An Efficient Blind Pseudo Turbo Equalizer with CMA and SAGMCMA for Single-Carrier System

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***Abstract*— Basically, conventional single-carrier system has an advantage in limited power communication systems. However, the performance of conventional single-carrier system is seriously degraded by ISI (inter-symbol interference). In order to overcome the ISI, communication systems are commonly used an equalizer. Turbo equalization technique was proposed to maximize system performance over ISI channel. Turbo equalizer can achieve impressive performance gains over ISI channel. Turbo equalizer performs iterative feedback from decoder to equalizer in receiver. Because of iterative feedback, turbo equalizer gets better system performance according to the iteration number. In this paper, we propose a novel turbo equalizer for compensating ISI, which is called as blind pseudo turbo equalizer. Blind means that the equalizer is used without any kinds of training sequence. And Pseudo turbo equalizer means modified structure of turbo equalizer. Pseudo turbo equalizer is included with feed-forward filter and feedback filter. Therefore, we evaluate the BER performance of blind pseudo turbo equalizer.**

Keywords- single-carrier system; ISI; blind pseudo turbo equalizer; iterative feedback.

I. INTRODUCTION

Single-carrier system has lower PAPR (peak to average power ratio) than OFDM (orthogonal frequency division multiplexing) system. So, single-carrier system has an advantage in limited power consumption systems. In particular, single-carrier system is much more efficient such as satellite communications.

However, the performance of conventional single-carrier system is affected by ISI (inter-symbol interference) from multipath channel. In order to overcome such as ISI, using complicated equalizer is general. In single-carrier system, equalization has a huge impact on overall system performance. In addition, most of the complexity depends on equalizer. So, design of equalization method is very important. There are many kinds of equalization method such as ZF (zero forcing) equalizer, MMSE (minimum mean square error) equalizer, adaptive equalizer and DFE (decision feedback equalizer). Especially, adaptive equalizer and DFE have several kinds of algorithm such as LMS (Least Mean Square), RLS (Recursive Least Square) and CMA (Constant Modulus Algorithm), etc. In addition, Viterbi equalizer and ML (Maximum Likelihood) based equalizer was proposed, also. These equalizers are good equalization performance, but very complicated.

Especially, turbo equalization was proposed in order to maximize system performance through iterations [1]-[3]. Turbo principle, which was developed for turbo code, was applied to the equalizer of receiver. Turbo equalization is combination with the SISO (soft input soft output) equalizer [4] and channel decoder in receiver side. So, turbo equalization technique performs feedback loop from decoder to equalizer iteratively like a turbo engine. Turbo equalization can achieve impressive performance gains for communication system over channels that require equalization. When we perform these steps, we use the extrinsic LLR (log-likelihood ratio) value. Finally, we can estimate the data bit through the hard decision.

On the other hand, blind equalizer was proposed for exception of any training sequence. In the data rate point of view, blind equalizer is very efficient because don't need to transmit training sequence. The CMA (constant modulus algorithm) is most basic algorithm of blind equalizer [5]. The CMA is used by constant modulus for calculating the error vector. SAGMCMA (stop-and-go modified constant modulus) is one of the modified algorithms [6]. In this paper, we propose blind pseudo turbo equalizer with CMA and SAGMCMA. We adopt blind and MMSE algorithms at feedforward and feedback filters, respectively [7]. Finally, we evaluate the BER performance of blind pseudo turbo equalizer.

II. SYSTEM MODEL

Fig. 1 shows transmitter system over ISI channel. Firstly, it generates data sequence randomly. Then, it performs encoding. After encoding, data passes interleaver and modulator. Finally, the signal is transmitted over ISI channel and added Gaussian noise. ISI channel can be modeled as follows

$$h(n) = \sum_{k=0}^{M-1} h_k \delta(n-k), \tag{1}$$

where h_k is filter coefficient and M is filter length.

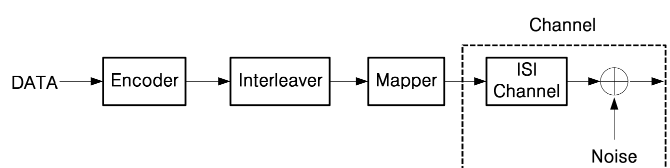

Figure 1. Block diagram of the transmitter system over ISI channel.

Fig. 2 shows structure of general turbo equalizer receiver. The received signal after passing the channel is equalized through equalizer. The equalized signal passes the de-mapper and de-interleaver. Then, the output of decoder, which is extrinsic information based on the LLR value, becomes input of equalizer after passing the interleaver and mapper. As below structure, if the system is performed depending on iterations repeatedly, it will be able to achieve better performance.

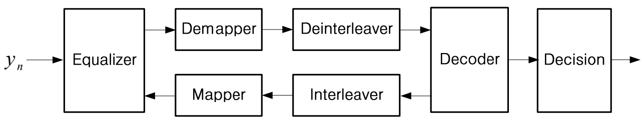

Figure 2. Block diagram of receiver of general turbo equalizer.

The LLR value is defined as

$$L(x) \triangleq \log \frac{P(x=+1)}{P(x=-1)}. \quad (2)$$

III. SISO-MMSE Based Turbo Equalizer

In this paper, we analysis linear SISO-MMSE (soft-input soft-output minimum mean square error) based turbo equalizer [3]. LLR output of SISO-MMSE equalizer is defined as

$$L_e(x_n) \triangleq \log \frac{P(x=+1|\hat{z}_n)}{P(x=-1|\hat{z}_n)} - \log \frac{P(x_n=+1)}{P(x_n=-1)}. \quad (3)$$

Estimated output of MMSE equalizer is written by

$$\hat{z}_n = \overline{\mathbf{x}}_n + v_n \mathbf{s}^H (\sigma_w^2 \mathbf{I}_N + \mathbf{H}\mathbf{V}_n\mathbf{H}^H)^{-1}(\mathbf{y}_n - \mathbf{H}\overline{\mathbf{x}}_n). \quad (4)$$

Where, $\mathbf{H}$ is $N \times (N+M-1)$ channel convolution matrix

$$\mathbf{H} \triangleq \begin{bmatrix} h_{M-1} & \dots & h_0 & 0 & \dots & & 0 \\ 0 & h_{M-1} & \dots & h_0 & 0 & \dots & 0 \\ & & \ddots & & & & \\ 0 & & \dots & 0 & h_{M-1} & \dots & h_0 \end{bmatrix} \quad (5)$$

and

$$\overline{\mathbf{x}}_n \triangleq \begin{bmatrix} \overline{x}_{n-M-N_2+1} & \overline{x}_{n-M-N_2+2} & \dots & \overline{x}_{n+N_1} \end{bmatrix}^T \quad (6)$$

$$\mathbf{V}_n \triangleq \mathrm{Diag}\left(v_{n-M-N_2+1} \quad v_{n-M-N_2+2} \quad \dots \quad v_{n+N_1} \right) \quad (7)$$

$$\mathbf{s} \triangleq \mathbf{H}\begin{bmatrix} \mathbf{0}_{1\times(N_2+M-1)} & 1 & \mathbf{0}_{1\times N_1} \end{bmatrix}^T. \quad (8)$$

Then, $\overline{x}_n$ and v_n can be obtained as

$$\begin{aligned} \overline{x}_n &= \sum_{x\in \mathrm{B}} x \cdot P(x_n = x) = P(x_n=+1) - P(x_n=-1) \\ &= \frac{e^{L(x_n)}}{1+e^{L(x_n)}} - \frac{1}{1+e^{L(x_n)}} = \tanh(L(x_n)/2) \end{aligned} \quad ,(9)$$

$$v_n = \sum_{x\in \mathrm{B}} |x - E(x_n)|^2 \cdot P(x_n = x) = 1 - |\overline{x}_n|^2. \quad (10)$$

So, (4) can be written by

$$\hat{z}_n = \sum_{k=-N_1}^{N_2} c_{n,k} \left(y_{n-k} - E(y_{n-k}) \right). \quad (11)$$

Where, the vector of coefficients is consequently set to

$$\mathbf{c}_n \triangleq \left(\sigma_w^2 \mathbf{I}_N + \mathbf{H}\mathbf{V}_n\mathbf{H}^H + (1-v_n)\mathbf{s}\mathbf{s}^H \right)^{-1} \mathbf{s}. \quad (12)$$

Therefore, LLR output of MMSE equalizer can be expressed by

$$L_e(x_n) = 2\mathbf{c}_n^H \left(\mathbf{y}_n - \mathbf{H}\overline{\mathbf{x}}_n + \overline{x}_n \mathbf{s} \right) / \left(1 - \mathbf{s}^H \mathbf{c}_n \right). \quad (13)$$

For the initial equalization step, we have $\overline{x}_n = 0$ and $v_n = 1$. So, we can obtain coefficient vector as $\mathbf{c}_n = \mathbf{c}_{NA}$.

$$\mathbf{c}_{NA} = \left(\sigma_w^2 \mathbf{I}_N + \mathbf{H}\mathbf{H}^H \right)^{-1} \mathbf{s}. \quad (14)$$

LLR output of MMSE equalizer can be changed by

$$L_e(x_n) = 2\mathbf{c}_{NA}^H \mathbf{y}_n / \left(1 - \mathbf{s}^H \mathbf{c}_{NA} \right). \quad (15)$$

IV. Proposed Blind Pseudo Turbo Equalizer

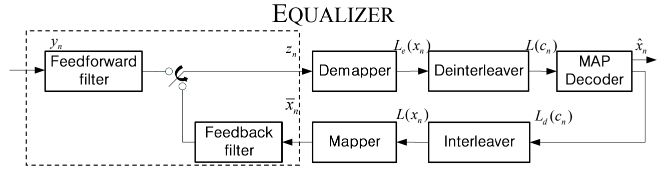

Figure 3. Block diagram of proposed blind pseudo turbo equalizer.

Fig. 3 shows detailed block diagram of proposed blind pseudo turbo equalizer. We adopt CMA (constant modulus algorithm) and SAGMCMA(stop-and-go modified constant modulus algorithm) at feedforward filter. For iterative equalization, we adopt linear SISO-MMSE algorithm at feedback filter. Proposed turbo equalizer is consist of combination of blind and MMSE algorithms like DFE (decision feedback equalizer). So, proposed turbo equalizer is called as blind pseudo turbo equalizer.

A. Constant Modulus Algorithm

$x(n)$ is transmitted signal and $y(n)$ is received signal after passing the channel.

$$y(n) = [y(n) \quad y(n-1) \quad \dots \quad y(n-N+1)]^T \quad (16)$$

The output of blind equalizer is expressed by

$$z(n) = f^T(n) \cdot y(n). \quad (17)$$

Where, the weighting vector of blind equalizer is defined by $f(n) = [f_0(n) \quad f_1(n) \quad \dots \quad f_{N-1}(n)]^T$.

Then, the cost function of CMA can be written by

$$J(n) = E\left[\left(|z(n)|^2 - R_2\right)^2\right]. \quad (18)$$

Where, $R_2 = \dfrac{E\left[|x(n)|^4\right]}{E\left[|x(n)|^2\right]}$.

The error vector of CMA can be obtained as

$$e(n) = z(n)\left[|z(n)|^2 - R_2\right]. \quad (19)$$

Therefore, the updating equation is defined as follows

$$f(n+1) = f(n) - \mu e(n) \cdot y(n). \quad (20)$$

Where, μ is step size.

B. Stop-and Go Modified Constant Modulus Algorithm

The error vector of SAGMCMA is written by

$$\begin{aligned} \widehat{e_R}(n) &= z_R(n)\left[z_R^{\,2}(n) - \hat{x}_R^{\,2}(n)\right] \\ \widehat{e_I}(n) &= z_I(n)\left[z_I^{\,2}(n) - \hat{x}_I^{\,2}(n)\right] \end{aligned} \quad (21)$$

Where, $\hat{x}(n)$ is decision symbol of $z(n)$. That is, $\hat{x}(n) = \hat{x}_R(n) + j\hat{x}_I(n)$.

Then, we can calculate the flag for updating as follows

$$fr_R = \begin{cases} 1 & if \ \mathrm{sgn}\,\hat{e}_R(n) = \mathrm{sgn}\,e_R(n) \\ 0 & if \ \mathrm{sgn}\,\hat{e}_R(n) \neq \mathrm{sgn}\,e_R(n) \end{cases}$$
$$fi_I = \begin{cases} 1 & if \ \mathrm{sgn}\,\hat{e}_I(n) = \mathrm{sgn}\,e_I(n) \\ 0 & if \ \mathrm{sgn}\,\hat{e}_I(n) \neq \mathrm{sgn}\,e_I(n) \end{cases}. \quad (22)$$

Therefore, we modify the updating equation as follows

$$f(n+1) = f(n) - \mu e'(n) \cdot x(n). \quad (23)$$

Where, $e'(n) = fr_R(n) \cdot e_R(n) + j \cdot fr_I(n) \cdot e_I(n)$.

V. Simulation Results

Table 1 shows simulation parameters for evaluating the BER performance of blind pseudo turbo equalizer. In this paper, we consider QPSK modulation, code rate of encoding is 1/2 and block size is 512. The equalizer length is 15-taps at feedforward and feedback filters, respectively. Also, we consider the AWGN and ISI channel. The ISI channel is Proakis channel B. Fig. 4 shows magnitude response of Proakis channel B. This ISI channel is dramatically fading channel.

TABLE I. Simulation Parameters

Parameters	Values
Modulation	QPSK
Code rate	1/2
Block size	512
Equalizer length	15 taps
Channel	ISI + AWGN
ISI channel (Proakis channel B)	[0.407 0.815 0.407]

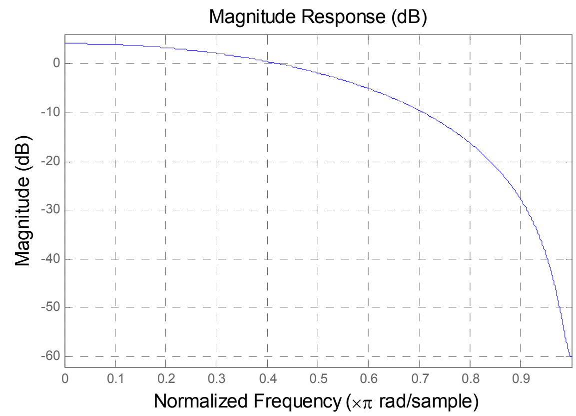

Figure 4. Magnitude response of Proakis channel B.

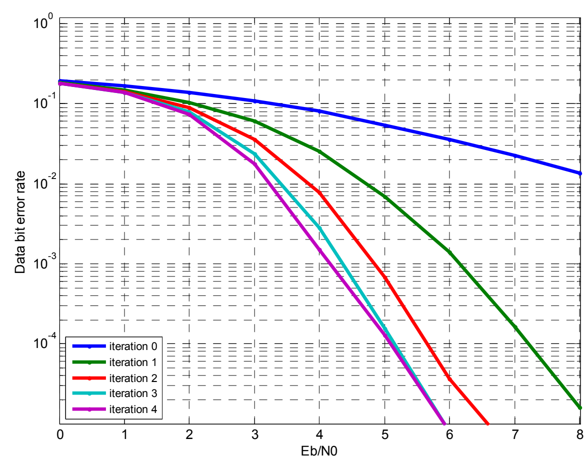

Figure 5. The BER performance of linear SISO-MMSE based turbo equalizer.

Fig. 5 shows the BER performance of linear SISO-MMSE based turbo equalizer. As a result of Fig. 5, it shows better BER performance according to iterations. However, the BER performance does not improve significantly more than 3 iterations.

Fig. 6 shows the BER performance of blind pseudo turbo equalizer adopted with CMA. As a result, Fig. 6 shows better BER performance according to the number of iterations. When the number of iterations is 4, the BER performance is satisfied as about 10^{-4} at 8dB.

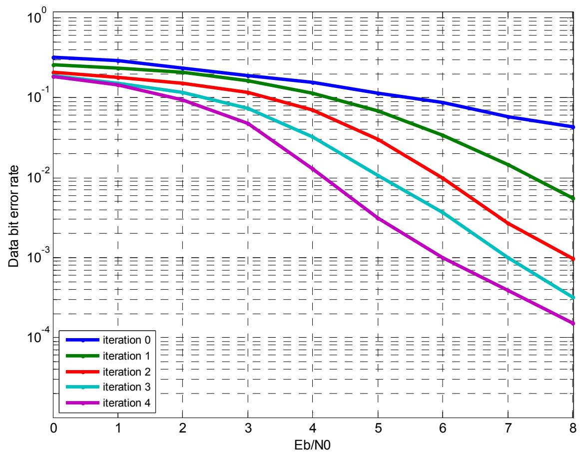

Figure 6. The BER performance of blind pseudo turbo equalizer adopted with CMA.

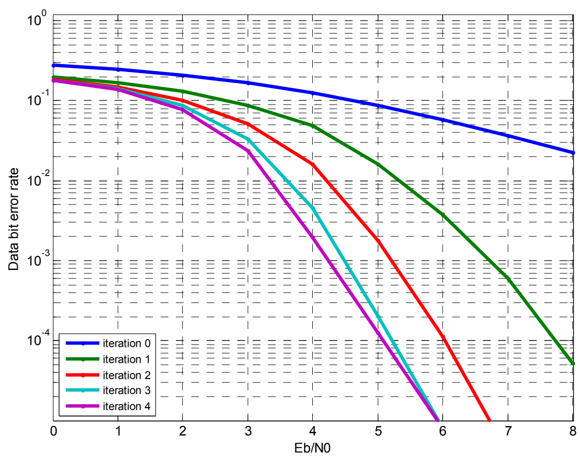

Figure 7. The BER performance of blind pseudo turbo equalizer adopted with SAGMCMA.

Fig. 7 shows the BER performance of blind pseudo turbo equalizer adopted with SAGMCMA. As a result, Fig. 7 shows better BER performance according to the number of iterations. When the number of iterations is 4, the BER performance is satisfied as about 10^{-4} at 5dB.

Fig. 8 shows the BER performance comparison between CMA and SAGMCMA. As a result, Fig. 8 shows better BER performance adopted with SAGMCMA. The proposed blind pseudo turbo equalizer adopted with SAGMCMA almost achieve on the BER performance of linear SISO-MMSE based turbo equalizer.

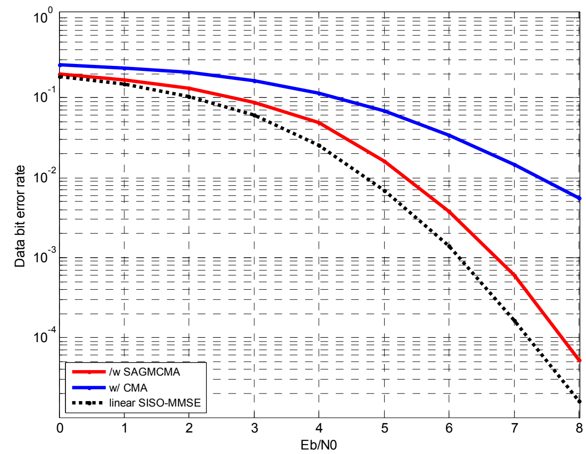

Figure 8. The BER performance comparison between CMA and SAGMCMA(# of iteration=1).

VI. CONCLUSION

In this paper, we analyze the conventional linear SISO-MMSE turbo equalizer principle. Then, we propose blind pseudo turbo equalizer. Finally, we evaluate the BER performance through the computer simulation. We adopt the CMA and SAGMCMA at feedforward filter. For iterative equalization, linear SISO-MMSE is adopted at feedback filter. That is, blind pseudo turbo equalizer means combination of blind and MMSE algorithms like the iterative DFE. The simulation results show better BER performance according to the number of iterations. As the result of Fig. 6, the BER performance adopted with CMA is satisfied about 10^{-4} at 8dB when the number of iterations is 4. As the result of Fig. 7, the BER performance adopted with SAGMCMA is satisfied about 10^{-4} at 5dB when the number of iterations is 4. Compare to results of Fig. 8, the BER performance of SAGMCMA is much better than CMA. Therefore, the proposed blind pseudo turbo equalizer adopted SAGMCMA can make the system performance to satisfy over ISI channel situations.

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REFERENCES

[1] R. Koetter, A.C. Singer, and M. Tüchler "Turbo Equalization," *IEEE Signal Processing Mag.*, vol. 21, no. 1, pp. 67–80, Jan. 2004.

[2] Douillard, Catherine; Jézéquel, Michel; Berrou, Claude, "Iterative Correction of Intersymbol-Interference: Turbo-Equalization," *European Transactions on Telecommunications*, vol. 6, no. 5, pp. 507-511, September-October 1995.

[3] M. Tücher, R. Koetter, and A.C. Singer, "Turbo Equalization: Principles and New Results," *IEEE Transactions on Communications*, vol. 50, no. 5, pp. 754-767, May 2002.

[4] Mauri Nissil¨a and Subbarayan Pasupathy, "Soft-Input Soft-Output Equalizers for Turbo Receivers: A Statistical Physics Perspective," *IEEE TRANSACTIONS ON COMMUNICATIONS*, VOL. 55, NO. 7, JULY 2007.

[5] C. R. Johnson Jr., P. Schniter, J. T. Endres, et al., "Blind equalization Using the Constant Modulus Criterion: A Review", Proceedings of the IEEE, Vol.86, No.10, pp.1927-1949, 1998.

[6] X. Wei, Y. Xiaoniu and Z. Zhaoyang, "A Stop and Go Blind Equalization Algorithm for QAM Signals", *6th International Conference on Wireless Communcations Networking and Mobile Computing(WiCOM), 2010*.

[7] H. Peng and J. Li, "Turbo Equalization in Blind Receiver", *International Conference on Communcations and Intelligence Information Security, 2010*.

[8] Won Young Yang, et al, "MATLAB / Simulink for Digital Communication", A-Jin Publishing, 1st edition, 2009.

[9] John G. Proakis, "Digital Communications", Mc Graw Hill, 4th edition, 2000.