Distribution Functions of Selection Combiner Output in Equally Correlated Rayleigh, Rician, and Nakagami-m Fading Channels

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***Abstract*—We develop a novel approach to derive the cumulative distribution functions (cdfs) of the selection-combining (SC) output signal-to-noise ratio (SNR) in equally correlated Rayleigh, Ricean, and Nakagami-m fading channels. We show that a set of equally correlated channel gains can be transformed into a set of conditionally independent channel gains. Single-fold integral expressions are, therefore, derived for the cdfs of the SC output SNR. Infinite series representations of the output cdfs are also provided. New expressions are applied to analyze the average error rate, the outage probability, and the output statistics of SC. Numerical and simulation results that illustrate the effect of fading correlation on the performance of L-branch SC in equally correlated fading channels are provided.**

***Index Terms*—Cumulative distribution function (cdf), diversity, equally correlated fading channel, outage probability, selection combining (SC).**

I. Introduction

CORRELATED fading among diversity branches can significantly degrade the performance of spatial diversity systems, such as maximal ratio combining (MRC), equal gain combining (EGC), and selection combining (SC). In practice, independent fading across diversity branches can rarely be achieved due to the insufficient separation between the antennas. Thus, quantifying the resultant degradation of the performance of diversity systems is a long standing problem of importance.

A. Equally Correlated Model

In this paper, we study the distribution of the SC output signal-to-noise ratio (SNR) in equally correlated fading channels. The equally correlated model may be valid for a set of closely placed antennas [1] and be used as a worst-case benchmark or as a rough approximation by replacing every $\rho_{ij}(i \neq j)$ in the correlation matrix with the average value of $\rho_{ij}(i \neq j)$. However, this model (unlike the exponentially correlated model) may have limited usefulness in representing a scenario with equally placed antennas. Experimental measurements have shown that for three equally placed antennas with $L = 3$ (a triangular), the fading correlation among them does not follow the equally correlated model [2].

B. Relation to the Previous Papers

While comprehensive theoretical results for the bit-error rate (BER) and the outage performance of MRC systems in various correlated fading channels are available, by comparison, such performance analysis for L-branch EGC and SC is not available. This is due to the lack of explicit expressions for the joint probability density function (pdf) of the branch SNRs for $L > 2$, when the branch signals are correlated. The SC performance has therefore been comprehensively treated for various *independent* fading models and modulation methods. For correlated fading, almost all results deal with two branches $(L = 2)$ or three branches $(L = 3)$ (see [3]–[11]).

Accordingly, in the extensive list of papers dealing with SC, we have been able to find only a few papers that address L-branch $(L > 3)$ SC in correlated fading. Exceptionally, Ugweje and Aalo [12] derive the pdf of the SC output SNR in correlated Nakagami fading as a multiple series of generalized Laguerre polynomials using [13]. However, the convergence properties of this series appear to be poor and the complexity of this approach increases rapidly for $L > 3$. Zhang and Lu [14] have derived a general approach for L-branch SC in correlated fading, which requires L-dimensional integration. For large $L\,(> 3)$, this method is also fairly complicated. Mallik and Win [15] analyze the generalized SC (GSC) in equally correlated Nakagami-m fading channels using its output characteristic function (chf). All of these methods utilize the joint chf of the branch SNRs and the complexity increases as L increases. Following Miller [16], Mallik [17] derives the joint pdf of the multivariate Rayleigh distribution. However, the pdf expression requires L-fold integration. Karagiannidis *et al.* derive the joint pdf of the exponentially correlated Nakagami-m distribution [18] and apply this result with the Green's matrix to approximate multivariate Nakagami-m distribution [19]. Although such an approximation is accurate for exponentially and linearly correlated models, it may not be good for the equally correlated model.

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C. Our Contributions

We thus develop a new approach to derive the cdfs of the L-branch SC output SNR in equally correlated fading channels and apply the new results to analyze the SC performance. The novel insight of this paper is that a set of independent Gaussian random variables (RVs) can be linearly combined to form a set of equally correlated complex Gaussian RV's. Using this insight, we translate the problem of the SC output cdf in equally correlated fading to the problem of the SC output cdf in a *conditionally independent* fading environment. This reformulation allows us to extend known results for independent fading to analyze the L-branch SC performance in correlated fading. It should be emphasized that this approach can be used to analyze not only SC, but also more general diversity combining schemes. Further, the new representations developed for equally correlated channel gains may be useful in other applications such as cochannel interference modeling and multiple-antenna systems.

This paper is organized as follows. Section II develops new representations for equally correlated Rayleigh, Rician, and Nakagami-m channel gains. Section III derives the cdfs of the SC output in three types of equally correlated fading channels. Section IV derives infinite series representations for the output cdfs. Section V uses our new expressions to evaluate the average error rates of various modulation schemes, the outage probability, and the output statistics of SC. Section VI presents some numerical and simulation results, and Section VII concludes the paper.

II. Representation of Equally Correlated Channel Gains

Rayleigh, Ricean, and Nakagami-m distributions are widely used to model the amplitude fluctuations of received signals from different multipath fading channels [20]–[22]. We develop new representations of equally correlated channel gains for these fading models, which can be used to evaluate the performance of various diversity combiners.

The following notation will be used throughout the paper. We denote the average, the complex conjugate, and the absolute value of X as $E(X)$, X^*, and $|X|$, respectively. We write $X \sim N(\mu, \sigma^2)$ to denote that X is Gaussian distributed with mean μ and variance σ^2. We let $Z \sim C(\mu, \sigma^2)$ denote that Z is complex Gaussian distributed with mean $\mu = E(Z)$ and covariance $\sigma^2 = (1/2)E[(Z-\mu)(Z-\mu)^*]$.

The noncentral chi-square distribution with n degrees of freedom and noncentrality parameter s^2 will be denoted by $\chi_n(s, \sigma^2)$. The central chi-square distribution with n degrees of freedom is denoted as $\chi_n(0, \sigma^2)$. The mth order Marcum Q-function is defined as [23]

$$Q_m(a,b) = \int_b^\infty x\left(\frac{x}{a}\right)^{m-1} \exp[-\left(\frac{x^2+a^2}{2}\right)]I_{m-1}(ax)dx \tag{1}$$

where $I_m(\cdot)$ is the mth-order modified Bessel function of the first kind. For brevity, we write $Q(a,b)$ to denote $Q_1(a,b)$. The Kronecker delta will be defined as $\delta_{ii} = 1$ and $\delta_{ij} = 0$ for $i \neq j$.

A. Correlated Rayleigh Envelopes

Rayleigh envelopes are frequently used to model the amplitudes of received signals in urban and suburban areas [20]. We represent the Rayleigh envelopes using a set of zero-mean complex Gaussian RVs given by

$$G_k = (\sqrt{1-\rho}X_k + \sqrt{\rho}X_0) + i(\sqrt{1-\rho}Y_k + \sqrt{\rho}Y_0) \tag{2}$$

for $k = 1, \ldots, L$, where $i = \sqrt{-1}$, $0 \leq \rho \leq 1$, and X_k, $Y_k \sim N(0, 1/2)$, $(k = 0, 1, \cdots, L)$, are independent. That is, for any $j, k \in \{0, \ldots, L\}$, $E(X_k Y_j) = 0$, and $E(X_k X_j) = E(Y_k Y_j) = (1/2)\delta_{kj}$. The validity of (2) for positive ρ only may at first seem a significant limitation. However, the entire range for ρ is between $-1/(L-1)$ and 1 (the lower limit follows from the positive-definiteness constraint on the covariance matrix). For large L, we may therefore ignore $-1/(L-1) \leq \rho < 0$. However, a more complicated representation than (2) can be developed to handle negative ρ values. For brevity, we omit the details and do not pursue it further.

Since $G_k \sim C(0, 1/2)$, $|G_k|$ is a set of Rayleigh envelopes with mean-square value $E(|G_k|^2) = 1$. The cross-correlation coefficient between any G_k and $G_j (k \neq j)$ equals $E(G_k G_j^*)/\sqrt{E(|G_k|^2)E(|G_j|^2)} = \rho$. This specifies the correlation (also known as power correlation) between two complex Gaussian samples. However, it is required to relate this to the envelope correlation (i.e., the correlation between two Rayleigh samples). The cross-correlation coefficient between the envelopes $|G_k|$ and $|G_j| (k \neq j)$ can be expressed in terms of power cross-correlation coefficient ρ as [24, eq. (1.5-26)]

$$\rho_e = \frac{(1+\rho)E_i\left(\frac{2\sqrt{\rho}}{1+\rho}\right) - \frac{\pi}{2}}{2 - \frac{\pi}{2}} \tag{3}$$

where $E_i(\eta)$ denotes the complete elliptic integral of the second kind with modulus η. For a given ρ_e, solving (3) yields ρ. Several solution methods are discussed in [25] and [26], and we also provide a new, more general solution in (10). Thus, using (2) and (3), we can readily represent a set of equally correlated Rayleigh envelopes with a specified value of envelope correlation.

Next, we introduce a "trick" that enables performance analysis. We consider $X_0 = x_0$ and $Y_0 = y_0$ to be fixed. Then, $G_k \sim C(\sqrt{\rho}(x_0 + iy_0), (1-\rho)/2)$. Consequently, the branch powers $|G_k|^2 \sim \chi_2(\sqrt{\rho(x_0^2 + y_0^2)}, (1-\rho)/2)$ are independent. Performance analysis can now be carried out in two steps. First, conditional performance results are obtained for the set of conditionally independent branch powers, and the conditional results are functions of $x_0^2 + y_0^2$. The second step is to average the conditional results over the distribution of $X_0^2 + Y_0^2$.

B. Correlated Ricean Envelopes

Ricean distribution is usually used where line-of-sight (LOS) propagation exists. We can also represent the Ricean fading envelopes by a set of complex Gaussian RVs

$$G_k = (\sqrt{1-\rho}X_k + \sqrt{\rho}X_0 + m_1) + i(\sqrt{1-\rho}Y_k + \sqrt{\rho}Y_0 + m_2) \tag{4}$$

for $k = 1, \ldots, L$, where $X_k, Y_k \sim N(0, 1/2), k = 0, 1, \cdots, L$, are independent, and $m_1 + im_2$ is the nonzero LOS component.

Since $G_k \sim C(m_1 + im_2, 1/2)$, $|G_k|$ is Ricean distributed with the Ricean factor $K = m_1^2 + m_2^2$ and the mean-square value $E(|G_k|^2) = 1 + K$. The power correlation between G_k and $G_j (k \neq j)$ is equal to $(E\{[G_k - E(G_k)][G_j - E(G_j)]^*\}/\sqrt{E[|G_k - E(G_k)|^2]E[|G_j - E(G_j)|^2]}) = \rho$. Thus, (4) can be used to represent the equally correlated Ricean fading envelopes.

C. Correlated Nakagami-m Envelopes

The Nakagami-m distribution is a versatile statistical distribution which can accurately model a variety of fading environments. It has greater flexibility in matching some empirical data than the Rayleigh, Lognormal, or Ricean distribution. It also includes the Rayleigh and the one-sided Gaussian distributions as special cases. Moreover, the Nakagami-m can closely approximate the Ricean distribution and the Hoyt distribution [27]. When the fading severity index (m) is an integer, a Nakagami-m envelope is the square root of the sum of squares of m independent Rayleigh variates. Hence, we can represent the Nakagami-m fading envelopes using a set of Lm zero-mean complex Gaussian RVs

$$G_{kj} = (\sqrt{1-\rho}X_{kj} + \sqrt{\rho}X_{0j}) + i(\sqrt{1-\rho}Y_{kj} + \sqrt{\rho}Y_{0j}) \quad (5)$$

for $k = 1, \ldots, L$ and $j = 1, \ldots, m$, where X_{kj} and $Y_{kj} \sim N(0, 1/2)$, $k = 0, 1, \ldots, L$ and $j = 1, \ldots, m$, are independent. That is, for any k, $l \in \{0, \ldots, L\}$, and j, $n \in \{1, \ldots, m\}$, $E(X_{kj}Y_{ln}) = 0$ and $E(X_{kj}X_{ln}) = E(Y_{kj}Y_{ln}) = (1/2)\delta_{kl}\delta_{jn}$.

The cross-correlation coefficient between G_{kj} and G_{ln} is

$$\rho_g = \frac{E\left(G_{kj}G_{ln}^*\right)}{\sqrt{E\left(|G_{kj}|^2\right)E\left(|G_{ln}|^2\right)}} = \begin{cases} \rho, & k \neq l \quad \text{and} \quad j = n \\ 0, & j \neq n. \end{cases} \quad (6)$$

Let R_k denote the summation of the absolute square of G_{kj}

$$R_k = \sum_{j=1}^{m} |G_{kj}|^2. \quad (7)$$

From (6), we can see that, for any fixed k, $G_{kj} \sim C(0, 1/2)$, $j = 1, \ldots, m$, are independent. Thus, $R_k \sim \chi_{2m}(0, 1/2)$ is the sum of squares of m independent Rayleigh envelopes with cross-correlation coefficient [27]

$$\rho_r = \frac{E(R_k R_l)}{\sqrt{E\left(R_k^2\right)E\left(R_l^2\right)}} = \rho^2, \quad k \neq l. \quad (8)$$

Therefore, $\sqrt{R_k}$ is a set of equally correlated Nakagami-m fading envelopes with mean-square value $E(R_k) = m$. The relationship between the power correlation ρ_r and the envelope correlation ρ_e is [27]

$$\rho_e = \frac{F\left(-\frac{1}{2}, -\frac{1}{2}; m; \rho_r\right) - 1}{\psi(m) - 1} \quad (9)$$

where $\psi(m) = \Gamma(m)\Gamma(m+1)/\Gamma^2(m+1/2)$ and $F(\cdot)$ is the hypergeometric function [28, eq. (15.1.1)]. Note that for $m = 1$, (9) reduces to (3). Since there is no closed-form solution, for a given ρ_e, we may use a polynomial approximation for ρ_r. Expressing the hypergeometric function in the form of Gauss series [28, eq. (15.1.1)] and using the reversion of power series [29, p. 138], we obtain the approximation for ρ_r as

$$\begin{aligned}\rho_r \approx\; & 4xm - \frac{2m^2x^2}{m+1} \\ & - \frac{2(2m+1)m^3x^3}{(m+2)(m+1)^2} - \frac{5(5m+3)(2m+1)m^4x^4}{2(m+3)(m+2)(m+1)^3} \\ & - \frac{(2m+1)(256m^3+843m^2+743m+198)m^5x^5}{2(m+4)(m+3)(m+2)^2(m+1)^4}\end{aligned} \quad (10)$$

where $x = \rho_e(\psi(m) - 1)$. Then, using (8), we can immediately obtain ρ.

III. Distribution Functions of the SC Output

In this section, we derive the expressions for the cdfs of the SC output SNR in equally correlated Rayleigh, Rician, and Nakagami-m fading channels. We assume that the received signals at different branches are identically distributed and equally correlated with each other. The noise components at different branches are assumed to be independent of the signal components and uncorrelated with each other.

A. Rayleigh Fading Channels

In SC, the branch with the largest instantaneous SNR is selected as the output as follows:

$$\gamma_{\text{sc}} = \max(\gamma_1, \gamma_2, \cdots, \gamma_L) \quad (11)$$

where L is the number of diversity branches. The instantaneous SNR of the kth branch is $\gamma_k = |G_k|^2(E_s/N_0), k = 1, 2 \cdots, L$, where E_s is the energy of the transmitted signals and N_0 is the noise power spectral density per branch. Note that, when the fading correlation $\rho = 1$, i.e., all of the branches experience the same fading, the L-branch SC reduces to the single-branch SC whose performance is well known. In our derivation, we will not consider this case.

Recall that, when X_0 and Y_0 are fixed, $(X_0^2 + Y_0^2 = T)$ and $|G_k|^2 \sim \chi_2(\sqrt{\rho(x_0^2 + y_0^2)}, (1-\rho)/2)$ are independent, whose cdf is given by [30, eq. (2-1-124)]

$$F_{|G_k|^2|T}(y|t) = \Pr\left(|G_k|^2 \le y|t\right) = 1 - Q\left(\sqrt{\frac{2\rho t}{1-\rho}}, \sqrt{\frac{2y}{1-\rho}}\right). \quad (12)$$

Evaluating the cdf of SC output for fixed $X_0^2 + Y_0^2 = t$, we obtain the conditional cdf

$$\begin{aligned} F_{\gamma_{sc}|T}(y|t) &= \Pr(\gamma_1 \le y, \ldots, \gamma_L \le y|t) \\ &= \Pr\left(|G_1|^2 \le \frac{y}{\bar{\gamma}_c}, \ldots, |G_L|^2 \le \frac{y}{\bar{\gamma}_c}\middle| t\right) \\ &= \left[1 - Q\left(\sqrt{\frac{2\rho t}{1-\rho}}, \sqrt{\frac{2y}{\bar{\gamma}_c(1-\rho)}}\right)\right]^L \end{aligned} \quad (13)$$

where $\bar{\gamma}_c = (E_s/N_0)E(|G_k|^2) = (E_s/N_0)$ is the average branch SNR. Notice that $T \sim \chi_2(0, 1/2)$, where $T = X_0^2 + Y_0^2$, and its pdf is given by a special case of [30, eq. (2-1-110)]

$$p_T(t) = e^{-t}, \quad t \ge 0. \quad (14)$$

Averaging the conditional cdf (13) over the distribution of T [see (14)], we obtain the cdf for the output of L-branch SC in equally correlated Rayleigh fading as

$$F_{\gamma_{sc}}(y) = \int_0^\infty F_{\gamma_{sc}|T}(y|t)p_T(t)dt = \int_0^\infty \left[1-Q\left(\sqrt{\frac{2\rho t}{1-\rho}}, \sqrt{\frac{2y}{\bar{\gamma}_c(1-\rho)}}\right)\right]^L e^{-t}dt. \quad (15)$$

Notice that Matlab provides the function NCX2CDF to compute the cdf of the noncentral chi-square distribution, which is in the form of the Marcum Q-function. Therefore, we can readily evaluate (15) numerically using Matlab.

To the best of our knowledge, this is a novel result. It reduces the L-dimensional integration [14, eq. (9)] necessary for the cdf of correlated SC output to a single-fold integral, enabling the analysis for L-branch SC in equally correlated Rayleigh fading. This new expression (15) for the output cdf reduces to the previous results for two special cases.

Case 1) Independent Rayleigh fading channel ($\rho = 0$): Using the relation $I_0(0) = 1$ and $Q(0, x) = e^{-x^2/2}$, the output cdf (15) simplifies to $F_{\gamma_{sc}}(y) = [1-e^{-y/\bar{\gamma}}]^L$, which is equivalent to the well-known result [3, eq. (10-4-12)].

Case 2) Dual-branch ($L = 2$) SC in equally correlated Rayleigh fading channels: Using integration by parts, we can show that the output cdf can now be written as

$$F_{\gamma_{\rm sc}}(y) = \int_0^\infty \left[1-Q\left(\sqrt{\frac{2\rho t}{1-\rho}}, \sqrt{\frac{2y}{\bar{\gamma}_c(1-\rho)}}\right)\right]^2 \exp[-t]dt = 1 - 2\exp\left[-\frac{y}{\bar{\gamma}_c}\right] Q\left(\sqrt{\frac{2y}{\bar{\gamma}_c(1-\rho^2)}}, \rho\sqrt{\frac{2y}{\bar{\gamma}_c(1-\rho^2)}}\right) + \exp\left[-\frac{2y}{\bar{\gamma}_c(1-\rho^2)}\right] I_0\left[\frac{2\rho y}{\bar{\gamma}_c(1-\rho^2)}\right]. \quad (16)$$

This result is equivalent to the well-known expression [3, eq. (10-10-8)].

Differentiating (15) yields the output pdf

$$p_{\gamma_{\rm sc}}(y) = \frac{L}{\bar{\gamma}_c} e^{-\frac{y}{\bar{\gamma}_c(1-\rho)}} \int_0^\infty e^{-t}\left[1-Q\left(\sqrt{2\rho t}, \sqrt{\frac{2y}{\bar{\gamma}_c(1-\rho)}}\right)\right]^{L-1} \times I_0\left(2\sqrt{\frac{\rho y t}{\bar{\gamma}_c(1-\rho)}}\right) dt. \quad (17)$$

There is no closed-form solution to this integral. However, in the case of $L = 2$, the output pdf can be written in terms of the Marcum Q-function

$$p_{\gamma_{\rm sc}}(y) = \frac{2}{\bar{\gamma}_c} e^{-\frac{y}{\bar{\gamma}_c}} \left[1-Q\left(\rho\sqrt{\frac{2y}{\bar{\gamma}_c(1-\rho^2)}}, \sqrt{\frac{2y}{\bar{\gamma}_c(1-\rho^2)}}\right)\right]. \quad (18)$$

This special case is already known [7, eq. (11)].

Using the new expression for output pdf (17), we can also derive the moment generating function (mgf) of the output SNR. The mgf is defined as the statistical average

$$\phi_{\rm sc}(s) = E\left[\exp(-s\gamma)\right] = L(1-\rho)\int_0^\infty e^{-[1+s\bar{\gamma}_c(1-\rho)]y} \int_0^\infty e^{-t} \times \left[1 - Q(\sqrt{2\rho t}, \sqrt{2y})\right]^{L-1} I_0(2\sqrt{\rho y t})dtdy. \quad (19)$$

In the special case of dual-branch SC, the mgf can be expressed in a closed-form [7] equation as shown in (20), at the bottom of the page.

B. Ricean Fading Channels

Using the representations of the equally correlated Ricean channel gains (4) and following the same approach, we can also derive the output cdf of SC in equally correlated Ricean channels.

Fix $(X_0 + m_1/\sqrt{\rho})^2 + (Y_0 + m_2/\sqrt{\rho})^2 = T$, $|G_k|^2 \sim \chi_2(\sqrt{(\sqrt{\rho}x_0 + m_1)^2 + (\sqrt{\rho}y_0 + m_2)^2}, (1-\rho)/2)$ are independent, whose cdf is given by [30, eq. (2-1-124)]. Also notice that $T \sim \chi_2(\sqrt{K/\rho}, 1/2)$, where $T = (X_0+m_1/\sqrt{\rho})^2+(Y_0+m_2/\sqrt{\rho})^2$, and its pdf is given by [30, eq. (2-1-118)]. Hence, the output cdf can be obtained as

$$F_{\gamma_{\rm sc}}(y) = e^{-\frac{K}{\rho}} \int_0^\infty \left[1-Q\left(\sqrt{\frac{2\rho t}{1-\rho}}, \sqrt{\frac{2(1+K)y}{\bar{\gamma}_c(1-\rho)}}\right)\right]^L \times e^{-t} I_0\left(2\sqrt{\frac{Kt}{\rho}}\right) dt \quad (21)$$

where K is the Ricean factor. To the best of our knowledge, (21) is also a new result. We are not aware of any study dealing with L-branch SC in correlated Ricean fading.

C. Nakagami-m Fading Channels

To derive the output cdf of SC in equally correlated Nakagami-m fading channels, we use the channel gain representations (5) and (7). Now we fix $X_{0j} = x_{0j}$ and $Y_{0j} = y_{0j}$, $j = 1, \ldots, m$. The branch power $R_k \sim \chi_{2m}(\sqrt{\rho\sum_{j=1}^m (x_{0j}^2 + y_{0j}^2)}, (1-\rho)/2)$ is independent, whose cdf is given by [30, eq. (2-1-124)]. Noticing that

$$\phi_{\rm sc}(s) = \frac{2}{1+s\bar{\gamma}_c} - \frac{4(1-\rho^2)}{[s\bar{\gamma}_c(1-\rho^2)+2]^2 - 4\rho^2 - s\bar{\gamma}_c(1-\rho^2)\sqrt{[s\bar{\gamma}_c(1-\rho^2)+2]^2 - 4\rho^2}} \quad (20)$$

$T \sim \chi_{2m}(0,1/2)$, where $T=\sum_{j=1}^{m}(x_{0j}^2+y_{0j}^2)$, and its pdf may be found as [30, eq. (2-1-110)], we can readily obtain the output cdf

$$F_{\gamma_{\rm sc}}(y)=\frac{1}{\Gamma(m)}\int_0^\infty\left[1-Q_m\left(\sqrt{\frac{2\rho t}{1-\rho}},\sqrt{\frac{2my}{\bar{\gamma}_c(1-\rho)}}\right)\right]^L \times t^{m-1}e^{-t}dt. \quad (22)$$

This novel result reduces to several well-known special cases. The cdf of SC in independent Nakagami-m fading channels can be obtained by setting $\rho=0$ in (22) to yield

$$F_{\gamma_{\rm sc}}(y)=\left[1-Q_m\left(0,\sqrt{\frac{2my}{\bar{\gamma}_c}}\right)\right]^L = \left[1-e^{-\frac{my}{\bar{\gamma}_c}}\sum_{k=0}^{M-1}\frac{1}{k!}\left(\frac{my}{\bar{\gamma}_c}\right)^k\right]^L \quad (23)$$

which is equivalent to the result [31]. As expected, when $m=1$, this new expression reduces to (15) for the output cdf of SC in equally correlated Rayleigh fading channels. For $L=2$, we have also checked (22) numerically against an infinite series for the cdf derived from [32, eq. (3)]. Both of the methods give exactly the same numerical values.

Differentiating (22) yields the output pdf of SC in equally correlated Nakagami-m fading channels. It can be shown that, for dual-branch $(L=2)$ SC, the output pdf reduces to

$$p_{\gamma_{\rm sc}}(y) = \frac{y^{m-1}m^m}{\Gamma(m)\bar{\gamma}_c^m} \times e^{-\frac{my}{\bar{\gamma}_c}}\left[1-Q_m\left(\rho\sqrt{\frac{2my}{\bar{\gamma}_c(1-\rho^2)}},\sqrt{\frac{2my}{\bar{\gamma}_c(1-\rho^2)}}\right)\right]. \quad (24)$$

This result is equivalent to that of [7]. These special cases reaffirm the rightness of (22).

IV. Infinite Series Representation

In this section, we provide infinite series representations for computation of the cdfs of SC output SNR in equally correlated Rayleigh, Ricean, and Nakagami-m fading channels. We start our derivation from the equally correlated Rayleigh fading case. Noting the property of the Marcum Q-function $Q(a,\infty)=1$, we may rewrite the output cdf (15) as

$$F_{\gamma_{\rm sc}}(y)=\int_0^\infty\left[\int_0^{\sqrt{\frac{2y}{\bar{\gamma}_c(1-\rho)}}} xe^{-\frac{\rho t}{1-\rho}-\frac{x^2}{2}}I_0\left(x\sqrt{\frac{2\rho t}{1-\rho}}\right)dx\right]^L e^{-t}dt. \quad (25)$$

Using an infinite series [28, eq. (9.6.10)] for $I_0(x)$ and interchanging the order of integration and summation, we obtain an infinite series representation of the output cdf after some algebra as

$$F_{\gamma_{\rm sc}}(y)=(1-\rho)\sum_{k=0}^{\infty}\frac{k!a_k(y)}{(\rho L+1-\rho)^{k+1}} \quad (26)$$

TABLE I

Number of Terms Required in (26) to Achieve Significant Five-Figure Accuracy ($\bar{\gamma}_c=1$)

	$L=3$			$L=6$		
ρ	$y=-5$dB	$y=0$dB	$y=5$dB	$y=-5$dB	$y=0$dB	$y=5$dB
0.2	6	8	12	7	10	17
0.4	7	11	19	8	14	28
0.6	9	15	30	10	20	46
0.8	14	27	59	16	37	96

where

$$a_k(y)=\begin{cases}\left[1-e^{-\frac{y}{\bar{\gamma}_c(1-\rho)}}\right]^L, & k=0\\ \dfrac{\sum_{n=1}^{k}\frac{(nL-k+n)}{n!}\rho^n a_{k-n}(y)P\left[n+1,\frac{y}{\bar{\gamma}_c(1-\rho)}\right]}{k\left[1-e^{-\frac{y}{\bar{\gamma}_c(1-\rho)}}\right]}, & k\geq 1\end{cases} \quad (27)$$

where $P(a,x)$ is the incomplete gamma function [28, eq. (6.5.4)], which is available in standard numerical softwares, such as Maple V and Matlab.

Compared with the integral expression (15) for the SC output cdf, the infinite series representation (26) may be more useful for computation. The number of terms required in (26) to achieve a target accuracy depends on several factors, including the fading correlation ρ, the normalized branch SNR $y/\bar{\gamma}_c$, and the diversity order L as well. Table I lists the results for a significant five-figure accuracy over a range of values of ρ, y, and L with the average branch SNR $\bar{\gamma}_c=1$. As the fading correlation ρ, the branch SNR y or the diversity order L increases, more terms are required in (26) to achieve the target accuracy.

Following the same approach as above, we obtain the infinite series representation for output cdfs in Ricean and Nakagami-m fading channels, respectively, as

$$F_{\gamma_{\rm sc}}(y)=(1-\rho)e^{-\frac{K}{\rho}}\sum_{k=0}^{\infty}\frac{k!}{(\rho L+1-\rho)^{k+1}} \times\sum_{j=0}^{k}\left[\frac{K(1-\rho)}{\rho}\right]^{k-j}\frac{a_j\left[(K+1)y\right]}{\left[(k-j)!\right]^2} \quad (28)$$

$$F_{\gamma_{\rm sc}}(y)=\frac{(1-\rho)^m}{\Gamma(m)}\sum_{k=0}^{\infty}\frac{\Gamma(k+m)c_k(y)}{(\rho L+1-\rho)^{k+m}} \quad (29)$$

where

$$c_k(y)=\begin{cases}\left[P\left(m,\frac{my}{\bar{\gamma}_c(1-\rho)}\right)\right]^L, & k=0\\ \dfrac{\sum_{n=1}^{k}\frac{(nL-k+n)}{n!}\rho^n c_{k-n}(y)P\left[m+n,\frac{my}{\bar{\gamma}_c(1-\rho)}\right]}{kP\left(m,\frac{my}{\bar{\gamma}_c(1-\rho)}\right)}, & k\geq 1.\end{cases} \quad (30)$$

V. Performance Analysis

Using the output cdfs (15), (21), and (22), we can readily evaluate the average error rate, the outage probability, and the output statistics of L-branch SC in equally correlated fading channels. We assume that channel fading is nonselective and changing slowly enough so that the channel parameters remain constant for the duration of the signaling interval.

A. Average Error Rate

The average BER or symbol-error rate (SER) is one of the most commonly used performance criterion of digital communication systems. Several methods can be used to obtain the error rates. Conventionally, the average error rate is obtained by integrating the conditional error probability (CEP) $P_e(\gamma)$, over the pdf of the SC output SNR $\gamma_{\rm sc}$ [see (17)] to yield

$$\bar{P}_e = \int_0^\infty P_e(\gamma) p_{\gamma_{\rm sc}}(\gamma) d\gamma. \tag{31}$$

Using (19), the mgf approach [33], [34] can be readily employed for evaluating the error-rate performance of SC. We can also express the average error probability in terms of the cdf of $\gamma_{\rm sc}$. This can readily be done using the integration-by-parts method as follows:

$$\bar{P}_e = \int_0^\infty -P_e'(\gamma) F_{\gamma_{\rm sc}}(\gamma) d\gamma \tag{32}$$

where $-P_e'(\gamma) = -(d/d\gamma)P_e(\gamma)$ denotes the negative derivative of the CEP. Next, we will present the BER or SER for various modulation schemes with SC in equally correlated Rayleigh fading channels according to their CEP forms.

1) $P_e(\gamma) = aQ(\sqrt{b\gamma})$*:* Binary phase-shift keying (BPSK) is used for reverse link in CDMA2000 due to its high power efficiency. The CEP for coherent binary frequency-shift keying (BFSK) and M-ary pulse amplitude modulation (PAM) is in the form of [30]

$$P_e(\gamma) = aQ(\sqrt{b\gamma}) \tag{33}$$

where $(a, b) = (1, 2)$ for BPSK, $(a, b) = (1, 1)$ for coherent BFSK, $(a, b) = (2(M-1)/M, 6\log_2 M/M^2 - 1)$ for M-ary PAM, and $Q(x)$ is the area under the tail of the Gaussian pdf and defined as [30, eq. (2-1-97)].

Substituting (33) into (32), we obtain the following expression for the average BER:

$$\bar{P}_e = \frac{a}{2}\sqrt{\frac{b}{2\pi}} \int_0^\infty \frac{e^{-\frac{b\gamma}{2}}}{\sqrt{\gamma}} \int_0^\infty \left[1 - Q\left(\sqrt{\frac{2\rho t}{1-\rho}}, \sqrt{\frac{2\gamma}{\bar{\gamma}_c(1-\rho)}}\right)\right]^L \times e^{-t} dt\, d\gamma. \tag{34}$$

Notice that there is a removable singularity at $\gamma = 0$ in (34). That is the integrand approaches 0 as $\gamma \to 0$, i.e.,

$$\lim_{\gamma \to 0} \frac{e^{-\frac{b\gamma}{2}}}{\sqrt{\gamma}} \left[1 - Q\left(\sqrt{\frac{2\rho t}{1-\rho}}, \sqrt{\frac{2\gamma}{\bar{\gamma}_c(1-\rho)}}\right)\right]^L = 0. \tag{35}$$

When a numerical quadrature technique such as Gaussian quadrature is used, this singularity can be readily handled. Common mathematical software provides both Gaussian quadrature techniques and also the straight forward techniques such as Newton–Cotes formulas. All such techniques can be readily adapted to handle all integral expressions derived in this section.

2) $P_e(\gamma) = a\exp(-b\gamma)$*:* The CEP of noncoherent BPSK (NCBFSK) and differential BPSK (DBPSK) can be expressed in the exponential form [30]

$$P_e(\gamma) = ae^{-b\gamma} \tag{36}$$

where $(a, b) = (0.5, 1)$ for DBPSK and $(a, b) = (0.5, 0.5)$ for NCBFSK.

Following the same procedure as the above, we obtain the BER as

$$\bar{P}_e = ab \int_0^\infty e^{-b\gamma} \int_0^\infty \left[1 - Q\left(\sqrt{\frac{2\rho t}{1-\rho}}, \sqrt{\frac{2\gamma}{\bar{\gamma}_c(1-\rho)}}\right)\right]^L \times e^{-t} dt\, d\gamma. \tag{37}$$

The CEP for noncoherent M-ary frequency-shift keying (MFSK) can be expressed as a sum of exponential forms [30, eq. (5-4-46)]. Thus, we can readily write down the SER for MFSK using (37) to yield

$$\bar{P}_e = \sum_{n=1}^{M-1} \frac{(-1)^{n+1} nk}{(n+1)^2} \binom{M-1}{n} \int_0^\infty e^{-\frac{nk\gamma}{n+1}} \times \int_0^\infty \left[1 - Q\left(\sqrt{\frac{2\rho t}{1-\rho}}, \sqrt{\frac{2\gamma}{\bar{\gamma}_c(1-\rho)}}\right)\right]^L e^{-t} dt d\gamma \tag{38}$$

where $k = \log_2 M$ is the number of bits per symbol.

3) $P_e(\gamma) = aQ(\sqrt{b\gamma}) - cQ^2(\sqrt{b\gamma})$*:* Since it is both easy to implement and fairly resistant to noise, quadrature phase-shift keying (QPSK) has been adopted in various third-generation (3G) standards, such as European Telecommunications Standards Institute (ETSI), Europe, and Association of Radio Industries and Business (ARIB), Japan. Quadrature-amplitude modulation (QAM) is also an attractive modulation scheme due to its spectral efficiency. Both 16-QAM and 64-QAM have been adopted in the IEEE 802.11a standard. The CEP for QPSK, QAM, minimum-shift keying (MSK) and coherent detected DPSK is in the following form [30]:

$$P_e(\gamma) = aQ(\sqrt{b\gamma}) - cQ^2(\sqrt{b\gamma}) \tag{39}$$

where $(a, b, c) = (2, 2, 1)$ for QPSK or MSK, $(a, b, c) = (2, 2, 2)$ for coherent detected DPSK and $(a, b, c) = ((4(\sqrt{M}-1)/\sqrt{M}), (3\log_2 M/M - 1), (4(\sqrt{M}-1)^2/M))$ for QAM.

The average SER is obtained as

$$\bar{P}_e = \sqrt{\frac{b}{2\pi}} \int_0^\infty \frac{e^{-\frac{b\gamma}{2}}}{\sqrt{\gamma}} \left(\frac{a}{2} - cQ(\sqrt{b\gamma})\right) \times \int_0^\infty \left[1 - Q\left(\sqrt{\frac{2\rho t}{1-\rho}}, \sqrt{\frac{2\gamma}{\bar{\gamma}_c(1-\rho)}}\right)\right]^L e^{-t} dt d\gamma. \tag{40}$$

4) $P_e(\gamma) = Q(a\sqrt{\gamma}, b\sqrt{\gamma}) - (1/2)I_0(ab\gamma) e^{-(a^2+b^2/2)\gamma}$*:* The performance analysis of $\pi/4$-shifted differentially encoded QPSK ($\pi/4$ DQPSK) has received considerable attention, owing to its adoption in the second generation of North American and Japanese digital cellular standards, such as the North American IS-54 and Japanese Personal Digital Cellular (PDC) (see [35] and its references). The CEP for $\pi/4$ DQPSK and noncoherent correlated BFSK can be written as [30], [36]

$$P_e(\gamma) = Q(a\sqrt{\gamma}, b\sqrt{\gamma}) - \frac{1}{2} I_0(ab\gamma) e^{-\frac{a^2+b^2}{2}\gamma} \tag{41}$$

where $(a,b) = (\sqrt{2-\sqrt{2}}, \sqrt{2+\sqrt{2}})$ for $\pi/4$ DQPSK and $(a,b) = (\sqrt{1-\sqrt{1-\lambda^2}/2}, \sqrt{1+\sqrt{1-\lambda^2}/2})$ for correlated NCBFSK, where λ is the correlation coefficient between the binary signals.

Substituting (41) into (31), we obtain the SER as

$$\begin{aligned}\bar{P}_e =& \frac{a^2-b^2}{4} \int_0^\infty e^{-\frac{(a^2+b^2)\gamma}{2}} I_0(ab\gamma) \\ &\times \int_0^\infty \left[1 - Q\left(\sqrt{\frac{2\rho t}{1-\rho}}, \sqrt{\frac{2\gamma}{\bar{\gamma}_c(1-\rho)}}\right)\right]^L e^{-t} dt d\gamma.\end{aligned} \tag{42}$$

B. Outage Probability

Outage probability is another standard performance measure of digital communication systems. It is defined as the probability that the output instantaneous SNR γ falls below a certain given threshold γ_{th}. Hence, evaluating the output cdfs (15), (21), and (22) at γ_{th}, we immediately obtain the outage probability for L-branch SC in equally correlated fading channels as follows:

$$P_{\text{out}} = \Pr(0 \le \gamma \le \gamma_{\text{th}}) = F_{\gamma_{sc}}(\gamma_{\text{th}}). \tag{43}$$

C. Output Quality Indicators

The mean output SNR of a diversity combiner is often used as a comparative performance measure. We only consider Rayleigh fading for brevity.

Since we have derived the output pdf [(17)] and output mgf [(19)], the moments of the output SNR can be determined. The mean output SNR is obtained as

$$\begin{aligned}\bar{\gamma}_{\text{sc}} =& E(\gamma_{\text{sc}}) \\ =& -\left.\frac{d\phi_{\text{sc}}(s)}{ds}\right|_{s=0} \\ =& L\bar{\gamma}_c(1-\rho)^2 \int_0^\infty x e^{-x} \int_0^\infty e^{-t} \left[1 - Q(\sqrt{2\rho t}, \sqrt{2x})\right]^{L-1} \\ &\times I_0(2\sqrt{\rho x t}) dt dx.\end{aligned} \tag{44}$$

More generally, we obtain higher order moments as

$$\bar{\gamma^n_{\text{sc}}} = E\left(\gamma_{\text{sc}}^n\right) = \int_0^\infty x^n p_{\gamma_{\text{sc}}}(x) dx = (-1)^n \left.\frac{d^n \phi_{\text{sc}}(s)}{ds^n}\right|_{s=0}. \tag{45}$$

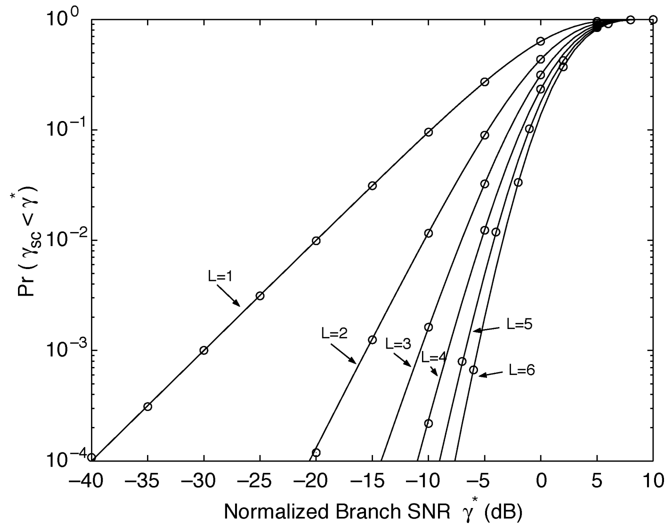

Fig. 1. CDF of the SC output SNR in equally correlated Rayleigh fading; $\rho = 0.5$. Circles denote simulation points.

Using the above expressions, we can readily derive other useful measures, such as the central moments, the skewness, the kurtosis, and the Karl Pearson's coefficient of variation (amount of fading) [37]. For brevity, we do not develop such results here.

VI. Numerical Results

Several numerical (represented by lines) and simulation (represented by circles and plus signs) results are given to illustrate the effect of correlation on the performance of SC in several fading channels. In all of the figures, ρ is the correlation among underlying Gaussian RVs, L is the diversity order, and $\gamma^* = \gamma/\bar{\gamma}_c$ is the normalized branch SNR. Note that semi-analytical simulation results are provided as an independent check of our analytical results. We use the Cholesky decomposition approach [38] to generate the equally correlated complex Gaussian variables and transform them to the Rayleigh, Ricean, and Nakagami-m envelopes.

Fig. 1 shows the effect of L on the output cdf $F_{\gamma_{\text{sc}}}(\gamma^*)$ of SC in equally correlated Rayleigh. The case of $L = 1$ represents a situation with no diversity. As expected, diversity gain can still be achieved even with correlated fading. The maximum additional diversity gain is achieved with dual-branch diversity. With increasing diversity order L, additional diversity gain diminishes, as is the case for independent fading.

Figs. 2–4 show the impact of ρ on the output cdf of four-branch SC in equally correlated Rayleigh, Ricean, and Nakagami-m fading channels, respectively. The case of $\rho = 0$ represents independent fading. The case of $\rho = 1$ represents a single-branch case. Observe that the maximum diversity gain will not be achieved when correlated fading exists. The diversity gain decreases as ρ increases. However, the diversity gain is still available even with high correlation. We can also see that, in the low correlation case, where ρ is small, the performance of SC is comparable to that in the independent case. However, in heavily correlated fading channels, where ρ

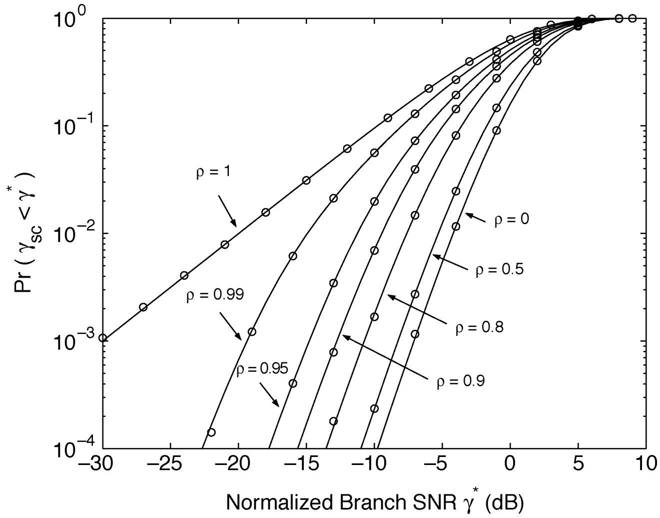

Fig. 2. Impact of fading correlation on the cdf of the SC output SNR in equally correlated Rayleigh fading; $L = 4$. Circles denote simulation points.

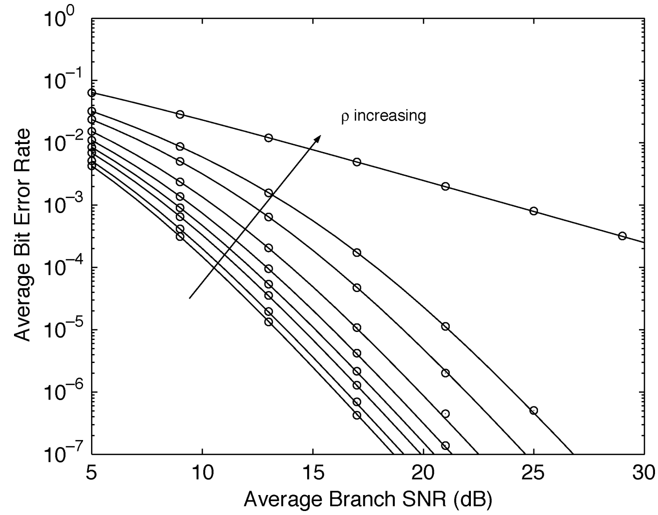

Fig. 5. Average BER of BPSK with four-branch SC in different equally correlated Rayleigh fading channels; $\rho \in \{0, 0.3, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 1\}$. Circles denote simulation points.

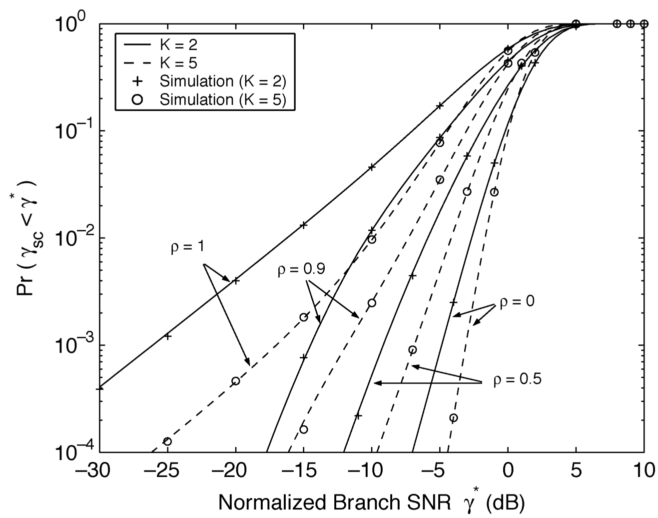

Fig. 3. Impact of fading correlation on the cdf of the SC output SNR in equally correlated Ricean fading; $L = 4$.

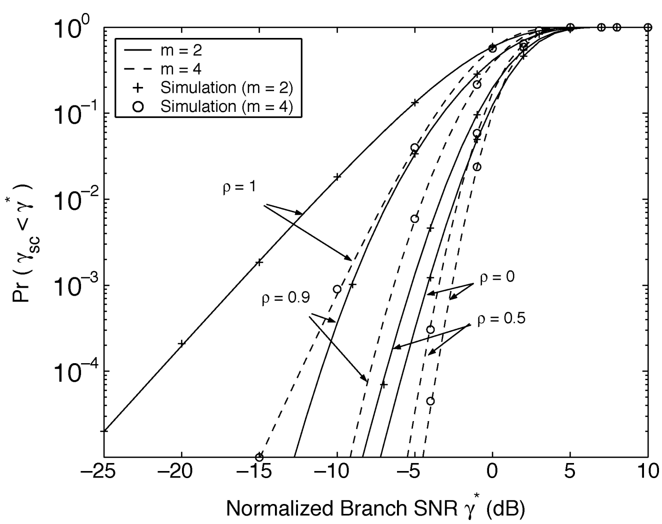

Fig. 4. Impact of fading correlation on the cdf of the SC output SNR in equally correlated Nakagami-m fading; $L = 4$.

tends to 1, a minute increase of ρ will cause severe degradation of the SC performance.

Fig. 5 shows the effect of ρ on the BER of BPSK with SC in equally correlated Rayleigh fading channels. The correlation among the available signals results in significant loss in performance. As an example, the BER of BPSK with SC increases from 3.6×10^{-6} to 1.9×10^{-4} when ρ increases from 0.3 to 0.9 at an average branch SNR of 15 dB.

VII. Conclusion

We have derived new representations for equally correlated Rayleigh, Ricean, and Nakagami-m fading gains. We showed that the cdfs of the SC output SNR can be represented as single-fold integral and derived infinite series representations. Consequently, unlike the other two existing methods [12], [14], any number of diversity branches can be handled as a single-fold integral (for the outage probability) or two-dimensional integral (for the error rate and the mean output SNR). The complexity of our approach does not increase with the number of diversity branches. These representations therefore resolve the long-standing open problem of SC performance in equally correlated fading channels. Consequently, we have derived SER expressions and outage and output statistics. Numerical results show that diversity benefits still exist in correlated fading channels, although the maximum diversity gain will not be achieved when fading is correlated. The representations developed in this paper can also be used to analyze EGC and other generalized SC schemes. These applications are currently being investigated. Finally, using bounds of the Marcum Q-function, we may develop performance bounds for L-branch SC in equally correlated fading channels.

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