Models and Theory for Relay Channels with Receive Constraints

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Abstract

Relay channels where terminals cannot receive and transmit at the same time are modeled as being memoryless with cost constraints. Cost functions are considered that measure the power consumed in each of three sleep-listen-or-talk (SLoT) modes, as well as the fraction of time the modes are used. It is shown that strategies that have the SLoT modes known ahead of time by all terminals are generally suboptimal. It is further shown that Gaussian input distributions are generally suboptimal for Gaussian channels. For several types of models and SLoT constraints, it is shown that multi-hopping (or decode-and-forward) achieves the information-theoretic capacity if the relay is geometrically near the source terminal, and if the fraction of time the relay listens to the source is lower bounded by a positive number. SLoT constraints for which the capacity claim might not be valid are discussed. Finally, it is pointed out that a lack of symbol synchronization between the relays has little or no effect on the capacity theorems if the signals are bandlimited and if independent input signals are optimal.

1 Introduction

It is known that multi-hopping, or decode-and-forward, achieves the capacity of wireless relay channels if the relay is near the source terminal and if the channel phase is random and known only locally [7]. This capacity result is also valid if the relay cannot transmit and receive at the same time, as long as the destination knows the source and relay operating modes, and the fraction of time the relay listens to the source is lower bounded by a positive number [8]. The latter situation occurs, e.g., when protocols or energy constraints restrict the amount of time the relay can transmit. The purpose of this paper is to take another look at the underlying assumptions, models and theory for channels where the relay has transmit *and* receive constraints.

Information theory for relays that cannot receive and transmit simultaneously has already been developed by several authors, e.g., [4, 5, 6, 11] and references therein. A common assumption is that there is a *fixed* slot structure, i.e., all terminals know at all times which mode (receive or transmit) every terminal is using. We drop this restriction. More precisely, we model the channels as being *memoryless* with cost constraints. We further consider the case where the terminals can be in one of three *sleep-listen-or-talk* (SLoT) modes. Two peculiar features of our model is that *random* SLoT strategies achieve better rates than fixed ones, and that non-Gaussian input distributions achieve better rates than Gaussian ones. We argue that these features can be exploited in practice.

This paper is organized as follows. In Section 2, we define the communication model and discuss some if its subtleties. In Section 3, we review capacity upper and lower bounds obtained from information theory. The lower bounds are based on the *decode-and-forward* (DF) strategy of [1, Thm. 1] and the *partial-decode-and-forward* (PDF) strategy of [1, Thm. 7] or [3]. In Section 4, we consider several examples of SLoT constraints, and discuss cases where the DF strategy achieves capacity. Section 5 briefly discusses symbol synchronization between the transmitters. Section 6 concludes the paper.

2 Model

A relay channel [1] has three terminals numbered $t = 1, 2, 3$, a message W, channel inputs X_{ti}, $t = 1, 2$, $i = 1, 2, \ldots n$, channel outputs Y_{ti}, $t = 2, 3$, $i = 1, 2, \ldots n$, and a message estimate $\hat{W}$. The source (terminal 1) transmits the sequence $X_1^n = X_1, X_2, \ldots, X_n$ that is a function of W. The relay (terminal 2) input X_{2i} is a function of the past outputs Y_2^{i-1} for $i = 1, 2, \ldots n$. The destination (terminal 3) computes $\hat{W}$ as a function of Y_3^n. For a memoryless channel, the joint probability distribution of the random variables $W, X_1^n, X_2^n, Y_2^n, Y_3^n, \hat{W}$ thus factors as

$$P_W(w)\,1(X_1^n = x_1^n(w))\left[\prod_{i=1}^{n} 1(X_{2i} = x_{2i}(y_2^{i-1}))P_{Y_2Y_3|X_1X_2}(y_{2i}, y_{3i}|x_{1i}, x_{2i})\right]1(\hat{W} = \hat{w}(y_3^n)) \tag{1}$$

where $P_W(\cdot)$ is the probability distribution of the random variable W, $1(\cdot)$ is the indicator function that is 1 if its argument is true, and is zero otherwise. Suppose $H(W) = B$ bits so the data rate is $R = B/n$ bits per channel use. The *capacity* C is the supremum of rates for which one can achieve $\Pr(\hat{W} \neq W) < \epsilon$ for any positive ϵ.

We specialize the model to Gaussian channels. Each terminal t is modeled as operating in one of three modes: sleep (S), listen (L) or talk (T). The terminal transmits $X_{ti} = 0$ if it is in mode S or L, and receives $Y_{ti} = 0$ if it is in mode S or T. We make this precise by considering the channel inputs to be vectors $\underline{x}_{ti} = [m_{ti}, x_{ti}]$ with alphabet

$$\underline{\mathcal{X}} = \{(S,0),(L,0)\} \cup \{\{T\} \times \mathbb{C}\} \tag{2}$$

where $\mathbb{C}$ is the set of complex numbers. Note that we have changed the notation of (1) and have written x_{ti} for the second component of the input $\underline{x}_{ti}$. We continue to follow this convention below. The Gaussian channel outputs are

$$Y_{ti} = \begin{cases} Z_{ti} + \sum_{s\neq t} \frac{A_{sti}}{d_{st}^{\alpha/2}} X_{si} & \text{if } M_{ti} = L \\ 0 & \text{if } M_{ti} \neq L \end{cases} \tag{3}$$

for $t = 2, 3$, where the Z_{ti} are independent, Gaussian, zero mean, unit variance, and have independent and identically distributed (i.i.d.) real and imaginary parts. The number d_{st} represents the distance between terminals s and t, and α is an attenuation exponent. The A_{sti} are fading random variables, and as in [7] we consider two kinds of fading:

- No fading: $A_{sti} = 1$ for all s, t, and i.
- Phase fading: $A_{sti} = e^{j\theta_{sti}}$ where θ_{sti} is uniformly distributed over $[0, 2\pi)$. The θ_{sti} are jointly independent of each other and all other random variables.

We further assume that terminal t knows only its *own* fading coefficients, i.e., terminal t knows A_{sti} for all s and i, but it does not know $A_{st'i}$ for $t' \neq t$. The full channel output of terminal t at time i is thus $\underline{Y}_{ti} = [\underline{A}_{ti}, Y_{ti}]$, where $\underline{A}_{ti}$ is the vector of A_{sti} for all s. We remark that the following theory also applies to other types of fading models (see [8, 13]).

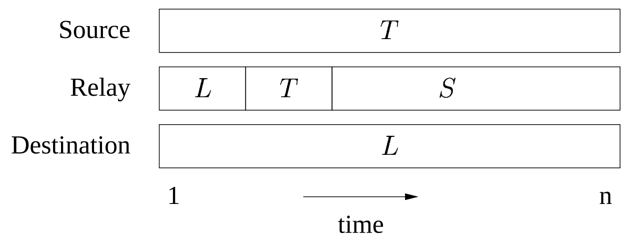

Figure 1: Example of a fixed SLoT strategy. The destination knows M_2.

2.1 Power Constraints

We introduce *cost functions* $f_t(\underline{x}_t)$ on the symbols in $\underline{\mathcal{X}}$:

$$f_t(\underline{x}_t) = f_t([m_t, x_t]) = \begin{cases} P_t(S) & \text{if } m_t = S \\ P_t(L) & \text{if } m_t = L \\ |x_t|^2 + P_t'(T) & \text{if } m_t = T. \end{cases} \tag{4}$$

where $P_t(m)$ is the power consumed in mode m, and where $P_t'(T)$ is some constant. One commonly imposes the following average *block* power constraints

$$\sum_{i=1}^{n} \frac{1}{n} \mathrm{E}\left[f_t(\underline{X}_{ti})\right] \le P_t, \quad t = 1, 2. \tag{5}$$

The constraints (5) let the source and relay distribute power across modes. However, to avoid having excessive power in any one mode, we add average *mode* power constraints

$$P_t(m) = \sum_{i:\, m_{ti}=m} \frac{1}{n_{tm}} \mathrm{E}\left[f_t([m, X_{ti}])\right] \le P_t^*, \quad t = 1, 2,\ m = S, L, T \tag{6}$$

where $P_t \le P_t^*$ and n_{tm} is the number of times that terminal t uses mode m. For simplicity, we have chosen P_t^* to be independent of m. Also, for our examples we assume the source always talks with $P_1'(T) = 0$, i.e.,

$$P_{M_1}(T) = 1 \quad \text{and} \quad \sum_{i=1}^{n} \frac{1}{n} \mathrm{E}\left[|X_{1i}|^2\right] \le P_1. \tag{7}$$

These constraints let us avoid optimizing $P_{M_1}(\cdot)$. We further assume that the destination always listens, i.e., $P_{M_3}(L) = 1$ (and $P_3 \ge P_3(L)$ so the power constraints are satisfied).

2.2 SLoT Constraints

A natural coding strategy is to choose a *fixed* SLoT structure, i.e., to specify ahead of time when every terminal should be in mode S, L or T. This is the approach taken in [4, 5, 6, 8, 11] and an example of such a strategy is depicted in Fig. 1. We call this a *fixed* or *deterministic SLoT strategy*. Alternatively, one might choose a *random SLoT strategy* as shown in Fig. 2. As we will show, a random strategy always performs as well as the fixed one, and usually better.

For both the fixed and random strategies, the following SLoT constraints seem natural:

- terminal t must be in sleep mode S at least a fraction β_t of the time,
- the relay must be in listen mode L at least a fraction γ_2 of the time,

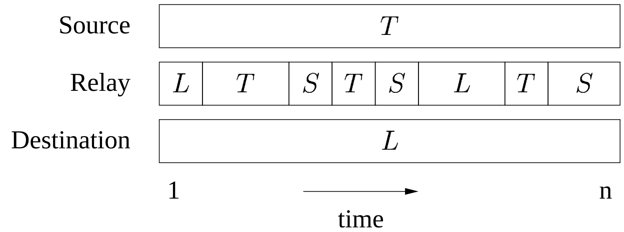

Figure 2: Example of a random SLoT strategy. The destination does not know M_2.

- terminals 1 and 3 never use modes L and T, respectively, i.e., $P_{M_1}(L) = P_{M_3}(T) = 0$.

The first constraint models the case where energy is at a premium. The second constraint ensures the relay receives sufficient symbols to permit decoding and coordination. Alternatively, this constraint reflects the fact that protocols sometimes restrict the fraction of time the relay can listen (or talk). Our capacity results depend on this constraint, i.e., for $\gamma_2 > 0$ we can sometimes prove that one achieves capacity. The third constraint is added because the source and destination have no channel output and input, respectively. We remark that one sometimes incurs a large power penalty when switching from mode S to modes L or T. For such cases, one might wish to use a *hybrid* strategy where the destination knows when $M_2 = S$.

3 Information Theory

3.1 Cut-set Upper Bound

The advantage of considering a *memoryless* model is that one can use all the existing theory on memoryless relay channels. In particular, we can apply the cut-set bound in [2, p. 445]:

$$\begin{aligned} C &\leq \max_{P_{\underline{X}_1 \underline{X}_2}(\cdot)} \min \left[I(\underline{X}_1; \underline{Y}_2 \underline{Y}_3 | \underline{X}_2),\ I(\underline{X}_1 \underline{X}_2; \underline{Y}_3) \right] \\ &= \max_{P_{\underline{X}_1 \underline{X}_2}(\cdot)} \min \left[I(X_1; Y_2 Y_3 | X_2 M_2 A_{12} A_{13} A_{23}),\ I(X_1 X_2; Y_3 | A_{13} A_{23}) \right]. \end{aligned} \tag{8}$$

where we have used the fact that $[M_1, M_2] - [X_1, X_2] - [\underline{Y}_2, \underline{Y}_3]$ forms a Markov chain. Note that (8) has no power or SLoT constraints associated with it. However, by using the concavity in $P_{\underline{X}_1 \underline{X}_2}(\cdot)$ of the minimum in (8), one can show that one can add the constraints

$$\begin{array}{ll} \mathrm{E}\left[f_t(\underline{X}_t)\right] \leq P_t, & t = 1,2 \\ \mathrm{E}\left[f_t(m_t, X_t)\right] \leq P_t^*, & t = 1,2,\ m_2 = S, L, T \end{array} \tag{9}$$

to (8). One can similarly show that one can add SLoT constraints to (8), e.g., $P_{M_t}(S) \geq \beta_t$, $t = 1, 2, 3$, $P_{M_2}(L) \geq \gamma_2$, and $P_{M_1}(L) = P_{M_3}(T) = 0$.

3.2 Achievable Rates: Decode-and-Forward

We apply [1, Thm. 1] to establish that the following rate is achievable:

$$\begin{aligned} R &= \max \min \left[I(\underline{X}_1; \underline{Y}_2 | \underline{X}_2), I(\underline{X}_1 \underline{X}_2; \underline{Y}_3) \right] \\ &= \max \min \left[I(X_1; Y_2 | X_2 M_2 A_{12}), I(X_1 X_2; Y_3 | A_{13} A_{23}) \right] \end{aligned} \tag{10}$$

where the maximization is over all $P_{\underline{X}_1 \underline{X}_2}(\cdot)$ satisfying the power and SLoT constraints. We call the strategy associated with this scheme a *decode-and-forward* strategy, or simply DF.

Suppose next that we use a *fixed* SLoT strategy. The achievable DF rate can be written as

$$R_F = \min\left[I(X_1;Y_2|X_2M_1M_2A_{12}), I(X_1X_2;Y_3|M_1M_2A_{13}A_{23})\right] \tag{11}$$

for some $P_{M_1M_2}(\cdot)\, p_{X_1X_2|M_1M_2}(\cdot)$. It is clear that Gaussian $p_{X_1X_2|M_1M_2}(\cdot)$ maximize (11). But if we use this $P_{M_1M_2}(\cdot)\, p_{X_1X_2|M_1M_2}(\cdot)$ in (10), we achieve

$$\begin{aligned} R = \min\big[&I(M_1;Y_2|X_2M_2A_{12}) + I(X_1;Y_2|X_2M_1M_2A_{12}), \\ &I(M_1M_2;Y_3|A_{13}A_{23}) + I(X_1X_2;Y_3|M_1M_2A_{13}A_{23})\big]. \end{aligned} \tag{12}$$

The rate (12) is at least as large as (11), and is usually larger. This means that a *random* SLoT strategy permits larger rates than the corresponding fixed SLoT strategy. Moreover, one sometimes achieves the largest rates with non-Gaussian $p_{X_1X_2|M_1M_2}(\cdot)$. The reason for the rate gain is that one can send information through the choice of operating modes. This gain should be feasible in practice, although the random SLoT strategies might require more sophisticated processing than the fixed ones.

3.3 Achievable Rates: Partial-Decode-and-Forward

The relay should not only listen, but also talk, which suggests that the relay should sometimes decode only *part* of the message. We can accommodate this by using [1, Thm. 7] as in [3], which establishes that the following rate is achievable:

$$\begin{aligned} R &= \max\,\min\left[I(Q;\underline{Y}_2|\underline{X}_2) + I(\underline{X}_1;\underline{Y}_3|Q\underline{X}_2), I(\underline{X}_1\underline{X}_2;\underline{Y}_3)\right] \\ &= \max\,\min\left[I(Q;Y_2|X_2M_2A_{12}) + I(X_1;Y_3|QX_2M_2A_{13}A_{23}), I(X_1X_2;Y_3|A_{13}A_{23})\right] \end{aligned} \tag{13}$$

where the maximization is over all $P_{Q\underline{X}_1\underline{X}_2}(\cdot)$ satisfying the power and SLoT constraints. We call the strategy associated with this scheme a *partial-decode-and-forward* strategy, or simply PDF. The reason for this choice of name is that the relay is decoding only that *part* of the message W represented by Q. The PDF strategy was used, e.g., in [5, Thm. 1] for relay channels and in [12] for multi-access relay channels. Such strategies have also been called *adaptive* DF protocols, e.g., in [9, Ch. 5]. We remark that one can make similar claims as in (11)–(12) when comparing fixed and random SLoT strategies.

4 Examples of SLoT Constraints

We consider several examples of SLoT constraints to illustrate the theory. For simplicity, we consider only strategies with $P_{M_1}(T) = P_{M_3}(L) = 1$. We further consider the geometry shown in Fig. 3, i.e., the source is at the origin, and the relay and destination are a distance of d and 1 to the right of the source, respectively.

4.1 Fixed SLoT Strategies without Fading

Consider a *fixed* SLoT strategy with no fading. As in [5, 6, 7], for DF we find that it is best to choose Gaussian X_1 and X_2 when conditioning on M_1 and M_2. We compute

$$\begin{aligned} R_F = \min\Bigg[&P_{M_2}(L)\log\left(1+\frac{P_1}{d_{12}^{\alpha}}\right), (P_{M_2}(S)+P_{M_2}(L))\log\left(1+\frac{P_1}{d_{13}^{\alpha}}\right) + \\ &P_{M_2}(T)\log\left(1+\frac{P_1}{d_{13}^{\alpha}}+\frac{P_2''(T)}{d_{23}^{\alpha}}+\frac{2\Re(\rho)\sqrt{P_1P_2''(T)}}{d_{13}^{\alpha/2}d_{23}^{\alpha/2}}\right)\Bigg] \end{aligned} \tag{14}$$

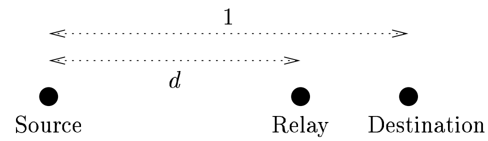

Figure 3: Geometry for a relay channel.

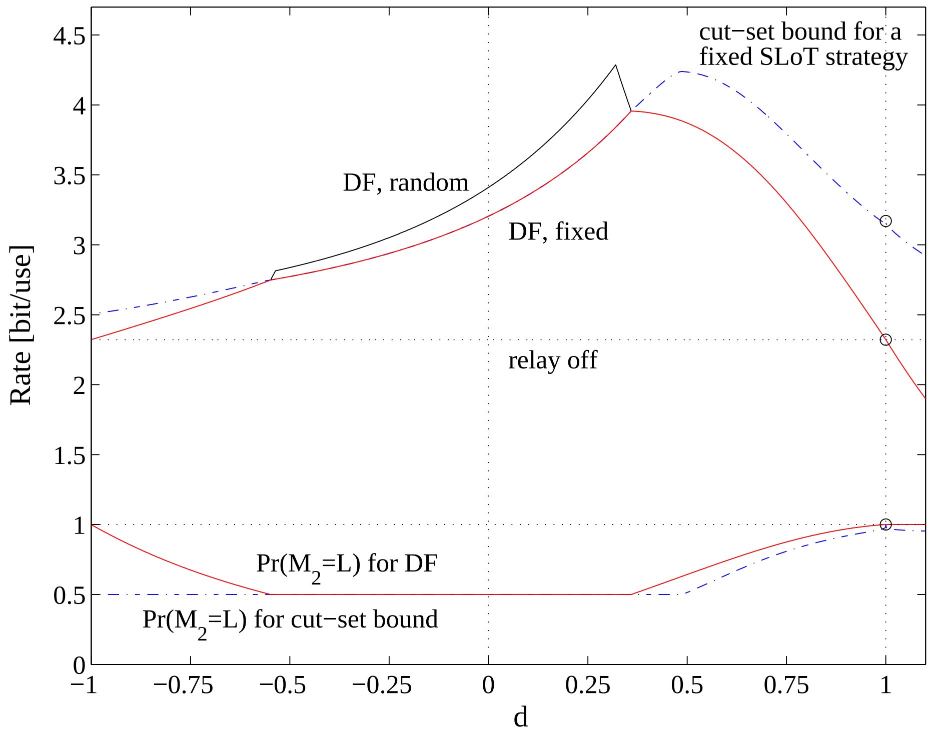

Figure 4: Rates for fixed and random SLoT strategies without fading.

where $\rho = \mathrm{E}\left[X_1 X_2^* | M_2 = T\right] / \sqrt{P_1 P_2''(T)}$, $\Re(\rho)$ is the real part of ρ, and

$$P_2''(T) = \min\left[P_2^*, \frac{P_2 - P_{M_2}(S)P_2(S) - P_{M_2}(L)P_2(L)}{P_{M_2}(T)}\right] - P_2'(T). \tag{15}$$

One clearly should use $\rho = 1$. Suppose $P_2(S) = P_2(L)$, in which case one should also set $P_{M_2}(S) = \beta_2$ (any $M_2 = S$ symbol should be made a $M_2 = L$ symbol). We optimize $P_{M_2}(L)$ numerically for the following example.

Example 1 *Consider the geometry of Fig. 3, and suppose the system parameters are*

$$\alpha = 4, \quad P_1 = P_2 = P_2^* = 4, \quad P_2(S) = P_2(L) = P_2'(T) = 0, \quad \beta_2 = 0, \gamma_2 = 0.5. \tag{16}$$

The DF rates are shown in Fig. 4 as the curve labeled "DF, fixed". The dash-dotted curve is the cut-set bound for a fixed SLoT strategy, and it is computed as (14) but with the first term inside the minimization of (14) replaced by (see [5, Thm. 1] or [6])

$$P_{M_2}(S)\log\left(1+\frac{P_1}{d_{13}^{\alpha}}\right) + P_{M_2}(L)\log\left(1+\frac{P_1}{d_{12}^{\alpha}}+\frac{P_1}{d_{13}^{\alpha}}\right) + P_{M_2}(T)\log\left(1+\frac{P_1(1-|\rho|^2)}{d_{13}^{\alpha}}\right). \tag{17}$$

For the cut-set bound and $P_2(S) = P_2(L)$, any $M_2 = S$ symbol should again be made a $M_2 = L$ symbol, so we set $P_{M_2}(S) = \beta_2$. The lower curves in Fig. 4 show the optimizing $P_{M_2}(L)$ for both DF and the cut-set bound.

The are several curious features about Fig. 4. First, as a notable difference to [7], we find that DF achieves capacity *without* fading if the relay is close to the source and if we are forced to use a *fixed* SLoT strategy. That is, DF achieves "capacity" for $-0.55 \leq d \leq 0.36$. Similar capacity results appear whenever $\gamma_2 > 0$. Second, the cut-set bound exhibits a sharp behavior near $d = 1$. Third, the DF strategy should not be used if $|d| \geq 1$. Finally, we remark that the PDF strategy can improve on the DF strategy (see [12]).

4.2 Random SLoT Strategies without Fading

Suppose we now use a *random* SLoT strategy as in Fig. 2. For the geometry and parameters of Example 1, for every d we simply choose the *same* distribution $P_{M_2X_1X_2}(\cdot)$ as for the "DF, fixed" curve, and insert this distribution into (10). The resulting DF rate is

$$R = \min\left[P_{M_2}(L)\log\left(1+\frac{P_1}{d_{12}^\alpha}\right), h(Y_3) - \log(\pi e)\right] \tag{18}$$

where we use the circular symmetry of Y_3 to write

$$h(Y_3) = \int_0^\infty -q(y)\log q(y)\; 2\pi y\, dy \tag{19}$$

$$q(y) = \frac{P_{M_2}(S)+P_{M_2}(L)}{\pi\sigma_1^2}\, e^{-y^2/\sigma_1^2} + \frac{P_{M_2}(T)}{\pi\sigma_2^2}\, e^{-y^2/\sigma_2^2} \tag{20}$$

$$\sigma_1^2 = 1+\frac{P_1}{d_{13}^\alpha}, \qquad \sigma_2^2 = 1+\frac{P_1}{d_{13}^\alpha}+\frac{P_2''(T)}{d_{23}^\alpha}+\frac{2\sqrt{P_1P_2''(T)}}{d_{13}^{\alpha/2}d_{23}^{\alpha/2}}. \tag{21}$$

The rates R are plotted in Fig. 4 as the uppermost curve labeled "DF, random". As expected, we find that $R \geq R_F$. Moreover, R is substantially *larger* than R_F in the interesting region where the relay is near the source. We can thus transmit at rates beyond the "capacity" of the fixed SLoT strategy.

It seems natural to suspect that DF achieves the true capacity of the relay channel defined by (2)–(6), as long as the relay is near the source and $\gamma_2 > 0$. To prove this, one must show that the maximizing distribution $P_{M_2X_1X_2}(\cdot)$ for the cut-set bound (8) is the same as the best $P_{M_2X_1X_2}(\cdot)$ for the DF rate (10).

4.3 Fixed SLoT Strategies with Phase Fading

Consider again a *fixed* SLoT strategy, but now with phase fading. As in [5, 7], for the DF strategy we find that it is best to choose Gaussian X_1 and X_2 that are statistically independent when conditioned on M_1 and M_2. We compute

$$\begin{aligned} R_F = \min\Bigg[&P_{M_2}(L)\log\left(1+\frac{P_1}{d_{12}^\alpha}\right), (P_{M_2}(S)+P_{M_2}(L))\log\left(1+\frac{P_1}{d_{13}^\alpha}\right) + \\ &P_{M_2}(T)\log\left(1+\frac{P_1}{d_{13}^\alpha}+\frac{P_2''(T)}{d_{23}^\alpha}\right)\Bigg] \end{aligned} \tag{22}$$

where $P_2''(T)$ is given by (15). Note that (22) is the same as (14) with $\rho = 0$. Suppose again that $P_2(S) = P_2(L)$, so that $P_{M_2}(S) = \beta_2$ is best. We optimize $P_{M_2}(L)$ numerically for the following example.

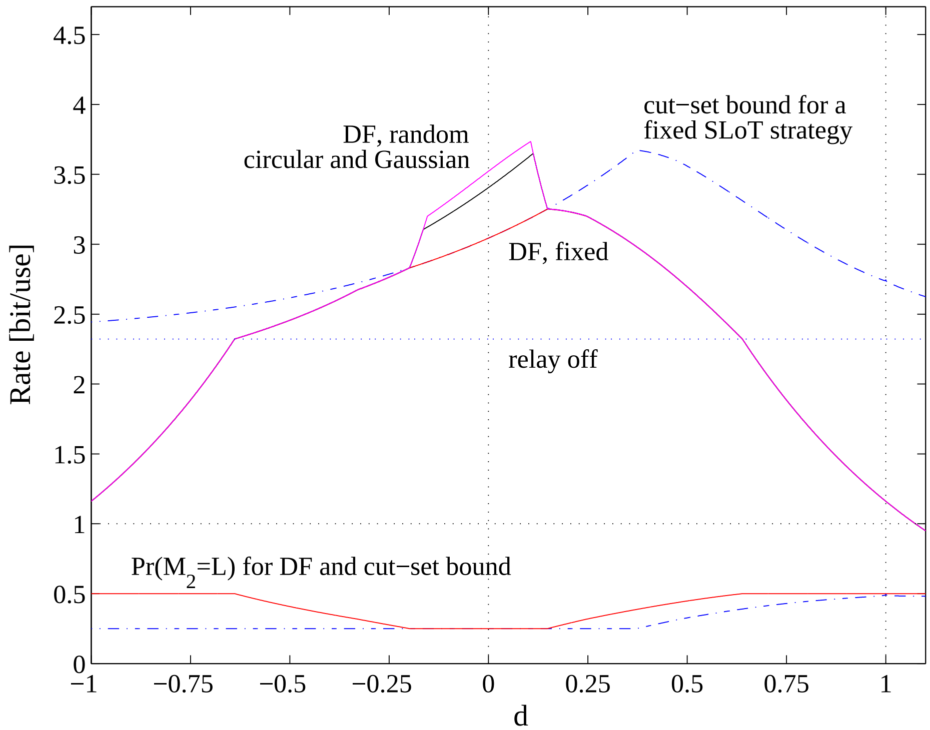

Figure 5: Rates for fixed and random SLoT strategies with phase fading.

Example 2 *Consider the geometry of Fig. 3, and the system parameters* $\alpha = 4$ *and*

$$P_1 = P_2 = 4, P_2^* = 40, \quad P_2(S) = P_2(L) = P_2'(T) = -4, \quad \beta_2 = 0.5, \gamma_2 = 0.25. \tag{23}$$

That is, the relay can make $P_2''(T)$ *as large as 40 while keeping its average power at* $P_2 = 4$*. The relay further collects energy in all modes, e.g., if it has a solar cell that refreshes energy. However, the relay must sleep for at least 1/2 of the time, and it must listen for at least 1/4 of the time. We find that* $P_{M_2}(S) = \beta_2$ *is best, and the optimum* $P_{M_2}(L)$ *are plotted as the lowermost curve in Fig. 5. The DF rates are plotted as the curve labeled "DF, fixed". The dash-dotted curve is the cut-set bound for a fixed SLoT strategy, and it is computed as (22) but with the first logarithm in (22) replaced by (17) with* $\rho = 0$*.*

Note that DF again achieves the "capacity" for a fixed SLoT strategy if the relay is near the source ($-0.2 \leq d \leq 0.15$) and $\gamma_2 > 0$. We also remark that the rates of Fig. 5 are not necessarily smaller than those in Fig. 4, even though $\rho = 0$ and the relay must sleep or listen for a larger fraction of time than before. The reason for this behavior is that the relay can talk with more power than before.

4.4 Random SLoT Strategies with Phase Fading

We now use *random* SLoT strategies for phase fading. For the parameters of Example 2, for every d we choose the same distribution $P_{M_2X_1X_2}(\cdot)$ as for the "DF, fixed" curve, and insert this distribution into (10). The DF rate is given by (18)–(20) and

$$\sigma_1^2 = 1 + \frac{P_1}{d_{13}^\alpha}, \qquad \sigma_2^2 = 1 + \frac{P_1}{d_{13}^\alpha} + \frac{P_2''(T)}{d_{23}^\alpha}. \tag{24}$$

The DF rate is plotted in Fig. 5 as the solid curve above the "DF, fixed" curve and below the uppermost solid curve. We again find that a random SLoT strategy achieves larger rates than the fixed one in the interesting region where the relay is close to the source.

Suppose next that for $M_2 = T$ we replace the Gaussian X_2 by $X_2 = \sqrt{P_2''(T)}\, e^{j\phi_2}$ where ϕ_2 is uniformly distributed over $[0, 2\pi)$. We call this a "circular" distribution for X_2. The resulting DF rate is given by (18), (19), (24), and with (20) replaced by

$$q(y) = \frac{P_{M_2}(S) + P_{M_2}(L)}{\pi\sigma_1^2}\, e^{-y^2/\sigma_1^2} + \frac{P_{M_2}(T)}{\pi\sigma_1^2}\, e^{-(y^2+\sigma_2^2-\sigma_1^2)/\sigma_1^2}\, I_0\left(2y\sqrt{\frac{\sigma_2^2-\sigma_1^2}{\sigma_1^2}}\right) \qquad (25)$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind of order zero. The circular DF rate is plotted in Fig. 5 as the uppermost solid curve. Note that a non-Gaussian input distribution for $M_2 = T$ achieves a larger rate than the Gaussian one. It again seems natural to suspect that DF achieves capacity as long as the relay is near the source and $\gamma_2 > 0$.

5 Symbol Synchronization

An important limitation of the model of Section 2 is that the network operates *synchronously*. The transmitting terminals might therefore need to be *symbol* synchronized, and this might be difficult to implement in wireless networks. However, we point out that as long as the signals are *bandlimited*, the DF and PDF strategies with *independent* X_1 and X_2 do not require symbol synchronization between terminals. This statement can be justified as follows. The filtered and sampled signal at the receiver contains sufficient statistics about the transmitted signals if the sampling rate is at or above the Nyquist rate. Further, both the DF and PDF strategies can be implemented with block-Markov encoders and joint decoders that can interpolate the Y_{3i} sequences of *different* receive blocks. This should permit decoding at the rates (10) or (13).

We remark that all three DF curves in Fig. 5 have independent X_1 and X_2. It remains to be seen whether independent inputs are capacity–achieving for the phase fading models considered here (this is currently known, e.g., for models where the relay can transmit and receive at the same time and in the same frequency band [7, 8, 13]).

6 Concluding Remarks

Relay channels where terminals cannot receive and transmit at the same time were modeled as being memoryless with cost constraints. It was shown that one should use random SLoT strategies to maximize information rates. Two interesting open problems are to find the capacity-achieving input distributions, and to determine whether one does in fact achieve capacity when the relay is near the source and $\gamma_2 > 0$. For phase fading, one might expect the capacity-achieving input distributions to have independent X_1 and X_2. If this is the case, then one will not need to phase or symbol synchronize the source and relay if the transmitted signals are bandlimited. Finally, we remark that many of the concepts of this paper will carry over to Rayleigh fading channels and vector channels (see [8, 13]), to multi-terminal channels (see [5, 8, 12]) and to channels with feedback (see [10]).

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