A Comparative Study of Interpolation Methods for Mapping Soil Properties

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ABSTRACT

The choice of an optimal interpolation technique for estimating soil properties at unsampled locations is an important issue in site-specific management. The objective of this study was to evaluate inverse distance (InvD) weighting, ordinary kriging (KO), and lognormal ordinary kriging (KO_{log}) to determine the optimal interpolation method for mapping soil properties. Relationships between statistical properties of the data and performance of the methods were analyzed using soil test P and K data from 30 agricultural fields. For InvD weighting, we used powers of 1, 2, 3, and 4. The numbers of the closest neighboring points ranged from 5 to 30 for the three methods. The results suggest that KO_{log} can improve estimation precision compared with KO for lognormally distributed data. The criteria helpful in deciding whether KO_{log} is applicable for the given data set were the Kolmogorov–Smirnov goodness-of-fit statistic, coefficient of variation, skewness, kurtosis, and the size of the data set. Careful choice of the exponent value for InvD weighting and of the number of the closest neighbors for both InvD weighting and kriging (KO or KO_{log}) significantly improved the estimation accuracy ($P \leq 0.05$). However, no a priori decision could be made about the optimal exponent and the number of the closest neighbors based on the statistical properties of the data. For the majority of the data sets, kriging with the optimal number of the neighboring points, a carefully selected variogram model, and appropriate log-transformation of the data performed better than InvD weighting. Correlation coefficients between experimental data and estimated results of kriging were higher than those of InvD for 57 out of a total of 60 data sets, kriging mean absolute errors were lower for 44 data sets, and kriging mean errors were lower than those of InvD weighting for 31 data sets.

Precision agriculture applies principles of farming according to the field variability, which creates new requirements for estimating and mapping spatial variability of soil properties. Improvement in estimation quality depends, first, on reliable interpolation methods for obtaining soil property values at unsampled locations and, second, on appropriate application of the methods with respect to data characteristics.

The interpolation techniques commonly used in agriculture include inverse distance weighting and kriging (Franzen and Peck, 1995; Weisz et al., 1995). Both methods estimate values at unsampled locations based on the measurements from the surrounding locations with certain weights assigned to each of the measurements. Inverse distance weighting is easier to implement, while kriging is more time-consuming and cumbersome; however, kriging provides a more accurate description of the data spatial structure, and produces valuable information about estimation error distributions. The accuracy of these two procedures has been compared in a number of studies. Creutin and Obled (1982) and Tabios and Salas (1985) compared kriging with several other interpolation techniques, including inverse distance, for annual precipitation distributions and found kriging to be superior to inverse distance weighting. Warrick et al. (1988) also reported kriging to be better than inverse distance weighting for mapping potato (*Solanum tuberosum* L.) yield and soil properties, such as percent of sand, Ca content, and infiltration rate. Laslett et al. (1987) obtained more accurate pH predictions by using kriging than by using inverse distance weighting. Leenaers et al. (1990) found kriging to be superior to inverse distance weighting for the majority of their soil Zn content data sets. Criteria for comparing the methods were mean squared error (Warrick et al., 1988), sum of squared errors (Laslett et al., 1987), and correlation coefficients between observed and estimated values (Leenaers et al., 1990).

Several other studies, however, found inverse distance weighting to be more accurate than kriging. Weber and Englund (1992) found that squared inverse distance weighting produced better interpolation results than any other method, including kriging. Wollenhaupt et al. (1994) compared inverse distance weighting and kriging for mapping soil P and K levels and found inverse distance to be relatively more accurate. Gotway et al. (1996) observed the best results in mapping soil organic matter contents and soil NO_3^- levels for several fields when inverse distance was used as an interpolation technique. The studies used mean squared error as a main criterion for comparison (Weber and Englund, 1992; Gotway et al., 1996).

Kriging performance can be significantly affected by variability and spatial structure of the data (Leenaers et al., 1990), and by the choice of variogram model, search radius, and the number of the closest neighboring points used for estimation. The above-mentioned studies by Weber and Englund (1992), Wollenhaupt et al. (1994), and Gotway et al. (1996) used a number of simplified assumptions in kriging applications. For example, the choice of the variogram model was limited to a spherical model, and a fixed number of the closest neighboring points was used for all the data sets. In a subsequent study, Weber and Englund (1994) noted that judicious selection of the variogram model and of the number of the closest neighbors used for the estimation led to significantly better estimation precision.

It has been observed that many of soil properties are lognormally rather than normally distributed. Numerous examples were reviewed by Parkin and Robinson (1992), including aggregate size, soil water flux, hydraulic conductivity, content of soil N, and concentration of

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Abbreviations: *D*, Kolmogorov–Smirnov goodness-of-fit statistic; *G*, goodness-of-prediction statistic; InvD, inverse distance; KO, ordinary kriging; KO_{log}, lognormal ordinary kriging; MAE, mean absolute error; ME, mean error; RI, relative improvement.

soil pore CO_2. Lognormal ordinary kriging has been proposed as an alternative to ordinary kriging for lognormally distributed data (Rendu, 1979; Journel, 1980; Rivoirard, 1990). It has been shown that if the distributions of the experimental data are clearly lognormal, log-transformation of the original data can result in better estimations (Journel and Huijbregts, 1978; Candela et al., 1988). However, lognormal ordinary kriging can be very sensitive to the back-transformation method and prone to produce biased estimation results, which makes its application a challenge (Journel, 1980; Deutsch and Journel, 1998). Our first objective was to compare inverse distance weighting, ordinary kriging, and lognormal ordinary kriging in order to determine the optimum method for mapping soil properties. The second objective was to analyze relationships between statistical properties of the data and performance of the interpolation technique.

MATERIALS AND METHODS

Data

We used soil test P and K data collected from 30 agricultural fields in Illinois, Indiana, and Iowa. The fields were sampled on a regular grid, as is often done in commercial soil sampling, even though there is no mathematical requirement for grid sampling. The distance between sampling locations varied from 25 to 100 m for different fields (Table 1). Statistical summaries of the P and K data, along with the number of data points for each field, are shown in Table 1.

Interpolation Techniques

Since detailed information about interpolation procedures can be found elsewhere in the literature (Journel and Huijbregts, 1978; Isaaks and Srivastava, 1989), we only briefly describe the methods used in the study. For both inverse distance (InvD) and kriging (KO or KO_{log}) interpolation methods, the value of variable Z at unsampled location x_0, $Z^*(x_0)$ is estimated based on the data from the surrounding locations, $Z(x_i)$, as

$$Z^*(x_0) = \sum_{i=1}^{n} w_i \, Z(x_i) \qquad [1]$$

where w_i are the weights assigned to each $Z(x_i)$ value and n is the number of the closest neighboring sampled data points used for estimation. The weights for the inverse distance method are

$$w_i = \frac{1/d_i^p}{\sum_{i=1}^{n} 1/d_i^p} \qquad [2]$$

where d_i is the distance between the estimated point and the sample and p is an exponent parameter. It has been shown that the choice of the exponent value can significantly affect estimation quality (Isaaks and Srivastava, 1989; Weber and Englund, 1994; Gotway et al., 1996). In our study, we compared InvD estimates with powers of 1, 2, 3, and 4. It is interesting to note that most of the commercial software that is available currently for the production of soil fertility maps for guidance of variable rate application equipment uses default exponent values of 2 or 4 without checking the appropriateness of those values (Agris, 1998, p. 147–171).

The other factor affecting the precision of InvD weighting is the number of the closest samples used for estimation. In this study, two cases were considered. In the first case, the number of the closest samples was fixed at 12, which is consistent with traditional approach to InvD weighting. In the second case, we applied InvD weighting with the number of the closest samples varying from 5 to 30. The search radius chosen

Table 1. Statistical summary of P and K contents, data collected from 30 agricultural fields in Illinois, Indiana, and Iowa.

			P				K			
Data set	Samples	Grid distance	Mean	CV	SK†	KT†	Mean	CV	SK	KT
	no.	m	kg ha^{-1}	%			kg ha^{-1}	%		
1	64	50	110	63	1.9	4.2	705	41	1.9	4.1
2	144	65	84	54	2.1	6.1	370	38	2.9	11.4
3	106	65	73	58	2.0	6.4	393	31	1.3	2.0
4	64	50	113	33	0.8	0.01	623	20	1.1	0.5
5	120	65	81	42	1.5	4.0	436	30	2.3	10.5
6	43	65	72	35	0.8	0.3	—	—	—	—
7	56	50	60	52	1.4	2.2	373	35	1.7	3.8
8	74	65	151	48	0.7	0.3	360	31	1.3	3.7
9	54	65	104	52	1.8	6.3	496	21	0.6	1.0
10	52	65	105	52	0.9	0.3	418	23	1.1	1.1
11	48	50	83	58	3.7	17.6	451	36	2.9	13.2
12	256	25	71	48	1.3	1.8	271	27	1.9	8.6
13	64	50	43	65	1.8	2.7	355	21	1.7	5.1
14	48	50	88	58	1.4	2.2	494	27	1.1	0.9
15	106	65	74	45	1.9	6.5	429	24	0.6	−0.2
16	48	65	110	39	1.4	2.8	497	23	0.8	0.9
17	36	65	130	42	1.0	1.5	271	31	1.0	0.7
18	74	65	119	68	1.6	3.0	431	33	2.6	10.1
19	390	65	105	53	1.7	5.2	488	30	1.5	3.6
20	111	65	72	43	1.7	3.7	386	23	1.1	1.2
21	1752	50	66	70	2.6	8.6	421	45	4.7	32.1
22	132	65	75	64	2.2	9.4	423	18	1.3	3.9
23	75	65	119	63	3.6	16.8	521	50	2.5	7.5
24	78	65	158	69	1.5	0.9	488	34	3.3	16.7
25	64	50	94	48	1.7	4.8	463	23	2.0	7.1
26	72	65	166	28	2.4	11.4	803	19	0.9	3.1
27	125	100	112	46	0.6	−0.3	418	38	0.4	0.03
28	61	100	110	26	0.5	0.1	576	24	1.1	2.3
29	64	100	108	38	0.7	−0.1	548	29	0.6	1.0
30	72	65	55	28	0.1	−0.6	614	18	−0.1	−0.8

† SK, skewness; KT, kurtosis.

was large enough to include the required number of the closest samples. Cross-validation was used to compare the results obtained with different number of the closest samples. Each value from the data set was eliminated in turn, and then estimated using information from the rest of the data (Kane et al., 1982; Goovaerts, 1997). The exponent value and the number of the closest neighboring points producing the best agreement between the measured data and the estimates were chosen as the optimal InvD weighting parameters.

Kriging calculates the values of w_i by estimating spatial structure of the variable's distribution represented by a sample variogram as

$$\gamma(h) = 1/2n \sum_{i=1}^{n} [Z(x_i + h) - Z(x_i)]^2 \quad [3]$$

where x_i and $x_i + h$ are sampling locations separated by a distance h, and $Z(x_i)$ and $Z(x_i + h)$ are measured values of the variable Z at the corresponding locations. The sample variogram is fitted with a variogram model and adequacy of the chosen model is tested using cross-validation. In this study, we considered spherical, Gaussian, and exponential models for the sample variogram fitting. The cross-validation was conducted with varying model parameter values and with numbers of the closest neighboring samples ranging from 5 to 30 until the highest estimation accuracy was reached. Accuracy of the selected variogram model was measured through the error between the measured data and the estimated values (Zhang et al., 1995). Cross-validation criteria used for sample variogram selection were (Myers, 1991) the correlation coefficient between measured and estimated values, mean error (ME),

$$\mathrm{ME} = 1/n \sum_{i=1}^{n} [Z^*(x_i) - Z(x_i)] \quad [4]$$

mean absolute error (MAE) (David, 1988),

$$\mathrm{MAE} = 1/n \sum_{i=1}^{n} [|Z^*(x_i) - Z(x_i)|] \quad [5]$$

and the reduced kriging variance (Zhang et al., 1992)

$$\mathrm{KRE} = \mathrm{MSE}/\mathrm{MKV} \quad [6]$$

where MSE is the mean squared error and MKV is the mean kriging variance for the studied data set. We selected the variogram model that resulted in the best values of the cross-validation criteria. Ordinary kriging (KO) was used to estimate weights based on the variogram model, and the variable values at unsampled locations were obtained using Eq. [1].

Lognormal ordinary kriging ($\mathrm{KO}_{\log}$) was performed similarly to the previously described ordinary kriging, except that natural logarithms were used rather than the actual data. Back-transformation of the lognormal estimates was performed using following equation (Rendu, 1979; Rivoirard, 1990; Weber and Englund, 1992):

$$Z(x_i) = \exp[Y(x_i) + 1/2V(x_i) - \lambda_i] \quad [7]$$

where $Y(x_i)$ is the lognormal kriging estimate, and $V(x_i)$ and λ_i are corresponding kriging variance and Lagrange multiplier. Kolmogorov–Smirnov goodness-of-fit test (Kanji, 1993) was used to analyze the data distributions.

To compare different interpolation techniques, we analyzed the errors between the measured data and the estimates using criteria such as mean error, mean absolute error, correlation coefficient, and the goodness-of-prediction statistic, G (Agterberg, 1984; Gotway et al., 1996):

$$G = \left\{1 - \sum_{i=1}^{n} [Z^*(x_i) - Z(x_i)]^2 / \sum_{i=1}^{n} [Z_m - Z(x_i)]^2\right\} \times 100 \quad [8]$$

where Z_m is the sample mean. Geostatistical analysis consisting of variogram calculation, cross-validation, and kriging was performed using the geostatistical software package GSLIB (Deutsch and Journel, 1998).

RESULTS AND DISCUSSION

A statistical summary of the P and K contents is presented in Table 1. Mean values of P and K contents varied for different fields, with maximums of 166 and 803 and minimums of 43 and 271 kg ha^{-1} for P and K, respectively. Variability of K content was less than the variability of P content for most of the studied fields, with an average coefficient of variation equal to 49% for P and 29% for K. The majority of the data sets had high positive skew and kurtosis values (Table 1). Histograms for P and K contents were constructed and plotted, along with theoretical normal and lognormal probability density functions. Examples of the histograms for four of the data sets are shown in Fig. 1. Kolmogorov–Smirnov D-statistics were calculated for fitting normal and lognormal distributions to the experimental data (Table 2). Since D-values vary with the histogram features, such as number of classes and class size, we chose the number of classes that produced the lowest D-values for fitting data with normal distribution and, hence, the closest correspondence between experimental and normal distributions. The same number of classes was also used for fitting lognormal distributions. For all of the data sets, D for lognormal distribution ($D_{\ln}$) was smaller than the appropriate table D-value at 0.01 significance level (Table 16 in Kanji, 1993), that indicated that at 0.01 significance level all of the data could be assumed to be lognormally distributed. For the majority of the data (21 P data sets and 24 K data sets), D-values from normal distribution (D_n) were lower than table values at 0.01 significance level as well, implying that either assumption of normal or lognormal distribution could be used for the data. Although the Kolmogorov–Smirnov test was not sufficient to distinguish between normal and lognormal distributions at $P \leq 0.01$, we still used D as a criterion for deciding which distribution, normal or lognormal, is the most appropriate for a given data set. Data sets were divided into two groups. Those with $D_n \leq D_{\ln}$ were placed into normally distributed group; those with $D_{\ln} \leq D_n$ were placed into lognormally distributed group.

Omnidirectional sample variograms for both original and log-transformed P and K contents were calculated using Eq. [3] and fitted with variogram models. Directional sample variograms were calculated in the south–north and east–west directions. For some of the data sets, there were not sufficient samples to produce a valid directional variogram; hence, only omnidirectional variograms were considered. For the data sets with sufficient samples, the directional variograms were inspected visually. Since there was no an underlying physical phenomenon to cause the data to be anisotropic, and there was no apparent anisotropy in directional variograms, only omnidirectional variograms were used in further analysis (Goovaerts, 1997). Most of the sample variograms were best fitted with spherical models. Gaussian models were used for 12 P and K data sets, and exponen-

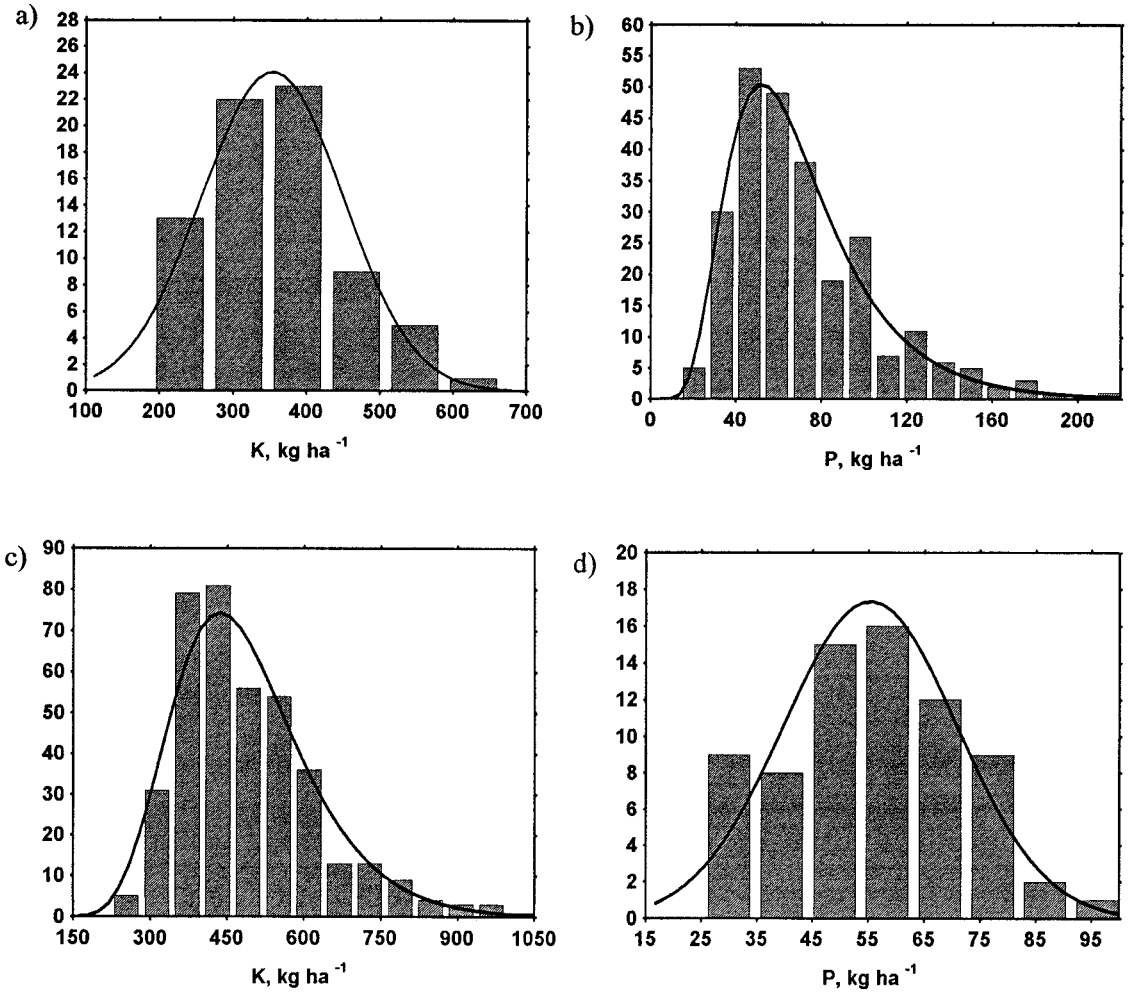

Fig. 1. Examples of the experimental histograms for (a) Data Set 8, (b) Data Set 12, (c) Data Set 19, and (d) Data Set 30 (see Table 1). The solid line represents either the theoretical normal distribution (Data Sets 8 and 30) or the theoretical lognormal distribution (Data Sets 12 and 19).

tial models were used for 4 data sets. Ranges of the selected variogram models are presented in Table 2.

Table 3 presents correlation coefficients, mean errors, mean absolute errors, and goodness-of-prediction *G*-values obtained by comparing measured data with estimates of ordinary kriging (KO) and lognormal ordinary kriging (KO_{log}), averaged for the data sets. For the data from the lognormally distributed group, KO_{log} produced overall better results than KO. Correlation coefficients and *G*-values from KO_{log} were higher then those from KO for 42 and 41 data sets, respectively (out of total 45 lognormally distributed P and K data sets). Mean absolute errors of KO_{log} were lower than those of KO for 30 out of 45 data sets. Mean errors of KO_{log} also were lower than or close to those of KO for the majority of the data, although some relatively high negative values were observed. For the data from the normally distributed group, KO produced better results than KO_{log}. Correlation coefficients for 9 out of total 14 normally distributed data sets were higher for KO. *G*-values from KO were higher for 8 normally distributed data sets. Mean absolute errors of KO were lower than those of the KO_{log} for all except 3 normally distributed data sets. The most significant difference was observed in mean error values. Large negative mean errors were obtained by applying KO_{log} to the data from the normally distributed group for 12 out of total 14 data sets, with the maximum observed mean error of −17.7. The negative bias in KO_{log} estimations was attributed to the deviation from lognormality in the data distributions (David, 1988). Link and Koch (1975) showed that negative bias is possible when lognormal transformation was used for positively skewed but not exactly lognormal distributions. For such data sets, either KO or KO_{log} would be appropriate, depending on the objectives of the investigation.

We observed that the improvement in estimation precision due to using lognormal kriging for the data from the lognormally distributed group could be related to the number of samples in the studied data sets. Three large data sets had positively skewed data that were significantly better fitted by lognormal than normal distribution (Data Sets 21, 19, and 12, with 1752, 390 and 256 samples, respectively; Table 1); however, KO_{log} did not considerably improve estimation precision compared with KO. The relative improvement in the corre-

Table 2. Kolmogorov–Smirnov *D*-statistics for fitting the soil P and K data with normal (D_n) and lognormal (D_{ln}) distributions, the inverse distance optimum exponent value (p), and the actual correlation length (R).

	P					K				
Data set	Dist.†	D_n	D_{ln}	p	R	Dist.†	D_n	D_{ln}	p	R
					m					m
1	ln	0.137	0.040	4	152	ln	0.173	0.091	2	182
2	ln	0.141	0.043	4	213	ln	0.147	0.062	4	213
3	ln	0.099	0.026	1	152	ln	0.108	0.047	1	253
4	ln	0.098	0.039	1	82	ln	0.091	0.061	4	182
5	ln	0.082	0.014	1	608	ln	0.062	0.027	2	395
6	ln	0.093	0.056	1	82		—	—	—	—
7	ln	0.090	0.055	1	91	ln	0.060	0.020	4	143
8	ln	0.073	0.061	4	228	n	0.030	0.052	4	167
9	ln	0.088	0.051	3	600	ln	0.044	0.040	3	182
10	ln	0.107	0.026	1	182	n	0.027	0.030	4	152
11	ln	0.208	0.100	4	91	ln	0.136	0.091	4	106
12	ln	0.106	0.019	4	304	ln	0.072	0.018	4	85
13	ln	0.176	0.054	4	144	ln	0.094	0.051	4	129
14	ln	0.104	0.039	4	182	n	0.021	0.036	4	91
15	ln	0.096	0.013	3	274	ln	0.033	0.016	2	274
16	ln	0.078	0.023	1	152	n	0.025	0.035	1	179
17	ln	0.062	0.031	1	182	ln	0.086	0.044	3	243
18	ln	0.099	0.030	1	426	ln	0.101	0.034	1	304
19	ln	0.057	0.011	3	608	ln	0.062	0.030	2	152
20	ln	0.111	0.030	1	82	ln	0.092	0.051	1	213
21	ln	0.137	0.022	3	395	ln	0.122	0.050	3	334
22	ln	0.073	0.021	4	219	ln	0.076	0.070	2	243
23	ln	0.182	0.058	4	198	ln	0.196	0.099	4	426
24	ln	0.256	0.155	4	122	n	0.076	0.098	4	122
25	ln	0.098	0.032	1	304	n	0.025	0.041	4	122
26	n	0.034	0.051	1	228	n	0.039	0.068	3	213
27	n	0.051	0.056	1	380	n	0.079	0.110	2	532
28	ln	0.040	0.033	4	137	n	0.029	0.042	1	258
29	ln	0.055	0.021	1	152	n	0.018	0.062	1	152
30	n	0.043	0.058	1	243	n	0.024	0.061	1	204

† Within P or K, the data set is assumed to normally (n) or lognormally (ln) distributed.

lation coefficient for these data seems to decrease as the number of samples in the data set increases. Relative improvement (RI) in the correlation coefficient value due to using lognormal kriging was calculated as

$$\mathrm{RI} = (R_{\mathrm{KOlog}} - R_{\mathrm{KO}})/R_{\mathrm{KO}} \quad [9]$$

where R_{KOlog} and R_{KO} are correlation coefficients between measured data and estimates of $\mathrm{KO}_{\log}$ and KO, respectively. Apparently, the influence of outliers that cause the data distribution being positively skewed decreases with an increase in the size of the data set and the data distribution behaves more like a normal distribution. Hence, the advantage of using log-transformation disappears with an increase in the size of the data set. The plot of RI versus the number of samples for three largest data sets, along with the average RI and average sample size for the remaining data sets, is shown in Fig. 2.

Data statistics such as coefficient of variation, skewness, and kurtosis can be used along with the Kolmogorov–Smirnov parameter D as an approximate indicator of how much better $\mathrm{KO}_{\log}$ will perform than KO. As a general trend for lognormally distributed data, larger RI values were observed for data with larger differences between the values of D_{n} and D_{ln}. Higher values of skewness and kurtosis also frequently corresponded to the higher RI values. Significant positive correlation ($R = 0.506$, $P = 0.05$) was observed between mean errors of $\mathrm{KO}_{\log}$ and coefficients of variation. Coefficients of variation for P and K data plotted versus mean error values are shown in Fig. 3. The plot suggests that $\mathrm{KO}_{\log}$ is more likely to produce biased estimates (large negative mean errors) for the data sets with low coefficients of variation, while more accurate estimates can be expected for those with high coefficients of variation.

After we chose the best estimation procedure (either

Table 3. Comparison of the ordinary kriging (KO) and lognormal ordinary kriging ($\mathrm{KO}_{\log}$) for normally and lognormally distributed data sets (P and K data collected from 30 agricultural fields).

Data sets with	N†	Corr. coef.		G‡		ME§		MAE¶	
		KO	$\mathrm{KO}_{\log}$	KO	$\mathrm{KO}_{\log}$	KO	$\mathrm{KO}_{\log}$	KO	$\mathrm{KO}_{\log}$
lognormal P	27	0.520	0.548	25.9	29.2*	0.48	−0.97*	28.2	27.7
normal P	3	0.544	0.541	33.1	31.9	0.56	−3.37*	24.8	25.3
lognormal K	18	0.532	0.554	27.8	30.7*	1.19	−3.08*	77.8	76.3
normal K	11	0.495	0.486	26.7	25.7	0.63	−8.64*	82.0	84.4

* Within statistics, the difference between KO and $\mathrm{KO}_{\log}$ was significant at the 0.05 probability level.
† Values are the average for N data sets.
‡ G, goodness-of-prediction statistic (Eq. [8]).
§ ME, mean error (Eq. [4]).
¶ MAE, mean absolute error (Eq. [5]).

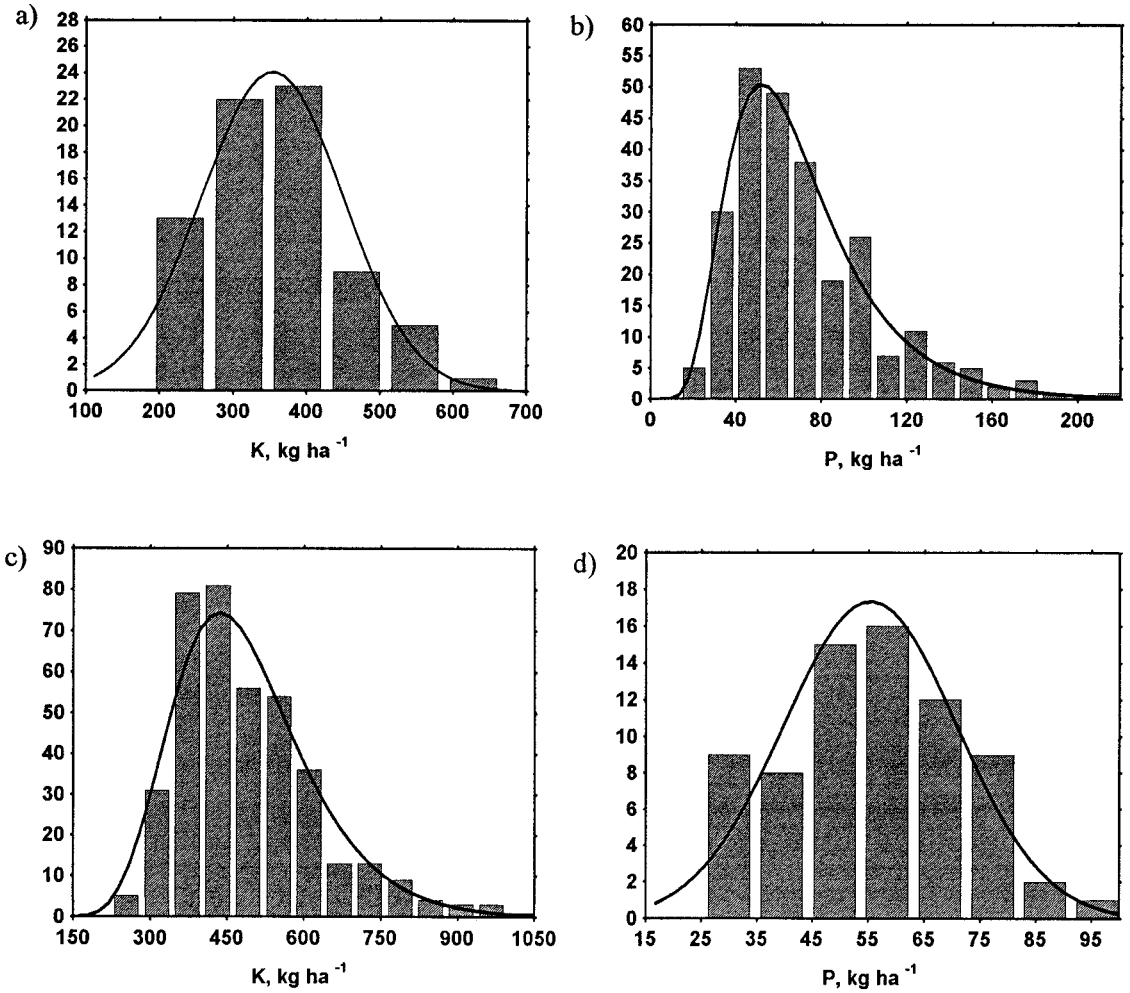

Fig. 2. Relative improvement (RI) in the correlation coefficient for lognormally distributed data as a function of data set size. The first point represents average sample size and average RI for the data sets, with the number of data points varying from 36 to 144. Vertical bars denote standard error of the RI.

KO or KO_{log}) for each data set, we compared it with the estimations produced by InvD weighting. Average values of *G* goodness-of-prediction criteria, correlation coefficients, mean errors, and mean absolute errors for the studied data are shown in Table 4. A noticeable difference in estimation precision was observed for different exponent values of InvD weighting and different numbers of the closest neighboring points. For the majority of the studied data, a power of 4 produced the best estimations, while for some data sets (22 out of total 60) a power of 1 was the best (Table 2). For 14 data sets, a power of 2 or 3 produced the most accurate results. Gotway et al. (1996) noted that the coefficient of variation of the data can be an indicator of which exponent value to use. They observed that InvD weighting with a power of 1 performed the best for data with high coefficients of variation, while InvD weighting with a power of 4 performed better for the less variable data. In this study, no significant correlation was found between the exponent value and the coefficient of variation of the studied data. Weber and Englund (1994) found that InvD weighting with a power of 1 resulted in higher estimation quality for the data with high skewness and kurtosis, while greater values for the exponent produced better estimations for data with low skewness and kurtosis. In this study, we observed that the majority of the data with high skewness (>2.5) were the best estimated with a power of 4 (5 out of 8 data sets). For most of the data with low skewness (<1), a power of 1 yielded the most accurate estimates (9 out of 15 data sets). The numbers of the closest neighboring points varied with field and exponent. No correlation was observed between the numbers of the closest neighbors and data statistics for the studied fields.

Optimum kriging (either KO or KO_{log}) with an optimal number of neighboring points produced significantly better results than the 12-point InvD weighting with a power of 4, which is the traditional approach to InvD applications. Correlation coefficients and *G*-values from kriging were higher than those of InvD weighting for all of the studied data sets, and kriging MAEs were lower than those of InvD weighting. Kriging MEs for the majority of the data were lower than those of InvD weighting; however, a few relatively large negative kriging mean errors were observed. Kriging with an optimal number of neighboring points and an optimal exponent also produced more accurate estimations than the InvD method for the majority of the studied data. Kriging correlation coefficients were higher than those of InvD weighting for 57 data sets, kriging *G*-values

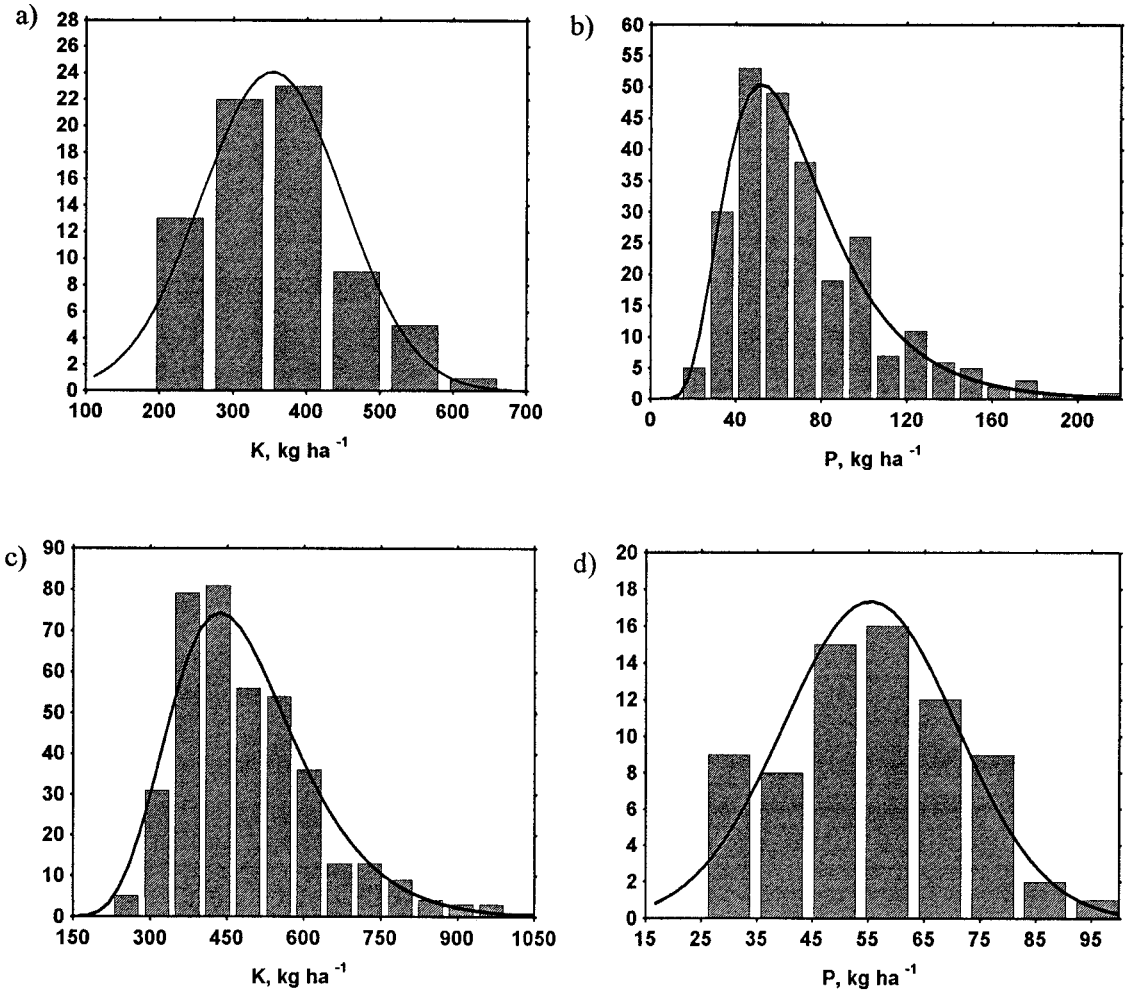

Fig. 3. Coefficients of variation for P and K data vs. mean errors between experimental data and estimates of lognormal ordinary kriging.

were higher for 59 data sets, kriging MAEs were lower for 44 data sets, and kriging MEs were lower for 31 data sets.

SUMMARY AND CONCLUSIONS

Data for P and K from 30 experimental fields were used to compare performance of three interpolation techniques: inverse distance weighting, ordinary kriging, and lognormal ordinary kriging. Our results indicate that lognormal ordinary kriging can be expected to produce overall better estimations for lognormally distributed data than ordinary kriging. For some data sets, however, lognormal ordinary kriging can result in biased estimations, with relatively high negative mean error between measured data and estimates. The Kolmogorov–Smirnov goodness-of-fit parameter D can be used as an indicator of which kriging procedure will result in better estimation quality. If D obtained by fitting data with lognormal distribution is lower than D from using normal distribution (i.e., $D_{ln} < D_n$), lognormal ordinary kriging can be expected to produce results with higher correlation coefficients and lower mean absolute error between measured data and estimates. In

Table 4. Comparison of the optimal kriging method (i.e., KO_{log} for the lognormally distributed data and KO for the normally distributed data), the inverse distance method with an exponent value of 4 and with 12 closest neighboring points ($InvD_{12}$), and the inverse distance method with an optimal exponent value and the optimal number of the closest neighboring points ($InvD_{opt}$). P and K data collected from 30 agricultural fields.

Data sets with	N†	Corr. coef.			G‡			ME§			MAE¶		
		Kriging	$InvD_{12}$	$InvD_{opt}$	Kriging	$InvD_{12}$	$InvD_{opt}$	Kriging	$InvD_{12}$	$InvD_{opt}$	Kriging	$InvD_{12}$	$InvD_{opt}$
lognormal P	27	0.547	0.483*	0.495*	29.2	20.6*	24.5*	−0.97	1.02*	0.96*	26.8	32.7*	30.6
normal P	3	0.544	0.493	0.547	33.1	26.4	30.9	0.50	0.54	0.30	22.1	26.3	24.8
lognormal K	18	0.554	0.432*	0.518	30.7	23.4*	25.6*	−3.08	2.01*	2.51*	76.3	79.8	78.2
normal K	11	0.495	0.440	0.460	26.7	18.4*	23.6	0.63	1.76	1.64	82.0	85.7	83.9

* Within statistics, the difference between optimal kriging and $InvD_{12}$ or $InvD_{opt}$ was significant at the 0.05 probability level.
† Values are the average for N data sets.
‡ G, goodness-of-prediction statistic (Eq. [8]).
§ ME, mean error (Eq. [4]).
¶ MAE, mean absolute error (Eq. [5]).

cases when D from normal distribution is smaller or the difference between D-values is relatively small, ordinary kriging should be preferred. Statistical properties of the data can also be useful in deciding between ordinary and lognormal ordinary kriging. High values of coefficient of variation, skewness, and kurtosis can be an indication that lognormal ordinary kriging will be a better choice. Lognormal ordinary kriging is more likely to produce large negative mean errors between measured and estimated values for data sets with relatively low coefficients of variation (CV $<$ 40%), than for those with high coefficients of variation. Ordinary kriging seems to be a safer choice of interpolation technique than lognormal ordinary kriging for data sets with more than 200 data points. For such data sets, regardless of whether data are normally or lognormally distributed, improvement in estimation precision due to lognormal ordinary kriging was negligible.

Comparing kriging with inverse distance weighting revealed that kriging with the optimal number of neighboring points, a carefully selected variogram model, and appropriate log-transformation of the data produces more accurate estimations than the inverse distance method for the majority of the data. If inverse distance weighting is used as an interpolation technique, a significant improvement in estimation precision can be achieved by selecting an optimal number of the closest neighboring points and an optimal exponent value. However, it seems that no exact recommendation about the choice of exponent value and the optimal number of neighboring points can be provided by analyzing the data statistics. Hence, the choice of the estimation parameters, such as optimal exponent and the number of the closest neighbors for inverse distance weighting, and the number of the closest neighbors and the variogram parameters for kriging, should be based on a comprehensive analysis of a wide range of parameter values.

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