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FEM in Plate Bending

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Abstract. The paper is concerned with the numerical method of determination bending force and calibration force in plate bending. For numeric procedure the finite element method is used. Calibration force is determined when bending force and calibration coefficient are known. Significant factors for determination of bending force are: material of the circular plate, bending radius circular plate, diameter of the circular plate, thickness of the circular plate and method of loading of the circular plate. The calibration coefficient is determined by experiment. The analysis of bending plate is limited to the facts and figures used so far in the fabrication of spherical tanks, i.e. for deformations up to 1 %.

1 The finite-element method

The finite-element equations are constructed by the condition that the first-order variation of the functional vanishes [1].

Rigid-plastic materials. The deformation process of the rigid-plastic materials is associated with the following boundary value problems. At a generic stage in the process of quasistatic distortion, the shape of the body, the internal distribution of temperature, the state of inhomogeneity, and the current values of material parameters are supposed to be given ot to have been determined already. The velocity vector v is prescribed on a part of the surface, S_v, together with traction f on the remainder of the surface, S_f.

This boundary value problem is dual to the variational problem where the functional is given by

$$\Phi = \int_V \vec{\sigma'} \cdot \vec{\dot{\varepsilon}} \cdot \mathrm{d}V - \int_{S_f} \vec{f} \cdot \vec{v} \cdot \mathrm{d}S + \int \lambda \dot{\varepsilon}_{ij} \mathrm{d}V \qquad (1)$$

Adding the incompressibility constraint, where $\dot{\varepsilon}$ is the strain-rate vector, and σ' is the deviatoric stress vector.

The first term of the equation (1) is

$$\int \sigma' \cdot \dot{\varepsilon} \mathrm{d}V = \int_V \bar{\sigma} \dot{\varepsilon} \mathrm{d}V \qquad (2)$$

where is

$$\bar{\sigma} = V\left[(3/2)\sigma'_{ij}\ \sigma'_{ij}\right] \cdot \dot{\bar{\varepsilon}} = V\left[(2/3)\dot{\varepsilon}_{ij}\ \dot{\varepsilon}_{ij}\right]$$

The Langragian multiplier, λ in equation (1) can be shown to be equal to the hydrostatic stress component σ_m. Another approach to removing the incompressibility constraint is the use of the penalty function method. The method introduces a very large positive constraint, K, as

$$\Phi = \int_v \overline{\sigma} \cdot \dot{\bar{\varepsilon}} \mathrm{d}V - \int_{S_f} \vec{f} \cdot \vec{v}\, \mathrm{d}S + \int_v \frac{1}{2} K(\dot{\varepsilon}_{ij})^2\, \mathrm{d}V \tag{3}$$

to penalize the dilatation strain. Comparing variations of functional (1) and (3), it can be seen that the constant K is the bulk modulus, since $\lambda = \sigma_{\mathrm{m}} = K\dot{\varepsilon}_{ij}$.

If the workpiece contains a rigid region, the stresses in the zone cannot be determined. This difficulty can be prevailed by considering an offset of the effective strain rate ($\dot{\varepsilon}_0$), which is several orders of magnitude smaller than the average strain rate in the deforming zone. The deviator stresses are assumed to vary linearly from zero to the flow stress if the effective strain rate is smaller than this offset value. This stress-strain rate relation in the "nearly rigid" zone is similar to Hook9s law, except for the use of strain rate instead of strain. It must be noted that in the nearly rigid zone the first term of the equation (3) must be modified according to the linear relationship between the deviator stress and the strain-rate.

2 Finite element analysis plane-strain sheet bending

Elasto-plastic materials. Various forms of the finite-element method dealing with large strain have been used by several investigators [2-5]. Among them the most elegant forms are the ones given by Hill [6]. In this paper we also utilize Hill´s formulation. Consider a body with volume V_0 and surface S_0 in a reference state. After a certain increment of time t, the body occupies the new position V and S. At the reference state each particle of the body is labeled by a set of coordinates (ξ^1, ξ^2, ξ^3,) which is imbedded in the material and moves with it [1]. Another coordinate system (x_1, x_2, x_3) which is fixed in space and not moving with the body will be chosen. Then, at any $t \geq t_0$, we have the following relations between the two coordinate systems

$$x_\alpha = x_\alpha(\xi^1, \xi^2, \xi^3, t).$$

The coordinate systems x_α is chosen to be rectangular Cartesian and at a generic moment t, the reference state coincides with the current state. The strain-rate $\dot{\varepsilon}_{ij}$ as a deformation measure is given by

$$\dot{\varepsilon}_{ij} = \frac{1}{2}(v_{ij} + v_{ji}) \tag{4}$$

and rotation rate as

$$\dot{w}_{ij} = \frac{1}{2}(v_{ij} - v_{ji}), \tag{5}$$

As a stress measure we use the contra variant component of Kirchhoff stress in the coordinated system which is defined as

$$\tau^{\alpha\beta} = \frac{\rho_0}{\rho}\sigma^{\alpha\beta}, \tag{6}$$

Where ρ_0 and ρ are are the material densities at the reference and the current states, respectively, and $\sigma^{\alpha\beta}$ is the Cauchy stress. The time derivative of the stress measure is the Jauman derivative, ($D\tau_{ij}$ / Dt), where time differentiation is carried out with the coordinate system which rotates but does not deform with the material.

Since the reference state is assumed to coincide with the current state, the two stress measure become identical, though their time rates have the following relation.

$$\frac{D\tau_{ij}}{Dt} = \frac{D\sigma_{ij}}{Dt} + \sigma_{ij}\dot{\varepsilon}_{kk} \tag{7}$$

It should also be noted that the material time derivative of the Cauchy stress, ($d\sigma_{ij}$ / dt), and the Jauman derivative of the Kirchhoff stress have the relationship

$$\frac{d\sigma_{ij}}{dt} = \frac{D\tau_{ij}}{Dt} - v_{k,k}\sigma_{ij} + \dot{w}_{ik}\,\sigma_{kj} + \dot{w}_{jk}\,\sigma_{ki} \tag{8}$$

Assuming that the Prandtl-Reuss equation holds for the Jauman derivative of the Kirchhoff stress and strain rates, the constitutive relation is given by

$$\frac{D\tau_{ij}}{Dt} = \frac{E}{1+\nu}\left[\delta_{ik}\delta_{jl} + \frac{\nu}{1-2\nu}\delta_{ij}\delta_{kl} - \alpha\frac{3\cdot\sigma_{ij}'\cdot\sigma_{kl}'(\frac{E}{1-\nu})}{2\bar{\sigma}^2(\frac{2}{3}h + \frac{E}{1+\nu})}\right]\dot{\varepsilon}_{kl}, \tag{9}$$

and its inverse becomes

$$\dot{\varepsilon}_{ij} = \frac{1+\nu}{E}\frac{D\tau_{ij}}{Dt} - \frac{\nu}{E}\delta_{ij}\frac{D\tau_{kk}}{Dt} + \alpha\frac{9\cdot\sigma_{kl}'\cdot\sigma_{ij}'}{4h\sigma^{-2}}\frac{D\tau_{kl}}{Dt}, \tag{10}$$

where $\alpha = 1$ for the plastic state and $\alpha = 0$ for the elastic state. In equations (9) and (10), E is the Young Modulus; ν poisson ratio; σ'_{ij}, the deviatoric stress; $\bar{\sigma}$, the effective stress; and h, the strain hardening rate. These equations can be regarded as a special case of those by Hill.

For the deformation process of elasto-plastic materials the velocity vector v_i is prescribed on a part, S of the surface, S_v, together with the nominal traction rate $\dot{t}_1$ on the remainder of the surface,. S_T Then, the deformation mode is characterized by a variational principle [6-9].

$$\delta\phi = \delta\left[\int_V U dV - \int_{S_T} t_i v_i dS\right] = 0, \tag{11}$$

Since we are only interested in stress and their rates on the basis of the x_1 coordinate system, the notational distinction between the contra variant and components will be abandoned there on where δ denotes the weak variation in the class of continuous differentiable velocity fields taking the prescribed values on S, and U is expressed by

$$U = \frac{1}{2}\left(\frac{D\tau_{ij}}{Dt}\dot{\varepsilon}_{ij} - 2\sigma_{ij}\dot{\varepsilon}_{ki}\dot{\varepsilon}_{kj} + \sigma_{ij}v_{k,i}v_{kj}\right). \tag{12}$$

Detailed derivations and discussions of the large-strain formulation of variation problems can be found elsewhere [9, 10].

2 Experiment Results

The existing increasing necessity for spherical tanks is easy understandable because it exist the possibility to store medium with minimal thickness of tank, small needed volume and minimum cost price. Spherical tanks are becoming fair more interesting with in creasing of their radius. The shell of spherical tank consists of steel sheet, while the segments are assembled by welding in whole at the actual place. The bending separate parts are achieved in several indentations on hydraulic press and can be seen from Figure 1.

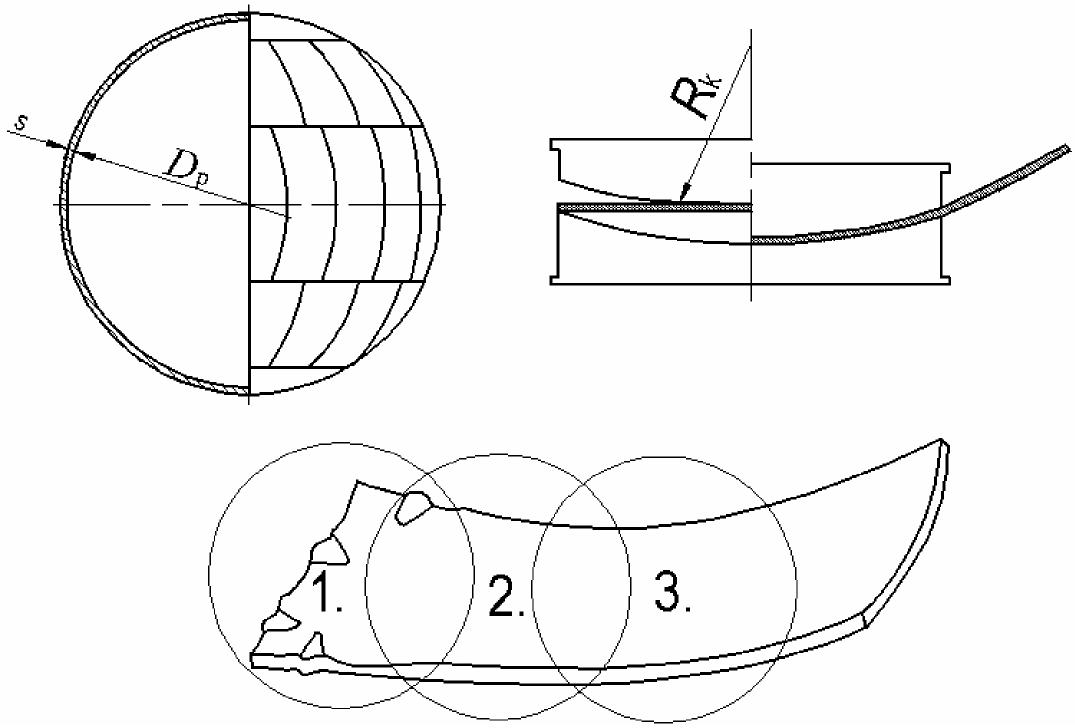

Figure 1. The bending of segment in tool

Figure 2 present tools Kt 5081 (R_k = 4925 mm) and Kt 5136 (R_k = 4500mm).

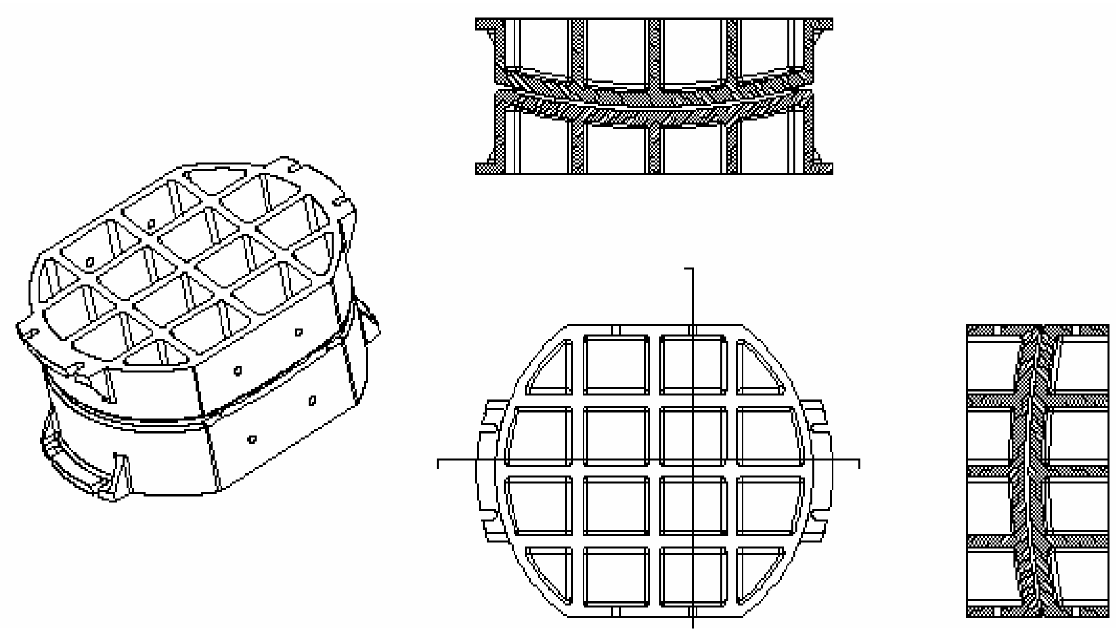
Figure 2. Tool Kt 5081 and Kt 5136

Table 1. Technical data

MATERIAL	Nioval 47 (≈ StE 500)
SHEET DIMENSION *s L B*	29x6000x1800(2000), and16x6000x1800(2000)
DIE NUMBER	Kt 5081(R_k = 4925 mm) and Kt 5136(R_k = 4500 mm)
UPPER DIE RADIUS	R_k = 4925 mm and R_k = 4500 mm
MEASUREMENT VALUE *H* FOR *b* = 1800	MEASUREMENT VALUE *H* FOR *l* = 1800

H^{b}_{p1}	H^{b}_{p2}	H^{b}_{p3}	H^{l}_{p1}	H^{l}_{p2}	H^{l}_{p3}
ARTIMETIC MEAN VALUE H^{b}_{p1}, H^{b}_{p2}, $H^{b}_{p3} \Rightarrow H^{b}_{p}$			ARTIMETIC MEAN VALUE H^{l}_{p1}, H^{l}_{p2}, $H^{l}_{p3} \Rightarrow H^{l}_{p}$		
CALCULATED VALUE FOR RADIUS OF PLATE RELATED TO H^{b}_{p} AND $b \Rightarrow \rho^{b}_{p}$			CALCULATED VALUE FOR RADIUS OF PLATE RELATED TO H^{l}_{p} AND $l \Rightarrow \rho^{l}_{p}$		
ARTIMETIC MEAN VALUE H^{b}_{p} i $H^{l}_{p} \Rightarrow H_{p}$					
ARTIMETIC MEAN VALUE ρ^{b}_{p} i $\rho^{l}_{p} \Rightarrow \rho_{p}$					
FORMING FORCE [MN]					
CALIBRATION FORCE [MN]					
FORMING TIME 10 s					
CALIBRATION TIME 3 s					

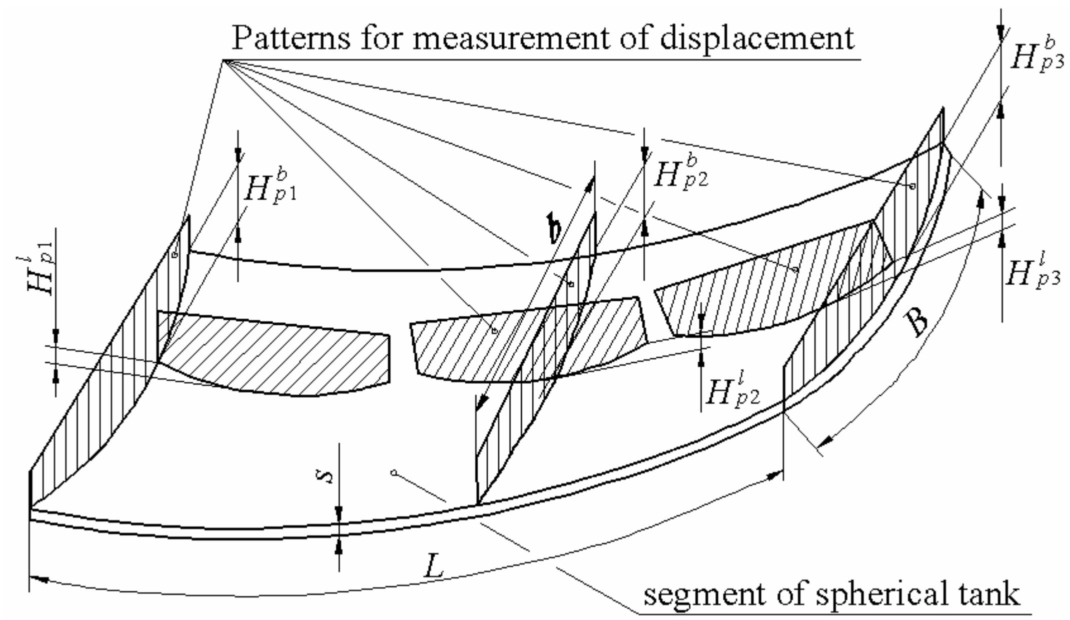

Figure 3: Pattern for displacement measurements

Table 2. Experimental results

DATA	EXPERIMENTAL RESULTS Kt 5-136 and Kt 5-081															
	II 16x1800 Kt5136	III Kt5136	IV 29x1800 Kt5136	V Kt5136	VI 16x1800 Kt5136	VII Kt5136	VIII 29x2000 Kt5136	IX Kt5136	X 16x1800 Kt5081	XI Kt5081	XII 29x1800 Kt5081	XIII Kt5081	XIV 16x2000 Kt5081	XV Kt5081	XVI 29x2000 Kt5081	XVII Kt5081
H^{b}_{p1}	66.2	67	66.4	67.7	75	76	76.9	76.6	50.5	51	52	51.6	58.2	57.1	58.9	58.1
H^{b}_{p2}	64.5	64	66.7	66.3	72	74	75.3	75.1	50.6	49.3	51.8	51.8	56	55	58.2	59.8
H^{b}_{p3}	66.7	65.5	66.4	67.9	75.3	75	76.4	78.4	52.2	51.5	51.6	52	57.2	56.1	57.2	58.2
H^{l}_{p1}	65	66.6	67.5	68.8	69.3	67.8	69.3	69.3	56.2	55.5	58.2	59.3	56.1	56.1	59.9	60.5
H^{l}_{p2}	67.5	68.5	69.5	70	68.5	69	70.5	70.4	58.1	58	60.1	59.7	59.8	59	61	61
H^{l}_{p3}	66.7	66.5	67.3	68.5	65.6	67.8	69.9	71.5	56.1	56	58.7	60.4	57.2	56.2	60	60.6
H^{b}_{p}	65.8	65.9	66.5	67.3	74.1	75.5	76.2	76.7	51.1	50.6	51.3	51.8	57.2	56.6	58.1	58.7
H^{l}_{p}	66.4	67.2	68.1	69.1	67.8	68.2	69.9	70.4	56.8	56.5	59	59.8	57.2	57.1	60.3	60.7
ρ^{b}_{p}	6188	6179	6123	6051	5503	5402	5353	5319	7951	8029	7920	7844	7109	7184	7000	6929
ρ^{l}_{p}	6133	6060	5981	5896	6007	5972	5829	5788	7159	7196	6894	6802	7048	7121	6746	6702
H_{p}	66.1	66.55	67.3	68.2	70.95	71.85	73.05	73.55	53.95	53.55	55.15	55.8	57.45	56.85	59.2	59.7
ρ_{p}	6160.5	6119	6052	5973.5	5755	5687	5591	5553.5	7555	7612.5	7407	7323	7078.5	7152.5	6873	6815.5

3 Numerical Method

The finite element approach consists of three parts: pre-processor, main program and machine related information and post-processor. Pre-processor for solving the main program MATCVO contains [11]: material data (modulus of elasticity, Poison's ratio, true stress or characteristic of plastic flow), the generating of structure, data according loading, connectivity and boundary conditions.
The material that takes place is Nioval 47, produced according to DIN 17102/83. The dimensions of specimens were B 6 x 50 according to DIN 50125. From the same table were taken sheet blanks ϕ 2000x16 mm for evaluation of the results obtained by FEM. The machine used for material data establishing used latterly in pre-processing is from firm "Siempelkamp". On the same machine were performed the experiments with the tool for bending of circular plate for evaluation the results achieved with FEM. By examination of the specimens the obtained results are:
- True stress or characteristic of material flow is $k_f = 588 + 700\varphi_v^{0.4}$
- Modulus of elasticity E=206112-206795 [N/mm^2]
- Poison's ratio ν=0.31 - 0.33

The main program is used for computing the displacements as well as the stresses.

4 Mathematical Interpretations of Results Obtained by FEM

Mathematical interpretation of bending force. In attempt to obtain equation for calculating bending force it has to be observed tables 9.10 till 9.14 that represent the results of FEM [12-19]. Analysing the expressions given for the diameters between ϕ1800 and ϕ2200 mm with thickness of 16- 29 mm and materials StE355, StE420, StE500 it can be written the general expression for bending force in the form:

$$F_{q00s} = \left[\left(s_1^2 / 6{,}75\right)\left(a_1 u_{z1} + 0{,}5\right)\right] R_m \ \text{[N]} \tag{13}$$

Figure 4 present numerical and experimental comparison the force depending of punch motion.

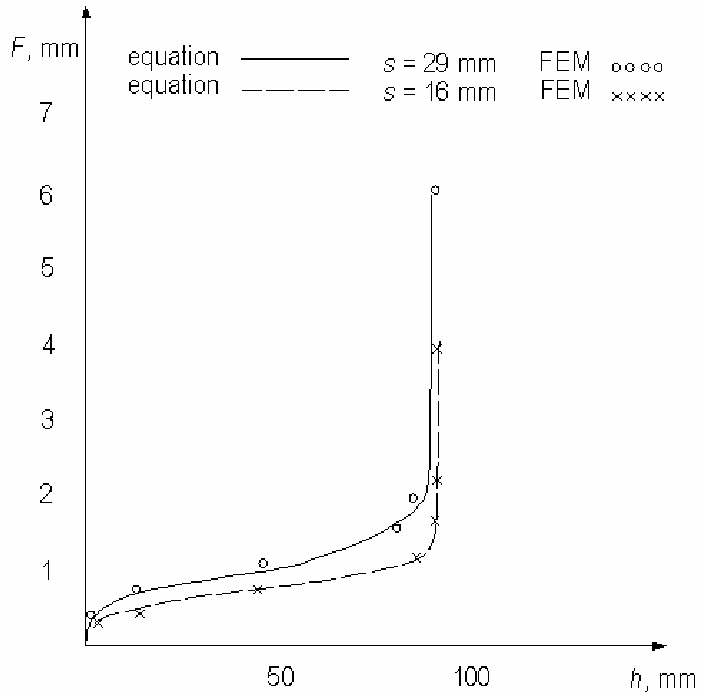

Figure 4. Numerical and experimental comparison the force F depending of punch motion *h*

The mathematical equation 13 obtained with the results of FEM can be accepted only for the tested area and examined materials as well as considered thickness among 15 and 40 mm. As in assembling the spherical tanks these are the commonly used parameters of thickness and materials it can be concluded that this expression can be used in projecting or planning the spherical tanks.

Experimental Obtained the Calibration Force. The experiments were conducted using the plates ϕ2000x16 mm of material NIOVAL 47. Using the equation 3 it can be find necessary bending force as F_{q00s} =1.185 MN. The plate bended with 1.185 MN had wrinkles at the edge of height up to 2 mm. Because of that reason the plate has to be calibrated. The calibration force can be calculated using the expression

$$F_k = k \cdot F_{q00s} \tag{14}$$

In order to compute the calibrating force it is necessary to obtain experimentally the amount of coefficient of calibration *k* for relevant dimensions. For the plate with dimensions ϕ2000 x 29 mm coefficient of calibration *k* is *k* = 2, and for the plate with dimensions ϕ2000 x 16 this value is *k* = 2.5. Comparing the values of coefficient of calibration *k* it can be seen that its value is greater when the plate thickness is smaller. It is because during the bending process the wrinkles tend to form themselves more if the thickness is smaller. On that way, for the plate with dimensions ϕ2000 x 20 it can be supposed using the above obtained experiments that *k* = 2.34.

Determination of the Mechanical Springback after Calibration. The factors that have the most significant influence on the amount of mechanical spring-back are: calibration forces, plate material, punch radius, punch motion, plate thickness and plate diameter. In order to research the influence of punch radius R_k, plate thickness and plate diameter, according to technological and productivity conditions it was used experiment′s factor plan. It where carried out the industrial experiments with scale dimension 1:1. Experiments were conducted using randomised distribution in attempt to avoid systematic errors. The considered factors were: punch radius 4500 and 4925 mm, plate thickness 16 and 29 mm, plate diameter 1800 and 2000 mm. Using all above mentioned considerations, it can be on the basis of experimental and industrial researching determined the law for coefficient of mechanical spring-back of calibrated plate with constant plate diameter K^{II}. It can be determined using expression [11-12]:

$$K^{II} = \frac{1}{0{,}768304 + 0{,}0906276\, R_p} \qquad R_k = K^{II} \frac{D_p}{2} \tag{15}$$

For technological usage it is recommended the calculation by using the equation (15). For instance, in the case of the plate with 2000 mm diameter, bending plate radius 7 m (shell diameter of spherical tank 14 m) and plate material NIOVAL 47 this coefficient of mechanical spring is 0,1406.

5 Conclusion

In metal forming processes, the cold axial-symmetrical bending process has a considerable great role. The production of axial-symmetrical bended plates with larger dimensions is industrial interesting for producers of steel tanks and similar products. FEM enables using program MATCVO and corresponding pre and post-processor the determination of displacements, stresses and mechanical spring-back of plates of various dimensions and related materials. In order to eliminate the wrinkles at plate edges that are formed during the process, the plates have to be calibrated after the bending.

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