

# An Adaptive Compress-and-Forward Scheme for the Orthogonal Multiple-Access Relay Channel

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**Abstract**—We consider a wireless relay network where two sources transmit independent information on mutually orthogonal channels to a common destination with the help of one relay. Based on the expression for the achievable rate in such a network for compress-and-forward relaying without Wyner-Ziv coding, we design mutual-information preserving quantizers for compression at the relay. In the proposed relaying scheme, both the relay and the destination share a fixed set of quantizers, among which the relay selects a suitable one depending on the channel quality on the source-relay links for compression of its received values. Simulations performed in Rayleigh block fading channels reveal that full diversity order of two can be achieved using that scheme. We also comment on the size of the quantizer set and the associated signaling overhead.

## I. INTRODUCTION

Diversity techniques have been widely studied as an effective means to combat multipath fading inherent in wireless communication channels. Whenever the application of time or frequency diversity techniques is precluded due to delay or bandwidth constraints, multiple transmit and/or receive antennas can often provide a form of spatial diversity. However, the placement of multiple antennas at mobile terminals of, e.g., a cellular communication network, is sometimes impeded by size limitations on the mobile devices. Cooperative diversity, first proposed in [1], [2], introduces spatial diversity without interfering with the size limitations of the terminals by allowing nodes to deliberately cooperate in facilitating their transmissions. One well known example is the relay channel with one source, one destination, and a single relay. However, under orthogonal transmission, the extra resources allocated to the relay result in a loss of spectral efficiency, which can be reduced by allowing several users to share one relay for joint processing. We therefore focus on the orthogonal multiple-access relay channel (MARC) displayed in Fig. 1 in the following, where two sources transmit independent information to a common destination via a single relay.

In related work, diversity achieving schemes are proposed for distributed antenna systems [3], and for the MARC using low-density parity-check codes [4] and distributed turbo codes [5], combined with network coding [6]. Common to that work is that the relay node is required to decode the source messages perfectly to allow for successful use of such a decode-and-forward scheme [7]. In contrast, for compress-and-forward

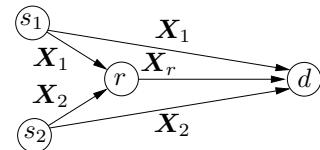


Fig. 1. The multiple-access relay channel.

[7], [8], the relay is not required to decode reliably; instead, it compresses its received symbols using Wyner-Ziv coding [9] to take advantage of the side information available at the destination before forwarding the compressed values. Design of compression schemes for compress-and-forward in the relay channel with a single source was studied in [10] and [11] with and without Wyner-Ziv coding, respectively. In more recent work [12], the authors combine network coding with analog forwarding of beliefs from the relay achieving notable gains in fixed additive white Gaussian noise (AWGN) channels, which can be improved upon by proper quantization at the relay [13].

Our goal is to introduce a practical, low-complexity, and yet diversity achieving compress-and-forward scheme for the MARC. To that end, we use a variational principle known as the information bottleneck method [14] to formulate the quantizer design problem and compute two-dimensional quantizers for symbol-by-symbol quantization of the received values at the relay [15]. In contrast to [15] however, the relay does not invoke soft decoders but quantizes the received values directly, which reduces the amount of delay introduced by the relay. The quantizers require little feedback from the destination to the relay, since they do not exploit the side information available to the destination for compression at the relay. Further, contrary to [15], we motivate the constraint function of the optimization problem characterizing the quantizer design using the expression for the achievable rates for compress-and-forward without Wyner-Ziv coding in the orthogonal MARC. Building upon these quantizers, we propose a scheme in which the relay adaptively selects a suitable quantizer from a fixed set based on the channel quality on its incoming links, and show by means of simulation that second order diversity can be achieved in Rayleigh block fading channels. Further, we compare the performance of our scheme with one where the relay forwards the quantized beliefs about the XOR of the

coded messages to the destination.

## II. SYSTEM MODEL

### A. Channel Model

Suppose that the relay's radio capabilities are limited by a half-duplex constraint, i.e., the relay cannot receive and transmit simultaneously in the same frequency band. Further, the transmissions from the sources and the relay are assumed to be orthogonalized either in frequency or in time. Despite the suboptimality of this constraint, the restriction to orthogonal channels eases practical implementation. Without loss of generality, we assume the orthogonality to be guaranteed by time division, so that a first slot is assigned to source 1, a second slot to source 2, and a third slot to the relay. Also, we restrict our study to a scheme in which an equal amount  $\alpha n$ ,  $0 < \alpha < 0.5$ , of the  $n$  total channel uses is allocated to each of the two sources, and the remaining  $(1 - 2\alpha)n$  uses of the channel are assigned to the relay node. Let  $\mathbf{X}_i \in \mathbb{M}_s^{\alpha n}$ ,  $i \in \{1, 2\}$ , and  $\mathbf{X}_r \in \mathbb{M}_r^{(1-2\alpha)n}$  be the transmitted vectors from the modulation alphabet  $\mathbb{M}_s$  at the sources and  $\mathbb{M}_r$  at the relay, respectively. For a path-loss coefficient  $p$  and distances  $d_{sr,i}, d_{sd,i}$ , and  $d_{rd}$  between the sources and the relay, the sources and the destination, and the relay and the destination, respectively, the received signals  $\mathbf{Y}_{ri} \in \mathbb{C}^{\alpha n}$ ,  $\mathbf{Y}_{di} \in \mathbb{C}^{\alpha n}$ , and  $\mathbf{Y}_{dr} \in \mathbb{C}^{(1-2\alpha)n}$  at the relay and at the destination read

$$\begin{aligned}\mathbf{Y}_{ri} &= \frac{h_{sr,i}}{\sqrt{d_{sr,i}^p}} \mathbf{X}_i + \mathbf{N}_{sr,i} \\ \mathbf{Y}_{di} &= \frac{h_{sd,i}}{\sqrt{d_{sd,i}^p}} \mathbf{X}_i + \mathbf{N}_{sd,i}\end{aligned}$$

in time slot  $i$ , and

$$\mathbf{Y}_{dr} = \frac{h_{dr}}{\sqrt{d_{rd}^p}} \mathbf{X}_r + \mathbf{N}_{dr}$$

in the last time slot, where  $h_{sr,i}$ ,  $h_{sd,i}$ , and  $h_{dr}$  are the complex channel fading coefficients satisfying  $E[|h_{sr,i}|^2] = E[|h_{sd,i}|^2] = E[|h_{dr}|^2] = 1$ , and the additive noise variables are independent proper complex Gaussian noise variables with zero mean and variance normalized to unity. The average values of the signal-to-noise ratio (SNR) are given as  $\rho_{sr,i} = P_i/d_{sr,i}^p$ ,  $\rho_{sd,i} = P_i/d_{sd,i}^p$ , and  $\rho_{rd} = P_r/d_{rd}^p$ , where  $P_i$  and  $P_r$  are the powers of the sources and the relay. Throughout, we make the common assumption that the receivers know the instantaneous SNR values  $\gamma_{sr,i} = |h_{sr,i}|^2 \rho_{sr,i}$ ,  $\gamma_{sd,i} = |h_{sd,i}|^2 \rho_{sd,i}$ , and  $\gamma_{rd} = |h_{rd}|^2 \rho_{rd}$  of their channels, and that the transmitters only possess knowledge about the average SNR. In particular, the relay is assumed to lack both average and instantaneous channel state information (CSI) of the source–destination channels due to their fading nature and limited signaling from the destination to the relay.

### B. Relay

The strategy at the relay is compress-and-forward [7], [8]. However, the relay cannot employ Wyner–Ziv coding to exploit the side information at the destination since instantaneous CSI about the source–destination channels (required to obtain

the statistics of  $\mathbf{Y}_{d1}$  and  $\mathbf{Y}_{d2}$ ) is not available at the relay, as explained before. Instead, it resorts to classical source coding, specifically, to two-dimensional symbol-by-symbol quantization. More formally, upon reception of  $\mathbf{Y}_{r1}$  and  $\mathbf{Y}_{r2}$ , the relay uses a two-dimensional quantizer with quantization rule  $q(y_{r1}, y_{r2})$ ,  $q : \mathcal{Y}_{r1} \times \mathcal{Y}_{r2} \rightarrow \mathcal{Z}$ , to jointly compress the pair of received sequences  $(\mathbf{Y}_{r1}, \mathbf{Y}_{r2})$  on a pairwise symbol-by-symbol basis yielding the compressed version  $\mathbf{Z} \in \mathcal{Z}^{\alpha n}$ , where  $\mathcal{Z}$  refers to the reproduction alphabet of the quantizer. Since the quantizer is two-dimensional and invariant for the entire block, the  $m$ -th component  $Z_m$  of  $\mathbf{Z}$  is given by  $Z_m = q(Y_{r1,m}, Y_{r2,m})$ . The design of the quantizer at the relay is related to [15], where, however, the relay invokes soft decoders before quantization of the likelihood-ratios at the soft decoder output. In contrast, we will motivate the quantizer design framework from the compress-and-forward rate region and summarize the design in Section IV. Given that the symbols  $\{z_1, z_2, \dots, z_{|\mathcal{Z}|}\}$  of the reproduction alphabet  $\mathcal{Z}$  are not uniformly distributed in general, the compression  $\mathbf{Z}$  is source encoded before encoding into the codeword  $\mathbf{X}_r$  for transmission from the relay.

### C. Destination

At the destination, the received sequences  $\mathbf{Y}_{d1}$ ,  $\mathbf{Y}_{d2}$ , and  $\mathbf{Y}_{dr}$  are used for joint decoding. Following the compress-and-forward strategy, first, an estimate  $\hat{\mathbf{Z}}$  of  $\mathbf{Z}$  is recovered using the received vector  $\mathbf{Y}_{dr}$  from the relay. Then, the source messages are decoded using  $\mathbf{Y}_{d1}$ ,  $\mathbf{Y}_{d2}$ , and  $\hat{\mathbf{Z}}$ .

## III. ACHIEVABLE RATES

Achievable rates for compress-and-forward relaying were established in [7] and extended to the MARC in [8]. However, the rates in [8, Eq. (14)] hold for a MARC model with a relay having a half-duplex constraint only, i.e., the two sources (and during the relay transmit state, the relay as well) are allowed to send simultaneously. Further, the relay makes use of Wyner–Ziv coding, using the side information at the destination. We specialize the rate region of [8] to the orthogonal MARC without Wyner–Ziv coding in the proposition below.

*Proposition 1:* For a time sharing parameter  $\alpha$ , the rate tuple  $(R_1, R_2)$  is achievable using compress-and-forward without Wyner–Ziv coding in the orthogonal MARC if

$$\begin{aligned}R_1 &\leq \alpha I(X_1; Y_{d1}, Z|X_2) \\ R_2 &\leq \alpha I(X_2; Y_{d2}, Z|X_1) \\ R_1 + R_2 &\leq \alpha I(X_1, X_2; Y_{d1}, Y_{d2}, Z),\end{aligned}\tag{1}$$

subject to the constraint

$$(1 - 2\alpha)I(X_r; Y_{dr}) \geq \alpha I(Y_{r1}, Y_{r2}; Z).\tag{2}$$

*Proof:* The constraint (2) ensures that the compression  $\mathbf{Z}$  can be perfectly recovered at the destination, since the required source coding rate  $\alpha I(Y_{r1}, Y_{r2}; Z)$  at the relay [16, Chapter 10] does not exceed the available rate  $(1 - 2\alpha)I(X_r; Y_{dr})$  on the relay–destination link. The rates (1) can then be shown to be achievable by standard arguments viewing the MARC as a channel with three outputs  $Y_{d1}$ ,  $Y_{d2}$ , and  $Z$ . ■

The rate region in Proposition 1 can be rewritten to serve as a basis to establish a framework for the design of quantizers at the relay, treated next.

#### IV. QUANTIZER DESIGN

We expand the expressions for the achievable rates in (1) in a term referring to the information obtained via the direct link and in a term corresponding to the information received via the relay, respectively, as

$$\begin{aligned} R_1 &\leq \alpha [I(X_1; Y_{d1}) + I(X_1; Z|X_2, Y_{d1})] \\ R_2 &\leq \alpha [I(X_2; Y_{d2}) + I(X_2; Z|X_1, Y_{d2})] \end{aligned} \quad (3)$$

$$R_1 + R_2 \leq \alpha [I(X_1, X_2; Y_{d1}, Y_{d2}) + I(X_1, X_2; Z|Y_{d1}, Y_{d2})].$$

Here, the rate gain offered by the transmission from the relay is reflected in the information expressions on the right hand sides of (3), namely  $I(X_1; Z|X_2, Y_{d1})$ ,  $I(X_2; Z|X_1, Y_{d2})$ , and  $I(X_1, X_2; Z|Y_{d1}, Y_{d2})$ . Consequently, to enlarge the set of achievable rates, the optimal quantization at the relay should be such that these mutual information expressions are maximal, bearing in mind the rate constraint on the relay-destination link. However, since our goal is not maximization of throughput but to supply reliability, we propose the following. Taking into account that statistics about the side information  $Y_{di}$ ,  $i \in \{1, 2\}$ , is not available at the relay in our setting, and to allow *opportunistic* allocation of source coding rate at the relay to the user with the stronger source-relay channel, the quantizer at the relay is designed such that  $I(X_1; Z|X_2) + I(X_2; Z|X_1)$  is large.

*Remark:* In [17], the authors study the relay channel with a single source, orthogonal transmissions, Gaussian codebooks, and compress-and-forward. They show that if the relay is unaware of the statistics of the side information, one can resort to classical source coding instead of the more complex Wyner-Ziv coding at the relay without significant performance loss in Rayleigh block fading channels. Although a similar result has yet to be found for the orthogonal MARC, the work in [17] suggests that including Wyner-Ziv coding at the relay might not be critical to increase reliability.

We will now describe the quantizer design in more detail. Let  $\gamma_{sr,1}$  and  $\gamma_{sr,2}$  be given, and therefore also the probability density functions  $p(y_{r1})$  and  $p(y_{r2})$  of the received values  $Y_{r1}$  and  $Y_{r2}$ . Since the quantization with quantization rule  $q(y_{r1}, y_{r2})$  is performed jointly over  $(Y_{r1}, Y_{r2})$  without account of any side information, we can also describe the two-dimensional quantizer by a probability density  $p(z|y_{r1}, y_{r2})$ , where  $p(z|y_{r1}, y_{r2})$  is a mapping taking only the values zero and one. Then, we formulate the optimization problem for the quantizer design as a minimization of the source coding rate  $I(Y_{r1}, Y_{r2}; Z)$  over all mappings  $p(z|y_{r1}, y_{r2})$ , with a constraint on  $I(X_1; Z|X_2) + I(X_2; Z|X_1)$  as

$$\begin{aligned} \min_{p(z|y_{r1}, y_{r2})} \quad & I(Y_{r1}, Y_{r2}; Z) \\ \text{s.t.} \quad & I(X_1; Z|X_2) + I(X_2; Z|X_1) \geq I, \end{aligned} \quad (4)$$

where  $I > 0$ . The problem of the form of (4) is known as a variational problem in the setting of the information

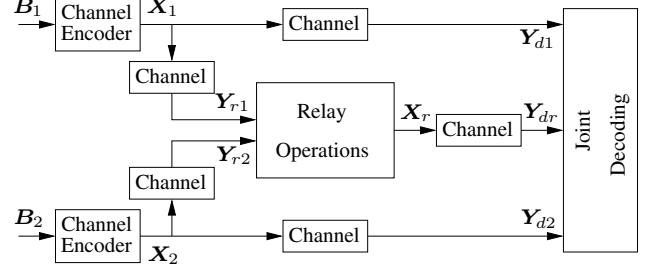


Fig. 2. System model.

bottleneck method [14]. Using a Lagrangian parameter  $\beta > 0$ , the Lagrangian  $\mathcal{L}(p(z|y_{r1}, y_{r2}), \beta)$  to (4) reads

$$\begin{aligned} \mathcal{L}(p(z|y_{r1}, y_{r2}), \beta) = & I(Y_{r1}, Y_{r2}; Z) \\ & - \beta (I(X_1; Z|X_2) + I(X_2; Z|X_1)), \end{aligned} \quad (5)$$

and despite being a nonconvex problem, an iterative algorithm similar to the Blahut-Arimoto algorithm [18] can be shown [14] to converge to a local minimum of (4). To ensure that the resulting mapping  $p(z|y_{r1}, y_{r2})$  does represent a two-dimensional quantizer, the parameter  $\beta$  has to satisfy  $\beta \gg 0$ ; otherwise,  $p(z|y_{r1}, y_{r2})$  would not correspond to a deterministic mapping. However, due to the nonconvexity of the design problem, that solution is not guaranteed to be the global optimum of (4). The interested reader is referred to [15] for more details on the design of quantizers with the information bottleneck method.

#### V. PRACTICAL IMPLEMENTATION

We now describe the practical implementation of the entire system and propose a compress-and-forward scheme based on quantizers designed as outlined in Section IV. Further, the reference systems used for comparison of the numerical results in Section VI are summarized.

##### A. System with the Relay

Throughout, we let the time sharing parameter  $\alpha = 1/3$  be fixed; optimization of  $\alpha$  is beyond the scope of this paper. At both sources, we assume  $k = 2000$  information bits, and take  $n = 12000$  total uses of the channel, so that  $R_1 = R_2 = k/n = 1/6$ . Fig. 2 shows the system model.

1) *Mobile Stations:* Each source  $i \in \{1, 2\}$  uses the turbo code of rate 1/2 specified in the UMTS standard [19] to encode its binary information word  $B_i \in \{0, 1\}^k$  into a codeword  $C_i \in \{0, 1\}^{2k}$ , which is then mapped to the corresponding vector of BPSK symbols  $X_i \in \{+1, -1\}^{\alpha n}$ .

2) *Relay:* The relay and the destination share a set of quantizers  $\mathcal{Q}$  designed with the framework introduced in Section IV. Given the realizations of the received sequences  $(y_{r1}, y_{r2})$  and of  $\gamma_{sr,1}$  and  $\gamma_{sr,2}$ , the relay selects the proper quantizer in  $\mathcal{Q}$ , interleaves<sup>1</sup> the vector  $y_{r2}$ , computes the sequence  $z$ , source encodes that sequence with an arithmetic encoder, and channel encodes using the UMTS turbo code

<sup>1</sup>Interleaving is necessary here to avoid short cycles in the iterative decoder at the destination, to be introduced later.

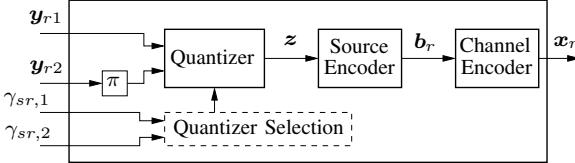


Fig. 3. Relay operations.

of appropriate rate, yielding the sequence  $\mathbf{x}_r \in \mathbb{M}_r^{(1-2\alpha)n}$ , where the modulation alphabet  $\mathbb{M}_r$  at the relay is chosen to be 16-QAM. For our numerical study, we compare two sets of quantizers. The first set  $\mathcal{Q}_1$  contains one quantizer with  $|\mathcal{Z}| = 5$  quantization regions for every pair of instantaneous SNR values  $\gamma_{sr,1}$  and  $\gamma_{sr,2}$  in the set

$$\mathcal{S}_1 = \{-9 \text{ dB}, -8 \text{ dB}, -7 \text{ dB}, -6 \text{ dB}, -5 \text{ dB}, -4 \text{ dB}, -3 \text{ dB}, -2 \text{ dB}, -1 \text{ dB}, 0 \text{ dB}, 1 \text{ dB}, 2 \text{ dB}, 3 \text{ dB}, 4 \text{ dB}, 5 \text{ dB}, 6 \text{ dB}, 7 \text{ dB}\}.$$

For this choice of  $\mathcal{S}_1$ , there are  $|\mathcal{Q}_1| = |\mathcal{S}_1|^2 = 289$  quantizers available at the relay. Consequently, signaling the relay's quantizer choice to the destination requires at most 9 bits; we assume this signaling to be perfect in the sequel. The other set  $\mathcal{Q}_2$  of quantizers consists of one quantizer with  $|\mathcal{Z}| = 5$  regions for every pair of  $\gamma_{sr,1}$  and  $\gamma_{sr,2}$  in the set  $\mathcal{S}_2 = \{-9 \text{ dB}, 0 \text{ dB}, 6 \text{ dB}\}$ , so that  $|\mathcal{Q}_2| = 9$ , resulting in 4 bits to signal the quantizer choice. Note that optimization of  $\mathcal{S}$  and  $\mathcal{Q}$  is not considered further in this work due to space constraints. The operations at the relay node are summarized in Fig. 3.

*3) Destination:* The destination uses the iterative decoder structure depicted in Fig. 4. In addition to two turbo decoders for the symbols  $y_{d1}$  and  $y_{d2}$  received over the direct links, the decoder contains the *relay check nodes* coupling the component turbo decoders, which are further detailed in Fig. 5. Note that the coupling of the component decoders at the destination is a result of the joint quantization at the relay. Essentially, if the information received via the relay is small, the decoder structure acts like two separate turbo decoders for  $y_{d1}$  and  $y_{d2}$ , since very little information can pass the check nodes. In contrast, if the information received from the relay is reliable, soft information is exchanged iteratively between the component decoders.

To avoid catastrophic error propagation in the variable-length arithmetic decoder in case of residual errors in  $\hat{b}_r$ , we assume a cyclic redundancy check (CRC) in the bits  $b_r$  sent by the relay, so that the information from the relay is discarded if  $\hat{z}$  cannot be reconstructed perfectly. The processing rules yielding the soft values  $\mathbf{L}_A^{(1)}, \mathbf{L}_A^{(2)}$  from  $\mathbf{L}_E^{(1)}, \mathbf{L}_E^{(2)}$ , and  $\hat{z}$  can be derived by applying the message passing rules for factor graphs to the function node  $p(x_1, x_2 | z)$  in the factor graph of the iterative decoder; see [15] for more details. Note that the density  $p(x_1, x_2 | z)$  depends on the choice of the quantizer and is among the outputs of the quantizer design algorithm.

#### B. Reference Systems

*1) Point-to-point link:* The point-to-point link without the aid of the relay serves as a first reference for the proposed

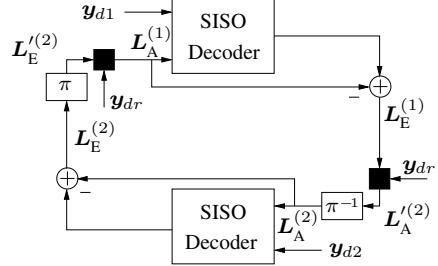


Fig. 4. Iterative decoder.

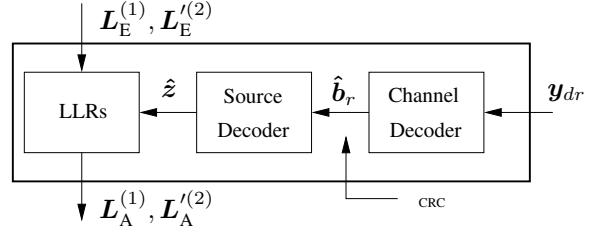


Fig. 5. Operations of the relay check node.

system. For a fair comparison, each source  $i$  encodes its information word  $\mathbf{B}_i \in \{0,1\}^k$  into a codeword  $\mathbf{C}_i \in \{0,1\}^{n/2}$  using the UMTS turbo code of rate 1/3. Again, BPSK modulation is used at the sources.

*2) Quantization of XOR at the relay:* Our second reference system includes the relay, but here, the relay does not perform two-dimensional quantization. Instead, it computes the likelihood ratios  $\mathbf{L} \in \mathbb{R}^{\alpha n}$  referring to the XOR  $\mathbf{x} = \mathbf{x}_1 \oplus \mathbf{x}'_2$  of the coded words based on its received vectors as in [13], where  $\mathbf{x}'_2$  is the interleaved version of  $\mathbf{x}_2$ . Following the computation of the XOR, the relay employs a scalar quantizer with quantization rule  $q(\ell)$  for compression of the block of likelihood ratios  $\mathbf{L}$ , so that the quantizer output is  $z \in \mathcal{Z}^{\alpha n}$  with  $z_m = q(L_m)$ ,  $m = 1, 2, \dots, \alpha n$ . The quantizer is obtained as a local optimum of the problem

$$\min_{p(z|\ell)} I(L; Z) \quad \text{s.t.} \quad I(X; Z) \geq D. \quad (6)$$

Note that although in [13], the relay performed soft decoding before quantization, the approach is conceptually related to the one used here without soft decoding. The remaining parameters are the same as in Section V-A; in particular,  $\mathcal{S}_{\text{XOR}} = \mathcal{S}_1$ , so that the quantizer set shared by the relay and the destination contains  $|\mathcal{Q}_{\text{XOR}}| = 289$  quantizers with  $|\mathcal{Z}| = 5$  quantization regions each.

## VI. NUMERICAL RESULTS

Throughout, we assume a symmetric network, i.e.,  $d_{sr} = d_{sr,1} = d_{sr,2}$  and  $d_{sd} = d_{sd,1} = d_{sd,2}$ . Further, the relay is placed closer to the destination than to the sources. In particular, we consider two cases:

- Case 1: the relay is placed between the sources and the destination, with  $d_{sr} = (9/10)d_{sd}$ .
- Case 2: the relay is placed behind the destination, with  $d_{sr} = (3/2)d_{sd}$ .

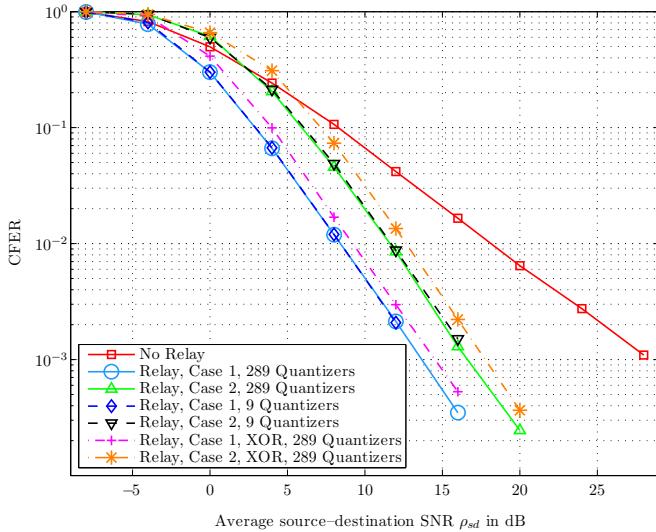


Fig. 6. Frame error rates for the MARC and the point-to-point link.

Due to the proximity of the destination and the relay, we take  $\rho_{rd} = \rho_{sd} + 15$  dB, and set  $p = 3.52$  for our study. The simulation results in Fig. 6 show the common frame error rate (CFER) of the reference systems and the system with the relay, for both network geometries. A common frame error is defined as the event that at least one information word  $u_i$  is decoded incorrectly. Based on these curves we observe that the schemes involving the relay achieve second order diversity for both case 1 and case 2, since the CFER decays proportional to  $\rho_{sd}^{-2}$ . However, if the relay processes the received values directly without going through the intermediate step of computing the likelihood ratios of the XOR of the coded vectors, considerably better performance is obtained. This is due to the fact that the reliability of the XOR at the relay is undesirably dominated by the weaker source–relay channel, a disadvantage avoided by joint quantization of the received values. Particularly, the system with the proposed two-dimensional quantizers at the relay gains more than 10 dB compared to the point-to-point link at relevant CFERs of  $10^{-3}$ . It is also important to note that the scheme involving joint quantization at the relay does not require the set of available quantizers at the relay to be prohibitively large. In fact, we observe that the system with 9 quantizers shared at the relay performs only marginally poorer than the one with a set of 289 quantizers, at considerable lower signaling overhead.

## VII. CONCLUSIONS

Starting from the rates achievable with compress-and-forward in the orthogonal MARC with classical source coding at the relay, we designed two-dimensional quantizers for compression of the received symbols at the relay. These quantizers are practical and of low complexity, since they process the pair of received sequences at the relay in a symbol-by-symbol manner. Further, the relay does not require knowledge of the side information at the receiver, a fact especially advantageous in wireless fading channels. In

a Rayleigh block fading environment, the relay chooses a suitable quantizer from a fixed set based on the SNR on the incoming links, and forwards its compressed estimate of the received sequences to destination. We observe from numerical results that full diversity order of two can be gained with this scheme. We emphasize that the observed diversity gain can be achieved with a practical compress-and-forward scheme of low complexity at the relay due the two-dimensional symbol-by-symbol quantization. Further, the scheme incurs small delay, since no decoding is performed at the relay node. Finally, we remark that the only overhead created through cooperation is due to the signaling of the quantizer choice at the relay to the destination.

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