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Advances in Hole-Drilling Residual Stress Measurements

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Abstract

Residual stress measurements by hole drilling have developed greatly in both sophistication and scope since the pioneering work of Mathar in the 1930s. Advances have been made in measurement technology to give measured data superior in both quality and quantity, and in analytical capability to give detailed residual stress results from those data. On the technology side, the use of multiple strain gauges, Moiré, Electronic Speckle Pattern Interferometry (ESPI) and Digital Image Correlation all provide prolific sources of high quality data. Modern analytical techniques using inverse methods provide effective ways of extracting reliable residual stress results from the mass of available data. This paper describes recent advances in both the measurement and analytical areas, and indicates some promising directions for future developments.

Introduction

The hole-drilling method is a widely used technique for measuring residual stresses. It has the advantages of good accuracy and reliability, standardized test procedures, and convenient practical implementation. The damage caused to the specimen is localized to the small drilled hole, and is often tolerable or repairable. For this reason, the method is sometimes described as “semi-destructive”.

The modern hole-drilling method has its roots in the pioneering work of Mathar in the 1930s [1]. It involves:

1. drilling a small hole in the specimen in the area of interest,
2. measuring the resulting deformations of the material around the hole, and
3. computing the corresponding residual stresses.

These three aspects of the hole drilling method have developed greatly since the time of Mathar. The low-speed drill that he used has been replaced by high-speed electric and air-turbine endmills, abrasive drilling and EDM machining. The original mechanical deformation measurement method has been replaced by the use of strain gauges and optical techniques such as Moiré, Electronic Speckle Pattern Interferometry (ESPI), and Digital Image Correlation. The early empirical stress computation procedures have been superseded by finite element calibrations and inverse calculations to accommodate the character and quantity of the newly available measured data. Procedural steps 2 and 3 described above, deformation measurement and stress computation, have greatly developed in sophistication and scope in recent years. This paper reviews these advances and suggests some promising directions for future developments.

Strain Gauge Measurements

Strain gauges were introduced for hole-drilling residual stress measurements in the 1950s and 1960s, e.g., [2,3]. Development of the measurement procedures has continued apace since then, leading to the introduction of ASTM Standard Test Method E837 [4] in 1981, several subsequent updates, and an extensive literature, e.g., [5,6,7]. A variant procedure, the Ring-Core method [8,9] has also been developed. Essentially, it is an “inside-out” version of the hole-drilling method. Hole-drilling involves cutting stressed material from the central area with the strains measured in the surrounding material, while the ring-core method has the rosette at the center with the surrounding stressed material being removed. The two methods are identical mathematically, and differ only in the numerical values used for the calibration constants. Hole-drilling is the more commonly used procedure because of its ease of use and lesser specimen damage.

The strain gauge hole-drilling method has seen developments in all three procedural steps identified in the Introduction. The first step, the practical mechanics of drilling a hole, is now well established [4, 5, 7,10]. The second step, the measurement of the surface strains, is strongly influenced by the geometry of the strain gauge rosette that is used. The standard Rendler and Vigness design [3] shown in Figure 1(a) is the most widely used style, and is suitable for general-purpose use. The three strain gauges that comprise the rosette are just sufficient to evaluate the three in-plane residual stresses σ_x, σ_y and τ_{xy}. Several other rosette geometries have been proposed over the years for specialized applications, for example an 8-gauge design [13] to improve measurement accuracy, 12-gauge [11] and 6-gauge [22] designs to provide thermal compensation and increased sensitivity, and 4-gauge [14] and 9-gauge [15] designs to allow consideration of plastic deformations. All these variant designs involve increased measurement complexity and rosette cost, and only the 6-gauge design is available commercially.

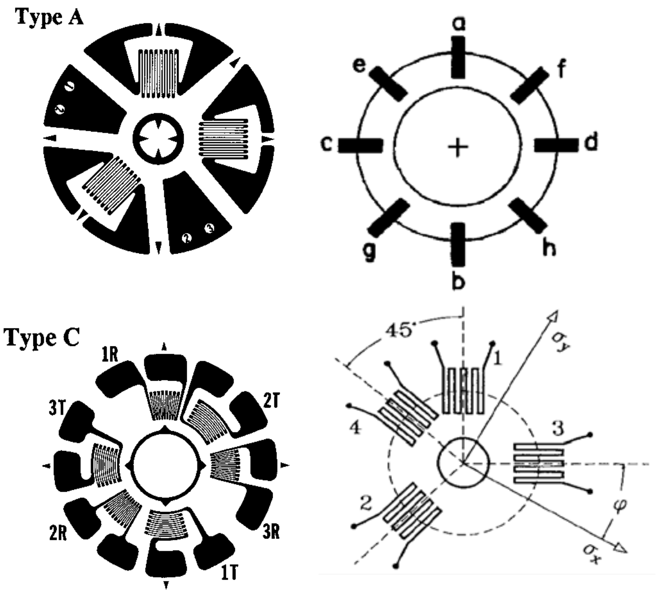

Fig. 1. Strain gauge rosettes used for hole-drilling residual stress measurements.
(a) ASTM style [3,14], (b) 8-gauge [13],
(c) 6-gauge [4,12], (d) 4-gauge [15].

Uniform Stress Measurements

For the third procedural step of hole-drilling measurements, the computation of the residual stresses, two possible cases are of interest. The first possibility occurs when the in-plane stresses do not vary with depth from the specimen surface (“uniform stresses”). In this case, the three in-plane residual stresses can be identified from three strain reliefs measured as the hole is directly drilled from zero to full depth. Such measurements use the minimum required strain data, and so, any measurement noise proportionally corrupts the computed residual stresses. This is a concern because, while the drilling relieves all the stress in the drilled hole, it relieves only about one third of the residual stresses at the strain gauge locations around the hole. Thus, the measured strains tend to be small, causing the relative effect of noise to be large.

A practical way to improve measurement accuracy is to make strain measurements at a series of small depth increments as the hole is drilled from zero to full depth [16]. All measured strain data can be considered, outliers identified and removed, and an averaging method used to minimize the effect of measurement noise. The use of eight hole depth increments is specified in ASTM E837 [4], and is an effective procedure for improving measurement quality [17].

Stress Profiling

In addition to their possible use for data averaging, strain measurements at a sequence of hole depth increments also provide the ability to determine the variation of residual stresses with depth. This process is often called “stress profiling”, and is the second possible case of interest when doing hole-drilling measurements. For this case, it is assumed that the variation of the in-plane stresses σ_x, σ_y and τ_{xy} occurs only in the depth direction, with no variation in the in-plane directions. Given the proximity to the free surface, the out-of-plane stresses are assumed to be zero.

Early methods for evaluating the stress profiles [18,19] relied on experimental calibrations of the strain vs. stress relationships. Of necessity, these methods were approximate because the experimental calibrations could not provide all the detailed calibration data needed. The subsequent development of finite element calculations provided the needed detailed calibrations [20]. They enabled the introduction of more accurate and reliable stress computation methods, notably the Integral and Power Series methods [21,22]. In addition, the finite element calculations provided greater accuracy and consistency. These features are particularly significant because the stress profile calculations are very sensitive to small errors. Detailed modeling of the strain gauges is necessary to achieve accurate results; it is not sufficient to assume that the strain sensitivity is uniform within each strain gauge area [23].

Although much more complex and error sensitive than uniform stress evaluations, stress profiling hole-drilling measurements are now widely used. The ASTM Standard Test Method E837 [4] is currently (in 2008) being revised to include a standardized procedure to evaluate residual stress vs. depth profiles.

Optical Techniques

In recent years, several optical techniques have been introduced for evaluating residual stresses by the hole-drilling method. These techniques have the advantage of providing full-field data, which are useful for data averaging, error checking and extraction of detailed information. Effectively, having full-field optical data is like having multi-element strain gauge rosettes of the type shown in Figure 1, but with many thousands of available gauges. In many ways, the optical techniques are complementary to the strain gauge technique, each approach having somewhat opposite advantages and disadvantages. Table 1 lists some features of strain gauge and optical measurements.

Strain Gauge Measurements	Optical Measurements
• Moderate equipment cost, high per-measurement cost	• High equipment cost, moderate per-measurement cost
• Significant preparation and measurement time	• Preparation and measurement time can be short
• Small number of very accurate and reliable measurements	• Large number of moderately accurate and reliable measurements
• Stress calculations are relatively compact	• Stress calculations can get quite large
• Modest capabilities for data averaging and self-consistency checking	• Extensive capabilities for data averaging and self-consistency checking
• Relatively rugged, suitable for field use	• Delicate, more suited to lab use
• Sensitive to hole-eccentricity errors	• Hole eccentricity can be corrected

Table 1. Features of Strain Gauge and Optical Measurements.

Moiré Interferometry

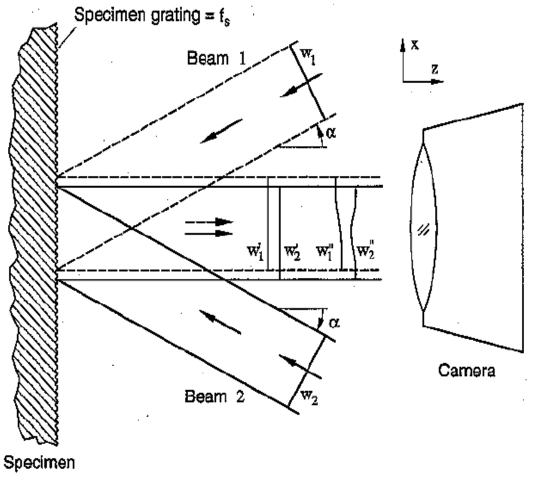

Fig. 2. Schematic arrangement used for Moiré interferometry (from Wu *et al.* [28]).

Moiré interferometry [24-30] provides a sensitive technique for measuring the small surface displacements that occur during hole drilling. Figure 2 schematically shows a typical optical arrangement [28]. Light from a single coherent laser source is split into two beams that illuminate the specimen surface with the symmetrical geometry shown in the diagram. A diffraction grating consisting of finely ruled lines, typically 600-1200 lines/mm, is replicated or made directly on the specimen surface. Diffraction of the beams creates a “virtual grating”, giving interference fringes consisting of light and dark lines. Figure 3 shows an example hole-drilling measurement [27]. Each light or dark line represents a contour line of in-plane surface displacement, in the x-direction in Figure 2. For typical optical arrangements, the in-plane displacement increment between fringe lines is about 0.5μm. The vertical lines are “carrier fringes” that are deliberately induced by slightly rotating one illumination beam. They correspond to a hypothetical uniform tensile or compressive strain in the x direction, and are added to enable the sign (tension or compression) of the surface displacements to be identified. These added strains are mathematically removed during the stress calculation.

The Moiré technique exemplifies the features of optical measurements summarized in Table 1. The full-field character of the measurements gives both an opportunity and a challenge. Potentially, large numbers of measurements can be obtained by extracting many individual points from within the field of view. Points close to the hole provide the most useful information. The associated challenge is to extract the data at those points efficiently, preferably with minimal human interaction, and to use the data within a compact and efficient numerical scheme to evaluate the corresponding residual stresses.


Fig. 3. Moiré fringe pattern created by hole drilling (from Nicoletto [27]).

A video image consisting of light and dark fringes, such as Figure 3, is difficult to interpret automatically. Fringe counting and interpolation can be challenging for complex fringe patterns, particularly in the presence of measurement noise. Automatic interpretation of light intensity data from fringe patterns can be difficult because any given light intensity could correspond to one of two possible phase angles. In addition, phase angle determination near the peaks of the light or dark fringes is sensitive to measurement noise because of the near zero slopes of the intensity vs. phase relationship in these areas. To address this issue, "phase-stepping" Moiré techniques [24,25] have been introduced, where the lengths of the optical paths are stepped using piezo actuators, with optical images measured at each step. Typically, four images are measured at 90° phase intervals. The optical phase at each image pixel can be determined from the pixel intensities in set of stepped images [31]. The phase is determined modulo 2π, so "unwrapping" [32] is needed to place the phase angles of all the pixels in correct sequence of fringe order. Ya *et al.* [30] describe an impressive example of phase-stepping Moiré measurements for hole-drilling residual stress evaluation.

The availability of "excess" data provides the possibility to improve stress evaluation accuracy and reliability by data averaging, and to be able to identify errors, outliers or additional features. This can be done visually, for example, the vertical non-symmetry of the fringes in Figure 3 shows that the residual stresses are non-uniform within plane. Alternatively, non-conforming data can be revealed by evaluating the "residuals", i.e., the difference between the actual measurements and the expected measurements based on the evaluated stresses.

Moiré measurements have the advantage of making useful measurements very near to the hole boundaries, much nearer than could be made by strain gauges. When using an attached diffraction grating, some minor delamination of the grating near the hole edge can limit the closeness of available measurements. The surface preparation to attach or form the diffraction grating on the surface is burdensome but not prohibitive.

ESPI

Electronic Speckle Pattern Interferometry (ESPI) [33,34] provides a further important method for measuring the surface displacements around a drilled hole. It has several similarities to the Moiré method and also involves measuring the interference pattern that is created when mixing two coherent light beams.

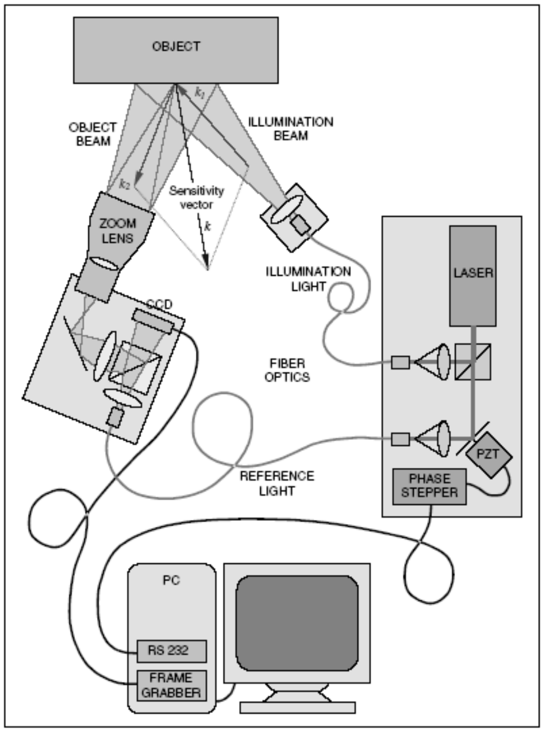

Fig. 4. Schematic arrangement used for ESPI measurements (from Steinzig [36]).

Figure 4 shows a typical ESPI arrangement [36]. The light from a coherent laser source is divided into two parts, one of which (the illumination/object beam) is used to illuminate the specimen surface so that it can be imaged by a CCD camera. The second (the reference beam) is fed directly to the CCD camera so that it creates an interference pattern on the CCD surface. The measured speckle pattern appears to be random noise, but each pixel in the image has a consistent phase relationship between the illumination/object and reference beams. This phase angle can be determined by using a piezo stepper to shift the phase of the reference beam in $\pi/2$ increments to create a set of four images. These four images can be combined to evaluate individually the local phase at every pixel.

The surface deformations caused by hole drilling alter the phase angles for each pixel in the illumination/object beam, which are then determined by measuring a second set of four images. These phase changes indicate the surface displacement in the direction of the "sensitivity vector", which for the arrangement in Figure 4 is in the direction of the bisector of the illumination and object beams. Figure 5(a) shows an example fringe pattern evaluated from the two sets of images, corresponding to the surface displacements around the drilled hole. This pictorial presentation is a mathematical construct designed to parallel the presentation provided by a photographic image when doing photographic holography. In ESPI, all measured images appear to contain just "speckle noise"; there is no directly measured image that shows a fringe pattern. The apparent elliptical shape of the hole in Figure 5 is caused by the oblique angle at which the hole was imaged. Only the area within the two dashed ellipses was used for the computation, the central area being too noisy, and the exterior area containing minimal deformations.

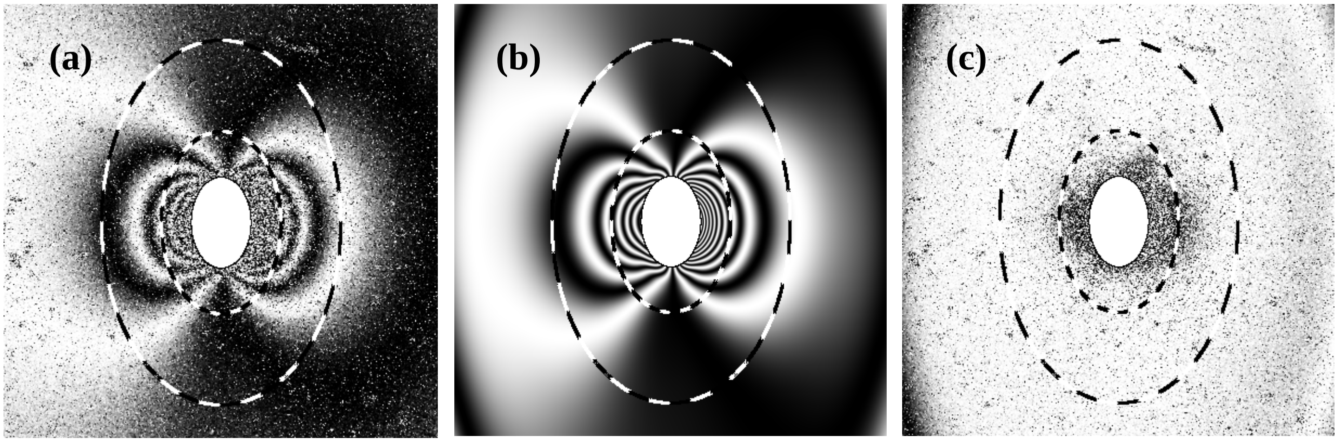

Fig. 5. ESPI hole-drilling measurements. (a) experimental data, (b) theoretical data, (c) misfit.

Developments in hole-drilling residual stress measurements using ESPI parallel those using Moiré measurements. Several different ESPI arrangements can be used, each with different capabilities. The arrangement shown in Figure 4 [35,36] measures surface displacements in the direction of the indicated "sensitivity vector". An optical arrangement similar to that shown in Figure 3, with phase stepping in one beam, is also useful for ESPI measurements [37,38]. In this case, the measured quantity is the in-plane displacement. Some further variations are possible, for example, the interesting radial in-plane arrangement in [39].

Shearography is another important class of ESPI measurements [40,41]. A Michelson interferometer is used to present two images of the specimen to a CCD camera, one image slightly shifted ("sheared") relative to the other. The two images interfere in the same way as the two beams shown in Figure 4, one of them acting as the illumination/object beam and the other as the reference beam. The resulting phase measurements give the differences in out-of-plane displacements of the paired points in the sheared images. These displacement differences in turn equal the mean surface slope between paired points, from which the residual stresses can be identified when doing hole-drilling measurements [42,43]. Shearography measurements tend to be more stable than displacement measurements because they are insensitive to rigid-body motions. However the inherent subtractions cause a tendency for the measured phase changes to be smaller.

A significant feature of ESPI is that it can work with a plain specimen surface, without attachment of the diffraction grating needed for Moiré measurements. This makes it possible to do ESPI measurements rapidly, and potentially to use the method as an industrial quality control tool. It also explicitly determines the phase at each point within the image area, as done with phase-stepped Moiré measurements. As for all the optical methods, ESPI equipment is delicate and expensive compared to strain gauge equipment, but the per-measurement cost is relatively low because no strain gauges need to be attached.

Digital Image Correlation

Digital Image Correlation [44,45,46] is a versatile optical technique for measuring surface displacements in two or three dimensions. The 2-D technique involves painting a textured pattern on the specimen surface and imaging the region of interest using a high-resolution digital camera. In some cases, for example, wood, the specimen may have sufficient natural texture not to require the addition of paint. The camera, which is set perpendicular to the surface, records images of the textured surface before and after deformation. The local details within the two images are then mathematically correlated, and their relative displacements determined. The algorithms used for doing this have become quite sophisticated, and with a well-calibrated optical system, displacements of +/- 0.02 pixel can be resolved.

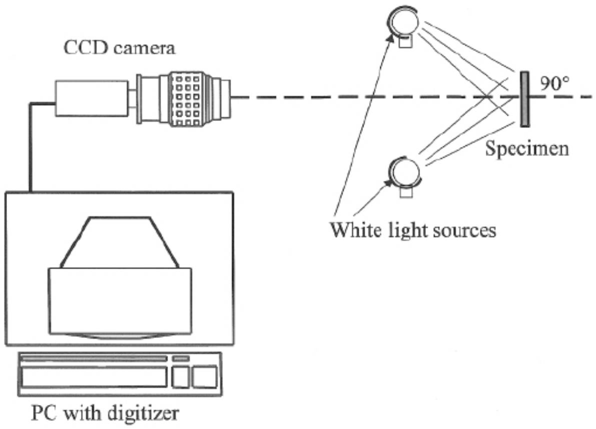

Fig. 6. Schematic arrangement used for 2-D Digital Image Correlation (from Sutton *et al.* [46]).

The 3-D technique involves imaging the region of interest with two cameras and using stereoscopic imaging to determine deformations in three dimensions [46]. The equipment is more complex than for the 2-D technique, and careful setup and calibration are required. Both the 2-D and 3-D techniques are less sensitive to environmental disturbances than Moiré or ESPI, and so are more suited to field use.

Digital Image Correlation has been successfully applied to residual stress measurements using hole-drilling. Both large [47] and small [48] specimens have been investigated. The challenge has been to find ways of using the available deformation data effectively. In principle, 1-D data are sufficient, and the use of some selected points can give reasonable residual stress results. As with computations with the other optical techniques, the residual stress evaluation benefits from the inclusion of a wider range of data, both in terms of number and type of data. The 2-D technique can evaluate two in-plane displacement components (horizontal and vertical, or radial and circumferential) from one pair of images. The use of such 2-D data can significantly improve the accuracy of the computed residual stresses.

Out-of-plane surface deformation data are additionally available using 3-D Digital Image Correlation [46]. These additional data can further improve the accuracy of residual stress evaluations from hole-drilling measurements. However, the out-of-plane displacements are much smaller and therefore less influential than the in-plane displacements. Thus, the major benefit is likely obtained by going from 1-D data to 2-D data. The further benefit of using 3-D data has yet to be evaluated in terms of the added cost and complexity of making the 3-D measurements.

Inverse Computation of Uniform Stresses

A defining characteristic of the hole-drilling method and almost all other destructive methods for measuring residual stresses is that they involve removal of stressed material in one area of the specimen and the measurement of deformations in a different nearby area [49]. This difference in the locations of the target stresses and the measured deformations creates a substantial computational challenge, particularly when stress vs. depth profiling is the objective. For the simpler “uniform stress” case, a straightforward stress calculation is possible. Minimally, there are just three strain data at the final hole depth and three in-plane stress components are to be determined. Even when data averaging is done using strain data from a series of hole depth increments [16], the required computation remains fairly straightforward.

The situation becomes more challenging when working with optical data, especially when it is in the form of light and dark fringes. Initial optical measurements for hole drilling used calculation methods parallel to those used for strain gauges [35,38,50]. Typically, they involved visually picking a small number of opportune points within the measured image, interpreting their fringe orders, and then doing a strain gauge style calculation. Although reasonable results are achieved, the performance of these methods can be significantly enhanced by including the contributions of the substantial quantity of additional data available beyond the few selected points.

Some desirable features of a residual stress computation method for use with optical data include:

- taking advantage of the wealth of data available within an optical image
- extracting the data from the image with a minimum of human interaction, preferably none
- using the available data in a compact and stable computation, preferably a linear one.

A typical spatial resolution for an image taken with a video camera is 640x480 pixels, giving a total of over 300,000 independent measurements. Even if only one third of the pixels are useful, over 100,000 data points remain, a very substantial number. The phase-stepped style of Moiré measurements, ESPI and Digital Image Correlation all give numerical deformation data at every pixel, and thus are well suited to meeting the above three computational objectives. The challenge is to use the large amount of available optical in an effective and compact way, without requiring long and complex computations. For this reason, linear computation methods, for example using a least-squares fit [51,52,53] are desirable. Non-linear procedures [54] can also be effective. However, they are much more computationally intensive, potentially less stable, and should be used only when essential.

When computing residual stresses from deformation data, it is important to note that stresses are not the only sources of measured deformations. Small rigid-body motions, as well as minor temperature variations, can cause shifts and tilts in the measured data. These are not problematic as long as they are also considered in the computations [51,53].

A computation using a large number of data to determine a small number of unknowns is over-determined, and no datum exactly fits the best-fit solution. The difference between the measured data and the theoretical data that fits the computed solution is the “misfit.” Ideally, the misfit should consist entirely of random noise with no apparent structure. Figure 5 shows an example ESPI measurement, the theoretical solution and the corresponding misfit. The misfit shows the desired random noise structure.

Inverse Computation of Stress Profiles

When computing residual stress vs. depth profiles, there is no longer a one-to-one relationship between the target stresses and the measured deformations (displacement or strain). Instead, a measured deformation depends on the contributions of the various stresses contained in all parts of the removed material. The relationship between the deformation measured at a point and the stresses causing it is an integral relationship, typically of the form:

$$d(h) = \int_0^h G(H,h)\ \sigma(H)\ dH \tag{1}$$

The deformation d(h) is measured after material to depth h has been removed, $\sigma(H)$ is the local stress at depth H originally within the removed material, and the kernel function G(H,h) defines the numerical relationship between the deformation measured when a depth h of material is removed, when a unit stress originally existed within the removed material. Equation (1) is called an Inverse Equation because the known quantity d(h) appears alone on the left, while the quantity to be calculated, $\sigma(H)$, appears enclosed within the integral on the right. If the stresses were known, it would be straightforward to perform the indicated integration to determine the corresponding displacements. However, the inverse calculation where the stresses are to be determined from the measured displacements is much more challenging. Equation (1) is classified as a Fredholm equation of the second kind, and requires the use of Inverse Methods [55,56] to determine a solution. Remarkably, the equations describing a very wide range of material-removal residual stress measurement methods have the form of equation (1), even though they are very different physically [57]. They can therefore be solved using the same general methods.

A common way of solving inverse equations such as equation (1) is to expand the stresses as a series

$$\sigma(H) = \sum_{j=1}^{n} c_j \, u_j(H) \qquad (2)$$

where $u_j(H)$ are basis functions and c_j are numerical coefficients to be determined. Providing that they span the model space, the basis functions $u_j(H)$ can be chosen freely, either for computational convenience or to fit the constraints of the physical system, for example, to enforce equilibrium. Common choices are pulse functions (Integral Method [22]), power series functions (Power Series Method [22]) and Legendre Polynomials [58]. Substitution of equation (2) into equation (1) gives a matrix equation

$$G_{ij} \, c_j = d_i \qquad (3)$$

where

$$G_{ij} = \int_0^{h_j} G(H, h_i) \, u_j(H) \, dH \qquad (4)$$

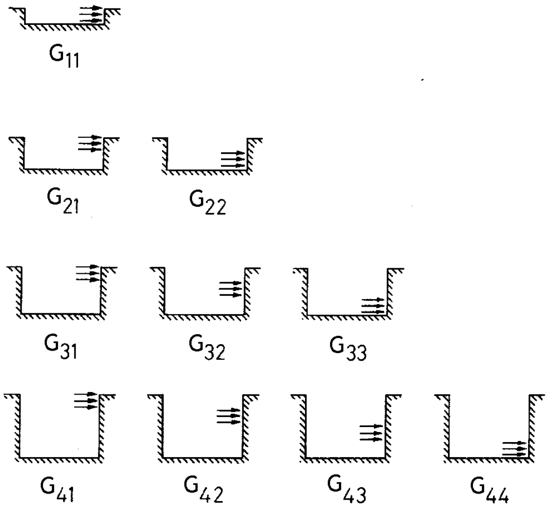

Fig. 6. Physical interpretation of matrix coefficients G_{ij} for the hole-drilling method (from Schajer [54]).

In the case of pulse functions, Figure 6 shows a graphical interpretation of matrix G_{ij} in equation (3) [59]. Coefficient G_{32} represents the deformation caused by a unit stress within step 2 of a hole 3 increments deep.

A common characteristic of inverse problems is that their numerical solution is ill conditioned. Small errors in the data cause proportionally larger errors in the calculated stresses. In the case of hole drilling, this behavior occurs because the strains are measured at the specimen surface, while the residual stresses of interest are in the interior. The ability to identify the interior stresses rapidly diminishes with distance from the measured surface, and disappears entirely for depths beyond about one hole diameter.

Another common characteristic of inverse problems is that their solution involves a balance between error sensitivity and spatial resolution. Thus, seeking a fine spatial resolution of the stress profile by doing measurements at a large number of small steps in hole depth causes the calculation to become increasingly unstable. For strain gage measurements, error sensitivity has been moderated by limiting the number and controlling the size of hole depth steps used to be fewer at larger hole depths [60,61]. This is effective, but it also limits the amount of data that can be used.

Another approach is to increase data content by making measurements at a large number of small steps in hole depth. Regularization, a form of smoothing, can then be used to stabilize the stress computation. Tikhonov regularization is a convenient choice [56,59,62]. When using regularization, the key issue is the choice of the amount of regularization to be used. Too little regularization gives stress solutions that are dominated by measurement noise, while excessive regularization distorts solutions by smoothing out real features. Optimal regularization minimizes the effects of measurement noise while preserving real features in the solution. The Morozov criterion [56] specifies that optimal regularization is achieved when the standard deviation of the misfit between the measured data and the theoretical data corresponding to the calculated solution equals the standard error of the measurements. In this way, the spatial resolution of stresses that can be achieved depends on both the quality and quantity of deformation data available.

Concluding Remarks

The introduction of full-field optical measurements of the deformations around a drilled hole has greatly expanded the scope of hole-drilling residual stress measurements. Data averaging and data consistency checking become feasible, and with careful choice of computation technique, the required handling of large quantities of data is not excessively burdensome, especially with modern computer equipment. The richness of the available data provide opportunities for more detailed analysis of the underlying residual stresses, in particular the evaluation of stresses that vary in-plane and stresses whose size approaches the yield stress. Both the latter cases have already attracted research attention, and are important issues for further exploration.

A present concern with the optical techniques is that their sensitive measurement capabilities typically require a sensitive measurement environment, for example, a climate-controlled laboratory with vibration-isolated optical benches. Work is underway to address this issue so that reliable measurements can be made in field conditions. This will be an important next step so that the full-field optical techniques can make the needed transition from a specialized laboratory device to a general-purpose measurement device. At present, strain gauges do this task very well, and their mature state of development and relatively low equipment cost will make them a tough contender in the competition among available residual stress measurement techniques. All techniques have their individual advantages and concerns, so likely all will continue to grow and develop.

Acknowledgments

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