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Rate Bounds for MIMO Relay Channels Using Precoding

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Abstract— **Relay channels will play a central role in next-generation multihop wireless systems. This paper considers the MIMO relay channel where multiple antennas are employed by each terminal. New lower bounds on the capacity of a Gaussian MIMO relay channel are derived under the assumption that the transmitter employs either superposition coding or dirty-paper coding. The proposed lower bounds improve on a previously proposed lower bound that arises from a simple transmit strategy.**

I. Introduction

Next-generation wireless systems must be able to provide improved coverage and throughput to an ever-increasing number of mobile users. Future network design will need to support communication in dense environments while minimizing infrastructure deployment costs. Multihop communication [1] has been developed to meet these objectives. A central feature of multihop communication is the use of intermediate helper nodes to relay data from a sender to a destination. An information-theoretic analysis of multihop communication, then, should incorporate relay channels. In a relay channel, the transmitter and a relay attempt to cooperate in some predetermined fashion to maximize the signaling rate to the receiver. The relay channel is an elementary component of a multihop network, and is thus an important building block of next-generation wireless systems.

Relay channels were first introduced in [2] and were further studied in [3]. An information-theoretic analysis of full-duplex relay channels was performed in [4], where upper and lower capacity bounds were derived for a general relay channel. In particular, it was found that the use of a relay, in many cases, is a direct improvement over a point-to-point channel. The work in [4] also introduced the concept of block-Markov encoding to achieve the capacity for certain types of relay channels.

The achievable data rates from the relaying techniques in [4] can be enhanced by applying concepts from multi-input multi-output (MIMO) signaling [5]–[7]. It has been shown that utilizing multiple antennas at the transmitter and/or receiver in a wireless system can result in sizable spectral efficiency gains [8]. This encouraging result has spurred studies of the capacity region for Gaussian MIMO multiple access (MAC) [9], [10] and broadcast (BC) [11], [12] channels.

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These studies of the MAC and BC channels form a good basis for an information-theoretic analysis of a MIMO relay channel. We note that MIMO relay channels, which offer the simultaneous benefits of multihop signaling and high data rates, have only recently been analyzed from an information-theoretic perspective [13]. The seminal work in [13] considers a Gaussian relay channel with multiple antennas at each terminal. Upper and lower capacity bounds are shown for both fixed channels and for channels undergoing Rayleigh fading.

We propose new lower capacity bounds for the Gaussian MIMO relay channel by having the transmitter employ precoding methods such as superposition coding and dirty-paper coding [14]. We consider dirty-paper coding in our analysis since it has been utilized in [11], [12] to derive the capacity region of the Gaussian MIMO BC channel; we also consider superposition coding as a simple example of dirty-paper coding. We find that our proposed lower bounds improve on the lower bounds from [13].

The remainder of this paper is organized as follows. In Section II we describe the system model. Section III contains the upper and lower capacity bounds from [13] for the Gaussian MIMO relay channel. In section IV, we first discuss superposition coding and dirty-paper coding and then apply them to a Gaussian MIMO relay channel. Numerical results for our precoding methods are given in Section V. We conclude the paper in Section VI.

II. System Model

We use boldface notation for matrices and vectors. $\mathbb{E}$ represents mathematical expectation. Re(x) denotes the real part of a complex number x. For a matrix $\mathbf{A}$, $\mathbf{A}^\dagger$, tr($\mathbf{A}$) and det($\mathbf{A}$) denote the transpose conjugate, trace, and determinant, respectively of $\mathbf{A}$ while $\mathbf{A} \succeq 0$ means that $\mathbf{A}$ is positive semi-definite. SNR represents signal-to-noise ratio. $\mathbf{I}_K$ denotes the $K \times K$ identity matrix. We use $\mathcal{CN}$($\mathbf{B}$, $\mathbf{C}$) to represent the circularly symmetric complex Gaussian distribution with mean $\mathbf{B}$ and covariance matrix $\mathbf{C}$. I($\mathbf{X}$;$\mathbf{Y}$) denotes the mutual information between $\mathbf{X}$ and $\mathbf{Y}$. All logarithms are base-2.

Consider the Gaussian MIMO relay channel illustrated in Fig. 1. Let $\mathbf{X}_1$ and $\mathbf{X}_2$ be the $M_t \times 1$ and $M_r \times 1$ transmitted signals from the transmitter and the relay. Let $\mathbf{Y}$ and $\mathbf{Y}_1$ be the $N_t \times 1$ and $N_r \times 1$ received signals at the receiver and the relay. Define $\mathbf{H}_1$, $\mathbf{H}_2$, and $\mathbf{H}_3$ as $N_r \times M_t$, $N_t \times M_t$ and $N_t \times M_r$ channel gain matrices. Also, $\mathbf{Z}$ and $\mathbf{Z}_1$ are independent $N_t \times 1$ and $N_r \times 1$ circularly-symmetric complex Gaussian noise vectors with distributions $\mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_t})$ and $\mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_r})$.

We define parameters related to the SNR at the receiver and at the relay as

$$\gamma_1 = \frac{SNR_1}{M_t}, \gamma_2 = \frac{SNR_2}{M_t}, \gamma_3 = \frac{SNR_3}{M_r} \tag{1}$$

where SNR_1 and SNR_2 are the expected SNR values for $\mathbf{X}_1$ at each receive antenna at the relay and the receiver, and SNR_3 is the expected SNR for $\mathbf{X}_2$ at each receive antenna at the receiver.

With these definitions, we can write the received signals at the relay and at the receiver as

$$\begin{aligned} \mathbf{Y}_1 &= \sqrt{\gamma_1}\mathbf{H}_1\mathbf{X}_1 + \mathbf{Z}_1 \\ \mathbf{Y} &= \sqrt{\gamma_2}\mathbf{H}_2\mathbf{X}_1 + \sqrt{\gamma_3}\mathbf{H}_3\mathbf{X}_2 + \mathbf{Z}. \end{aligned} \tag{2}$$

We assume that $\mathbb{E}(\mathbf{X}_1^\dagger\mathbf{X}_1) \le M_t$ and $\mathbb{E}(\mathbf{X}_2^\dagger\mathbf{X}_2) \le M_r$. We also assume that the relay has two sets of antennas, with one set for the receiver and one for the transmitter, so it operates in a full-duplex mode. The relay also cancels out interference from its transmitter array at its receiver array. We assume that all channel matrices are fixed and known at all three terminals and that $\mathbf{Z}$ and $\mathbf{Z}_1$ are uncorrelated with $\mathbf{X}_1$ and $\mathbf{X}_2$.

III. Background

It was shown in [4] that

$$C^G \le \max_{p(x_1,x_2)} \min\{I(\mathbf{X}_1;\mathbf{Y},\mathbf{Y}_1|\mathbf{X}_2), I(\mathbf{X}_1,\mathbf{X}_2;\mathbf{Y})\} \tag{3}$$

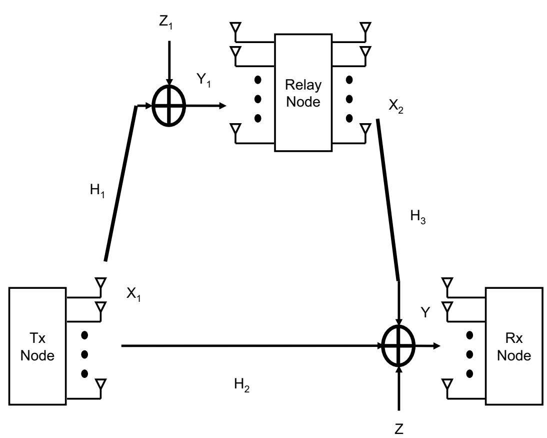

Fig. 1. Gaussian MIMO Relay Channel.

where the first term is the rate from the transmitter to the relay and the receiver, and the second term is the rate from the transmitter and the relay to the receiver.

Now let $\mathbf{X}_1$ and $\mathbf{X}_2$ be random vectors with mean zero and covariance matrices $\Sigma_{ij} = \mathbb{E}(\mathbf{X}_i\mathbf{X}_j^\dagger)$. In [13], the authors established the following capacity upper bound and lower bound for the case where the channel gains are fixed and known at each terminal.

Theorem 3.1: [13] An upper bound on the capacity of the MIMO relay channel is given by

$$C^G \le C^G_{upper} = \max_{0\le\rho\le1,\Sigma_{11},\Sigma_{22}} \min(C_1^G, C_2^G) \tag{4}$$

where tr(Σ_{11}) $\le M_t$, tr(Σ_{22}) $\le M_r$ and

$$\begin{aligned} C_1^G &\triangleq \log[\det(\mathbf{I}_{M_t} + \\ &\quad \left(1-\rho^2\right)\begin{bmatrix}\sqrt{\gamma_1}\mathbf{H}_1\\ \sqrt{\gamma_2}\mathbf{H}_2\end{bmatrix}\Sigma_{11}\begin{bmatrix}\sqrt{\gamma_1}\mathbf{H}_1\\ \sqrt{\gamma_2}\mathbf{H}_2\end{bmatrix}^\dagger\Big)\Big] \\ C_2^G &\triangleq \inf_{a>0}\log[\det(\mathbf{I}_{N_t} + (\gamma_2 + \tfrac{\rho^2}{a}\sqrt{\gamma_2\gamma_3})\mathbf{H}_2\Sigma_{11}\mathbf{H}_2^\dagger \\ &\quad +(\gamma_3 + a\sqrt{\gamma_2\gamma_3})\mathbf{H}_3\Sigma_{22}\mathbf{H}_3^\dagger)]. \end{aligned} \tag{5}$$

Theorem 3.2: [13] A lower bound on the capacity of the Gaussian MIMO relay channel is given by

$$C^G \ge C^G_{lower} = \max(C_d^G, \min(C_3^G, C_4^G)), \tag{6}$$

where

$$\begin{aligned} C_d^G &\triangleq \max\nolimits_{\Sigma_{11}} \log[\det(\mathbf{I}_{N_t} + \gamma_2\mathbf{H}_2\Sigma_{11}\mathbf{H}_2^\dagger)] \\ C_3^G &\triangleq \max\nolimits_{\Sigma_{11}} \log[\det(\mathbf{I}_{N_r} + \gamma_1\mathbf{H}_1\Sigma_{11}\mathbf{H}_1^\dagger)] \\ C_4^G &\triangleq \max\nolimits_{\Sigma_{22}} \log[\det(\mathbf{I}_{N_t} + \\ &\quad \gamma_3\mathbf{H}_3\Sigma_{22}\mathbf{H}_3^\dagger(\mathbf{I}_{N_t} + \gamma_2\mathbf{H}_2\Sigma_{11}^*\mathbf{H}_2^\dagger)^{-1})] \end{aligned} \tag{7}$$

with

$$\Sigma_{11}^* \triangleq \arg\max_{\Sigma_{11}\succeq 0} \log[\det(\mathbf{I}_{N_r} + \gamma_1\mathbf{H}_1\Sigma_{11}\mathbf{H}_1^\dagger)]. \tag{8}$$

Our objective is to use superposition coding and dirty-paper coding to improve upon the bound in Theorem 3.2. We outline these techniques in the next section.

IV. Precoding Methodology

Next we describe the precoding strategy employed in this paper. We divide the transmit message into two components, denoted by the random variables $\mathbf{S}$ and $\mathbf{C}$. $\mathbf{S}$ is the message that is decoded by the relay and is thus cooperatively sent by the transmitter-relay to the receiver. $\mathbf{C}$, however, is intended to be decoded only by the receiver, and thus is a source of "interference" at the relay that is known a-priori at the receiver.

We consider two classes of transmission strategies with this setup. The first is superposition coding, where codebooks for $\mathbf{S}$ and $\mathbf{C}$ are determined separately and then simply superposed at the transmitter. The second strategy is to utilize dirty-paper coding at the transmit end, where the transmitter attempts to mitigate the interference caused by $\mathbf{C}$ to the desirable signal corresponding to $\mathbf{S}$ at the relay.

Note that the receiver must determine both $\mathbf{S}$ and $\mathbf{C}$ to detect its message. Thus, if R_s denotes the rate for the codebook corresponding to $\mathbf{S}$ and R_c that for $\mathbf{C}$, the net achievable rate for all of our schemes is $R = R_s + R_c$. Assuming the receiver successively determines $\mathbf{S}$ and $\mathbf{C}$, the order in which they are determined impacts their rates. In this paper, we use both decoding orders and choose the order that maximizes the overall rate.

Let $\mathbf{U}$ and $\mathbf{V}$ be auxiliary variables representing the contribution of $\mathbf{S}$ and $\mathbf{C}$, respectively to $\mathbf{X}_1$. Define Σ_s, Σ_c and Σ_{X_2} to be the covariance matrices of $\mathbf{S}$, $\mathbf{C}$ and $\mathbf{X}_2$ respectively. Also, define

$$\mathbf{A} = \begin{bmatrix} \Sigma_s & \mathbb{E}(\mathbf{U}\mathbf{X}_2^\dagger) \\ \mathbb{E}(\mathbf{X}_2\mathbf{U}^\dagger) & \Sigma_{X_2} \end{bmatrix}$$

and $\mathbf{B} = [\,\mathbf{H}_2 \quad \mathbf{H}_3\,]$. In this case, $\mathbb{E}(\mathbf{U}\mathbf{U}^\dagger) = \Sigma_s$. In addition, define $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{U}$ and $\mathcal{V}$ as the finite alphabets for $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{U}$ and $\mathbf{V}$, respectively.

A. Superposition Coding

Consider the system illustrated in Fig. 2. Assume that $\mathbf{S}$ is decoded first at the receiver. Let $R_{sup,1}$ be the achievable rate for this case. It is proven in [15] that

$$R_{sup,1} = \sup_{p(x_1,x_2,u,v)} R_{sup,1s} + R_{sup,1c} \quad (9)$$

where

$$\begin{aligned} R_{sup,1s} &= \min(I(\mathbf{U};\mathbf{Y}_1|\mathbf{X}_2), I(\mathbf{U},\mathbf{X}_2;\mathbf{Y})) \\ R_{sup,1c} &= I(\mathbf{V};\mathbf{Y}|\mathbf{U},\mathbf{X}_2) \end{aligned} \quad (10)$$

and the supremum is taken over all joint distributions

$$p(x_1,x_2,u,v) = p(x_2)p(u|x_2)p(v|x_2)p(x_1|u,v)$$

on $\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{U} \times \mathcal{V}$. For the Gaussian MIMO relay channel, Gaussian input optimality can be shown. Thus

$$I(\mathbf{U};\mathbf{Y}_1|\mathbf{X}_2) = \log\left(\frac{\det\left(\mathbf{I} + \mathbf{H}_1(\Sigma_s + \Sigma_c)\mathbf{H}_1^\dagger\right)}{\det\left(\mathbf{I} + \mathbf{H}_1\Sigma_c\mathbf{H}_1^\dagger\right)}\right), \quad (11)$$

which is the maximum signaling rate for $\mathbf{S}$ over the transmitter-to-relay link,

$$I(\mathbf{U},\mathbf{X}_2;\mathbf{Y}) = \log\left(\frac{\det\left(\mathbf{I} + \mathbf{H}_2\Sigma_c\mathbf{H}_2^\dagger + \mathbf{B}\mathbf{A}\mathbf{B}^\dagger\right)}{\det\left(\mathbf{I} + \mathbf{H}_2\Sigma_c\mathbf{H}_2^\dagger\right)}\right), \quad (12)$$

representing the maximum signaling rate for $\mathbf{S}$ over the effective multiple-access channel from the transmitter and relay to the receiver, and

$$I(\mathbf{V};\mathbf{Y}|\mathbf{U},\mathbf{X}_2) = \log(\det(\mathbf{I} + \mathbf{H}_2\Sigma_c\mathbf{H}_2^\dagger)), \quad (13)$$

which is the maximum signaling rate for $\mathbf{C}$ over the transmitter-to-receiver link.

Now assume that $\mathbf{C}$ is decoded first at the receiver. Let $R_{sup,2}$ be the achievable rate for this case. It is proven in [15] that

$$R_{sup,2} = \sup_{p(x_1,x_2,u,v)} R_{sup,2s} + R_{sup,2c} \quad (14)$$

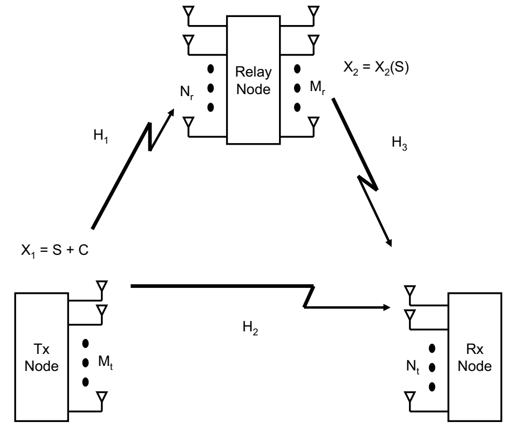

Fig. 2. Gaussian MIMO relay channel with superposition coding.

where

$$\begin{aligned} R_{sup,2c} &= I(\mathbf{V};\mathbf{Y}) \\ R_{sup,2s} &= \min(I(\mathbf{U};\mathbf{Y}_1|\mathbf{X}_2), I(\mathbf{U},\mathbf{X}_2;\mathbf{Y}|\mathbf{V})) \end{aligned} \quad (15)$$

and the supremum is also taken over all joint distributions

$$p(x_1,x_2,u,v) = p(x_2)p(u|x_2)p(v|x_2)p(x_1|u,v)$$

on $\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{U} \times \mathcal{V}$. In this case Gaussian input optimality yields

$$I(\mathbf{V};\mathbf{Y}) = \log\left(\frac{\det\left(\mathbf{I} + \mathbf{H}_2\Sigma_c\mathbf{H}_2^\dagger + \mathbf{B}\mathbf{A}\mathbf{B}^\dagger\right)}{\det\left(\mathbf{I} + \mathbf{B}\mathbf{A}\mathbf{B}^\dagger\right)}\right), \quad (16)$$

which is analogous to the rate in (13) and

$$I(\mathbf{U},\mathbf{X}_2;\mathbf{Y}|\mathbf{V}) = \log(\det(\mathbf{I} + \mathbf{B}\mathbf{A}\mathbf{B}^\dagger)), \quad (17)$$

which is analogous to the rate in (12) while I($\mathbf{U};\mathbf{Y}_1|\mathbf{X}_2$) is the same as in (11).

The objective is to choose the decoding order that yields a higher overall rate. We now state the following result.

Proposition 4.1: Let R_{sup} be the maximum signaling rate for the Gaussian MIMO relay channel employing superposition coding at the transmitter. Then

$$R_{sup} = \max(R_{sup,1}, R_{sup,2}) \geq C^G_{lower} \quad (18)$$

where C^G_{lower} is given in Theorem 3.2.

Proof: See [15] for a detailed proof. ∎

B. Dirty-Paper Coding

Assume that $\mathbf{S}$ is decoded first at the receiver. Let $R_{dpc,1}$ be the achievable rate for this case. It is proven in [15] that

$$R_{dpc,1} = \sup_{p(x_1,x_2,u,v)} R_{dpc,1s} + R_{dpc,1c} \quad (19)$$

where

$$\begin{aligned} R_{dpc,1s} &= \min(I(\mathbf{U};\mathbf{Y}_1|\mathbf{X}_2) - I(\mathbf{U};\mathbf{V}|\mathbf{X}_2), \\ &\qquad I(\mathbf{U},\mathbf{X}_2;\mathbf{Y})) \\ R_{dpc,1c} &= I(\mathbf{V};\mathbf{Y}|\mathbf{U},\mathbf{X}_2) \end{aligned} \quad (20)$$

and the supremum is taken over all joint distributions

$$p(x_1, x_2, u, v) = p(x_2)p(u, v|x_2)p(x_1|u, v)$$

on $\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{U} \times \mathcal{V}$. For the Gaussian MIMO relay channel, Gaussian input optimality can be shown and thus we have

$$I(\mathbf{U}; \mathbf{Y}_1|\mathbf{X}_2) - I(\mathbf{U}; \mathbf{V}|\mathbf{X}_2) = \log(\det(\mathbf{I} + \mathbf{H}_1\Sigma_s\mathbf{H}_1^\dagger)), \quad (21)$$

which is analogous to the rate in (11); I($\mathbf{U}, \mathbf{X}_2; \mathbf{Y}$) and I($\mathbf{V}; \mathbf{Y}|\mathbf{U}, \mathbf{X}_2$) are the same as in (12) and (13) respectively.

Now assume that $\mathbf{C}$ is decoded first at the receiver. Let $R_{dpc,2}$ be the achievable rate for this case. It is proven in [15] that

$$R_{dpc,2} = \sup_{p(x_1,x_2,u,v)} R_{dpc,2s} + R_{dpc,2c} \quad (22)$$

where

$$\begin{aligned} R_{dpc,2c} &= I(\mathbf{V}; \mathbf{Y}) \\ R_{dpc,2s} &= \min(I(\mathbf{U}; \mathbf{Y}_1|\mathbf{X}_2) - I(\mathbf{U}; \mathbf{V}|\mathbf{X}_2), \\ &\quad I(\mathbf{U}, \mathbf{X}_2; \mathbf{Y}|\mathbf{V})) \end{aligned} \quad (23)$$

and the supremum is taken over all joint distributions

$$p(x_1, x_2, u, v) = p(x_2)p(u, v|x_2)p(x_1|u, v)$$

on $\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{U} \times \mathcal{V}$. In this case Gaussian input optimality results in I($\mathbf{V}; \mathbf{Y}$), I($\mathbf{U}; \mathbf{Y}_1|\mathbf{X}_2$) − I($\mathbf{U}; \mathbf{V}|\mathbf{X}_2$) and I($\mathbf{U}, \mathbf{X}_2; \mathbf{Y}|\mathbf{V}$) being the same as in (16), (21), and (17) respectively.

The objective is to choose the decoding order that yields a higher overall rate. We now state the following result.

Proposition 4.2: Let R_{dpc} be the maximum signaling rate for the Gaussian MIMO relay channel employing dirty-paper coding at the transmitter. Then

$$R_{dpc} = \max(R_{dpc,1}, R_{dpc,2}) \geq R_{sup} \quad (24)$$

where R_{sup} is given in Proposition 4.1.

Proof: See [15] for a detailed proof. ∎

C. Example Calculation

We investigate the performance of our precoding methods for a particular channel configuration. Consider a MIMO relay channel where the transmitter has two antennas, while the relay and receiver each have one antenna and

$$\mathbf{H}_1 = [0\ 10], \mathbf{H}_2 = [1\ 0], \mathbf{H}_3 = 1.$$

1) Upper Bound: We substitute these values for the channels into the expressions in Theorem 3.1. The γ_j parameters must be accounted for here since the transmitter has more than one antenna. We can implicitly ignore the γ_j parameters by defining all of the covariance matrices, especially Σ_{11}, to have a trace constraint of one.

Here, we find that the upper bound

$$C_{upper}^G \approx 2.17878 \text{ bits/s/Hz}.$$

2) Lower Bound: We again substitute these values for the channels into the expressions in Theorem 3.2. Again, we can implicitly ignore the γ_j parameters.

We find that the lower bound

$$C_{lower}^G = \log(2) = 1 \text{ bits/s/Hz}.$$

3) Our Achievable Rate: We define

$$\Sigma_s = \begin{bmatrix} a & b \\ b^* & d \end{bmatrix} \text{ and } \Sigma_c = \begin{bmatrix} g & q \\ q^* & r \end{bmatrix}.$$

For superposition coding, if we decode $\mathbf{S}$ first,

$$I(\mathbf{U}; \mathbf{Y}_1|\mathbf{X}_2) = \log\left(1 + \frac{100d}{1+r}\right), \quad (25)$$

$$I(\mathbf{U}, \mathbf{X}_2; \mathbf{Y}) = \log\left(1 + \frac{a + 2Re(x) + k}{1+g}\right), \quad (26)$$

and

$$I(\mathbf{V}; \mathbf{Y}|\mathbf{U}, \mathbf{X}_2) = \log(1 + g). \quad (27)$$

We choose Σ_c, Σ_s, and Σ_{X_2} to maximize $R_{sup,1s}$ and $R_{sup,1c}$ subject to tr(Σ_c + Σ_s) ≤ 1 and tr(Σ_{X_2}) ≤ 1 due to the SNR-normalizing γ_j variables. We find that the optimal values are

$$\Sigma_s = \begin{bmatrix} 0.754 & 0 \\ 0 & 0.184 \end{bmatrix}, \Sigma_c = \begin{bmatrix} 0.0197 & 0.0287 \\ 0.0287 & 0.0419 \end{bmatrix},$$

$$\Sigma_{X_2} = [1], \text{ and } \mathbb{E}(\mathbf{U}\mathbf{X}_2^\dagger) = [\,0.869 \quad 0\,].$$

Thus

$$R_{sup,1} = R_{sup,1s} + R_{sup,1c} \approx 2.17363 \text{ bits/s/Hz}.$$

We can also decode $\mathbf{C}$ first for superposition coding, so I($\mathbf{U}$;$\mathbf{Y}_1$|$\mathbf{X}_2$) is the same as in (25),

$$I(\mathbf{U}, \mathbf{X}_2; \mathbf{Y}|\mathbf{V}) = \log(1 + a + 2Re(x) + k), \quad (28)$$

and

$$I(\mathbf{V}; \mathbf{Y}) = \log\left(1 + \frac{g}{1 + a + 2Re(x) + k}\right). \quad (29)$$

We find that the optimal values are

$$\Sigma_s = \begin{bmatrix} 0.73 & 0 \\ 0 & 0.215 \end{bmatrix}, \Sigma_c = \begin{bmatrix} 0.0033 & 0.0131 \\ 0.0131 & 0.0522 \end{bmatrix},$$

$$\Sigma_{X_2} = [1], \text{ and } \mathbb{E}(\mathbf{U}\mathbf{X}_2^\dagger) = [\,0.854 \quad 0\,].$$

Thus

$$R_{sup,2} = R_{sup,2s} + R_{sup,2c} \approx 2.15087 \text{ bits/s/Hz}.$$

Comparing the achievable rate for both cases, we choose to decode $\mathbf{S}$ first and so

$$R_{sup} \approx 2.17363 \text{ bits/s/Hz}.$$

Thus, we see that by using superposition coding, we have outperformed the lower bound from [13].

For dirty-paper coding, if we decode $\mathbf{S}$ first,

$$I(\mathbf{U}; \mathbf{Y}_1|\mathbf{X}_2) - I(\mathbf{U}; \mathbf{V}|\mathbf{X}_2) = \log(1 + 100d), \quad (30)$$

and I($\mathbf{U}$,$\mathbf{X}_2$;$\mathbf{Y}$) and I($\mathbf{V}$;$\mathbf{Y}$|$\mathbf{U}$,$\mathbf{X}_2$) are the same as in (26) and (27), respectively. We find that the optimal values are

$$\Sigma_s = \begin{bmatrix} 0.609 & 0 \\ 0 & 0.0326 \end{bmatrix}, \Sigma_c = \begin{bmatrix} 0.348 & -0.042 \\ -0.042 & 0.0104 \end{bmatrix},$$

$$\Sigma_{X_2} = [1], \text{ and } \mathbb{E}(\mathbf{U}\mathbf{X}_2^\dagger) = [\,0.78 \quad 0\,].$$

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Thus

$$R_{dpc,1} = R_{dpc,1s} + R_{dpc,1c} \approx 2.17563 \text{ bits/s/Hz}.$$

We can also decode $\mathbf{C}$ first for dirty-paper coding. Therefore $I(\mathbf{U};\mathbf{Y}_1|\mathbf{X}_2) - I(\mathbf{U};\mathbf{V}|\mathbf{X}_2)$, $I(\mathbf{U},\mathbf{X}_2;\mathbf{Y}|\mathbf{V})$ and $I(\mathbf{V};\mathbf{Y})$ are the same as in (30), (28) and (29) respectively. We find that the optimal choices are the same as for the case where $\mathbf{S}$ is decoded first for dirty-paper coding and thus

$$R_{dpc,2} = R_{dpc,2s} + R_{dpc,2c} = 2.17563 \text{ bits/s/Hz}.$$

Therefore, we can decode either $\mathbf{C}$ or $\mathbf{S}$ first and so

$$R_{dpc} \approx 2.17563 \text{ bits/s/Hz}.$$

By using dirty-paper coding, we have also outperformed the lower bound from [13].

V. Numerical Results

We employ a simple example to demonstrate how precoding at the transmitter outperforms the bounds in [13].

We choose $\mathbf{H}_2 = [1\ 0]$ and $\mathbf{H}_3 = 1$. We also choose $\mathbf{H}_1 = [x\ y]$, where $x, y \in \mathbb{R}$, and constrain $\|\mathbf{H}_1\| = 10$. By considering $\mathbf{H}_1$ and $\mathbf{H}_2$ as two-dimensional vectors, we can define an "angle" $\Theta(\mathbf{H}_1, \mathbf{H}_2)$ between them. We vary $\Theta(\mathbf{H}_1, \mathbf{H}_2)$ over the range $[0, \pi]$, where $\Theta(\mathbf{H}_1, \mathbf{H}_2)$ is expressed in radians. As $\Theta(\mathbf{H}_1, \mathbf{H}_2) \rightarrow \pi/2$, the gain between the second transmit antenna and the relay's antenna, or y, increases. Note that the norm constraint on $\mathbf{H}_1$ causes the gain between the first transmit antenna and the relay's antenna, or x, to decrease.

We observed that the lower bound from [13] is 1 bits/s/Hz for all values of $\Theta(\mathbf{H}_1, \mathbf{H}_2)$; this results from our fixing $\mathbf{H}_2$ at [1 0]. Thus we did not plot the lower bound in Fig. 3 to better illustrate the comparison between our precoding approaches and the upper bound from [13].

Fig. 3 shows that the upper bound decreases as $\Theta(\mathbf{H}_1, \mathbf{H}_2) \rightarrow \pi/2$ radians. Note that as $\Theta(\mathbf{H}_1, \mathbf{H}_2) \rightarrow \pi/2$, the transmitter would want to put more power on its second transmit antenna to exploit the rate benefits on the transmitter-to-relay link. This strategy, though, results in a loss of rate on the direct link since $\mathbf{H}_2$ is fixed at [1 0]. This leads to a monotonic decrease in the upper bound as $\Theta(\mathbf{H}_1, \mathbf{H}_2) \rightarrow \pi/2$.

We see that the achievable rates via superposition coding and dirty-paper coding always outperform the lower bound of 1 bits/s/Hz. Also, we see that the achievable rate from dirty-paper coding is never less than the achievable rate from superposition coding. In addition, note that our precoding methods yield rates that are close to capacity; the upper bound from [13] and the achievable rate from superposition coding differ by at most 0.01 bits/s/Hz in Fig. 3.

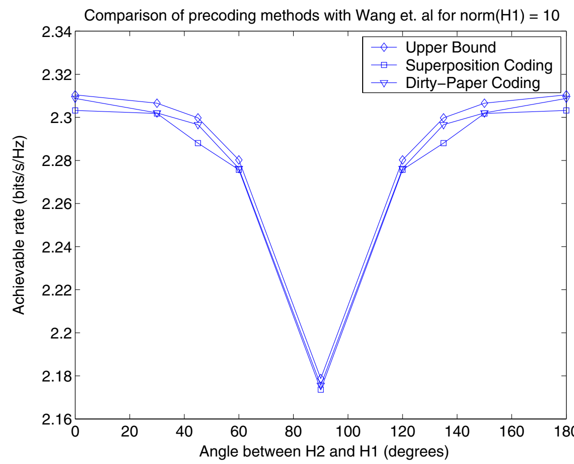

Fig. 3. Comparison of achievable rate via superposition coding and dirty-paper coding with upper bound from [13].

VI. Conclusion

We derived new lower capacity bounds for a Gaussian MIMO relay channel by employing precoding techniques such as superposition coding and dirty-paper coding at the transmitter. Our proposed bounds improve upon the lower bounds that were introduced in [13]. In particular, our results show the benefits of employing the relay's assistance via precoding at the transmitter, especially when the transmitter-to-relay channel is strong relative to the transmitter-to-receiver and/or relay-to-receiver channels. Our results suggest that transmit precoding should be an integral part of communication over MIMO relay channels.

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