MMSE DETECTION OF MULTI-CARRIER CDMA

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***Abstract*- Minimum mean-squared error (MMSE) detection of multi-carrier code-division multiple-access (CDMA) signals is investigated in this paper. The performance of two different design strategies for MMSE detection is compared. In one case, the MMSE filters are designed separately for each carrier while in the other case the optimization of the filters is done jointly. Naturally, the joint optimization produces a better receiver, but the difference in performance is shown to be substantial. A mechanism is then developed to track the channel fading parameters for all the users' signals so that joint optimization of the receiver filters is possible in a time-varying channel. Simulation results show that the performance of this receiver is close to ideal theoretical results for moderate vehicle speeds, those for which the normalized Doppler rate is below one percent.**

I. INTRODUCTION

In this paper we investigate the use of minimum mean-squared error (MMSE) detection techniques for a multi-carrier code-division multiple-access (CDMA) system. Our motivation for this work is the results of previous studies which have shown that practical implementation of the MMSE receiver can be problematic in a frequency-selective fading channel [1]-[3]. In a frequency-selective channel, the dimensionality of each interfering user's signal can be quite large unless the receiver is able to accurately track the channel fading parameters for *all* of the users' signals. This would be done implicitly with an adaptive implementation of the MMSE receiver, but studies have shown that typical adaptive algorithms are not able to track the changing channel conditions except at very low vehicle speeds [1]. The alternative would be to explicitly track the channel conditions and then use that information to form the MMSE receiver filter. This approach would work better at higher vehicle speeds but at the cost of much greater complexity.

In a multi-carrier CDMA system, the number of carriers is typically chosen to be large enough so that the signal on each subcarrier is propagated through a channel which behaves in a non-selective manner [4]. By designing an MMSE receiver for each subcarrier, it seems that we could circumvent the problems encountered on a frequency-selective channel. However, in this paper we show that such an approach would not be fruitful and will lead to severe degradation in performance. We then design an MMSE receiver which uses explicit channel tracking for all users' signals. The receiver along with the channel tracking mechanism is shown to perform close to the level promised by the ideal MMSE receiver.

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II. IDEAL MMSE PERFORMANCE OF MULTI-CARRIER CDMA

In this section, we will investigate the performance of a multi-carrier CDMA system on a Rayleigh fading channel. In a multi-carrier system, an information symbol is transmitted simultaneously as a CDMA signal on each carrier, so as to realize diversity in a fading channel. The received signal on the system's reverse link on the *m*th carrier is given by

$$r_m(t) = Re\left[\sum_{i=-\infty}^{\infty}\sum_{k=1}^{K}\sqrt{\frac{2P_k}{M}}\gamma_{k,m}(i)d_k(i)c_{k,m}(t-iT_b-\tau_k)e^{j\omega_m t}\right] + n_m(t) \quad (1)$$

where K is the number of CDMA users, M is the number of carriers, T_b is the bit time, $\tau_k \in (0, T_b)$ is the *k*th user's delay, ω_m is the carrier frequency of the *m*th carrier, and P_k is the composite average power of the *k*th user, which is divided equally amongst the M carriers. Also, $\gamma_{k,m}(i)$ is the complex Gaussian fading process of the *k*th user on the *m*th carrier during the *i*th bit interval. The fading processes of a given user on different carriers are assumed independent, which requires sufficient frequency spacing between carriers with respect to the channel's coherence bandwidth. The fading processes of different users are taken to be independent for the reverse link of the system. Also in equation (1), $d_k(i)$ is the *i*th data bit of the *k*th user. The spreading waveform of the *k*th user employed on the *m*th carrier is $c_{k,m}(t)$, which spans $(0, T_b)$, and consists of unit-amplitude pulses of duration $MT_c = T_b/(N/M)$, where N is the composite processing gain. In other words, a single-carrier system occupying the same bandwidth as the multi-carrier system would use a spreading waveform with N chips/bit and chips of duration T_c. Finally, $n_m(t)$ is a white Gaussian noise process with spectral height $N_0/2$, appearing on the *m*th carrier's CDMA signal, and the $n_m(t)$ processes on different carriers are independent.

The received signal is processed with a chip-matched filter, which consists of an integrator with duration MT_c. The samples are stored for one bit interval, giving a column vector of length N/M

$$\boldsymbol{r}_m(i) = \sum_{k=1}^{K} \sqrt{\frac{P_k}{P_1}} \gamma_{k,m}(i) \left[d_k(i) \boldsymbol{f}_{k,m} + d_k(i-1) \boldsymbol{g}_{k,m} \right] + \boldsymbol{n}_m(i) \quad (2)$$

where $\boldsymbol{f}_{k,m}$ and $\boldsymbol{g}_{k,m}$ depend on the left- and right-cyclic shifts of $\boldsymbol{c}_{k,m}$, the spreading code of the kth user on the mth carrier, and $\boldsymbol{n}_m(i)$ is a vector of independent complex Gaussian noise samples, with the real and imaginary parts also independent, and each with variance $\sigma^2 = N/(2E_b/N_0)$. Note that matrices and vectors will be written in boldface type throughout this paper.

A block diagram of a general linear receiver is shown in Figure 1. Each of the M received vectors is processed with a receiver filter $\boldsymbol{w}_m(i)$ to form a statistic $Z_m(i) = \boldsymbol{w}^H{}_m(i)\boldsymbol{r}_m(i)$, for $m = 1, 2, \ldots, M$. Note the time dependence of the filters in the time-varying fading channel. The individual statistics are summed to form an overall decision statistic

$$Z(i) = \sum_{m=1}^{M} Z_m(i) \quad (3)$$

Equivalently, we can define an overall receiver filter as $\boldsymbol{w}(i) = (\boldsymbol{w}_1^T(i), \boldsymbol{w}_2^T(i), \ldots, \boldsymbol{w}_M^T(i))^T$ and define an overall received vector as $\boldsymbol{r}(i) = (\boldsymbol{r}_1^T(i), \boldsymbol{r}_2^T(i), \ldots, \boldsymbol{r}_M^T(i))^T$, giving $Z(i) = \boldsymbol{w}^H(i)\boldsymbol{r}(i)$.

We next consider two different design strategies for performing MMSE detection. The best performance is obtained when the filters $\boldsymbol{w}_1(i), \boldsymbol{w}_2(i), \ldots, \boldsymbol{w}_M(i)$ are designed jointly so as to minimize the composite mean-squared error $J = \mathrm{E}[|d_1(i) - \boldsymbol{w}^H(i)\boldsymbol{r}(i)|^2]$. This gives the well-known Wiener solution $\boldsymbol{w}(i) = \boldsymbol{R}^{-1}(i)\boldsymbol{p}(i)$, with $\boldsymbol{R}(i) = \mathrm{E}[\boldsymbol{r}(i)\boldsymbol{r}^H(i)]$ and $\boldsymbol{p}(i) = \mathrm{E}[d_1{}^*(i)\boldsymbol{r}(i)]$ the correlation matrix and steering vector, respectively. These can be further decomposed as

$$\boldsymbol{R}(i) = \begin{bmatrix} \boldsymbol{R}_{1,1}(i) & \boldsymbol{R}_{1,2}(i) & \ldots & \boldsymbol{R}_{1,M}(i) \\ \boldsymbol{R}_{2,1}(i) & \boldsymbol{R}_{2,2}(i) & \ldots & \boldsymbol{R}_{2,M}(i) \\ & & & \\ \boldsymbol{R}_{M,1}(i) & \boldsymbol{R}_{M,2}(i) & \ldots & \boldsymbol{R}_{M,M}(i) \end{bmatrix} \quad (4)$$

where the individual sub-matrices are defined as $\boldsymbol{R}_{m,n}(i) = \mathrm{E}[\boldsymbol{r}_m(i)\boldsymbol{r}_n^H(i)]$, and $\boldsymbol{p}(i) = (\boldsymbol{p}_1^T(i), \boldsymbol{p}_2^T(i), \ldots, \boldsymbol{p}_M^T(i))^T$ with $\boldsymbol{p}_m(i) = \mathrm{E}[d_1{}^*(i)\boldsymbol{r}_m(i)]$.

An alternative sub-optimal approach is to design the M filters separately by choosing each of the filters $\boldsymbol{w}_m(i)$ for $m = 1, 2, \ldots, M$ to minimize the individual mean-squared error quantities $J_m = \mathrm{E}[|d_1(i) - \boldsymbol{w}_m^H(i)\boldsymbol{r}_m(i)|^2]$, which leads to $\boldsymbol{w}_m(i) = \boldsymbol{R}_{m,m}^{-1}(i)\boldsymbol{p}_m(i)$. Together with the previous notation, an overall filter using this design strategy can then be written as $\boldsymbol{w}(i) = \boldsymbol{R}_B^{-1}(i)\boldsymbol{p}(i)$ where

$$\boldsymbol{R}_B(i) = \begin{bmatrix} \boldsymbol{R}_{1,1}(i) & \boldsymbol{0} & \ldots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}_{2,2}(i) & \ldots & \boldsymbol{0} \\ & & & \\ \boldsymbol{0} & \boldsymbol{0} & \ldots & \boldsymbol{R}_{M,M}(i) \end{bmatrix} \quad (5)$$

We now investigate the performance of these two MMSE detection strategies on the Rayleigh fading channel. It has been demonstrated previously that MMSE detection will not work if the receiver is unable to at least track the fading processes of the desired user, as the steering vector, and hence the Wiener filter, would be zero-vectors [5]. Given that the desired user's fading processes are tracked, it is thus feasible to consider coherent detection, giving

$$\boldsymbol{p}(i) = ([\gamma_{1,1}(i)\boldsymbol{c}_{1,1}^T], [\gamma_{1,2}(i)\boldsymbol{c}_{1,2}^T], \ldots, [\gamma_{1,M}(i)\boldsymbol{c}_{1,M}^T])^T \quad (6)$$

Because the receiver is assumed to be synchronous with the desired user, the component vectors in equation (2) for the desired user are $\boldsymbol{f}_{1,m} = \gamma_{1,m}\boldsymbol{c}_{1,m}$ and $\boldsymbol{g}_{1,m} = \boldsymbol{0}$. We can then write the overall received vector as the sum of a signal component plus a composite noise-plus-interference component as

$$\boldsymbol{r}(i) = d_1(i)\boldsymbol{p}(i) + \tilde{\boldsymbol{r}}(i) \quad (7)$$

By using the matrix-inversion lemma, and invoking a Gaussian approximation on the vector $\tilde{\boldsymbol{r}}(i)$, the probability of bit error, with the decision on the ith data bit made according to $\hat{d}_1(i) = \mathrm{sign}(\mathrm{Re}[Z(i)])$, is approximately

$$P_e \approx \mathrm{Q}\left[\sqrt{2\boldsymbol{p}^H(i)\tilde{\boldsymbol{R}}^{-1}(i)\boldsymbol{p}(i)}\right] \quad (8)$$

with $Q(x) = \int_x^\infty (1/\sqrt{2\pi})\exp(-u^2/2)du$. Also in equation (8), we have defined an interference correlation matrix as $\tilde{\boldsymbol{R}}(i) = \mathrm{E}[\tilde{\boldsymbol{r}}(i)\tilde{\boldsymbol{r}}^H(i)]$, which may be decomposed as $\boldsymbol{R}(i)$ was in equation (4), with submatrices given by

$$\tilde{\boldsymbol{R}}_{m,n}(i) = \mathrm{E}[\tilde{\boldsymbol{r}}_m(i)\tilde{\boldsymbol{r}}_n^H(i)] = 2\sigma^2 \boldsymbol{I}_{\frac{N}{M}} \delta_{m,n} + \ldots$$
$$\sum_{k=2}^{K} \left(\frac{P_k}{P_1}\right) \mathrm{E}\left[\gamma_{k,m}(i)\gamma^*{}_{k,n}(i)\right] (\boldsymbol{f}_{k,m}\boldsymbol{f}_{k,m}^T + \boldsymbol{g}_{k,m}\boldsymbol{g}_{k,m}^T) \quad (9)$$

where $\boldsymbol{I}_x$ is an identity matrix of dimension x, and $\delta_{m,n} = 1$ if $m = n$ and 0 otherwise. The quantity $\mathrm{E}[\gamma_{k,m}(i)\gamma^*{}_{k,n}(i)]$

depends on the assumptions made about the receiver's ability to track the fading processes. We will look at two cases.

We first assume that the channel changes slowly enough so that the fading on all of the carriers of all of the users can be tracked. The fading processes are then deterministic quantities during the bit interval of interest, meaning that

$$E[\gamma_{k,m}(i)\gamma^*_{k,n}(i)] = \gamma_{k,m}(i)\gamma^*_{k,n}(i) \tag{10}$$

The probability of bit error in this case is found by combining equations (6), (9), and (10) with equation (8).

Results are shown in Figure 2 for a CDMA system with a composite processing gain of $N = 32$ chips/bit, $E_b/N_0 = 17$ dB, and for $M = 1, 2, 4$ carriers, where it is assumed that the Wiener solution is formed. The probability of bit error refers to the average probability calculated by equation (8), taken over 10000 trials, in which the fading processes, the spreading codes, the delays, and the powers of the users (which were taken as lognormal with a standard deviation of 1.5 dB) were varied for each trial. There is a substantial amount of diversity gained by the multi-carrier system. For example, if an average probability of bit error of 10^{-3} is desired, and assuming E_b/N_0 is fixed at 17 dB, the single-carrier system cannot reach this level, even for a single user. The 2-carrier system can handle about 9 users while the 4-carrier system can handle about twice that number, 18 users. The incremental diversity advantage between the two multi-carrier systems decreases noticeably as the loading increases up to 30 users.

We next consider the case in which the desired user's fading processes are tracked, but the other users' processes are not trackable. Then the fading processes are complex Gaussian random variables as opposed to constants, and for $k \geq 2$,

$$E[\gamma_{k,m}(i)\gamma^*_{k,n}(i)] = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases} \tag{11}$$

as the fading processes are assumed to have unit average power. The performance is shown in Figure 2 for comparison to the case in which all users' fading processes are tracked exactly. In the flat fading case, the knowledge of all of the interfering users' fading processes improves performance very little. With 2 and 4 carriers, however, there is a substantial loss realized due to the lack of knowledge of the interfering users' fading processes, a conclusion also reached in [2].

It also would be useful to compare these results for a multi-carrier system to those obtained when diversity is realized inherently in a frequency-selective fading channel. To conserve space, those results are not included here, but can be found in [6].

III. Tracking the Fading Processes

In this section, an algorithm is proposed for tracking the fading processes of the CDMA users. If such a tracking algorithm can be formulated, then an estimate of the exact Wiener solution can be formed, in contrast to traditional approaches such as the LMS or RLS algorithms, in which the channel is tracked implicitly. It has been shown that these algorithms cannot track the channel models discussed here unless the channel changes very slowly.

First recall the received vector on the *m*th carrier, from equation (2), and note that it may be written as

$$\boldsymbol{r}_m(i) = (\boldsymbol{F}_m\boldsymbol{D}(i) + \boldsymbol{G}_m\boldsymbol{D}(i-1))\boldsymbol{P}\Gamma_m(i) + \boldsymbol{n}_m(i) \tag{12}$$

where the matrices in this expression are defined as

$$\boldsymbol{F}_m = \left[\boldsymbol{f}_{1,m}\ \boldsymbol{f}_{2,m}\ \ldots\ \boldsymbol{f}_{K,m}\right] \qquad \boldsymbol{D}(i) = \begin{bmatrix} d_1(i) & & & \\ & d_2(i) & & \\ & & \ldots & \\ & & & d_K(i) \end{bmatrix}$$

$$\boldsymbol{G}_m = \left[\boldsymbol{g}_{1,m}\ \boldsymbol{g}_{2,m}\ \ldots\ \boldsymbol{g}_{K,m}\right] \qquad \boldsymbol{P} = \begin{bmatrix} 1 & & & \\ & \sqrt{P_2/P_1} & & \\ & & \ldots & \\ & & & \sqrt{P_K/P_1} \end{bmatrix} \tag{13}$$

and the column vector of fading coefficients on the *m*th carrier, which is to be tracked, is

$$\Gamma_m(i) = \left[\gamma_{1,m}(i)\ \gamma_{2,m}(i)\ \ldots\ \gamma_{K,m}(i)\right]^T \tag{14}$$

Also in equation (12), $\boldsymbol{n}_m(i)$ is a column vector of independent, complex Gaussian noise samples, with the real and imaginary parts independent from each other and each with variance $\sigma^2 = N/(2E_b/N_0)$. Equation (12) can be rewritten as

$$\boldsymbol{r}_m(i) = \boldsymbol{A}_m(i)\Gamma_m(i) + \boldsymbol{n}_m(i) \tag{15}$$

with

$$\boldsymbol{A}_m(i) = (\boldsymbol{F}_m\boldsymbol{D}(i) + \boldsymbol{G}_m\boldsymbol{D}(i-1))\boldsymbol{P} \tag{16}$$

For estimation purposes, we assume the fading processes to be essentially constant over an *L*-bit window, giving

$$\boldsymbol{r}_m^{(L)}(i) = \boldsymbol{A}_m^{(L)}(i)\Gamma_m(i) + \boldsymbol{n}_m^{(L)}(i) \tag{17}$$

where matrices from L bit intervals have been concatenated to form

$$\begin{aligned} \boldsymbol{r}_m^{(L)}(i) &= \left[\boldsymbol{r}_m^T(i-(L-1))\ \boldsymbol{r}_m^T(i-(L-2))\ \dots\ \boldsymbol{r}_m^T(i)\right]^T \\ \boldsymbol{A}_m^{(L)}(i) &= \left[\boldsymbol{A}_m^T(i-(L-1))\ \boldsymbol{A}_m^T(i-(L-2))\ \dots\ \boldsymbol{A}_m^T(i)\right]^T \\ \boldsymbol{n}_m^{(L)}(i) &= \left[\boldsymbol{n}_m^T(i-(L-1))\ \boldsymbol{n}_m^T(i-(L-2))\ \dots\ \boldsymbol{n}_m^T(i)\right]^T \end{aligned} \tag{18}$$

We can then formulate a maximum-likelihood estimation problem for the fading processes. It is assumed that the receiver has knowledge of the data bits of all of the users. This would be reasonable either when the receiver is in training mode, or when decision feedback is used. In this case, the maximum-likelihood estimate of the vector $\Gamma_m(i)$ minimizes the cost function $C(\Gamma_m(i)) = \left\|\boldsymbol{r}_m^{(L)}(i) - \boldsymbol{A}_m^{(L)}(i)\Gamma_m(i)\right\|^2$, giving the least-squares solution

$$\begin{aligned} \Gamma_{m,ML}(i) &= ([\boldsymbol{A}_m^{(L)}(i)]^T[\boldsymbol{A}_m^{(L)}(i)])^{-1}([\boldsymbol{A}_m^{(L)}(i)]^T\boldsymbol{r}_m^{(L)}(i)) \\ &= \boldsymbol{Q}_m^{-1}(i)\boldsymbol{S}_m(i) \end{aligned} \tag{19}$$

where we have defined

$$\begin{aligned} \boldsymbol{Q}_m(i) &= [\boldsymbol{A}_m^{(L)}(i)]^T[\boldsymbol{A}_m^{(L)}(i)] = \sum_{l=0}^{L-1} \boldsymbol{A}_m^T(i-l)\boldsymbol{A}_m^T(i-l) \\ \boldsymbol{S}_m(i) &= [\boldsymbol{A}_m^{(L)}(i)]^T\boldsymbol{r}_m^{(L)}(i) = \sum_{l=0}^{L-1} \boldsymbol{A}_m^T(i-l)\boldsymbol{r}_m^T(i-l) \end{aligned} \tag{20}$$

This form of the matrices suggests recursive estimates using exponentially-weighted windows,

$$\begin{aligned} \boldsymbol{Q}_m(i) &= \lambda\boldsymbol{Q}_m(i-1) + \boldsymbol{A}_m^T(i)\boldsymbol{A}_m^T(i) \\ \boldsymbol{S}_m(i) &= \lambda\boldsymbol{S}_m(i-1) + \boldsymbol{A}_m^T(i)\boldsymbol{r}_m^T(i) \end{aligned} \tag{21}$$

where $0 \le \lambda \le 1$ is the forgetting factor. Once the fading has been estimated according to this procedure, an estimate of the true Wiener solution of the tap weights may be formed.

To gain insight into the operation of this channel estimator, consider a single-bit estimator, i.e. with $\lambda = 0$, giving

$$\begin{aligned} \Gamma_{m,ML}(i) &= [\boldsymbol{A}_m^T(i)\boldsymbol{A}_m(i)]^{-1}\boldsymbol{A}_m^T(i)\boldsymbol{r}_m(i) \\ &= \Gamma_m(i) + [\boldsymbol{A}_m^T(i)\boldsymbol{A}_m(i)]^{-1}\boldsymbol{A}_m^T(i)\boldsymbol{n}_m(i) \end{aligned} \tag{22}$$

Thus the estimate of $\Gamma_m(i)$ is equal to the true value plus a term due only to the thermal noise, and independent of the multi-access interference.

Furthermore, if the system were synchronous, then the matrix $\boldsymbol{G}_m$ would be a zero-matrix, and the estimate of $\Gamma_m(i)$ could be written as

$$\Gamma_{m,ML}(i) = \boldsymbol{D}^{-1}(i)[(\boldsymbol{F}_m\boldsymbol{P})^T(\boldsymbol{F}_m\boldsymbol{P})]^{-1}(\boldsymbol{F}_m\boldsymbol{P})^T\boldsymbol{r}_m(i) \tag{23}$$

The estimator could then be visualized as in Figure 3. The received signal is processed first with a matched filter bank. The MAI is then removed with a decorrelator. Finally, the data is removed, leaving an unbiased estimate of $\Gamma_m(i)$.

Note that here the matrix $(\boldsymbol{F}_m\boldsymbol{P})^T(\boldsymbol{F}_m\boldsymbol{P})$ will be invertible only if the columns of $\boldsymbol{F}_m$ are linearly independent. This condition will be violated as the number of users surpasses N/M, and the decorrelator will not exist. However, as the observation window is increased, which would obviously be done in order to get good estimates of the fading processes, the existence of the decorrelator would be almost certain. A similar interpretation of the estimator would still apply, that is matched filter/decorrelator/data removal.

The performance of this algorithm was tested via simulation for a multi-carrier CDMA system, with a data rate of 10 kHz, a carrier frequency of 900 MHz, a processing gain of 32 chips/bit, and an E_b/N_0 of 17 dB. With 15 users present, results for the average probability of bit error are shown in Figure 4 as a function of the vehicle speed, which was varied from 20 mph up to 200 mph. Single-carrier, 2-carrier, and 4-carrier systems were considered. Results for the Wiener solution are shown for comparison. It is seen that the performance is very close to ideal for vehicle speeds below about 80 mph, or a normalized Doppler frequency under 1%. In Figure 5, the identical system was simulated, this time with a fixed vehicle speed of 80 mph, and with the number of users varying between 1 and 30, corroborating the results of Figure 4.

IV. Conclusion

In this paper, the performance of multi-carrier CDMA systems employing MMSE detection has been investigated. It has been shown that the receiver must track the fading processes of all of the users on all of the carriers. A tracking algorithm was then presented, and its performance evaluated. It was shown that performance close to that of the ideal Wiener solution could be obtained with this algorithm, provided that the normalized Doppler frequency caused by the motion of the mobile was less than about 1%.

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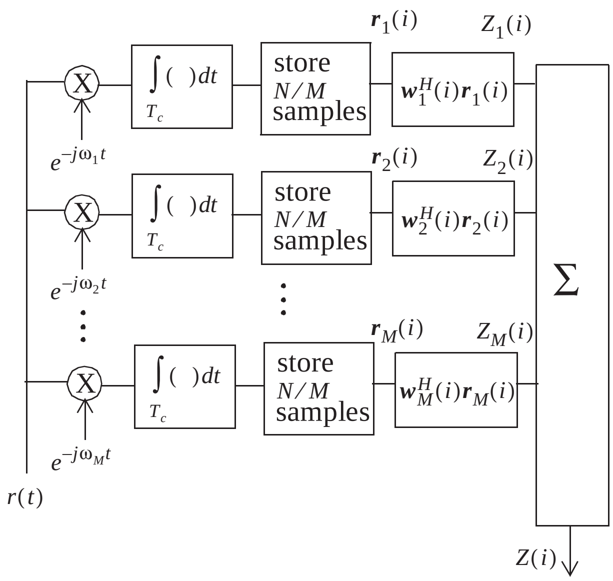

Figure 1: General receiver for multi-carrier CDMA.

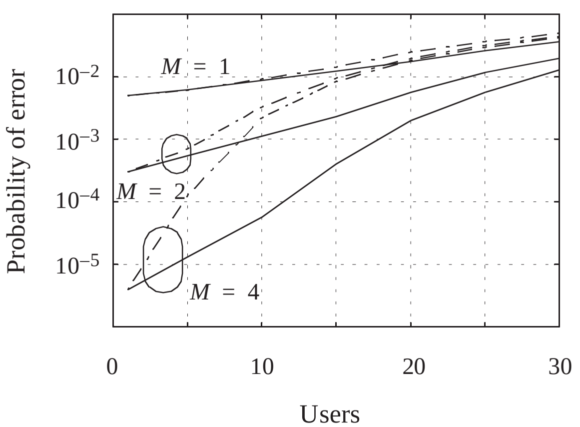

Figure 2: Probability of error vs. number of users for multi-carrier CDMA system, $E_b/N_0 = 17$ dB, composite processing gain is 32 chips/bit, M carriers used. Solid line indicates all users' fading tracked, dashed line indicates only desired users' fading tracked, both cases forming the Wiener solution.

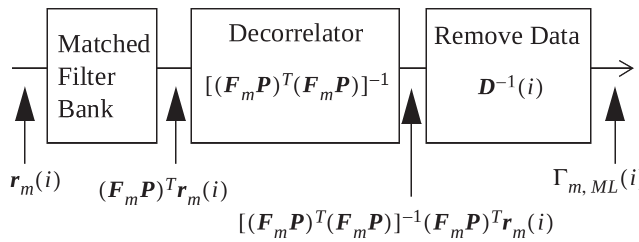

Figure 3: Block diagram of fading estimator using a single-bit observation window.

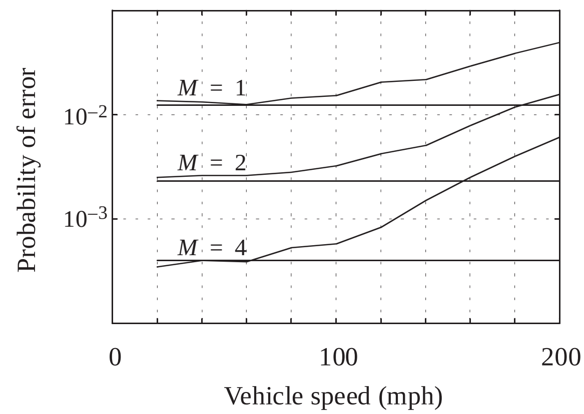

Figure 4: Probability of error vs. vehicle speed for multi-carrier CDMA system with $E_b/N_0 = 17$ dB, composite processing gain of 32 chips/bit, 15 asynchronous users, bit rate of 10000 bits/sec, and carrier frequency of 900 MHz, and M carriers. Curves show performance for tracking algorithm, solid straight lines show Wiener solution.

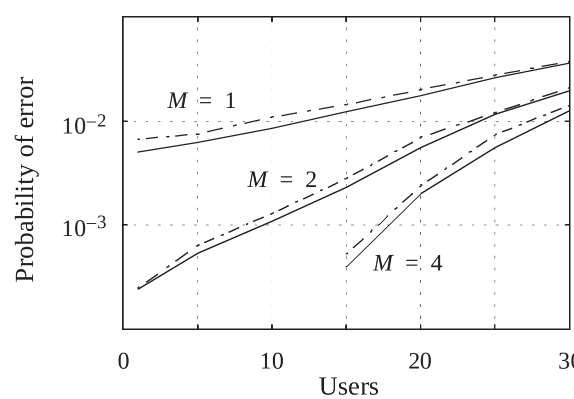

Figure 5: Probability of error vs. number of users for multi-carrier CDMA system with vehicle speed of 80 mph, and same parameters as in Figure 4. Dashed curves show performance of tracking algorithm, solid lines show Wiener solution.