Diagonal Principal Component Analysis for Face Recognition

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Abstract

In this paper, a novel subspace method called diagonal principal component analysis (DiaPCA) is proposed for face recognition. In contrast to standard PCA, DiaPCA directly seeks the optimal projective vectors from *diagonal face images* without image-to-vector transformation. While in contrast to 2DPCA, DiaPCA reserves the correlations between variations of rows and those of columns of images. Experiments show that DiaPCA is much more accurate than both PCA and 2DPCA. Furthermore, it is shown that the accuracy can be further improved by combining DiaPCA with 2DPCA.

Keywords: Principal component analysis (PCA); diagonal PCA; 2-Dimensional PCA; face recognition

1 Introduction

Principal component analysis (PCA), also known as eigenfaces [3], is one of the state-of-the-art methods in face recognition. Especially, for the *single sample per person problem* where only one image per person is available for training, most face recognition methods such as linear discriminant analysis (LDA), discriminant eigenfeatures and fisherfaces fail, while variants of PCA still work [1]. This is one of the reasons that why face recognition based on PCA is still very active, although it has been investigated for decades. In the original PCA-based face recognition, the 2D face images must be transformed into 1D vectors column by column or row by row. However, concatenating the entries in 2D matrices into 1D vectors often leads to a high-dimensional vector space, where it is difficult to evaluate the covariance matrix accurately due to the large size and the relatively small number of training samples [4]. Furthermore, computing the eigenvectors of a large size covariance matrix is very time-consuming.

Recently, a new technique called 2-dimensional principal component analysis (2DPCA) was proposed to solve the above problems [4]. The main idea behind 2DPCA is to find the optimal

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projective vectors in the row direction of images without the image-to-vector transformation. That is, it first constructs the so-called *image covariance matrix* from rows of images and then computes its eigenvectors as the projection vectors. Since the size of the image covariance matrix is equal to the width of images, which is quite small compared with the size of the covariance matrix used in PCA, 2DPCA evaluates the image covariance matrix more accurately and computes the corresponding eigenvectors more efficiently than PCA.

However, the projective vectors of 2DPCA only reflect variations between rows of images, while the omitted variations between columns of images are usually also useful for recognition. In that case, 2DPCA can hardly obtain improved accuracy. In this paper, a novel method called diagonal principal component analysis (DiaPCA) is proposed. In contrast to 2DPCA, DiaPCA seeks the optimal projective vectors from *diagonal face images* and therefore the correlations between variations of rows and those of columns of images can be kept. Experimental results on a subset of FERET database show that DiaPCA is much more accurate than both PCA and 2DPCA. Furthermore, it is shown that the accuracy can be further improved by combining DiaPCA and 2DPCA together.

The rest of this paper is organized as follows. Section 2 presents the proposed DiaPCA method. Section 3 introduces the combination of DiaPCA and 2DPCA. Section 4 reports on the experimental results. Finally, Section 5 concludes.

2 Diagonal Principal Component Analysis

Our motivation for developing the DiaPCA method originates from an essential observation on the recently proposed 2DPCA [4]. That is, 2DPCA can be seen as the row-based PCA, which has been pointed out in [5]. So 2DPCA only reflects the information between rows, which implies some structure information (e.g. regions of a face like eyes, nose, etc.) cannot be uncovered by it. We attempt to solve that problem by transforming the original face images into corresponding *diagonal face images*. Because the rows (columns) in the transformed diagonal face images simultaneously integrate the information of rows and columns in original images, it can reflect both information between rows and those between columns. Through the entanglement of row and column information, it is expected that DiaPCA may find some useful block or structure information for recognition in original images.

Suppose that there are M training face images, denoted by m by n matrices $\mathbf{A}_k (k = 1, 2, ..., M)$. For each training face image, define the corresponding *diagonal face image* as follows:

1) If the height m is equal to or smaller than the width n, use the method illustrated in Fig. 1(a) to generate the diagonal image $\boldsymbol{B}$ for the original image $\boldsymbol{A}$.

2) If the height m is bigger than the width n, use the method illustrated in Fig. 1(b) to generate the diagonal image $\boldsymbol{B}$ for the original image $\boldsymbol{A}$.

Without loss of generalization, assume that the width n is no smaller than the height m. For each training face image $\mathbf{A}_k$, derive the corresponding diagonal face $\mathbf{B}_k$ using the method illustrated in Fig. 1(a). Note that $\mathbf{B}_k$ s are of the same size of $\mathbf{A}_k$ s.

Based on the diagonal faces, define the *diagonal covariance matrix* as

$$\mathbf{G} = \frac{1}{M}\sum_{k=1}^{M}(\mathbf{B}_k - \overline{\mathbf{B}})^T(\mathbf{B}_k - \overline{\mathbf{B}}), \tag{1}$$

where $\overline{\mathbf{B}} = \frac{1}{M}\sum_k \mathbf{B}_k$ is the mean diagonal face. According to Eq. (1), the projective vectors $\mathbf{X}_1,\ldots,\mathbf{X}_d$ can be obtained by computing the d eigenvectors corresponding to the d biggest eigenvalues of $\mathbf{G}$. Since the size of $\mathbf{G}$ is only n by n, computing its eigenvectors can be efficient.

Let $\mathbf{X} = [\mathbf{X}_1,\ldots,\mathbf{X}_d]$ denote the projective matrix, projecting training faces $\mathbf{A}_k$ s onto $\mathbf{X}$, yielding m by d *feature matrices*

$$\mathbf{C}_k = \mathbf{A}_k\mathbf{X}. \tag{2}$$

Given a test face image $\mathbf{A}$, first use Eq. (2) to get the feature matrix $\mathbf{C} = \mathbf{A}\mathbf{X}$, then a nearest neighbor classifier can be used for classification. Here the distance between $\mathbf{C}$ and $\mathbf{C}_k$ is defined as $d(\mathbf{C},\mathbf{C}_k) = \|\mathbf{C} - \mathbf{C}_k\| = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{d}\left(\mathbf{C}^{(i,j)} - \mathbf{C}_k^{(i,j)}\right)^2}$.

3 DiaPCA+2DPCA

Suppose the n by d matrix $\mathbf{X} = [\mathbf{X}_1,\ldots,\mathbf{X}_d]$ is the projective matrix of DiaPCA. Let $\overline{\mathbf{A}} = \frac{1}{M}\sum_k \mathbf{A}_k$ denote the mean training face, the projective matrix $\mathbf{Y} = [\mathbf{Y}_1,\ldots,\mathbf{Y}_q]$ of 2DPCA is computed as follows. When the height m is equal to the width n, $\mathbf{Y}$ is gotten by computing the q eigenvectors corresponding to the q biggest eigenvalues of the image covariance matrix $\frac{1}{M}\sum_{k=1}^{M}(\mathbf{A}_k - \overline{\mathbf{A}})^T(\mathbf{A}_k - \overline{\mathbf{A}})$. On the other hand, when the height m is not equal to the width n, $\mathbf{Y}$ is gotten by computing the q eigenvectors corresponding to the q biggest eigenvalues of the alternative image covariance matrix $\frac{1}{M}\sum_{K=1}^{M}(\mathbf{A}_k - \overline{\mathbf{A}})(\mathbf{A}_k - \overline{\mathbf{A}})^T$.

Projecting training faces $\mathbf{A}_k$ s onto $\mathbf{X}$ and $\mathbf{Y}$ together, yielding the q by d feature matrices

$$\mathbf{D}_k = \mathbf{Y}^T \mathbf{A}_k \mathbf{X} . \tag{3}$$

Given a test face image $\mathbf{A}$, first use Eq. (3) to get the feature matrix $\mathbf{D} = \mathbf{Y}^T \mathbf{A}\mathbf{X}$, then a nearest neighbor classifier can be used for classification.

4 Experiments

The proposed methods are tested using a subset of the FERET face database [2], [5]. It comprises 400 gray-level frontal view face images from 200 persons, each of which is cropped with the size of 60×60. There are 71 females and 129 males; each person has two images (**fa** and **fb**) with different facial expressions. The **fa** images are used as gallery for training while the **fb** images as probes for test. PCA (eigenfaces), 2DPCA and the proposed DiaPCA and DiaPCA+2DPCA methods are used for feature extraction, and then a nearest neighbor classifier is employed for classification. All the experiments are carried out on a PC with P4 1.7GHz CPU and 256MB memory.

Table 1 presents the comparisons of the four methods on top recognition accuracy, the corresponding dimensions of feature vector (for PCA) or feature matrices (for the other three methods), and the running time costs. Table 1 shows that the best accuracy of DiaPCA (90.5%) is significantly higher than the best accuracy of PCA and 2DPCA (both 85.5%). Table 1 also shows that by combining DiaPCA and 2DPCA together, the accuracy is further improved to 91.5%. Note that the best recognition accuracy of two recent face recognition methods designed for the single training sample per person problem, i.e. Mat-FLDA [1] and KSOM-face [2], are 86.5 [1] and 89.5% [2], respectively, on the same face dataset. Moreover, it can be found from Table 1 that the running time costs of 2DPCA, DiaPCA and DiaPCA+2DPCA are comparable, much smaller than that of PCA. This is not difficult to understand because the size of covariance matrices used in the former three methods are much smaller than that used in PCA, and therefore the corresponding eigenvectors can be computed more efficiently. Finally, Fig. 2 depicts the cumulative match scores [2] (top k match) of the four methods. It is impressive that the proposed methods consistently outperform PCA and 2DPCA no matter which rank is considered.

5 Conclusion

A novel face recognition method called diagonal principal component analysis (DiaPCA) is proposed in this paper. The essential idea of the proposed method is to generate the *diagonal face images* from the original training images, from which the optimal projective vectors are sought, therefore the correlations between variations of rows and those of columns of images can be reserved. Experimental results on a subset of FERET database show that DiaPCA is much more accurate than both PCA and 2DPCA. It is shown that the accuracy can be further improved by combining DiaPCA and 2DPCA together.

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Table 1 Comparison on top recognition accuracy (%), the corresponding dimensions of feature vectors or matrices, and the running time costs (second).

Method	Accuracy (%)	Dimension	Time (s)
PCA	85.50	16	7.77
2DPCA	85.50	60×4	0.79
DiaPCA	90.50	60×5	0.87
DiaPCA +2DPCA	91.50	16×5	0.73

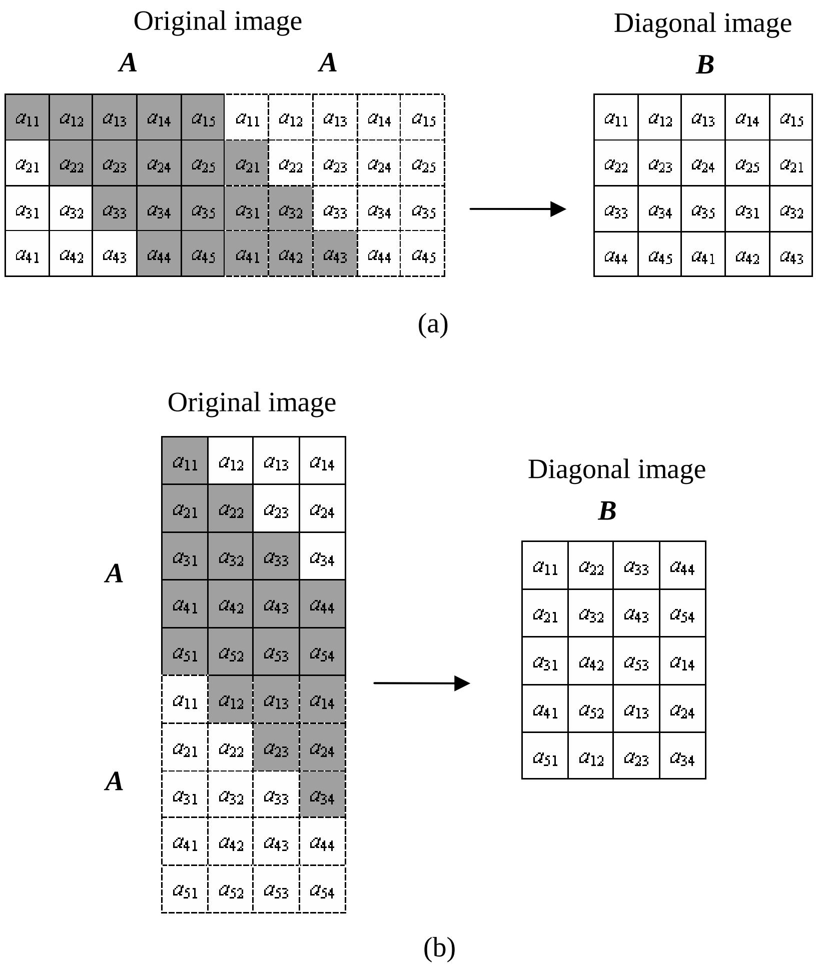

Fig. 1 Illustration of the ways for deriving the diagonal face images.

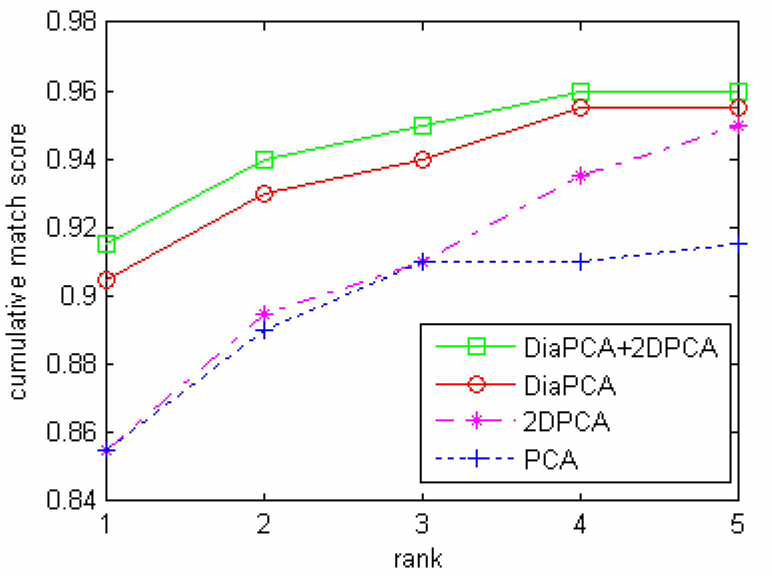

Fig. 2 Cumulative match scores of the four methods.