Game Theoretical Issues in Optical Networks

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Abstract

In this paper we focus on the problem in optical networks in which selfish or non-cooperative users can configure their communications so as to minimize the cost paid for the service. Such a cost depends on the personal configuration and on the one of the other users. During a series of time steps, at each of which only one user can move to a better configuration, a Nash equilibrium is eventually reached, that is a situation in which no user can select an improved solution and thus is interested in further modifications. In such a setting, the network provider must determine suitable payment functions covering the network costs that induce Nash equilibria with the best possible global performances. We first present results in the classical scenario in which we are interested in optimizing the optical spectrum, that is in minimizing the total number of used wavelengths. We then outline possible settings in which the approach can be eventually applied to minimize the cost of optical routing due to specific hardware components such as ADMs or filters, that are typical examples of expensive elements whose price can be shared among different lightpaths under specific constraints.

I. INTRODUCTION

All-optical networks are widely considered to be the future of the state of the art communication networks due to the possibility of managing thousand of users, covering wide areas and providing a bandwidth which is orders of magnitude faster than traditional networks. Such high performances elect optical as the leading technology in many applications such as video conferencing, scientific visualization and high-speed distributed computing. The key to high speeds in optical networks is to maintain the signal in optical form, thereby avoiding the prohibitive overhead of conversion to and from the electrical form at the intermediate nodes. The high bandwidth provided by optical networks can be partitioned by means of the *wavelength-division multiplexing* (WDM) [6] in order to obtain a large number of parallel high speed channels along a same optical fiber (see [2], [12] for a survey of the main related results).

We study routing problems in optical networks from a non-cooperative point of view, i.e. analyzing a game in which selfish agents want to maximize their benefit. In particular, we are interested in Nash equilibria, i.e. solutions of the games in which no agent gains unilaterally changing its strategy, and, given a social function measuring the social goodness of a solution, in bounding the *price of anarchy* or *coordination ratio* [17], [18], [22], i.e. the ratio between the social cost of the worst Nash equilibrium and the social optimum. Several games have been investigated in the literature [9], [10], [19], [24] and shown to possess pure Nash equilibria or to converge to a pure Nash equilibrium independently from their starting state.

In this paper, we first present results concerning the scenario considered in [4] and [3], in which a service provider has to satisfy a given set of point-to-point communication requests, charging each of them a cost depending on its wavelength and on the wavelengths of the other requests met along its path in the network. Each request is issued by a non-cooperative agent who is interested only in the minimization of his own cost. Under this assumption any request is willing to be rerouted each time it may be served by a cheaper path in the network and the evolution of the network can be modelled as a multi-player game. A routing solution, that is an assignment of paths and colors to the requests, in which no request can lower its cost by choosing a different strategy is a Nash equilibrium.

We then outline other possible settings in which we are interested in the minimization of the network hardware cost due to routing elements. More precisely, we address two types of expensive hardware components: the Add-Drop-Multiplexer (*ADMs*), needed at the endpoints of optical communication paths, and the pass-band *filters*, capable of directing *wavebands* or intervals of contiguous wavelengths. The cost of such hardware components is equally shared among the agents using them, and a Nash equilibrium is a solution in which no agent can decrease its payment by adopting a different routing strategy.

The paper is organized as follows. In the next section we give the basic definitions and notation. In Section III we show the above mentioned results concerning the scenario in which the objective is to minimize the number of used wavelengths. In Section IV we present the other above mentioned settings coping with the minimization of hardware components. Finally, in Section V, we give some conclusive remarks and discuss some open questions.

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II. The Model

We model an optical network as an undirected graph $G = (V, E)$ where nodes in V represent sites and undirected edges in E bidirectional optical fiber links between the sites.

Given any two nodes $x, y \in V$, we denote a communication request between x and y as $\{x, y\}$. A communication instance in G is a multiset of requests I eventually containing multiple requests between the same pairs of nodes. A path system P for an instance I in G is a set of paths containing a distinguished simple connecting path in G for each request in I. A solution $\mathcal{R}(G, I)$ for an instance I in G, $\mathcal{R}$ for short, is a pair $(P_{\mathcal{R}}, c_{\mathcal{R}})$ in which $P_{\mathcal{R}}$ is a path system for I and $c_{\mathcal{R}} : I \rightarrow W$ (with $W = I\!N^+$ being the set of wavelengths) is a function associating a wavelength or color to each request in I. A point-to-point communication requires to establish a uniquely colored path between the two nodes whose color is different from the colors of all the other paths sharing one of its edges.

Let $\omega_{\mathcal{R}}(G, I)$ be the number of colors used by the routing $\mathcal{R}$ for I in G and $\omega(G, I) = \min_{\mathcal{R}} \omega_{\mathcal{R}}(G, I)$ be the minimum number of colors that can be used by any routing for I.

In order to model our non-cooperative environment, we assume that each communication request $\{x, y\} \in I$ is issued and handled by an agent α that for the sake of simplicity we consider as coincident with the request, that is $\alpha = \{x, y\}$. A payment function $price_{\mathcal{R}} : I \rightarrow I\!R^+$ is a function associating to each agent $\alpha \in I$ the price he has to pay to the network provider in order to obtain the asked service if the routing $\mathcal{R}$ is adopted. Let $price$ denote the collection of the functions $price_{\mathcal{R}}$ for all the possible routings $\mathcal{R}$. In the first considered scenario the payment function is properly chosen by the network provider with the purpose of optimizing the optical spectrum, while in the other ones (minimization of the number of hardware components) it is given by equally sharing the costs of the used ADMs or filters.

A routing $\mathcal{R}$ is at Nash equilibrium if and only if for any agent α and routing $\mathcal{R}'$ differing from $\mathcal{R}$ only for the path and/or the color associated to α, it holds $price_{\mathcal{R}}(\alpha) \leq price_{\mathcal{R}'}(\alpha)$.

A game $\mathcal{G} = (G, I, price)$ among the $|I|$ agents belonging to I on the network G induced by the collection of pricing functions $price$ has $P_\alpha \times W$ as the set of strategies for each agent α, where P_α is the set of connecting paths for agent α. Denoted as $\mathcal{N}$ the set of the routings at Nash equilibrium, the coordination ratio or price of anarchy ρ of the game $\mathcal{G}$ is defined as the ratio between the social cost of the worst Nash equilibrium and the social optimum. For instance, in the classical scenario in which the social function measures the number of used wavelengths, $\rho(\mathcal{G}) = sup_{\mathcal{R} \in \mathcal{N}} \frac{\omega_{\mathcal{R}}(G,I)}{\omega(G,I)}$.

A game $\mathcal{G}$ is said to be convergent if, starting from a generic configuration, and letting at each stage an agent to move to a better configuration, a Nash equilibrium is always reached. Notice that, conversely, a non convergent game might admit a Nash equilibrium.

III. Optical Spectrum Minimization

In order to represent the increasing cost incurred by the network provider to implement a routing using up to a given wavelength and to give to our payment functions a higher degree of generality, we assume the existence of a non-decreasing function $f : W \rightarrow I\!R^+$ associating a (positive) cost to every color.

Let the function $\sigma_{\mathcal{R}} : E \rightarrow 2^W$ associating to every edge $e \in E$ the set of the wavelengths currently used along e be the edge state of the network G induced by a routing $\mathcal{R}$ for I. We first propose suitable cost functions defined on the edges that will be used as building blocks for the definition of the agents' payment functions:

- $col(e, \alpha) = f(c(\alpha))$: the amount charged to α on the edge e is the cost, according to f, of the color he uses.
- $max(e, \alpha) = \max_{k \in \sigma_{\mathcal{R}}(e)} f(k)$: the amount charged to α on the edge e is the cost of the highest color used along e (considering also the other agents).
- $sum(e, \alpha) = \sum_{k \in \sigma_{\mathcal{R}}(e)} f(k)$: the amount charged to α on the edge e is the sum of the costs of all the colors used along e.
- $avmax(e, \alpha) = \max_{k \in \sigma_{\mathcal{R}}(e)} \frac{f(k)}{|\sigma_{\mathcal{R}}(e)|}$: the amount charged to α on the edge e is the cost of the highest color used along e, averaged or shared among all the agents traversing e.
- $avsum(e, \alpha) = \sum_{k \in \sigma_{\mathcal{R}}(e)} \frac{f(k)}{|\sigma_{\mathcal{R}}(e)|}$: the amount charged to α on the edge e is the sum of the costs of all the colors used along e, averaged on all the agents traversing e.

Starting from any edge cost function $cost$, it is possible to define the following payment functions:

- $max-cost(\alpha) = \max_{e \in p(\alpha)} cost(e, \alpha)$: the price asked to α is the maximum cost, according to $cost$, of an edge used by α.
- $sum-cost(\alpha) = \sum_{e \in p(\alpha)} cost(e, \alpha)$: the price asked to α is the sum of the costs of the edges used by α.

The combination of the introduced edge cost functions with the above two strategies, that is maximization or summation, gives rise to ten possible payment functions. In all the cases, since the function f is non decreasing, agents have an incentive to choose small colors so as to possibly minimize the overall number of used colors.

Unfortunately, the results of [4], [3] show that these payment functions either are not convergent or yield the worst possible price of anarchy, i.e. they converge to an equilibrium in which each agent uses a different wavelength. More precisely, it is possible to prove the following theorem.

Theorem 1: The functions $max - col$, $max - max$, $sum - col$ and $max - sum$ are convergent, but induce games $\mathcal{G} = (G, I, price)$ with price of anarchy $\rho = \frac{|I|}{w(G,I)}$; the function $sum - sum$ is in general non convergent, even if the existence of Nash equilibria in the corresponding games is an open question. Finally, no Nash equilibria exist for the games induced by the payment functions

1) $sum - max$ when the pricing function f is unbounded;
2) $max - avmax$ and $sum - avmax$ when f is such that $\exists k : f(k) > 2f(1)$;
3) $max - avsum$ and $sum - avsum$ when the f is such that $\exists k : f(k) > f(1)$, that is f is non constant.

Since the results obtained for generic networks are not fully satisfactory, it is worth considering networks having specific topologies, like chains (nodes connected along a line), rings (cycles of nodes) and trees.

Let us first consider the payment function $price(\alpha) = c(\alpha)$; it induces a game in which a routing $\mathcal{R}$ at Nash equilibrium can be seen as a solution of the classical ***First-Fit*** algorithm for the all-optical routing problem that assigns to each request the smallest available color. In particular, such a solution is the one returned by ***First-Fit*** when requests are considered in non decreasing order of color in $\mathcal{R}$, which uses at most $25.72\omega(G, I)$ colors in chains [16] and $\mathcal{O}((\log |I|)\omega(G, I))$ ones in trees [1]. Concerning rings, by properly inducing the agents not to use a distinguished chosen edge, the First-Fit arguments can be applied on the resulting induced chain. Thus, is possible to prove the following theorem.

Theorem 2: There exist payment functions inducing converging games with a price of anarchy 25.72 in chains, 51.44 in rings and $\mathcal{O}(\log |I|)$ in trees, all converging in $\omega_{\mathcal{R}}(G, I)^2$ steps from any initial routing $\mathcal{R}$.

Finally, it is possible to improve the above results for rings and chains by forcing the agents to simulate the behavior of the online algorithm proposed by Slusarek [23]. In particular, the following theorem holds.

Theorem 3: There exists payment functions inducing a converging games having price of anarchy 6 in rings and 3 in chains.

IV. Hardware Components Minimization

When the various parameters comprising the switching mechanism in optical networks became clearer, the focus of studies shifted, and today a large portion of research concentrates with the total hardware cost. In this section we present two scenarios according to the specifically addressed hardware components.

A. ADMs Components

Add-Drop-Multiplexers (ADMs) [13], [8], [7] are costly hardware components that convert the signal from optical to electronic form and viceversa. The key point here is that each lightpath uses two ADMs, one at each endpoint. If two adjacent (having a common endpoint) lightpaths are assigned the same wavelength, then they can use the same ADM. An ADM may be shared by at most two lightpaths.

In studying the ADMs cost, the issue of *grooming* [14] became central. This problem stems from the fact that the network usually supports traffic that is at rates which are lower than the full wavelength capacity, and therefore the network operator has to be able to put together (= groom) low-capacity demands into the high capacity fibers. In graph-theoretical terms, this can viewed as assigning colors to the lightpaths so that at most g of them (g being the *grooming factor*) can share one edge. Each lightpath still uses two ADMs, one at each endpoint, but g lightpaths with the same wavelength terminating at a given node through the same incoming edge can all use the same ADM (thus saving $g - 1$ ADMs). Moreover, all the same colored paths ending at the node through two given incident edges can share the same ADM.

In the game induced by this scenario, each agent has to pay for the ADMs it uses, eventually equally sharing their cost if they are also used by other agents. The social cost is given by the total number of ADMs used in the network.

B. Filters Components

Filters [11], [21] are costly hardware components that are located in the sites of the network in order to allow or not allow the forwarding of a signal with a particular wavelength through one of the other fiber link connected to the site.

In each site of the network there is one set of filters at each incoming fiber. Each filter has an interval of wavelengths or colors (*waveband*) and a *destination edge* associated to it. Since filters are of *band-pass* type, when a filter is reached by a signal, it forwards the signal through its destination edge if and only if the color of the signal is inside the interval of colors associated with the filter.

Similarly to the precedent scenario, in the induced game, each agent has to pay for the filters it uses, eventually equally sharing their cost if they are also used by other agents. The social cost is given by the total number of filters used in the network.

V. Conclusion and Open Problems

We have described different scenarios for game theoretical issues in optical networks. For the optical spectrum minimization scenario we have presented results about the convergence to Nash equilibria and the price of anarchy. For the hardware cost minimization scenarios, we have outlined worth investigating issues which are current object of our research.

Moreover, observe that payments must be computed by agents in order to establish their costs in a strongly distributed non-cooperative environment. Thus, a critical issue is the one of determining the achievable price of anarchy according to the level information they have about the network, that is to the pricing functions they can evaluate. In fact, the knowledge about the current state of the network can be limited by technological constraints as well as privacy policies carried out by the service provider or simply enforced by the law, so that not all pricing mechanisms might be feasible.

Finally, recent works [20], [15], [5] focused on the speed of convergence to Nash equilibria and on the social cost obtained after a limited number of selfish moves, not necessary yielding a Nash equilibrium. It would be interesting to pursue this research direction also with respect to the optical scenarios presented in this paper.

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