A New Class of Smith Predictors for Network Congestion Control¹

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Abstract

A new class of time-variant Smith predictors for buffer set point control over communication networks is proposed. The new Smith predictor uses two different types of time-variant network delay models, i.e. the forward delay (modeling delay effects on the data stream) and the return delay (modeling delay effects on the buffer occupancy information.) The proposed control scheme tracks the desired buffer level, even under large delay uncertainties, abrupt delay changes and additional disturbances such as unknown buffer depletion rates. The question of stability of the resulting system will also be analyzed. Applications of the proposed scheme range from load balancing in distributed computing to congestion control in WAN networks.

1 INTRODUCTION

Classical Smith predictors are used to eliminate known, constant delays from feedback control loops [1], thus allowing an increased loop gain before instabilities occur. However, if the delays are uncertain or even time-variant, classical Smith predictor performance deteriorates and the control system can easily become unstable. A particularly important case arises, if the plant to be controlled is an integrator. This is due to the fact that the relationship between the net input rate into a buffer and the occupancy level of the buffer is given by integration. It is therefore not surprising that the problem of controlling an integrator plant (buffer) that is connected through time - delays to a controller (variable bit rate source) has recently attracted quite some attention in the networking and the control community [2, 3]. In all but a handful of papers [4-6], constant network delays were assumed. The validity of such a constant delay assumption is limited and breaks down for dynamic networks or when network traffic is bursty. Of course, any Smith predictor Kamal Premaratne

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that is to work well under all traffic conditions must model the network connections as time-variant delays. Based on this idea, this paper presents a queue control mechanism that is based on time-variant Smith predictors. The design insures tracking of the desired buffer set point under a number of non-ideal situations such as delay mismatches in the forward and return path as well as unknown buffer depletion rates.

2 PRELIMINARIES 2.1 The classical Smith predictor

Classical Smith predictors [1] are used to remove potentially destabilizing delays from the feedback loop by employing a "loop cancellation" technique:



Figure 1: Classical Smith predictor

In Figure 1a) the classical Smith predictor is shown. The idea is to design the controller such that the outside loop (that includes the delay τ_1) is canceled through the use of an internal loop. Once the loops cancel, the system in Figure 1a) is equivalent to the systems in Figure 1b) and c). The net result is that in the equivalent system in Figure 1c) the delay τ_1 is not longer in the loop. This allows to design a high performance controller C(z) using the classical control tools. The fundamental assumptions for this

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technique are a constant, and exactly know delay τ_1 .

2.2 The forward and backward delays

The system under investigation is shown in Figure 2. The controller SP(z) is transmitting data to a queue (modeled here as an ideal buffer). The data undergoes a time variant delay that needs to be modeled using a Variable Bit Rate (VBR) interface (see [7]). The length of the queue at the switch y(n) is fed back to the controller and also encounters a timevariant delay. We assume that the controller always uses the most recently available sample and thus an Hold Freshest Sample (HFS) [7] interface is appropriate. The input u(n) represents the desired buffer occupancy level. At equilibrium, we have y(n) = u(n).



Figure 2: Network Embedded Control System

2.3 Problem formulation

In the light of the previous results discussed in sections 2.1 and 2.2, this paper will analyze the benefits of a time-variant Smith predictor for network control systems similar to the one shown in Figure 3. In particular, the case with delay uncertainty in $\tau_1(n)$ and $\tau_2(n)$ is a key focus of this work. This problem is of great importance in congestion control mechanism for Wide Area Networks (WANs).



Figure 3: The studied system when a proportional gain is considered

3 MAIN RESULT 3.1 The time-variant Smith predictor

Figure 4 depicts the proposed time-variant Smith predictor for the congestion control of wide area networks.

The quantity y_{out} represents the amount of data that is depleted from the buffer at each time step. The same amount of data is, ideally, known and accounted for at the controller y_{out}^* .

If the controller and the plant have synchronized clocks it is possible for the controller to measure the



Figure 4: Time-variant Smith predictor for congestion control

delays in the return path $\tau_2(n)$ and to match them exactly in the controller model. However, the forward delays cannot be measured or predicted; at the best the forward delays can be measured at the plant (again assuming synchronization) and transmitted at the controller together with the feedback information.

3.2 A modified design for minimizing rise times

Figure 5 shows a modified Smith predictor. In the case of perfect delay matching both the forward and the return path the transfer function of the system is equal to one (plus a delay). Key difference to the design in Figure 5 is the elimination of the top feedback loop, containing the buffer plant. This results in overall system, that perfectly cancels the plant dynamics and the rise time is affected only by the forward delay. Since in this case we rely on perfect plant/disturbance information this feedback system will behave less robustly then the one in Figure 4.



Figure 5: Modified Time-variant Smith predictor for congestion control

3.3 Properties of the designs

In this subsection analyze the properties of the designs, such as stability, tracking of the input, rise time, maximum allowable proportional controller gain, etc.

We will analyze side by side the proportional controller, the classical Smith predictor with fixed delays, the time-variant Smith predictor presented in section 3.1 and the modified Smith predictor presented in section 3.2

For all the systems we assume bounded delays both in the forward and in the return paths:



Figure 6: a) The result of the simulations for the four systems. b) Detailed view of the simulation results at the second step

$$0 \le \tau_1(n) \le \bar{\tau} = 5 \tag{1}$$
$$0 \le \tau_2(n) \le \bar{\tau} = 5 \tag{2}$$

For the time-variant Smith predictor and for the modified Smith predictor we assumed that that the integrator plant and the controller are synchronized and that the feedback packets sent by the plant have time-stamps such that it is possible to measure the delays $\tau_2(n)$ and compensate them in the Smith predictor: $\tau_2^*(n) = \tau_2(n)$.

The time-variant stability analysis of either of the considered systems is either NP hard to perform or conservative. Moreover it was shown in [5, 8] that the presence of time-variant delays in the forward path causes the system not to have an equilibrium point. In [9] the simulations indicated that, for this kind of systems (i.e. proportional controller, time-variant delays, integrator plant), the time-variant stability range is almost identical to the time-invariant stability range. It was shown that for a first order system with delays in the feedback path the time-variant and time-invariant stability regions match closely.

Therefore, in the simulations to follow we will consider the controller gains k that will result in the largest loop-gain while keeping the arising time-invariant systems stable.

For the system depicted in Figure 3 with the two delays bounded as in the equations (1),(2) a time invariant analysis results in a stability range for the gain k:

A classical Smith predictor with fixed delays $\tau_1^* = \tau_2^* = 2$ has a time-invariant stable gain range given by:

The system in Figure 4, if we fix $\tau_1^* = 2$ and assume that we can match the delays in the return path $\tau_2^* = \tau_2$ has a time-invariant stability range given by:

The system in Figure 5, if we fix $\tau_1^* = 2$ and assume that we can match the delays in the return path $\tau_2^* = \tau_2$ has a time-invariant stability range given by:

We simulated the four systems for a total of 1000 time steps. In all simulations we used time-variant delays in the forward and return paths. For each system we used the same delay trace. Since the typical input for such systems is a superposition of step inputs, we used two step inputs (as it can be seen in Figure 6.

Table 1 shows some characteristics of the two step responses presented in Figure 6.

As it can be seen from table 1 the time-variant Smith predictor and the modified version typically perform better than the proportional controller and the classical Smith predictor. One drawback of the modified time-variant Smith predictor is the inability to adapt if there is a disturbance in the depletion rate of the buffer.

Consider the case of a mismatch in the depletion rate at the buffer y_{out} and the depletion rate known by the controller y_{out}^* (i.e. $y_{out} \neq y_{out}^*$). It is known that the Smith predictor used for an integrator plant is unable to completely eliminate a constant disturbance [10]. In Figure 7 the depletion rate at the buffer is doubled $(y_{out} = 2y_{out}^*)$. With the exception of the modified Smith predictor, the other three controllers still track (with a small steady state error) the input.

	Proportional	Classical	Time-variant	Modified
	Controller	Smith predictor	Smith predictor	Smith predictor
input-output MSE	5.1589	4.5592	4.1787	3.6562
Rise Time	10	10	10	6
at first step				
Rise Time	14	12	14	2
at second step				
95% Settling Time	24	30	26	24
at first step				
95% Settling Time	23	19	18	7
at second step				
% Overshoot	49%	78%	51%	51%
at first step				
% Overshoot	11%	17%	6%	1%
at second step				

 Table 1: Step response characteristics for the simulation in Figure 6



Figure 7: The result of the simulations for the four systems in the case of mismatch in the depletion rates $(y_{out} \neq y_{out}^*)$

4 CONCLUSION

This paper provides a simulation based comparison of the performance of various Smith predictor based network control systems. The simulation shows that the proposed control schemes (the timevariant Smith predictor and the modified Smith predictor) feature better tracking and faster rise and settling time. This performance improvement is accomplished using a time-variant Smith predictor that actually models the time-variant forward and return delays.

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