OFDM Waveform Design Compromising Spectral Nulling, Side-lobe Suppression and Range Resolution

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***Abstract*—Motivated by dual use of OFDM signal format for communications and radar in ever-worsening Electromagnetic (EM) coexistence environments, this paper deals with transmit waveform design problem considering multiple design objectives. Spectral nulling is a typical way for friendly coexistence with narrow band systems. However, a Non-Contiguous Orthogonal Frequency-Division Multiplexing (NC-OFDM) waveform generated by turning off the interfering sub-carriers does not lead to satisfactory results. In this paper a convex optimization based waveform design framework is used to achieve deep spectral nulling while retaining low waveform autocorrelation side lobes and good range resolution. Because of dual use of the waveform, the data blocks to transmit are either unknown or chosen from a known dataset. Optimal sub-carrier weights are obtained for given transmission data blocks. In addition, waveform design for unknown data blocks are discussed and examined.**

Key words: NC-OFDM, spectral nulling, waveform design, convex optimization, Semi-Definitive Programming (SDP).

I. Introduction

OFDM (Orthogonal Frequency-Division Multiplexing) is a widely adopted modulation format for communications. Because of the wide bandwidth nature of the waveform, it can be used for high range resolution radar as well [1]–[9], and dual use of OFDM waveform for communication and ranging in a single system is possible [1]–[4], [8]. One advantage of such type of waveforms is that the individual sub-carriers can be flexibly adjusted or turned off for better spectrum utilization and coexistence with narrow band systems. An OFDM waveform with muted sub-carriers is called Non-Contiguous OFDM (NC-OFDM) waveform [10]. By turning off a small percentage of sub-carriers, the NC-OFDM waveform still retains the wide-bandwidth nature and good range resolution. However, turning off sub-carriers does not always lead to sufficient nulling at some concerned frequencies. A desired transmit waveform for ranging has to compromise multiple objectives like deep nulling (frequency domain requirement), low autocorrelation side lobes (time domain requirement), and good range resolution (narrow autocorrelation main lobe, time domain requirement). With multiple design objectives–they usually contradict each other–are involved, achieving a satisfactory design is quite difficult.

A large body of work in transmit waveform design has been reported in literature [5]–[7], [9], [11]–[17]. Sparse frequency transmit waveform design work reported in [15]–[17], though not being restricted to OFDM waveforms, is related to the work introduced in this paper. Different from previous work, this paper considers a scenario where ranging and communications perform ***simultaneously***, so the data blocks carried by the OFDM waveform can be uncertain or restrict to a known dataset. This implies that robustness should be considered to generate waveforms that perform well over a set of given transmission data blocks. It is found that the waveform characteristic depends largely on how the sub-carriers are weighted. Thus focus should be on how the sub-carrier weights are synthesized, taking into account multiple optimization objectives and computational complexity. The waveform design is formulated as a convex optimization problem. Specifically, Semi-Definitive Programming (SDP) [18], [19] is employed to synthesize the weights.

Major work and contributions reported in this paper can be summarized as follows:

1) it is found the NC-OFDM waveform can be largely improved simply by refining ~~the~~ the sub-carrier weights; and in particular, the weights at the lowest and highest sub-carriers play more important role, which motivates an easy-use yet effective method, called edge-tone amplification, to modify traditional NC-OFDM waveforms;
2) to compromising spectral nulling, side-lobe suppression, and ranging resolution, an SDP-based optimal OFDM waveform design technique for a single known data block is proposed and examined;
3) robust waveform design for unknown data blocks is discussed, and a practical robust design technique is proposed and evaluated.

The rest part of this paper is organized as follows. Mathematical formulation for OFDM waveform optimization is provided in the next section. Waveform design for unknown data blocks is presented in section III. Performance evaluation of proposed techniques is given in section IV, followed by concluding remarks in section V.

II. Problem Formulation

Consider a baseband OFDM waveform segment that carries a data block. Suppose there are N sub-carrier frequencies staring from F_0 and separated by Δf, and each sub-carrier carries a stream of M digital-modulated symbols with symbol duration $T = 1/\Delta f$. Parameter M is called OFDM waveform length in symbol. The transmitted waveform of MT seconds can be expressed in baseband as

$$x(t) = \sum_{n=0}^{N-1} s_n(t) e^{j2\pi(F_0+n\Delta f)t}, \quad t \in (0, MT] \tag{1}$$

where $s_n(t)$ is a weighted modulated waveform of M symbols on the n-th sub-carrier. Denoting by $b_{n,m} \in \mathbb{C}$ the m-th sub-carrier symbol on the n-th sub-carrier, $a_n \in \mathbb{C}$ the corresponding sub-carrier weight, and letting $p(t)$ be a symbol-level rectangular pulse with pulse width T and height 1, $s_n(t)$ can be expressed as

$$s_n(t) = a_n \sum_{m=0}^{M-1} b_{n,m} p(t - mT), \quad t \in (0, MT], \;\; n = 0, 1, 2, \cdots, N-1 \tag{2}$$

Note that the data block carried by an OFDM waveform segment is $\mathbf{b} \triangleq \{b_{n,m}, \; n = 0, 1, \cdots, N-1, \; m = 0, 1, \cdots, M-1\}$. When multiple ($G$) data blocks $\mathbf{b}^{(g)}, \; g = 0, 1, \cdots, G-1$, are involved, it is desired to find a robust design based on the dataset $\mathcal{U} = \left\{\mathbf{b}^{(g)}, \; 0 \le g \le G-1\right\}$. For the sake of simplicity, the channel effect is not considered in this paper. The performance of designed waveform can be judged in time domain and frequency domain. The autocorrelation of $x(t)$, denoted by $\gamma(\tau), \; \tau \ge 0$, is typically considered for ranging purpose, and sharp autocorrelation main lobe as well as low side lobes are desired. The baseband OFDM frequencies can be divided into two sets: Ω_1 for the allowed in-bnad frequencies and Ω_0 corresponding to the nulling sub-band. Let $X(f)$ be Fourier transform of $x(t)$, the transmit energies within Ω_0 and Ω_1 are $\int_{\Omega_0} |X(f)|^2 df$ and $\int_{\Omega_1} |X(f)|^2 df$, respectively. A straightforward description of waveform optimization with a known data block can be

$$\begin{aligned} \boldsymbol{a}_{opt} = \arg\max_{\boldsymbol{a}} & \left\{ \frac{\int_{\Omega_1} |X(f)|^2 df}{\int_{\Omega_0} |X(f)|^2 df} \right\} \\ s.t. \;\; & \frac{|\gamma(\tau_0)|}{\gamma(0)} \le \alpha_0 \\ & \frac{|\gamma(\tau)|}{\gamma(0)} \le \alpha, \tau > \tau_0 \\ & \boldsymbol{a}^H \boldsymbol{a} \le P \end{aligned} \tag{3}$$

where $\boldsymbol{a} = (a_0, a_1, a_2, \cdots, a_{N-1})^T$, $\{\tau > \tau_0\}$ is the support of autocorrelation side lobes, α_0 is a positive constant used to achieve a narrow autocorrelation main lobe, α ($< \alpha_0$) represents an allowed peak relative value of autocorrelation side-lobe, and $P > 0$ is a positive number used to prevent the sub-carrier weight from being unbounded..

In order to use convex optimization, ratio maximization in (4) is expressed alternatively and the original formulation is changed to

$$\begin{aligned} \boldsymbol{a}_{opt} = \arg\max_{\boldsymbol{a}} & \left\{ \int_{\Omega_1} |X(f)|^2 df \right\} \\ s.t. \;\; & \int_{\Omega_0} |X(f)|^2 df < \epsilon \\ & |\gamma(\tau_0)| - \alpha_0 \gamma(0) \le 0 \\ & |\gamma(\tau)| - \alpha \gamma(0) \le 0, \tau > \tau_0 \\ & \boldsymbol{a}^H \boldsymbol{a} \le P \end{aligned} \tag{4}$$

where ϵ is a real constant for transmit power nulling over Ω_0.

To take advantage of powerful numerical optimization tools, a discrete-time analytical framework is adopted to handle optimization (5). Assume each symbol pulse is sampled Q times. Let $t_s = T/Q$ be the sampling interval, $K = MT/t_s = MQ$ be the total number of sample of the entire waveform, and $\boldsymbol{v}_n$ be a $K \times 1$ vector defined as

$$\begin{aligned} \boldsymbol{v}_n = \Big(& b_{n,0}, \; b_{n,1} e^{j2\pi(F_0+n\Delta f)t_s}, \cdots, b_{n,\lfloor k/Q \rfloor} e^{j2\pi(F_0+n\Delta f)kt_s}, \\ & \cdots, b_{n,\lfloor (K-1)/Q \rfloor} e^{j2\pi(F_0+n\Delta f)(K-1)t_s} \Big)^T, \\ & 0 \le n \le N-1 \end{aligned} \tag{5}$$

where $b_{n,\lfloor k/Q \rfloor}$ represents the k-th sample of the sub-data-stream on the n-th sub-carrier, and function $\lfloor \xi \rfloor$ rounds the element to the nearest integer. By defining a $K \times 1$ vector

$$\boldsymbol{x} = (x_0, x_1, \cdots, x_{K-1})^T \tag{6}$$

and a $K \times N$ matrix

$$V = (\boldsymbol{v}_0, \boldsymbol{v}_1, \cdots, \boldsymbol{v}_{N-1}), \tag{7}$$

the waveform $x(t)$ can be expressed in a matrix format:

$$\boldsymbol{x} = V\boldsymbol{a} \tag{8}$$

To derive discrete-time autocorrelation function, define a delay operator: for a $K \times 1$ vector $\boldsymbol{x}$ and $p \ge 0$, $\mathcal{D}(\boldsymbol{z}, p)$ is given by

$$\mathcal{D}(\boldsymbol{x}, p) = (\underbrace{0, 0, \cdots, 0}_{p \; 0's}, x_0, x_1, \cdots, x_{K-1-p})^T \tag{9}$$

The delay version of matrix V is defined as

$$\mathcal{D}(V, p) = (\mathcal{D}(\boldsymbol{v}_0, p), \mathcal{D}(\boldsymbol{v}_1, p), \cdots, \mathcal{D}(\boldsymbol{v}_{N-1}, p)) \tag{10}$$

With this delay operator, $\gamma(\tau)$ in sampled version can be written as

$$\begin{aligned} \gamma_p &= (\mathcal{D}(\boldsymbol{x}, p))^H \boldsymbol{x} \\ &= \boldsymbol{a}^H (\mathcal{D}(V, p))^H V\boldsymbol{a} \end{aligned} \tag{11}$$

and the support of side lobes may be expressed as $p \ge p_0$.

Now we can have convex versions of the other two constraints:

$$\begin{aligned} & |\gamma_{p_0}| - \alpha_0 \gamma_0 \\ = & \left| \boldsymbol{a}^H (\mathcal{D}(V, p_0))^H V\boldsymbol{a} \right| - \alpha_0 \left(\boldsymbol{a}^H V^H V \boldsymbol{a} \right) \le 0 \end{aligned} \tag{12}$$

and

$$\begin{aligned}&|\gamma_p| - \alpha\gamma_0\\&= \left|\boldsymbol{a}^H \left(\mathcal{D}(V,p)\right)^H V\boldsymbol{a}\right| - \alpha\left(\boldsymbol{a}^H V^H V\boldsymbol{a}\right) \le 0,\\&\quad p = p_0+1, p_0+2, \cdots, K-1\end{aligned} \tag{13}$$

To apply discrete Fourier transform (DFT) to equation (8), let $\tilde{\boldsymbol{x}}$ and $\tilde{\boldsymbol{v}}_n$ be the DFTs of $\boldsymbol{x}$ and $\boldsymbol{v}_n$, and denote $\tilde{V} = (\tilde{\boldsymbol{v}}_0, \tilde{\boldsymbol{v}}_1, \cdots, \tilde{\boldsymbol{v}}_{N-1})$. The waveform in discrete frequency domain is given by

$$\tilde{\boldsymbol{x}} = \tilde{V}\boldsymbol{a} \tag{14}$$

Denote $\mathcal{L}_0$ and $\mathcal{L}_1$ the discrete frequency index sets corresponding to Ω_0 and Ω_1. Each row of $\tilde{V}$ corresponds to a frequency index. Let $\tilde{V}_1$ be a matrix modified from $\tilde{V}$ by copying all rows of $\tilde{V}$ to $\tilde{V}_1$ and zeroing those rows corresponding to $\mathcal{L}_0$. Similarly, create matrix $\tilde{V}_0$ by zeroing those rows corresponding to $\mathcal{L}_1$ in $\tilde{V}$. Hence, the transmitted energies over $\mathcal{L}_0$ and $\mathcal{L}_1$ can be represented (omitting a scale) by $\boldsymbol{a}^H\tilde{V}_0^H\tilde{V}_0\boldsymbol{a}$ and $\boldsymbol{a}^H\tilde{V}_1^H\tilde{V}_1\boldsymbol{a}$, respectively.

Finally, the original optimization formulation (5) is converted to

$$\begin{aligned}\boldsymbol{a}_{opt} = \arg\max_{\boldsymbol{a}} &\left\{\boldsymbol{a}^H\tilde{V}_1^H\tilde{V}_1\boldsymbol{a}\right\}\\ s.t.\;\; &\boldsymbol{a}^H\tilde{V}_0^H\tilde{V}_0\boldsymbol{a} < \epsilon\\ &\left|\boldsymbol{a}^H\left(\mathcal{D}(V,p_0)\right)^H V\boldsymbol{a}\right| - \alpha_0\left(\boldsymbol{a}^H V^H V\boldsymbol{a}\right) \le 0\\ &\left|\boldsymbol{a}^H\left(\mathcal{D}(V,p)\right)^H V\boldsymbol{a}\right| - \alpha\left(\boldsymbol{a}^H V^H V\boldsymbol{a}\right) \le 0,\\ &\quad p = p_0+1, p_0+2, \cdots, K-1\\ &\boldsymbol{a}^H\boldsymbol{a} \le P\end{aligned} \tag{15}$$

This formulation can be expressed in a more compact way by denoting $A_1 = \tilde{V}_1^H\tilde{V}_1$, $A_0 = \tilde{V}_0^H\tilde{V}_0$, and $B_p = \left(\mathcal{D}(V,p)\right)^H V$:

$$\begin{aligned}\boldsymbol{a}_{opt} = \arg\max_{\boldsymbol{a}} &\left\{\boldsymbol{a}^H A_1\boldsymbol{a}\right\}\\ s.t.\;\; &\boldsymbol{a}^H A_0\boldsymbol{a} < \epsilon\\ &\left|\boldsymbol{a}^H B_{p_0}\boldsymbol{a}\right| - \alpha_0\left(\boldsymbol{a}^H B_0\boldsymbol{a}\right) \le 0\\ &\left|\boldsymbol{a}^H B_p\boldsymbol{a}\right| - \alpha\left(\boldsymbol{a}^H B_0\boldsymbol{a}\right) \le 0,\\ &\quad p = p_0+1, p_0+2, \cdots, K-1\\ &\boldsymbol{a}^H\boldsymbol{a} \le P\end{aligned} \tag{16}$$

The optimization problem in (16) is a Quadratic Constraint Quadratic Program (QCQP) problem known as being NP-hard [20], and SDP can be used as a relaxation means to obtain a suboptimal solution. For the principle and details about the SDP, refer to [18], [19]. The formulation (16) can be equivalently expressed as

$$\begin{aligned}\boldsymbol{a}_{opt} = \max_{\boldsymbol{a}} &\, tr\left(A_1\boldsymbol{a}\boldsymbol{a}^H\right)\\ s.t.\;\; &tr\left(A_0\boldsymbol{a}\boldsymbol{a}^H\right) < \epsilon\\ &\left|tr\left(B_{p_0}\boldsymbol{a}\boldsymbol{a}^H\right)\right| - \alpha_0\cdot tr\left(B_0\boldsymbol{a}\boldsymbol{a}^H\right) \le 0\\ &\left|tr\left(B_p\boldsymbol{a}\boldsymbol{a}^H\right)\right| - \alpha\cdot tr\left(B_0\boldsymbol{a}\boldsymbol{a}^H\right) \le 0,\\ &\quad p = p_0+1, p_0+2, \cdots, K-1\\ &tr\left(\boldsymbol{a}\boldsymbol{a}^H\right) \le P\end{aligned} \tag{17}$$

Let W be a $N \times N$ semi-definite Hermitian matrix. By changing the optimization variable from $\boldsymbol{a}$ to W and replacing $\boldsymbol{a}\boldsymbol{a}^H$ with W, the above QCQP problem can be converted into a SDP problem formulated below:

$$\begin{aligned}\tilde{W} = \max_{W} &\, tr\left(A_1 W\right)\\ s.t.\;\; &tr\left(A_0 W\right) < \epsilon\\ &\left|tr\left(B_{p_0}W\right)\right| - \alpha_0\cdot tr\left(B_0 W\right) \le 0\\ &\left|tr\left(B_p W\right)\right| - \alpha\cdot tr\left(B_0 W\right) \le 0,\\ &\quad p = p_0+1, p_0+2, \cdots, K-1\\ &tr\left(W\right) \le P\\ &W\ is\ Hermitian\\ &W \succ= 0\end{aligned} \tag{18}$$

where $\tilde{W}$ is the solution to the SDP problem, $tr(\cdot)$ is matrix trace operator, and notation $W \succ= 0$ means that W is a positive semidefinite matrix. The leading eigenvector of $\tilde{W}$, denoted by $\tilde{\boldsymbol{a}}$, is the resultant sub-carrier weight vector (omit a scale). If the rank of $\tilde{W}$ is one, then $\tilde{W} = \lambda\tilde{\boldsymbol{a}}\tilde{\boldsymbol{a}}^H$ with λ being a constant, and $\tilde{\boldsymbol{a}}$ is actually the optimal solution to the original QCQP problem in (16); otherwise, it is a suboptimal solution.

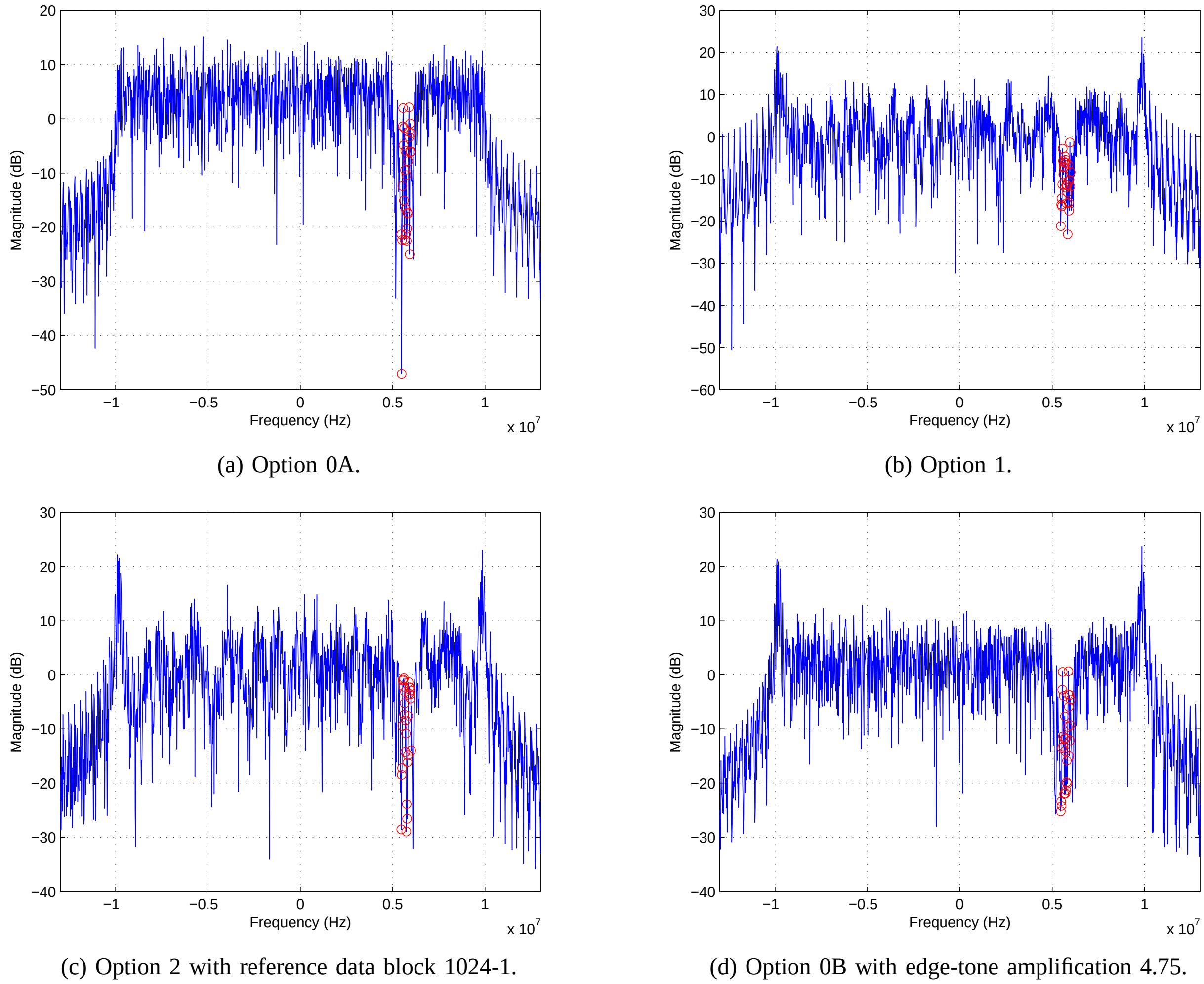
III. Design of Waveforms for Unknown Data Blocks

Sub-carrier muting is probably the simplest way to reduce interference in the sub-band shared with other systems, and it can be viewed as a kind of robust NC-OFDM waveform design that does not depend on any data block. However, this simple method usually does not lead to sufficient spectral nulling, since adjacent sub-carriers can have strong energy leaking in the nulling sub-band. The proposed SDP-based method is a more general waveform design technique that is able to compromise multiple design objectives. If a dataset $\mathcal{U} = \left\{\mathbf{b}^{(g)},\ 0 \le g \le G-1\right\}$ is unknown during the design, we need to find a set of sub-carrier weights that are good in general, or robust to these data blocks $\mathbf{b}^{(g)}$'s.

It is found by simulation that the following robust design approach for an unknown dataset works fine: replace the data-dependent matrices by matrices made from a reference data block with $M \gg 1$, then with the specified matrices, run the SDP algorithm and use the solution $\tilde{\boldsymbol{a}}$ as a "robust" design. As an example, a robust design for a dataset with $M = 16$ may be obtained by using a reference data block with $M = 1024$. Its robustness can be verified using simulation.

IV. Numerical Results

The proposed waveform design techniques are examined using computer simulation, with regular OFDM and NC-OFDM based waveforms as benchmarks in measuring spectral nulling depth etc. Three parameters are used to measure the waveform performance: nulling depth, range resolution, and side-lobe suppresion. The Nulling depth is defined as in-band energy ratio of the regular OFDM waveform to the proposed waveform, where the in-band energy is measured within the concerned sub-band and the regular waveform is generated



(a) Option 0A.

(b) Option 1.

(c) Option 2 with reference data block 1024-1.

(d) Option 0B with edge-tone amplification 4.75.

Fig. 1. Spectra for different options with data block 16-1.

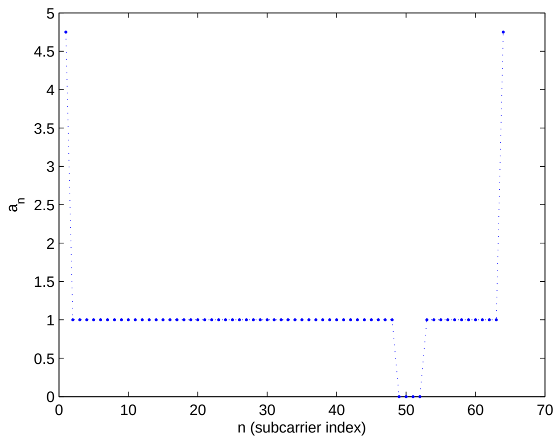
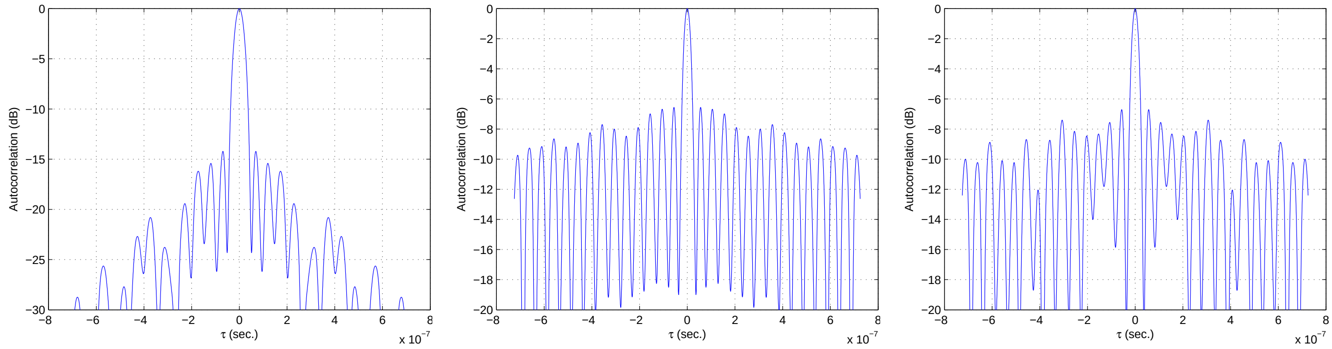


Fig. 2. Weight vector for Option 0B with amplification 4.75.

with data block 1024-1. Range resolution is defined as $c \cdot \Delta\tau$, where c is the speed of light and $\Delta\tau$ is the autocorrelation main lobe width calculated at 60% of the main lobe height (4.437 dB lower from the top). Side-lobe suppression is defined as the magnitude ratio of the autocorrelation main lobe to the highest side lobe. QPSK is used to modulate the data stream on each sub-carrier. 64 OFDM sub-carriers over a 20-MHz band are considered, and a single 500-kHz notch sub-band is centered at 5.75-MHz offset above the center of the 20-MHz band. Matlab CVX is used to implement the SDP-based algorithms. In generating random data blocks with Matlab, waveform length (M) in conjunction with random seed number are used to identify a data block. For instance, data block 16-2 has a length $M = 16$ and is generated with random seed number 2.

As observed from the optimal designs, the lowest and highest sub-carriers are weighted much heavily than others, which motivates an easy-use yet effective method to refine traditional NC-OFDM waveforms. This heuristic method, called edge-tone amplification, can be used to trade side-lobe suppression for range resolution. The following waveform design options are considered:

1) Option 0A – turning off frequency tones (traditional NC-OFDM)



(a) Option 0A. (b) Option 0B. (c) Option 1.

Fig. 3. Waveform autocorrelations based on data block 16-1; Option 0A has a wider mainlobe and a worse range resolution.

TABLE I
RESULTS FOR $N = 64$ OVER DATA BLOCKS 8-1 AND 16-1. THE VALUE OF EDGE-TONE AMPLIFICATION IS 4.75 FOR OPTION 0B.

		Nulling (dB)		Resolution (m)		Suppression (dB)	
	M=	8	16	8	16	8	16
Option	0A	12.00	11.21	15.68	15.83	14.40	14.31
	0B	12.30	10.73	11.33	11.48	6.54	6.53
	1	46.57	32.03	11.33	11.33	7.34	7.29
	2	29.05	29.08	12.08	12.08	7.45	7.19

TABLE II
RESULTS OF OPTION 2 FOR $M = 8, 16$, $N = 64$, OVER DATASETS $\{$8-1, 8-3, 8-5, 8-7$\}$ AND $\{$16-1, 16-3, 16-5, 16-7$\}$. THE REFERENCE DATA BLOCK USED IS 1024-1.

	Nulling (dB)		Resolution (m)		Suppression (dB)	
M=	8	16	8	16	8	16
Block 1	29.05	29.08	12.08	12.08	7.45	7.19
Block 3	29.43	30.72	12.08	12.08	7.64	7.54
Block 5	32.07	32.73	11.93	12.08	7.49	7.44
Block 7	30.64	31.77	11.93	12.08	7.61	7.58

2) Option 0B – turning off frequency tones plus edge-tone amplification
3) Option 1 – SDP optimization for a given data block
4) Option 2 – SDP optimization with the the required matrices generated with a randomly selected large data block

Fig.1 shows the spectra for four design options. It can be seen from Fig.1(b) and Fig.1(c) that SDP optimization leads to higher weights at the lower and upper corners of the signal band, which motivates Option 0B that sets heavier weights at the first and the last sub-carriers of an NC-OFDM waveform. Fig.1(d) is the resultant spectrum corresponding to the modified weight vector shown in Fig.2. The major impact of edge-tone amplification is a narrower autocorrection main lobe and increased side lobes, which can be observed in Fig.3.

Edge-tone amplification is a simple technique to trade side-lobe suppression for range resolution, but it does not really have influence on spectral nulling. From the results in Table I we can clearly see that the optimization methods (Option 1 and Option 2) perform better in compromising the three performance indicators. The cost of robustness can also be seen from the tests on blocks 8-1 and 16-1: overall speaking, the result of robust optimization (Option 2) is slightly worse than that of the non-robust optimization (Option 1).

Robustness may be judged by testing the performance over different data blocks. Table II shows the results of Option 2 over datasets $\{$8-1, 8-3, 8-5, 8-7$\}$ and $\{$16-1, 16-3, 16-5, 16-7$\}$, and a large reference data block 1024-1 is used to generate the estimates of the data-dependent matrices A_1, A_0 and B_p. Option 2 may not perform as good as the optimal Option 1 on a specific data block, but it perform quite well evenly over different data blocks.

V. CONCLUSIONS

Motivated by dual use of the OFDM waveform format for communications and radar in the ever-worsening EM coexistence environments, this paper proposes and verifies a few effective OFDM waveform design techniques to enable dual use under coexistence condition. The feasibility to achieve deep spectral notch while maintaining good range resolution and sufficient side-lobe suppression is demonstrated. Depending on if the data blocks for communications are known, a proper design method can be employed to synthesize the sub-carrier weights. In particular, robust designs over different data blocks can be obtained, so that less data-dependent designs can be employed for ranging and communications at the same time. The proposed work can be extended to consider other per-sub-carrier modulation schemes and more design constraints like Peak-to-Average Power Ratio (PAPR) which is commonly considered in OFDM waveform design [11]–[14], [21], [22].

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