

# Hybrid Choice Models: from Static to Dynamic

Moshe Ben-Akiva<sup>1</sup>, Maya Abou-Zeid, and Charisma Choudhury

Extended abstract prepared for presentation at TRISTAN VI, Thailand, June 2007

## The Hybrid Choice Model

The gap between discrete choice models and behavioral theory has spurred various developments that attempted to enrich the behavioral realism of discrete choice models by explicitly modeling one or more components of the “black box” of decision-making (e.g. accounting for attitudes and perceptions) or employing more flexible error structures in the specification of the utility function (see for example McFadden, 2001; Morikawa et al., 2002; Ben-Akiva et al.; 2002b; Train, 2003). The most general framework that has been proposed to date has been the Hybrid Choice Model (HCM) (Ben-Akiva et al., 2002a; Walker and Ben-Akiva, 2002) which integrates latent variable and latent class models with discrete choice methods to model the influence of latent variables and classes on the choice process. Latent variable models capture the formation and measurement of latent psychological factors, such as attitudes and perceptions, which explain unobserved individual heterogeneity. Latent class models also capture unobserved heterogeneity by modeling latent segments of the population that could differ in their choice sets or decision protocols for example.

## Plans as Hidden Decision Layers

While the HCM is general enough to model the effect of any type of latent variable, discrete or continuous, no attempt has been made so far at explicitly incorporating multi-stage decision-making processes. One exception is the modeling of the effects of latent plans on subsequent actions (Ben-Akiva et al., 2006). In many situations, individual behavior comes as a result of a conscious planning process. People plan ahead several aspects of their lives: their daily travel behavior, their weekly activity participation patterns, their next job or residential location, and so on. They then select actions to execute their plans. The plans themselves are difficult to observe. It is the actions that result from the planning process that are observed. For example, in the case of residential relocation, the actual moves by households are observed, but the planning process that the household employed to come up with a move is unobserved. Yet it is important to model these plans because they determine the action choice set and guide the actions. In the residential relocation example, if the household had not considered a move from its current residence, then the action choice set consists of one action only, which is not to move, and the household is unaware of the attributes of alternative residential locations.

---

<sup>1</sup> Corresponding author. Massachusetts Institute of Technology, 77 Massachusetts Avenue, Room 1-181, Cambridge, MA 02139. E-mail: mba@mit.edu

More generally, this sequential decision process involving the choice of a plan and an action can be represented as shown in Figure 1, where the upper level represents the choice of plan and the lower level represents the choice of action.  $p$  denotes a plan,  $j$  denotes an action, and  $J_p$  denotes the number of alternative actions under plan  $p$ . This latent choice model can be expressed as the product of two choice models: the choice of a plan and the choice of an action for a given plan.

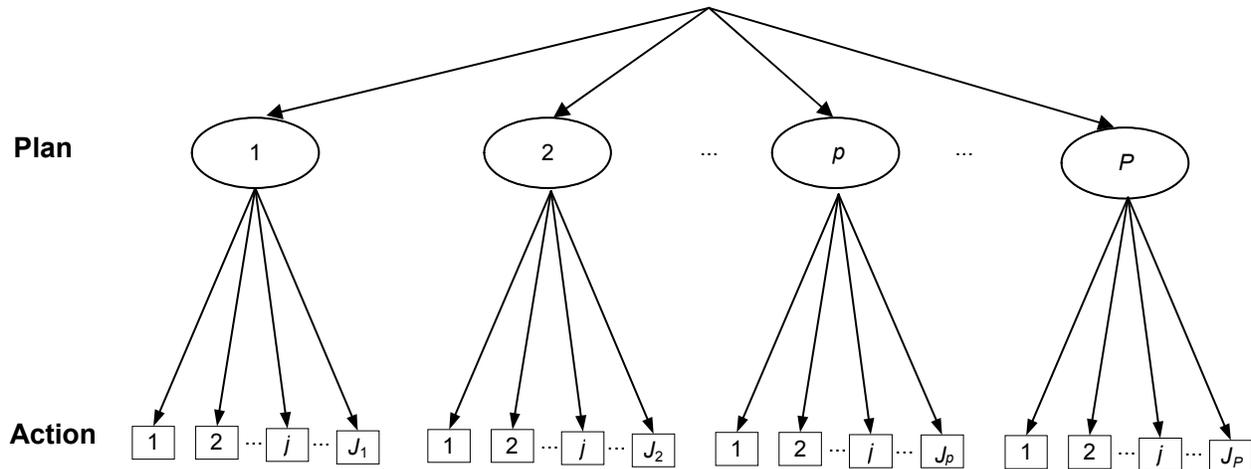


Figure 1. Plans and actions.

### Behavioral Dynamics

The HCM has been developed to model choice in a static context. However, many choice situations take place in a dynamic environment and involve interdependencies among decisions made at different points in time. For example, individuals' plans and actions, as well as external conditions, are subject to change. People may consider several alternatives to come up with a plan, but the actions that they end up executing might be different from what they initially planned. This evolution in plans could be due to several factors. First, situational constraints, contextual changes, or acquisition of information might lead one to revise one's plan. For example, an unusual level of congestion might lead a traveler to revise his planned time of travel or route. Second, people's current plans are influenced by their past experiences so that as their history changes, their plans could change as well. For example, the choice of an action with an unfavorable outcome might lead one to abandon the plan that led to this action in future choice situations. Third, people might eventually adapt to conditions in their environment so that they might exhibit inertia in the choice of their plans and actions. An example of this effect is the decision to stay in the same residential location for several years due to adaptation to and satisfaction with the surrounding environment and housing conditions.

Capturing such evolutions of plans and resulting actions is key to understanding behavioral dynamics. The major hypothesis of our approach is that behavioral dynamics are best explained by a sequence of latent variables (e.g. latent plans) that follow a Markovian process (to capture state dependencies). This hypothesis motivates the integration of discrete choice analysis with a

Hidden Markov Model (HMM) (Baum and Petrie, 1966; Baum, 1972). Our previous work in this area (Ben-Akiva et al., 2006; Choudhury et al., 2007) involved modeling driving behavior on congested freeways, where complex merging phenomena are observed, including normal, courtesy, and forced merging. By explicitly modeling the dynamics of these unobserved merging tactics (plans) and their effect on gap acceptance decisions (actions), the latent plan model was shown to consistently perform better than the model which does not include planning in terms of both statistical performance and realism of the simulated behavior.

The dynamic HCM framework applied to plans and actions is shown in Figure 2. In the upper panel of the figure, the upper level follows a Markovian process representing the evolution in plans. The lower level represents the actions over the time horizon, allowing for feedback mechanisms from an action in a given time period to the plan in the next time period. The lower panel focuses in on the choice behavior at a given point in time.

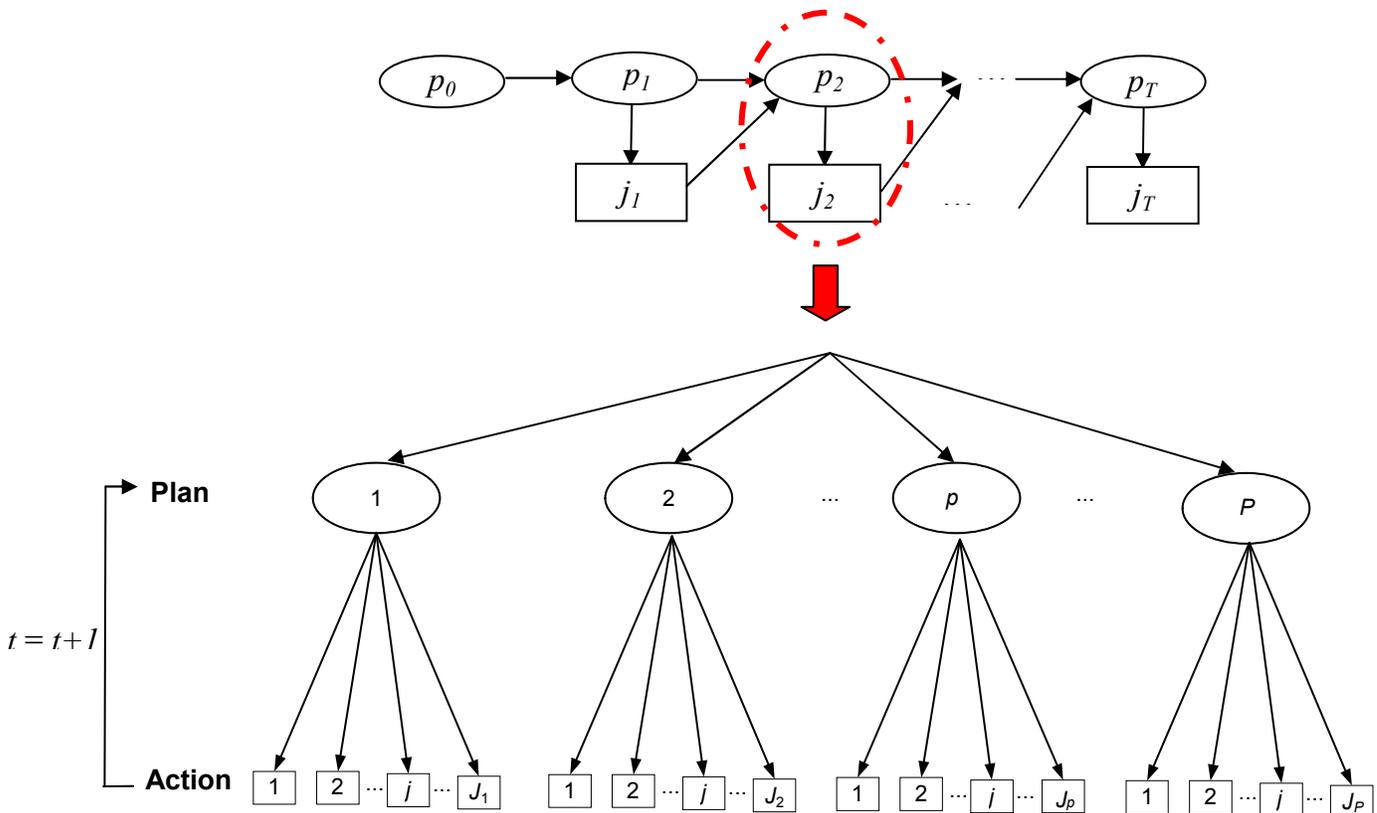


Figure 2. Dynamic plans and actions.

The HMM assumptions are: (1) the latent variables in time period  $t$  are determined only by the latent variables in the previous time period (first-order Markov model) and might be affected by the choice observed in the previous time period (experience) and (2) conditional on the latent variables, the choice observed at a given time period is independent of the choices observed at previous time periods and is only dependent on the current latent variables.

It can be shown that in calculating the joint probability of a trajectory of actions, the computational advantage of the HMM assumptions is the reduction in the number of summations from  $|P|^T$  to  $|P|T$ , where  $|P|$  is the number of possible plans and  $T$  is the number of time periods.

## **Conclusion**

These new developments in discrete choice modeling aim at bridging the gap with behavioral theory. Our approach is based on the Hybrid Choice Model (HCM) which has been proposed to integrate latent variable models, latent class models, flexible error structures, and multiple data sources with discrete choice methods.

We extend the HCM in two major ways. First, we argue that observed individual actions are preceded by plans and show how to model these unobserved plans. Second, we extend the HCM framework to model dynamics of individual behavior. Our approach is based on the integration of Hidden Markov Models and discrete choice methods. We introduce state dependence and interactions between plans and actions into choice models involving hidden decision layers.

The extended HCM makes behavioral models more realistic and their predictions more accurate. We have already applied the dynamic HCM as a two-layer decision hierarchy involving plans and actions to driving behavior modeling with resulting significant improvements in statistical performance and realism of the simulated behavior.

## References

- Baum, L.E. (1972): An inequality and associated maximization technique in statistical estimation for probabilistic functions of Markov processes. *Inequalities*, Vol. 3, pp. 1-8.
- Baum, L.E. and Petrie, T. (1966): Statistical inference for probabilistic functions of finite state Markov chains. *Annals of Mathematical Statistics*, Vol. 37, No. 6, pp. 1554-1563.
- Ben-Akiva, M., Choudhury, C., and Toledo, T. (2006): Modeling latent choices: application to driving behavior. Paper presented at the 11<sup>th</sup> International Conference on Travel Behaviour Research, Kyoto, Japan, August 16-20, 2006.
- Ben-Akiva, M., McFadden, D., Train, K., Walker, J., Bhat, C., Bierlaire, M., Bolduc, D., Boersch-Supan, A., Brownstone, D., Bunch, D.S., Daly, A., de Palma, A., Gopinath, D., Karlstrom, A., and Munizaga, M.A. (2002a): Hybrid choice models: progress and challenges. *Marketing Letters*, Vol. 13, No. 3, pp. 163-175.
- Ben-Akiva, M., Walker, J., Bernardino, A.T., Gopinath, D., Morikawa, T., and Polydoropoulou, A. (2002b): Integration of choice and latent variable models. In *Perpetual Motion: Travel Behaviour Research Opportunities and Application Challenges*, Ed. Mahmassani, H.S., Elsevier, pp. 431-470.
- Choudhury, C.F., Ben-Akiva, M., Toledo, T., Lee, G., and Rao, A. (2007): Modeling cooperative lane changing and forced merging behaviour. Presented at the 86<sup>th</sup> Annual Meeting of the Transportation Research Board, Washington, DC.
- McFadden, D. (2001): Economic choices. *American Economic Review*, Vol. 91, No. 3, pp. 351-378.
- Morikawa, T., Ben-Akiva, M., and McFadden, D. (2002): Discrete choice models incorporating revealed preferences and psychometric data. *Econometric Models in Marketing, Advances in Econometrics*, Vol.16, Eds. Franses, P.H. and Montgomery, A.L., Elsevier, pp. 29-55.
- Train, K. (2003): *Discrete Choice Methods with Simulation*, Cambridge University Press.
- Walker, J. and Ben-Akiva, M. (2002): Generalized random utility model. *Mathematical Social Sciences*, Vol. 43, No. 3, pp. 303-343.