THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
345 E. 47th St., New York, N.Y. 10017

96-GT-332

DYNAMIC MODELING OF SINGLE-SHAFT INDUSTRIAL GAS TURBINE

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ABSTRACT

In the paper the dynamic non-linear model of single shaft industrial gas turbine was developed as the first stage of a methodology aimed at the diagnosis of measurement and control sensors and gas turbine operating conditions.

The model was calibrated by means of reference steady-state condition data of a real industrial gas turbine and was used to simulate various machine transients.

The model is modular in structure and was carried out in simplified form, but not so as to compromise its accuracy, to reduce the calculation time and thus make it more suitable for on-line simulation.

The comparison between values of working parameters obtained by the simulations and measurements during some transients on the gas turbine in operation provided encouraging results.

NOMENCLATURE

A	Area
C	Torque
c_p	Specific heat at constant pressure
c_v	Specific heat at constant volume
D	Hydraulic diameter
F	Friction force
Fm	$= \frac{m\sqrt{T/T_{ref}}}{p/p_{ref}}$ Mass flow function
FN	$= N/\sqrt{T/T_{ref}}$ Rotational speed function
g	Gravity acceleration
h	Enthalpy
J_g	Moment of inertia of rotating masses connected to the gas turbine shaft reduced to the shaft speed
k	$= c_p/c_v$
L	Length
l	Linear coordinate
LHV	Lower Heating Value
m	Mass flow rate
N	Rotational speed
P	Power
p	Pressure
R	Gas constant
1/s	Integrator
T	Stagnation temperature
t	Time
V	Volume
v	Velocity
x	Variable
z	Altitude
β	Pressure Ratio
η	Efficiency
λ	Friction coefficient
ρ	Density

Presented at the International Gas Turbine and Aeroengine Congress & Exhibition
Birmingham, UK — June 10-13, 1996
This paper has been accepted for publication in the Transactions of the ASME
Discussion of it will be accepted at ASME Headquarters until September 30, 1996

Subscripts

o	initial steady-state condition value
c	compression, compressor
cc	combustor (combustion chamber)
e	expansion
el	electric
f	fuel
i	i-section/module
is	isentropic
ot	outlet turbine section
r	resisting
ref	reference value
t	turbine

Acronyms

C	Compressor
CC	Combustor (Combustion Chamber)
CM	Compressor Map
ED	Exhaust Duct
EG	Electric Generator
G	Gearbox
ID	Intake Duct
IGV	Inlet Guide Vanes
PID	Proportional-Integral-Derivative controller
T	Turbine
TM	Turbine Map

INTRODUCTION

The analysis of gas turbine transient conditions is a normal practice in the field of turbojet engines because they work in unsteady conditions during important operating situations (DeHoff and Hall, 1978; Szuch, 1978).

Recently, this kind of analysis is utilized more and more frequently on industrial gas turbines (Schobeiri, 1987; Krikelis and Papadakis, 1988; Blotenberg, 1993) also thanks to reduction of both calculation times required for dynamic simulations and costs of computer systems.

The implementation of industrial gas turbine dynamic models can make it possible:

- to predict machine transient conditions due to components of different kind, volume and time constant, reducing testing costs;
- to design gas turbine control system;
- to generate time series of transient condition data.

The last makes it possible to have a large number of data otherwise difficultly to be available for industrial gas turbines that work mainly in steady-state conditions.

The time series of transient condition data can be used to set-up simple I/O linear models of gas turbines by means of appropriate mathematical techniques (Ogata, 1987). These linear models, implemented on integrated circuits, are suitable to diagnose measurement and control sensors or to detect faults in gas turbine components (Merrington, 1989; Gertler and Singer, 1990; Gertler, 1991; Merrington et al., 1991) with low system costs. This process seems particularly suitable to develop low cost on-condition diagnostic systems that are therefore of interest also in the field of small/medium power size industrial gas turbines.

In the diagnostic applications, the machine dynamic model can also be used to simulate operating conditions of gas turbines with faults in components and/or measurement and control sensors, in order to set-up the diagnostic methodology.

In this paper, the dynamic non-linear model of single-shaft industrial gas turbine is developed as the first stage of a methodology aimed at the diagnosis of measurement and control sensors and gas turbine operating conditions.

BASIC MODELING ASSUMPTION

The dynamic non-linear gas turbine model was developed, similar to the previous authors' work for static modeling (Benvenuti et al., 1993), by splitting the machine into elementary modules corresponding to its main components, and carrying out a dynamic model for each of them. The overall representation of a specific gas turbine is carried out by identifying the necessary modules and connecting them appropriately by means of thermodynamic and mechanic links.

The dynamic behavior of each module is described by means of equations representing the thermodynamic transformations, the mass and momentum balance.

The mass and momentum balance equations are used in differential unsteady one-dimensional form, in the hypothesis of assimilating each block to a constant section duct (Blotenberg, 1993). Within these hypotheses, the above equations take on the following forms, respectively (Shapiro, 1954):

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial l} = 0 \qquad \text{mass balance} \qquad (1)$$

$$\rho\frac{\partial v}{\partial t}+\rho v\frac{\partial v}{\partial l}=-\left(\frac{\partial p}{\partial l}+F+\rho g\frac{dz}{dl}\right) \quad \text{momentum balance} \quad (2)$$

Equations (1) and (2) are integrated considering that the change in fluid density takes place according to an isentropic transformation.

The equations representing the thermodynamic transformations are used instead in stationary form, since the fluid thermal inertia is considered negligible in comparison to the mechanical inertia.

From a thermodynamic standpoint the fluid is considered as a perfect gas and in each modules are used mean values of specific heat at constant pressure and at constant volume depending on the temperature between module input and output and on the fluid composition. The use of mean specific heats in dynamic simulations, where changes in thermodynamic and performance values are analyzed with reference to an initial steady-state condition, does not significantly affect the result accuracy, but considerably reduces both the model complexity and the calculation time (Blotenberg, 1993).

The mass flow rates bled on the compressor are calculated without considering the dynamic effects on them in transient conditions, and considering that the mass flow function of air bled at the outlet of each elementary compressor module is constant in all operating conditions (Benvenuti et al., 1993).

The effect on thermodynamic cycle of turbine nozzle and blade row cooling flows, calculated starting from the mass flow rates bled on the compressor, is assessed by splitting the total cooling flow appropriately into two parts and assuming that one is mixed upstream and the other downstream from the turbine module, causing a reduction of the main flow total temperature and then, a reduction of the available enthalpy drop (Benvenuti et al., 1993).

In addition to the equations describing the various modules, equations are used that represent the dynamic balance of shafts and rotating masses of the machine connected to them.

The simulation of the gas turbine working was carried out by integrating the differential equations and solving the static equations with the variable values calculated at each time.

Since the intention was to limit calculation and system costs in the subsequent diagnostic stage as well, the dynamic model must be able to be used on a PC and solved using commercially available software. For this reason, to integrate the differential equations was used SIMULINK (MathWorks, 1991) of MATLAB (MathWorks, 1990), which is a flexible and widespread software.

COMPONENT MODELING

The splitting of the gas turbine into elementary modules makes it possible to model machines of any configuration, linking the various modules appropriately.

The description of how the main modules were modeled is found below.

__Duct.__ This term refers to the modules of the gas turbine in which no thermodynamic transformations occur, but only a mass flow. In order to carry out dynamic simulations theirs modeling is especially important because the "duct" modules are the site of inertial phenomena due to the mass of fluid they contain. These phenomena are described by means of mass and momentum balance equations (1) and (2).

In particular, these equations take on the form (Isermann, 1984; Blotenberg, 1993):

$$\frac{dp_i}{dt}=\frac{kRT_i}{V_i}\left(m_i-m_{(i+1)}\right) \quad \text{mass balance} \quad (3)$$

$$\frac{dm_i}{dt}=\frac{A_i}{L_i}\left(p_{(i-1)}-p_i\right)-\frac{\lambda_i kRT_{(i-1)}m_i^2}{A_iD_i\left(p_{(i-1)}+p_i\right)}$$

momentum balance (4)

in the hypotheses that the duct, whatever its geometry, may be assimilated with a constant section pipe and the change in fluid density takes place according to an isentropic transformation.

In the case of intake duct, the integration of (3) and (4) make it possible to calculate the outlet pressure and air mass flow rate for given input conditions and duct geometry.

In the case of exhaust duct, where the outlet pressure as well as the input conditions are known, it is sufficient to integrate equation (4) alone in order to calculate the outlet gas mass flow rate.

As an example, fig. 1 shows the model of "intake duct" (indicated with ID in fig. 3) using SIMULINK blocks. Note that the equations (3) and (4) (in which i=1) were solved as an analog computer was used, using the SIMULINK blocks and transport signal lines in place of the computer components and physical links among the various components, respectively.

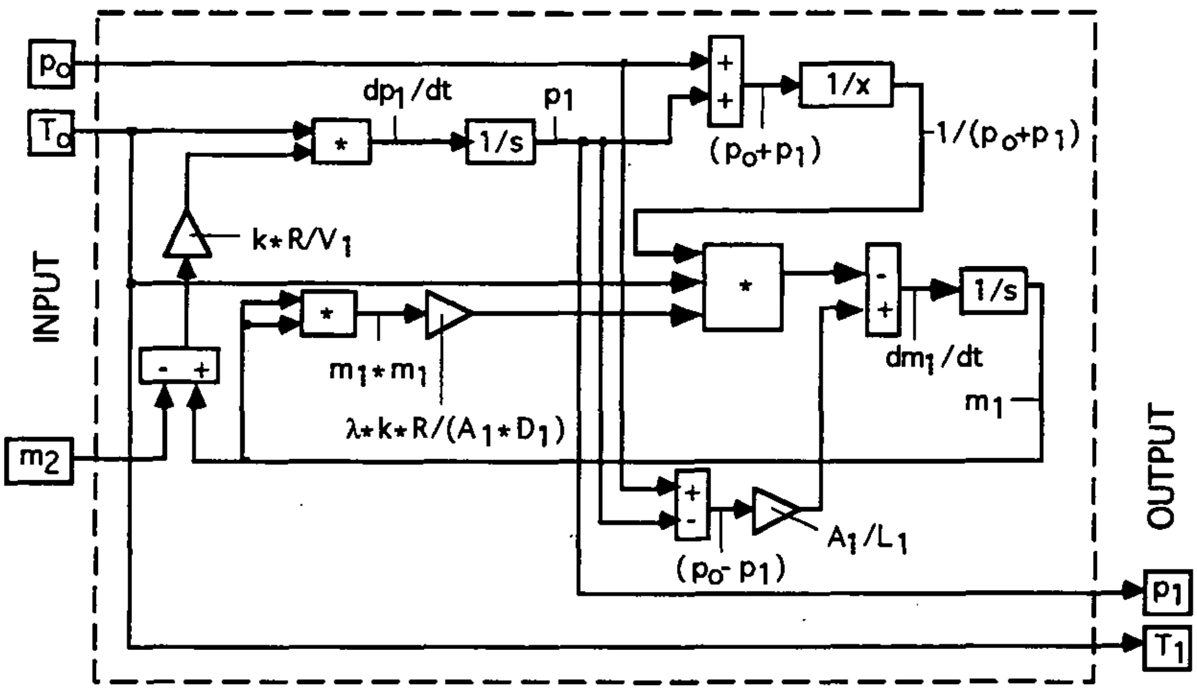

Fig. 1: Model of intake duct (ID) using SIMULINK blocks.

Compressor. The elementary "compressor" module is represented by the portion of compressor between two air bleed points. The mass flow rate that passes through the module and the corresponding isentropic compression efficiency are determined using the performance maps of the particular compressor, when the pressure ratio, rotational speed function, and angle of variable compressor IGV, if they exist, are known.

Equations (5) and (6) make it possible to calculate, respectively, outlet compressor temperature and compression power, which provides shaft torque if rotational speed is known:

$$T_i = T_{(i-1)} + T_{(i-1)}\left[\left(\frac{p_i}{p_{(i-1)}}\right)^{\frac{k-1}{k}} - 1\right]\frac{1}{\eta_{isc}} \qquad (5)$$

$$P_c = m_i c_p T_{(i-1)}\left[\left(\frac{p_i}{p_{(i-1)}}\right)^{\frac{k-1}{k}} - 1\right]\frac{1}{\eta_{isc}} \qquad (6)$$

The outlet compressor pressure is determined by integrating the mass balance equation (3) written for the compressor, where "V" represents the volume of fluid contained in the compressor and in the downstream diffuser.

Combustor. The fluid dynamic modeling of "combustor" is carried out, as already seen for the other modules, using the mass and momentum balance equations (3) and (4) which, when integrated, make it possible to calculate the pressure and gas mass flow rate at the combustor outlet for given input conditions and geometry.

The gas temperature at the combustor outlet is calculated using the following balance equation, in the hypothesis that the combustion and release of heat are instantaneous, since the thermal inertia has been neglected with respect to the mechanical inertia:

$$T_i = \frac{m_{(i-1)}c_p T_{(i-1)} + (LHV\,\eta_{cc} + h_f)m_f}{m_i c_p} = \approx T_{(i-1)} + \frac{(LHV\,\eta_{cc})m_f}{m_i c_p} \qquad (7)$$

Turbine. In the elementary "turbine" module, the expansion is assumed to be adiabatic and with no variation in the gas mass flow rate. Mixing between the main flow and cooling flows are therefore concentrated upstream and downstream from the module.

The expansion isentropic efficiency is determined using the performance map of the particular turbine when the expansion pressure ratio and rotational speed function are known.

To calculate the gas mass flow rate through the turbine, it was deemed sufficiently approximate to consider the mass flow function at the turbine inlet to be constant in all operating conditions. This assumption is realistic since the transient model is used to simulate working conditions, without considering machine start-up and shut-down.

Similar to the compressor, equations (8) and (9) make it possible to calculate turbine exhaust temperature and power which provides shaft torque if rotational speed is known:

$$T_i = T_{(i-1)} - T_{(i-1)}\eta_{ise}\left[1-\left(\frac{p_i}{p_{(i-1)}}\right)^{\frac{k-1}{k}}\right] \quad (8)$$

$$P_t = m_i c_p T_{(i-1)}\eta_{ise}\left[1-\left(\frac{p_i}{p_{(i-1)}}\right)^{\frac{k-1}{k}}\right] \quad (9)$$

The integration of mass balance equation (3) written for the turbine makes it possible to calculate machine outlet pressure.

GLOBAL GAS TURBINE MODEL

Once the elementary module models have been set-up, the overall model of the particular gas turbine was obtained by:

- appropriately linking the modules of which it is composed;
- carrying out the control logic;
- providing the values of constants that are in the various equations.

Block diagram

In this work the model for simulating a single-shaft industrial gas turbine, with variable compressor IGV angle and first turbine nozzle cooled alone, working in parallel with electric mains was carried out. Fig. 2 shows the schematic lay-out and fig. 3 the simplified block diagram of the machine. These highlight boundary and control inputs and output variables, the compressor and turbine maps, and direct and feedback main links among the various modules.

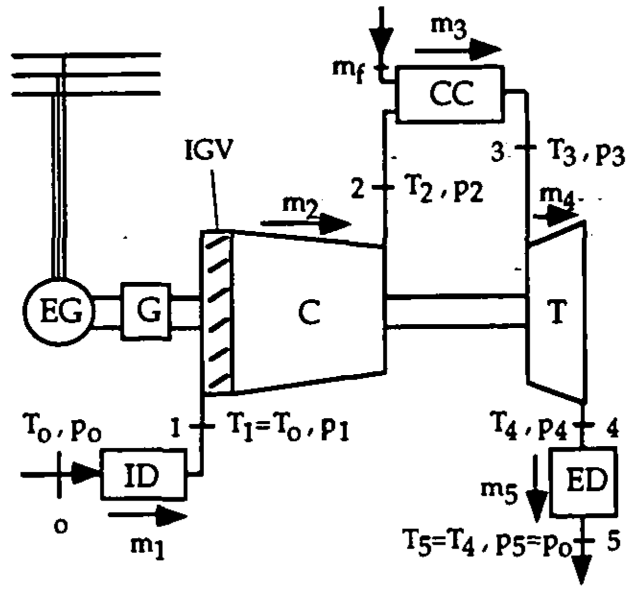

Fig. 2: Lay-out of the single-shaft gas turbine.

Control logic

The machine load adjustment is performed by means of fuel flow rate control and varying the IGV angle with the logic of keeping the turbine outlet temperature constant. This logic is especially suited for optimum heat recover steam generator operation in cogenerative applications.

To simulate this type of load control (by adjusting the IGV angle to keep the turbine outlet temperature constant) it was considered that the IGV angle at each time is obtained using a feedback "PID" controller on turbine outlet temperature, as shown in fig. 3.

Since the need was to simulate operation of a single-shaft gas turbine in parallel with electrical mains, it was not necessary to create a model of the rotational speed controller. In this case, the torque offered by the electric generator to the gas turbine adapts almost instantaneously to the torque delivered by the machine, thereby keeping the gas turbine rotational speed constant and equal to the synchronism speed. Therefore the equation expressing the dynamic balance of rotating masses connected to the shaft:

$$J_g \frac{2\pi}{60}\frac{dN}{dt} = C_t - C_c - C_r \quad (10)$$

becomes static and makes it possible to calculate the delivered torque C_r and thus the electrical power produced.

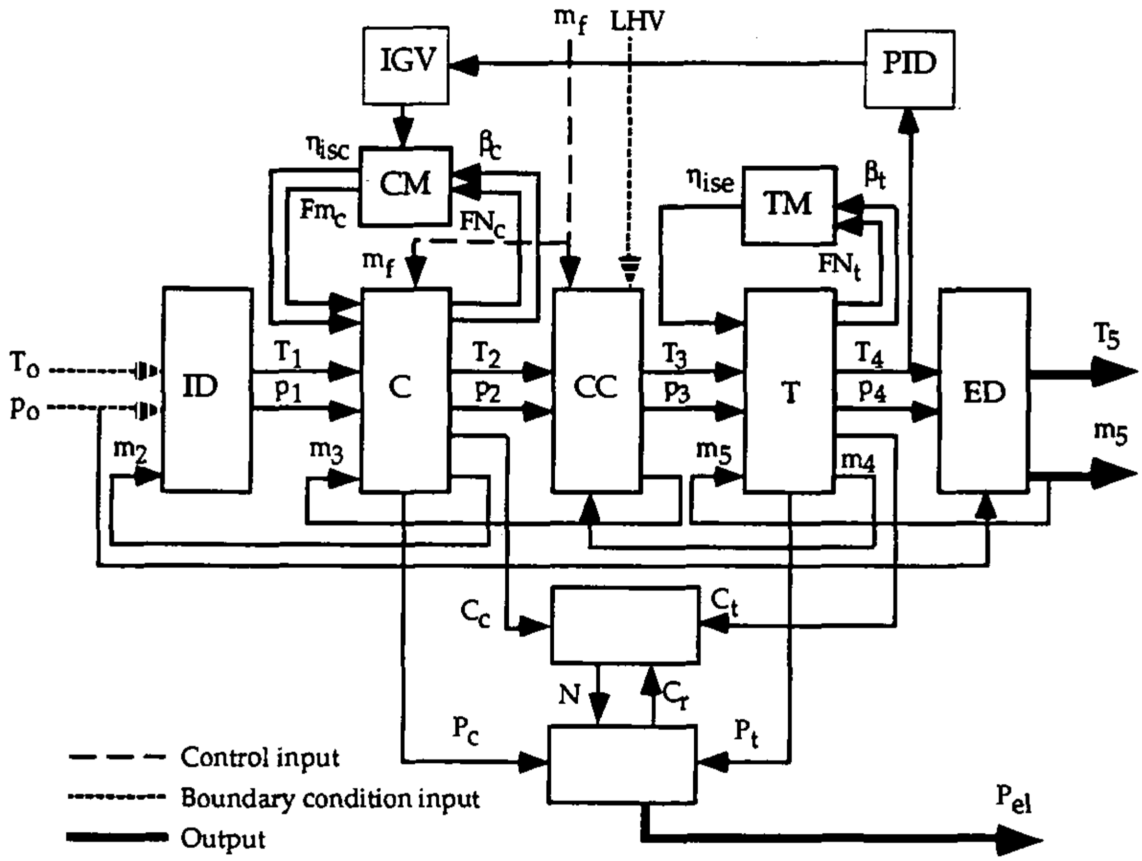

Fig. 3: Block diagram of the single-shaft gas turbine.

Simulation parameter values

In order to complete the overall gas turbine model, it is necessary to provide the characteristic constants of the particular machine which are in the various equations.

The constants may be classified as:

- geometrics, such as characteristic volumes, areas and lengths;
- thermodynamics and fluid-dynamics, mainly represented by the mean specific heats at constant pressure and at constant volume, and by the friction coefficients of ducts.

Before simulation, these may be read from a startup data file and processed to calculate the constants that are in the model equations.

In addition to the mentioned constants, at the start of the simulation it is necessary to know all values in the initial steady state condition. These initial values may, for example, be calculated by a stationary program that uses the same equations of the dynamic program, and which makes it possible to calculate the cycle in the initial steady state condition.

In place of this solution, which basically requires the use in series of two programs, one static and one dynamic, it was preferred to provide the dynamic program with the initial values of a particular reference operating condition as constants. If the reference operating condition is different from the one in which the simulation must start, it is possible to go in the steady-state condition relative to the desired boundary conditions by means of an initial adjustment transient.

For this reason the model may accept as input, in addition to the control variable represented by the fuel flow rate, the variables representing the boundary conditions, such as ambient pressure and temperature and fuel Lower Heating Value.

SIMULATION AND COMPARISON WITH EXPERIMENTAL DATA

In order to assess the validity of the dynamic model, it was decided to compare the results obtained from the simulation of transient conditions with measurements taken on a gas turbine working in a cogeneration plant.

Load reductions on the machine in operation were carried out by the control system in two ways:

- reducing the fuel flow rate and closing the IGV to keep the turbine outlet temperature constant;
- reducing the fuel flow rate alone, after that the IGV reached the total closure position.

In both cases the electrical power, fuel flow rate and turbine outlet temperature during the two transients were recorded. The measurements for the two load reduction operations are shown in figures 4 and 5 respectively, all values referred to the initial steady-state condition.

In order to correctly simulate the load adjustment transient caused by fuel flow rate reduction and IGV closing, the control system characteristics, as delays and "PID" constants, must be known.

In the case examined, as the control system characteristics were not known, they were determined in order to reproduce, during the simulation, the electrical power and turbine outlet temperature curves experimentally recorded, using as input the best reproduction of fuel flow rate curve shown in fig. 4.

In this way, once delays and "PID" constants were determined, the simulation provides the electrical power, fuel flow rate and turbine outlet temperature curves shown in fig. 6 in comparison to the points obtained by sampling, with regular time intervals, the curves in fig. 4. The agreement between the above curves on the one hand shows how the determined characteristics of the control system make it possible to reproduce its real behavior, and on the other provide a first confirmation of the validity of the dynamic model developed.

The simulation of load reduction transient due to the only reduction of the fuel flow rate provides the electrical power, fuel flow rate and turbine outlet temperature curves shown in fig. 7, in comparison to recorded data. As input the best reproduction of fuel flow rate curve shown in fig. 5 was used. The agreement between the above curves provides an additional confirmation of the model validity, since in this case the curves achieved by the simulation are not in any way affected by unknown features of the control system, but depend only on the model adopted.

In both simulations, the transient end values obtained for the various variables differ only negligibly from those measured.

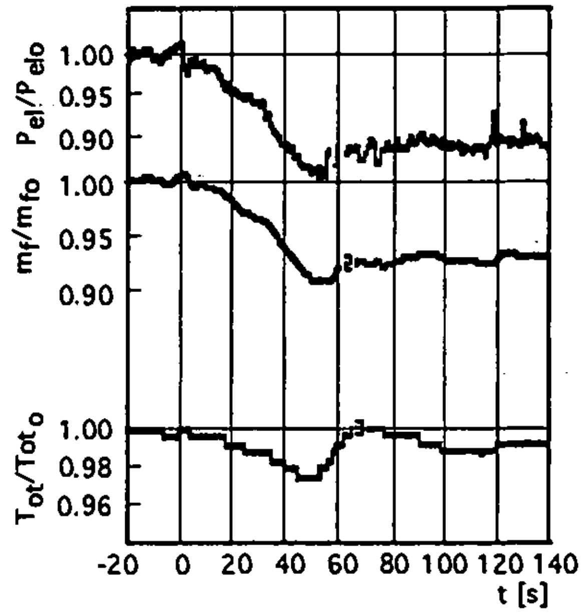

Fig. 4: Electrical power, fuel flow rate and turbine outlet temperature measured in the case of load reduction performed reducing the fuel flow rate and closing the IGV.

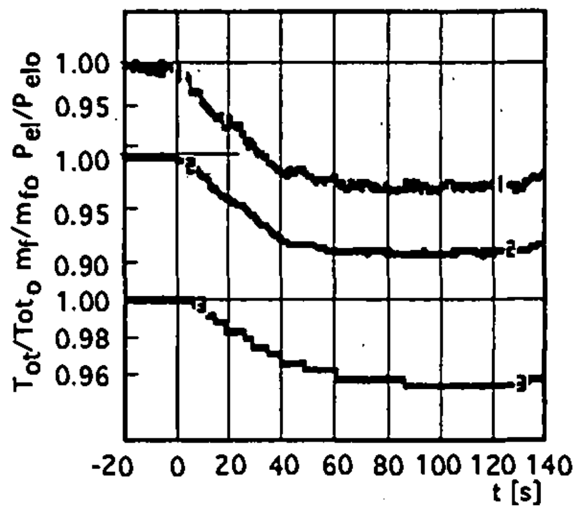

Fig. 5: Electrical power, fuel flow rate and turbine outlet temperature measured in the case of load reduction performed reducing the fuel flow rate alone.

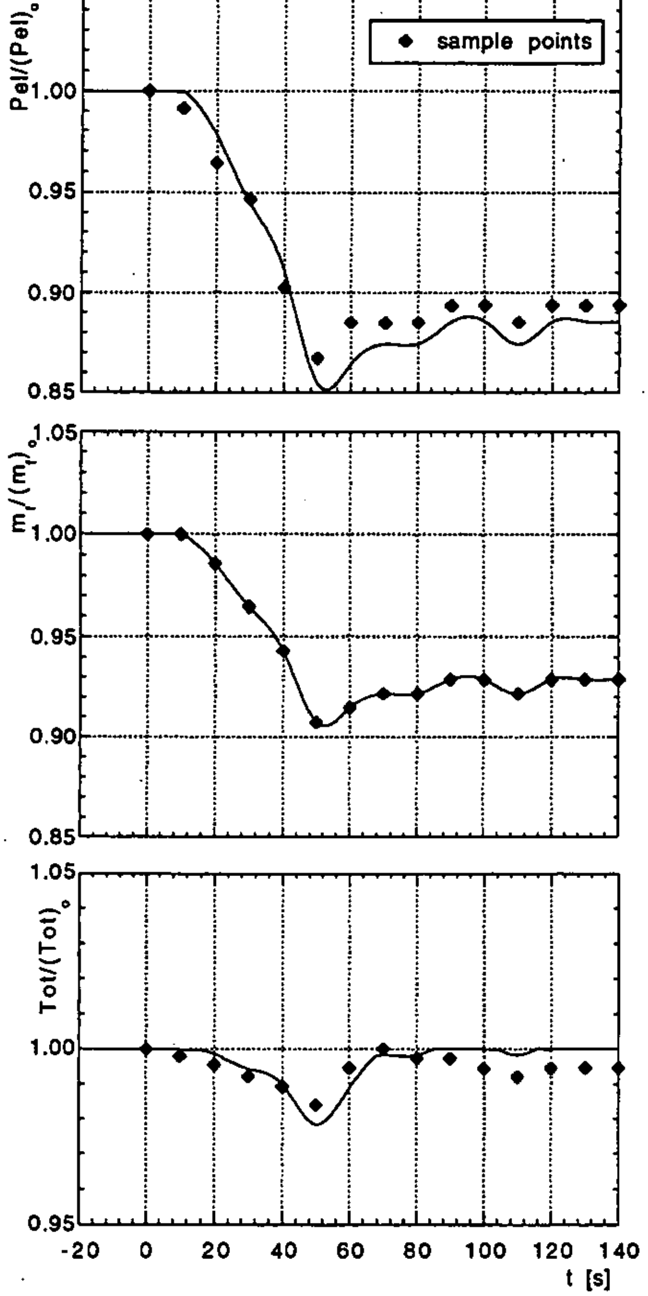

Fig. 6: Electrical power, fuel flow rate and turbine outlet temperature in the case of load reduction performed reducing the fuel flow rate and closing the IGV.

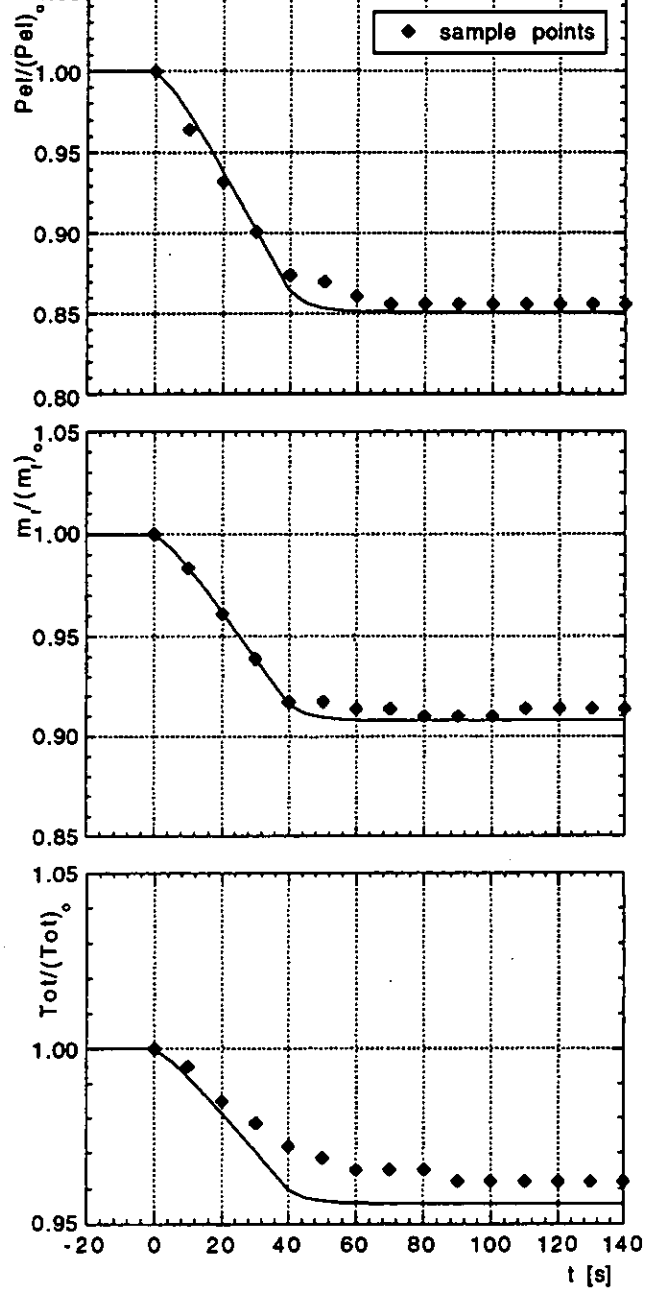

Fig. 7: Electrical power, fuel flow rate and turbine outlet temperature in the case of load reduction performed reducing the fuel flow rate alone.

CONCLUSION

The development of a modular non-linear dynamic gas turbine model has made it possible to set-up a computer program that may be used to simulate the operation of single-shaft machines, but may easily be extended to gas turbines having a more complex configuration.

In order to evaluate the validity of the model, the results of the simulation were compared to measurements taken during load reduction transients on a single-shaft industrial gas turbine in operation.

In particular, in the case of load reduction performed by the control system reducing the fuel flow rate and closing the IGV, the mean square differences between the values obtained by the simulation and those experimentally measured (with reference to comparison points shown in the figures) are about 0.011 for electrical power, 10^{-5} for fuel flow rate and 0.004 for turbine outlet temperature.

Similarly, in the case of load reduction performed by reducing the fuel flow rate alone, the mean square differences are about 0.008 for electrical power and turbine outlet temperature and 0.004 for fuel flow rate.

Regarding the percentage difference between calculated and measured transient end values, they are about -0.9 % for electrical power, 0.001 % for fuel flow rate and 0.5 % for turbine outlet temperature, in the case of load reduction performed by fuel flow rate reduction and IGV closing, and about -0.6 % for all three variables, in the case of load reduction performed by fuel flow rate reduction alone.

The results obtained therefore appear to provide a first confirmation of the validity of the set-up dynamic model, also since its simplified formulation appears suitable for use as a generator of time series of transient condition data, which are necessary in order to develop a methodology to diagnose gas turbine operation and measurement and control sensors.

ACKNOWLEDGMENTS

The Authors thank the Company Nuovo Pignone for having permitted the tests on gas turbine and the publication of results.

The work was carried out with contribution of the M.U.R.S.T. (Italian Ministry of University and Scientific & Technological Research).

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