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Experimental Analysis and Control of a Chaotic Pendubot

Abstract

Applying attractor reconstruction techniques and other chaotic measurements, it is shown that the long-term dynamics of a vertical, underactuated, two-degrees-of-freedom robot called Pendubot may exhibit complex dynamics including chaotic behavior. These techniques use only the measurement of some available variable of the system, and the resulting reconstruction allows us to identify unstable periodic orbits embedded in the chaotic attractor. In this paper, we also propose a parameter-perturbation-like control algorithm to stabilize the behavior of the Pendubot to force its dynamics to be periodic. We control this device using only the measurement of one of its angular position coordinates and consider that the system may be seen as five-dimensional (a non-autonomous, four-dimensional system), taking the amplitude of a sinusoidal external torque as the perturbation parameter. We change this parameter to stabilize one of the equilibrium points in the so-called Lorenz map. The main advantage of the method proposed here is that it can be implemented directly from time series data, irrespective of the overall dimension of the phase space. Also, reconstructions of the attractor based on the measurements are shown, as well as some experimental results of the controlled system.

KEY WORDS—Pendubot, chaos control, underactuated robot, delay coordinates

1. Introduction

Chaotic dynamics have been widely studied in several disciplines during the last decades. A chaotic signal is generated

by a deterministic dynamical system, but because of its sensitivity to initial conditions, it is long-term unpredictable. Some methods have been developed to analyze and calculate some important parameters of a chaotic system; Lyapunov exponents and some other geometrical characteristics (Parker and Chua 1989) are of great use in analyzing this complex behavior. Some of these techniques need a mathematical model of the system. In practice, it is frequent that a model is not available, and the only information is given by some measurements obtained directly from the system from which the fundamental dynamical parameters must be calculated. However, this is not a trivial problem.

In this sense, some techniques have been proposed to reconstruct an attractor from a time series obtained from the measurement of a system variable. One of these techniques that is often used, giving interesting and useful results, is the so-called delayed coordinates method. The application of this technique to analyze the complex behavior of a real system, an underactuated mechanical manipulator, is the first of two objectives of this paper.

Chaotic oscillations are a particular kind of irregular and unpredictable behavior commonly considered as undesirable; this is the reason for trying to eliminate them from the overall behavior of a system. Stabilizing a periodic motion in a chaotic system does not require a great amount of energy for the control action since the attractor contains an infinite number of unstable periodic orbits embedded in it. There exist some applications in which the main goal is to transform an irregular motion into a regular one, for instance, in power electronics devices (Baillieul, Brockett, and Washburn 1980). Chaos suppression has been achieved in several experimental preparations for controlling some medical pathologies such as cardiac arrhythmia (Garfinkel et al. 1992) or in regulating

voltage peaks in the brain in epileptic crisis (Gluckman et al. 1996).

For mechanical systems, irregular oscillations are particularly undesirable because of the possibility of mechanical damage. Another reason to stabilize periodic orbits is that, in some robotics applications, it is required to follow a desired periodic trajectory, for example in walking robots (Canudas de Wit, Roussel, and Goswami 1997).

Underactuated manipulators arise in a number of important applications such as free-flying space robots, hyper-redundant manipulators, snake-like robots, and manipulators with structural flexibility, among others. Previous work on modeling and control of such manipulators can be found in Chirikjian and Burdick (1991), Jain and Rodríguez (1991), Rodríguez, Kreutz-Delgado, and Jain (1991). An important goal in manipulator control is to perform tasks involving the exact tracking of some desired trajectory. The exact tracking depends on the nature of the designed control algorithm after a proper analysis of the particular device dynamics. The absence of an actuator transforms the robot into an underactuated device that may lead to a malfunction of the system, to an erroneous tracking of the desired trajectory and in some cases to instability. Moreover, underactuated robots may exhibit rich dynamical behaviors including chaos (Alvarez-Gallegos, Alvarez, and González-Hernández 1997; González-Hernández, Alvarez-Gallegos, and Alvarez 2001).

Earlier works on stabilization of periodic orbits can be found in Chen and Dong (1998), Ott, Grebogi, and Yorke (1990a), and Hunt (1992); however, most of them were designed for systems described by three differential equations yielding two-dimensional Poincaré or Lorenz maps (Lorenz 1963; Peitgen, Jürgens, and Saupe 1992). For higher-dimensional systems there are some approaches (Ott, Grebogi and Yorke 1990b; Ding et al. 1996), but a general framework has not been established yet; moreover, these methods are only capable of imposing a particular dynamics for the stabilization. The method developed in this paper deals with both problems: control of higher-dimensional systems, and imposing a desired dynamics for the stabilization.

In order to stabilize periodic orbits in a chaotic attractor, it is necessary to approximate the dynamics of the system in a lower-dimensional space where a periodic orbit may be seen as a fixed point; Ott, Grebogi, and Yorke (1990a) suggest the use of a Poincaré map for finding this fixed point. In this paper, we report on the use of Lorenz maps, instead of Poincaré maps, for the local identification of the dynamics around the periodic unstable orbits of the attractor. It has been proposed earlier (Alvarez, Alvarez-Gallegos, and González-Hernández 1999) that the use of this type of map allows better identification schemes and avoids ill-conditioned identification problems.

The second part of this paper shows the application of the proposed parameter perturbation method to control the complex dynamics of the Pendubot. Given a time series obtained from some measured variable of the Pendubot, which displays

an irregular behavior, the objective is to force its dynamics to a periodic motion.

The main contributions in this paper are the analysis of the dynamics of an underactuated robot and the proposal of a new method to stabilize periodic orbits embedded in the chaotic attractor of this device. The analysis and the control action were implemented on the real system.

The characterization of the dynamical behavior of the system is accomplished without an explicit dynamical system model, using only a time series generated from the measurements of one of the variables of the system. First, the largest Lyapunov exponent of the time series is computed, and then the fractal dimension and the frequency spectrum for determining the chaotic nature of the measured signal are obtained.

The analysis is accomplished by using an embedded coordinates method, again using only a time series. We use the average mutual information (AMI; Abarbanel et al. 1993) in order to choose the delay time factor for the coordinates and the percentage of false nearest neighbors (PFNN) (Abarbanel and Kennel 1993) to find the embedding dimension of the attractor to be reconstructed.

Finally, in this paper we propose a framework for designing flexible control laws for the stabilization of periodic orbits embedded in higher-dimension chaotic attractors from system measurements. In this way, flexibility means the capability to impose arbitrary dynamics with the proposed control law. This control law is applied directly to a real system.

The paper is organized as follows. In Section 2 we give some useful concepts on dynamical systems and some tools to analyze chaotic behavior from time series. In Section 3 we analyze the dynamics of the Pendubot. Section 4 deals with the control strategy. In Section 5 experimental results are shown and, finally, Section 6 contains some concluding remarks.

2. Analysis of Chaotic Systems

There are no analytical solutions for equations describing chaotic phenomena; even an approximate solution is not easy to find. Some analysis techniques for this type of system involve perturbation methods (Naifeh and Balachandran 1995) for setting approximate solutions. Once we have these approximate solutions it is possible to compute some other descriptors of their behavior. We call chaotic descriptors to those quantities used to measure the “degree of chaotic behavior” of the system. Among others, Lyapunov exponents, Lorenz maps, and local dimension of the attractor are some descriptors that give information about the dynamical behavior of the system.

Let us consider a system described by

$$\dot{x} = f(t, x, \mu) \quad (1)$$

where $x \in \mathbb{R}^n$ is the state, $f : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^q \rightarrow \mathbb{R}^n$ is a C^r ($r \geq 1$) vector field, and μ denotes the system parameters.

The solution of eq. (1) is some vector function $x = x(t)$ that describes the trajectories in the state space constructed with its coordinates. Depending on the parameter values, some systems may display different steady states, ranging from equilibrium points to chaotic attractors.

DEFINITION 1. (Chaotic Attractor) (Wiggins 1990). Consider a C^r ($r \geq 1$) autonomous vector field on \mathfrak{N}^n , which defines a system like that given by eq. (1). Denote the flow generated by eq. (1) as $\phi(t, x)$, and assume that $\Lambda \subset \mathfrak{N}^n$ is a compact set, invariant under $\phi(t, x)$. Then Λ is said to be chaotic if it has the following properties.

1. Sensitive dependence on initial conditions. There exists $\epsilon > 0$ such that, for any $x \in \Lambda$ and any neighborhood U of x , there exists $y \in U$ and some $t > 0$ such that $|\phi(t, x) - \phi(t, y)| > \epsilon$.
2. Topological transitivity. For any two open sets $U, V \subset \Lambda$, there exists some $t \in \mathfrak{R}$ such that $\phi(t, U) \cap V \neq \emptyset$.

Systems that show this behavior are called chaotic. They are not easy to analyze due to the absence of tools allowing a good understanding of the phenomena. In recent years, many techniques have been developed for the analysis of the dynamics of this type of system. Some of these techniques are given below, but before we give some definitions and a useful theorem.

DEFINITION 2. (Correlation Dimension). Let A be a bounded subset of \mathfrak{N}^n and $C(r)$ a function proportional to the probability that two arbitrary points on the orbit in state space are separated no more than r (for constructing $C(r)$; see, for example, Parker and Chua 1989). Then, the correlation dimension is defined by (Grassberger and Procaccia 1983):

$$D_c(A) = \lim_{r \rightarrow 0} \frac{\log C(r)}{\log(r)}. \tag{2}$$

Typically, this quantity is not an integer number for a chaotic attractor A . When this situation occurs it is said that A is a fractal set.

THEOREM 1. (Whitney’s Theorem) (Guillemin and Pollack 1974). Let A be a compact C^r ($r > 2$) manifold with dimension $D_c(A)$ as defined in Definition (2). Then there is a C^r embedding of A in $\mathfrak{N}^{\text{ceil}(2D_c(A)+1)}$.

Paraphrasing, such $D_c(A)$ -dimensional manifolds may be diffeomorphically mapped to the Euclidian space $\mathfrak{N}^{\text{ceil}(2D_c(A)+1)}$. Hence, the Euclidian space $\mathfrak{N}^{\text{ceil}(2D_c(A)+1)}$ is large enough to contain a diffeomorphic copy of these $D_c(A)$ -dimensional manifolds. Actually, this is a maximum bound; particularly it is possible to embed a chaotic attractor in a smaller dimension.

DEFINITION 3. (Embedding Dimension). The dimension d_E , called the embedding dimension, is the dimension for which

the attractor is fully unfolded, i.e., the dimension in which two points far away from each other in the original space are not projected near each other in the observation space. Generically, $d_E \leq \text{ceil}(2D_c(A) + 1)$.

It is not always possible to find mathematical models like eq. (1) to describe the behavior of a chaotic system, but it is usually possible to measure at least one of the variables involved in its evolution. Some methods for analyzing the chaotic phenomena use time series obtained from these measurements. Based on the embedding theorem, these methods reconstruct the attractor, and calculate some important system parameters (Eckmann and Ruelle 1985; Fraser and Swinney 1986; Kennel, Brown, and Abarbanel 1992; Abarbanel and Kennel 1993; Abarbanel et al. 1993). By applying this theorem, it is possible to reconstruct the attractor if the embedding dimension is previously determined. Therefore, a first problem is to find this dimension from a time series. A method to solve this problem, called delayed coordinates, is discussed in what follows.

1.1. Attractor Reconstruction

Let us consider a system modeled by eq. (1). We assume that a time series obtained by sampling some signal s is available from the system. This signal may depend indirectly on the state x , via a map g , therefore $s = h(g(x))$ with $h : \mathfrak{N} \rightarrow \mathfrak{R}$ a function that relates the state with the variable which is being measured. If g is a diffeomorphism, then $\xi = g(x)$ can be used as a system state. Since h is a scalar function, then s measures only a projection of x onto a one-dimensional space. In order for s to be useful for the reconstruction of a topological equivalent attractor, the map $h \circ g$ should be a submersion from \mathfrak{N}^n to \mathfrak{R} , which means that the state x should be observable through the measurement s .

Let us now consider a time series $s(t)$ from a system variable obtained at regular time intervals of length τ_s from an initial time t_0 to a final time $t_f = t_0 + N\tau_s$; let us denote this time series by $s(k) \stackrel{\text{def}}{=} s(t_0 + k\tau_s)$, $k = 0, 1, \dots, N$. Theorem 1 tells us that if s can be given by a submersion $h \circ g$, then the geometric structure of the system dynamics can be reconstructed using scalar measurements $h(g(x(k)))$ in a state space built by vectors of the form:

$$y = [h \circ g^0(x), h \circ g^1(x), \dots, h \circ g^{d-1}(x)] \tag{3}$$

where $g^j = g^{j-1} \circ g$, $g^0 = id$. This reconstruction can be implemented with arbitrary smooth functions h and g . A typical selection for g is the time-delay operator, $g^i(x(k)) \stackrel{\text{def}}{=} x(k - iT)$. Since the time series is obtained at regular time intervals, then $T_i = iT$, $T \in \mathbb{Z}^+$ and $h \circ g^i(x(k)) = h(x(k - iT)) = s(k - iT)$, then

$$y(k) = [s(k), s(k - T), \dots, s(k - (d - 1)T)] \tag{4}$$

where T is the time delay and d is the dimension of the reconstruction. If the system has an attractor with dimension $d_E(A)$, then d is finite and by Theorem 1, is not greater than $ceil(2D_c(A) + 1)$. A successful reconstruction depends on an appropriate selection of parameters T and $d = d_E$. In the following sections some tools for computing these parameters are described.

1.2. Average Mutual Information

Before formally describing the idea of mutual information, we have to consider some restrictions. Theoretically, provided exact and noise free information and a large enough value of d , attractor reconstruction can be implemented with an arbitrary value of the time delay T . However, this is not a realistic situation and we need an appropriate tool in order to compute the value of the time delay T . First, if the T period is too short, both signals (the original time series and its delayed version) would not be independent enough to be considered as coordinates and, secondly, if T is too large, every connection between these coordinates would be numerically subject to be random like one with respect to the other. A useful measure for determining this parameter is the AMI.

The mutual information (MI) between a measurement a_i , taken from a measurements set $A = \{a_i\}_{i=0}^N$ and a measurement b_j , taken from another set $B = \{b_j\}_{j=0}^N$ is defined as (Abarbanel et al. 1993):

$$IM(a_i, b_j) = \log_2 \left[\frac{P_{AB}(a_i, b_j)}{P_A(a_i) P_B(b_j)} \right] \quad (5)$$

where P_{AB} is a joint probabilistic density, and P_A and P_B are individual probability densities. Therefore, the average mutual information is defined as

$$PIM_{AB} = \sum_{i,j} P_{AB}(a_i, b_j) IM(a_i, b_j). \quad (6)$$

Applying this definition over the time series $s(k)$ and its delay $s(k - T)$,

$$PIM(T) = \sum_{k=0}^N P(s(k), s(k - T)) P_m(k, T) \quad (7)$$

where

$$P_m(k, T) = \log_2 \left[\frac{P(s(k), s(k - T))}{P(s(k)) P(s(k - T))} \right]. \quad (8)$$

We propose to use a value of T which is near to the minimum of the AMI (Abarbanel et al. 1993). Due to the multiple minima of this function we suggest taking the first one.

1.3. Global False Nearest Neighbors

Attractor reconstruction is based upon the idea that the measured variable is a projection of an unknown system. If this is

a continuous and finite-dimensional dynamical system, then any observed intersection of the trajectory in the projection can be removed by increasing the dimension of the space in which the signal is represented. This can be done by choosing an appropriate value for the dimension d , i.e., the embedding dimension d_E . A proposed method to compute this dimension is the technique called false nearest neighbors (FNN).

The nearest points to the reconstruction vector (4) in a d -dimensional space is the nearest vector to $y(k)$ and it will be denoted as

$$y_{NN}(k) = [s_{NN}(k), s_{NN}(k - T), \dots, s_{NN}(k - (d - 1)T)]. \quad (9)$$

If $y_{NN}(k)$ is a true neighbor of $y(k)$, then it will be in the neighborhood of this point due to the dynamics of the system. On the other hand, if it is a false neighbor, it appears in the neighborhood due to a projection onto the d -dimensional space from a higher-dimensional space. In order to determine if $y_{NN}(k)$ is a false neighbor of $y(k)$, the reconstruction dimension is increased to $d + 1$, and the normalized increment in the Euclidian distance between eq. (4) and eq. (9) in the new space, with respect to the distance in the d -dimensional space is given by

$$\delta(k, d) = \frac{|s(k - dT) - s_{NN}(k - dT)|}{R_d(T)} \quad (10)$$

where

$$R_d^2(k) = \sum_{j=0}^{d-1} [s(k - jT) - s_{NN}(k - jT)]^2. \quad (11)$$

If $y_{NN}(k)$ is a true neighbor then this increment must be small; if it is not, then it is a false neighbor and d must be increased. Another criterion, which has to be under consideration is the relative increment of the Euclidian distance with respect to the nominal size of the attractor R_A , in the case when this size is known approximately,

$$\delta_A(k, d) = \frac{|s(k - dT) - s_{NN}(k - dT)|}{R_A}. \quad (12)$$

If the combination of this criteria is not satisfied, then the real value for d must be increased (Abarbanel et al. 1993).

The PFNN in the d -dimensional space is given by

$$PFNN(d) = \frac{\text{Number of false neighbors}}{\text{Total number of reconstructed vectors}} \times 100\%. \quad (13)$$

When d is large enough to unfold the trajectory, $PFNN(d)$ is very small. Therefore, the embedding dimension d_E is the minimum value of d such that $PFNN(d) \approx 0$.

Although other criteria have been proposed for the calculation of these couple of important parameters (Abarbanel

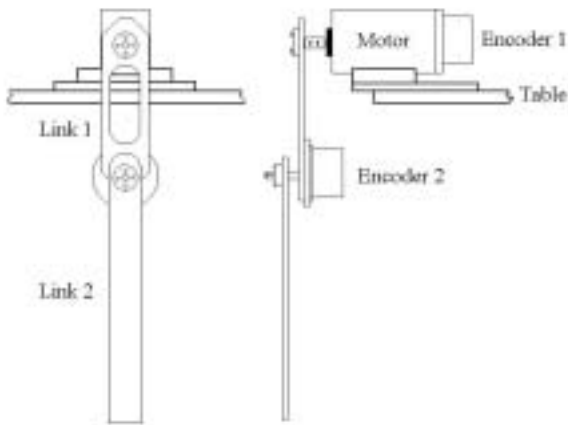


Fig. 1. The Pendubot.

et al. 1993), those described here, in our opinion, offer the best compromise between easy computation and accurate results. In later sections the applications of these methods to the system under study are illustrated. The computation of these parameters was performed using Matlab® and applied directly to measurements of the system.

2. The Pendubot

In this section we show the application of the method mentioned above to an electromechanical underactuated system. We have chosen an underactuated mechatronic system that presents a wide variety of behaviors. The system, called Pendubot (Spong and Block 1995), consists of two rigid links. Link 1 is directly coupled to the shaft of a 90 V permanent magnet DC motor mounted to the end of a table; this motor is the only actuator of the system. Link 2 is coupled to link 1 and is moved only by the motion of link 1. The angular position of both links is monitored to a computer via optical encoders, as shown in Figure 1.

All of our computations were performed on a personal computer with a D/A card and an encoder interface card. The control algorithms were programmed directly in C. The voltage to the DC motor is supplied via a servo amplifier. There is a relationship between the supplied voltage and the applied torque to the DC motor; this relationship was experimentally found

$$\tau = (l_1 w_2 + l_{c1} w_1) \cos \left[\frac{\pi}{2} - (0.2763V + 0.0335) \right], \quad (14)$$

where τ is the applied torque and V is the voltage supplied to the power amplifier. The parameters used here were measured

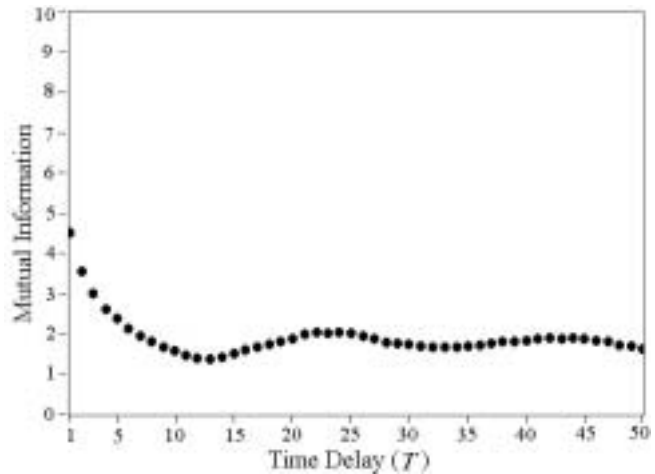


Fig. 2. AMI for an amplitude $p = 1.91$.

directly from the device: $l_1 = 0.26987$ m, $w_1 = 5.1885$ N, $l_{c1} = 0.13494$ m and $w_2 = 3.2824$ N.

We measured the angular position of the second link (the one without an actuator) while a sinusoidal voltage input of the form

$$V = p \sin(\omega t) \quad (15)$$

was applied, where p is an available parameter, which changes the dynamics of the overall system, and V is given in volts. The frequency used was $\omega = 9$ rad s⁻¹ and we have varied the voltage amplitude from 0.1 to 2.9 V. In the following, we show an example of attractor reconstruction applying the average mutual information criterion for finding a suitable T , and the false neighbors idea to find the embedding dimension d_E . The angular position of the second link of the Pendubot has been sampled every 16 ms, from where we have taken the transients off.

2.1. Example: Chaotic Attractor

We have analyzed the Pendubot dynamics for an input signal with amplitude $p = 1.91$. Figure 2 shows the AMI as a function of the time delay T . By applying the criterion previously described, we found $T = 13$. Using the FNN technique, the embedding dimension turns out to be $d_E = 4$; this is shown in Figure 3. Figure 4 shows the reconstructed attractor.

In order to show that this behavior is chaotic, we have obtained the correlation dimension (Grassberger and Procaccia 1983; Grassberger 1988) of the attractor, obtaining $D_c(A) = 3.4713$ and the largest Lyapunov exponent (LE; Wolf et al. 1985) as $\lambda_1 = 1.41$. These fractal dimensions and positive exponent λ_1 indicate that the time series is chaotic.

Although positive LEs also describe noisy signals, we can say that the behavior of the system for the parameter value

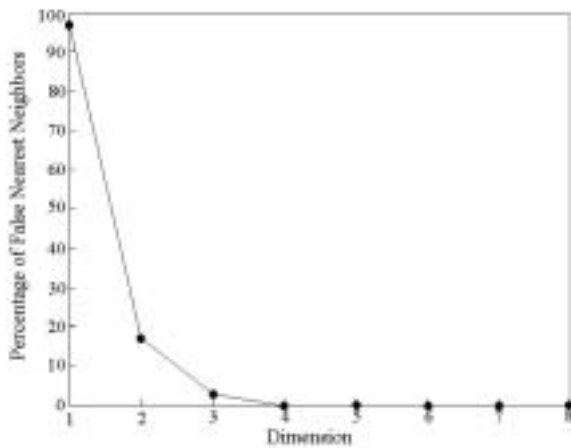


Fig. 3. PFNN for an amplitude $p = 1.91$ and a time delay $T = 13$.

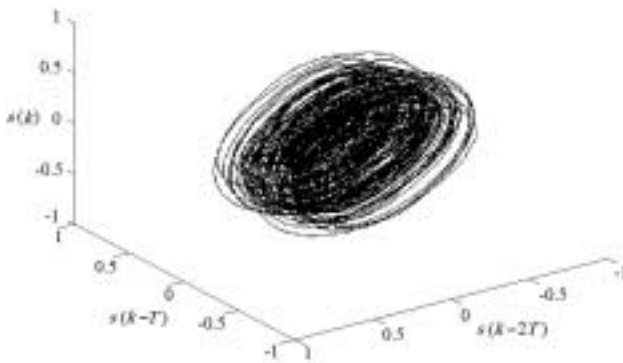


Fig. 4. Projection of the reconstructed attractor for $p = 1.91$.

$p = 1.91$ is chaotic because: (a) the AMI of the signal has a strict local minimum; (b) the PFNN falls to zero in some finite dimension (for noise signals, this percentage never falls to zero); (c) the geometry of the reconstructed attractor (Figure 4); (d) the signal has a broad frequency spectrum (Figure 5).

3. Chaos Control Strategy

Recently, control and anticontrol of chaotic systems have been taken into account in a wide variety of problems arising wherever there are complex behaviors appearing in some physical or biological system. The different approaches can be grouped mainly into four categories (Chen and Dong 1998): chaos control via external action, control engineering approaches, in-

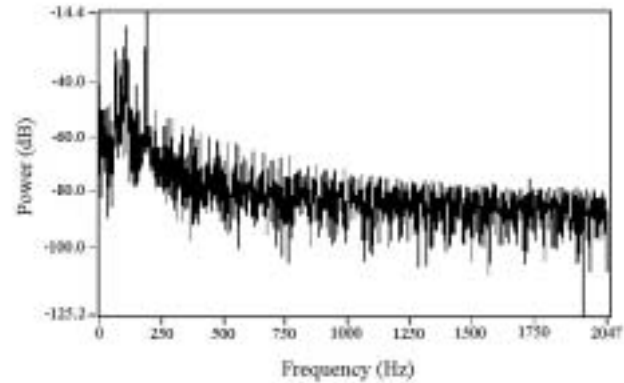


Fig. 5. Frequency spectrum for $p = 1.91$.

telligent computation approaches, and parameter perturbation methods.

Our proposal can be considered as a parameter perturbation method. Most of the existing methods were designed to be implemented in systems described by three differential equations (Chen and Dong 1998), yielding two-dimensional Poincaré or Lorenz maps. Ott, Grebogi, and Yorke (1990a) recommended the use of the Poincaré map instead of the Lorenz map (Lorenz 1963), but in some cases the use of these maps leads to an ill-conditioned identification procedure of the local dynamics (Alvarez, Alvarez-Gallegos, and González-Hernández 1999). Although there are few approximations for higher-dimensional systems (Ott, Grebogi, and Yorke 1990b; Ding et al. 1996), there is no general framework; moreover, these methods are only capable of imposing certain dynamics for the stabilization. The method developed here deals with both problems: higher-dimensional systems and imposing desired dynamics for the stabilization.

3.1. Method

The main advantage of parameter perturbation methods is that it is possible to implement them without any prior knowledge about the system equations. This feature makes them very popular for the experimental control of chaotic systems. Since a chaotic attractor can be considered as the closure of an infinite number of unstable periodic orbits (UPOs), the idea behind these methods is to stabilize one of these UPOs. For the implementation of the method proposed here it is necessary to identify an unstable periodic orbit of the attractor to locally characterize its dynamics, to determine the response of the system, and to calculate the change of the attractor to external stimulus.

The process of identifying a UPO is done using a map constructed with delayed versions of a time series obtained by sampling the consecutive local maxima (or minima) of the system's output signal. This map is called the Lorenz map

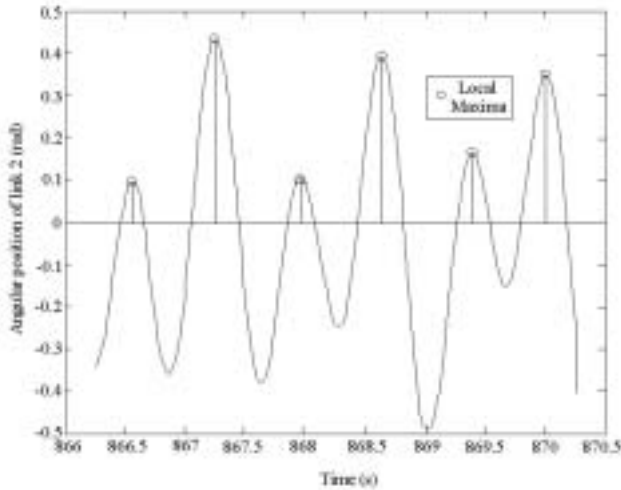


Fig. 6. Local maxima used to construct the Lorenz map.

(Lorenz 1963). It provides information about the dynamics of the original system in a discrete space. Figure 6 shows the process of sampling local maxima. Some authors (Peitgen, Jürgens, and Saupe 1992) call this map the “one-dimensional return map”, because it is reconstructed from only one dimension (a measured local maxima); since this map may be m -dimensional, we prefer to call it the Lorenz map in order to avoid possible confusion.

It can be seen from Figure 6 that, if the measured signal has a limit cycle behavior, consecutive local maxima will have constant values for a stable periodic orbit. In the Lorenz map, this behavior will be seen as a stable m -dimensional fixed point. Conversely, due to the chaotic nature of the measured variable, we expect the appearance of unstable fixed points in the Lorenz map, corresponding to unstable periodic orbits in the reconstructed space.

Once we have determined about which unstable fixed point in the Lorenz map (corresponding to an UPO) we wish to regulate the behavior, the motion of consecutive points is observed. These points occasionally approach the neighborhood of the chosen unstable fixed point. Moreover, the unstable fixed point also moves according to the fixed set of parameters used. If some parameter in this set is changed, then the location of the unstable fixed point changes as well. Then, in order to determine the attractor response to external stimulus it is necessary to introduce a perturbation in the system by varying one of these parameters. We may distinguish three stages for the method:

1. perturbation of a fixed point;
2. linearization about the fixed point;
3. determination of the value of the control parameter.

3.2. Fixed Point Perturbation

The Lorenz map is obtained from sampling consecutive local maxima of the measured variable, as shown in Figure 6. Let us denote this sampled signal as $\{\zeta\}_{n=1}^N$. The explicit nature of the dynamics of the Lorenz map is unknown; therefore, a reconstruction of this map is required in order to characterize the local dynamics around the selected fixed point. Likewise for attractor reconstruction (4), we apply the delay operator to this sampled signal to form a vector of $m = d_E - 1$ delayed coordinates. Now let us define a new vector $z \in \mathfrak{R}^m$ for denoting the discrete time evolution of consecutive local maxima. This vector is related to ζ as

$$z = \begin{bmatrix} z_n^{(1)} \\ z_n^{(2)} \\ \vdots \\ z_n^{(m)} \end{bmatrix} = \begin{bmatrix} \zeta(n) \\ \zeta(n-1) \\ \vdots \\ \zeta(n-(m-1)) \end{bmatrix}. \quad (16)$$

Then, although explicitly unknown, the Lorenz map associated with a specific value of a parameter p can be expressed by

$$z_{n+1} = G(z_n, p). \quad (17)$$

This map describes the dynamics of the (maximal) sampled values of the original measured signal and it has an m -dimensional fixed point (related to a periodic orbit in the original space). The location of this point depends also on the parameter p :

$$\bar{z} = G(\bar{z}, p). \quad (18)$$

If the parameter changes with every iteration, the fixed point location also changes:

$$\bar{z}(p_n) = G(\bar{z}(p_n), p_n). \quad (19)$$

The change in the location of the fixed point due to a parameter perturbation can be estimated with a shift vector b :

$$b \equiv \frac{d}{dp} \bar{z}(p^*) \approx \frac{\bar{z}(p_{n+1}) - \bar{z}(p_n)}{p_{n+1} - p_n}. \quad (20)$$

Later in this paper, we show the experimental process for finding the fixed point location and its change.

3.3. Linearization about a Fixed Point

We assume that the behavior of the Lorenz map in a neighborhood of the fixed point is linear. Thus, it is possible to find an m -dimensional Jacobian matrix A of the form:

$$A = D_{\bar{z}}G(z_n, p_n) = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ a^{(m)} & a^{(m-1)} & a^{(m-2)} & \dots & a^{(1)} \end{bmatrix}. \quad (21)$$

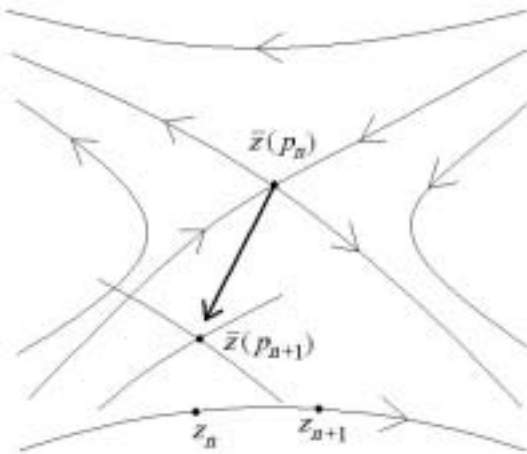


Fig. 7. Displacement of the unstable fixed point due to a change in a parameter.

Notice that the canonical form of matrix A is due to the nature of the reconstructed vector (16). On the other hand, a vector $B = D_{p^*}G(z_n, p_n) = [0 \ 0 \ \dots \ b]^T$ also can be obtained, yielding an equation representing the dynamics about \bar{z} , which can be represented as

$$(z_{n+1} - \bar{z}) = A(z_n - \bar{z}) + B(p_n - p^*), \quad (22)$$

where p^* is the nominal value for parameter p at the chaotic regime. Because of the chaotic nature of the system, this linearized version is expected to be unstable, so that matrix A will have at least one unstable eigenvalue. Parameters $a^{(1)}, a^{(2)}, \dots, a^{(m)}$ and b should be estimated from experimental data.

3.4. Determination of the Value of the Control Parameter

Once the required parameters are determined, an approximation of the local dynamics around the unstable periodic orbit in the Lorenz map is available. Equation (22) describes a non-autonomous discrete time linear system in which the value of parameter p in the n th iteration is the actuating external action.

Now we impose the desired behavior via a state feedback control law in order to assign new poles to the linearized controlled discrete system.

$$p_n = p^* - K^T(z_n - \bar{z}) \quad (23)$$

with $K \in \mathbb{R}^m$ being a constant vector chosen appropriate such that $\det(\lambda I - A + BK)$ be strictly Hurwitz. Stabilizing the fixed point in this map corresponds to stabilizing an unstable periodic orbit in the reconstructed space. Other approaches,

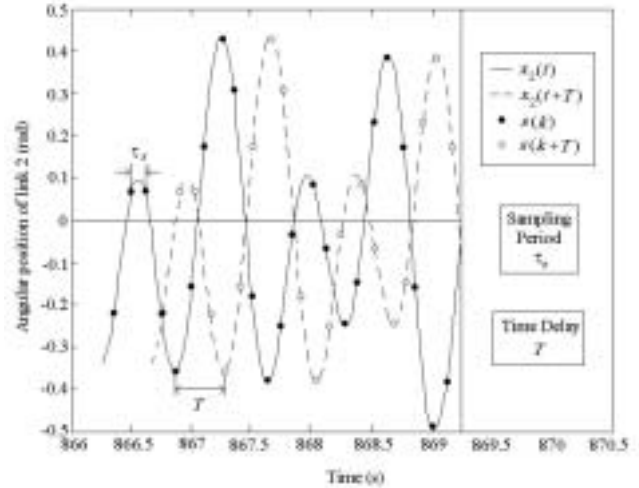


Fig. 8. Construction of the Lorenz map for the Pendubot.

such as those reported in Ott, Grebogi, and Yorke (1990b) and Ding et al. (1996), deal with this type of problem, but they do not provide flexibility for choosing a desired behavior. This new idea allows us to implement more robust algorithms for the stabilization of periodic orbits.

4. Experimental Results

As mentioned earlier, the Pendubot is an underactuated, two-degrees-of-freedom (2-DOF) robotic system whose unique available actuator is located in the first link. In this section we show the application of the proposed method to this robotic system. The input signal used was a sinusoidal torque via a voltage applied to the servoamplifier (see eqs. (14) and (15)), with a frequency $\omega = 9 \text{ rad s}^{-1}$ where the amplitude plays the role of control parameter p . The available variable is the angular position of the second link and it is denoted by $x_2(t)$. All the experiments were carried out using a PC 486DX2/50 with a Keithley Metrabyte DAC-02 output data card and a TECH 80-5312B system for input data from the optical encoders.

We have found a chaotic regime by selecting $p = p^* = 0.2589$; this value was taken as the nominal parameter, in such a way that the control objective will be to find a suitable control action p_n to stabilize an unstable periodic orbit of the attractor.

Following the procedure described before, the first step is to find the embedding dimension of the reconstructed attractor. This embedding dimension was calculated previously in Section 3 as $d_E = 4$; therefore, the dimension of the Lorenz map turns out to be $m = 3$. The signals used for the Lorenz map construction are shown in detail in Figure 8.

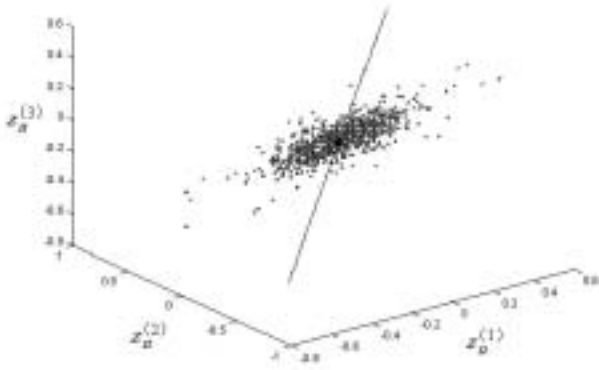


Fig. 9. Lorenz map for the Pendubot from experimental data.

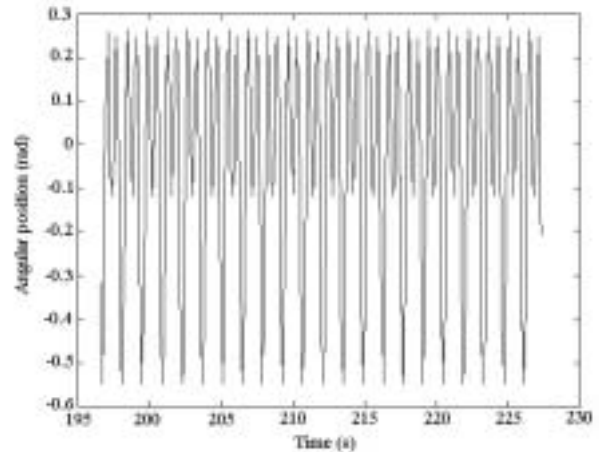


Fig. 10. Controlled variable $x_2(t)$ in steady state.

4.1. Fixed Points of the Lorenz Map

A fixed point in the Lorenz map should satisfy

$$\bar{z} = G(\bar{z}, p^*). \tag{24}$$

Because of the use of delayed coordinates in the Lorenz map, the fixed point has the form $\bar{z} = [\bar{z}_{eq}, \bar{z}_{eq}, \dots, \bar{z}_{eq}]^T$, i.e., it belongs to the line $z_1 = z_2 = z_3$ in the three-dimensional Lorenz map shown in Figure 9. Experimentally, we have found $\bar{z}_{eq} = 0.2589$.

4.2. Local Dynamics

In order to obtain the local stability of the periodic orbit, the local dynamics around the associated fixed point is estimated. This procedure consists of finding points close to the fixed point in the Lorenz map and using a standard mean squared algorithm in order to identify the local dynamics (22). Results for the Pendubot using experimental data, considering $m = 3$, are given in what follows. Matrix A was estimated to be

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2.3901 & -1.4653 & 0.9799 \end{bmatrix} \tag{25}$$

and $B = [0 \ 0 \ 0.2279]^T$. The resulting eigenvalues for the linearization around \bar{z} were $\lambda(A) = \{-0.8157, 0.8978 \pm j1.4574\}$; this denotes instability of the fixed point.

4.3. Control Law

The control law (23) for the parameter p is directly applied in eq. (15). In order to show the flexibility of the proposed method, we have arbitrarily chosen a local dynamics such that $\lambda(A - BK) = \{-0.1, -0.2, -0.3\}$; then, the calculated gain

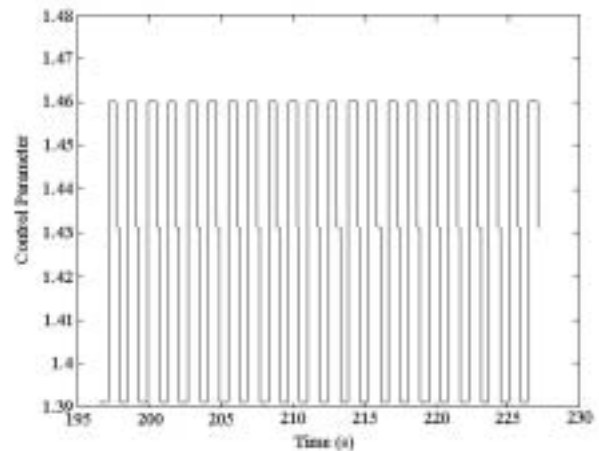


Fig. 11. Control parameter p_n .

vector was $K = [2.3841 \ 1.3553 \ 0.5799]$. This control law is enabled when z_n is in the neighborhood of the fixed point to be stabilized. In Figure 10, the controlled angular position variable is shown in steady state. It can be seen that the values of the local maxima of the controlled variable are very close to the calculated fixed point in the Lorenz map ($\bar{z}_{eq} = 0.2589$). Figure 11 shows the corresponding time evolution of the control parameter p_n in steady state. Finally, the reconstructed controlled attractor, which is a double-period oscillation, is shown in Figure 12.

It is worth mentioning that the design of the control law is made off-line and about an unstable fixed point of the Lorenz map of the nonlinear system, but once the appropriate values for the feedback gain vector are chosen, the control law (23) is applied directly to the nonlinear system.

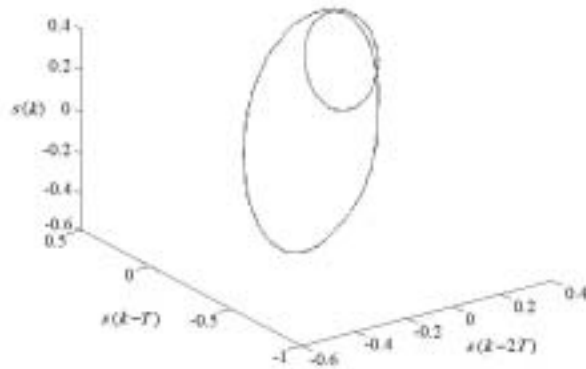


Fig. 12. Projection of the reconstructed attractor for the controlled system.

5. Concluding Remarks

In this paper we have proposed a procedure to analyze the complex behavior of an underactuated robot as a first stage for designing a UPO's stabilizing control method. These methods help us to analyze the dynamics of any system requiring only a measurement of one of its variables. Particularly, they are useful for extracting information from the system; furthermore, they are the first stage in the application of the method used here to control chaotic behavior. After the analysis of several experiments, applying a delayed coordinates method via AMI and PFNN, Lyapunov exponents, and fractal dimension techniques to the time series of the measured variable, we conclude that the Pendubot exhibits chaotic motion.

On the other hand, the proposed method for stabilizing periodic orbits offers a way to control chaotic systems without any prior knowledge of the system dynamics. It needs a measure of only one of the system variables and the availability of a system parameter which can play the role of a control input. Another advantage of the method is that it is possible to impose a desired dynamics for the stabilization of the UPO, allowing the application of some other control techniques such as optimal, robust or adaptive control over the discrete time linearization of the local dynamics in the Lorenz map.

The design of the control law was made off-line. A possible extension of the method for an on-line implementation is currently under investigation.

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