MMSE Based Transceiver Designs in Closed-Loop Non-Regenerative MIMO Relaying Systems

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Abstract

In this paper, we propose a new design strategy based on the minimum mean-squared error (MMSE) in closed-loop non-regenerative multiple-input multiple-output (MIMO) relaying systems. Instead of conventional singular value decomposition based methods, we address the problem for joint MMSE design in a different approach using the Wiener filter solution which leads to simple derivations of the optimal MMSE designs. First, allowing the channel state information (CSI) at the source, we provide a closed form solution for a source-relay-destination joint MMSE design by extending existing relay-destination joint MMSE designs. Second, for the limited feedback scenario, we address a codebook design criteria for the multiple streams precoding design with respect to the MMSE criterion. From our design strategy, we observe that compared to conventional non-regenerative relaying systems, the source or the destination only needs to know the CSI corresponding to its own link such as the source-to-relay or the relay-to-destination in view of the MMSE. Simulation results show that the proposed design gives about 7.5dB gains at a bit error rate (BER) of 10^{-4} over existing relay-destination joint MMSE schemes and we can get close to the optimal unquantized schemes with only a few feedback bits.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) wireless systems have been widely studied to increase communication reliability and spectral efficiency [1]–[4]. Recently, wireless relaying techniques also attracts great attention since the communication range and coverage can be extended by supporting shadowed areas where there are strong shadowing effects. These benefits make MIMO relaying techniques an important component for next generation wireless networks. A general information theoretic analysis of the relay channel was first reported in [5] and [6]. Furthermore, recently, researches on the capacity of MIMO relaying systems have been studied in [7]–[9].

MIMO relaying systems can be designed as either regenerative or non-regenerative. The regenerative scheme, also known as the decode-and-forward (DF) scheme, indicates that the relay decodes the original information from the previous node before it retransmits the information to the subsequent node. In contrast, the non-regenerative relay, or the amplify-and-forward (AF) scheme, implies that the relay node does not decode the signal while only a linear weighting process is performed. In practical relay systems, a non-regenerative method shows advantages of simple implementation and small processing delay compared to the regenerative relay systems. For these reasons, this paper focuses on the non-regenerative relaying system.

Currently many schemes have been developed based on the AF relaying in [10]–[18] Several studies have shown that proper linear operations can improve the system performance remarkably. For example, [12]–[14] have demonstrated that in a AF relaying network, a linear technique based on singular value decomposition (SVD) is optimal for maximizing the system performance. Especially allowing perfect channel state information (CSI) of both the source-to-relay link and the relay-to-destination link at the relay and the destination, the authors in [12] and [14] proposed the relay-destination joint optimization scheme with respect to minimum mean square error (MMSE) and maximum channel capacity (MCC), respectively. More recently, when the perfect CSI of both sides is available at all nodes, optimal source-relay-destination joint designs were developed with respect to the MCC [15], quality-of-service (QoS)

[16] and MMSE criteria [17], where all solutions are found by an iterative method. In this paper, we refer to the case where the CSI of both sides of links is available as global CSI, and the case where the CSI of its own link is available at the source or destination as local CSI.

In this paper, we provide a new MMSE design strategy to achieve the optimal performance for closed-loop relaying systems. Instead of conventional SVD based methods, we address the problem in a different approach using the Wiener filter solution [19]. Then we prove that the error covariance matrix for the MMSE relaying system is decomposable into a sum of two individual error matrices, which leads to easy derivations of the joint MMSE designs. Although the global CSI is required for achieving the optimal performance, our design strategy also demonstrate that the local CSI incurs little performance loss in terms of MMSE.

As an extended structure of the existing relay-destination joint optimal scheme [14], we first provide a new source-relay-destination joint MMSE design. In contrast to the iterative solutions in [15]–[17], our joint MMSE design strategy generates a general closed form solution and does not require the source and destination to know about the CSI of the other side of link. Although a high signal-to-noise ratio (SNR) approximation is employed in the derivation of the proposed solution, it can be confirmed by numerical results that it provides little performance loss in all SNR range in comparison to the optimal designs with much reduced complexity.

Second, from our proposed design, it becomes readily provable that for the limited feedback (or quantized feedback) scenario, the Grassmannian codebook [20]–[22] is efficient for quantizing the optimal precoders at the source and relay. The single stream optimal beamforming design in [23] is a special case of our solution. Simulation results show that the proposed joint MMSE scheme provides about 7.5dB and 5dB gains at a bit error rate (BER) of 10^{-4} for 4QAM and 16QAM cases, respectively, over the existing relay-destination joint optimized scheme, and we can get close to the optimal unquantized case with only a few feedback bits.

The remainder of this paper is organized as follows: In Section II, we first describe the system model.

Then in Section III, we show that from the MMSE point of view, the error covariance matrix can be decomposed into a sum of two individual error covariance matrices. Section IV proposes the optimal joint MMSE design and then the quantized precoding design is presented in Section V. In Section VI, we show that the proposed MMSE designs do not require the source-to-relay channel information at the destination. In Section VII, Monte Carlo simulations are performed. Finally, the conclusion is given in Section VIII.

Throughout this paper, normal letters represent scalar quantities, boldface letters indicate vectors and boldface uppercase letters designate matrices. The superscripts $(\cdot)^T$, $(\cdot)^\dagger$ and $(\cdot)^*$ stand for transpose, conjugate transpose, and element-wise conjugate, respectively. $E[\cdot]$ denotes the expectation operator, $\mathbf{I}_N$ indicates an $N \times N$ identity matrix, and $\mathrm{tr}\,(\mathbf{A})$ represents the trace of a matrix $\mathbf{A}$. Furthermore $\lambda_{min}(\mathbf{A})$ and $\lambda_{max}(\mathbf{A})$ indicate a minimum and maximum singular value of a matrix $\mathbf{A}$. Accounting for a complex matrix $\mathbf{A}$, we denote the real part of $\mathbf{A}$ by $\Re\{\mathbf{A}\}$ and the stacked columns of $\mathbf{A}$ by $\mathrm{vec}(\mathbf{A})$.

II. System Description

In this section, we consider a system model for non-regenerative MIMO relaying channels as shown in Figure 1. The source node transmits data information to the destination node through one relay node which helps the communication between two nodes. We assume that the source, relay and destination node use N_t, N_r and N_d antennas, respectively, and no direct path is assumed due to a large path loss between the source and the destination. As we consider a spatial multiplexing (SM) system which transmits N_s data streams simultaneously, we assume $N_s \leq \min\{N_t, N_r, N_d\}$. Furthermore, in this paper, we basically assume the half-duplex system, where data transmission in the relay system occurs in two separate time slots since in the full duplex mode, the power of the transmitted signal at the relay typically overshadows that of the desired signal at the relay.

In the first time slot, the symbol vector $\mathbf{x}$ is precoded by the N_t by N_s precoding matrix $\mathbf{F}$, and transmitted to the relay node. As shown in Figure 1, the N_r dimensional received signal vector $\mathbf{y}_R$ at the relay node is given as $\mathbf{y}_R = \mathbf{HFx} + \mathbf{n}_1$, where $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ denotes the first hop channel matrix of the source-to-relay link, $\mathbf{x}$ represents the N_s dimensional transmit signal vector with $E[\mathbf{xx}^\dagger] = \sigma_x^2 \mathbf{I}_{N_s}$ and $\mathbf{n}_1$ indicates the additive complex Gaussian noise vector at the relay node with zero mean and the covariance matrix $E[\mathbf{n}_1\mathbf{n}_1^\dagger] = \sigma_{n_1}^2 \mathbf{I}_{N_r}$. Then the received signal $\mathbf{y}_R$ at the relay is multiplied by the $N_s \times N_r$ relay receiver $\mathbf{L}_R$. Assuming tr$(\mathbf{FF}^\dagger) = N_s$, we define the total source transmit power as $P_T \triangleq E[\|\mathbf{Fx}\|^2] = \sigma_x^2 N_s$.

In the second time slot, the relay signal $\mathbf{y} = \mathbf{L}_R\mathbf{y}_R = \mathbf{L}_R(\mathbf{HFx}+\mathbf{n}_1)$ is precoded by the $N_r \times N_s$ relay precoder $\gamma\mathbf{B}$ and transmitted to the destination node. Here $\mathbf{L}_R$, $\mathbf{B}$ and the scaling parameter γ which compose the relay filter $\mathbf{Q}$ will be explained later. The N_d dimensional signal $\mathbf{y}_D$ at the destination is given as

$$\begin{aligned}\mathbf{y}_D &= \mathbf{GQHFx} + \mathbf{GQn}_1 + \mathbf{n}_2 \\ &= \gamma\mathbf{GBL}_R\mathbf{HFx} + \gamma\mathbf{GBL}_R\mathbf{n}_1 + \mathbf{n}_2,\end{aligned}$$

where $\mathbf{G} \in \mathbb{C}^{N_d \times N_r}$ indicates the second hop channel matrix of the relay-to-destination link and $\mathbf{n}_2$ is the zero mean complex Gaussian noise vector at the destination node with $E[\mathbf{n}_2\mathbf{n}_2^\dagger] = \sigma_{n_2}^2 \mathbf{I}_{N_d}$. In this case, $\mathbf{Q}$ needs to satisfy the total relay transmit power P_R as $E[\|\mathbf{Qy}_R\|^2] = P_R$. Then the received signal $\mathbf{y}_D$ is multiplied by the linear receiver $\mathbf{W}_D$ at the destination node, and we have the final observation $\tilde{\mathbf{y}}_D$ as $\tilde{\mathbf{y}}_D = \mathbf{W}_D\mathbf{y}_D$.

The SNR between each channel link is defined as $\text{SNR}_1 \triangleq P_T/(N_s\sigma_{n_1}^2)$ and $\text{SNR}_2 \triangleq P_R/(N_s\sigma_{n_2}^2)$, respectively. In addition, we define the following singular value decompositions as

$$\mathbf{H} = \mathbf{U}_h\mathbf{\Phi}\mathbf{V}_h^\dagger \quad \text{and} \quad \mathbf{G} = \mathbf{U}_g\mathbf{\Omega}\mathbf{V}_g^\dagger,$$

where $\mathbf{\Phi}$ and $\mathbf{\Omega}$ represent $N_r \times N_t$ and $N_d \times N_r$ matrices with singular values $\phi_i \in \mathbb{R}$ for $i = 1, 2, \ldots, \min(N_t, N_r)$ and $\omega_i \in \mathbb{R}$, for $i = 1, 2, \ldots, \min(N_r, N_d)$, in a descending order on the main diagonal, respectively. Furthermore N_s dimensional square diagonal matrices $\overline{\mathbf{\Phi}}$ and $\overline{\mathbf{\Omega}}$ are defined as $\overline{\mathbf{\Phi}} \triangleq diag\{\phi_1, \phi_2, \ldots, \phi_{N_s}\}$ and $\overline{\mathbf{\Omega}} \triangleq diag\{\omega_1, \omega_2, \ldots, \omega_{N_s}\}$, respectively. We denote the matrix constructed by the first N_s columns of a unitary matrix $\mathbf{U}_h$, $\mathbf{V}_h$, $\mathbf{U}_g$ and $\mathbf{V}_g$ as $\overline{\mathbf{U}}_h$, $\overline{\mathbf{V}}_h$, $\overline{\mathbf{U}}_g$ and $\overline{\mathbf{V}}_g$, respectively.

III. Decomposition of the Error Covariance Matrix

In this section, we show that from the MMSE point of view, the error covariance matrix can be decomposed into a sum of two individual error covariance matrices where each of them corresponds to the first hop channel $\mathbf{H}$ and second hop channel $\mathbf{G}$.

A. Optimum Destination Receiver

First we derive the optimal receive filter $\hat{\mathbf{W}}_D$ at the destination. Throughout this paper, unless specified otherwise, we assume that the MMSE optimal receive filter $\hat{\mathbf{W}}_D$ is adopted at the destination and refer to this filter as the destination Wiener filter (D-WF). Defining the error vector as $\mathbf{e} \triangleq \gamma^{-1}\tilde{\mathbf{y}}_D - \mathbf{x} = \mathbf{W}_D\mathbf{y}_D^{'} - \mathbf{x}$ where $\mathbf{y}_D^{'} = \gamma^{-1}\mathbf{y}_D$, the problem can be mathematically formulated as

$$\hat{\mathbf{W}}_D = \arg\min_{\mathbf{W}_D} E\left[\|\mathbf{e}\|^2\right]. \tag{1}$$

Here a scaling parameter γ^{-1} is introduced for simplification of the derivation for the relay filter $\mathbf{Q}$ in the following subsection. For given specific $\mathbf{Q}$, $\mathbf{F}$ and γ, the optimal receive filter $\hat{\mathbf{W}}_D$ is easily obtained as [24]

$$\begin{aligned}
\hat{\mathbf{W}}_D &= \mathbf{R}_{\mathbf{x},\mathbf{y}_D^{'}}(\mathbf{R}_{\mathbf{y}_D^{'},\mathbf{y}_D^{'}})^{-1} \\
&= \gamma\mathbf{R}_x\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{Q}^\dagger\mathbf{G}^\dagger\left(\mathbf{G}\mathbf{Q}\mathbf{H}\mathbf{F}\mathbf{R}_x\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{Q}^\dagger\mathbf{G}^\dagger + \mathbf{R}_n\right)^{-1}, \\
&= \gamma\left(\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{Q}^\dagger\mathbf{G}^\dagger\mathbf{R}_n^{-1}\mathbf{G}\mathbf{Q}\mathbf{H}\mathbf{F} + (1/\sigma_x^2)\mathbf{I}_{N_s}\right)^{-1}\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{Q}^\dagger\mathbf{G}^\dagger\mathbf{R}_n^{-1},
\end{aligned} \tag{2}$$

where $\mathbf{R}_n = \sigma_{n_1}^2\mathbf{G}\mathbf{Q}\mathbf{Q}^\dagger\mathbf{G}^\dagger + \sigma_{n_2}^2\mathbf{I}_{N_d}$. Note that for the derivation of (2), we have used a matrix inversion lemma. Then using the result in (2), we obtain the error covariance matrix $\mathbf{R}_E \triangleq E[\mathbf{e}\mathbf{e}^\dagger]$ as

$$\begin{aligned}
\mathbf{R}_E &= E[(\mathbf{W}_D\mathbf{y}_D^{'} - \mathbf{x})(\mathbf{W}_D\mathbf{y}_D^{'} - \mathbf{x})^\dagger] \\
&= \left(\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{Q}^\dagger\mathbf{G}^\dagger\mathbf{R}_n^{-1}\mathbf{G}\mathbf{Q}\mathbf{H}\mathbf{F} + (1/\sigma_x^2)\mathbf{I}_{N_s}\right)^{-1}.
\end{aligned} \tag{3}$$

Note that $E\left[\|\mathbf{e}\|^2\right] = \mathrm{tr}(\mathbf{R}_E)$.

B. Optimum Relay Transceiver

Next, in view of the relay node, the problem for minimizing the mean square error (MSE) under the relay power constraint is written as

$$\begin{aligned} &\min_{\mathbf{Q}} E\left[\|\mathbf{e}\|^2\right] \\ \text{s.t.} \quad &\text{tr}\left(\mathbf{Q}(\sigma_x^2\mathbf{H}\mathbf{F}\mathbf{F}^\dagger\mathbf{H}^\dagger + \sigma_{n_1}^2\mathbf{I}_{N_r})\mathbf{Q}^\dagger\right) = P_R. \end{aligned} \tag{4}$$

Note that it can be easily verified that the MSE function in (4) is convex over the relay transceiver filter $\mathbf{Q}$ when the source transmit filter $\mathbf{F}$, the destination receiver filter $\mathbf{W}_D$ and γ are given, and is also convex with respect to γ if $\mathbf{F}$, $\mathbf{W}_D$ and $\mathbf{Q}$ are fixed. Then we can find necessary conditions for $\mathbf{Q}$ and γ by constructing the cost function $\mathcal{C}$ with the Lagrangian multiplier λ as

$$\mathcal{C} = E\left[\|\mathbf{e}\|^2\right] + \lambda\{\text{tr}\left(\mathbf{Q}(\sigma_x^2\mathbf{H}\mathbf{F}\mathbf{F}^\dagger\mathbf{H}^\dagger + \sigma_{n_1}^2\mathbf{I}_{N_r})\mathbf{Q}^\dagger\right) - P_R\}. \tag{5}$$

In the following Lemma, using the cost function $\mathcal{C}$ in (5), we provide the MMSE optimal relay filter $\hat{\mathbf{Q}}$.

Lemma 1: For given $\mathbf{F}$ and $\mathbf{W}_D$, the optimal relay transceiver $\hat{\mathbf{Q}}$ for minimizing the MSE has the following form as $\hat{\mathbf{Q}} = \gamma\tilde{\mathbf{Q}} = \gamma\mathbf{B}\mathbf{L}_R$, where γ, $\mathbf{B}$ and $\mathbf{L}_R$ are computed as

$$\begin{aligned} \gamma &= \sqrt{P_R/\text{tr}(\tilde{\mathbf{Q}}(\sigma_x^2\mathbf{H}\mathbf{F}\mathbf{F}^\dagger\mathbf{H}^\dagger + \sigma_{n_1}^2\mathbf{I}_{N_r})\tilde{\mathbf{Q}}^\dagger)}, \\ \mathbf{B} &= \mathbf{G}^\dagger\mathbf{W}_D^\dagger(\mathbf{W}_D\mathbf{G}\mathbf{G}^\dagger\mathbf{W}_D^\dagger + \frac{\text{tr}(\sigma_{n_2}^2\mathbf{W}_D\mathbf{W}_D^\dagger)}{P_R}\mathbf{I}_{N_s})^{-1} \qquad (6) \\ \text{and} \quad \mathbf{L}_R &= (\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F} + (\sigma_{n_1}^2/\sigma_x^2)\mathbf{I}_{N_s})^{-1}\mathbf{F}^\dagger\mathbf{H}^\dagger. \qquad (7) \end{aligned}$$

Proof: See Appendix A. ■

In Lemma 1, we can identify that $\mathbf{B}$ and $\mathbf{L}_R$ correspond to the relay transmit Wiener filter for the second hop channel $\mathbf{G}$ and the relay receive Wiener filter (R-WF) for the first hop channel $\mathbf{H}$, respectively. The scaling parameter γ indicates the relay power normalizing coefficient. Note that only from the relay point of view, the optimum $\mathbf{B}$ has the Wiener filter solution as in (2) or (6), while from the joint optimization perspective with $\mathbf{F}$ and $\mathbf{W}_D$, the optimal $\hat{\mathbf{B}}$ follows the channel diagonalizing structure as will be shown in Section IV.

C. Decomposition of $\mathbf{R}_E$

In the following lemma, we will show that from the MMSE point of view, the error covariance matrix $\mathbf{R}_E$ is composed of two individual error covariance matrices.

Lemma 2: Given the optimal relay transceiver $\hat{\mathbf{Q}} = \gamma \mathbf{B}\mathbf{L}_R$ and the destination receiver $\hat{\mathbf{W}}_D$, the overall error matrix $\mathbf{R}_E$ in (3) can be decomposed into a sum of two individual error matrices as

$$\mathbf{R}_E(\mathbf{F}, \mathbf{B}) = \sigma_{n_1}^2 \left(\mathbf{F}^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{F} + \frac{\sigma_{n_1}^2}{\sigma_x^2} \mathbf{I}_{N_s} \right)^{-1} + \sigma_{n_2}^2 \left(\gamma^2 \mathbf{B}^\dagger \mathbf{G}^\dagger \mathbf{G} \mathbf{B} + \sigma_{n_2}^2 \mathbf{R}_y^{-1} \right)^{-1}, \tag{8}$$

where $\mathbf{R}_y$ stands for the covariance matrix of the relay signal $\mathbf{y} = \mathbf{L}_R(\mathbf{H}\mathbf{F}\mathbf{x} + \mathbf{n}_1)$.

Proof: See Appendix B. ■

Note that the R-WF $\mathbf{L}_R$ in (7) is completely a function of $\mathbf{F}$. Thus we need to optimize only $\mathbf{F}$ and $\mathbf{B}$ in (8).

IV. Joint MMSE Design Strategy

In this section, we investigate joint MMSE designs assuming that the perfect CSI is available at all nodes. Our designs are based on the constraint which bounds the expected norm of the transmit power as $E\left[\|\mathbf{F}\mathbf{x}\|^2\right] = \text{tr}(\mathbf{F}\mathbf{F}^\dagger)\sigma_x^2 = P_T$ and $E\left[\|\mathbf{B}\mathbf{y}\|^2\right] = \text{tr}(\mathbf{B}\mathbf{R}_y\mathbf{B}^\dagger) = P_R$. It should be noted that in this section, the relay power normalizing coefficient γ is assumed to be included in the relay precoder $\mathbf{B}$. Nevertheless, such a norm constraint (NC) does not restrict the peak power at the output. Hence in this section, we also provide the optimal solution based on the maximum eigenvalue constraint (MVC) [25] given as $L_T = \lambda_{max}(\sigma_x^2 \mathbf{F}\mathbf{F}^\dagger)$ and $L_R = \lambda_{max}(\mathbf{B}\mathbf{R}_y\mathbf{B}^\dagger)$. From the fact that $\text{tr}(\mathbf{A}) \leq \lambda_{max}(\mathbf{A})N$ for any M by N ($M \geq N$) matrix $\mathbf{A}$, the MVC limits the norm power as well while imposing a limit on the peak power of the transmit vectors.

A. Source-Relay-Destination Joint MMSE Design

For the case where the relay and the destination nodes know the full CSI of both $\mathbf{H}$ and $\mathbf{G}$, the jointly optimized filter at the relay-destination was derived in [14]. We refer to this as the relay-destination joint MMSE design with NC (RD-NC). In this section, assuming the CSI of the channel $\mathbf{H}$ is available additionally at the source node, we provide the source-relay-destination joint MMSE filter design. Note that in contrast to existing schemes in [15]–[17], the CSI of the channel $\mathbf{G}$ is not required at the source in our design criteria.

1) Norm Power Constraint: As we have shown in the previous section, the optimal relay transceiver $\hat{\mathbf{Q}}$ allows us to decompose the composite relay channel as two individual MIMO channels. Now using Lemma 2, we can set up the problem for minimizing the MSE as

$$\min_{\{\mathbf{F},\mathbf{B}\}} \operatorname{tr}\left(\mathbf{R}_E(\mathbf{F},\mathbf{B})\right)$$
$$\text{s.t.} \quad \operatorname{tr}\left(\sigma_x^2 \mathbf{F}\mathbf{F}^\dagger\right) = P_T \;\; \text{and} \;\; \operatorname{tr}\left(\mathbf{B}\mathbf{R}_y\mathbf{B}^\dagger\right) = P_R. \tag{9}$$

In principle, the jointly optimal source and the relay precoder $\hat{\mathbf{F}}$ and $\hat{\mathbf{B}}$ which satisfy the problem (9) should be computed iteratively as in [17] because two matrices are mutually connected. However an iterative method may not be desirable in practical implementation. Instead here we propose an approximated method to obtain a closed form solution. From equation (28) in Appendix B, it can be easily checked that under the practical assumption of $\frac{\sigma_x^2}{\sigma_{n_1}^2} \gg 1$, $\mathbf{R}_y$ rapidly approaches the identity matrix $\sigma_x^2 \mathbf{I}_{N_s}$. In this case, the source precoding matrix $\mathbf{F}$ can be determined independent of $\mathbf{B}$ because it does not have any effect on the second term of (8) with this assumption. Numerical results in Section VII show that this practical assumption causes almost no performance loss even in the low SNR_1 region.

Accordingly, the original problem in (9) can be separated as

$$\min_{\mathbf{F}} \operatorname{tr}\left(\sigma_{n_1}^2 (\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F} + \frac{\sigma_{n_1}^2}{\sigma_x^2}\mathbf{I}_{N_s})^{-1}\right) \qquad \min_{\mathbf{B}} \operatorname{tr}\left(\sigma_{n_2}^2 \left(\mathbf{B}^\dagger\mathbf{G}^\dagger\mathbf{G}\mathbf{B} + \sigma_{n_2}^2\mathbf{R}_y^{-1}\right)^{-1}\right)$$
$$\text{s.t.} \;\; \operatorname{tr}\left(\sigma_x^2\mathbf{F}\mathbf{F}^\dagger\right) = P_T \qquad\qquad \text{s.t.} \;\; \operatorname{tr}\left(\mathbf{B}\mathbf{R}_y\mathbf{B}^\dagger\right) = P_R. \tag{10}$$

It is remarkable that in this situation, there is no need for the source node to know about the channel information of $\mathbf{G}$. Each problem in (10) is the same as the optimization problem in conventional MIMO systems derived in [26] and [27]. Especially in [27], it was shown that the channel diagonalizing structure is optimal for minimizing a trace function. When the objective function $f(\cdot)$ is Schur-concave, it is known that the following bound holds as

$$f\left(\boldsymbol{\delta}(\mathbf{R}_E)\right) \leq f\left(\boldsymbol{d}(\mathbf{R}_E)\right), \tag{11}$$

where $\boldsymbol{\delta}(\mathbf{R}_E)$ and $\boldsymbol{d}(\mathbf{R}_E)$ denote vectors which consist of the eigenvalues and diagonal elements of $\mathbf{R}_E$ in a decreasing order, respectively. The equality in (11) holds when the matrix $\mathbf{R}_E$ has a diagonalized structure and the trace function is a representative Schur-concave function. Therefore, without loss of generality, we can assume that the optimal solutions of (10) are expressed in the form of $\hat{\mathbf{F}} = \overline{\mathbf{V}}_h\boldsymbol{\Delta}_f$ and $\hat{\mathbf{B}} = \overline{\mathbf{V}}_g\boldsymbol{\Delta}_b$ where the power loading matrices $\boldsymbol{\Delta}_f$ and $\boldsymbol{\Delta}_b$ are defined as $\boldsymbol{\Delta}_f \triangleq diag\{f_1, f_2, \ldots, f_{N_s}\}$ and $\boldsymbol{\Delta}_b \triangleq diag\{b_1, b_2, \ldots, b_{N_s}\}$, respectively.

These problems in (10) can be solved efficiently using the Lagrangian multipliers ν and τ, and we obtain the water-pouring like solutions as

$$|f_i|^2 = \frac{1}{\phi_i^2\sigma_x^2}\left(\sqrt{\frac{\sigma_{n_1}^2\phi_i^2\sigma_x^2}{\nu}} - \sigma_{n_1}^2\right)^+ \quad \text{and} \quad |b_i|^2 = \frac{1}{\omega_i^2 r_i}\left(\sqrt{\frac{\sigma_{n_2}^2\omega_i^2 r_i}{\tau}} - \sigma_{n_2}^2\right)^+. \tag{12}$$

where $(x)^+$ is defined as $\max(x,0)$ and $r_i = \sigma_x^4\phi_i^2 f_i^2/(\sigma_x^2\phi_i^2 f_i^2 + \sigma_{n_1}^2)$ stands for the i-th diagonal element of $\mathbf{R}_y$. Here ν and τ are chosen to meet the power constraint (9). Note that if $r_i = 0$, then $b_i = 0$. From this result, we have $\mathbf{L}_R = (\boldsymbol{\Delta}_f^2\overline{\boldsymbol{\Phi}}^2 + (\sigma_{n_1}^2/\sigma_x^2)\mathbf{I}_{N_s})^{-1}\boldsymbol{\Delta}_f\overline{\boldsymbol{\Phi}}\overline{\mathbf{U}}_h^\dagger$.

Then we obtain a closed form solution for $\hat{\mathbf{F}}$ and $\hat{\mathbf{Q}}$ as

$$\hat{\mathbf{F}} = \overline{\mathbf{V}}_h\boldsymbol{\Delta}_f \quad \text{and} \quad \hat{\mathbf{Q}} = \overline{\mathbf{V}}_g\boldsymbol{\Theta}\overline{\mathbf{U}}_h^\dagger,$$

where the elements of $\boldsymbol{\Delta}_f$ are given in (12) and $\boldsymbol{\Theta}$ indicates an $N_s \times N_s$ diagonal matrix as $\boldsymbol{\Theta} = diag\{\theta_1, \cdots, \theta_{N_s}\}$. Here θ_i is computed from

$$|\theta_i|^2 = \frac{1}{\omega_i^2(\sigma_x^2\phi_i^2 f_i^2 + \sigma_{n_1}^2)}\left(\sqrt{\frac{\sigma_x^4\sigma_{n_2}^2\omega_i^2\phi_i^2 f_i^2}{\tau(\sigma_x^2\phi_i^2 f_i^2 + \sigma_{n_1}^2)}} - \sigma_{n_2}^2\right)^+.$$

Note that only amplitudes of f_i, b_i and θ_i are given, since the phase does not affect the MSE. We refer to this design criterion as the source-relay-destination joint MMSE design with NC (SRD-NC).

In what follows, we briefly address the bit error rate (BER)-based criteria. Until now, we have studied the optimization based on the MMSE criterion. However the optimization in the MMSE sense may not lead to a solution which minimizes the BER. Recently, it was shown in [27] that by applying a discrete Fourier transform (DFT) matrix or a Hadamard matrix to the diagonalized error covariance matrix, we can make all diagonal elements of the error covariance matrix have the same value while maintaining the MSE. This is equivalent to minimizing the maximum MSE in each stream, and is called the average BER-based criteria or ARITH-BER [27]. This approach can be directly applied to relaying systems by setting $\hat{\mathbf{F}} = \overline{\mathbf{V}}_h \mathbf{\Delta}_f \mathbf{Z}$ and $\hat{\mathbf{B}} = \overline{\mathbf{V}}_g \mathbf{\Delta}_b \mathbf{Z}$, where $\mathbf{Z}$ indicates the $N_s \times N_s$ DFT matrix, and from this we obtain the BER optimized precoders. We refer to this design criterion as the BER-based source-relay-destination joint design with NC (SRD-BNC).

2) Maximum Eigenvalue Constraint: In the following, we determine $\hat{\mathbf{F}}$ and $\hat{\mathbf{B}}$ which minimize the MSE based on the MVC constraint. As shown previously, the problems for $\mathbf{F}$ and $\mathbf{B}$ are given separately as

$$\begin{aligned} &\min_{\mathbf{F}} \operatorname{tr}\left(\sigma_{n_1}^2 (\mathbf{F}^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{F} + \frac{\sigma_{n_1}^2}{\sigma_x^2} \mathbf{I}_{N_s})^{-1}\right) && \min_{\mathbf{B}} \operatorname{tr}\left(\sigma_{n_2}^2 \left(\mathbf{B}^\dagger \mathbf{G}^\dagger \mathbf{G} \mathbf{B} + \sigma_{n_2}^2 \mathbf{R}_y^{-1}\right)^{-1}\right) \\ &\quad \text{s.t.} \ \ \lambda_{max}(\sigma_x^2 \mathbf{F}\mathbf{F}^\dagger) = L_T && \quad \text{s.t.} \ \ \lambda_{max}(\mathbf{B}\mathbf{R}_y\mathbf{B}^\dagger) = L_R. \end{aligned} \tag{13}$$

Each problem corresponds to the conventional MIMO systems with the source transmit vector $\mathbf{x}$ and the relay receiver output signal $\mathbf{y}$. Especially the first problem in (13) is the exactly same one in [25] whose solution is given as $\hat{\mathbf{F}} = \sqrt{\frac{L_T}{\sigma_x^2}} \overline{\mathbf{V}}_h$.

For the second problem in (13), denoting $\text{MSE}_{\mathcal{G}} \triangleq \sigma_{n_2}^2 \left(\mathbf{B}^\dagger \mathbf{G}^\dagger \mathbf{G} \mathbf{B} + \sigma_{n_2}^2 \mathbf{R}_y^{-1}\right)^{-1}$, we have $\operatorname{tr}(\text{MSE}_{\mathcal{G}})$ $= \sum_{i=1}^{N_s} \sigma_{n_2}^2 r_i (\sigma_{n_2}^2 + r_i |\omega_i b_i|^2)^{-1}$. Then from the constraint $\lambda_{max}(\mathbf{B}\mathbf{R}_y\mathbf{B}^\dagger) = \max_i (r_i |b_i|^2) = L_R$, we obtain the following inequality as

$$\operatorname{tr}(\text{MSE}_{\mathcal{G}}) \leq \sum_{i=1}^{N_s} \sigma_{n_2}^2 r_i (\sigma_{n_2}^2 + L_R |\omega_i|^2)^{-1} \tag{14}$$

where the equality holds when $\mathbf{\Delta}_b \mathbf{R}_y \mathbf{\Delta}_b^\dagger = L_R \mathbf{I}_{N_s}$. Therefore we can compute the optimal relay precoder $\hat{\mathbf{B}} = \overline{\mathbf{V}}_g \mathbf{\Delta}_b$ where the i-th element of $\mathbf{\Delta}_b$ is given as $b_i = \sqrt{\frac{L_R}{r_i}}$. We refer to this as the source-relay-destination joint MMSE design with MVC (SRD-MV). It should be emphasized that $\hat{\mathbf{B}}$ approaches $\sqrt{\frac{L_R}{\sigma_x^2}}\overline{\mathbf{V}}_g$ as SNR_1 increases, since r_i comes close to σ_x^2.

B. Optimal Relay-Destination Joint MMSE Design

As mentioned earlier, it was shown in [14] that the jointly optimal relay filter $\mathbf{Q}$ for RD-NC has a canonical form of $\overline{\mathbf{V}}_g\mathbf{\Lambda}\mathbf{U}_h^\dagger$ where $\mathbf{\Lambda}$ is a $N_s(=N_t)$ by N_r diagonal matrix. Although the approach in [14] provides a solution for the open-loop case where the source does not have any information on the channel, the destination is required to know about both the channel $\mathbf{H}$ and $\mathbf{G}$ for achieving the optimal performance. In this section, we provide a new approach for the jointly optimized designs at the relay-destination with two different constraints and show that the existing RD-NC [14] is a special case of our design strategy. From this result, we will describe in Section VI that the RD-NC [14] can also be applied to the case with the local CSI where only the information of the relay-to-destination link is allowed at the destination.

1) Norm Power Constraint: We consider a system where the perfect CSI is allowed only at the relay and the destination node. In this case, we cannot determine the source precoder $\mathbf{F}$, and thus $\mathbf{F}$ is set to $\mathbf{F}=\mathbf{I}_{N_t}$. Then, the optimization problem (9) is rephrased as

$$\min_{\mathbf{B}} \text{tr}\left(\text{MSE}_{\mathcal{G}}\right) \qquad \text{s.t.} \quad \text{tr}\left(\mathbf{B}\mathbf{R}_y\mathbf{B}^\dagger\right) = P_R. \tag{15}$$

Here the SVD of $\mathbf{R}_y$ and $\mathbf{L}_R$ is given as $\mathbf{R}_y = \mathbf{V}_h\mathbf{\Sigma}\mathbf{V}_h^\dagger$ and $\mathbf{L}_R = \mathbf{V}_h\widetilde{\mathbf{\Sigma}}\mathbf{U}_h^\dagger$, respectively, where $\mathbf{\Sigma} \triangleq \sigma_x^4\mathbf{\Phi}^2(\sigma_x^2\mathbf{\Phi}^2+\sigma_{n_1}^2\mathbf{I}_{N_s})^{-1}$ and $\widetilde{\mathbf{\Sigma}} \triangleq \sigma_x^{-2}\mathbf{\Sigma}$. Note that the covariance matrix $\mathbf{R}_y$ is a positive definite matrix, and is not a diagonal matrix.

Without loss of generality, from (11), we can assume that the optimal $\hat{\mathbf{B}}$ has a form of $\hat{\mathbf{B}} = \overline{\mathbf{V}}_g\mathbf{\Xi}\mathbf{V}_h^\dagger$ where $\mathbf{\Xi}$ indicates an N_s by N_s diagonal matrix. Then we obtain

$$\begin{aligned} \min_{\mathbf{B}} \text{tr}\left(\text{MSE}_{\mathcal{G}}\right) &= \min_{\mathbf{\Xi}} \text{tr}\left(\sigma_{n_2}^2(\mathbf{V}_h\mathbf{\Xi}^\dagger\overline{\mathbf{V}}_g^\dagger\mathbf{G}^\dagger\mathbf{G}\overline{\mathbf{V}}_g\mathbf{\Xi}\mathbf{V}_h^\dagger + \sigma_{n_2}^2\mathbf{V}_h\mathbf{\Sigma}\mathbf{V}_h^\dagger)^{-1}\right) \\ &= \min_{\mathbf{\Xi}} \text{tr}\left(\sigma_{n_2}^2(\mathbf{\Xi}^\dagger\overline{\mathbf{\Omega}}^2\mathbf{\Xi} + \sigma_{n_2}^2\mathbf{\Sigma})^{-1}\right) \end{aligned}$$

and finally the simple convex problem is determined as

$$\min_{\boldsymbol{\Xi}} \mathrm{tr}\left(\sigma_{n_2}^2(\boldsymbol{\Xi}^\dagger\overline{\boldsymbol{\Omega}}^2\boldsymbol{\Xi}+\sigma_{n_2}^2\boldsymbol{\Sigma})^{-1}\right) \qquad \text{s.t.} \quad \mathrm{tr}\left(\boldsymbol{\Xi}\boldsymbol{\Sigma}\boldsymbol{\Xi}^\dagger\right)=P_R. \tag{16}$$

This problem can be solved very efficiently using the Lagrangian multiplier τ. Denoting ξ_i as the i-th diagonal element of $\boldsymbol{\Xi}$, we obtain the optimal solution as $|\xi_i|^2=\frac{1}{\omega_i^2\Sigma_i}\left(\sqrt{\frac{\sigma_{n_2}^2\omega_i^2\Sigma_i}{\tau}}-\sigma_{n_2}^2\right)^+$, where $\Sigma_i=\sigma_x^4\phi_i^2/(\sigma_x^2\phi_i^2+\sigma_{n_1}^2)$ designates the i-th diagonal element of $\boldsymbol{\Sigma}$, and τ is the water-level chosen to satisfy the power constraint. As a result, the optimal relay transceiver $\hat{\mathbf{Q}}=\mathbf{B}\mathbf{L}_R$ is given as $\hat{\mathbf{Q}}=\overline{\mathbf{V}}_g\boldsymbol{\Lambda}\mathbf{U}_h^\dagger$, where $\boldsymbol{\Lambda}=diag\{\Lambda_1,\cdots,\Lambda_{N_t}\}$ is defined as an $N_s\times N_r$ diagonal matrix. Here Λ_i is obtained from

$$|\Lambda_i|^2=\frac{1}{\omega_i^2(\sigma_x^2\phi_i^2+\sigma_{n_1}^2)}\left(\sqrt{\frac{\sigma_x^4\sigma_{n_2}^2\omega_i^2\phi_i^2}{\tau(\sigma_x^2\phi_i^2+\sigma_{n_1}^2)}}-\sigma_{n_2}^2\right)^+. \tag{17}$$

Note that the solution in (17) is the same as the result in [14].

2) Maximum Eigenvalue Constraint: Considering the MVC, we can rewrite the problem in (16) as

$$\min_{\boldsymbol{\Xi}} \mathrm{tr}\left(\sigma_{n_2}^2(\boldsymbol{\Xi}^\dagger\overline{\boldsymbol{\Omega}}^2\boldsymbol{\Xi}+\sigma_{n_2}^2\boldsymbol{\Sigma})^{-1}\right) \qquad \text{s.t.} \quad \lambda_{max}\left(\mathbf{B}\mathbf{R}_y\mathbf{B}^\dagger\right)=L_R.$$

Then using a similar approach, we can compute the optimal precoder $\hat{\mathbf{B}}=\overline{\mathbf{V}}_g\boldsymbol{\Xi}\mathbf{V}_h^\dagger$, where ξ_i is calculated as $\xi_i=\sqrt{\frac{L_R}{\Sigma_i}}$. We refer to this as the relay-destination joint optimal design with MVC (RD-MV). It should also be noted that $\hat{\mathbf{B}}$ approaches $\sqrt{\frac{L_R}{\sigma_x^2}}\overline{\mathbf{V}}_g\mathbf{V}_h^\dagger$ as SNR_1 increases.

V. Quantized Precoding Design

So far, we have assumed still ideal cases which allow the perfect CSI of the channel $\mathbf{H}$ or $\mathbf{G}$ at the transmit sides (i.e., source or relay). This may be realized in time division duplex (TDD) systems with channel reciprocity, while in frequency division duplex (FDD) systems, a proper channel identification process is required. One efficient solution which minimizes the required feedback overhead is to share a codebook of precoding matrices among all nodes and to send back the index of the codeword to the transmit sides. For the case of conventional MIMO Gaussian channels, it has been shown that the codebook design is associated with Grassmannian packing problems [20]–[22]. In this section, we briefly show that the Grassmannian codebook is also efficient for quantizing the optimal precoding matrices in MMSE-based relaying systems.

We assume that the source precoding matrix $\mathbf{F}$ belongs to a codebook $\mathcal{U}_F(N_t, N_s)$ shared between the source and the relay. Similarly, the relay precoding matrix $\mathbf{B}$ is determined by a possibly different codebook $\mathcal{U}_B(N_r, N_s)$, shared between the relay and the destination. Denoting $\mathbf{F}_i \in \mathcal{U}_F(N_t, N_s)$ and $\mathbf{B}_j \in \mathcal{U}_B(N_r, N_s)$ where i and j indicate the codeword indices, we further assume that $\mathbf{F}_i$ and $\mathbf{B}_j$ are unitary matrices consisting of N_s orthogonal unit vectors. Note that this assumption is not especially restrictive since it follows a form of optimal precoders with the MVC illustrated in the previous section.

From Lemma 2, the unconstrained problem for minimizing the MSE is given as

$$\begin{aligned} & \min_{\{i,j\}} \ \mathrm{tr}\left(\mathbf{R}_E(\mathbf{F}_i, \mathbf{B}_j)\right) \\ \approx \ & \min_{\{i,j\}} \ \mathrm{tr}\left(\sigma_{n_1}^2 \left(\mathbf{F}_i^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{F}_i\right)^{-1} + \sigma_{n_2}^2 \left(\frac{P_R}{N_s \sigma_x^2} \mathbf{B}_j^\dagger \mathbf{G}^\dagger \mathbf{G} \mathbf{B}_j\right)^{-1}\right) & (18) \\ \leq \ & \min_i \ \frac{N_s \sigma_{n_1}^2}{\lambda_{min}^2(\mathbf{H}\mathbf{F}_i)} + \min_j \ \frac{N_s^2 \sigma_x^2 \sigma_{n_2}^2}{P_R \lambda_{min}^2(\mathbf{G}\mathbf{B}_j)} \ , & (19) \end{aligned}$$

where we used a high SNR assumption in (18) and a fact that $\lambda_{max}(\mathbf{A}^{-1}) = \lambda_{min}^{-1}(\mathbf{A})$ in (19). Note that in high SNR, γ approaches a constant $\sqrt{P_R/(N_s\sigma_x^2)}$ under the unitary precoding assumption. Thus maximizing the $\lambda_{min}^2(\mathbf{H}\mathbf{F}_i)$ and $\lambda_{min}^2(\mathbf{G}\mathbf{B}_j)$ is an approximated method for minimizing the MSE.

The authors in [22] have shown that the Grassmannian codebook designed for maximizing the minimum projection 2-norm distance [21] among codewords is most efficient for maximizing the minimum singular value $\lambda_{min}^2(\mathbf{H}\mathbf{F}_i)$ or $\lambda_{min}^2(\mathbf{G}\mathbf{B}_j)$. Therefore, from the result in (19), we can see that the projection 2-norm distance based Grassmannian codebook is also efficient for quantizing the optimal source and the relay precoding matrices. It is remarkable that quantizing the relay precoder $\mathbf{B}$ with the unquantized R-WF is more suitable for the MMSE criterion than directly quantizing the relay transceiver $\mathbf{Q}$. It should be also noted that the optimal and the quantized beamforming design in [23] is a special case of our solutions with $N_s = 1$.

We refer to this quantized precoding design as SRD-Q. Alternatively, with the same manner in Section

IV-B, if no CSI is available at the source (i.e., $\mathbf{F} = \mathbf{I}_{N_s}$), we can assume that only the codeword index for the relay precoder is conveyed from the destination to the relay, which is referred as RD-Q. Numerical results in Section VII illustrate that we can get close to the optimal designs with only a few feedback bits.

VI. CSI REQUIREMENT FOR DESTINATION

In general, for achieving the optimal performance, conventional AF relaying systems require the global CSI at the destination. However, in practical implementation, this may be undesirable because of a large system overhead. In this section, we show that our design strategy can be well suited to systems with the local CSI at the destination where only the CSI of the relay-to-destination link is available at the destination. From the MMSE point of view, we can obtain a simplified form of D-WF in (2) in the following Lemma.

Lemma 3: Given the optimal relay transceiver $\hat{\mathbf{Q}} = \gamma \mathbf{B}\mathbf{L}_R$, the D-WF in (2) can be simplified as

$$\hat{\mathbf{W}}_D = \left(\mathbf{B}^\dagger \mathbf{G}^\dagger \mathbf{G}\mathbf{B} + (\sigma_{n_2}^2/\gamma^2)\mathbf{R}_y^{-1}\right)^{-1} \mathbf{B}^\dagger \mathbf{G}^\dagger. \tag{20}$$

Proof: See Appendix C. ∎

Recall that $\mathbf{R}_y$ can be approximated as $\sigma_x^2 \mathbf{I}_{N_s}$ in the practical situation of $\frac{\sigma_x^2}{\sigma_{n_1}^2} \gg 1$. Hence from Lemma 3, we can obtain the suboptimal D-WF referred to as D-Sub as

$$\mathbf{W}_{D-Sub} = (\mathbf{B}^\dagger \mathbf{G}^\dagger \mathbf{G}\mathbf{B} + \frac{\sigma_{n_2}^2}{\sigma_x^2 \gamma^2}\mathbf{I}_{N_s})^{-1} \mathbf{B}^\dagger \mathbf{G}^\dagger. \tag{21}$$

This indicates that the destination node does not need to know about the channel information of $\mathbf{H}$. Besides, as we have studied previously, the source precoder $\mathbf{F}$ does not require any knowledge of the second hop channel $\mathbf{G}$. In other words, for all our MMSE designs, the source or the destination only needs to know the local CSI corresponding to its own link such as the source-to-relay or the relay-to-destination. Note that in the case of RD-NC and RD-MV, the optimal relay precoder $\mathbf{B}$ includes $\overline{\mathbf{V}}_h$, so the CSI of the effective channel $\widetilde{\mathbf{G}} \triangleq \mathbf{G}\mathbf{B}$ is required at the destination. In the following section, simulation results will show that the proposed D-Sub has almost no performance loss even in the low SNR region compared to the optimal case. The required CSI comparison is described in Table I. Here k_F and k_B designate the codebook size of $\mathcal{U}_F(N_t, N_s)$ and $\mathcal{U}_B(N_r, N_s)$, respectively.

VII. Simulation Results

In this section, Monte Carlo simulations are performed to illustrate the BER and MSE performance of our proposed schemes in flat fading channels. We assume that $\sigma_{n_1}^2 = \sigma_{n_2}^2 = 1$ and $N_r = 4$. The total transmit power to noise ratio is defined as $\text{SNR}_o \triangleq P/\sigma_{n_1}^2$, assuming that $P_T = P/2$ and $P_R = P/2$, where P denotes the total transmit power. We use the notation $N_t \times N_r \times N_d$ to denote an system with N_t source antennas, N_r relay antennas and N_d destination antennas. For all simulations, we set the same peak eigenvalue $L_T = P_T/N_s$ and $L_R = P_R/N_s$ so that the norm power constraints are satisfied as $\sigma_x^2 \text{tr}(\mathbf{F}\mathbf{F}^\dagger) = P_T$ and $\text{tr}(\mathbf{B}\mathbf{R}_y\mathbf{B}^\dagger) = P_R$. Moreover, we assume that elements of the channel $\mathbf{H}$ and $\mathbf{G}$ have an independent and identically distributed (i.i.d.) complex Gaussian distribution with zero mean and unit variance.

From Figure 2 to 5, we provide the performance of the proposed joint MMSE designs. The AF w/ D-WF indicates the most simple scheme where no filtering operation is performed at the relay ($\mathbf{Q} = \gamma\mathbf{I}_{N_r}$) while the optimal receiver D-WF is adopted at the destination to exhibit the performance lower bound. Note that the MCC [12] [15] and the QoS [16] schemes are not included in the simulation results since their design criteria are different from our MMSE based design strategy.

In Figure 2, we show the MSE performance of various schemes. Note that we have normalized the MSE by σ_x^2 when obtaining the MSE curve. Clearly the proposed SRD-NC outperforms the RD-NC because all nodes of the source, the relay and the destination are jointly optimized while RD-NC is optimized only for the relay and the destination. It is remarkable that the proposed designs show almost no performance loss even when only the local CSI is allowed at the destination. We also provide the MSE performance of the optimal design based on the minimum arithmetic sum of MSE (MA-MSE) criterion which has been solved by an iterative method in [17]. Note that MA-MSE follows the same criterion as our proposed design SRD-NC. From this plot, we can confirm that except for a little MSE loss in the low SNR region, our approximated method almost achieves the performance of the optimal designs with substantially reduced complexity and smaller CSI requirement.

In Figure 3, we provide the BER performance of the optimal design which minimizes the maximum MSE (MM-MSE) [17] and the proposed design SRD-BNC in various antenna configurations. MM-MSE follows the same design criterion as SRD-BNC. From this plot, we can check that compared to the optimal solution founded by an iterative method, the proposed approximation scheme shows little performance loss in all SNR range under the BER-based criterion.

The BER performance comparison with suboptimal designs in 4QAM and 16QAM constellations are presented in Figures 4 and 5, respectively. From these plots, we can check that the proposed SRD-BNC exhibits about 7.5dB and 5dB gains over the RD-NC at a BER of 10^{-4} for 4QAM and 16QAM cases, respectively. It is also worthwhile to note that in the local CSI situation at the destination, the proposed designs with D-Sub almost achieves the optimal performance. Therefore in comparison to conventional schemes [10]–[17], our design strategy is more practical in terms of saving the cost related to the CSI.

In Figures 6, 7 and 8, we present the MSE and BER performance of the Grassmannian quantization schemes comparing to the optimal designs. We simulate two streams precoding on $2 \times 4 \times 2$ and $4 \times 4 \times 2$ wireless relaying systems. The codebooks are designed for maximizing the minimum projection two-norm distance between any pair of the codeword matrix column space, defined as $d_{proj}(\mathbf{A}_1, \mathbf{A}_2) \triangleq \|\mathbf{A}_1\mathbf{A}_1^\dagger - \mathbf{A}_2\mathbf{A}_2^\dagger\|_2^2$. The codebook sizes k_F and k_B are chosen from 4, 16 or 64 which correspond to 2, 4 or 6 required bits. For all simulations, the codewords $\mathbf{F}_i$ and $\mathbf{B}_j$ are selected independently at the relay and the destination in accordance with the criteria $\mathbf{F}_i = \arg\min_i \ \text{tr}\left(\sigma_{n_1}^2(\mathbf{F}_i^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F}_i + \frac{\sigma_{n_1}^2}{\sigma_x^2}\mathbf{I}_{N_s})^{-1}\right)$ and $\mathbf{B}_j = \arg\min_j \ \text{tr}\left(\sigma_{n_2}^2(\frac{P_R}{\sigma_x^2 N_s}\mathbf{B}_j^\dagger\mathbf{G}^\dagger\mathbf{G}\mathbf{B}_j + \frac{\sigma_{n_2}^2}{\sigma_x^2}\mathbf{I}_{N_s})^{-1}\right)$, respectively. Figure 6 exhibits the MSE performance of the Grassmannian precoding designs both at the source and the relay. As the plot shows, we can get very close to the optimal scheme SRD-MV with only a few feedback bits. Also, the MSE and BER performance of the relay only precoding design with quantization can be seen from Figures 7 and 8. The system with only 6 feedback bits almost achieves the performance within 1dB of the optimal unquantized schemes RD-MV and RD-NC at a BER of 10^{-4}. Although not presented in this paper, the proposed MMSE based quantizing schemes with D-Sub also have no performance loss when only the local CSI is allowed at the destination.

VIII. Conclusion

In this paper, we have shown that from the MMSE point of view, the optimal relay filter is composed of two individual receive and transmit Wiener filters and then the overall error covariance matrix can be decomposed into a sum of two individual error covariance matrices. From this approach, we have easily formulated and solved the two optimal joint MMSE schemes for an relay-destination and a source-relay-destination joint design. Also for the limited feedback scenario, we have addressed the codebook design and the selection criteria for multiple streams precoding design in the relaying system. Finally, we have illustrated that for all our MMSE designs, the source or the destination only needs to know the CSI corresponding to its own link such as the source-to-relay or the relay-to-destination. The analytical results are verified by comparing the performance of the optimal and suboptimal schemes under different scenarios.

Appendix A

Optimal Relay Transceiver

We first set the derivations of the cost function (5) to zero as [24]

$$\begin{aligned}\frac{\partial \mathcal{C}}{\partial \mathbf{Q}^*} &= \gamma^{-2}\sigma_x^2\mathbf{G}^\dagger\mathbf{W}_D^\dagger\mathbf{W}_D\mathbf{G}\mathbf{Q}\mathbf{H}\mathbf{F}\mathbf{F}^\dagger\mathbf{H}^\dagger - \sigma_x^2\gamma^{-1}\mathbf{G}^\dagger\mathbf{W}_D^\dagger\mathbf{F}^\dagger\mathbf{H}^\dagger \\ &\quad +\sigma_{n_1}^2\gamma^{-2}\mathbf{G}^\dagger\mathbf{W}_D^\dagger\mathbf{W}_D\mathbf{G}\mathbf{Q} + \lambda\sigma_x^2\mathbf{Q}\mathbf{H}\mathbf{F}\mathbf{F}^\dagger\mathbf{H}^\dagger + \sigma_{n_1}^2\lambda\mathbf{Q} = 0 \qquad (22)\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \mathcal{C}}{\partial \gamma} &= \mathrm{tr}(\sigma_x^2\mathbf{W}_D\mathbf{G}\mathbf{Q}(\rho^2\mathbf{H}\mathbf{F}\mathbf{F}^\dagger\mathbf{H}^\dagger + \sigma_{n_1}^2\mathbf{I}_{N_r})\mathbf{Q}^\dagger\mathbf{G}^\dagger\mathbf{W}_D^\dagger + \sigma_{n_2}^2\mathbf{W}_D\mathbf{W}_D^\dagger) \\ &\quad -\mathrm{tr}\left(\gamma\sigma_x^2\mathbf{W}_D\mathbf{G}\mathbf{Q}\mathbf{H}\mathbf{F}\right) = 0. \qquad (23)\end{aligned}$$

These results can be verified using some rules such as $d\mathrm{tr}\,(\mathbf{Y}) = \mathrm{tr}\,(d\mathbf{Y})$, $\mathrm{vec}(d\mathbf{X}) = d\mathrm{vec}(\mathbf{X})$, $\mathrm{tr}(\mathbf{X}^T\mathbf{Y}) = \mathrm{vec}(\mathbf{X})^T\mathrm{vec}(\mathbf{Y})$ (For detail, see [28]). When deriving (23), we have used a fact that $\mathrm{tr}\,(\Re\{\sigma_x^2\mathbf{W}_D\mathbf{G}\mathbf{Q}\mathbf{H}\mathbf{F}\}) = \mathrm{tr}\,(\sigma_x^2\mathbf{W}_D\mathbf{G}\mathbf{Q}\mathbf{H}\mathbf{F})$.

Combining the power constraint in (4) and (22), we obtain $\hat{\mathbf{Q}} = \gamma\tilde{\mathbf{Q}}$ with

$$\tilde{\mathbf{Q}} = (\mathbf{G}^\dagger\mathbf{W}_D^\dagger\mathbf{W}_D\mathbf{G} + \lambda\gamma^2\mathbf{I}_{N_r})^{-1}(\mathbf{G}^\dagger\mathbf{W}_D^\dagger\sigma_x^2\mathbf{F}^\dagger\mathbf{H}^\dagger)(\sigma_x^2\mathbf{H}\mathbf{F}\mathbf{F}^\dagger\mathbf{H}^\dagger + \sigma_{n_1}^2\mathbf{I}_{N_r})^{-1}, \tag{24}$$

where $\gamma = \sqrt{P_R/\mathrm{tr}\left(\tilde{\mathbf{Q}}\left(\sigma_x^2\mathbf{H}\mathbf{F}\mathbf{F}^\dagger\mathbf{H}^\dagger + \sigma_{n_1}^2\mathbf{I}_{N_r}\right)\tilde{\mathbf{Q}}^\dagger\right)}$. It should be noted that directly evaluating $\tilde{\mathbf{Q}}$ and γ is formidable since $\tilde{\mathbf{Q}}$ and the scalar value γ are inter-related. Letting $\mu = \lambda\gamma^2$, we can overcome this difficulty as in [19].

Defining $\mathcal{A}$ and $\mathcal{D}$ as $\mathcal{A} \triangleq (\mathbf{G}^\dagger\mathbf{W}_D^\dagger\mathbf{W}_D\mathbf{G} + \mu\mathbf{I}_{N_r})^{-1}$ and $\mathcal{D} \triangleq (\sigma_x^2\mathbf{H}\mathbf{F}\mathbf{F}^\dagger\mathbf{H}^\dagger + \sigma_{n_1}^2\mathbf{I}_{N_r})^{-1}$, respectively, the last term of equation (23) can be modified as

$$\begin{aligned}
\mathrm{tr}\left(\gamma^2\sigma_x^2\mathbf{W}_D\mathbf{G}\tilde{\mathbf{Q}}\mathbf{H}\mathbf{F}\right) &= \mathrm{tr}\left(\gamma^2\sigma_x^2\mathbf{W}_D\mathbf{G}\mathcal{A}(\mathbf{G}^\dagger\mathbf{W}_D^\dagger\sigma_x^2\mathbf{F}^\dagger\mathbf{H}^\dagger)\mathcal{D}\mathbf{H}\mathbf{F}\right)\\
&= \mathrm{tr}\left(\gamma^2(\mathbf{G}^\dagger\mathbf{W}_D^\dagger\sigma_x^2\mathbf{F}^\dagger\mathbf{H}^\dagger)\mathcal{D}\mathbf{H}\mathbf{F}\sigma_x^2\mathbf{W}_D\mathbf{G}\mathcal{A}\right)\\
&= \mathrm{tr}\left(\gamma^2\mathcal{A}^{-1}\mathcal{A}(\mathbf{G}^\dagger\mathbf{W}_D^\dagger\sigma_x^2\mathbf{F}^\dagger\mathbf{H}^\dagger)\mathcal{D}\mathcal{D}^{-1}\mathcal{D}\mathbf{H}\mathbf{F}\sigma_x^2\mathbf{W}_D\mathbf{G}\mathcal{A}\right)\\
&= \mathrm{tr}\left(\gamma^2\mathcal{A}^{-1}\tilde{\mathbf{Q}}\mathcal{D}^{-1}\tilde{\mathbf{Q}}^\dagger\right)\\
&= \mathrm{tr}\left(\gamma^2(\mathbf{G}^\dagger\mathbf{W}_D^\dagger\mathbf{W}_D\mathbf{G} + \mu\mathbf{I}_{N_r})(\tilde{\mathbf{Q}}(\sigma_x^2\mathbf{H}\mathbf{F}\mathbf{F}^\dagger\mathbf{H}^\dagger + \sigma_{n_1}^2\mathbf{I}_{N_r})\tilde{\mathbf{Q}}^\dagger)\right).
\end{aligned} \tag{25}$$

Substituting this result in (25) into equation (23), we compute $\mu = \mathrm{tr}\left(\sigma_{n_2}^2\mathbf{W}_D\mathbf{W}_D^\dagger\right)/P_R$. Finally, using the matrix inversion lemma, the closed form solution for the optimal relay transceiver $\hat{\mathbf{Q}}$ is given as

$$\hat{\mathbf{Q}} = \gamma\mathbf{G}^\dagger\mathbf{W}_D^\dagger\left(\mathbf{W}_D\mathbf{G}\mathbf{G}^\dagger\mathbf{W}_D^\dagger + \frac{\mathrm{tr}(\sigma_{n_2}^2\mathbf{W}_D\mathbf{W}_D^\dagger)}{P_R}\mathbf{I}_{N_r}\right)^{-1}\left(\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F} + \frac{\sigma_{n_1}^2}{\sigma_x^2}\mathbf{I}_{N_s}\right)^{-1}\mathbf{F}^\dagger\mathbf{H}^\dagger.$$

Appendix B

Decomposition of $\mathbf{R}_E$

By Lemma 1, we can rewrite the error covariance matrix $\mathbf{R}_E$ in (3) as

$$\mathbf{R}_E = \left(\gamma^2\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{L}_R^\dagger\mathbf{B}^\dagger\mathbf{G}^\dagger\mathbf{R}_n^{-1}\mathbf{G}\mathbf{B}\mathbf{L}_R\mathbf{H}\mathbf{F} + (1/\sigma_x^2)\mathbf{I}_{N_s}\right)^{-1}, \tag{26}$$

where $\mathbf{R}_n = \gamma^2\sigma_{n_1}^2\mathbf{G}\mathbf{B}\mathbf{L}_R\mathbf{L}_R^\dagger\mathbf{B}^\dagger\mathbf{G}^\dagger + \sigma_{n_2}^2\mathbf{I}_{N_d}$. Let the effective channel $\mathcal{H}$ of the overall source-to-destination link be $\mathcal{H} = \gamma\mathbf{G}\mathbf{B}\mathbf{L}_R\mathbf{H}\mathbf{F}$. Then using the matrix inversion lemma, the $\mathbf{R}_E$ in (26) becomes

$$\begin{aligned}\mathbf{R}_E &= \sigma_x^2\mathbf{I}_{N_s} - \sigma_x^4\mathcal{H}^\dagger\left(\mathbf{R}_n + \sigma_x^2\mathcal{H}\mathcal{H}^\dagger\right)^{-1}\mathcal{H} \\ &= \sigma_x^2\mathbf{I}_{N_s} - \sigma_x^4\mathcal{H}^\dagger\left(\gamma^2\mathbf{G}\mathbf{B}\mathbf{R}_y\mathbf{B}^\dagger\mathbf{G}^\dagger + \sigma_{n_2}^2\mathbf{I}_{N_d}\right)^{-1}\mathcal{H}, \end{aligned}\tag{27}$$

where $\mathbf{R}_y$ is calculated as

$$\begin{aligned}\mathbf{R}_y &= \mathbf{L}_R(\sigma_x^2\mathbf{H}\mathbf{F}\mathbf{F}^\dagger\mathbf{H} + \sigma_{n_1}^2\mathbf{I}_{N_r})\mathbf{L}_R^\dagger \\ &= \sigma_x^2\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F}\left(\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F} + (\sigma_{n_1}^2/\sigma_x^2)\mathbf{I}_{N_s}\right)^{-1}. \end{aligned}\tag{28}$$

Applying the matrix inversion lemma again on equation (27) and using simple calculations, we can obtain

$$\begin{aligned}\mathbf{R}_E &= \sigma_x^2\mathbf{I}_{N_s} - \frac{\sigma_x^4}{\sigma_{n_2}^2}\mathcal{H}^\dagger\mathcal{H} + \frac{\sigma_x^4\gamma^2}{\sigma_{n_2}^4}\mathcal{H}^\dagger\mathbf{G}\mathbf{B}\left(\frac{\gamma^2}{\sigma_{n_2}^2}\mathbf{B}^\dagger\mathbf{G}^\dagger\mathbf{G}\mathbf{B} + \mathbf{R}_y^{-1}\right)^{-1}\mathbf{B}^\dagger\mathbf{G}^\dagger\mathcal{H} && (29)\\ &= \sigma_x^2\mathbf{I}_{N_s} - \frac{\sigma_x^4}{\sigma_{n_2}^2}\mathcal{H}^\dagger\mathcal{H} + \frac{\sigma_x^4\gamma}{\sigma_{n_2}^2}\mathcal{H}^\dagger\mathbf{G}\mathbf{B}\left(\frac{\gamma^2}{\sigma_{n_2}^2}\mathbf{B}^\dagger\mathbf{G}^\dagger\mathbf{G}\mathbf{B} + \mathbf{R}_y^{-1}\right)^{-1} \\ &\quad\times\left(\frac{\gamma^2}{\sigma_{n_2}^2}\mathbf{B}^\dagger\mathbf{G}^\dagger\mathbf{G}\mathbf{B} + \mathbf{R}_y^{-1} - \mathbf{R}_y^{-1}\right)\mathbf{L}_R\mathbf{H}\mathbf{F} \\ &= \sigma_x^2\mathbf{I}_{N_s} - \frac{\sigma_x^4}{\sigma_{n_2}^2}\mathcal{H}^\dagger\mathcal{H} + \frac{\sigma_x^4}{\sigma_{n_2}^2}\mathcal{H}^\dagger\mathcal{H} - \frac{\sigma_x^2\gamma}{\sigma_{n_2}^2}\mathcal{H}^\dagger\mathbf{G}\mathbf{B}\left(\frac{\gamma^2}{\sigma_{n_2}^2}\mathbf{B}^\dagger\mathbf{G}^\dagger\mathbf{G}\mathbf{B} + \mathbf{R}_y^{-1}\right)^{-1}\mathcal{J} && (30)\\ &= \sigma_x^2\mathbf{I}_{N_s} - \sigma_x^2\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{L}_R^\dagger\left(\frac{\gamma^2}{\sigma_{n_2}^2}\mathbf{B}^\dagger\mathbf{G}^\dagger\mathbf{G}\mathbf{B} + \mathbf{R}_y^{-1} - \mathbf{R}_y^{-1}\right)\left(\frac{\gamma^2}{\sigma_{n_2}^2}\mathbf{B}^\dagger\mathbf{G}^\dagger\mathbf{G}\mathbf{B} + \mathbf{R}_y^{-1}\right)^{-1}\mathcal{J} \\ &= \sigma_x^2\mathbf{I}_{N_s} - \sigma_x^2\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{L}_R^\dagger\mathcal{J} + \sigma_{n_2}^2\mathcal{J}^\dagger\left(\gamma^2\mathbf{B}^\dagger\mathbf{G}^\dagger\mathbf{G}\mathbf{B} + \sigma_{n_2}^2\mathbf{R}_y^{-1}\right)^{-1}\mathcal{J}\end{aligned}$$

where we have used a fact that $\frac{\gamma^2}{\sigma_{n_2}^2}\mathbf{B}^\dagger\mathbf{G}^\dagger\mathbf{G}\mathbf{B} = \frac{\gamma^2}{\sigma_{n_2}^2}\mathbf{B}^\dagger\mathbf{G}^\dagger\mathbf{G}\mathbf{B} + \mathbf{R}_y^{-1} - \mathbf{R}_y^{-1}$ in the last term of (29) and (30), and $\mathcal{J}$ is defined as $\mathcal{J} \triangleq \sigma_x^2\mathbf{R}_y^{-1}\mathbf{L}_R\mathbf{H}\mathbf{F}$. Note that $\mathcal{J}$ is equivalent to an identity matrix $\mathbf{I}_{N_s}$. Then finally it follows

$$\begin{aligned}\mathbf{R}_E &= \sigma_x^2\mathbf{I}_{N_s} - \sigma_x^2\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{L}_R^\dagger + \sigma_{n_2}^2\left(\gamma^2\mathbf{B}^\dagger\mathbf{G}^\dagger\mathbf{G}\mathbf{B} + \sigma_{n_2}^2\mathbf{R}_y^{-1}\right)^{-1} \\ &= \sigma_{n_1}^2\left(\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F} + \frac{\sigma_{n_1}^2}{\sigma_x^2}\mathbf{I}_{N_s}\right)^{-1} + \sigma_{n_2}^2\left(\gamma^2\mathbf{B}^\dagger\mathbf{G}^\dagger\mathbf{G}\mathbf{B} + \sigma_{n_2}^2\mathbf{R}_y^{-1}\right)^{-1}.\end{aligned}$$

APPENDIX C

SIMPLIFICATION OF D-WF

Using the matrix inversion lemma, the original D-WF in (2) can be reformed as

$$\hat{\mathbf{W}}_D = \gamma^2\sigma_x^2\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{L}_R^\dagger\mathbf{B}^\dagger\mathbf{G}^\dagger(\gamma^2\sigma_x^2\mathbf{G}\mathbf{B}\mathbf{L}_R\mathbf{H}\mathbf{F}\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{L}_R^\dagger\mathbf{B}^\dagger\mathbf{G}^\dagger + \mathbf{R}_n)^{-1}.$$

Then after some manipulations, we have

$$\begin{aligned}
\hat{\mathbf{W}}_D &= \gamma^2\sigma_x^2\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{L}_R^\dagger\mathbf{B}^\dagger\mathbf{G}^\dagger(\gamma^2\mathbf{G}\mathbf{B}\mathbf{R}_y\mathbf{B}^\dagger\mathbf{G}^\dagger + \sigma_{n_2}^2\mathbf{I}_{N_d})^{-1} \\
&= \gamma^2\sigma_x^2\mathbf{F}^\dagger\mathbf{H}^\dagger\left(\mathbf{H}\mathbf{F}(\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F} + \frac{\sigma_{n_1}^2}{\sigma_x^2}\mathbf{I}_{N_s})^{-1}\right)\mathbf{B}^\dagger\mathbf{G}^\dagger(\gamma^2\mathbf{G}\mathbf{B}\mathbf{R}_y\mathbf{B}^\dagger\mathbf{G}^\dagger + \sigma_{n_2}^2\mathbf{I}_{N_d})^{-1} \\
&= \gamma^2\mathbf{R}_y\mathbf{B}^\dagger\mathbf{G}^\dagger(\gamma^2\mathbf{G}\mathbf{B}\mathbf{R}_y\mathbf{B}^\dagger\mathbf{G}^\dagger + \sigma_{n_2}^2\mathbf{I}_{N_d})^{-1} \\
&= \gamma^2(\gamma^2\mathbf{B}^\dagger\mathbf{G}^\dagger\mathbf{G}\mathbf{B} + \sigma_{n_2}^2\mathbf{R}_y^{-1})^{-1}\mathbf{B}^\dagger\mathbf{G}^\dagger, \qquad (31)
\end{aligned}$$

where we have used the results of (7) and (28).

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TABLE I

COMPARISON OF THE REQUIRED CSI

	Scheme	Source	Relay	Destination
Conventional designs	QoS [16] / MA-MSE [17] / MM-MSE [17]	$\mathbf{H}$, $\mathbf{G}$	$\mathbf{H}$, $\mathbf{G}$	$\mathbf{H}$, $\mathbf{G}$
	RD-NC [14] / MCC [12]	None	$\mathbf{H}$, $\mathbf{G}$	$\mathbf{H}$, $\mathbf{G}$
	R-MMSE [29] / R-ZF [29]	None	$\mathbf{H}$, $\mathbf{G}$	None
Proposed designs w/ D-Sub	SRD-NC / SRD-BNC / SRD-MV	$\mathbf{H}$	$\mathbf{H}$, $\mathbf{G}$	$\mathbf{G}$
	RD-NC / RD-MV	None	$\mathbf{H}$, $\mathbf{G}$	$\widetilde{\mathbf{G}}$
	SRD-Q	$(\log_2 k_F)$ bits	$\mathbf{H}$, $(\log_2 k_B)$ bits	$\mathbf{G}$
	RD-Q	None	$\mathbf{H}$, $(\log_2 k_B)$ bits	$\mathbf{G}$

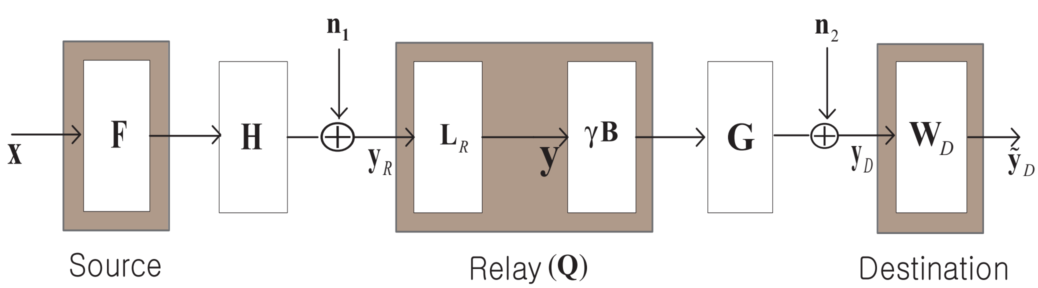

Fig. 1. System description for the non-regenerative MIMO relaying system

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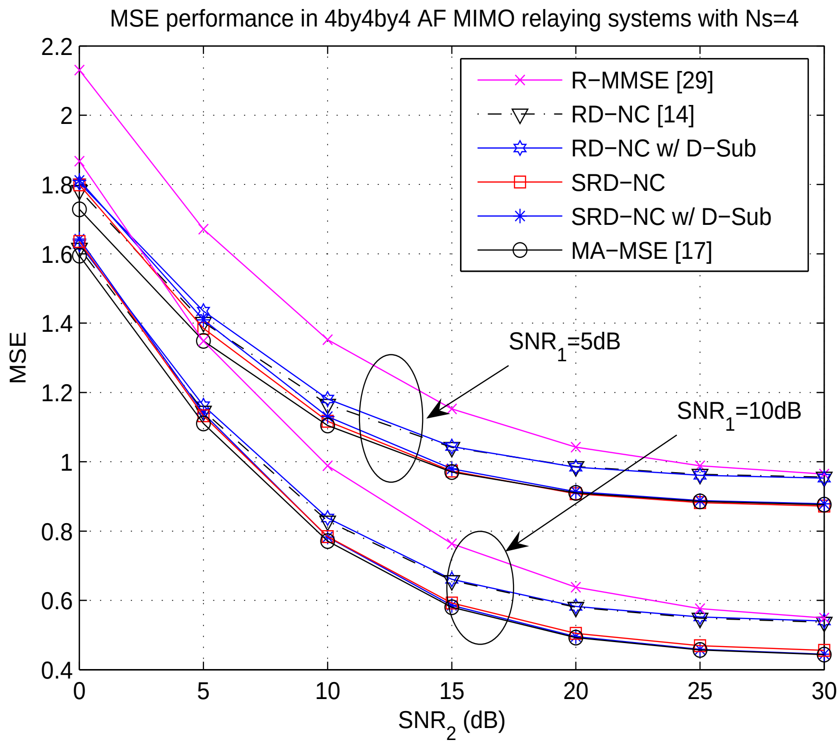

Fig. 2. MSE performance of various designs as a function of SNR_2

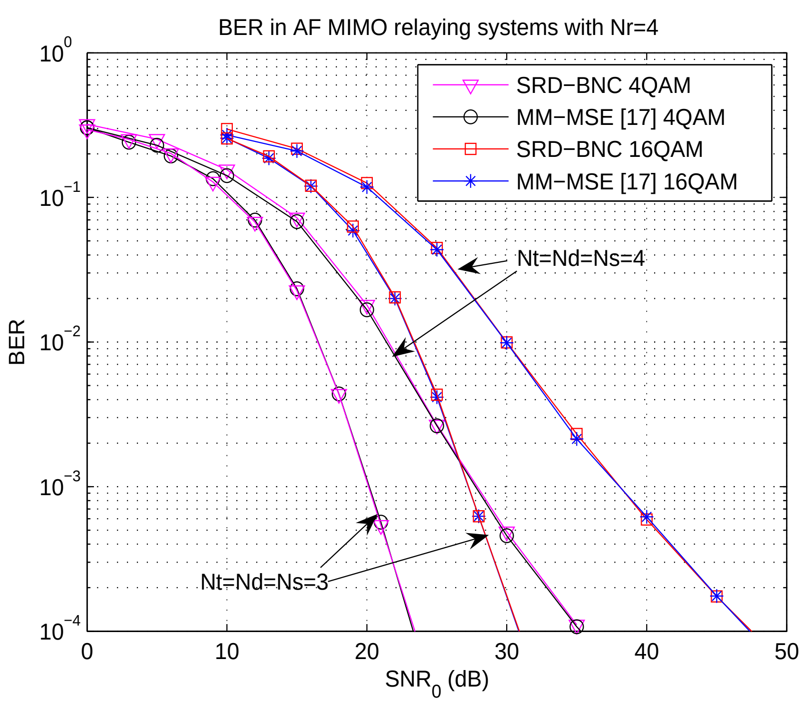

Fig. 3. BER performance comparison as a function of SNR_0

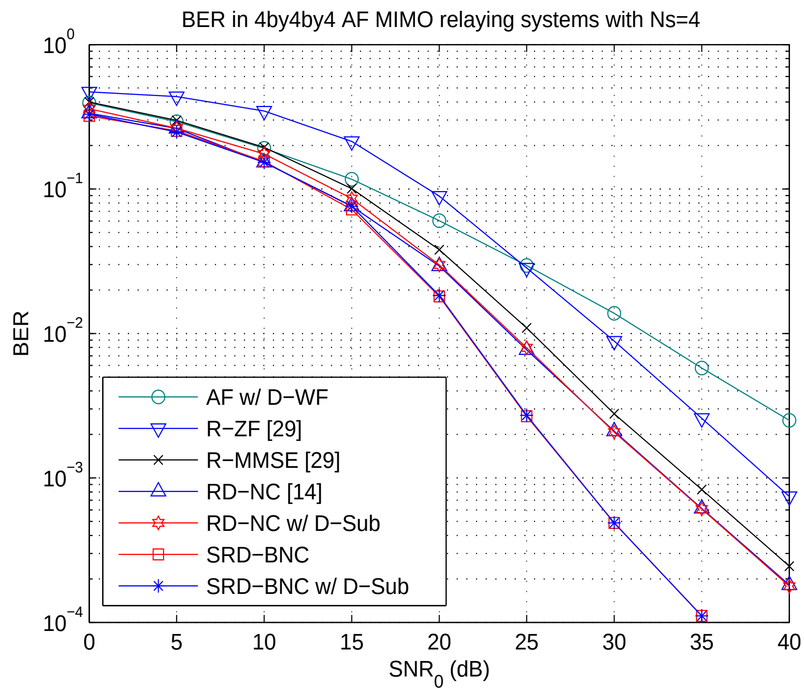

Fig. 4. BER performance comparison as a function of SNR_0 with 4QAM

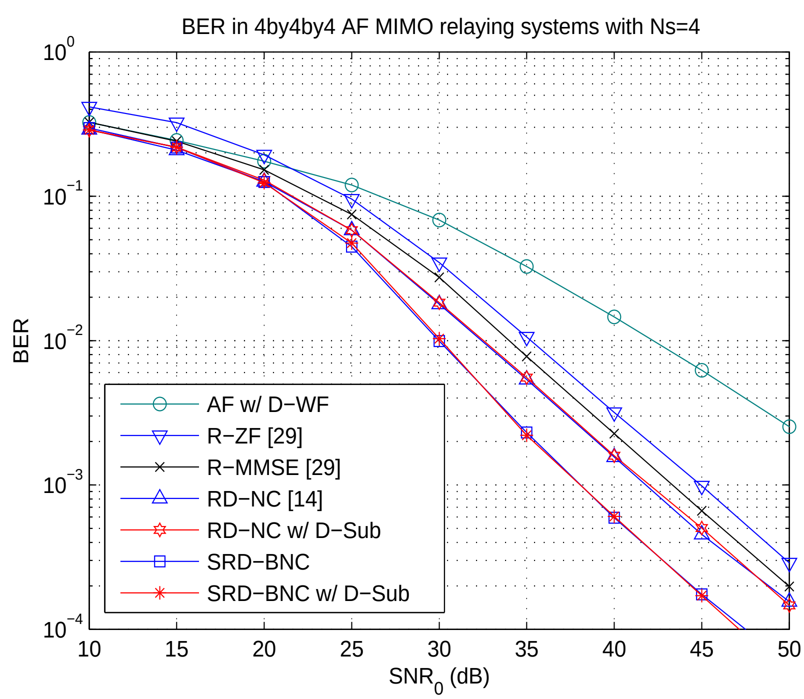

Fig. 5. BER performance comparison as a function of SNR_0 with 16QAM

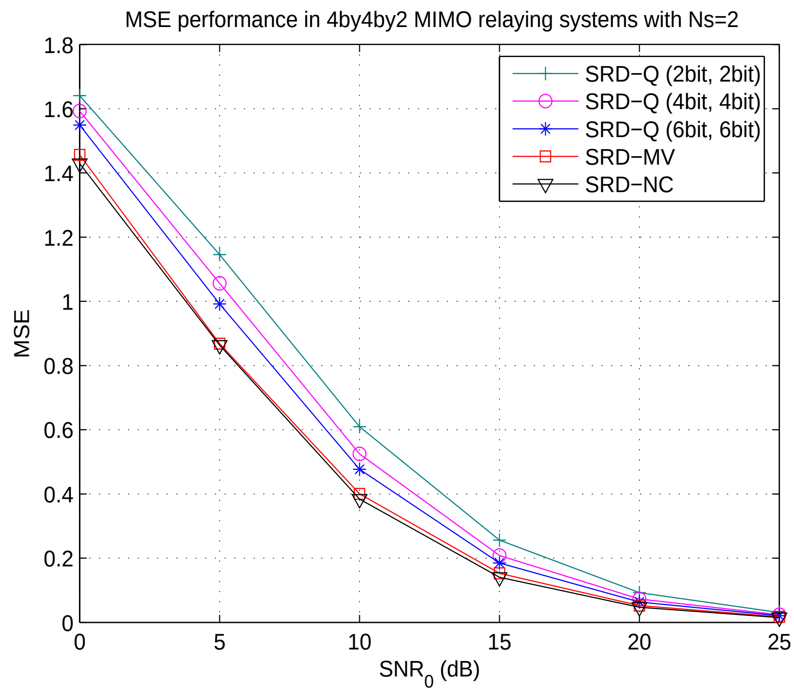

Fig. 6. MSE performance of various designs as a function of SNR_0

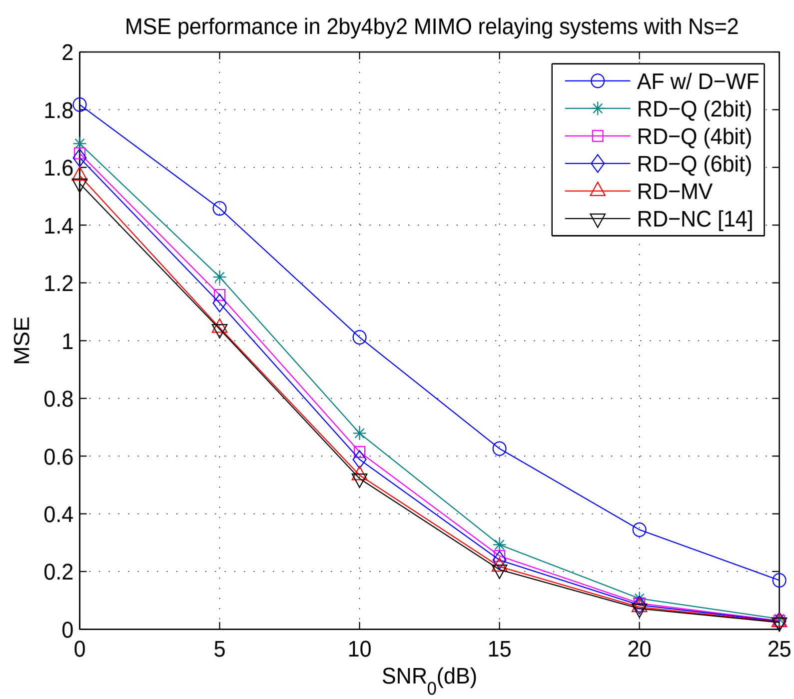

Fig. 7. MSE performance of various designs as a function of SNR_0

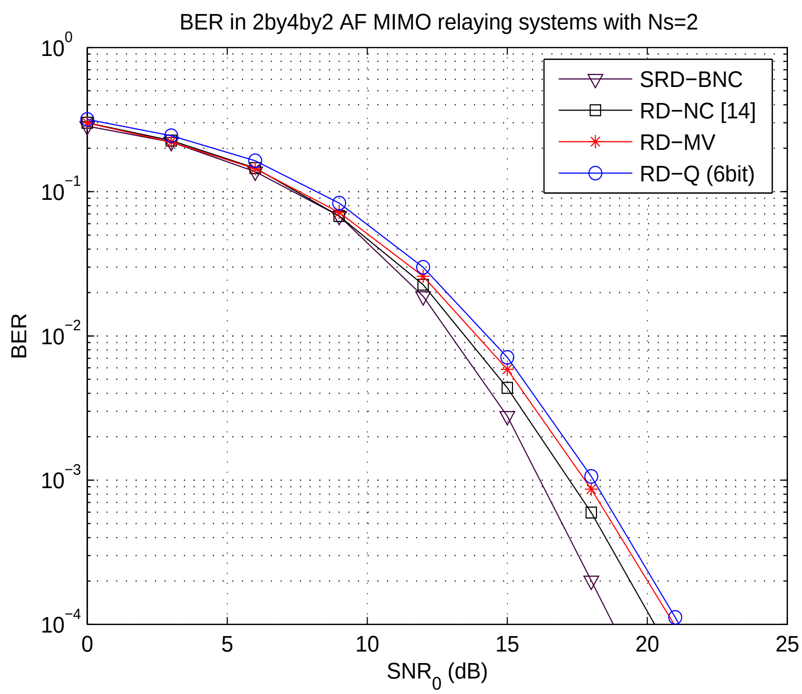

Fig. 8. BER performance of various designs as a function of SNR_0 with 4QAM