

# Power Adaptation for Joint Switched Diversity and Adaptive Modulation Schemes in Spectrum Sharing Systems

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**Abstract**—Under the scenario of an underlay cognitive radio network, we propose in this paper an adaptive scheme using transmit power adaptation, switched transmit diversity, and adaptive modulation in order to improve the performance of existing switching efficient schemes (SES) and bandwidth efficient schemes (BES). Taking advantage of the channel reciprocity principle, we assume that the channel state information (CSI) of the interference link is available to the secondary transmitter. This information is then used by the secondary transmitter to adapt its transmit power, modulation constellation size, and used transmit branch. The goal of this joint adaptation is to minimize the average number of switched branches and the average system delay given the fading channel conditions, the required error rate performance, and a peak interference constraint to the primary receiver. We analyze the proposed scheme in terms of the average number of branch switching, average delay, and we provide a closed-form expression of the average bit error rate (BER). We demonstrate through numerical examples that the proposed scheme provides a compromise between the SES and the BES schemes.

**Index Terms**—Adaptive modulation, performance analysis, power adaptation, spectrum sharing, and switched diversity.

## I. INTRODUCTION

WIRELESS communication systems are continuously evolving and growing, leading to an increasing need for spectrum resources. Although different spectrum bands are allocated to specific services, it was identified that these bands are unoccupied most of the time or partially used by the primary users. This underutilization of the frequency spectrum and the solutions to promote its efficient use gave the concept of cognitive radio more importance. First introduced by Mitola and Maguire [1], the basic idea of cognitive radio is to allow a primary (licensed) and secondary (unlicensed) users to coexist in the same frequency spectrum. In underlay cognitive radio systems, the primary and secondary users can transmit simultaneously as long as the interference of the secondary to the primary link stays below a predetermined threshold [2].

In underlay cognitive radio, the performance of the secondary link can be badly affected because of the interference

constraint to the primary user. In such scenarios, Zhang *et al.* show in [3] that beamforming is the optimal strategy to maximize the capacity of the secondary transmitter (ST) when dealing with a multiple-input single-output (MISO) secondary user's channel. While the authors in [3] assume that channels from the secondary transmitter to the secondary and primary receivers are perfectly known at the ST, we propose a more practical system using power adaptation and transmit diversity techniques. These techniques are very helpful in improving the performance of the secondary link while respecting the interference constraint to the primary user [4]. Dual-branch switch and stay (SSC) is one of these techniques that received a great deal of attention [5–7]. In this technique, the current branch is used as long as the signal-to-noise ratio (SNR) is above a predetermined threshold, otherwise the transmitter switches and uses the second branch. Switch and examine (SEC) has been proposed as an alternative to SSC to take advantage of the additional diversity branches [8]. The scan and wait (SWC) technique was then proposed in [9] in order to improve the performance of SEC and other traditional techniques at the expense of some time delay. In the SWC technique, the transmitter buffers data for a channel coherence time whenever none of the available diversity paths is able to reach a predetermined minimum quality requirement. The process of scanning and waiting is repeated indefinitely until an acceptable path is found.

While in [10], [11] rate adaptation is used with switched transmit diversity, the authors in [12] use joint rate and power adaptation in a typical spectrum-sharing system in order to maximize the secondary user's capacity. Considering that the ST is power limited and using joint power and rate adaptation as in [12], we extend the work done in [10], [11] to a more practical scenario. Using the reciprocity principle [13], we assume that the full interference channel state information (CSI) is available to the ST. As such, the channel coefficients for the forward and the reverse directions can be assumed to be similar and the CSI experienced by the reverse link can be used for the forward link especially that the ST and the primary receiver (PR) operate in the same frequency band.

In this paper, we derive expressions of the cumulative distribution function (CDF) and the probability density function (PDF) of the output SNR at the secondary receiver (SR). We use these statistical results to analyze the average number of branch switching, the average spectral efficiency, and the bit error rate (BER) of the proposed scheme. Using transmit power adaptation at the ST, we minimize the average system delay and the number of branch switching experienced by existing switching efficient schemes (SES) and bandwidth efficient schemes (BES). These delay and switching performance improvements come at the expense of a reduction in the spectral efficiency because of the power limitation taken

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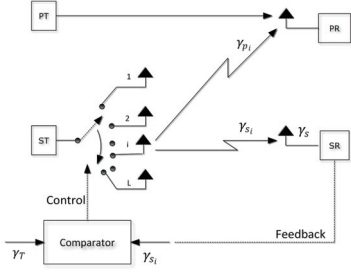


Fig. 1. System model.

into consideration in the proposed scheme.

The remainder of the paper is organized as follows. First, Section II presents the system and channel models. Next, Section III gives the mode of operation and the performance of the proposed scheme and Section IV illustrates this performance via numerical examples. Finally, Section V concludes the paper.

## II. MODELS AND ADAPTIVE MODULATION

### A. System and Channel Models

We consider the underlay cognitive system model shown in Fig. 1. A secondary user is allowed to share the spectrum with a primary link as long as an interference constraint to the PR with a peak value  $Q$  is respected. Equivalent to the downlink of a cellular system, the ST is equipped with  $L$  antennas and the SR has a single antenna. Let  $h_{s_i}$ ,  $i \in \{1, 2, \dots, L\}$  be the channel coefficient between the  $i$ th branch of the ST and the SR, and  $h_{p_i}$  be the channel coefficient between the  $i$ th branch of the ST and the PR. We assume that  $h_{s_i}$  and  $h_{p_i}$  are zero-mean complex Gaussian random variables with variances  $\sigma_s^2$  and  $\sigma_p^2$ , respectively. Assuming a transmit power  $P^t$  below a maximum power level  $P_{\max}$  and assuming a Gaussian noise with zero mean and variance  $N_0$  on both the secondary and the interference channels, the received SNRs from the  $i$ th antenna at the SR and the PR are given respectively by  $\gamma_{s_i} = \frac{P^t |h_{s_i}|^2}{N_0}$  and  $\gamma_{p_i} = \frac{P^t |h_{p_i}|^2}{N_0}$ . While our proposed scheme and analytical framework are applicable to generic fading scenarios, we assume for our study that the received signal from each diversity branch experiences independent identically distributed (i.i.d.) Rayleigh fading.

### B. Adaptive Transmission System

We consider the constant-power variable-rate uncoded  $M$ -ary quadrature amplitude modulation ( $M$ -QAM) [14] as an adaptive modulation system. With this adaptive modulator, the SNR range is divided into  $N + 1$  fading regions and the constellation size  $M = 2^n$  is assigned to the  $n$ th region  $n = 0, 1, \dots, N$ . The BER of coherent  $2^n$ -QAM with two-dimensional Gray coding over an additive white Gaussian noise (AWGN) channel with a SNR of  $\gamma$  can be well approximated as [14]

$$P_{b_n}(\gamma) \simeq \frac{1}{5} \exp\left(\frac{-3\gamma}{2(2^n - 1)}\right). \quad (1)$$

Given a target instantaneous BER equal to  $P_{b_0}$ , the adaptive modulator switching thresholds  $\gamma_n$  for  $n = 0, 1, \dots, N$  are given by

$$\gamma_n = -\frac{2}{3} \ln(5 P_{b_0})(2^n - 1); \quad n = 0, 1, \dots, N. \quad (2)$$

## III. JOINT POWER-ADAPTED SWITCHED DIVERSITY, AND ADAPTIVE MODULATION SCHEME

In this section, we first give the mode of operation of the proposed scheme, characterize the statistics of its output SNR, and then analyze its performance in terms of the average spectral efficiency, the average number of branch switching, the average delay, and the BER.

### A. Mode of Operation

Depending on the used SWC threshold  $\gamma_k$ ,  $k \in 1, \dots, N$ , we present different variations of the proposed scheme. While the case of  $k = 1$  presents a variation of the SES scheme offering the best delay and switching performance, the case of  $k = N$  is a variation of the BES scheme improving the delay performance and maintaining the spectral efficiency above a required level. In the beginning of each data burst, the secondary base station transmits a training sequence using its maximum power level  $P_{\max}$ . Using SWC, the ST cyclically switches between the  $L$  antennas in order to find the antenna that both verifies the interference constraint and reaches the required modulation threshold. The first antenna is then tested for the interference constraint  $Q$ ; if this constraint is not respected then the transmit power is adapted to the interference channel and is set to  $P^t = \frac{Q N_0}{|h_{p_1}|^2}$ . This branch is selected if its received SNR at SR is above  $\gamma_k$  else the next branch is tested for the same process. If none of the  $L$  branches satisfies constellation size  $k$ , the branch with the highest  $\gamma_{s_j}$  is selected from  $S$ , where  $S$  is the set of branches having an output SNR at the SR above  $\gamma_1$ . If  $S$  is empty, then the base station buffers the data and waits for channel coherence time before going again through a scanning of the available branches.

### B. Statistics of the Output SNR

Let  $\gamma_s$  denote the output SNR at the SR and let  $P_k = \Pr[\gamma_s < \gamma_k]$  be the probability that the received SNR from a given branch is below  $\gamma_k$ . The probability that no transmission occurs during a certain time slot is then given by  $P_1^L$ . Applying the mode of operation of this scheme as described above, the CDF of  $\gamma_s$  can be shown to be given by

$$F_{\gamma_s}(\gamma) = \begin{cases} \sum_{i=1}^L \frac{P_k^{i-1}}{1-P_k} (F_{\gamma_{s_i}}(\gamma) - F_{\gamma_{s_i}}(\gamma_k)) \\ + \sum_{i=1}^L \frac{P_k^{i-1}}{1-P_k} (F_{\gamma_{s_i}}(\gamma_k) - F_{\gamma_{s_i}}(\gamma_1)) F_{\gamma_{s_i}}^{L-i}(\gamma_k), & \gamma \geq \gamma_k; \\ \sum_{i=1}^L \frac{P_k^{i-1}}{1-P_k} (F_{\gamma_{s_i}}(\gamma) - F_{\gamma_{s_i}}(\gamma_1)) F_{\gamma_{s_i}}^{L-i}(\gamma), & \gamma_1 \leq \gamma < \gamma_k; \\ 0, & \gamma < \gamma_1, \end{cases} \quad (3)$$

where  $F_{\gamma_{s_i}}(\cdot)$  is given by

$$F_{\gamma_{s_i}}(\gamma) = \Pr[\gamma_{s_i} \leq \gamma] = \Pr\left[\min\left(P_{\max}, \frac{Q N_0}{|h_{p_i}|^2}\right) \frac{|h_{s_i}|^2}{N_0} \leq \gamma\right] \\ = 1 - \left(1 - \frac{\gamma}{\gamma + Q} e^{-\frac{\gamma}{Q}}\right) e^{-\frac{\gamma}{\gamma_1}}, \quad (4)$$

where  $\bar{\gamma} = \frac{P_{\max}\sigma_s^2}{N_0} = \frac{P_{\max}\sigma_p^2}{N_0}$  is the common average faded SNR for the secondary and interference links.

Differentiating (3) with respect to  $\gamma$ , the PDF of the received SNR for the proposed scheme is obtained as

$$f_{\gamma_s}(\gamma) = \begin{cases} \sum_{i=1}^L \frac{P_1^{i-1}}{1-P_1^L} f_{\gamma_{s_i}}(\gamma), & \gamma \geq \gamma_k; \\ \sum_{i=1}^L \frac{P_1^{i-1}}{1-P_1^L} \left( (L-i+1)F_{\gamma_{s_i}}^{L-i}(\gamma) \right. \\ \left. - (L-i)F_{\gamma_{s_i}}(\gamma)F_{\gamma_{s_i}}^{L-i-1}(\gamma) \right) f_{\gamma_{s_i}}(\gamma), & \gamma_1 \leq \gamma < \gamma_k; \\ 0, & \gamma < \gamma_1, \end{cases} \quad (5)$$

where  $f_{\gamma_{s_i}}(\cdot)$  is the PDF of the received SNR from one branch.

### C. Average Number of Branch Switching

The power consumption and the processing complexity of a given switched diversity system can be quantified in terms of the average number of branch switching. Based on the mode of operation of the proposed scheme, a further branch needs to be checked if the previous one fails to meet the modulation requirement. Using the same steps as in [9], the average of the number of switched branches  $N_s$  can be expressed as

$$\bar{N}_s = \sum_{n=0}^{\infty} \sum_{l=1}^L (nL+l) \left[ P_1^L \right]^n P_k^{l-1} (1-P_k) + \frac{LF_{\gamma_s}(\gamma_k)}{1-P_1^L}, \quad (6)$$

where  $nL+l$  branches are switched if data is buffered for  $n$  consecutive coherence times and the  $l$ th tested branch in the slot time number  $n+1$  has an acceptable quality. Using the mode of operation of the power-adapted scheme, the average number of switched branches simplifies to  $\frac{\sum_{l=0}^{L-1} P_k^l}{1-P_1^L}$ .

### D. Average Spectral Efficiency

A general expression of the average spectral efficiency of an adaptive modulation system is given in [14, Eq. (33)] by  $\eta = \sum_{n=1}^N n p_n$ , where  $p_n = F_{\gamma_s}(\gamma_{n+1}) - F_{\gamma_s}(\gamma_n)$  denotes the probability that the  $n$ th constellation is used. The average spectral efficiency is then given by

$$\eta = N - \sum_{n=1}^N F_{\gamma_s}(\gamma_n). \quad (7)$$

### E. Average BER

The general expression of the average BER for an adaptive modulation system is given in [14, Eq.(35)] as  $\bar{P}_b = \frac{1}{\eta} \sum_{n=1}^N n \bar{P}_{b_n}$ , where  $\bar{P}_{b_n}$  is the average BER for constellation size  $n$ , and is given by

$$\bar{P}_{b_n} = \int_{\gamma_n}^{\gamma_{n+1}} P_{b_n}(\gamma) f_{\gamma_s}(\gamma) d\gamma \quad (8)$$

Applying integration by parts and using (1) and (3), (8) can be obtained in closed-form as follows

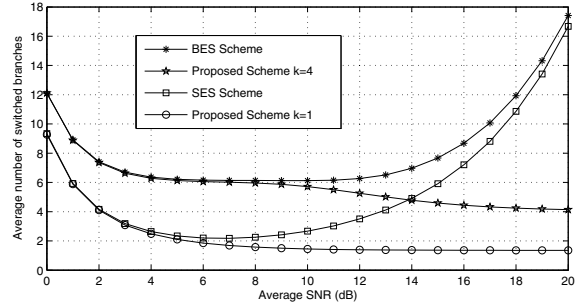


Fig. 2. Average number of switched branches versus average SNR.

$$\bar{P}_{b_n} = \begin{cases} \frac{\sum_{i=1}^L P_1^{i-1}}{1-P_1^L} \left( P_{b_n}(\gamma_{n+1})(P_{n+1}-1) - P_{b_0}(P_n-1) \right. \\ \left. - \frac{(e^{-\beta_n \gamma_n} - e^{-\beta_n \gamma_{n+1}})(1-e^{-Q/\bar{\gamma}})}{5b_n/\alpha_n} + \frac{\alpha_n Q}{5} e^{\alpha_n Q} \right) \times \left( E_i[-\beta_n(\gamma_n+Q)] - E_i[-\beta_n(\gamma_{n+1}+Q)] \right), & n \geq k; \\ P_{b_n}(\gamma_{n+1}) \left( \sum_{i=1}^L P_1^{i-1} P_{n+1}^{L-i} (P_{n+1}-P_1) \right) \\ - P_{b_0} \left( \sum_{i=1}^L P_1^{i-1} P_n^{L-i} (P_n-P_1) \right) \\ + \alpha_n \left( \sum_{i=1}^L \frac{P_1^{i-1}}{1-P_1^L} I_{L-i+1} - \sum_{i=1}^L \frac{P_1^i}{1-P_1^L} I_{L-i} \right), & n < k, \end{cases} \quad (9)$$

where  $\alpha_n = -3/(2(2^n-1))$ ,  $\beta_n = \alpha_n + 1/\bar{\gamma}$ ,  $E_i$  is the exponential integral function, and  $I_K$  is defined as

$$I_K = \int_{\gamma_n}^{\gamma_{n+1}} P_{b_n}(\gamma) F_{\gamma_{s_i}}^K(\gamma) d\gamma. \quad (10)$$

Applying binomial expansion to (4), (10) can be rewritten as

$$I_K = \frac{1}{5} \sum_{l=0}^K \sum_{m=0}^{K-l} \sum_{j=1}^l \binom{K}{l} \binom{K-l}{m} \binom{l}{j} (-1)^{m+j} Q^j e^{-\frac{l*Q}{\bar{\gamma}}} e^{\zeta_n Q} \\ \times \left( \frac{E_j[\zeta_n(\gamma_n+Q)]}{(\gamma_n+Q)^{j-1}} - \frac{E_j[\zeta_n(\gamma_{n+1}+Q)]}{(\gamma_{n+1}+Q)^{j-1}} \right) \\ + \sum_{l=0}^K \sum_{m=0}^{K-l} \binom{K}{l} \binom{K-l}{m} \frac{(e^{\zeta_n \gamma_n} - e^{\zeta_n \gamma_{n+1}}) e^{-\frac{l*Q}{\bar{\gamma}}}}{5\zeta_n}, \quad (11)$$

where  $\zeta_n = \alpha_n + (m+l)/\bar{\gamma}$ , and  $E_j(\cdot)$  is the exponential integral function of order  $j$ .

The expression of the average BER can be finally obtained in closed-form by replacing  $\bar{P}_{b_n}$  given in (9) in the following equation

$$\bar{P}_b = \frac{1}{\eta} \sum_{n=1}^N n \bar{P}_{b_n}. \quad (12)$$

## IV. NUMERICAL EXAMPLES

The performance of the proposed scheme is illustrated in this section with some selected numerical results for different values of  $k$ . For these examples, we set  $L = 6$ ,  $N = 4$ ,  $P_{\max}^{\text{dB}} = 2$  dB, and the BER constraint as  $P_{b_0} = 10^{-3}$ .

In Fig. 2, we depict the average number of switched branches as a function of  $\bar{\gamma}$  for  $Q_{\text{dB}} = 10$  dB for the power adapted scheme and the existing schemes. While more branches need to be tested in SES and BES especially in the high average SNR range because of breaking the interference



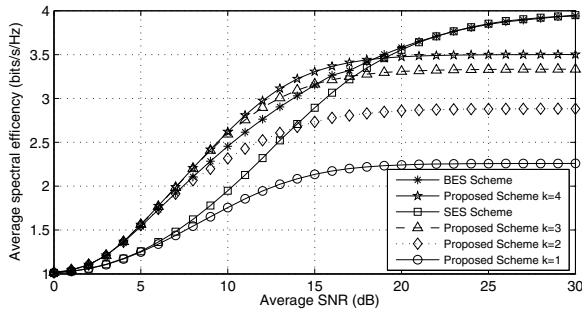


Fig. 3. Average spectral efficiency versus average SNR.

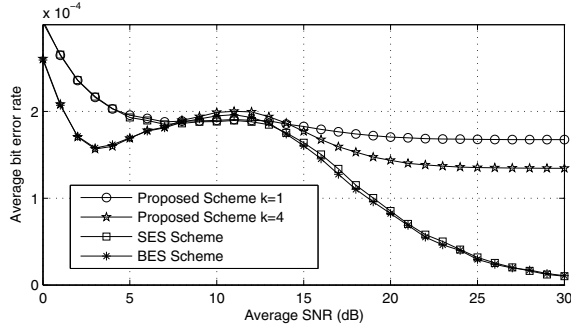


Fig. 4. Average BER versus average SNR.

constraint, power adaptation in the new scheme reduces the average number of switched branches. The power adapted scheme with  $k = 4$  shown in the figure is an improvement of the BES scheme with the use of power adaptation. The same is true for the proposed scheme with  $k = 1$  and the SES scheme.

The average system delay introduced by the SWC technique experiences the same behavior as the average number of switched branches and it is minimized for the proposed schemes. Indeed, thanks to the power adaptation process, the probability that a branch fails to be selected is minimized and in the high average SNR range most of the branches will reach the buffering threshold  $\gamma_1$  which minimizes the average system delay compared to existing schemes.

The spectral efficiency of the proposed schemes is presented in Fig. 3 for  $Q_{dB} = 10$  dB as a function of  $\bar{\gamma}$ . The better switching efficiency of the proposed scheme comes at the expense of a low spectral efficiency especially for  $k = 1$ . The higher  $k$  the better the spectral efficiency experienced by the proposed scheme and we can see that it outperforms the spectral efficiency of the BES scheme for  $k = 4$  in the medium average SNR range. The advantage of the proposed scheme is that it improves the delay and the switching efficiency while we guarantee a spectral efficiency above a certain threshold by a good choice of  $k$ . For example, if we want to keep the spectral efficiency above 3 bits/s/Hz, both  $k = 3$  and  $k = 4$  can be used.

In Fig. 4 we present the BER performance for the proposed scheme for  $k = 1$  and  $k = 4$  and for the SES and the BES schemes for  $Q_{dB} = 10$  dB as a function of  $\bar{\gamma}$ . Since the BES scheme aims at reaching a higher constellation size, it looks for a good quality branch. This explains the best BER performance of the BES scheme. In the low average SNR

range the proposed scheme with  $k = 4$  has the same BER as the BES scheme. In the high average SNR range, the power limitation in the ST makes the BER saturate but still verifying the required BER. Similarly, for  $k = 1$ , compared to the SES scheme, the same BER performance is experienced in the low average SNR range and it saturates in the high average SNR range for the proposed scheme with  $k = 1$ .

## V. CONCLUSION

We proposed in this paper a power adaptive scheme that aims at reducing the average delay and the average number of switched branches experienced by the SES and BES schemes while maintaining a high spectral efficiency and complying with the BER requirements. The proposed power adaptation process complies with practical considerations by both taking into consideration the power limitation in the ST, and by benefiting from the reciprocity principle in the interference channel in order to satisfy the interference constraint to the primary receiver. While for low switching threshold, the spectral efficiency of the proposed scheme is not satisfactory, the proposed scheme gives much better results for higher SWC switching thresholds. The proposed scheme then provides a compromise between delay performance and spectral efficiency performance which is also a compromise between the SES and BES schemes.

## REFERENCES

- [1] J. Mitola and G. Maguire, "Cognitive radio: making software radios more personal," *IEEE Personal Commun.*, vol. 6, no. 4, pp. 13–18, Aug. 1999.
- [2] M. Gastpar, "On capacity under receive and spatial spectrum-sharing constraints," *IEEE Trans. Inf. Theory*, vol. 53, no. 2, pp. 471–487, Feb. 2007.
- [3] R. Zhang and Y.-C. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 88–102, Feb. 2008.
- [4] H. Wang, J. Lee, S. Kim, and D. Hong, "Capacity enhancement of secondary links through spatial diversity in spectrum sharing," *IEEE Trans. Wireless Commun.*, vol. 9, no. 2, pp. 494–499, Feb. 2010.
- [5] M. A. Blanco and K. J. Zdunek, "Performance and optimization of switched diversity systems for the detection of signals with Rayleigh fading," *IEEE Trans. Commun.*, vol. 27, pp. 1887–1895, Dec. 1979.
- [6] C. Tellambura, A. Annamalai, and V. K. Bhargava, "Unified analysis of switched diversity systems in independent and correlated fading channels," *IEEE Trans. Commun.*, vol. 49, pp. 1955–1965, Nov. 2001.
- [7] Y.-C. Ko, M.-S. Alouini, and M. K. Simon, "Analysis and optimization of switched diversity systems," *IEEE Trans. Veh. Technol.*, vol. 49, no. 5, pp. 1813–1831, Sep. 2000.
- [8] H.-C. Yang and M.-S. Alouini, "Performance analysis of multibranch switched diversity systems," *IEEE Trans. Commun.*, vol. 51, no. 5, pp. 782–794, May 2003.
- [9] H.-C. Yang, M. K. Simon, and M.-S. Alouini, "Scan and wait combining (SWC): a switch and examine strategy with a performance-delay tradeoff," *IEEE Trans. Wireless Commun.*, vol. 5, no. 9, pp. 2477–2483, Sep. 2006.
- [10] K. A. Qaraqe, Z. Boudia, M. Abdallah, and M.-S. Alouini, "Joint switched transmit diversity and adaptive modulation in spectrum sharing systems," in *Proc. 2011 International ICST Conference on Cognitive Radio Oriented Wireless Networks and Communications*.
- [11] Z. Boudia, M. Abdallah, K. Qaraqe, and M.-S. Alouini, "Spectrally efficient switched transmit diversity for spectrum sharing systems," in *Proc. 2011 IEEE VTC – Fall*.
- [12] V. Asghari and S. Aissa, "Adaptive rate and power transmission in spectrum-sharing systems," *IEEE Trans. Wireless Commun.*, vol. 9, no. 10, pp. 3272–3280, Oct. 2010.
- [13] J. R. Carson, "A generalization of the reciprocal theorem," *Bell Syst. Tech. J.*, vol. 3, pp. 393–399, 1924.
- [14] M.-S. Alouini and A. J. Goldsmith, "Adaptive modulation over Nakagami fading channels," *Wireless Personal Commun.*, vol. 13, pp. 119–143, May 2000.