

An On-Line Robust and Adaptive T-S Fuzzy-Neural Controller for More General Unknown Systems

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Abstract

This paper proposes a novel method of on-line modeling via the Takagi-Sugeno (T-S) fuzzy-neural model and robust adaptive control for a class of general unknown nonaffine nonlinear systems with external disturbances. Although studies about adaptive T-S fuzzy-neural controllers have been made on some nonaffine nonlinear systems, little is known on the more complicated and general nonlinear systems. Compared with the previous approaches, the contribution of this paper is an investigation of the more general unknown nonaffine nonlinear systems using on-line adaptive T-S fuzzy-neural controllers. Instead of modeling these unknown systems directly, the T-S fuzzy-neural model approximates a so-called virtual linearized system (VLS), with modeling errors and external disturbances. We prove that the closed-loop system controlled by the proposed controller is robust stable and the effect of all the unmodeled dynamics, modeling errors and external disturbances on the tracking error is attenuated under mild assumptions. To illustrate the effectiveness and applicability of the proposed method, simulation results are given in this paper.

Keywords: *fuzzy-neural model, on-line modeling, general unknown systems.*

1. Introduction

Most physical systems are described by a set of differential equations. Research has focused on the development of various design techniques for controllers of these systems. The existence of a mathematical model of the system is assumed for model-based control. Controllers are designed to modify the behavior of the system and achieve some desired performance [1]. To this purpose, a systematic way to construct a model mapping the inputs to the outputs is needed. Fuzzy models are usually

used in the case where the model structure and parameters are unknown [2]. There are two fuzzy model structures, Takagi-Sugeno (T-S) and Mamdani. T-S fuzzy systems are nonlinear systems described by a set of IF-THEN rules. Such a model can approximate a wide class of nonlinear systems. In [3-5], the authors proved that the T-S fuzzy system can approximate any continuous function to any precision.

By using well-known off-line tuning algorithms for unknown nonlinear systems, an initial fuzzy-neural model with adjustable parameters can be constructed. However, the derived fuzzy-neural model with the off-line tuned parameters cannot cope with parameter changes arising from some external disturbance [6]. Thus, off-line algorithms cannot be applied to situations where real-time processing is required such as adaptive control and signal processing. In these situations, the adjustable parameters must be tuned on-line during operation to compensate for undesirable effects. The objective of adaptive control is to maintain consistent performance of a system in the presence of uncertainties. Ideally, then, a fuzzy-neural controller would be adaptive [6-8]. Further issues are stability analysis and controller design of T-S fuzzy-neural controlled systems [9-10]. These have been extensively investigated in the literature. The existence of a common positive definite matrix for a set of Lyapunov inequalities is a sufficient condition for stabilization [11-14]. However, this is very difficult to achieve by using an on-line approach, even using the well-known linear matrix inequalities (LMIs) method [11-14]. Therefore, in this paper, adaptive schemes are used for simultaneous online modeling and controller design, instead of off-line modeling. Moreover, stability analysis of the adaptive T-S fuzzy-neural controlled systems is easier than that of the LMIs method.

The stabilization problem for the systems represented in T-S fuzzy-neural models has been addressed, e.g. [12, 15], but studies concerning tracking controller design based on T-S fuzzy-neural models for unknown nonlinear systems are relatively few. Tracking control designs for unknown nonlinear systems are important issues for practical applications. In [6], the authors only consider the stabilization problem for affine systems. In this paper, we apply the on-line adaptive T-S fuzzy-neural modeling approach to the design of robust tracking controllers for

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Manuscript received 31 Jan. 2008.

the more general unknown nonaffine nonlinear systems.

On the whole, this paper deals with the Takagi-Sugeno (T-S) fuzzy-neural model because of their capability to approximate dynamic nonlinear systems [16-19]. Although studies about adaptive T-S fuzzy-neural controllers have been made on some nonaffine nonlinear systems, little is known about the more complicated and general nonlinear systems. In [23, 26], the authors proposed that the nonaffine nonlinear system possesses a strong relative degree and then they transformed it into the state-space form. In [27], the authors considered that the nonaffine problems have an input nonlinearity which is algebraically invertible with respect to the available control action. Compared with the previous approaches [19, 23, 26, 27], the contribution of this paper is an investigation of the more general uncertain nonaffine nonlinear systems using adaptive T-S fuzzy-neural controllers. Instead of modeling these unknown systems directly, the T-S fuzzy-neural model approximates a so-called virtual linearized system (VLS), with modeling errors and external disturbances. We propose an on-line identification algorithm for the T-S fuzzy-neural model and put significant emphasis on the robust tracking controller design using the adaptive scheme for a class of general uncertain nonaffine nonlinear systems.

The rest of the paper is organized as follows. Section 2 reviews T-S fuzzy-neural model and fuzzy-neural networks. Section 3 introduces the T-S fuzzy-neural model for the virtual linearized system (VLS). Section 4 presents a controller design for online modeling and robust tracking. In Section 5, simulation results are presented to confirm the effectiveness and applicability of the proposed method. Finally, conclusions are given in Section 6.

2. T-S Fuzzy-Neural Model

Figure 1 shows the configuration of the T-S fuzzy-neural model [18], which is a typical T-S fuzzy inference system [16] constructed from a neural network structure. It has a total of six layers. The T-S fuzzy-neural model is essentially a multi-model approach in which a set of linear models are combined to describe the global behavior of the system [11, 17, 18]. Based on this idea, the T-S fuzzy-neural model is appropriate for developing fuzzy-neural controllers because many systems can be expressed locally in some form of mathematical model. The T-S fuzzy-neural model can approximate a wide class of nonlinear systems. In [3-5], the authors proved that the T-S fuzzy-neural system can approximate any continuous function to any precision.

The T-S fuzzy-neural model approximates a nonlinear system with a combination of several linear systems. It is

formed by fuzzy partitioning of the input space. The premise of a fuzzy implication indicates a fuzzy subspace of the input space and each consequent expresses a local input-output relation in the subspace corresponding to the premise part [6]. The T-S fuzzy-neural model defined is

$$R^{(i)} : \text{If } z_1 \text{ is } F_1^i \text{ and } \dots z_n \text{ is } F_n^i \text{ and } z_{n+1} \text{ is } F_{n+1}^i \quad (1)$$

$$\text{Then } \bar{y}_l = p_{l1}^i z_1 + p_{l2}^i z_2 + \dots + p_{l(n+1)}^i z_{n+1}$$

where $\mathbf{z} = [z_1 \ z_2 \ \dots \ z_{n+1}]^T \in R^{n+1}$ is a state vector, y is the system output, F_j^i ($j = 1, 2, \dots, n+1$) are fuzzy sets, and p_{lk}^i ($i = 1, 2, \dots, h, k = 1, 2, \dots, n+1, l = 1, 2, \dots, n$) are adjustable parameters. The T-S fuzzy-neural model can be described by the fuzzy-neural network shown in Fig. 1.

3. T-S Fuzzy-Neural Model for Virtual Linearized System (VLS)

Suppose that the general unknown nonaffine nonlinear system is

$$\begin{aligned} \dot{x}_1 &= f_1(\mathbf{x}) + d_{d1} \\ \dot{x}_2 &= f_2(\mathbf{x}) + d_{d2} \\ &\vdots \\ \dot{x}_n &= f_n(\mathbf{x}, u) + d_{dn} \\ y &= x_1 \end{aligned} \quad (2)$$

where $f_1, f_2, \dots, f_{n-1} : R^n \rightarrow R$, and $f_n : R^{n+1} \rightarrow R$ are unknown functions which define a smooth mapping on the open sets R^n and R^{n+1} , respectively, $\mathbf{x} = [x_1 \ x_2 \ \dots \ x_n]^T$ is a state vector, u and y are the control input and system output, respectively, and $\mathbf{d}_d = [d_{d1} \ d_{d2} \ \dots \ d_{dn}]^T$ represents external disturbances. Without loss of generality, we assume a solution for (2) exists.

Assumption 1: Assume that the general unknown nonaffine nonlinear system (2) can be piece-wise linearized. If we use Taylor series expansion of the unknown nonaffine nonlinear system in (2) around time varying states, $[\mathbf{x}_o^T(t), u_o(t)]^T$, there is a virtual linearized system (VLS):

$$\dot{\mathbf{x}}_\delta = \mathbf{A}\mathbf{x}_\delta + \mathbf{B}u_\delta + \mathbf{d} = \mathbf{A}\mathbf{x}_\delta + \mathbf{b}_e b u_\delta + \mathbf{d} \quad (3)$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} = \begin{bmatrix} d_{h1} + d_{d1} + f_1(\mathbf{x}_o) \\ d_{h2} + d_{d2} + f_2(\mathbf{x}_o) \\ \vdots \\ d_{hn} + d_{dn} + f_n(\mathbf{x}_o, u_o) \end{bmatrix} \quad (4)$$

where d_{hj} , $j = 1, 2, \dots, n$, stands for higher order terms,

u_o is an operating input, $u_\delta = u - u_o$ is an input deviation, $\mathbf{x}_o = [x_{o1} \ x_{o2} \ \dots \ x_{on}]^T$ is a vector of operating states, $\mathbf{x}_\delta = [x_{\delta 1} \ x_{\delta 2} \ \dots \ x_{\delta n}]^T = \mathbf{x} - \mathbf{x}_o$ is a vector of state deviations, and $\mathbf{b}_e = [0 \ 0 \ \dots \ 1]^T$,

$$\mathbf{A} = \begin{bmatrix} a_{11}(\mathbf{x}_o) & a_{12}(\mathbf{x}_o) & \dots & a_{1n}(\mathbf{x}_o) \\ a_{21}(\mathbf{x}_o) & a_{22}(\mathbf{x}_o) & \dots & a_{2n}(\mathbf{x}_o) \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}(\mathbf{x}_o, u_o) & a_{n2}(\mathbf{x}_o, u_o) & \dots & a_{nn}(\mathbf{x}_o, u_o) \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b(\mathbf{x}_o, u_o) \end{bmatrix}$$

$$a_{ij} = \frac{\partial f_i}{\partial x_j} \Big|_{(\mathbf{x}_o, u_o)}, \text{ and } b = \frac{\partial f_n}{\partial u} \Big|_{(\mathbf{x}_o, u_o)}, \quad i=1,2,\dots,n, \\ j=1,2,\dots,n. \quad \blacklozenge$$

Remark 1: Instead of modeling the unknown systems (2) directly, the T-S fuzzy-neural model in (1) is used to approximate the virtual linearized system (VLS) in (3).

The T-S fuzzy-neural model can be described by the fuzzy-neural network shown in Fig. 1. The coefficient, p_{lk} , of the fuzzy-neural network is

$$p_{lk} = \frac{\sum_{i=1}^h p_{lk}^i \left(\prod_{j=1}^{n+1} \mu_{F_j^i}(z_j) \right)}{\sum_{i=1}^h \left(\prod_{j=1}^{n+1} \mu_{F_j^i}(z_j) \right)} \quad (5)$$

where $\mu_{F_j^i}(z_j)$ is the value of the membership function.

For the tuning of the weighting factors p_k^i , we define

$$w^i \equiv \frac{\prod_{j=1}^{n+1} \mu_{F_j^i}}{\sum_{i=1}^h \prod_{j=1}^{n+1} \mu_{F_j^i}}, \quad i=1,2,\dots,h. \quad (6)$$

Assumption 2: The antecedent part of the fuzzy implication describes the conditions of the operation states $[\mathbf{x}_o^T, u_o]^T$. The consequent part of the fuzzy implication represents the linearization of the general nonaffine nonlinear system (2). \blacklozenge

Based on the above assumptions, for the purpose of approximating the virtual linearized system (VLS) in (3), the i th fuzzy implication (1) can be described as

$$R^{(i)} : \text{If } x_{\delta 1} \text{ is } F_1^i \text{ and...and } x_{\delta n} \text{ is } F_n^i \text{ and } u_\delta \text{ is } F_{n+1}^i \quad (7)$$

$$\text{Then } \dot{\mathbf{x}}_\delta = \hat{\mathbf{A}}^i \mathbf{x}_\delta + \hat{\mathbf{B}}^i u_\delta$$

where

$$\hat{\mathbf{A}}^i = \begin{bmatrix} p_{11}^i & p_{12}^i & \dots & p_{1n}^i \\ p_{21}^i & p_{22}^i & \dots & p_{2n}^i \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1}^i & p_{n2}^i & \dots & p_{nn}^i \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{a}}_1^{iT} \\ \hat{\mathbf{a}}_2^{iT} \\ \vdots \\ \hat{\mathbf{a}}_n^{iT} \end{bmatrix}$$

and

$$\hat{\mathbf{B}}^i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ p_{n(n+1)}^i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \hat{b}^i \end{bmatrix}.$$

After applying some commonly used defuzzification strategies, we can obtain

$$\dot{\mathbf{x}}_\delta = \sum_{i=1}^h w^i \left\{ \hat{\mathbf{A}}^i \mathbf{x}_\delta + \hat{\mathbf{B}}^i u_\delta \right\} \\ = \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1n} \\ p_{21} & p_{22} & \dots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \dots & p_{nn} \end{bmatrix} \mathbf{x}_\delta + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ p_{n(n+1)} \end{bmatrix} u_\delta \quad (8)$$

where p_{ij} ($i=1,2,\dots,n, j=1,2,\dots,n$) and $p_{n(n+1)}$ are used to approximate a_{ij} ($i=1,2,\dots,n, j=1,2,\dots,n$) and b of the virtual linearized system (VLS) in (3), respectively. The virtual linearized system (VLS) is derived from the unknown nonlinear system (2).

4. Controller Design for Online Modeling and Robust Tracking

To design a robust controller for (2), the following assumptions are required.

Assumption 3: Let \mathbf{x}_δ and u_δ belong to compact sets \mathbf{U}_x and \mathbf{U}_u , respectively, where

$$\mathbf{U}_x = \{ \mathbf{x} \in \mathbb{R}^n : \|\mathbf{x}\| \leq m_x < \infty \}$$

$$\mathbf{U}_u = \{ u \in \mathbb{R} : |u| \leq m_u < \infty \}$$

and m_x , m_u are design parameters. We define $\phi_{lj} = [p_{lj}^1 \ p_{lj}^2 \ \dots \ p_{lj}^h]$, $l=1,2,\dots,n$ $j=1,2,\dots,n$, and $\phi_{n(n+1)} = [p_{n(n+1)}^1 \ p_{n(n+1)}^2 \ \dots \ p_{n(n+1)}^h]$. It is known that the optimal adjustable parameters ϕ_{lj}^* and $\phi_{n(n+1)}^*$ lie in some convex regions

$$\mathbf{M}_{\phi_{lj}} = \{ \phi_{lj} \in \mathbb{R}^h : \|\phi_{lj}\| \leq m_{\phi_{lj}} \},$$

$$l=1,2,\dots,n, \quad j=1,2,\dots,n$$

and

$$\mathbf{M}_{\phi_{n(n+1)}} = \{ \phi_{n(n+1)} \in \mathbb{R}^h : \|\phi_{n(n+1)}\| \leq m_{\phi_{n(n+1)}} \}$$

where the radii $m_{\phi_{lj}}$ and $m_{\phi_{n(n+1)}}$ are constant, and

$$\phi_{lj}^* = \arg \min_{\phi_{lj} \in \mathbf{M}_{\phi_{lj}}} \left[\sup_{\mathbf{x}_\delta \in \mathbf{U}_x, u_\delta \in \mathbf{U}_u} \left| p_{lj}^i(\mathbf{x}_\delta, u_\delta) - \hat{p}_{lj}^i(\mathbf{x}_\delta, u_\delta | \phi_{lj}) \right| \right],$$

$$l = 1, 2, \dots, n \quad j = 1, 2, \dots, n$$

and

$$\hat{\phi}_{n(n+1)}^* = \arg \min_{\phi_{n(n+1)} \in \mathbb{M}_{\phi(n+1)}} \left[\sup_{\mathbf{x}_\delta \in \mathbb{U}_x, u_\delta \in \mathbb{U}_u} |p_{n(n+1)}^i(\mathbf{x}_\delta, u_\delta) - \hat{p}_{n(n+1)}^i(\mathbf{x}_\delta, u_\delta | \phi_{n(n+1)})| \right].$$

According to assumption 3, we define the optimal adjustable matrices as

$$\hat{\mathbf{A}}^{i*} = \begin{bmatrix} p_{11}^{i*} & p_{12}^{i*} & \cdots & p_{1n}^{i*} \\ p_{21}^{i*} & p_{22}^{i*} & \cdots & p_{2n}^{i*} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1}^{i*} & p_{n2}^{i*} & \cdots & p_{nn}^{i*} \end{bmatrix}$$

$$\hat{\mathbf{B}}^{i*} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ p_{n(n+1)}^{i*} \end{bmatrix}.$$

Assumption 4 [20]: The parameter vector $\phi_{n(n+1)}$ is chosen such that $p_{n(n+1)} = \sum_{i=1}^h w^i \hat{b}^i$ is bounded away from

zero. ♦

Lemma 1 [21]: Suppose that a matrix $\Lambda \in R^{n \times n}$ is given. For every symmetric positive definite matrix $\mathbf{Q} \in R^{n \times n}$, the Lyapunov matrix equation $\Lambda^T \Gamma + \Gamma \Lambda = -\mathbf{Q}$ has a unique solution for $\Gamma = \Gamma^T > 0$ if and only if $\Lambda \in R^{n \times n}$ is a Hurwitz matrix. ♦

Lemma 2 [22]: If both $\mathbf{e}(t)$ and $\dot{\mathbf{e}}(t) \in L_\infty^n$, and $\mathbf{e}(t) \in L_p^n$, for some $p \in [1, \infty)$, then $\lim_{t \rightarrow \infty} \|\mathbf{e}(t)\| = 0$. ♦

We define the reference signal vector $\mathbf{r} = [r_1 \ r_2 \ r_3 \ \cdots \ r_n]^T$. Thus the error vector is $\mathbf{e} = \mathbf{x} - \mathbf{r} = [e_1 \ e_2 \ \cdots \ e_n]^T$. Let $\omega_j = \dot{r}_j - \lambda_j e_j$, $j = 1, 2, \dots, n$. In order to compensate for the total effect of unmodeled dynamics $\mathbf{d}_h = [d_{h1} \ d_{h2} \ \cdots \ d_{hn}]^T$, external disturbances $\mathbf{d}_d = [d_{d1} \ d_{d2} \ \cdots \ d_{dn}]^T$, and modeling error $\mathbf{d}_f = [d_{f1} \ d_{f2} \ \cdots \ d_{fn}]^T$, we redefine the fuzzy-neural approximator in (7) including total error

$$\tilde{\mathbf{d}} = \begin{bmatrix} \tilde{d}_1 \\ \tilde{d}_2 \\ \vdots \\ \tilde{d}_n \end{bmatrix} = \begin{bmatrix} d_{h1} + d_{d1} + f_1(\mathbf{x}_o, u_o) + d_{f1} \\ d_{h2} + d_{d2} + f_2(\mathbf{x}_o, u_o) + d_{f2} \\ \vdots \\ d_{hn} + d_{dn} + f_n(\mathbf{x}_o, u_o) + d_{fn} \end{bmatrix}$$

as follows:

$$\dot{\mathbf{x}}_\delta = \sum_{i=1}^h w^i \{ \hat{\mathbf{A}}^i \mathbf{x}_\delta + \hat{\mathbf{B}}^i u_\delta \} + \tilde{\mathbf{d}} = \sum_{i=1}^h w^i \{ \hat{\mathbf{A}}^i \mathbf{x}_\delta + \mathbf{b}_e \hat{b}^i u_\delta \} + \tilde{\mathbf{d}}. \quad (9)$$

According to (9) and assumption 4, a fuzzy-neural controller can be chosen as

$$\mathbf{b}_e u_\delta = \frac{\dot{\mathbf{x}}_\delta - \sum_{i=1}^h w^i \hat{\mathbf{A}}^i \mathbf{x}_\delta - \tilde{\mathbf{d}}}{\sum_{i=1}^h w^i \hat{b}^i} \equiv \frac{-\sum_{i=1}^h w^i \hat{\mathbf{A}}^i \mathbf{x}_\delta + \omega - \mathbf{b}_e u_s}{\sum_{i=1}^h w^i \hat{b}^i} \quad (10)$$

where $\omega = [\omega_1 \ \omega_2 \ \cdots \ \omega_n]^T$, and u_s is an error compensator. Define a coefficient matrix

$$\Lambda = \begin{bmatrix} -\lambda_1 & 0 & \cdots & 0 \\ 0 & -\lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -\lambda_n \end{bmatrix} \quad (11)$$

where the coefficients, $\lambda_1, \lambda_2, \dots, \lambda_n$, are selected such that the matrix Λ is a Hurwitz matrix. From (9) and (10), we obtain

$$u = \frac{-\sum_{i=1}^h w^i \hat{\mathbf{a}}^{iT} \mathbf{x}_\delta + \dot{r}_n - \lambda_n e_n - u_s}{\sum_{i=1}^h w^i \hat{b}^i} + u_o. \quad (12)$$

Now, using $\dot{\mathbf{e}} = \dot{\mathbf{x}} - \dot{\mathbf{r}}$ and the virtual linearized system (VLS) in (3), the error dynamic equation of the VLS is

$$\dot{\mathbf{e}} = \Lambda \mathbf{e} + \sum_{i=1}^h w^i \tilde{\mathbf{A}}^i \mathbf{x}_\delta + \mathbf{b}_e \sum_{i=1}^h w^i \tilde{b}^i u_\delta - \mathbf{b}_e u_s + \tilde{\mathbf{d}} \quad (13)$$

where $\tilde{\mathbf{A}}^i = \hat{\mathbf{A}}^{i*} - \hat{\mathbf{A}}^i$, $\tilde{b}^i = \hat{b}^{i*} - \hat{b}^i$. We define u_s (the error compensator) and e_Δ as

$$u_s = \text{sign}(e_\Delta) k \quad (14)$$

and

$$e_\Delta = \mathbf{e}^T \Gamma \mathbf{b}_e \quad (15)$$

where $\Gamma > 0$ is a Lyapunov matrix and k is the control gain.

Assumption 5: The approximated system (9) has an error term $\mathbf{e}^T \Gamma \tilde{\mathbf{d}} \leq \bar{k}$, where \bar{k} is some positive constant.

There exists a positive value of k , such that $k > \frac{\bar{k}}{|e_\Delta|}$. ♦

On the basis of the above discussion, the following theorem can be obtained.

Theorem 1: Consider the general unknown nonaffine nonlinear system (2), which is approximated as (9) and satisfies Assumptions 1 and 3 to 5. If the controller is designed as (12) with update laws

$$\dot{\hat{\mathbf{A}}}^i = \eta_1 w^i \mathbf{e} \mathbf{x}_\delta^T \quad (16)$$

$$\dot{\hat{b}}^i = \eta_2 \mathbf{e}^T \Gamma \mathbf{b}_e w^i u_\delta, \quad i = 1, 2, \dots, h \quad (17)$$

and the control gain k is selected such that

$$|e_\Delta| k > \bar{k}$$

where η_1 and η_2 are positive constants, then the closed-loop system is robust stable and $\lim_{t \rightarrow \infty} \|\mathbf{e}(t)\| = 0$. ♦

Proof:

Consider the Lyapunov-like function candidate

$$v = \frac{1}{2} \mathbf{e}^T \mathbf{\Gamma} \mathbf{e} + \frac{1}{2\eta_1} \sum_{i=1}^h \text{tr}(\tilde{\mathbf{A}}^{iT} \mathbf{\Gamma} \tilde{\mathbf{A}}^i) + \frac{1}{2\eta_2} \sum_{i=1}^h \tilde{b}^i{}^2. \quad (18)$$

Differentiating (18) with respect to time, we get

$$\begin{aligned} \dot{v} = & \frac{1}{2} \dot{\mathbf{e}}^T \mathbf{\Gamma} \mathbf{e} + \frac{1}{2} \mathbf{e}^T \mathbf{\Gamma} \dot{\mathbf{e}} + \frac{1}{2\eta_1} \sum_{i=1}^h \text{tr}(\dot{\tilde{\mathbf{A}}}^{iT} \mathbf{\Gamma} \tilde{\mathbf{A}}^i) \\ & + \frac{1}{2\eta_1} \sum_{i=1}^h \text{tr}(\tilde{\mathbf{A}}^{iT} \mathbf{\Gamma} \dot{\tilde{\mathbf{A}}}^i) + \frac{1}{\eta_2} \sum_{i=1}^h \tilde{b}^i \dot{\tilde{b}}^i. \end{aligned} \quad (19)$$

Inserting (13) and (14) in the above equation yields

$$\begin{aligned} \dot{v} = & \frac{1}{2} \mathbf{e}^T (\mathbf{\Lambda}^T \mathbf{\Gamma} + \mathbf{\Gamma} \mathbf{\Lambda}) \mathbf{e} + \mathbf{e}^T \mathbf{\Gamma} \sum_{i=1}^h w^i \tilde{\mathbf{A}}^i \mathbf{x}_\delta \\ & + \mathbf{e}^T \mathbf{\Gamma} \mathbf{b}_e \sum_{i=1}^h w^i \tilde{b}^i u_\delta + \frac{1}{\eta_1} \sum_{i=1}^h \text{tr}(\tilde{\mathbf{A}}^{iT} \mathbf{\Gamma} \tilde{\mathbf{A}}^i) \\ & + \frac{1}{\eta_2} \sum_{i=1}^h \tilde{b}^i \dot{\tilde{b}}^i + \mathbf{e}^T \mathbf{\Gamma} \tilde{\mathbf{d}} - \mathbf{e}^T \mathbf{\Gamma} \mathbf{b}_e u_s. \end{aligned} \quad (20)$$

From Lemma 1, substituting $\mathbf{\Lambda}^T \mathbf{\Gamma} + \mathbf{\Gamma} \mathbf{\Lambda} = -\mathbf{Q}$ for (20), we have

$$\begin{aligned} \dot{v} = & -\frac{1}{2} \mathbf{e}^T \mathbf{Q} \mathbf{e} + \mathbf{e}^T \mathbf{\Gamma} \sum_{i=1}^h w^i \tilde{\mathbf{A}}^i \mathbf{x}_\delta + \mathbf{e}^T \mathbf{\Gamma} \mathbf{b}_e \sum_{i=1}^h w^i \tilde{b}^i u_\delta \\ & + \frac{1}{\eta_1} \sum_{i=1}^h \text{tr}(\tilde{\mathbf{A}}^{iT} \mathbf{\Gamma} \dot{\tilde{\mathbf{A}}}^i) + \frac{1}{\eta_2} \sum_{i=1}^h \tilde{b}^i \dot{\tilde{b}}^i + \mathbf{e}^T \mathbf{\Gamma} \tilde{\mathbf{d}} \\ & - \mathbf{e}^T \mathbf{\Gamma} \mathbf{b}_e u_s \\ = & \Delta + \mathbf{e}^T \mathbf{\Gamma} \tilde{\mathbf{d}} - \mathbf{e}^T \mathbf{\Gamma} \mathbf{b}_e \text{sign}(e_\Delta) k \end{aligned} \quad (21)$$

where

$$\begin{aligned} \Delta = & -\frac{1}{2} \mathbf{e}^T \mathbf{Q} \mathbf{e} + \mathbf{e}^T \mathbf{\Gamma} \sum_{i=1}^h w^i \tilde{\mathbf{A}}^i \mathbf{x}_\delta + \mathbf{e}^T \mathbf{\Gamma} \mathbf{b}_e \sum_{i=1}^h w^i \tilde{b}^i u_\delta \\ & + \frac{1}{\eta_1} \sum_{i=1}^h \text{tr}(\tilde{\mathbf{A}}^{iT} \mathbf{\Gamma} \dot{\tilde{\mathbf{A}}}^i) + \frac{1}{\eta_2} \sum_{i=1}^h \tilde{b}^i \dot{\tilde{b}}^i \\ = & -\frac{1}{2} \mathbf{e}^T \mathbf{Q} \mathbf{e} + \text{tr} \left(\sum_{i=1}^h w^i \tilde{\mathbf{A}}^{iT} \mathbf{\Gamma} \mathbf{e} \mathbf{x}_\delta^T - \sum_{i=1}^h \frac{\tilde{\mathbf{A}}^{iT} \mathbf{\Gamma} \dot{\tilde{\mathbf{A}}}^i}{\eta_1} \right) \\ & + \left(\mathbf{e}^T \mathbf{\Gamma} \mathbf{b}_e \sum_{i=1}^h w^i \tilde{b}^i u_\delta - \frac{1}{\eta_2} \sum_{i=1}^h \tilde{b}^i \dot{\tilde{b}}^i \right). \end{aligned}$$

If we select $\hat{\mathbf{A}}^i$ and \hat{b}^i , as (16) and (17) and from assumption 5, (21) becomes

$$\dot{v} \leq -\frac{1}{2} \mathbf{e}^T \mathbf{Q} \mathbf{e} + \bar{k} - |e_\Delta| k. \quad (22)$$

Choose the value of k , such that $|e_\Delta| k > \bar{k}$, then

$$\dot{v} = -\frac{1}{2} \mathbf{e}^T \mathbf{Q} \mathbf{e} \leq 0. \quad (23)$$

Equations (18) and (23) only guarantee that $\mathbf{e}(t) \in L_\infty$, but not that it converges. The boundedness of $\mathbf{e}(t)$ implies the boundedness of $\mathbf{x}(t)$. Since the operating states are finite, \mathbf{x}_δ is bounded. Based on Assumption 1 and

the boundedness of \mathbf{x}_δ , u_δ is bounded. Therefore, $\dot{\mathbf{e}}(t)$ is bounded, i.e. $\dot{\mathbf{e}}(t) \in L_\infty$. Integrating both side of (23) yields

$$v(t) - v(0) \leq -\frac{1}{2} \lambda_{\min}(\mathbf{Q}) \int_0^t \|\mathbf{e}(\tau)\|^2 d\tau \quad (24)$$

where $\lambda_{\min}(\mathbf{Q}) > 0$ is the minimum eigenvalue of \mathbf{Q} .

When t tends to infinity, (24) becomes

$$\int_0^\infty \|\mathbf{e}(\tau)\|^2 d\tau \leq \frac{v(0) - v(\infty)}{\frac{1}{2} \lambda_{\min}(\mathbf{Q})}. \quad (25)$$

Since the right side of (25) is bounded, $\mathbf{e} \in L_2$. Therefore, by using Lemma 2, $\|\mathbf{e}(t)\| \rightarrow 0$ as $t \rightarrow \infty$. This completes the proof. \blacklozenge

Algorithm 1:

- 1) Select the coefficients, $\lambda_1, \lambda_2, \dots, \lambda_n$, such that the matrix $\mathbf{\Lambda}$ is a Hurwitz matrix.
- 2) Choose an appropriate value k in (14) such that $|e_\Delta| k > \bar{k}$. In order to remedy control chattering, (14) can be modified as

$$u_s = \begin{cases} k & \text{if } e_\Delta \geq 0 \text{ and } |e_\Delta| > \alpha \\ -k & \text{if } e_\Delta < 0 \text{ and } |e_\Delta| > \alpha \\ \frac{ke_\Delta}{\alpha} & \text{if } |e_\Delta| < \alpha \end{cases}$$

where α is a positive constant.

- 3) Choose an appropriate matrix \mathbf{Q} . Then, solve the Lyapunov matrix equation in Lemma 1.
- 4) Construct fuzzy sets for \mathbf{x}_δ and u_δ .
- 5) Obtain the control law (12) and update laws (16) and (17).

Remark 2: If we review theorem 1, choosing an appropriate value of k , such that $|e_\Delta| k > \bar{k}$ (or (22)) implies (23), is very important. That is $\mathbf{e}^T \mathbf{\Gamma} \tilde{\mathbf{d}} - \mathbf{e}^T \mathbf{\Gamma} \mathbf{b}_e \text{sign}(e_\Delta) k$ must be a negative number. Otherwise the effect of all the unmodeled dynamics, and external disturbances on the tracking error can not be efficiently attenuated by the proposed controller.

Figure 2 shows the overall scheme of the T-S fuzzy-neural controller proposed in this paper.

5. Simulation Results

This section presents the simulation results of the proposed on-line T-S fuzzy modeling and robust adaptive fuzzy control for general nonaffine uncertain systems to illustrate that the tracking error of the closed-loop system can be made arbitrarily small. In addition, the simulation results confirm that the effect of all the unmodeled T-S fuzzy system dynamics, modeling

errors and external disturbances on the tracking error is attenuated efficiently by the proposed controller.

Example 1: Consider a nonaffine system [23]:

$$\begin{aligned}\dot{x}_1 &= x_2 + d_{d1} \\ \dot{x}_2 &= 0.2(1 + e^{x_1 x_2})(2 + \sin(x_2))(u + e^u - 1) + d_{d2} \\ y &= x_1\end{aligned}$$

where u is the control input, and both d_{d1} and d_{d2} are external disturbances which are assumed to be values randomly in the interval $[-0.1, 0.1]$ (cases 1, 2) or a square wave with the amplitude ± 0.1 and the period 2π (case 3). In this example, three different cases for the operation states \mathbf{x}_o , u_o and reference signals are simulated. The three cases are shown in Table I.

Five fuzzy sets over the interval $[-6, 6]$ are defined for $\mathbf{x}_\delta = [x_{\delta 1}, x_{\delta 2}]^T$ with the term sets (PB, PS, Z, NS, NB) and three fuzzy sets over the interval $[-1400, 1400]$ for u_δ . The design parameters are selected as $\eta = 0.002$, $\lambda_1 = 2$, $\lambda_2 = 2$ and $\mathbf{Q} = [2 \ 1; 1 \ 2]$. The initial states of system are assumed to be $\mathbf{x}(0) = [0.4, 0.6]^T$ (case 1), $\mathbf{x}(0) = [0.5, 0.5]^T$ (case 2) and $\mathbf{x}(0) = [0.8, 0.3]^T$ (case 3). We use the proposed control law in (12) to control the state x_1 of the system to track the reference signal $r_1 = \sin(t)$ (cases 1, 2), $1.5 - e^{-0.5t}$ (case 3) and the state x_2 of the system to track the reference signal $r_2 = \cos(t)$ (cases 1, 2), $0.5e^{-0.5t}$ (case 3). Figs. 3 and 4 (case 1), Figs. 6 and 7 (case 2), and Figs. 9 and 10 (case 3) show that the curves of the states x_1 and x_2 of the closed-loop system, respectively. The responses of control input u are shown in Fig. 5 (case 1), Fig. 8 (case 2) and Fig. 11 (case 3). The simulation results indicate that the effect of all the unmodeled dynamics, and external disturbances on the tracking error is attenuated efficiently by the proposed controller.

Example 2: Consider a more complicated nonlinear system [24]:

$$\begin{aligned}\dot{x}_1 &= -x_1 + 0.072(1 - x_1)e^{\frac{-1}{x_2+20}} + d_{d1} \\ \dot{x}_2 &= -1.3x_2 + 0.576(1 - x_1)e^{\frac{-1}{x_2+20}} + 0.3(u + 0.5 \cos(10\pi t)) + d_{d2} \\ y &= x_2\end{aligned}$$

where u is the control input, and both d_{d1} and d_{d2} are external disturbances which are assumed to be values randomly in the interval $[-0.1, 0.1]$.

Five fuzzy sets over the interval $[-6, 6]$ are defined

for $\mathbf{x}_\delta = [x_{\delta 1}, x_{\delta 2}]^T$ with the term sets (PB, PS, Z, NS, NB) and three fuzzy sets over the interval $[-1400, 1400]$ for u_δ . The design parameters are selected as $\eta = 0.0003$, $\lambda_1 = 8$, $\lambda_2 = 10$ and $\mathbf{Q} = [1 \ 0; 0 \ 1]$. The initial states of system are assumed to be $\mathbf{x}(0) = [1, 1.2]^T$. We use the proposed control law in (12) to control the state x_1 of the system to track the reference signal $r_1 = 0$, and the state x_2 of the system to track the reference signal $r_2 = 0.5t$, when $t \leq 2$ and $r_2 = 1$, when $t > 2$. Figures 12 and 13 show that the curves of the states x_1 and x_2 of the closed system, respectively. The response of control input u is shown in Fig. 14. The simulation results indicate that the effect of all the unmodeled dynamics, and external disturbances on the tracking error is attenuated efficiently by the proposed controller.

Example 3: Consider a third order nonlinear system [25] described by :

$$\begin{aligned}\dot{x}_1 &= x_3 + d_{d1} \\ \dot{x}_2 &= 5x_1 - 10x_2 + d_{d2} \\ \dot{x}_3 &= -8x_2 + 8 \left(\frac{e^{-0.5x_3} - 1}{e^{-0.5x_3} + 1} \right) + \left(4 - e^{-0.15x_1^2 + 1.1} \right) u + d_{d3} \\ y &= x_1\end{aligned}$$

where u is the control input, and both d_{d1} , d_{d2} and d_{d3} are external disturbances which are assumed to be random values in the interval $[-0.1, 0.1]$.

Five fuzzy sets over the interval $[-6, 6]$ are defined for $\mathbf{x}_\delta = [x_{\delta 1}, x_{\delta 2}, x_{\delta 3}]^T$ with the term sets (PB, PS, Z, NS, NB) and three fuzzy sets over the interval $[-1400, 1400]$ for u_δ . The design parameters are selected as $\eta = 0.002$, $\lambda_1 = 15$, $\lambda_2 = 20$, $\lambda_3 = 40$ and $\mathbf{Q} = [0.6 \ 0 \ 0; 0 \ 0.6 \ 0; 0 \ 0 \ 0.6]$. The initial states of system are assumed to be $\mathbf{x}(0) = [1, 1, -1]^T$. We use the proposed control law in (12) to control the states, x_1 , x_2 , and x_3 of the system to track the reference signal $r=0$. Figures 15, 16 and 17 show the curves of the states x_1 , x_2 and x_3 of the closed system, respectively. The response of the control input u is shown in Fig. 18. The simulation results indicate that the effect of all the unmodeled dynamics, and external disturbances on the tracking error is attenuated efficiently by the proposed controller.

6. Conclusions

We propose a novel approach of on-line T-S

fuzzy-neural modeling and robust adaptive control for a class of general nonaffine nonlinear systems to achieve the efficient attenuation of the unmodeled dynamics, modeling errors and external disturbances. The initial values of the parameters of the fuzzy-neural model are tuned to their true values through update laws. Instead of modeling the unknown systems directly, the T-S fuzzy-neural model approximates the virtual linearized system (VLS), with modeling errors and external disturbances. Theorem 1 proved that although the bound of unmodeled dynamics, modeling errors and external disturbances are unknown, the tracking error of the closed-loop system can be made arbitrarily small. Simulation results support the theoretical arguments about the T-S fuzzy-neural modeling and the tracking performance of the design algorithms under the adaptive tuning methods.

Acknowledgment

This work was supported by the National Science Council, Taiwan, under Grant NSC 95-2221-E-030-011. The authors would like to thank B. Schack for his useful comments and suggestions on improving this paper.

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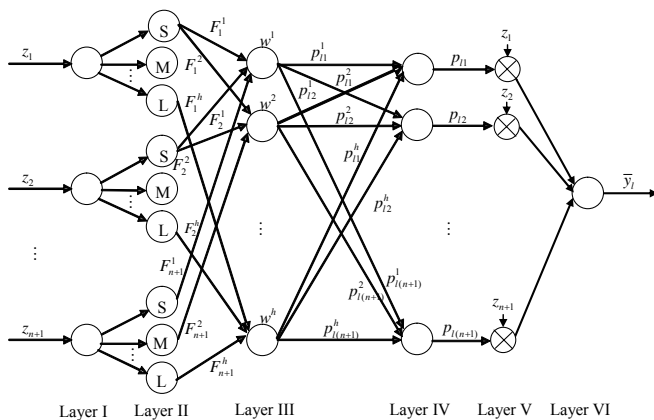


Figure 1. Configuration of the T-S fuzzy-neural network.

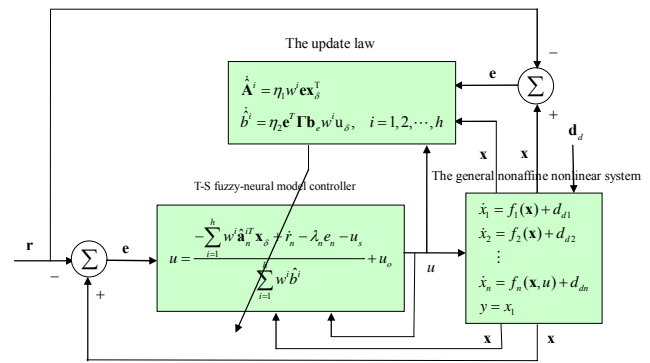


Figure 2. The overall scheme of the T-S fuzzy-neural controller.

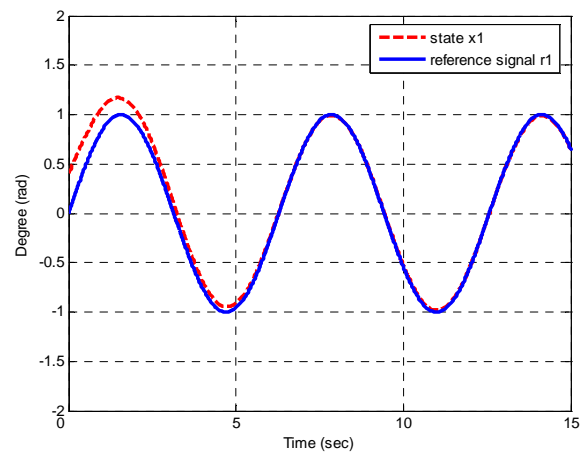


Figure 3. Curve of the state x_1 of the tracking control (case 1) in example 1.

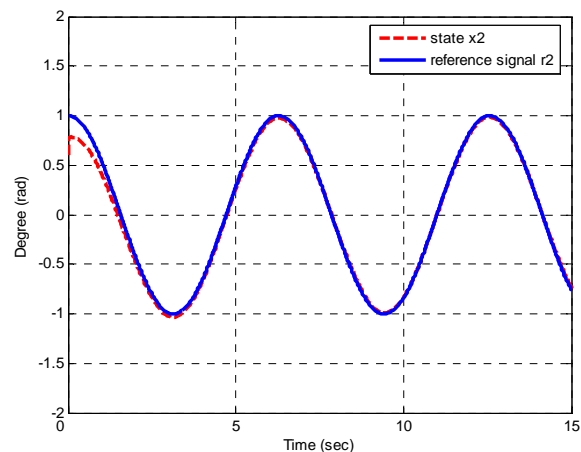


Figure 4. Curve of the state x_2 of the tracking control (case 1) in example 1.

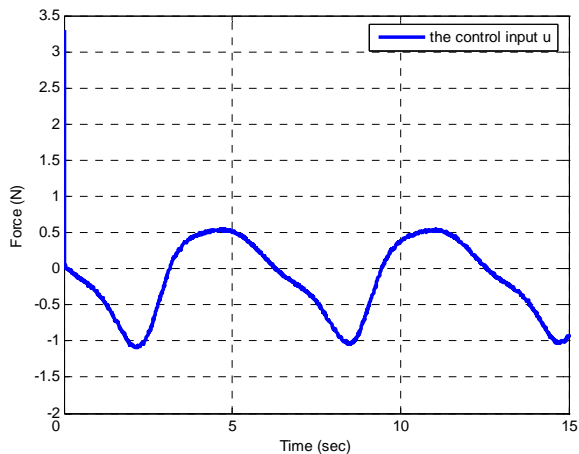


Figure 5. Response of control input u (case 1) in example 1.

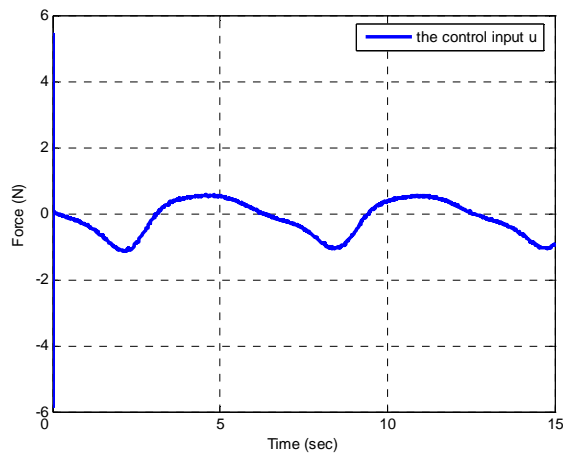


Figure 8. Response of control input u (case 2) in example 1.

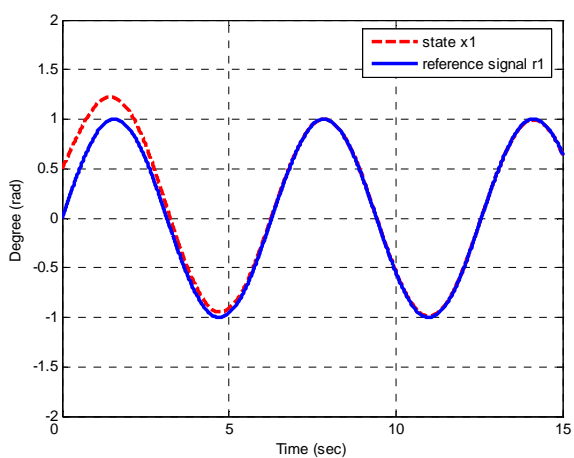


Figure 6. Curve of the state x_1 of the tracking control (case 2) in example 1.

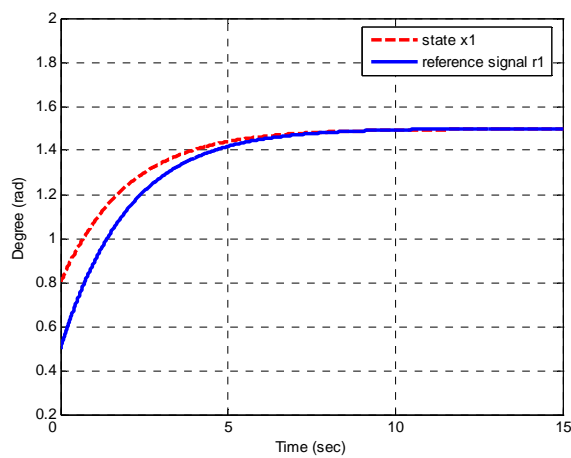


Figure 9. Curve of the state x_1 of the tracking control (case 3) in example 1.

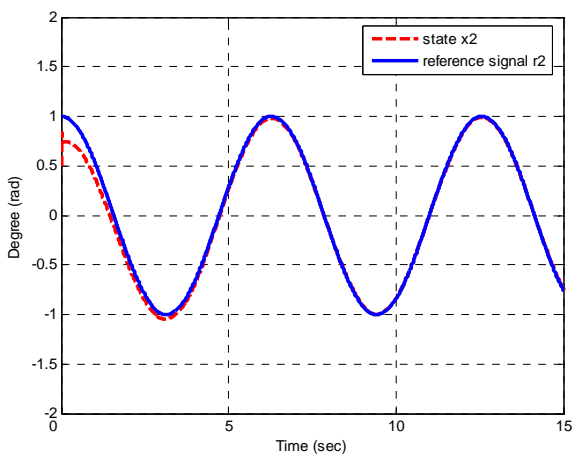


Figure 7. Curve of the state x_2 of the tracking control (case 2) in example 1.

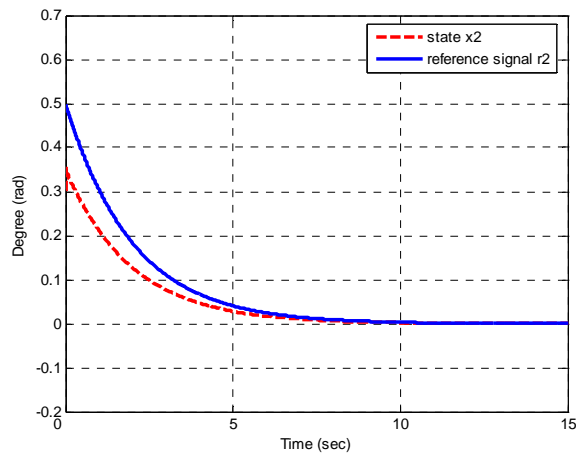


Figure 10. Curve of the state x_2 of the tracking control (case 3) in example 1.

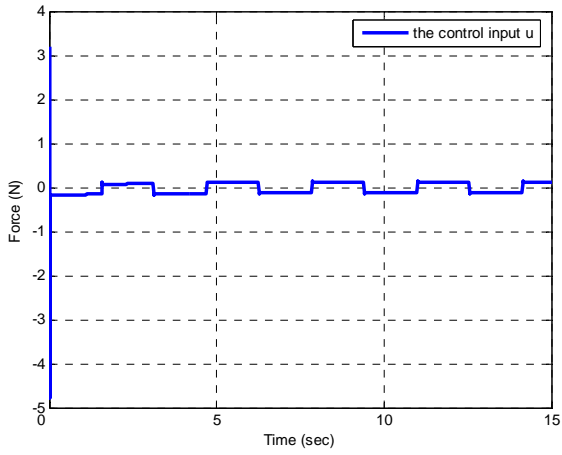


Figure 11. Response of control input u (case 3) in example 1.

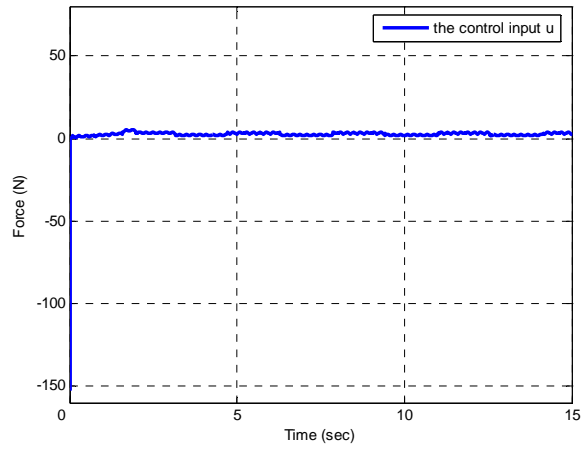


Figure 14. Response of control input u in example 2.

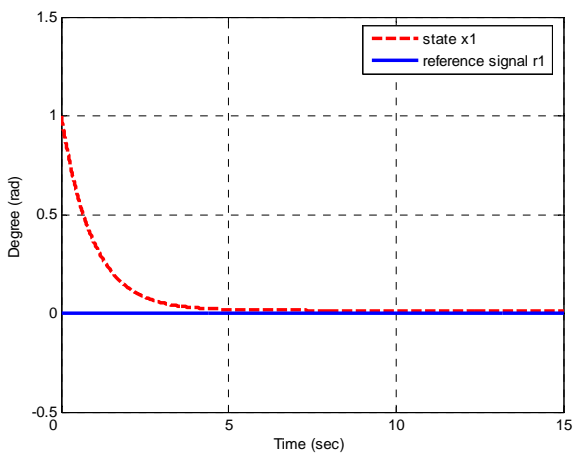


Figure 12. Curve of the state x_1 of the tracking control in example 2.

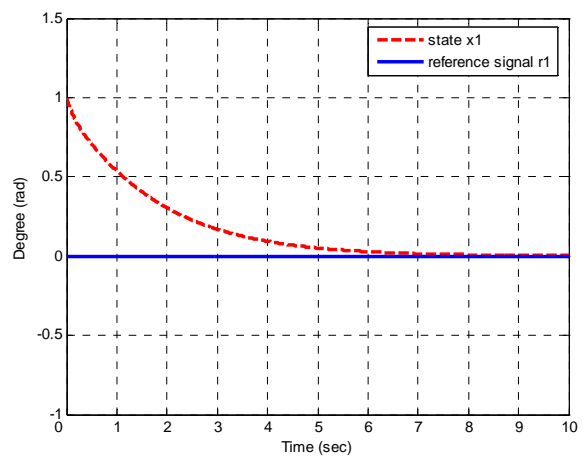


Figure 15. Curve of the state x_1 of the tracking control in example 3.

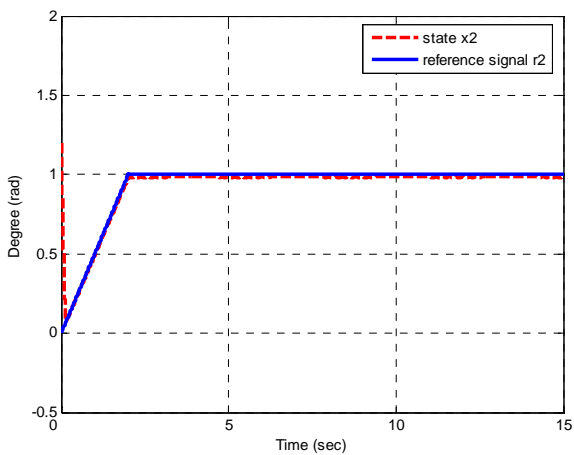


Figure 13. Curve of the state x_2 of the tracking control in example 2.

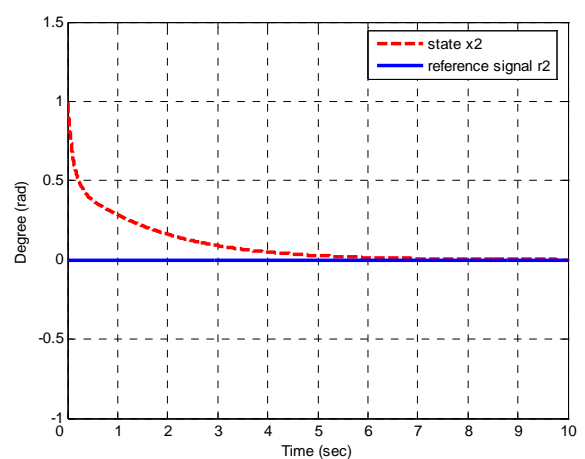


Figure 16. Curve of the state x_2 of the tracking control in example 3.

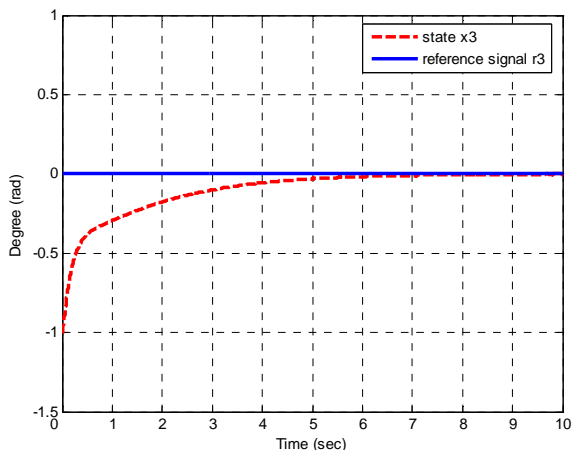


Figure 17. Curve of the state x_3 of the tracking control in example 3.

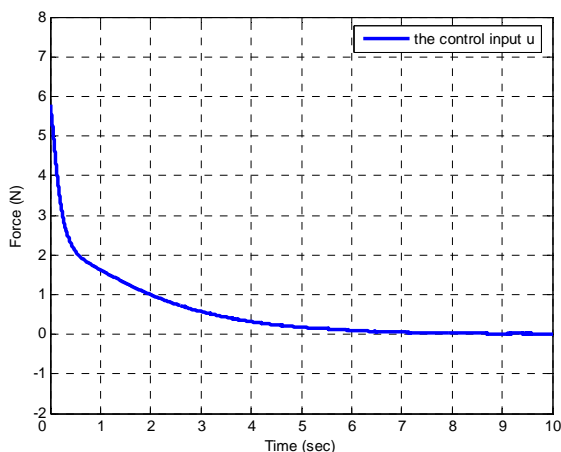


Figure 18. Response of control input u in example 3.



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