D_S$ Total route delay encountered along the route S, reflecting the cumulative amount of non-travel time (interchange time, queuing time, parking time etc) spent in transit enroute between all activities situated on the route.

$w_S$ Total facility wait time encountered along the route S, reflecting the cumulative amount of time spent waiting for the schedule related opportunities to open.

$W_S$ Total activity wait time encountered along the route S, reflecting the cumulative amount of time spent waiting to commence schedule related activities.

$T_S$ Total activity participation time utilized along the route S, reflecting the cumulative amount of time an individual can spend in undertaking activities present within his/her schedule.

$a_S$ Total attractiveness of facilities utilized for activity participation along the route S.

It accordingly follows that equation 41 simplifies to:

$$BM_{RIS_{1...m}} = \oplus_1 \left[\exp\left(\left(\alpha_l \ln a_s + \beta_l \ln T_s\right) - \left(\lambda_l\, t_s + \mu_l\, D_s + \gamma_l\, w_s + \eta_l\, W_s\right)\right)\right] \qquad 48$$

Comparing the above expression with equation 23 and considering Burns' (5) generic locational and route benefit measures outlined earlier in equations 1 and 2 respectively, we observe that the above expression is a route benefit/route opportunity measure entirely consistent with Burns' proposition. This interesting result enables us to develop a series of space-time utility accessibility measures for activity schedules using the above family of route benefit measures.

Rearranging and replacing the benefit measure annotation BM with that of utility U and removing the primary binary operation notation $\oplus_1$ (the binary operation will as before be reintroduced in the subsequent discussions), it can thus be shown that equation 48 can be expressed as:

$$U_{RIS_{1...m}} = \exp\left[\left(\lambda_l\, \mu_l\, \gamma_l\, \eta_l\right)\left(\left(\frac{\alpha_l \ln a_s + \beta_l \ln T_s}{\lambda_l\, \mu_l\, \gamma_l\, \eta_l}\right) - \left(\frac{t_S}{\mu_l\, \gamma_l\, \eta_l} + \frac{D_{S_1}}{\lambda_l\, \gamma_l\, \eta_l} + \frac{w_S}{\lambda_l\, \mu_l\, \eta_l} + \frac{W_S}{\lambda_l\, \mu_l\, \gamma_l}\right)\right)\right] \qquad 49$$

Adopting similar principles to those outlined earlier in the section (namely consumer welfare maximisation, consumer welfare aggregation and utility maximizing choice behavior within a random utility framework) it accordingly becomes possible to develop the following activity schedule based space-time utility accessibility measures:

$$STUAM_{RIS_{1...m}} = \max_{S_{1...m}}\left[\exp\left[\left(\lambda_l\, \mu_l\, \gamma_l\, \eta_l\right)\left(\left(\frac{\alpha_l \ln a_s + \beta_l \ln T_s}{\lambda_l\, \mu_l\, \gamma_l\, \eta_l}\right) - \left(\frac{t_s}{\mu_l\, \gamma_l\, \eta_l} + \frac{D_s}{\lambda_l\, \gamma_l\, \eta_l} + \frac{w_s}{\lambda_l\, \mu_l\, \eta_l} + \frac{W_s}{\lambda_l\, \mu_l\, \gamma_l}\right)\right)\right]\right] \qquad 50$$

$$STUAM_{RIS_{1...m}} = \sum_{S=1}^{m}\left[\exp\left[\left(\lambda_l\, \mu_l\, \gamma_l\, \eta_l\right)\left(\left(\frac{\alpha_l \ln a_s + \beta_l \ln T_s}{\lambda_l\, \mu_l\, \gamma_l\, \eta_l}\right) - \left(\frac{t_s}{\mu_l\, \gamma_l\, \eta_l} + \frac{D_s}{\lambda_l\, \gamma_l\, \eta_l} + \frac{w_s}{\lambda_l\, \mu_l\, \eta_l} + \frac{W_s}{\lambda_l\, \mu_l\, \gamma_l}\right)\right)\right]\right] \qquad 51$$

$$STUAM_{RIS_{1...m}} = \frac{1}{\lambda_l\, \mu_l\, \gamma_l\, \eta_l} \times \ln \sum_{S=1}^{m} \exp\left(\exp\left[\left(\lambda_l\, \mu_l\, \gamma_l\, \eta_l\right)\left(\left(\frac{\alpha_l \ln a_s + \beta_l \ln T_s}{\lambda_l\, \mu_l\, \gamma_l\, \eta_l}\right) - \left(\frac{t_s}{\mu_l\, \gamma_l\, \eta_l} + \frac{D_s}{\lambda_l\, \gamma_l\, \eta_l} + \frac{w_s}{\lambda_l\, \mu_l\, \eta_l} + \frac{W_s}{\lambda_l\, \mu_l\, \gamma_l}\right)\right)\right]\right) \qquad 52$$

Use of the homogenous route benefit measure assumption inherent in equations 48 and 49 poses a number of interesting practical challenges. Principal amongst these is how best to define an attractiveness term that is transferable across different types of activity. While Burns (5) acknowledged that the *'…notion of attribute aggregation…'* was equally applicable to a single activity location as well as a series of distinct activity types, the challenges associated with attribute aggregation increase as the range of activities to which it is to be applied increases.

While it has been mathematically convenient to assume that the marginal utility of attractiveness and activity participation time, as well as the travel, route delay, facility and activity temporal impedance parameters are homogenous (i.e., they do not vary with activity or activity location) it is unlikely that these parameters will in fact remain constant.

The original route benefit measure presented in equation 33, with its heterogeneous attractiveness terms, marginal utility of attractiveness and activity participation time, in addition to heterogeneity in the travel, route delay, facility and activity temporal impedance parameters, can be utilized in this original form, to develop a similar series of space-time utility accessibility measures for individual activity schedules, addressing a number of the difficulties outlined in the foregoing discussions.

The use of binary addition as the second binary operation ($\oplus_2$) in place of multiplication has an advantage. In cases where it is not possible to undertake an activity (i.e. the activity participation time $T_k$ is zero) or the attractiveness of the activity location is zero, then a route benefit measure of the form outlined in equation 34 will generate a user benefit measure and an overall accessibility of zero. The corresponding binary addition based measures will also yield a user benefit of zero for the activity/activity location in question, but will allow the accessibility measure as a whole to be positive in magnitude due to the successful undertaking of other activities present within the activity schedule.

## Incorporating the Utility of Mandatory Activities

The homogeneous and heterogeneous formulations of the individual activity schedule space-time utility measure of accessibility outlined in equations 50-52 and in the proceeding discussions can be extended to incorporate the utility associated with participation in upstream and downstream mandatory activities. This allows the assessment of whether an individual is socially excluded by the transport/land-use system. In North American terminology this is equivalent to a lack of equity in how transportation decisions may affect various social or ethnic groups, alternatively known as "transportation justice". Research undertaken to date on space-time user benefit measures or space-time based utility measures of accessibility have essentially considered the user benefit associated with participation in a single discretionary activity undertaken between coupling or constraining mandatory activities. However, it can reasonably be argued that mandatory activities are by their nature more important to the individual than discretionary activities. It follows that participation in mandatory activities should form the basis of any robust assessment of the likely level of social exclusion (or transportation justice) arising from an individual's inability to undertake his/her activity schedule in its entirety.

## Space-Time Accessibility Measures and Financial Constraints

In the following discussion we outline how income factors can be incorporated within space-time utility accessibility measures. To date these measures have excluded consideration of financial constraints, which in practice serve to limit the range of land-use-transport options and associated activity schedules available to the individual.

Odoki (24) and Odoki *et al* (14) have to date been the only researchers to directly incorporate income aspects of the individual within Burns' broader space-time locational benefit measure. Odoki et al proposed the following form of spatial/temporal separation impedance/deterrence parameter:

$$\lambda = \omega\left(\frac{c}{\rho I} + \frac{1}{v}\right) \qquad 53$$

Which forms part of a negative exponential deterrence function of the form:

$$g(t) = \exp(-\lambda t) \qquad 54$$

Where:

- $c$ Monetary cost of travel per unit distance of travel.
- $I$ Income/monetary benefit or utility expected for the individual as a result of undertaking the activity undertaken downstream of the travel episode under consideration.
- $\rho$ Parameter which varies according to the activity/journey purpose, mode of travel, travel time and which decreases as a function of income$^{0.25}$ as proposed by Goodwin (25).
- $\rho I$ Denotes the value of travel time to the individual under consideration.
- $\omega$ Denotes the relative effort of the individual to travel using the transport mode in question ($\omega \geq 1$).
- $t$ Temporal separation between the origin and destination points.
- $v$ Average speed of travel for the transport mode in question between the origin and destination points.

Particular properties of the spatial/temporal separation impedance parameter $\lambda$ are that as individual income falls the velocity or speed of travel is less of a factor compared to the cost of travel which will dominate. In addition, as income rises the cost of travel is less of a factor with velocity or speed of travel being the determining factor. This is entirely consistent with the notion of diminishing marginal utility of income present within microeconomic consumer theory, in which the monetary cost of travel becomes less of a factor as income rises and conversely becomes more of a factor as income declines.

This form of approach can be easily implemented within the proposed family of space-time user benefit and space-time utility accessibility measures outlined earlier in which the travel time, route delay, the facility wait time and the activity wait time parameters, denoted by $\lambda$, $\mu$, $\gamma$ and $\eta$ respectively are essentially value of time parameters which vary with the individual's income and activity type. Low income groups will generally have lower travel time, route delay, facility wait time and activity wait time parameters than high income groups, reflecting the increased deterrent effect of non-activity time on spatial interaction and activity benefit as perceived by higher income groups. For lower income groups the travel time, route delay, facility wait time and activity wait time parameters will generally be lower for activities such as shopping and leisure which in general require the expenditure of income, in comparison to income generating activities such as employment/work which in the majority of instances does not involve any expenditure other than that incurred during travelling.

## Practical Implementation of Activity Chain Utility Measures of Accessibility

The family of space-time accessibility measures can be implemented with a little care using existing travel or activity diary data. In the majority of instances travel/activity diary datasets do not contain explicit information on the temporal timings of constraining mandatory activities defining the vertices of the space-time prism. It is, for example, possible to identify one vertex of the constraining space-time prism, such as the arrival/departure time at/from work or school, however the location of the corresponding upstream/downstream vertex is unidentifiable. Thus the determination of the time budget available for discretionary activity participation may be a subjective exercise. Stochastic frontier models utilized by Kitamura *et al* (26,27) and Pendyala *et al* (28) can be used to identify the relative location of the temporal vertices of the space-time prism. The technique of stochastic frontier models involves the use of observed trip start and end times as the dependent variables within the model, together with a series of socio-economic, demographic, individual and household attribute data utilized as the independent variables. The stochastic frontier model of prism vertices is used to identify the approximate temporal location of the unobserved frontier, namely the upstream or downstream vertex.

The practical implementation of space-time user benefit measures and utility accessibility measures is dependent upon deriving suitable estimates for the activity attractiveness, activity participation, travel time, route delay, facility wait and activity wait parameters. A future avenue of research will investigate the potential for estimating these parameters through the use of two techniques, stated preference techniques and the use of revealed preference techniques used in conjunction with travel/activity diary datasets.

It is anticipated that the use of revealed preference techniques will require that for each person type segment analysed (i.e. segmentation by individual type and income) that the travel/activity diary datasets contain detailed spatial and temporal information pertaining to the occurrence of all delays encountered en-route between adjacent activity locations as well as the spatial location of these activity locations. The principle difficulty envisaged with this approach is that traditional travel/activity diary datasets do not explicitly capture this information and while it is feasible to require respondents to specify both the instance in time and duration for which a delay was encountered, this places an additional burden on the respondent which is likely to reduce the number of travel/activity diaries successfully completed.

This coupled with the difficulty in obtaining spatial information on locations at which activities were actually performed in addition to those at which delays were encountered means that in the absence of the development of portable handheld electronic travel/activity diaries, facilitating the capture of this information, means that stated preference techniques will have to be applied to the population segments in question. This is quite appealing from a number of aspects. First, it avoids the need for capturing spatial and temporal locations of activities and delays occurring en-route between adjacent activities as in the former case. Second, the use of stated preference techniques to estimate the parameters means that in contrast to the revealed preference approach, the parameters not only reflect the individual's responses to existing land-use-transport constraints present within the local environment but also encapsulate aspects of how the individual's travel/activity behavior would vary in the absence of such constraints.

It is anticipated that the travel time and route delay parameters will vary by mode of transport as can the facility and activity wait time parameters. Similar variations are also anticipated in these parameters as well as the activity attractiveness and activity participation parameters as a consequence of relative change in the sequence in which particular activities are performed. Such variations are more easily captured through stated preference techniques than revealed preference approaches.

## SUMMARY AND CONCLUSIONS

The methodology outlined in this paper for determining the activity schedule based space-time utility accessibility of an individual, represents a considerable advance on existing measures of accessibility. These measures have in the main tended to consider single disjointed activity/trip episodes often analysed at an aggregate level and neglected the constraining effect of time and income on individual accessibility as well as the utility of activity participation.

In particular the incorporation of route delay, facility wait and activity wait temporal terms within the underlying locational and route benefit measures facilitates the determination of more robust and realistic space-time accessibility measures.

Researchers, due to the perceived complexity of the task, have hitherto not undertaken the development of space-time utility accessibility measures based upon an extension of Burns' route benefit measure to activity schedules. This is despite the behavioral attractiveness of the route benefit approach.

However despite the benefits of the proposed approach for determining individual space-time utility accessibility, there are a number of inherent assumptions and avenues for future research that can be pursued in relation to the proposed methodology:

- The technique while satisfying a number of axioms present within Weibull's axiomatic framework (22,23) also violates several axioms. While Weibull's axiomatic framework provides a useful mechanism for ensuring that accessibility measures are both internally and externally consistent, the framework does, as previously noted by Miller (8), exclude a number of observed forms of spatial choice behavior. Principally among these are activity schedules encompassing multi-stop travel and imperfectly informed and hierarchical information processing strategies. A fruitful area of future research could involve the extension of Weibull's axiomatic framework to encompass these forms of spatial choice behavior.
- The route benefit based logsum space-time utility accessibility measure presented in equation 52 is based upon a multinomial logit decision making process and consequently by the very nature of activity schedules and their relation to the land-use environment is likely to violate the axiom of independence from irrelevant alternatives (IIA). In addition the existence of non-linear income effects present within the travel time, route delay, facility wait and activity wait parameters as well as potentially being present within the activity attractiveness and activity participation parameters, violates the requirement identified by McFadden (29) for log-sum benefit measures to be a linear function of income if they are to be interpreted as a measure of Marshallian consumer surplus or Hickesian compensating variation. The mixed multinomial logit (30) and the more general GEV model, offer the potential for incorporating complex income effects within welfare measures without violating the theoretical assumptions underpinning the associated choice model. A worthwhile area of research is to apply the family of route benefit measures outlined herein to alternative choice mechanisms for instance generalisation of the random utility framework beyond IID (30) or the competing destinations choice model (31).
- Difficulty of the technique in its current form to trade off an early or late start/end time and duration changes of mandatory and discretionary activities.

## REFERENCES


1. Morris, J.M., Dumble, P.L. and Wigan, M.R. 'Accessibility indicators for transport planning', *Transportation Research*, **13A**(2), pp 91-109, 1979.
2. Vickerman, R.W. 'Accessibility, attraction and potential: A review of some concepts and their use in determining mobility', *Environment and Planning*, **6A**, pp 675-691, 1974.
3. Ingram, D.R. 'The concept of accessibility: A search for an operational form', *Regional Studies*, **5**, pp 101-107, 1971.
4. Hansen, W.G. 'How accessibility shapes land use', *Journal of The American Institute Of Planners*, **25**, pp 73-76, 1959.
5. Burns, L.D. 'Transportation, temporal and spatial components of accessibility', Lexington Books, D.C. Heath and Company, Lexington, 1979.
6. Pirie, G.H. 'Measuring accessibility: A review and proposal', *Environment and Planning*, **11A**, pp 299-312, 1978.

7. Jones, S.R. ‘Accessibility measures: A literature review’, TRRL Laboratory Report 967, 1981.
8. Miller, H.J. ‘Measuring space-time accessibility benefits within transportation networks: Basic theory and computational procedures’, *Geographical Analysis,* **31**, pp 187-212, 1999.
9. Ben-Akiva, M. and Lerman, S.R. ‘Disaggregate travel and mobility choice models and measures of accessibility’, In *Behavioral Travel Modelling*, (Eds) D. Hensher and P. Stopher, pp 654-679, 1979, Croom-Helm, London.
10. Hanson, S. ‘Spatial diversification and multi-purpose travel: Implications for choice theory’, *Geographical Analysis*, **12**, pp 245-257, 1980.
11. Kitamura, R., Nishii, K. and Goulais, K. ‘Trip chaining behavior by central city commuters: A causal analysis of time-space constraints’, In *Developments In Dynamic and Activity-Based Approaches To Travel Analysis*, (Eds) P.M. Jones, pp 145-170, Oxford Studies In Transport, Avebury, 1990, Aldershot.
12. Miller, H.J. ‘GIS software for measuring space-time accessibility in transportation planning and analysis’, A Paper Prepared For The International Workshop On Geographical Information Systems For Transportation and Intelligent Transportation Systems, Chinese University Of Hong Kong, Shatin, New Territory, Hong Kong, April 26-28, 1999.
13. Wu, Y, and Miller, H.J. ‘Computational tools for measuring space-time accessibility within dynamic flow transportation networks’, *Journal Of Transportation Statistics, Special Issue On Methodological Issues In Accessibility*, **4**, **2/3**, pp 1-14, 2001.
14. Odoki, J.B., Kerali, H.R. and Santorini, F. ‘An integrated model for quantifying accessibility benefits in developing countries’, *Transportation Research*, **35A**, pp 601-623, 2001.
15. Kwan, M.P. ‘Space-time and integral measures of individual accessibility: A comparative analysis using a point-based framework’, *Geographical Analysis*, **30**, pp 273-289, 1998.
16. Kwan, M.P. and Hong, X.D. ‘Network based constraints oriented choice set formulation using GIS’, *Geographical Systems*, **5**, pp 139-162, 1998.
17. Wilson, A.G. ‘Retailer’s profits and consumer’s welfare in a spatial interaction shopping model’, In I. Masser (Ed), *Theory and Practice In Regional Science*, Papers in Regional Science 6, pp 42-59, 1976, Pion.
18. Wilson, A.G. ‘A family of spatial interaction models & associated developments’, *Environment & Planning* **3**, pp 1-32, 1971.
19. Hägerstrand, T. ‘What about people in regional science ?’, *Papers Of The Regional Science Association*, **14**, pp 7-21, 1970.
20. Lenntorp, B. ‘A time-geographic study of movement possibilities of individuals’, *Lund Studies In Geography, Human Geography,* **44**, The Royal University Of Lund, 1976.
21. Miller, H.J. ‘Modelling accessibility using space-time prism concepts within geographical information systems’, *International Journal Of Geographical Systems*, **5**, pp 287-301, 1991.
22. Weibull, J.W. ‘An axiomatic approach to the measurement of accessibility’, *Regional Science and Urban Economics*, **6**, pp 357-379, 1976.
23. Weibull, J.W. ‘On the measurement of accessibility’, *Environment and Planning* **12A**, pp 53-67, 1980.
24. Odoki, J.B. ‘Accessibility benefits analysis as a tool for transportation planning in developing countries’, PhD Thesis, University Of Trieste, Polytechnic Of Milan, Italy, 1992.

25. Goodwin, P.B. ‘Value of time’, European Conference Of Ministers Of Transport, Paris, France, 1976.
26. Kitamura, R., Yamamoto, T., Kishizawa, K. and Pendyala, R.M. ‘Stochastic frontier models of prism vertices’, *Transportation Research Record* **1718**, pp 18-26, 2000.
27. Kitamura, R., Akiyama, T., Yamamoto, T. and Golob, T.F. ‘Accessibility in a metropolis: Toward a better understanding of land-use’, *Transportation Research Record* **1780**, pp 64-75, 2001.
28. Pendyala, R.M., Yamamoto, T., Kitamura, R. ‘On the formulation of time-space prisms to model constraints on personal activity-travel engagement’, *Transportation*, **29**(1), pp 73-94, 2002.
29. McFadden, D. ‘Measuring willingness-to-pay for transportation improvements’ In *Theoretical Foundations of Travel Choice Modelling,* (Eds) T. Garling, T. Aaititla and K. Westin, Elsevier Science, Amsterdam, 1998.
30. McFadden, D. and K. Train, ‘Mixed MNL models for discrete response’ *Journal of Applied Econometrics* **15**(5) 447-70, 2000.
31. Fotheringham, A.S. and O’Kelly, M.E. ‘Spatial interaction models: Formulations and applications’, *Studies In Operation Regional Science*, Kluwer Academic Publishers, 1989.

**Figure 1: Space-Time Prism For An Individual Constrained By Two Coupling Events**

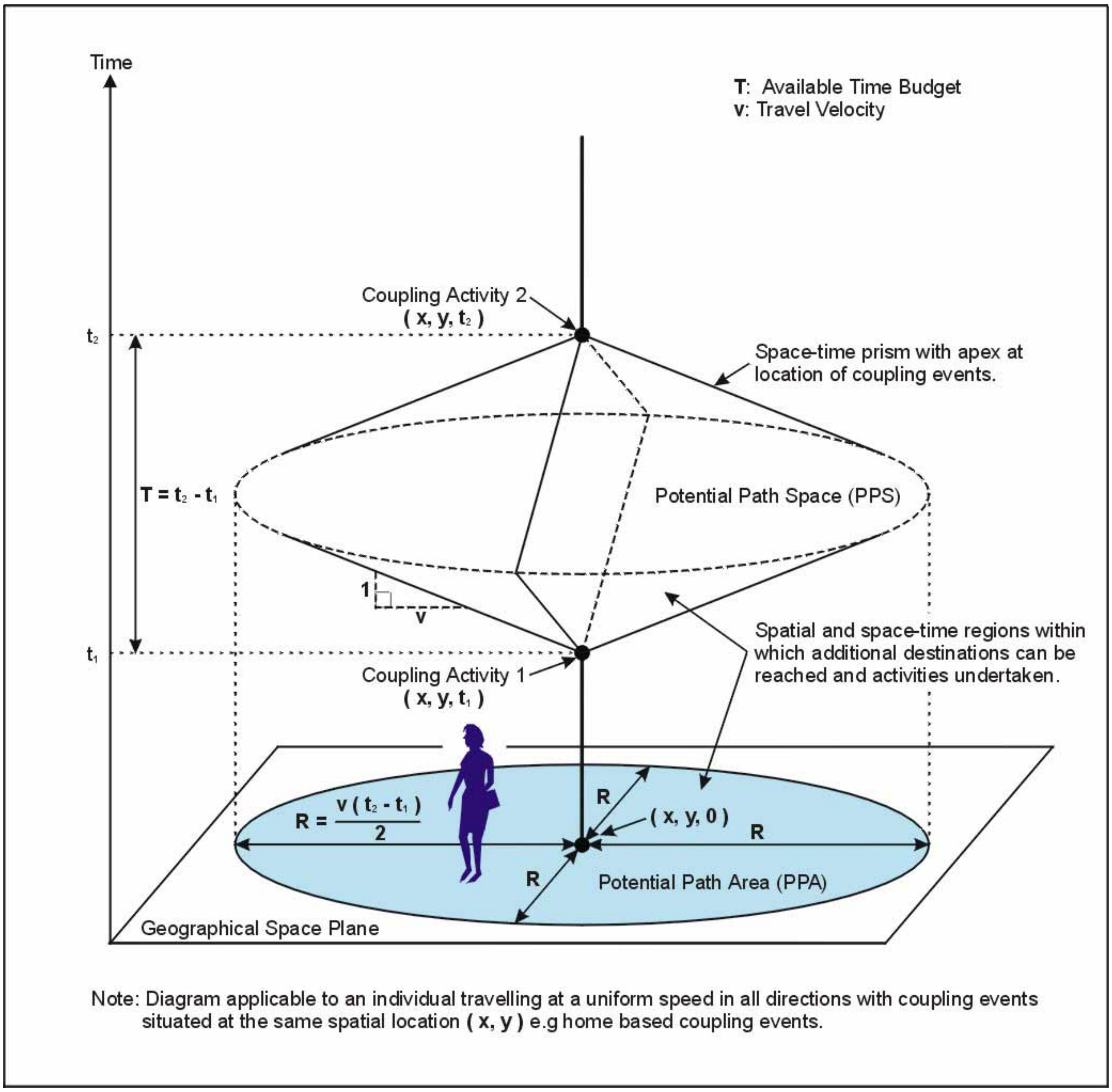
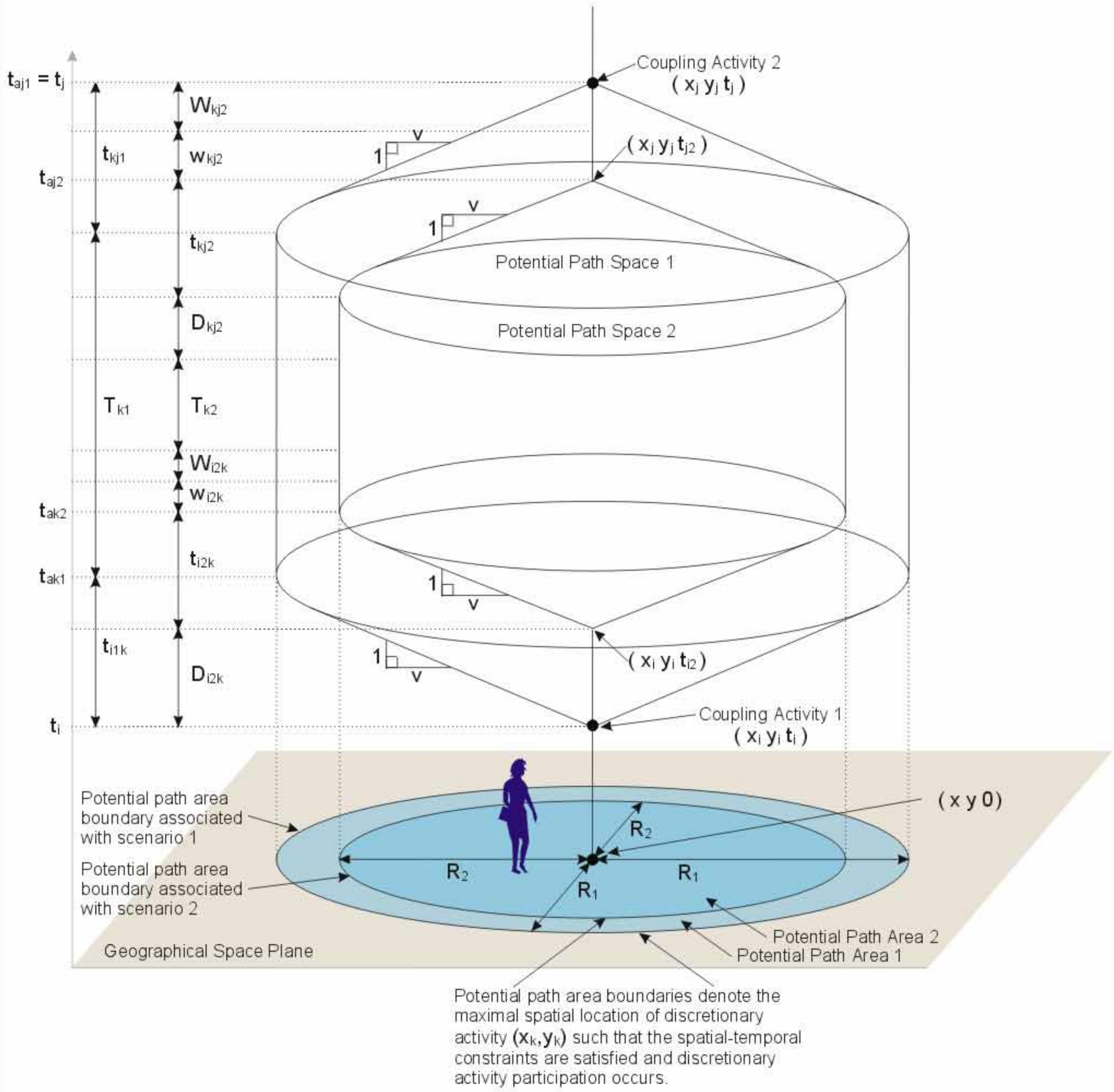


**Figure 2: Space-Time Prism For An Individual Undertaking A Discretionary Activity Constrained By Two Coupling Events**




Note: Diagram applicable to an individual travelling at a uniform speed in all directions who undertakes a discretionary activity of duration T, whilst satisfying his/her coupling constraints, which are situated at the same spatial location (x,y) e.g. a home or office based coupling event.

Where:

$$D_{i1k} = w_{i1k} = W_{i1k} = D_{kj1} = w_{kj1} = W_{kj1} = 0$$

$$T_{k1} > T_{k2} > T_{lmin}$$

$$R_1 = \frac{v\,(t_j - t_i - T_{k1})}{2}$$

$$R_2 = \frac{v\,(t_j - t_i - D_{i2k} - w_{i2k} - W_{i2k} - T_{k2} - D_{kj2} - w_{kj2} - W_{kj2})}{2}$$

**Figure 3: The Variation In Total Utility During The Course Of The Day For A Constrained Discretionary Activity**

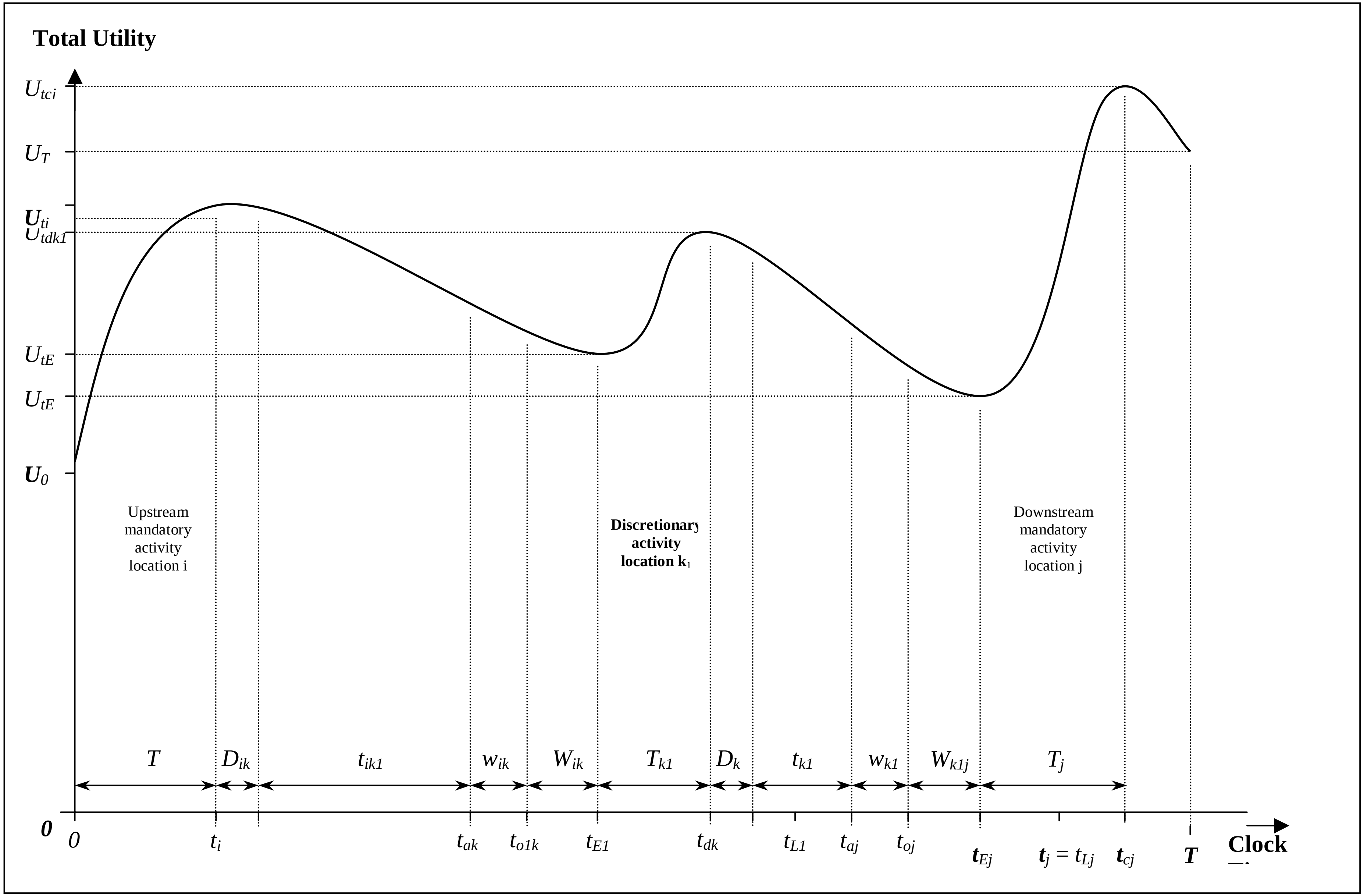


**Figure 4: The Effect Of A Variety Of Deterrence Functions On The Utility Of A General Activity**