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On Optimal Relay Placement and Sleep Control to Improve Energy Efficiency in Cellular Networks

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***Abstract*—We consider the joint optimization of relay station (RS) placement and RS sleep/active probability in order to enhance the energy efficiency of a one-dimensional cellular network. When the RSs are always active, we derive the conditions for optimal RS placement that minimizes transmission power of base stations (BSs) and RSs, subject to a user rate requirement. The closed-form placement solution is obtained for a path-loss exponent of two, and a simple numerical method for the solution under general values of the path-loss exponent is proposed and evaluated. When the circuit power consumption of active RSs is considered, RSs should enter sleep mode to save power. To be tractable, and since the random user arrivals are not predicted by the network, we consider a basic scheme where each RS randomly switches between sleep and active modes with a certain probability. An algorithm based on the projected Newton method is proposed to jointly optimize the RS placement and active probability. It is shown via numerical examples that the benefit of implementing RSs and optimizing RS placement is substantial and increases with the path-loss exponent. We also demonstrate the interaction between RS placement and sleep control when the total power consumption, including both the transmission power and the circuit power, is minimized.**

I. Introduction

Green solutions for information and communication technology (ICT), which is responsible for 2% to 10% of world wide energy consumption [1], provide a new dimension for design optimization of wireless systems. For cellular networks, it is shown in [1] that the base stations (BSs) consume nearly 60-70% of the total network energy. Therefore, improving the energy efficiency of BSs plays an important role in providing green mobile access solutions. Without modifying current cellular structures, one promising way to reduce the energy consumption of BSs is to use low power relay stations (RSs) [2] [3].

By deploying RSs in the network, the transmission power can be saved due to decreased distances between transmitter-receiver pairs. The transmission power saving depends on how these RSs are placed. On the other hand, implementing RSs in the network introduces other costs. In terms of energy consumption, the circuit power [4] that RSs consume should be considered. Especially when the traffic load of the network is low, long "idle" times can lead to substantial circuit power consumption [5]. Hence it is not efficient to keep RSs active all the time, even though they can potentially save transmission power. Thus, a mechanism to put RSs to sleep should be developed that minimizes overall energy consumption in the network. That is the mechanism we investigate in this paper. We model the RS sleep control as deciding the probability that an RS is in active mode. Intuitively, the optimal active probability for a given relay station will depend on its position. Thus, minimal energy consumption requires joint optimization of the active probabilities and placements of the RSs. It is this joint optimization that we address in this paper.

Early studies on energy efficient node placement mainly focus on sensor nodes [6]. For relays, [7] investigates the joint RS placement and initial energy allocation problem to maximize the sensor network lifetime. Other works on RS placement are proposed to maximize the network capacity [8], or to minimize the average probability of error [9]. Unlike these previous works, we optimize the RS placement jointly with RS sleep behavior to minimize the total power consumption, including the transmission power and the circuit power. The downlink of a one-dimensional highway cellular network is considered, and the load is modeled as the vehicle arrival rate. We exploit the projected Newton method [13] to solve the non-linear optimization problem of joint RS placement and active probability allocation. For the special case of minimizing the transmission power only, i.e., RSs are always active since we neglect circuit energy consumption, a closed-form expression for optimal RS placement is obtained when the path-loss exponent is two, and conditions for optimal RS placement with general values of path-loss exponent are also provided. Notably, our work serves as an initial effort on the combination of network planning with operation control, especially with respect to energy efficient wireless communications.

The remainder of the paper is organized as follows. In Section II, the one-dimensional highway cellular system with decode-and-forward (DF) relaying is introduced. In Section III, the problem of RS placement that minimizes the transmission power only is investigated under various values of path-loss exponent. In Section IV, our general method for solving the joint RS placement and active probability allocation problem is presented. Numerical examples are provided in Section

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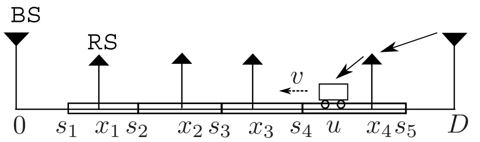


Fig. 1. One-dimensional highway cellular network with $M = 4$ RSs between two BSs. A vehicle at position u is traveling with speed v, served by two-hop DF relaying. Each rectangular box represents the coverage of an RS.

V, after which Section VI concludes the paper.

II. System Model and Problem Formulation

Consider the downlink of a one-dimensional cellular network corresponding to the highway scenario [10] as shown in Fig. 1. For simplicity, we assume that different frequency bands are used in adjacent BSs so that inter-cell interference is ignored. Therefore, we focus on the region between two adjacent BSs with length D, and we assume that M RSs are deployed along this region. The coordinates of the two BSs are denoted by 0 and D, and the positions of the RSs are denoted by a vector $\mathbf{x} = \{x_i\}_{i=1}^M$, where $0 \le x_1 \le x_2 \le \cdots \le D$. Each vehicle corresponding to one mobile user passes through this region with speed v m/s, and requires data service of constant rate r bits/s. A user can either directly communicate with the BS or utilize two-hop half-duplex DF relaying[1]. Based on a path-loss channel model, for a user at position $0 \le u \le D$, if direct transmission is adopted, the transmit power P_b from the BS that guarantees a given required rate r must satisfy the rate equation $r = W \log_2(1 + \frac{\eta_0 P_b}{d_0^\alpha})$. We then have

$$P_b = \frac{(2^{r/W} - 1) d_0^\alpha}{\eta_0}, \tag{1}$$

where α is the path-loss exponent; $\eta_0 = G_0/N_0$ includes the effects of the antenna gain G_0 from the BS to the user and thermal noise N_0; W is the channel bandwidth; and $d_0 = \min\{u, D-u\}$ is the distance from the BS to the user. If DF relaying is adopted via RS i, the BS transmission power P_b is

$$P_{bi} = \frac{(2^{2r/W} - 1) d_1^\alpha}{\eta_1}, \tag{2}$$

and the transmission power P_i from RS i is

$$P_i = \frac{(2^{2r/W} - 1) d_2^\alpha}{\eta_2}, \tag{3}$$

where $d_1 = \min\{x_i, D - x_i\}$ is the distance from the BS to RS i; $d_2 = |u - x_i|$ is the distance from RS i to the user; and η_1 and η_2 are defined similar to η_0 corresponding to the link from BS to RS and RS to user, respectively. We remark that half-duplex relaying requires two time slots, therefore the rate for each hop is $2r$. The criterion for selecting RS or direct transmission is to minimize the transmission power

$$i(u) = \arg\min \left\{ P_b, \{P_{bi} + P_i\}_{i=1}^M \right\}, \tag{4}$$

[1]A simple multi-hop relay requires large per-hop rate due to the half-duplex constraint, and thus leads to large per-hop transmission power. This provides marginal benefits on the total energy efficiency. Thus we consider the two-hop relay only.

where $i(u) = 0$ and $i(u) = M + 1$ represent the left and the right BS, respectively. We further define the *coverage* of RS i as the region in which the user will select RS i.

In addition to the transmission power, the power consumption associated with the RF circuits should be considered [4]. We assume that whenever the RS is active, the circuit power is P_{on}, regardless of whether the RS is transmitting or idle. To minimize the total power consumption, it is not optimal to keep all the RSs active, especially when the car arrivals are very sparse so that the energy consumption associated with P_{on} will outweigh the saving of transmission power via relaying. Therefore, we consider the case that the RSs can switch to sleep mode independently[2]. The active probabilities of RSs are denoted by a vector $\boldsymbol{\beta} = \{\beta_i\}_{i=1}^M$, where $0 \le \beta_i \le 1$, $\forall i$. The sleep coordination among RSs is not considered. A user can only select an active RS, and thus the expected transmission power for a user at position u is a function of u, $\mathbf{x}$ and $\boldsymbol{\beta}$:

$$P(u, \mathbf{x}, \boldsymbol{\beta}) = \mathbb{E}\left\{ \min\left\{ P_b, \{P_{bi} + P_i\}_{\text{RS } i \text{ is active}} \right\} \right\}. \tag{5}$$

Assume that users in each cell are served with orthogonal channels so that the transmission power for them can be added up. The average total power consumption is then given by

$$P_{\text{tot}} := \frac{\lambda}{v} \int_0^D P(u, \mathbf{x}, \boldsymbol{\beta}) \mathrm{d}u + P_{\text{on}} \boldsymbol{\beta}^T \mathbf{e}, \tag{6}$$

where λ is the vehicle arrival rate, i.e., the number of vehicles that enter the region of interest in a second; $\mathbf{e}$ is a vector with all elements equalling to 1. We aim to minimize P_{tot} by optimizing the RS placement $\mathbf{x}$ and RS active probability $\boldsymbol{\beta}$, i.e. we have the following optimization problem

$$\begin{aligned} \min \quad & P_{\text{tot}} \\ \text{s.t.} \quad & 0 \le x_i \le D, \quad i = 1, \ldots, M \\ & 0 \le \beta_i \le 1, \quad i = 1, \ldots, M. \end{aligned} \tag{7}$$

The optimization problem in (7) is hard to solve analytically, since (5) is a complicated piecewise function. To get more insight, we first investigate the RS placement to minimize the transmission power, i.e., $\boldsymbol{\beta} = \mathbf{e}$, for which a closed-form solution exists for $\alpha = 2$. Then an efficient numerical algorithm to jointly optimize $\mathbf{x}$ and $\boldsymbol{\beta}$ is presented.

III. Optimal RS Placement with $\boldsymbol{\beta} = \mathbf{e}$

Without sleep control, i.e., RSs are always active with $\boldsymbol{\beta} = \mathbf{e}$, the objective is to minimize the transmission power

$$\min \int_0^D P(u, \mathbf{x}, \boldsymbol{\beta}) \mathrm{d}u \quad \text{s.t. } 0 \le x_i \le D, \ i = 1, \ldots, M \tag{8}$$

The solution of (8), i.e., the optimal RS placement, satisfies the following theorems. All the proofs are omitted here due to space limits, and can be found in the extended paper [14].

Theorem 1: With optimal RS placement $\mathbf{x}$, the coverage of RSs is consecutive, i.e., there is no BS coverage between the coverage of any two adjacent RSs.

[2]To ensure network coverage, here we assume that the BSs cannot go into sleep mode. Joint coordination of BS and RS sleep modes is discussed in [3].

Due to Theorem 1, it is sufficient to denote the left border of the coverage of RS i as s_i, and s_{M+1} denotes the right border of RS M, as shown in Fig. 1. Suppose there are M_1 RSs on the left of $D/2$, $M_2 = M - M_1$ RSs on the right. We also have:

Theorem 2: The optimal RS placement $\mathbf{x}$ satisfies: $M_1 = M_2$ if M is an even number, otherwise $|M_1 - M_2| = 1$ if M is an odd number.

For brevity, we first provide some notations:

$$k_0 = \frac{(2^{r/W} - 1)\eta_2}{(2^{2r/W} - 1)\eta_0}, \quad k_1 = \frac{\eta_2}{\eta_1}. \tag{9}$$

We now provide the necessary condition for optimal RS placement:

Theorem 3: The optimal placement $\mathbf{x} = \{x_i\}_1^M$ and the corresponding coverage borders $\{s_i\}_1^{M+1}$ satisfy

$$k_0 s_1^\alpha = k_1 x_1^\alpha + |s_1 - x_1|^\alpha, \tag{10}$$

$$k_0(M - s_{M+1})^\alpha = k_1(D - x_M)^\alpha + |s_{M+1} - x_M|^\alpha, \tag{11}$$

$$k_1 x_i^\alpha + |s_{i+1} - x_i|^\alpha = k_1 x_{i+1}^\alpha + |s_{i+1} - x_{i+1}|^\alpha, \quad i = 1, \ldots, M-1 \tag{12}$$

$$\alpha k_1 x_i^{\alpha-1}(s_{i+1} - s_i) + |s_i - x_i|^\alpha = (s_{i+1} - x_i)^\alpha, \quad i = 1, \ldots, M_1 \tag{13}$$

$$\alpha k_1 (D - x_i)^{\alpha-1}(s_{i+1} - s_i) + |s_{i+1} - x_i|^\alpha = (s_i - x_i)^\alpha, \quad i = M_1 + 1, \ldots, M \tag{14}$$

Based on the above optimal conditions, the closed-form optimal RS placement for $\alpha = 2$ is obtained, and an efficient numerical method to solve the optimal condition is proposed for general values of α.

A. Path-loss exponent $\alpha = 2$

For highway cellular networks, very few obstacles exist and directional antennas are often deployed for better road coverage. As a result, the line-of-sight path dominates, which means the free-space path-loss model with $\alpha = 2$ is a good approximation for propagation. Denote

$$\xi = k_0(k_1 + 1) - k_1. \tag{15}$$

Assume that $k_0 > \frac{k_1}{k_1+1}$, otherwise it can be shown that putting RSs into the system will not save transmission power no matter how they are placed. For an even number of RSs, we have:

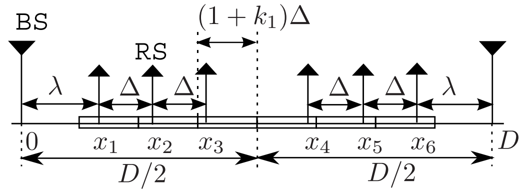
Theorem 4: If $\boldsymbol{\beta} = \mathbf{e}$, $\alpha = 2$ and M is an even number, the optimal RS placement $\mathbf{x}$ satisfies

$$x_i = \begin{cases} \lambda + (i-1)\Delta, & \text{if } i \le M/2, \\ D - \lambda - (M-i)\Delta, & \text{if } i > M/2, \end{cases} \tag{16}$$

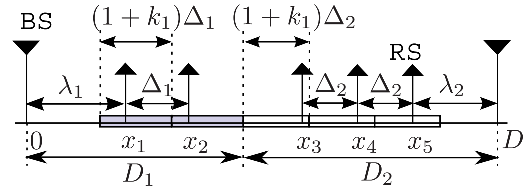
where

$$\lambda = \frac{(\sqrt{\xi} + 1)D}{2(1 + k_1)(1 + \sqrt{\xi}M)}, \Delta = \frac{\sqrt{\xi}D}{(1 + k_1)(1 + \sqrt{\xi}M)}. \tag{17}$$

The illustration of the optimal placement is shown in Figure 2(a). The optimal placement of the RSs is symmetric with respect to the point $D/2$. Also, for the coverage area of each RS, the following proposition holds:



(a) Even number $M = 6$ of RSs



(b) Odd number $M = 5$ of RSs with $M_1 = 2$ and $M_2 = 3$.

Fig. 2. Illustration of the optimal RS placement with $\alpha = 2$.

Proposition 1: If $\alpha = 2$ and M is an even number, under the optimal RS placement, the coverage is of the same length $(1 + k_1)\Delta$ for all RSs.

For an odd number of RSs, the following theorem holds.

Theorem 5: If $\alpha = 2$ and M is an odd number, the optimal RS placement $\mathbf{x}$ satisfies

$$x_i = \begin{cases} \lambda_1 + (i-1)\Delta_1, & \text{if } i \le (M-1)/2, \\ D - \lambda_2 - (M-i)\Delta_2, & \text{if } i > (M-1)/2, \end{cases} \tag{18}$$

where

$$\begin{cases} \lambda_j = \frac{(\sqrt{\xi}+1)D_j}{(1+k_1)(1+\sqrt{\xi}(M \odot 1))}, \\ \Delta_j = \frac{2\sqrt{\xi}D_j}{(1+k_1)(1+\sqrt{\xi}(M \odot 1))}, \end{cases} \tag{19}$$

and

$$\begin{cases} \text{operator } \odot \text{ is } -, \text{ if } j = 1, \\ \text{operator } \odot \text{ is } +, \text{ if } j = 2, \end{cases} \tag{20}$$

and D_1 and D_2 is the solution of

$$\begin{cases} \frac{D_1}{D_2} = \frac{(1+\sqrt{\xi}(M-1))}{(1+\sqrt{\xi}(M+1))} \sqrt{\frac{\xi + k_1(1+\sqrt{\xi}(M+1))^2}{\xi + k_1(1+\sqrt{\xi}(M-1))^2}} \\ D_1 + D_2 = D \end{cases} \tag{21}$$

Due to symmetry, $x_i' = D - x_{M+1-i}$, $i = 1, \ldots, M$ is also an optimal placement.

The corresponding optimal placement is illustrated in Figure 2(b). Surprisingly, for an odd number of RSs, the placement is not symmetric, i.e., the $\{(M + 1)/2\}$th RS is not necessarily located at $D/2$. In fact, we find that if one RS is at $D/2$, the optimal placement of the rest of the $M - 1$ RSs is exactly the same as the case with only $M - 1$ RSs, and the RS in the middle will never be selected by users anywhere. In other words, to make the additional RS valuable, it should not be in the middle. Also, the following proposition holds for the coverage:

Proposition 2: If $\alpha = 2$ and M is an odd number, under the optimal RS placement, the coverage is of the same length for the left $(M-1)/2$ (or $(M+1)/2$) RSs with $(1+k_1)\Delta_1$ (or $(1+k_1)\Delta_2$), and is of the same length for the right $(M+1)/2$ (or $(M-1)/2$) RSs with $(1 + k_1)\Delta_2$ (or $(1 + k_1)\Delta_1$).

Algorithm 1 Reference RS Placement for One Sided Region

Initialization: Initialize $x_1^{(r)}$, and set $i = 1$.

Calculate $s_1^{(r)}$ according to (10)
repeat
  Decide $x_{i+1}^{(r)}$ and $s_{i+1}^{(r)}$ according to (12) and (13)
  $i = i + 1$
until i equals to $M^{(r)}$
Decide the region border $D^{(r)}$ according to (11)

B. Path-loss exponent $\alpha > 2$

In this case, we do not have an analytical solution for optimal placement. However, we propose an efficient numerical method to find the optimal RS placement based on the conditions in Theorem 3. We present some propositions of the optimal placement $\mathbf{x}$ that are required by the numerical method. First, define $D_1 = s_{M_1+1}$, we have:

Proposition 3: With optimal RS placement $\mathbf{x}$, the positions of the left M_1 RSs is the optimal placement for the region $[0, D_1]$ with one BS on the left border, while the positions of the right M_2 RSs is the optimal placement for the region $[D_1, D]$ with one BS on the right border.

Therefore, we can now focus on the optimal RS placement for a one sided region, without loss of generality, having one BSs on the left border. The optimal RS placement of the one sided region has the following properties:

Proposition 4: For a one sided region $[0, D_1]$ with M_1 RSs, the optimal RS placement is $\mathbf{x}$ and the corresponding coverage borders are stacked in vector $\mathbf{s}$. Then, $\forall m \leq M$, $\{x_i\}_1^m$ is the optimal placement for the one sided region $[0, s_{m+1}]$ with m RSs. In addition, for the scaled one sided region $[0, cD_1]$, the optimal RS placement is $\mathbf{x}' = c\mathbf{x}$. Denote the transmission power for a user at position u as P_u, and then $P'_{cu} = c^\alpha P_u$ for the scaled RS placement.

The above proposition can be inferred from Theorem 3, based on which a method is proposed to get a set of reference positions of $M^{(r)}$ RSs for a one sided region, as shown in Algorithm 1. We use fast numerical methods like the bi-section search to calculate $x_i^{(r)}$, $x_i^{(r)}$ and $D^{(r)}$, and they are selected based on their sorting relations, so it can be shown that the *uniqueness* of the reference positions is guaranteed.

Then the optimal placement for the original two sided region $[0, D]$ is decided by Algorithm 2, where $\lfloor a \rfloor$ returns the largest integer no larger than a, and the values of M_1 and M_2 are based on Theorem 2. When M is an odd number, one can also select $M_1 = (M + 1)/2$ and $M_2 = (M - 1)/2$.

IV. Joint Optimization of RS Placement $\mathbf{x}$ and Active Probability $\boldsymbol{\beta}$

This section aims at solving problem (7), which is a constrained non-linear optimization problem. Because the problem is generally non-convex, the sub-gradient method is not guaranteed to converge or to find the optimal solution. We therefore resort to the Newton method to find the zero-gradient solutions. The original Newton method cannot handle the constraints, so we use the projected Newton method [13] to deal with the constraints, while at the same time preserve the fast convergence.

Algorithm 2 RS Placement Optimization

Initialization: Use **Algorithm 1** to generate reference placement $\mathbf{x}^{(r)}$ for $M^{(r)} = \lfloor (M + 1)/2 \rfloor$ RSs

if M is even **then**
  Decide the optimal placement $\mathbf{x}$

$$x_i = \begin{cases} \left(\frac{D}{2D^{(r)}}\right) x_i^{(r)} & i = 1, \ldots, M^{(r)} \\ D - \left(\frac{D}{2D^{(r)}}\right) x_{M-i+1}^{(r)} & i = M^{(r)} + 1, \ldots, M. \end{cases}$$

else
  Let $M_1 = (M - 1)/2$ and $M_2 = (M + 1)/2$
  Solve for the lengths D_1 and D_2 so that

$$\begin{cases} \left(\frac{D_1}{s_{M_2}}\right)^\alpha P_{s_{M_1}}^{(r)} = \left(\frac{D_2}{D^{(r)}}\right)^\alpha P_{D^{(r)}}^{(r)} \\ D_1 + D_2 = D \end{cases}$$

  Decide the optimal placement $\mathbf{x}$

$$x_i = \begin{cases} \left(\frac{D_1}{s_{M_2}}\right) x_i^{(r)} & i = 1, \ldots, M_1 \\ D - \left(\frac{D_2}{D^{(r)}}\right) x_{M-i+1}^{(r)} & i = M_1 + 1, \ldots, M. \end{cases}$$

end if

Our method proceeds as follows. First, we aggregate the variables $\mathbf{x}$ and $\boldsymbol{\beta}$ into a $2M \times 1$ vector $\boldsymbol{\theta} = [\mathbf{x}^T, \boldsymbol{\beta}^T]^T$. We write the total power consumption $P_{\text{tot}}(\boldsymbol{\theta})$ as the function of the aggregate variable vector. Since (6) has no closed-form expression, we approximate the gradient $\nabla P_{\text{tot}}(\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$. First, $P_{\text{tot}}(\mathbf{x})$ is approximated with N randomly generated user positions $\mathbf{u} = \{u_j\}_{j=1}^N$, and the partial derivative with respect to RS position x_i is calculated as

$$\frac{\partial P_{\text{tot}}(\boldsymbol{\theta})}{\partial x_i} \approx \frac{\sum_{j=1}^N [P(u_j, \mathbf{x} + \epsilon \mathbf{e}_i, \boldsymbol{\beta}) - P(u_j, \mathbf{x} - \epsilon \mathbf{e}_i, \boldsymbol{\beta})]}{2N\epsilon}, \tag{22}$$

where vector $\mathbf{e}_i$ equals 1 on the ith element and zero elsewhere and ϵ is a small constant. Since we also have the RS position constraint that $0 \leq x_i \leq D$, the approximation becomes

$$\frac{\sum_{j=1}^N [P(u_j, \mathbf{x} + (D - x_i)\mathbf{e}_i, \boldsymbol{\beta}) - P(u_j, \mathbf{x} - \epsilon \mathbf{e}_i, \boldsymbol{\beta})]}{N(D - x_i + \epsilon)},$$

if $x_i + \epsilon > D$. Or if $x_i + \epsilon > D$, it becomes

$$\frac{\sum_{j=1}^N [P(u_j, \mathbf{x} + \epsilon \mathbf{e}_i, \boldsymbol{\beta}) - P(u_j, \mathbf{x} - x_i \mathbf{e}_i, \boldsymbol{\beta})]}{N(x_i + \epsilon)}.$$

Notice that when we increase N and make ϵ very small, the approximation can be fairly precise. As for the partial derivative with respective to β_i, one should replace the constraints D by 1 in the two equations following (22). Then $\boldsymbol{\theta}$ is updated iteratively according to

$$\boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_t - \boldsymbol{\Sigma}_t \mathbf{A}_t \nabla P_{\text{tot}}(\boldsymbol{\theta}), \tag{23}$$

where $\boldsymbol{\Sigma}_t$ is a $2M \times 2M$ diagonal matrix, and the $2M \times 2M$ matrix $\mathbf{A}_t$ is the key to the projected Newton method.

A. Structure of the update matrix $\mathbf{A}_t$

Iteration index t is omitted in this sub-section for brevity. Define a function that outputs a subset of indices $I(\boldsymbol{\theta}) \subset \{M+1, M+2, \ldots, 2M\}$ as

$$I(\boldsymbol{\theta}) = \left\{ M+i \left| \left.\frac{\partial P_{\text{tot}}(\boldsymbol{\theta}^{+})}{\partial \beta_i}\right|_{\beta_i=0} > 0; \text{ or } \left.\frac{\partial P_{\text{tot}}(\boldsymbol{\theta}^{-})}{\partial \beta_i}\right|_{\beta_i=0} < 0 \right.\right\}, \tag{24}$$

where we use right and left derivatives because P_{tot} is not differentiable with respect to β_i when β is on the boundary. Notice that only the indices corresponding to $\boldsymbol{\beta}$ are considered for $I(\boldsymbol{\theta})$ because it can be shown that the partial derivative with respect to x_i never satisfies the conditions in (24) on the border. Correspondingly, the complement of $I(\boldsymbol{\theta})$ is given by

$$\bar{I}(\boldsymbol{\theta}) = \{ i \,|\, i \notin I(\boldsymbol{\theta}),\, i = 1, 2, \ldots, 2M \}. \tag{25}$$

As a result, the variables in $\boldsymbol{\theta}$ are divided into two parts. The basic intuition is to use the gradient method to update the variables in $I(\boldsymbol{\theta})$, and to use the traditional Newton method for the rest. Define $\mathbf{A}(I)$ as the sub-matrix of $\mathbf{A}$ with rows and columns corresponding to the indices in I, i.e.,

$$[\mathbf{A}(I)]_{i,j} = A_{[I]_i,[I]_j}. \tag{26}$$

Then the matrix $\mathbf{A}$ has the property

$$\left[\mathbf{A}(\bar{I}(\boldsymbol{\theta}))^{-1}\right]_{i,j} = \frac{\partial^2 P_{\text{tot}}(\boldsymbol{\theta})}{\partial \theta_{[\bar{I}(\boldsymbol{\theta})]_i} \partial \theta_{[\bar{I}(\boldsymbol{\theta})]_j}}, \tag{27}$$

$$\mathbf{A}(I(\boldsymbol{\theta})) = \mathbf{I}, \tag{28}$$

where $\mathbf{I}$ is the identity matrix. For the variables in $\bar{I}(\boldsymbol{\theta})$, the equivalent update matrix is the inverse Hessian matrix of the corresponding variables, i.e.,

$$\mathbf{A}(\bar{I}(\boldsymbol{\theta})) = \mathbf{H}(\bar{I}(\boldsymbol{\theta}))^{-1}, \tag{29}$$

where $\mathbf{H}(\bar{I}(\boldsymbol{\theta}))$ is the original Hessian matrix with respect to $\boldsymbol{\theta}$; For the variables in $I(\boldsymbol{\theta})$, the update matrix is diagonal, so essentially they are updated with the gradient method.

B. Algorithm

Since direct calculation of the Hessian matrix is not applicable, we use the quasi-Newton method [11] with to update the approximated inverse Hessian $\hat{\mathbf{A}}_t(\bar{I}(\boldsymbol{\theta}_{t+1}))$ as

$$\hat{\mathbf{A}}_{t+1}(\bar{I}(\boldsymbol{\theta}_{t+1})) = \mathbf{B}_t + \frac{(\mathbf{z}_t^{(\bar{I})} - \mathbf{B}_t \mathbf{y}_t^{(\bar{I})})(\mathbf{z}_t^{(\bar{I})} - \mathbf{B}_t \mathbf{y}_t^{(\bar{I})})^T}{(\mathbf{z}_t^{(\bar{I})} - \mathbf{B}_t \mathbf{y}_t^{(\bar{I})})^T \mathbf{y}_t^{(\bar{I})}}, \tag{30}$$

where

$$\begin{cases} \mathbf{B}_t = \hat{\mathbf{A}}_t(\bar{I}(\boldsymbol{\theta}_{t+1})), \\ \mathbf{z}_t^{(\bar{I})} = -\left[\boldsymbol{\Sigma}_t \hat{\mathbf{A}}_t \nabla P_{\text{tot}}(\boldsymbol{\theta}_t)\right] \left(\bar{I}(\boldsymbol{\theta}_{t+1})\right), \\ \mathbf{y}_t^{(\bar{I})} = \left[\nabla P_{\text{tot}}(\boldsymbol{\theta}_{t+1}) - \nabla P_{\text{tot}}(\boldsymbol{\theta}_t)\right] \left(\bar{I}(\boldsymbol{\theta}_{t+1})\right). \end{cases}$$

Note that $\mathbf{B}_t$, $\mathbf{z}_t^{(\bar{I})}$ and $\mathbf{y}_t^{(\bar{I})}$ are all corresponding to the indices of $\bar{I}(\boldsymbol{\theta}_{t+1})$. For $\boldsymbol{\Sigma}_t$, we use a constant σ for those diagonal elements corresponding to $I(\boldsymbol{\theta}_t)$. i.e., $\boldsymbol{\Sigma}_t(I(\boldsymbol{\theta}_t)) = \sigma \mathbf{I}$ as the original Newton method. The diagonal elements corresponding to $\bar{I}(\boldsymbol{\theta}_t)$ are set as $\boldsymbol{\Sigma}_t(\bar{I}(\boldsymbol{\theta}_t)) = \kappa^t \mathbf{I}$, where κ is a small positive real number. Also, since we use noisy estimation of the gradient, and use this noisy estimation to update the Hessian, the iteration is a stochastic analog of the Newton method [12][3]. The the convergence under general conditions is discussed in [12] [13]. We can now specify the iterative algorithm that solves (7) shown in Algorithm 3, where $[\theta_i]^{\#}$ returns the closest number to θ_i in the feasible region. Because the Newton method can only find the local optimums, we run Algorithm 3 with different initial values, and select the solution with the minimum total power consumption.

Algorithm 3 Joint RS Placement and Active Probability Optimization

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Initialization: Set $t = 0$, $\mathbf{A}_0 = \mathbf{I}$. Initialize $\mathbf{x}_0$ with random RS positions within $[0, D]$, and $\boldsymbol{\beta}_0$ with random values within $[0, 1]$. Aggregate $\mathbf{x}_0$ and $\boldsymbol{\beta}_0$ into $\boldsymbol{\theta}_0$.

Generate N random user positions $\mathbf{u}$
Approximate $\nabla P_{\text{tot}}(\boldsymbol{\theta}_0)$ according to (22)
repeat
  $t = t + 1$
  Determine $I(\boldsymbol{\theta}_{t-1})$ and $\bar{I}(\boldsymbol{\theta}_{t-1})$ according to (24) and (25)
  Update $\boldsymbol{\theta}_t = \left[\boldsymbol{\theta}_{t-1} - \boldsymbol{\Sigma}_{t-1} \hat{\mathbf{A}}_{t-1} \nabla P_{\text{tot}}(\boldsymbol{\theta}_{t-1})\right]^{\#}$
  Generate N random user positions $\mathbf{u}$
  Approximate $\nabla P_{\text{tot}}(\boldsymbol{\theta}_t)$ according to (22)
  Set $\hat{\mathbf{A}}_t(I(\boldsymbol{\theta}_t))$ according to (28)
  Update $\hat{\mathbf{A}}_t(\bar{I}(\boldsymbol{\theta}_t))$ according to (30)
until $\|\boldsymbol{\theta}_t - \boldsymbol{\theta}_{t-1}\| < \varepsilon$

V. Numerical Results

In this section, we set $D = 2000$m as for the typical macro-cells [10]. Parameter η_0, r and W are set so that the transmission power with direct transmission to satisfy the rate of the user at $D/2$ is 1W, while $\eta_2 = \eta_0$ and $\eta_1 = 4\eta_0$.

A. Transmission Power Saving

We first observe the saving on transmission power obtained by RS placement optimization only, i.e., we assume that $P_{\text{on}} = 0$ so that RSs are always active with $\boldsymbol{\beta} = \mathbf{e}$. We normalize the results by the transmission power of the BSs when no RS is deployed. It is shown in Fig. 3 that implementing RSs can greatly reduce the transmission power, and the power efficiency improvement increases with the path-loss exponent α. In addition, by optimizing the RS placement, the transmission power can be substantially decreased compared to even placement, whereby $x_i = \frac{D}{M+1} i$, $i = 1, \ldots, M$, and this gain also becomes more significant when α increases.

B. Total Power Consumption Saving

In this set of numerical examples, we set the RS circuit power consumption $P_{\text{on}} = 50$mW. The vehicle speed is

[3] In this case, the convergence actually requires that $\lim_{t\to\infty} \sigma_t = 0$ [12]. The rate that σ_t approaches zero actually depends on the approximation precision of ∇P_{tot}, i.e., the choice of N. In this paper, we choose large N to have precise estimation of the gradient, so that we can choose σ_t to be a constant and have fast convergence.

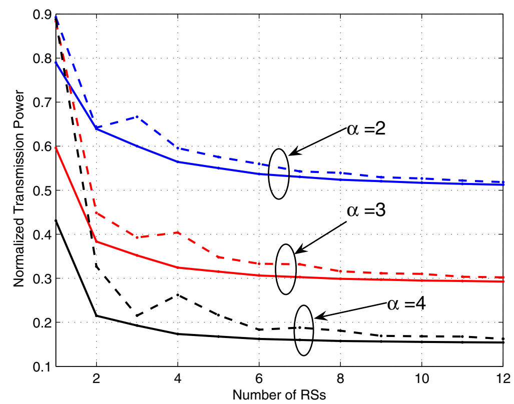

Fig. 3. Normalized transmission power. Dashed lines correspond to evenly placed RSs, while solid lines correspond to the optimal placement.

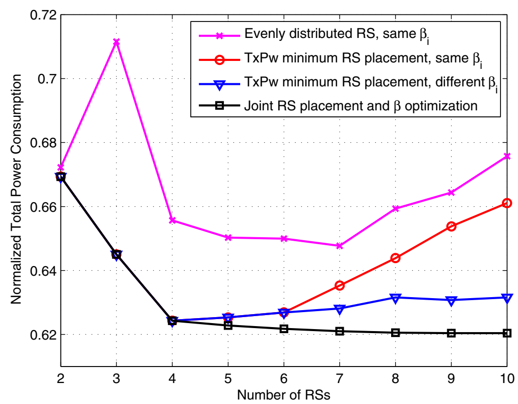

Fig. 4. Normalized total power consumption with $\alpha = 2$.

70km/h, and the arrival rate $\lambda = 0.1\text{s}^{-1}$. We also normalize the results by the transmission power consumption of the BSs when no RS is implemented, because the BSs are not allowed to sleep in order to guarantee coverage. We compare the normalized total power consumptions of four cases with $\alpha = 2$:

Case 1: RSs are evenly distributed, and $\boldsymbol{\beta} = \beta_0 \mathbf{e}$, where β_0 is optimized to minimize the total power consumption.

Case 2: RSs are optimally placed to minimize transmission power as described in Section III, and $\boldsymbol{\beta} = \beta_0 \mathbf{e}$, where β_0 is optimized to minimize the total power consumption.

Case 3: RSs are optimally placed to minimize transmission power as described in Section III, and $\boldsymbol{\beta}$ is optimized according to Algorithm 3 with fixed RS placement $\mathbf{x}$. The result may have different values of β_i, depending on the positions of RSs.

Case 4: The RS placement and the RS active probability $\boldsymbol{\beta}$ are jointly optimized by Algorithm 3.

From Fig. 4, as expected, case 4 achieves the best power saving performance. Actually, as shown in (5) and (7), the optimal RS placement in case 4 depends on v and λ. On the other hand, case 3, which fixes the RS placement that minimizes the transmission power and optimizes $\boldsymbol{\beta}$ when $\frac{\lambda}{v}$ changes, performs very close to case 4, and thus it is a promising choice for practical systems. Moreover, as shown in Fig. 4, when the circuit power is counted, deploying more RSs may not provide power saving gain. In this setting, five to six RSs are enough to achieve the optimal performance.

VI. Conclusion

We have investigated the benefit of improving energy efficiency with relaying for cellular networks. Two issues are jointly considered: RS placement and RS sleep control. The detailed structure of the optimal RS placement for pure transmission power minimization is provided. We also present an algorithm to deal with the interaction between RS placement and RS sleep/active probability allocation, which aims at minimizing the total power consumption, including the transmission power and the circuit power of active RSs. We have found that the power saving gain led by placing RSs optimally increases with the path-loss exponent. It has also been shown that fixing the RS placement that minimizes the transmission power and optimizing the RS active probability according to load conditions can be a good tradeoff between practicality and performance.

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