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Design of dynamic controller for neutral differential systems with delay in control input

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Abstract

In this paper, the design problem of dynamic output feedback controller for asymptotic stabilization of a class of neutral systems have been considered. A criterion for the existence of such controllers is derived based on the linear matrix inequality (LMI) approach combined with the Lyapunov method. A parameterized characterization of the controllers is given in terms of the feasible solutions to the LMIs, which can be solved by various convex optimization algorithms. A numerical example is given to illustrate the proposed design method.

1. Introduction

Time-delays often occur in many dynamic systems such as electrical networks, biological systems, chemical systems, economy, and other disciplines. The time-delays are a source of instability and poor performance. Considerable effort has been done on different aspects of time-delay systems over the decades; e.g. see [1] and the references therein. In recent years, more attention is focused on the stability analysis and stabilization problem of neutral delay-differential systems, which are the general form of delay systems and contain delay on the derivatives of some system variables [2–10]. However, all the works in the literature are restricted to the static state feedback control schemes for stabilization of unstable neutral systems although output measurement based control is a necessary prerequisite for practical control problems. Furthermore, in some situation, there is a strong need to construct dynamic controller instead of static controller in order to obtain better performance and dynamical behavior of state response. To the best knowledge of authors, the topic of dynamic output feedback control for neutral systems with delay in control input has not been investigated yet.

This paper is concerned with the design problem of output dynamic feedback controller for a class of neutral systems with delay in control input. Using the Lyapunov functional stability theory combined with the LMI technique, a stabilization criterion for the existence of the controller is derived in terms of LMIs, and theirs solutions provide a parameterized representation of the controller. The LMIs can be easily solved by various efficient convex optimization algorithms [11]. Finally, a numerical example is given to illustrate the proposed design method.

Through the paper, $\Re^n$ denotes n-dimensional Euclidean space, $\Re^{n\times m}$ is the set of all $n \times m$ real matrices, I denotes identity matrix of appropriate order, and $\star$ represents the elements below the main diagonal of a symmetric block matrix. $\|\cdot\|$ denotes Euclidean norm of a given vector and its induced norm of a matrix. $\lambda_{\mathrm{m}}(\cdot)$ and $\rho(\cdot)$ denote the minimum eigenvalue and spectral radius of the matrix $(\cdot)$, respectively. diag$\{\cdot\}$ denotes the block diagonal matrix. The

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notation $W > 0$ ($\geqslant, <, \leqslant 0$) denotes a symmetric positive definite (positive semidefinite, negative, negative semidefinite) matrix W.

2. Problem statement and controller design

Consider a class of neutral delay-differential system of the form:

$$\begin{cases} \dot{x}(t) = Ax(t) + E\dot{x}(t-h) + B_0u(t) + B_1u(t-h), \\ y(t) = Cx(t), \end{cases} \tag{1}$$

with the initial condition function

$$x(t_0 + \theta) = \phi(\theta), \quad \forall \theta \in [-h, 0], \tag{2}$$

where $x(t) \in \Re^n$ is the state vector, $A, E \in \Re^{n\times n}$, $B_0, B_1 \in \Re^{n\times m}$ and $C \in \Re^{l\times n}$ are constant system matrices, $u(t) \in \Re^m$ is the control input, $y(t) \in \Re^l$ is the measured output, h is a positive constant time-delay, and $\phi(\cdot) \in \mathscr{C}_0$ is the initial vector, where $\mathscr{C}_0$ is a set of all continuous differentiable function on $[-h, 0]$ to $\Re^n$.

In this paper, we consider the following dynamic output feedback controllers in order to stabilize system (1):

$$\begin{aligned} \dot{\xi}(t) &= A_c\xi(t) + B_cy(t), \\ u(t) &= C_c\xi(t), \end{aligned} \tag{3}$$

where $\xi(t) \in \Re^n$ is the controller state vector, and A_c, B_c, and C_c are gain matrices with appropriate dimensions to be determined later. Applying this controller (3) to system (1) results in the closed-loop system

$$\dot{z}(t) = \bar{A}_0z(t) + \bar{A}_1z(t-h) + \bar{A}_2\dot{z}(t-h), \tag{4}$$

where

$$z(t) = \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix}, \quad \bar{A}_0 = \begin{bmatrix} A & B_0C_c \\ B_cC & A_c \end{bmatrix}, \quad \bar{A}_1 = \begin{bmatrix} 0 & B_1C_c \\ 0 & 0 \end{bmatrix}, \quad \bar{A}_2 = \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}.$$

Before proceeding further, we need a well-known fact and a lemma.

Fact 1 (*Schur complement*). Given constant symmetric matrices$\Sigma_1, \Sigma_2, \Sigma_3$ where $\Sigma_1 = \Sigma_1^{\mathrm{T}}$ and $0 < \Sigma_2 = \Sigma_2^{\mathrm{T}}$, then $\Sigma_1 + \Sigma_3^{\mathrm{T}}\Sigma_2^{-1}\Sigma_3 < 0$ if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^{\mathrm{T}} \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0, \quad \text{or} \quad \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^{\mathrm{T}} & \Sigma_1 \end{bmatrix} < 0.$$

Lemma 2. [12]. *For any constant symmetric positive-definite matrix Θ, a positive scalar σ, and the vector function $\omega : [0, \sigma] \to \Re^m$ such that the integrations in the following are well defined, then*

$$\sigma \int_0^\sigma \omega^{\mathrm{T}}(s)\Theta\omega(s)\,\mathrm{d}s \geqslant \left(\int_0^\sigma \omega(s)\,\mathrm{d}s\right)^{\mathrm{T}} \Theta \left(\int_0^\sigma \omega(s)\,\mathrm{d}s\right).$$

Now, we define a new operator $\mathscr{D}(z_t) : \mathscr{C}_0 \to \Re^n$ as

$$\mathscr{D}(z_t) = z(t) + \bar{A}_1 \int_{t-h}^t z(s)\,\mathrm{d}s - \bar{A}_2z(t-h). \tag{5}$$

With the above operator, the transformed systems is

$$\dot{\mathscr{D}}(z_t) = (\bar{A}_0 + \bar{A}_1)z(t). \tag{6}$$

Remark 3. The well-known criterion [1] for stability of the operator $\mathscr{D}(z_t)$ given in (5) is

$$\rho(h|\bar{A}_1| + |\bar{A}_2|) < 1. \tag{7}$$

Now, we establish a criterion in terms of matrix inequalities, for dynamic output feedback controller of the neutral delay-differential system (1) using the Lyapunov stability theory.

Theorem 4. *For given scalar $h > 0$, suppose that $\rho(|E|) < 1$. Then, there exists an dynamic output feedback controller* (3) *for the system* (1) *if there exist a positive scalars ε and δ, positive-definite matrices S, Y, X, and matrices $\hat{A}, \hat{B}, \hat{C}$ satisfying the following LMIs*:

$$\begin{bmatrix} \Omega_1 & \varepsilon hYE^{\mathrm{T}} & \delta\hat{C}^{\mathrm{T}}B_1^{\mathrm{T}} & \Omega_2 & \Omega_3 & -\hat{A}^{\mathrm{T}} & -h\cdot\Omega_3 & h\hat{A}^{\mathrm{T}} \\ \star & -hI & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & -\delta I & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \Omega_4 & -A^{\mathrm{T}} & \Omega_5 & hA^{\mathrm{T}} & -h\cdot\Omega_5 \\ \star & \star & \star & \star & -hX & -\varepsilon hY & 0 & 0 \\ \star & \star & \star & \star & \star & -\varepsilon hI & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -X & -\delta Y \\ \star & \star & \star & \star & \star & \star & \star & -\delta I \end{bmatrix} < 0, \tag{8}$$

$$\begin{bmatrix} Y & I \\ I & S \end{bmatrix} > 0, \tag{9}$$

where

$$\begin{aligned} &B = B_0 + B_1, \\ &\Omega_1 = AY + YA^{\mathrm{T}} + B\hat{C} + \hat{C}^{\mathrm{T}}B^{\mathrm{T}} + 2Y, \\ &\Omega_2 = \hat{A}^{\mathrm{T}} + A + \varepsilon hYE^{\mathrm{T}}E + I, \\ &\Omega_3 = -(YA^{\mathrm{T}} + \hat{C}^{\mathrm{T}}B^{\mathrm{T}}), \\ &\Omega_4 = SA + A^{\mathrm{T}}S + \hat{B}C + C^{\mathrm{T}}\hat{B}^{\mathrm{T}} + \varepsilon hE^{\mathrm{T}}E, \\ &\Omega_5 = -(A^{\mathrm{T}}S + C^{\mathrm{T}}\hat{B}^{\mathrm{T}}). \end{aligned} \tag{10}$$

Proof. Let us consider the following legitimate Lyapunov functional candidate [1]:

$$V = \mathscr{D}^{\mathrm{T}}(z_t)P\mathscr{D}(z_t) + \frac{1}{h}\int_{t-h}^{t}(s-t+h)z^{\mathrm{T}}(s)\bar{A}_1^{\mathrm{T}}W\bar{A}_1z(s)\,\mathrm{d}s + h\int_{t-h}^{t}z^{\mathrm{T}}(s)\bar{A}_2^{\mathrm{T}}R\bar{A}_2z(s)\,\mathrm{d}s, \tag{11}$$

where $P > 0, W > 0$ and $R > 0$.

Taking the time derivative of V along the solution of (6), we have

$$\begin{aligned} \frac{\mathrm{d}V}{\mathrm{d}t} = {}& 2z^{\mathrm{T}}(t)\Big(\bar{A}_0 + \bar{A}_1\Big)^{\mathrm{T}}P\left(z(t) + \int_{t-h}^{t}\bar{A}_1z(s)\,\mathrm{d}s - \bar{A}_2z(t-h)\right) + z^{\mathrm{T}}(t)\bar{A}_1^{\mathrm{T}}PW\bar{A}_1z(t) - \frac{1}{h} \\ & \times\int_{t-h}^{t}z^{\mathrm{T}}(s)\bar{A}_1^{\mathrm{T}}W\bar{A}_1z(s)\,\mathrm{d}s + hz^{\mathrm{T}}(t)\bar{A}_2^{\mathrm{T}}R\bar{A}_2z(t) - hz^{\mathrm{T}}(t-h)\bar{A}_2^{\mathrm{T}}R\bar{A}_2z(t-h). \end{aligned} \tag{12}$$

By Lemma 2, a bound of the term $-\int_{t-h}^{t}z^{\mathrm{T}}(s)\bar{A}_1^{\mathrm{T}}W\bar{A}_1z(s)\,\mathrm{d}s$ of right-hand side of (12) can be obtained as

$$-\int_{t-h}^{t}z^{\mathrm{T}}(s)\bar{A}_1^{\mathrm{T}}W\bar{A}_1z(s)\,\mathrm{d}s \leqslant -\left(\frac{1}{h}\int_{t-h}^{t}\bar{A}_1z(s)\,\mathrm{d}s\right)^{\mathrm{T}}(hW)\left(\frac{1}{h}\int_{t-h}^{t}\bar{A}_1z(s)\,\mathrm{d}s\right). \tag{13}$$

Substituting (13) into (12) gives that

$$\begin{aligned} \frac{\mathrm{d}V}{\mathrm{d}t} \leqslant {}& z^{\mathrm{T}}(t)\Big(P(\bar{A}_0 + \bar{A}_1) + (\bar{A}_0 + \bar{A}_1)^{\mathrm{T}}P + \bar{A}_1^{\mathrm{T}}W\bar{A}_1 + h\bar{A}_2^{\mathrm{T}}R\bar{A}_2\Big)z(t) + 2z^{\mathrm{T}}(t)(\bar{A}_0 + \bar{A}_1)^{\mathrm{T}}P\int_{t-h}^{t}\bar{A}_1z(s)\,\mathrm{d}s \\ & - 2z^{\mathrm{T}}(t)(\bar{A}_0 + \bar{A}_1)^{\mathrm{T}}P\bar{A}_2z(t-h) - \left(\frac{1}{h}\int_{t-h}^{t}\bar{A}_1z(s)\,\mathrm{d}s\right)^{\mathrm{T}}W\left(\frac{1}{h}\int_{t-h}^{t}\bar{A}_1z(s)\,\mathrm{d}s\right) \\ & - hz^{\mathrm{T}}(t-h)\bar{A}_2^{\mathrm{T}}R\bar{A}_2z(t-h) \equiv \mathscr{Z}^{\mathrm{T}}(t)\Sigma\mathscr{Z}(t) \end{aligned} \tag{14}$$

where

$$\mathscr{X}(t)=\begin{bmatrix} z(t) \\ \bar{A}_2 z(t-h) \\ \frac{1}{h}\int_{t-h}^{t}\bar{A}_1 z(s)\,\mathrm{d}s \end{bmatrix},$$

$$\Sigma=\begin{bmatrix} 2P(\bar{A}_0+\bar{A}_1)+\bar{A}_1^{\mathrm{T}}W\bar{A}_1+h\bar{A}_2R\bar{A}_2 & -(\bar{A}_0+\bar{A}_1)^{\mathrm{T}}P & h(\bar{A}_0+\bar{A}_1)^{\mathrm{T}}P \\ \star & -hR & 0 \\ \star & \star & -W \end{bmatrix}.$$

Thus, if the inequality $\Sigma<0$ holds, there exists a positive scalar γ such that

$$\frac{\mathrm{d}V}{\mathrm{d}t}\leqslant-\gamma\|z(t)\|^2. \tag{15}$$

In the matrix Σ, the matrices $P>0$, $W>0$ and $R>0$ and the controller parameters A_c, B_c and C_c, which included in the matrix $\bar{A}_0$, are unknown and occur in nonlinear fashion. Hence, the inequality $\Sigma<0$ cannot be considered as an linear matrix inequality problem. In the following, we will use a method of changing variables such that the inequality can be solved as convex optimization algorithms [13].

First, partition the matrix P and its inverse as

$$P=\begin{bmatrix} S & N \\ N^{\mathrm{T}} & U \end{bmatrix},\quad P^{-1}=\begin{bmatrix} Y & M \\ M^{\mathrm{T}} & W \end{bmatrix}, \tag{16}$$

where $S, Y\in\Re^{n\times n}$ are positive-definite matrices, and M and N are invertible matrices. Note that the equality $P^{-1}P=I$ gives that

$$MN^{\mathrm{T}}=I-YS. \tag{17}$$

Define

$$F_1=\begin{bmatrix} Y & I \\ M^{\mathrm{T}} & 0 \end{bmatrix},\quad F_2=\begin{bmatrix} I & S \\ 0 & N^{\mathrm{T}} \end{bmatrix}. \tag{18}$$

Then, it follows that

$$PF_1=F_2,\quad F_1^{\mathrm{T}}PF_1=F_1^{\mathrm{T}}F_2=\begin{bmatrix} Y & I \\ I & S \end{bmatrix}>0. \tag{19}$$

Now, postmultiplying and premultiplying the matrix inequality, $\Sigma<0$, by the matrix $\mathrm{diag}\{F_1^{\mathrm{T}},F_1^{\mathrm{T}},F_1^{\mathrm{T}}\}$ and by its transpose, respectively, gives

$$\begin{bmatrix} \begin{pmatrix} F_2^{\mathrm{T}}(\bar{A}_0+\bar{A}_1)F_1+F_1^{\mathrm{T}}(\bar{A}_0+\bar{A}_1)^{\mathrm{T}}F_2 \\ +F_1^{\mathrm{T}}\bar{A}_1^{\mathrm{T}}W\bar{A}_1F_1+hF_1^{\mathrm{T}}\bar{A}_2R\bar{A}_2F_1 \end{pmatrix} & -F_1^{\mathrm{T}}(\bar{A}_0+\bar{A}_1)^{\mathrm{T}}F_2 & hF_1^{\mathrm{T}}(\bar{A}_0+\bar{A}_1)^{\mathrm{T}}F_2 \\ \star & -hF_1^{\mathrm{T}}RF_1 & 0 \\ \star & \star & -F_1^{\mathrm{T}}WF_1 \end{bmatrix}<0. \tag{20}$$

Here, define the matrices W and R as

$$W=\mathrm{diag}\{\delta I,Q\},\quad R=\mathrm{diag}\{\varepsilon I,Q\},$$

where δ and ε is positive scalars, and Q is the positive-definite matrix to be chosen later.

By utilizing the relation (16)–(19), it can be easily obtained that the inequality (20) is equivalent to

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ \star & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ \star & \star & (3,3) & -h\varepsilon Y & 0 & 0 \\ \star & \star & \star & -h\varepsilon I & 0 & 0 \\ \star & \star & \star & \star & (5,5) & -\delta Y \\ \star & \star & \star & \star & \star & -\delta I \end{bmatrix}<0, \tag{21}$$

where

$$
\begin{aligned}
(1,1) &= AY + YA^{\mathrm{T}} + BC_cM^{\mathrm{T}} + MC_c^{\mathrm{T}}B^{\mathrm{T}} + \varepsilon hYE^{\mathrm{T}}EY + 2Y + \delta MC_c^{\mathrm{T}}B_1^{\mathrm{T}}B_1C_cM^{\mathrm{T}},\\
(1,2) &= A + YA^{\mathrm{T}}S + MC_c^{\mathrm{T}}B^{\mathrm{T}}S + YC^{\mathrm{T}}B_c^{\mathrm{T}}N^{\mathrm{T}} + MA_c^{\mathrm{T}}N^{\mathrm{T}} + \varepsilon hYE^{\mathrm{T}}E + I,\\
(1,3) &= -(YA^{\mathrm{T}} + MC_c^{\mathrm{T}}B^{\mathrm{T}}),\\
(1,4) &= -(YA^{\mathrm{T}}S + YC^{\mathrm{T}}B_c^{\mathrm{T}}N^{\mathrm{T}} + MC_c^{\mathrm{T}}B^{\mathrm{T}}S + MA_c^{\mathrm{T}}N^{\mathrm{T}}),\\
(1,5) &= -h\cdot(1,3),\\
(1,6) &= -h\cdot(1,4),\\
(2,2) &= SA + NB_cC + A^{\mathrm{T}}S + C^{\mathrm{T}}B_c^{\mathrm{T}}N^{\mathrm{T}} + \varepsilon hE^{\mathrm{T}}E,\\
(2,3) &= -A^{\mathrm{T}},\\
(2,4) &= -(A^{\mathrm{T}}S + C^{\mathrm{T}}B_c^{\mathrm{T}}N^{\mathrm{T}}),\\
(2,5) &= -h\cdot(2,3),\\
(2,6) &= -h\cdot(2,4),\\
(3,3) &= -h(\varepsilon YY + MQM^{\mathrm{T}}),\\
(5,5) &= -(\delta YY + MQM^{\mathrm{T}}).
\end{aligned}
$$

By defining a new set of variables as follows:

$$
\begin{aligned}
X &= MQM^{\mathrm{T}},\\
\hat{A} &= SAY + SB_0\hat{C} + \hat{B}CY + NA_cM^{\mathrm{T}},\\
\hat{B} &= NB_c,\\
\hat{C} &= C_cM^{\mathrm{T}},
\end{aligned}
\tag{22}
$$

the inequality (21) is simplified to the following inequality:

$$
\Pi = \begin{bmatrix}
\begin{pmatrix} \Omega_1 + \varepsilon hYE^{\mathrm{T}}EY \\ +\delta\hat{C}^{\mathrm{T}}B_1^{\mathrm{T}}B_1\hat{C} \end{pmatrix} & \Omega_2 & \Omega_3 & -\hat{A}^{\mathrm{T}} & -h\cdot\Omega_3 & h\hat{A}^{\mathrm{T}} \\
\star & \Omega_4 & -A^{\mathrm{T}} & \Omega_5 & hA^{\mathrm{T}} & -h\cdot\Omega_5 \\
\star & \star & -hX - \varepsilon hYY & -\varepsilon hY & 0 & 0 \\
\star & \star & \star & -\varepsilon hI & 0 & 0 \\
\star & \star & \star & \star & -X - \delta YY & -\delta Y \\
\star & \star & \star & \star & \star & -\delta I
\end{bmatrix} < 0,
\tag{23}
$$

where Ω_1, Ω_2, and Ω_3 are defined in (10).

Note that the inequality $\Pi < 0$ holds if

$$
\Pi + \mathrm{diag}\{0, 0, \varepsilon hYY, 0, \delta YY, 0\} < 0.
\tag{24}
$$

By Fact 1 (*Schur complement*), the inequality (24) is equivalent to the inequality (8). Also, the operator $\mathscr{D}(z_t)$ is stable if the condition (7) holds. Note that the condition (7) is equivalent to $\rho(|E|) < 1$. Therefore, by Theorem 9.8.1 (p. 292–3) of Hale and Lunel [1] with the stable operator $\mathscr{D}(z_t)$ and (15), we conclude that system (1) and (4) are both asymptotically stable. This completes the proof. □

Remark 5. The problem of Theorem 4 is to determine whether the problem is feasible or not. It is called the feasibility problem. The solutions of the problem can be found by solving generalized eigenvalue problem in S, Y, X, $\hat{A}$, $\hat{B}$, $\hat{C}$, δ, and ε, which is a quasiconvex optimization problem. Note that a locally optimal point of a quasiconvex optimization problem with strictly quasiconvex objective is globally optimal [11]. Various efficient convex optimization algorithms can be used to check whether the matrix inequalities (8) and (9) is feasible. In this paper, in order to solve the matrix inequality, we utilize Matlab's LMI Control Toolbox [14], which implements state-of-the-art interior-point algorithms, which is significantly faster than classical convex optimization algorithms [11].

Remark 6. Given any solution of the matrix inequalities (8) and (9) in Theorem 4, a corresponding controller of the form (3) will be constructed as follows:

- Using the solution X and selecting any positive-definite matrix Q, compute the the invertible matrices M satisfying the relation $X = MQM^{\mathrm{T}}$.
- Using the matrix M, computer the invertible matrix N satisfying (17).
- Utilizing the matrices M and N obtained above, solve the system of equations (22) for B_c, C_c and A_c (in this order).

To illustrate the design procedure, we give a numerical example.

Example 7. Consider the following neutral delay-differential system of the form:

$$\begin{cases} \dot{x}(t) = Ax(t) + E\dot{x}(t-h) + B_0 u(t) + B_1 u(t-h), \\ y(t) = Cx(t), \end{cases} \tag{25}$$

where

$$A = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}, \quad E = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 10 \\ 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C = [4 \quad 2], \quad h = 1.$$

Here, we want to construct a suitable dynamic output feedback controller of the form (3) for system (25), which guarantees the asymptotic stability of the closed-loop system. First, let's check the stability of the operator $\mathscr{D}(z_t)$ given in (7). Since $\rho(|E|) = 0.2 < 1$, the operator is stable.

Next, by applying Theorem 4 to system (25) and checking the feasibility of LMIs (8) and (9), we can find that the LMIs are feasible and obtain the solutions of the inequalities:

$$S = \begin{bmatrix} 26.4302 & -24.8644 \\ -24.8644 & 56.1020 \end{bmatrix}, \quad Y = \begin{bmatrix} 3.3366 & -0.1049 \\ -0.1049 & 0.3400 \end{bmatrix}, \quad X = \begin{bmatrix} 160.2574 & -2.9441 \\ -2.9441 & 115.8740 \end{bmatrix},$$
$$\hat{A} = \begin{bmatrix} -4.8214 & 0.6177 \\ -0.6376 & -0.4602 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} -11.1971 \\ -4.1317 \end{bmatrix}, \quad \hat{C} = [-4.1751 \quad -0.3207], \quad \varepsilon = 169.3747, \quad \delta = 170.0186.$$

Now, let's choose $Q = 10I$. Then in light of Remark 6, two invertible matrices M, N are

$$M = \begin{bmatrix} 4.0030 & -0.0397 \\ -0.0397 & 3.4038 \end{bmatrix}, \quad N = \begin{bmatrix} -22.4012 & 3.0365 \\ 22.1366 & -5.8182 \end{bmatrix},$$

and the corresponding positive-definite matrix P is

$$P = \begin{bmatrix} 26.4302 & -24.8644 & -22.4012 & 3.0365 \\ -24.8644 & 56.1020 & 22.1366 & -5.8182 \\ -22.4012 & 22.1366 & 19.2249 & -2.6770 \\ 3.0365 & -5.8182 & -2.6770 & 0.6435 \end{bmatrix}.$$

Then, we can get the stabilizing dynamic output feedback controller for the system (25):

$$A_c = \begin{bmatrix} -17.2071 & -1.7205 \\ -24.0423 & -3.7014 \end{bmatrix}, \quad B_c = \begin{bmatrix} 1.2310 \\ 5.3936 \end{bmatrix}, \quad C_c = [-1.0440 \quad -0.1064].$$

3. Concluding remarks

This paper concerned with the problem of stabilization for a class of neutral differential systems with delay in control input. Then we have designed a dynamic output feedback controller which guarantees the asymptotic stability of the systems, and derived stabilization criterion in terms of LMIs which can be easily solved by various efficient convex optimization algorithms. Finally a numerical example is shown to illustrate the design procedure.

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