Price Spikes in Electricity Markets: A Strategic Perspective

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Abstract

This paper aims to analyze the issue of price spikes in electricity markets through the lens of noncooperative game theory. The case we consider is Australia's long established National Electricity Market (NEM). Specifically, we adapt von der Fehr and Harbord's [26] multi-unit auction model to settings that more closely reflect the structure of the NEM, showing that price spikes can be related to a specifiable threshold in demand.

JEL Classification Codes: C72; Q40; Q47

Keywords: Electricity Markets, Spot Price Behaviour, Non-Cooperative Game Theory.

1 Introduction

The frequency and severity of price spikes in electricity markets, is an issue with increasing traction in domestic politics [21, 23]. This has clearly been the case in Australia for some time. As the economics of resources and sustainability receives greater public attention, the issue of energy prices has become a genuine policy concern. In particular, an inspection into the root causes of price spikes has far reaching implications for competition and climate change policy.

The issue we address in this paper is the tendency for spot prices in Australia's National Electricity Market (NEM) to spike drastically (above \$1000/MWh). In addition to this, it is

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common for daily prices to peak at a level ($100-$1000/MWh) that can be described as irregularly high. This behaviour tends to affect the pricing of electricity in the long-run, ultimately raising prices in the retail market. It is no surprise then that this issue is important when it comes to policy-making. Figures 1.1 and 1.2 track the price behaviour that we wish to explain. Figure 1.1 shows the demand and price profiles for New South Wales in January 2007. The most distinct behavioural property, and the one on which we focus, is the tendency for price to spike as demand peaks. The other critical observation is that such spot price behaviour is highly non-linear. Figure 1.2, an undated scatter plot of the same price-demand data points, shows us just how random spiking behaviour can appear relative to demand.

The NEM is Australia's largest electricity market, with five states all interconnected via a combination of high voltage transmission lines. The NEM was established in 1998 and fully interconnected in 2005 and it represents the worlds longest interconnected power system which runs over 5000 km. The market itself was established as a pool based energy only market to meet instantaneous demand by perform 5 minute auctions which settle on every half hour. Generators must submit price and supply bids to the market operator the day before for each 5 minute dispatch period for the ensuing day, so that the market operator can meet demand. Generation assets have historically been government owned with more private investment in generation over the years since the markets establishment. The main reason for selecting this market for examination is the availability and clarity of data available. All 5 minute dispatch and 30 min settlement periods have an enormous amount of data available, including bids, interconnector flows and generation asset behaviour. Furthermore, the NEM also represents a a fairly unique market with a diverse range of generation assets and its energy only settlement structure.

Supply auctions for electricity are a common market design. As such, there is a healthy body of literature dedicated to strategic analysis of price and bidding behaviour in this setting. In particular, Klemperer and Meyer's [19] Supply Function Equilibrium (SFE) model has been used extensively for analysis of electricity markets. For example Green and Newbery [15] have used this approach to argue that, with capacity constraints, the supply function equilibrium is a Cournot solution. Further work in this area has yielded solutions in cases of completely inelastic demand, oligopoly and capacity constraints in various combinations [4, 17, 24]. These results have been used as a basis for the analysis of price behaviour and modeling. Extending beyond closed form

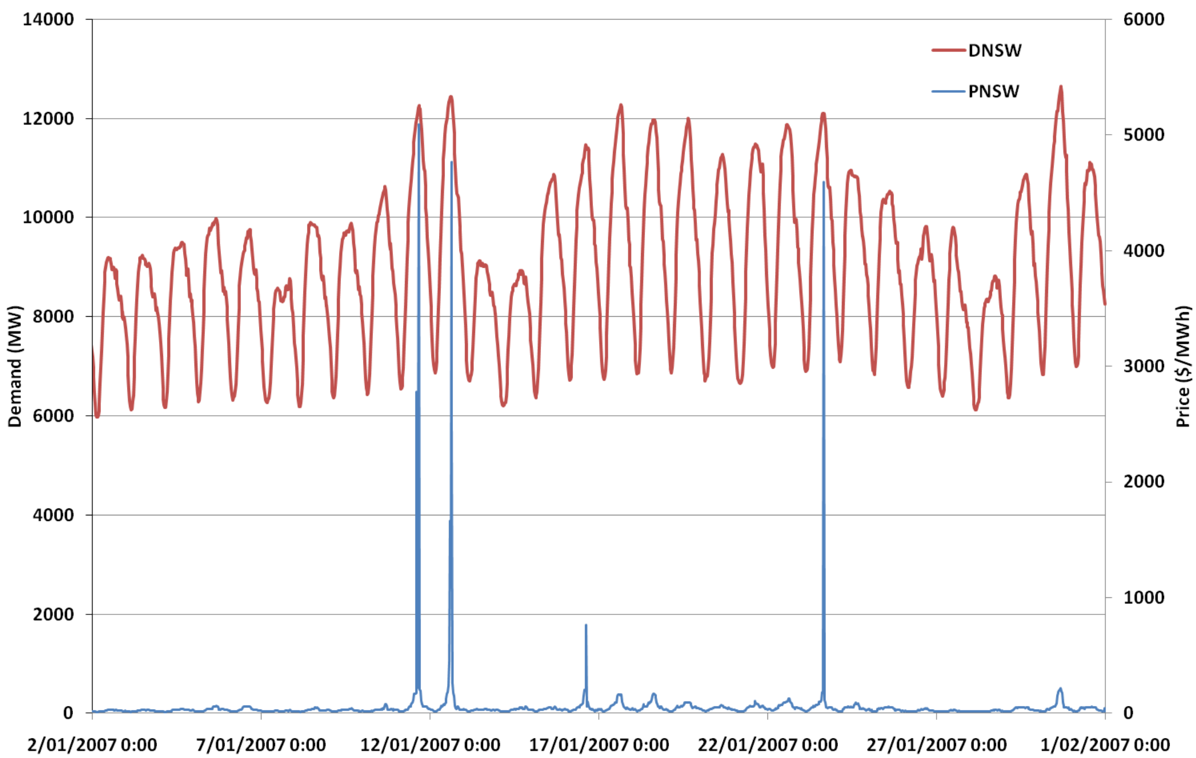

Figure 1.1: NSW demand and price profile for January 2007 (Demand Red and Price Blue)

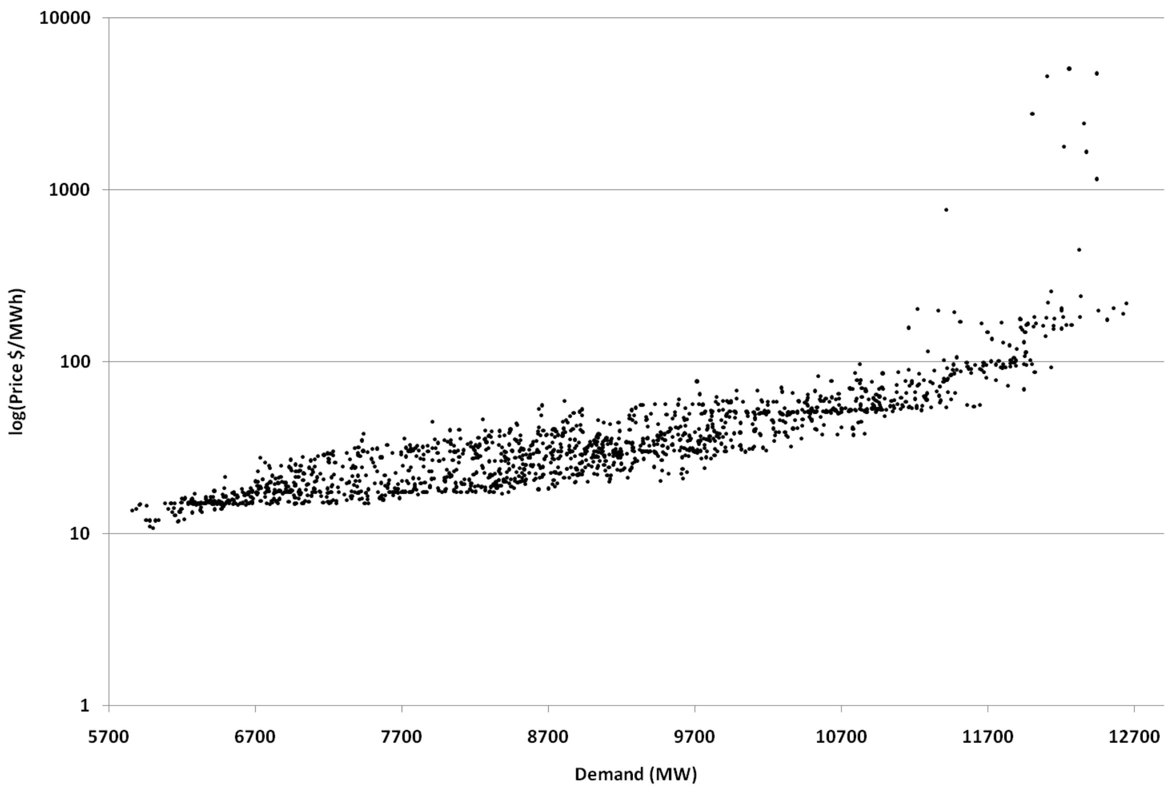

Figure 1.2: Demand-Price Scatter for NSW, January 07

solutions, there is a body of work that uses numerical SFE solutions to address the complexities that arise from network and operational constraints [5], different cost structures [3] and options contracts [1, 2].

The other major modeling approach in the analysis of electricity markets is the multi-unit auction model, first used in this context by von der Fehr and Harbord [26]. It has since been extended to a broader strategic setting by Fabra, von der Fehr and Harbord [12], but the thrust of the key results is very much the same. Both the supply function equilibrium and multi-unit auction models suggest that serious price effects will arise in the presence of capacity constraints [1]. More specifically, they are likely to arise when one firm faces periods of completely inelastic residual demand. All of the modeling done herein works towards this simple yet critical insight into how electricity markets function. As a theoretical tool, the multi-unit auction model is a far more accurate representation of the Australia's' NEM market design. In particular, it lends itself to a concise equilibrium analysis and is far more representative of the strategic choices faced by NEM generators. As such, we opt to use the multi-unit auction model for our analysis. In addition to the work already mentioned, some theoretical extensions have been made. For example, Dechenaux and Kovenock [10] offer a complete characterization in oligopoly with elestic demand, while Garca-Daz and Marin [14] have extended to the case of heterogeneous unit costs.

On the empirical side, there is a growing literature on the use of strategic models, predominately the SFE and multi-unit approaches, as a structural means to analyze bidding behaviour in electricity markets. Using NEM data from 1997 and access to proprietary contract information, Wolak [29] has used optimal bid restrictions to estimate the marginal costs for a Victorian NEM generator. In a similar spirit to this approach, Sweeting [25] and Hortasu and Puller [18] have analysed bid behaviour in the England and Wales, and Texas power markets. Wolfram [31] has used von der Fehr and Harbord's mixed strategy results as the basis of a regression model for the British electricity spot market, finding statistically significant evidence in support of the conjecture that generators in this market strategically manipulate their bids. Using a similar modelling framework to the multi-unit auction, Crawford, Crespo and Tauchen [8] have defined and characterised "bid function equilibria", which they use to derive testable structural equations.

Brennan and Melanie [6], offered analysis, prior to the creation of the NEM in 1998, on the

[1] In fact it can be shown that a supply function equilibrium can be constructed from the limit of the mixed strategy equilibrium in the n-unit auction as $n \to \infty$ [22]

likely price behaviour in such a market. Motivated by the insights of von der Fehr and Harbord [26] and Green and Newbery [15], they used load duration curves to show that generators in NSW - Macquarie, Delta and Pacific - face significant periods of completely inelastic residual demand. They argued that the price effects of this, as suggested by the theory, could be serious. There has not been a great deal of strategic analysis of the NEM since then and its is our objective here to revisit their argument, with 10 years of data and a range of insights covering the working of the NEM.

2 The Model

The model we adopt was first introduced by von der Fehr and Harbord [26] and subsequently extended by Fabra, von der Fehr and Harbord [12]. In it's simplest form, two generators engage in a uniform price supply auction. We assume that marginal costs, c_i for generator i, are constant. Denoting k_i for plant i's capacity, p_i as its price bid and Q as (completely inelastic) system demand, the price and dispatch rules are as follows (assuming $p_i < p_j$):

$$p = \begin{cases} p_i & \text{if } Q < k_i \\ p_j & \text{if } k_i < Q \end{cases} \tag{2.1}$$

$$q_i = \begin{cases} Q & \text{if } Q < k_i \\ k_i & \text{if } k_i < Q \end{cases} \tag{2.2}$$

$$q_j = \begin{cases} 0 & \text{if } Q < k_i \\ Q - k_i & \text{if } k_i < Q \end{cases} \tag{2.3}$$

In addition to this, we assume that, in the case of a tie, dispatch is awarded with priority to the lowest marginal cost producer. Additionally, given that demand Q is treated as completely inelastic, there is a market maximum $\overline{p}$ for price. In the NEM this stands as the \$10,000 Value of Lost Load (VOLL). For relevance we assume that $Q < k_1 + k_2$ in all cases. Von der Fehr and Harbord have analysed the equilibrium properties of this game which, allowing for the possession of multiple generation units, is named the multi-unit auction. We can summarise the first set of key results of their paper [26] in the following proposition.

Theorem 2.1. *For completely inelastic demand, Q:*

- *If* $\Pr(Q < \min\{k_1, k_2\}) = 1$ *then there exist pure strategy equilibria in all of which the system marginal price is equal to the marginal cost of the least efficient generator and only one generator produces.*

- *If* $\Pr(Q > \max\{k_1, k_2\}) = 1$ *then all pure strategy equilibria are given by offer price pairs* (p_1, p_2) *with either* $p_1 = \overline{p}$ *and* $p_2 < \overline{p}$ *or* $p_1 < \overline{p}$ *and* $p_2 = \overline{p}$. [2]

Proof. This is a very intuitive result. Let us assume, without loss of generality, that firm 1 is the most efficient generator ($c_1 < c_2$). When $Q < \min\{k_1, k_2\}$, either firm will find it profitable to undercut the other when $p > c_2$. This places an upper bound on p of c_2. Firm 1 will find it profitable to undercut firm 2 with a bid, p_1, that is marginally less than c_2, to which firm 2 cannot profitably respond. Now let $Q > \max\{k_1, k_2\}$. If we suppose that both $p_1 > \overline{p}$ and $p_2 < \overline{p}$, it is clear that this cannot be an equilibrium, as the marginal firm may increase profit unambiguously by increasing its bid and receiving the same dispatch, due to perfectly inelastic demand. Let $p_1 = \overline{p}$, then the optimal response for firm 2 is any $p_2 < \overline{p}$ that is sufficiently low as to avoid any incentive for firm 1 to undercut its bid. The same logic applies in the case where $p_2 = \overline{p}$. Either outcome is a pure strategy equilibrium. □

The critical issue in a market of this design is capacity. Once demand exceeds the collective capacity of all but one generator, this one generator faces perfectly inelastic residual demand. The best response in this situation is naturally to bid the highest possible price $\overline{p}$, assuming there is no greater profit to be made from undercutting the current price. Here, we seek to demonstrate that the general thrust of this theoretical result holds in oligopoly for a market design with multi-step bids and multiple regions.

The second aspect of the model that one may consider is uncertainty in demand. Von der Fehr and Harbord [26] show in the case where both the states $Q > \max\{k_1, k_2\}$ and $Q < \min\{k_1, k_2\}$ have a positive probability of occurrence, no pure strategy equilibrium can exist. In addition, the mixed strategy equilibrium strikes a balance between bidding low, hence increasing the probability of being dispatched, and bidding high, which increases expected profit in the event that this bid becomes system marginal price.

[2] Refer to von der Fehr and Harbord [26] for further detail on these results.

3 Extending the Model

Having established the critical theoretical properties of the multi-unit auction, we wish to show how they extend to a market similar to the NEM. We desire an n-firm model with multi-step bids and multiple regions. Fabra, von der Fehr and Harbord [12] have already established important results in oligopoly and multi-step bids, which we will state and refine, before giving some results.

3.1 Multi-step bids

Our results thus far have assumed a duopoly with a single unit bid and so we must now look at how this extends to non-trivial multi-unit auctions in oligopoly. This is not as involved as it may seem, and in fact it has been asserted [12, 10] that the pure strategy equilibrium outcomes that we have detailed are independent of the number of steps allowed in players' bid functions. We offer proof of this claim. Let us establish some notation. Let I be the set of players for a multi-unit auction of order n [3], the strategy choice for generator i is $b_i = (p_i, q_i)$, where $p_i,\ q_i \in \mathbb{R}^n$ are price and quantity vectors. We denote the combined bid profile $b = \{b_j \mid j \in I\}$ and $b_{-i} = \{b_j \mid j \neq i\}$. Under this construction, von der Fehr, Harbord and Fabra [12] and Dechenaux and Kovenock [10] asserted that the set of pure strategy equilibrium outcomes in the multi-unit auction is independent of the number of steps in the bid function. Their results are based on the following reasoning. Let the multi-unit auction be of order n. Assume generator i is faced with some bid profile b_{-i}. Given b_{-i}, only one of i's bids will determine profit, and thus i's best response can be obtained with only one bid. As every generator can formulate their best response with a single unit bid, the set of pure strategy equilibria is unaffected by the value of n.

We contend however that proofs by this argument are insufficient. They show only that, in this game, players' best responses to one particular opponent profile are unaffected by the number of allowable bid steps. No proof is offered, however, to show how this relates to the equilibria. We wish to fill out this argument with a very simple lemma, necessary for a complete proof of the claim.

Lemma 3.1. *Suppose we have two games $G = (X_i, U_i)_{i=1}^N$ and $H = (Y_i, U_i)_{i=1}^N$ with $Y \subset X$, where*

[3] That is, n steps in the bid function

X and Y are arbitrary spaces and $U_i : X \mapsto \mathbb{R}$ defines the pay-offs for players in both G and H. If:

$$\max_{x_i \in X_i} U_i(x_i, x_{-i}) = \max_{y_i \in Y_i} U_i(y_i, x_{-i}), \quad \forall i, \; \forall x_{-i}, \tag{3.1}$$

and letting y_i^*, x_i^* be the best responses in each case, for all $j \neq i$;

$$\arg\max_{x_j \in X_j} U_j(x_j, x_i^*, x_{-i,j}) = \arg\max_{x_j \in X_j} U_j(x_j, y_i^*, x_{-i,j}), \tag{3.2}$$

with $U_j(x_j^*, x_i^*, x_{-i,j}) = U_j(x_j^*, y_i^*, x_{-i,j})$ then all equilibria in H and G are equivalent in terms of pay-offs. Additionally, there exists a mapping $\psi : X \mapsto Y$ such that for all equilibria x in G, $U(x) = U(\psi(x))$ and $\psi(x)$ is an equilibrium in H.

Proof. We construct ψ explicitly by $\psi_i(x) = \arg\max_{y_i \in Y_i} U_i(y_i, x_{-i})$. To see that fixed points are preserved under some best response correspondence, take any equilibrium x^* in $X \setminus Y$. Condition (3.1) guarantees us that any player can achieve the same profit with a best response in Y, while (3.2) ensures that the other player's best responses are unchanged from such a simplification in strategies. Note it suffices to show that for any $j \neq i$,

$$\arg\max_{y_j \in Y_j} U_j(y_j, x_{-j}^*) = \arg\max_{y_j \in Y_j} U_j(y_j, \psi_i(x^*), x_{-i,j}^*), \tag{3.3}$$

as we then have by induction on the number of players that

$$\arg\max_{y_i \in Y_i} U_i\left(y_i, \psi_{-i}(x^*)\right) = \psi_i(x^*), \qquad \forall i. \tag{3.4}$$

This can be shown quite easily, as (3.2) requires that j's best response in X_j does not change under the inclusion of $\psi(x_i^*)$ in the profile, while (3.1) guarantees that a payoff equivalent best response can be found in Y_j, for which (3.2) holds as $Y \subset X$. Finally, condition (3.1) guarantees that profits are the same for x^* and $\psi(x^*)$ by the same process of induction on n. $\square$

Having this result greatly simplifies elements of our analysis where we need to consider the effects of including multi-unit bids. We are guaranteed now that the pure strategy equilibrium outcomes will be unaffected by such an extension, as we shall later see. This result a simple exposition of how equilibrium outcomes can be preserved under a permutation to the game via some form of best response equivalence. This allows us from hereon to treat only the results for the single unit auction, in the knowledge that any multi-unit results are equivalent. The critical result is the following:

Proposition 3.2. *The equilibrium results in any multi-unit auction of order n are equivalent in terms of price and pay-offs to the multi-unit auction of single order.*

Proof. Let B_i denote the strategy space for the single unit auction, and B_i^n denote the strategy space for the auction of order n. We need only to show that conditions (3.1) and (3.2) hold and we can invoke Lemma 3.1. Suppose that some firm i faces a bid profile b_{-i}^n. Denote any best response in B_i^n as b_i^{n*}. If i becomes the marginal bidder then it's profit is determined only by one $(p_i^k, q_i^k) \in b_i^{n*}$. In this case then a bid $b_i^* = (p_j^k, q_j^k) \in B_i$ would clearly be pay-off equivalent. If i is not the marginal generator it is easy to see that profit is determined only by the highest step (p_i^k, q_i^k) such that $p_i^k < p$ and the same argument applies. Thus condition (3.1) is satisfied. In addition, it is easy to see that such a change in strategies has no effect on the profits of the other firms. If i is the marginal generator, the profits of every other firm are determined only by the one bid step b_i^* and so their best responses are unaffected. If i is not the marginal generator, the profit functions for any of the remaining firms are similarly unaffected by the transformation in strategies and therefore, trivially, neither are their best responses.

Thus, by Lemma 3.1, any equilibrium profile b^{*n} can be mapped to a payoff equivalent equilibrium b^* in the single unit auction. Finally, because quantity Q is fixed and we have a uniform price p, we must necessarily have also that the uniform price outcome p in equilibrium is unchanged. □

Proposition 3.2 permits us to analyse only the single unit auction, in the knowledge that the price outcomes for auctions of any higher order are equivalent.

3.2 Oligopoly

Now suppose that the supply auction is expanded to incorporate an n-firm oligopoly. Fabra et al [12] have shown that the basic properties of the duopoly model are preserved. We will illustrate this in a result similar to Proposition 4 in Fabra et al [12], with the only difference being that we assume complete asymmetry in marginal costs. We label each firm $I = \{1, 2, ..., n\}$ in ascending order of marginal costs (assuming complete asymmetry) and capacities $k_1, k_2, ...$ accordingly. We denote $K_{-i} = \sum_{j \neq i} k_j$ as the sum of capacities less generator i.

To establish some intuition for this result, let us suppose that $Q < K_{-i}$ for all i. Where might the equilibrium lie in this case? Suppose that some generator k is currently the marginal bidder at

p_k. Any generator i with $c_i < p_k$ can then guarantee themselves a profit by bidding less than p_k. Thus, we can only have an equilibrium once there is no longer an opportunity for any generator to profitably undercut. So in this case if $Q < \sum_{i=1}^{k-1} k_i$, k cannot be the marginal bidder in equilibrium. This condition establishes that price in equilibrium must be equal to the marginal cost of some generator, which we shall call a competitive outcome. Let π_i^c be the profit to generator i from such a competitive outcome. Let us also define:

$$Q_i = \sum_{j=1}^{i} k_j \tag{3.5}$$

$$\hat{Q} = \min_{i \in I} \left(\frac{\pi_i^c}{\overline{p} - c_i} + K_{-i} \right) \tag{3.6}$$

We can now state the following.

Proposition 3.3. *In the $n-firm$ multi-unit auction, price in strategic equilibrium can be completely described as:*

$$p(Q) = \begin{cases} c_{i+1} & \textit{if } Q \in (Q_{i-1}, Q_i],\ \forall\ Q_i < \hat{Q} \\ c_{j+1} & \textit{if } Q \in (Q_{j-1}, \hat{Q}],\ Q_j = \min_i \{Q_i \mid Q_i \geq \hat{Q}\} \\ \overline{p} & \textit{if } Q > \hat{Q} \end{cases} \tag{3.7}$$

Proof. Suppose that $Q \in (Q_{i-1}, Q_i]$ and, for contradiction, that $p > c_{i+1}$. In this case, generators 1 to $i+1$ can guarantee themselves a profit by bidding below p. If the marginal bidder was any $j > i+1$, they will be undercut. If the marginal bidder is any $j \leq i+1$ they will also be undercut by any of the generators not currently dispatched. Thus p cannot be greater than c_{j+1} in equilibrium. Conversely, if $p < c_{i+1}$ then naturally we expect only generators 1 to i will be dispatched as it is not profitable for any $j > i$. This is also not an equilibrium as the marginal bidder can profitably raise its bid until $p = c_{i+1}$, leaving this as the only viable equilibrium candidate. Now suppose that $Q > \hat{Q}$. This implies that for some generator j:

$$(\overline{p} - c_j)(Q - K_{-j}) > \pi_j^c \tag{3.8}$$

Therefore j at least will find it profitable to bid $\overline{p}$ and this becomes the market price. It is easy to see that this equilibrium outcome price will maintain for any $Q > \hat{Q}$. □

3.3 Multiple Regions

Our focus in this section is to analyse, in terms of our model, the effect of inter-regional competition on the critical threshold we have established. A simple extension of the equilibrium analysis in

Proposition 3.3 allows us to see that the increase in competition can potentially constrain the effective markups for generators in equilibrium. In this paper however we are more interested in pinning down the threshold effects.

We define inter-regional competition by assuming an agent with transfer capacity T, who performs arbitrage on the difference between the prices in connected regions. In the absence of transmission losses or transmission constraints, this would be equivalent to combining the two markets and applying the same equilibrium properties described in Proposition 3.3. Let us illustrate this argument. Suppose that there exist two regions, A and B with demand Q^A and Q^B. For each region we order generators in terms of marginal costs $c_1^A, c_2^A, .., c_m^A$ and $c_1^B, c_2^B, ..., c_n^B$. We start by letting regional prices p_A, p_B be determined as in Proposition 3.3. By consequence then we also have an equivalent series of demand thresholds for each region, $Q_1^A, Q_2^A, ...$ and $Q_1^B, Q_2^B, ...$ If we take $Q = Q^A + Q^B$ we can locate where total system demand lies in terms of the marginal generator. Suppose without loss of generality that $Q \in (Q_{j-1}, Q_j)$ and that j belongs to region 1. From a strategic perspective we can see that the system marginal price will be c_{j+1}, regardless of the region to which $j+1$ belongs. Let generator i_2 be the highest in our ordering below j but belonging to region B and let t^{A-B} denote the flow of electricity from region A to B. It is clear that

$$t^{A-B} = (Q - Q_{j-1}) + Q_{j-1}^A - Q_A, \tag{3.9}$$

which is equivalent to regional dispatch less regional demand. This condition holds as long as $|t^{A-B}| \leq T$.

Without loss of generality, let us consider the case for generators in region A. We denote $j = \arg\max_{i \in A} k_i$ as the largest generator. Now consider if demand Q^A is greater than the threshold $\hat{Q}$ defined in Proposition 3.3. For j to bid $\overline{p}$, as in the single region case, is no longer a guaranteed equilibrium as its bid may be undercut by any generator in B. In fact we can say unequivocally that such a bid *will* be undercut until transmission is constrained either by the interconnector or by B's generation capacity. Thus we find that the critical threshold for region A becomes:

$$\hat{Q}^A = \frac{\pi_j^c}{\overline{p} - c_j} + K_{-j} + \min\{T, K^B - Q^B\}. \tag{3.10}$$

Generally speaking, we can see that the effect of inter-regional competition is to increase the critical threshold by an amount equivalent to the import capacity of the region in question. For more complicated systems we can make a recursive definition. In a network of indeterminate size and topology, let ω be a region with a set $\mathcal{N}(\omega)$ of neighbours. We define import capacity as the following:

$$I^{\omega} = \sum_{i \in \mathcal{N}(\omega)} \min\{T^{i\omega}, K^{i} - Q^{i} + I^{i-\omega}\} \tag{3.11}$$

Where:

$$I^{i-\omega} = I^{i} - \min\{T^{\omega i}, K^{i} - Q^{i} + I^{\omega - i}\} \tag{3.12}$$

There are no problems with this recursive definition so long as we are guaranteed the existence of a terminal node or region in the network. In the NEM the terminal nodes are Queensland, Tasmania and South Australia. Using this definition of import capacity, we have the following.

Proposition 3.4. *For a region ω with neighbours $\mathcal{N}(\omega)$, there exists*

$$\hat{Q}^{\omega} = \min_{i \in \omega} \left(\frac{\pi_i^c}{\overline{p} - c_i} + K_{-i} \right) + I^{\omega}, \tag{3.13}$$

such that the price in equilibrium is equal to the market maximum $\overline{p}$ when $Q \geq \hat{Q}^{\omega}$ and equal to c_{j+1} when $Q < \hat{Q}^{\omega}$, $Q \in (Q_{i-1}, Q_i]$ and regional demand is less than $\hat{Q}^{\lambda}$ for every other region λ.

Proof. If $Q > \hat{Q}^{\omega}$ it is easy to see that there exists j such that:

$$(\overline{p} - c_j)[Q - K_{-j} - I^{\omega}] > \pi_j^c \tag{3.14}$$

And therefore j will find it profitable to bid its capacity at $\overline{p}$. Finally, because transmission (I^{ω}) and the capacity of the remaining generators (K_{-i}) are constrained, j cannot be undercut and $\overline{p}$ becomes the market price. Now suppose that $\min_{i \in \omega} \left(\frac{\pi_i^c}{\overline{p} - c_i} + K_{-i} \right) < Q < \hat{Q}^{\omega}$. It is no longer an equilibrium for any generator j to bid $\overline{p}$ as there is an incentive for any of ω's interconnectors to undercut this bid, which persists as long as $Q < K_{-j} + I^{\omega}$ and the local threshold in any connected region λ has not been exceeded. ◻

We have seen then that the basic properties of our equilibrium analysis persist in the multi-region case, with only a necessary redefinition of the critical threshold $\hat{Q}$.

3.4 Uncertainty

To this point our equilibrium analysis has assumed that the level of demand is known with certainty. However von der Fehr and Harbord [26] have shown that, if any more than one generator sets the uniform price with positive probability, no pure strategy equilibrium can exist. It is important then to consider the consequences of uncertainty in demand on our model. We will refer to the density function of demand as $g : [\underline{Q}, \overline{Q}] \mapsto \mathbb{R}$, which we assume to be continuous. Let G be the cumulative distribution. We imagine then that uncertainty in demand arises in the form of such a stationary interval that increases and decreases with expected demand. In addition, we preserve all of the assumptions that we have previously made, and refer to the set of thresholds characterised in Proposition 3.3 as $\mathbf{Q}^T$.

Chiefly we wish to demonstrate that the generator with the most market power has an incentive, as demand approaches the threshold, to gradually shift capacity to it's highest, or possibly a higher price band. We will illustrate this with one particular example in pure strategies. Suppose that $\mathbf{Q}^T \cap \operatorname{supp} g = \{\hat{Q}\}$, that Q_j is the threshold immediately before $\hat{Q}$ and let m be the largest generator, with $m < j$, such that m will be dispatched in either case. We therefore have two generators with positive probability of setting market price, however we will find that there can still exist an equilibrium in pure strategies for this unique case. As $Q_j < [\underline{Q}, \overline{Q}] < Q_{j+1}$, the equilibrium strategy for $j+1$, as defined by Proposition 3.3, is to bid capacity in at c_{j+2}. This maximises the profit that can be obtained without providing an incentive for $j+2$ to undercut. We suppose for now that this is the strategy pursued by $j+2$. Now consider m's expected profit across the support of g. As $Q_m, Q_{m-1} \notin \operatorname{supp} g$, m's best response is to bid some capacity at $p_m^1 < c_{j+2}$, and the rest at $\overline{p}$. Note that it is neither profitable for m to use any extra bid steps, nor for it to bid any price between c_{j+2} and $\overline{p}$. Let $b_m = \big((p_m^1, q_m^1), (\overline{p}, k_m)\big)$. Expected profit is now

$$\pi(b_m, b_{j+1}) = G(K_{-m} + q_m^1)[c_{j+2} - c_m]q_m^1 + \int_{K_{-m}+q_m^1}^{\overline{Q}} [\overline{p} - c_m](Q - K_{-m})dG(Q). \tag{3.15}$$

Thus, m's optimal response is to choose $q_m^{1*} = \arg\max_{q_m^1} \pi(b_m, b_{j+1})$. In previous cases, as in von der Fehr and Harbord's standard multi-unit model, the probability of m being dispatched at this higher price yielded an incentive for the other effective bidder ($j+1$ in this case) to also raise its bid, which increased the incentive for m to lower its bid *ad infinitum.* In this case however, if $j+1$ were to raise it's bid above c_{j+2}, this would only serve to bring firm $j+2$ into dispatch, thereby decreasing $j+1$'s profits. We therefore have a Nash equilibrium as long as $\overline{Q} \leq Q_{j+1} - (k_j - q_m^{1*})$. If this condition does not hold, then $j+2$ has a positive probability of being the marginal bidder, and so there is no longer an equilibrium in pure strategies. If we note m's optimal bid condition:

$$
\begin{aligned}
&(c_{j+2} - c_m)\left(G(K_{-m} + q_m^{1*}) - q_m^{1*} g(K_{-m} + q_m^{1*})\right) \\
&\qquad + (\overline{p} - c_m)\left[\left(1 - G(K_{-m} + q_m^{1*})\right) - q_m^{1*} g(K_{-m} + q_m^{1*})\right] = 0,
\end{aligned}
\tag{3.16}
$$

we can see that the optimal choice of q_m^1 will decrease as the support of g shifts uniformly upward, increasing the likelihood that m will face perfectly inelastic residual demand, thereby decreasing the marginal benefit from bidding q_m^1 competitively (the first term on the left hand side) and increasing the marginal benefit from moving capacity to the higher band (the second term). Accordingly, we have $\lim_{\underline{Q} \to \hat{Q}} q_m^1 = 0$.

It is important to note that this modified situation is one in which Lemma 3.1 does not apply. In this case Generator m can in fact guarantee itself a greater profit with a two step bid than with a single step bid. In other words, condition (3.1) no longer applies for m, as both steps of its bid may determine profit with a positive probability. We can see still though that conditions (3.1) and (3.2) apply for any generator $i \neq m$. We have demonstrated here that, when facing uncertainty in demand, the largest generator has an incentive to shift capacity to the highest price band as the probability of threshold exceedance increases.

4 Analysis

Our equilibrium analysis has led us to the following conclusions regarding the behaviour of spot price in the National Electricity Market. Firstly, that there exists a specifiable threshold in demand where one generator has a strategic incentive to shift its capacity to the highest possible price band. Secondly, that, when faced with uncertainty in demand, the equilibrium strategy for this generator is to gradually shift capacity into higher price bands as demand approaches the threshold. Naturally, the next step is to see if we can find behaviour that fits these general properties in the data.

Figures 4 and 4.2 show the general daily property we are seeking; that the spot price tends to peak rapidly at a certain point in demand. This allows us to identify such a behaviour as exhibiting a threshold property. We wish to discover if the specific demand threshold that we derived in our theoretical analysis is close to this identified threshold.

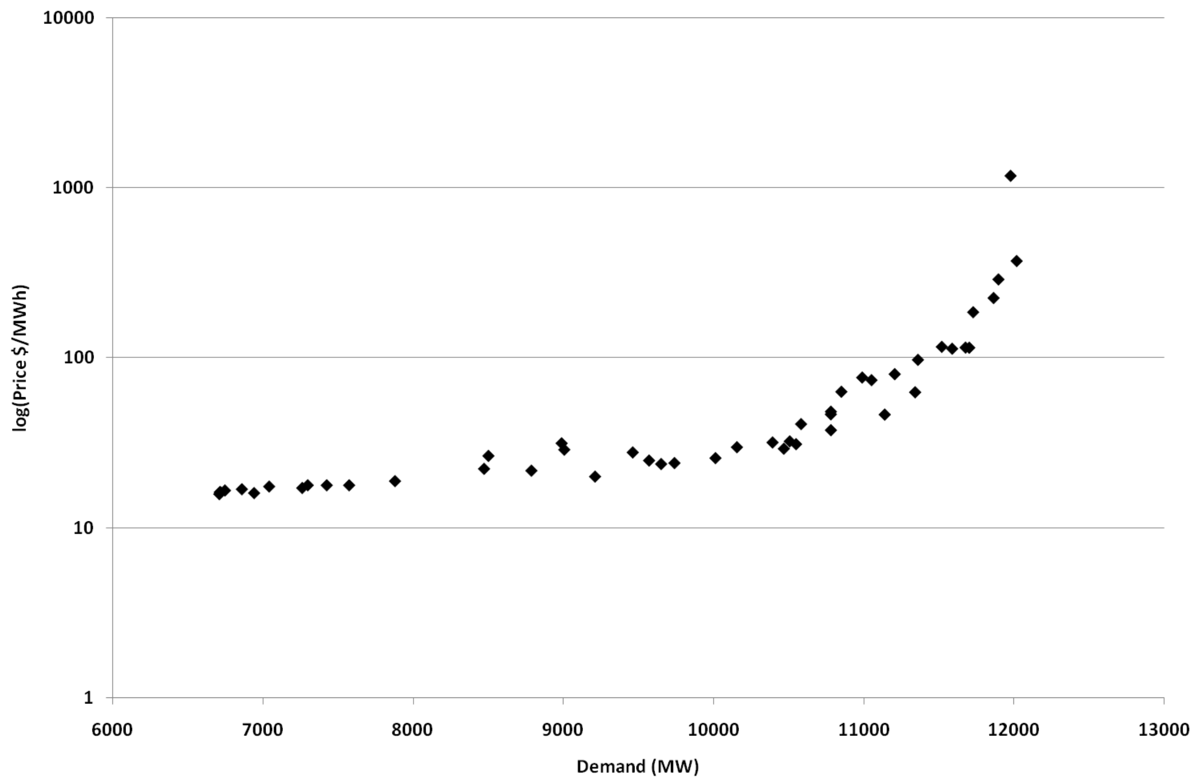

Figure 4.1: Scatter of Demand and Price, NSW, 5th February 2007

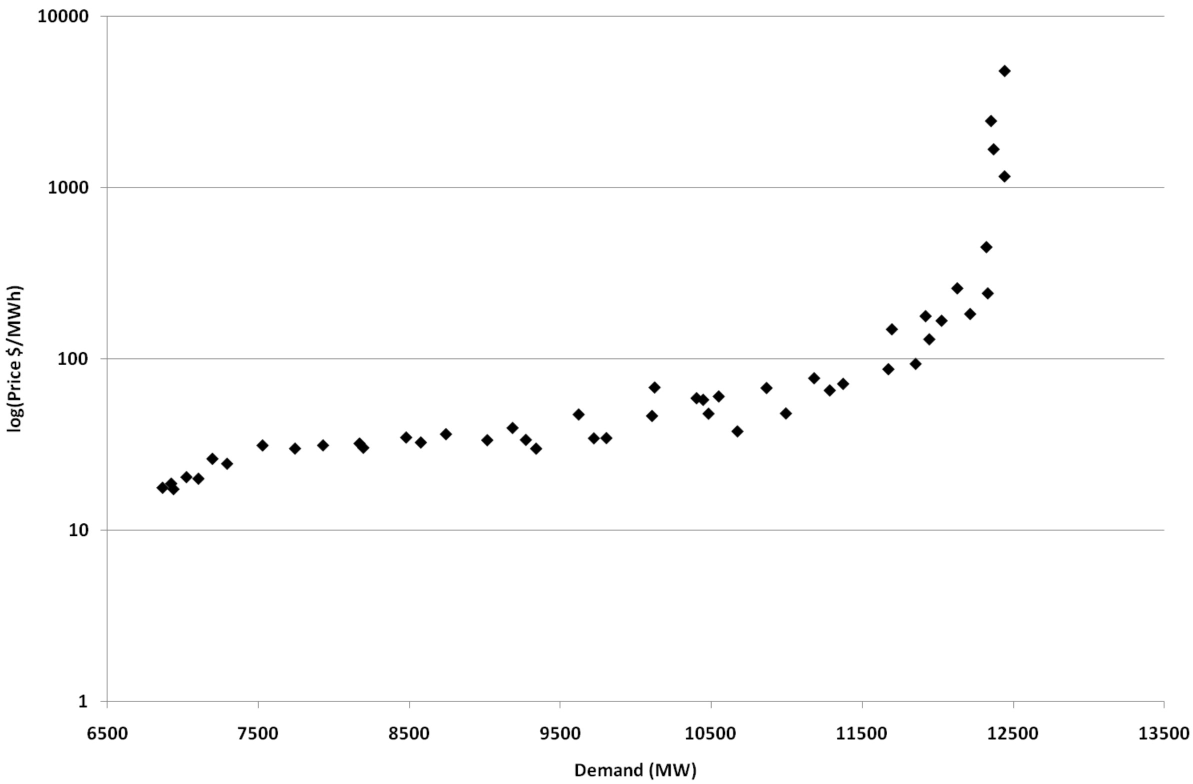

Figure 4.2: Scatter of Demand and Price, NSW, 12th January 2007

4.1 Specifying the Threshold

We define the thresholds for each region according to that presented in Proposition 3.4, with the following exception. We find that for any generator i, the value of $\frac{\pi_i^c}{\overline{p}-c_i}$ is sufficiently close to 0 [4] to exclude it from the specification. Thus for example we have excluded Loy Yang A[5] station for Victoria and we get the following:

$$\begin{aligned} QVIC = K^{VIC} - K^{LOY\ YANG\ A} \\ + \min\{T^{SNOWY-VIC}, \max\{K^{NSW} - DNSW, -T^{VIC-SNOW}\}\} \\ + \min\{T^{SA-VIC}, \max\{K^{SA} - DSA, -T^{VIC-SA}\}\} \\ + \min\{T^{TAS-VIC}, \max\{K^{TAS} - DTAS, -T^{VIC-TAS}\}\} \end{aligned} \tag{4.1}$$

Our goal is to see how this threshold specification holds in particular price events.

4.2 Price Event Analysis

4.2.1 New South Wales, January 12, 2007

Our first case study offers a simple exposition of our central idea. Remembering always that the changes we expect will come about in a probabilistic or gradual fashion due to uncertainty, we can see that, in this case, the behaviour predicted by our model seems to hold. Figure 4.3 illustrates how price behaves as demand approaches then exceeds the threshold. We notice how price begins to rapidly ascend as demand approaches $QNSW$, becoming more severe as firms know $DNSW > QNSW$ with certainty. Figure 4.4 shows this process over time, with the same result. Additionally we can see that the shoulders of the price spike, at roughly 10:00am and 6:00pm respectively, correspond approximately well to the approach and retreat of demand from the critical threshold $QNSW$. In terms of the model we can interpret this as the gradual shifting of capacity to the maximum price band.

[4]It is usually less than 1.

[5]Loy Yang A is Victoria's largest generating station which is rated at 2000MW.

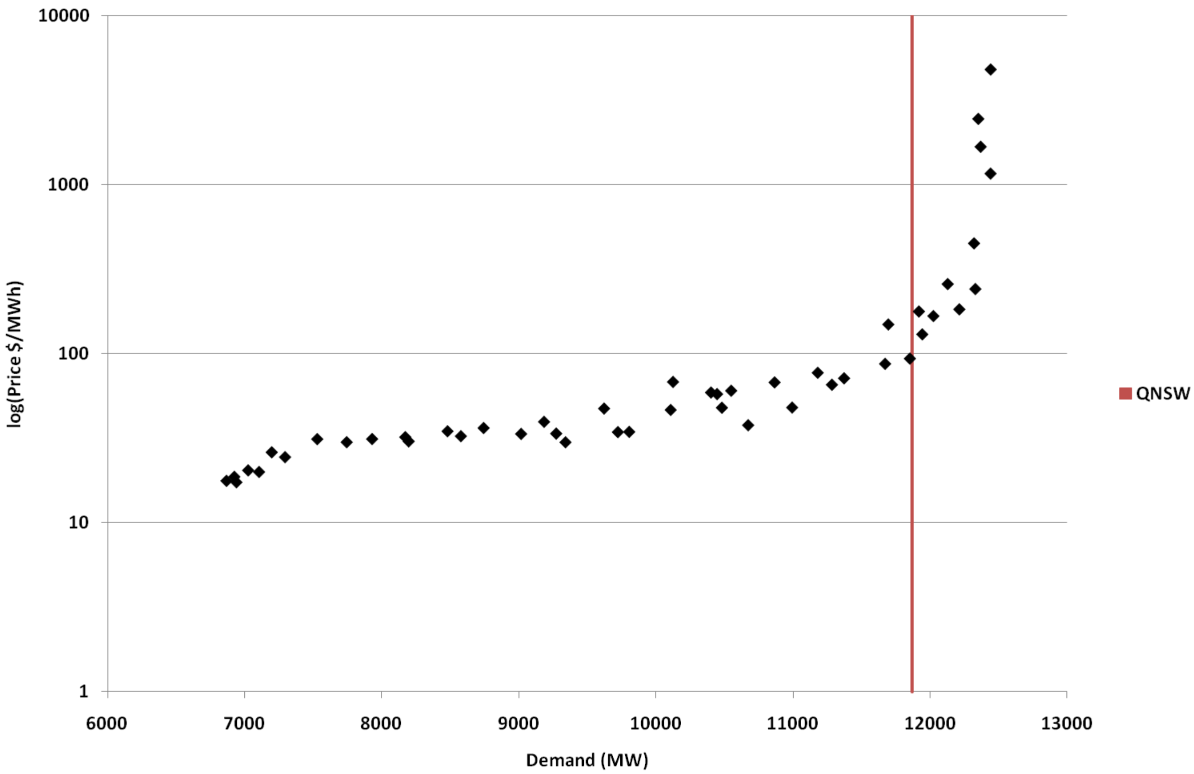

Figure 4.3: Price with QNSW (Red), January 12, 2007

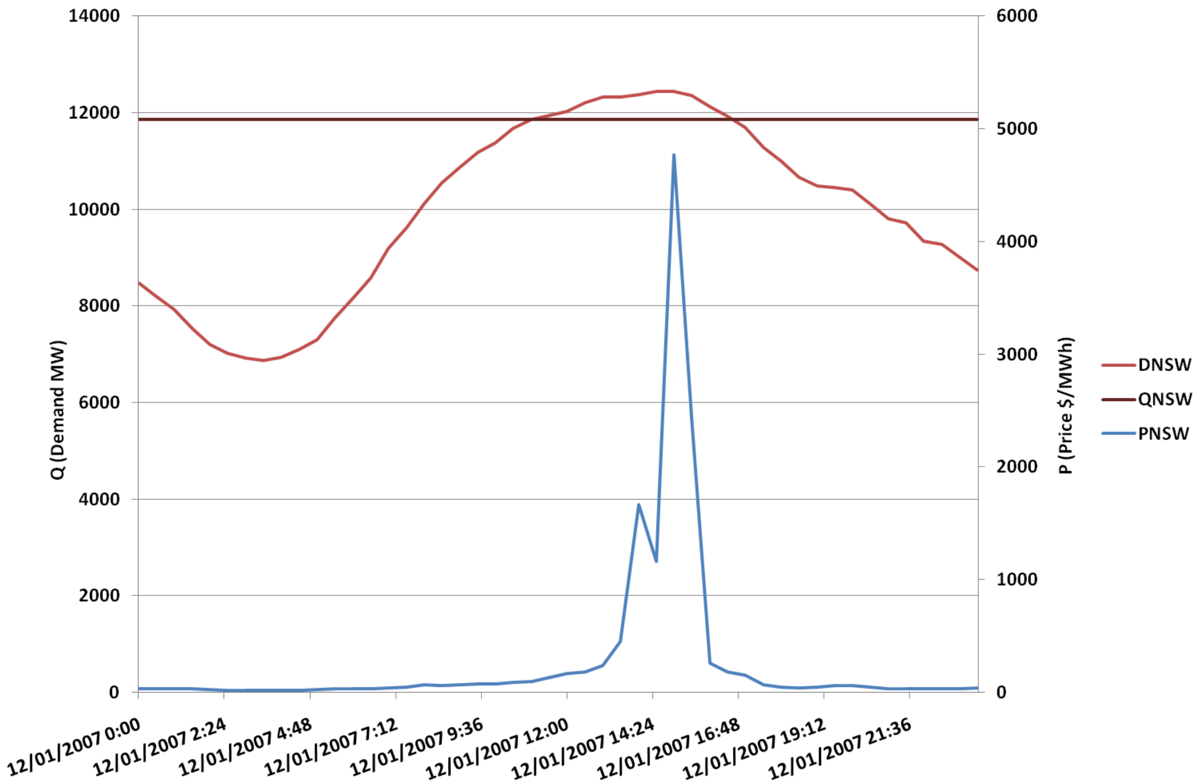

Figure 4.4: NSW, January 12, 2007

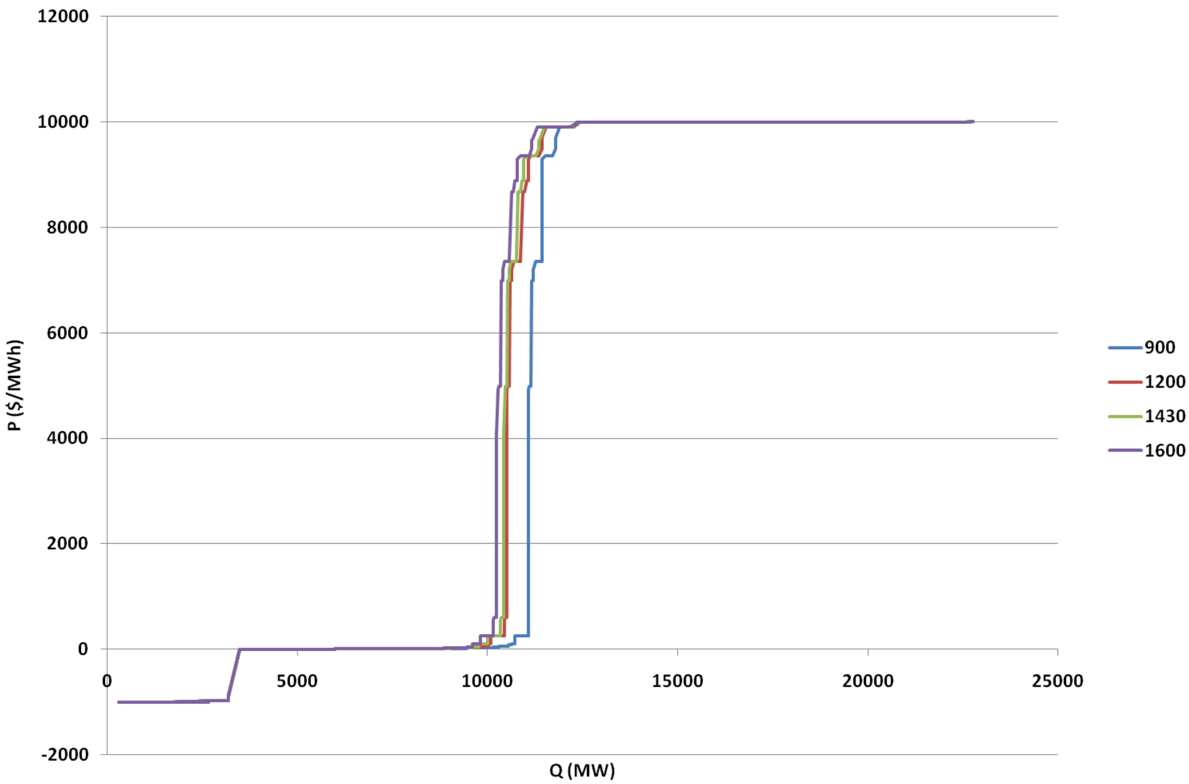

Figure 4.5: NSW Aggregate Supply, January 12, 2007

This is due to the fact that, as the probability of the state $DNSW > QNSW$ increases, the expected marginal benefit from pricing at $\overline{p}$[6] increases. We can in fact explicitly observe this gradual shifting in capacity to higher bands by examining the change in the aggregate bid stack over time. Figure 4.5 shows the aggregation of NSW generators' supply bids at 9:00 am, 12:00 pm, 2:30 pm and 4:00 pm. As we can see, between these times aggregate supply has shifted gradually to the left as demand approaches, then travels beyond the threshold. This is again predicted by the model. To finalise our test we can look at how the bids for the largest generator in NSW, Macquarie Generation, change over this time period. Figure 4.6 shows that there is indeed a gradual shifting of capacity to the maximum level over the period, which results in a shift to the left in the supply curve. One specific issue with this type of analysis is that our theory predicts that once demand is beyond the threshold with certainty, the largest generator will price *all* their capacity at the market maximum. Figure 4.6 suggests such an outcome is a long way from realisation. A corollary to this observation is that, given Macquarie is not shifting all of it's capacity to the highest band, the price spike must be supported by other generators holding capacity above the regional price, keeping it from dispatch. This is not predicted by the model under certainty, however it does still fit with our characterization of the mixed strategy equilibria in uncertainty. We will discuss additional reasons for such behaviour in §5.

4.2.2 New South Wales and Victoria, February 5, 2007

We look now at a particular case where demand exceeded the critical threshold for both New South Wales and Victoria. We make the same observations as we did before regarding the timing of the spike; when the demand series $DVIC$ is sufficiently beyond the threshold $QVIC$ and simultaneously, $DNSW$ has breached $QNSW$. Additionally we can see that the shoulders of the price spike are timed fairly well with the approach and exceedance of the threshold level. It seems then that we can successfully reapply our analysis for the previous case in terms of these characteristics. Figure 4.7 however requires some extra comment. Firstly, it illustrates nicely the critical importance of the

[6] And more generally, shifting capacity to higher price bands

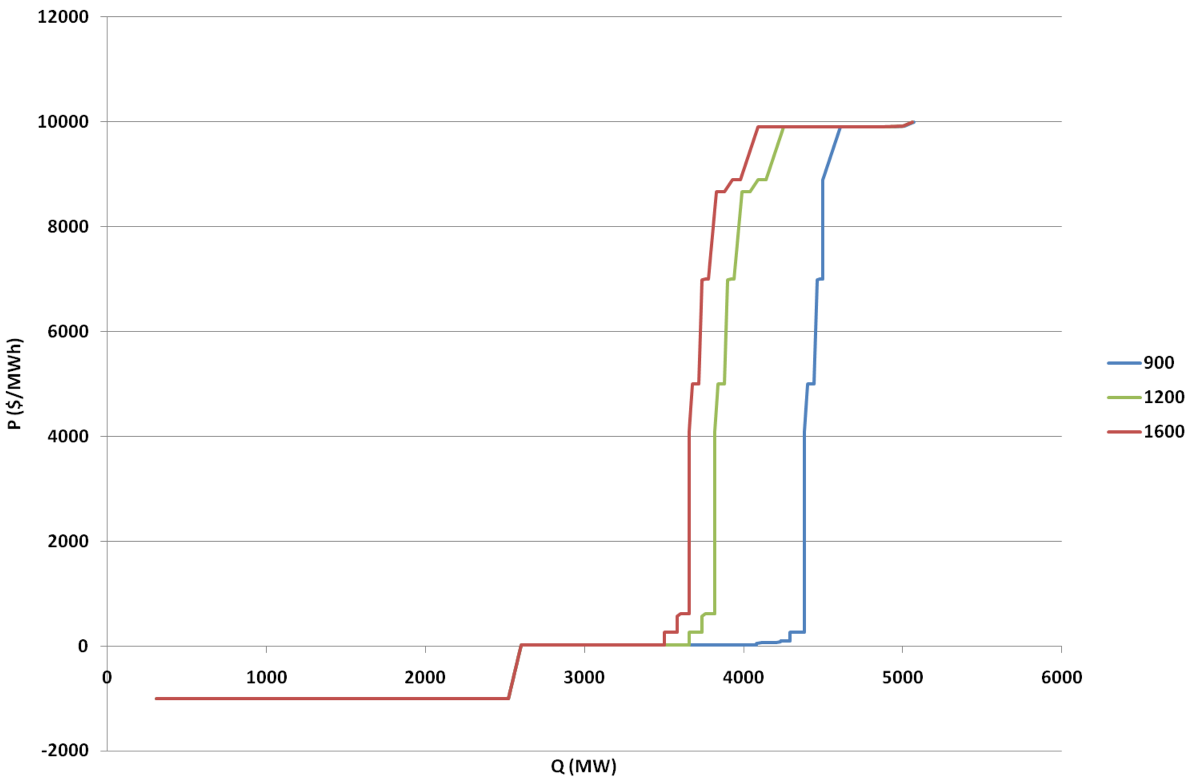

Figure 4.6: Macquarie Generation Aggregate Supply, January 12, 2007

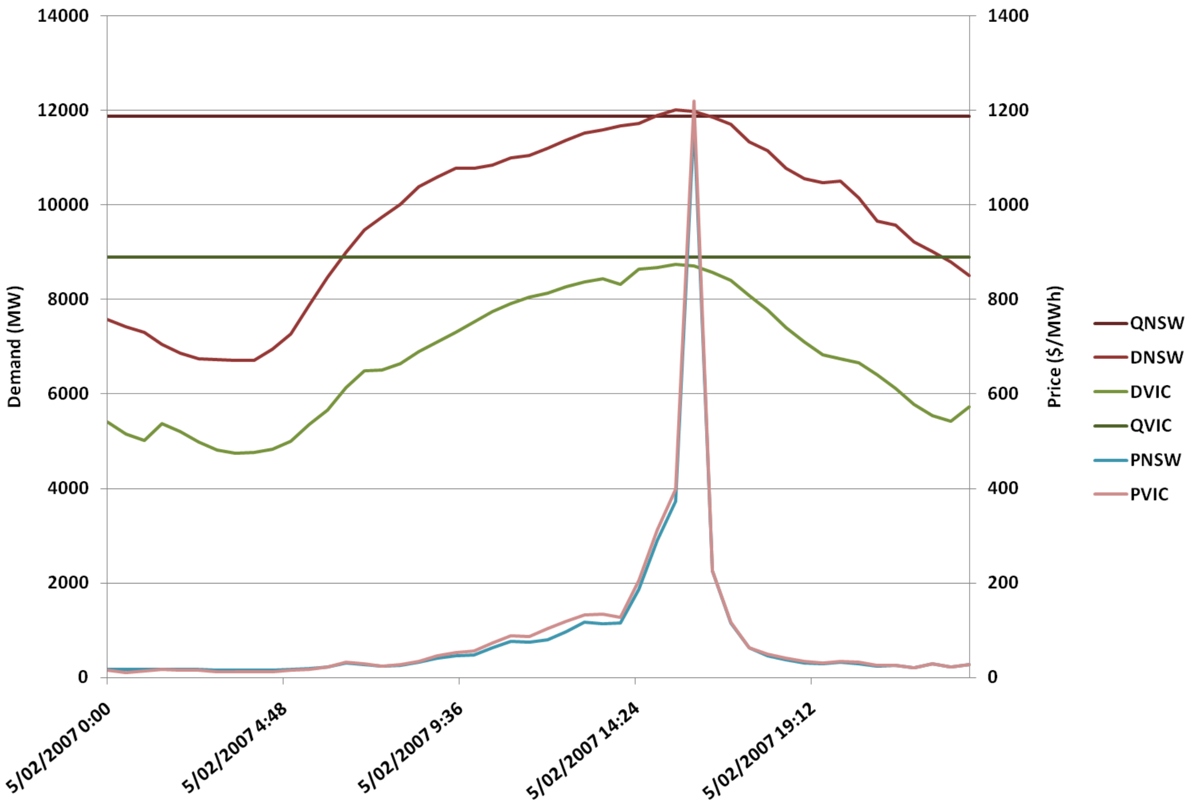

Figure 4.7: NSW and VIC, February 5, 2007

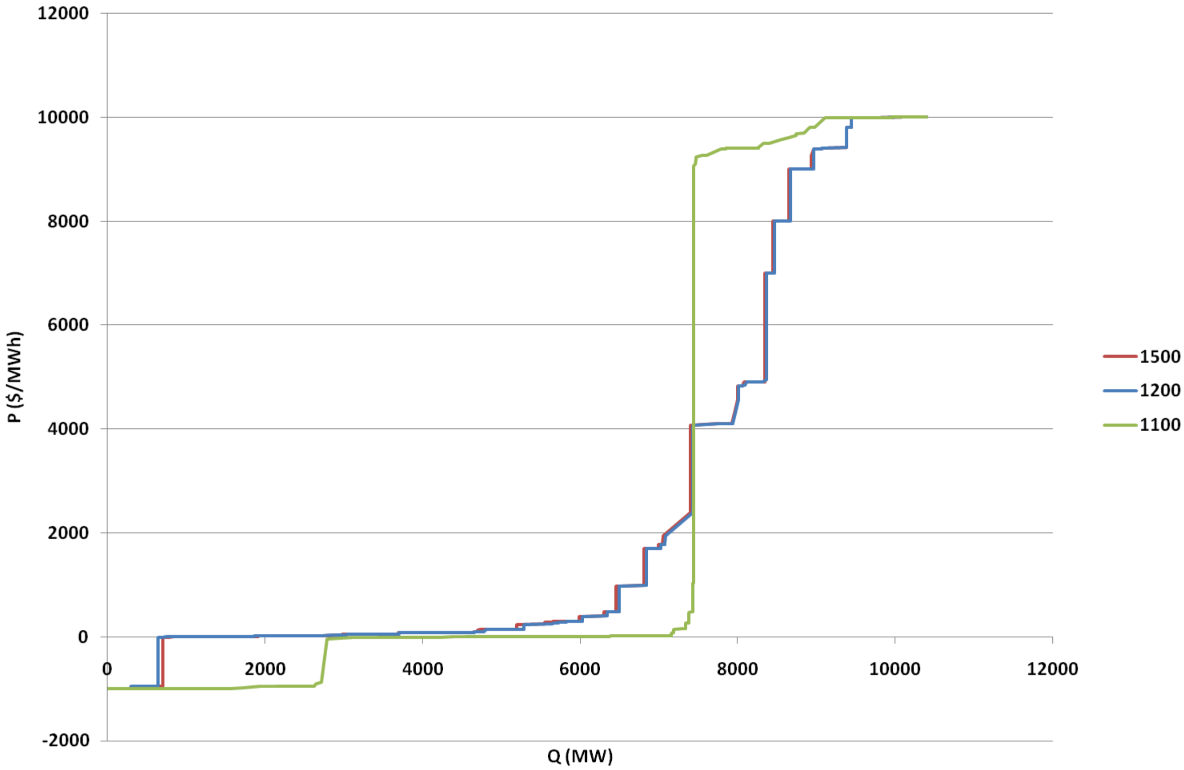

Figure 4.8: VIC Aggregate Supply, February 5, 2007

connectedness between regions. Specifically we can see that in this case $PVIC$ would have been less likely to spike if the local threshold had not fallen between 9:00 am and 7:00 pm, bottoming out at 3:30 pm, the time of the spike, thereby allowing demand to exceed it. The reason behind this was exceptionally high levels of demand across the NEM, which necessarily constrained Victoria's import capacity. In accordance with the way we have defined it, this change served to force down the critical level of demand. This particular case provides a good illustration of the importance of regional import capacity and particularly, the way that above average levels of demand across the NEM can contribute to local price spike events.

Turning to bid analysis we find that, although the movement in supply for New South Wales is identical to that on January 12, supply changes in Victoria are more distinct. Figure 4.8 indicates that the shift in Victorian supply is more of a flattening than the parallel shift that we observe in

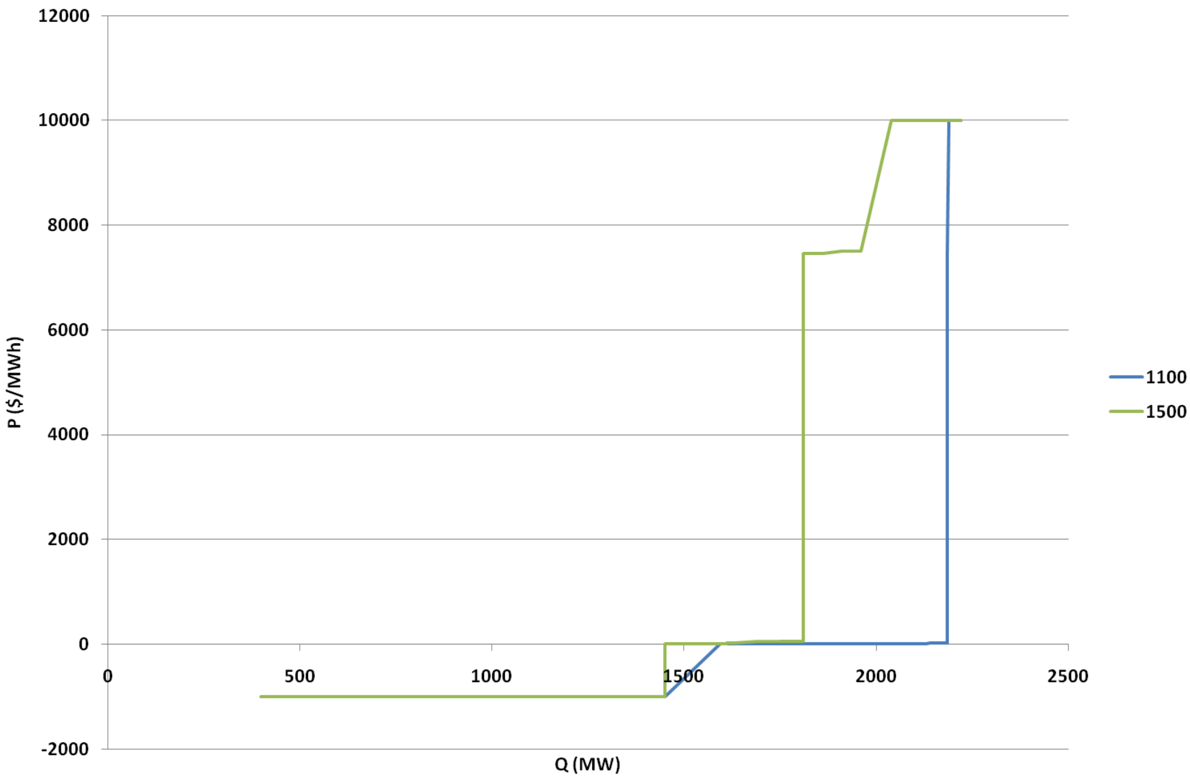

Figure 4.9: Loy Yang A Aggregate Supply, February 5, 2007

New South Wales.

Figure 4.9 confirms our expectations that Loy Yang A, the firm with the greatest combined capacity, will shift this capacity to the highest allowable price as demand approaches the threshold. However in contrast to observed behaviour in New South Wales on January 12, some firms appear to have shifted more capacity into lower price bands, which causes the flattening we see. It should be noted that our expectations regarding changes in industry supply are entirely dependent on the degree of uncertainty in demand. In this case, for example, if it is known with greater certainty that demand will reach beyond the threshold, generators know with greater certainty that price will spike regardless of their bids, increasing the marginal benefit to bidding low and guaranteeing a greater level of dispatch.

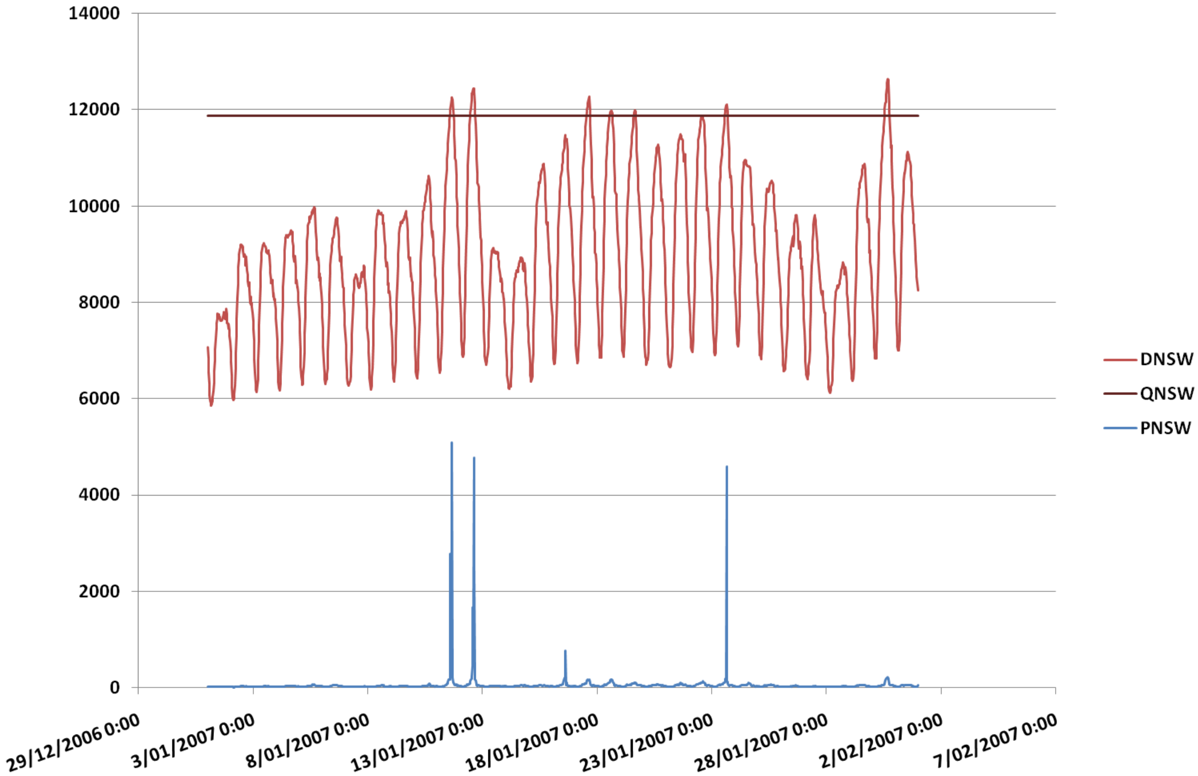

Figure 4.10: Threshold Analysis: NSW and VIC, January 2007

4.3 General Analysis

One issue with our case study method is that it cannot support any notion that the principles of the model hold in general. An intuitive response to this problem is simply to extend the breadth of cases as succinctly as possible. Figures 4.10 and 4.11 display movements in price and demand in New South Wales and Victoria relative to their local thresholds for January 2007.

In the spirit of our observations thus far, we can note the high level of coincidence in both regions between exceedance of the threshold and the presence of a price spike. In addition, we can see that any time there is a price spike unaccompanied by a breaking of the local threshold, it can be explained by a breaking of the threshold in the other region. The 17th and 23rd of January

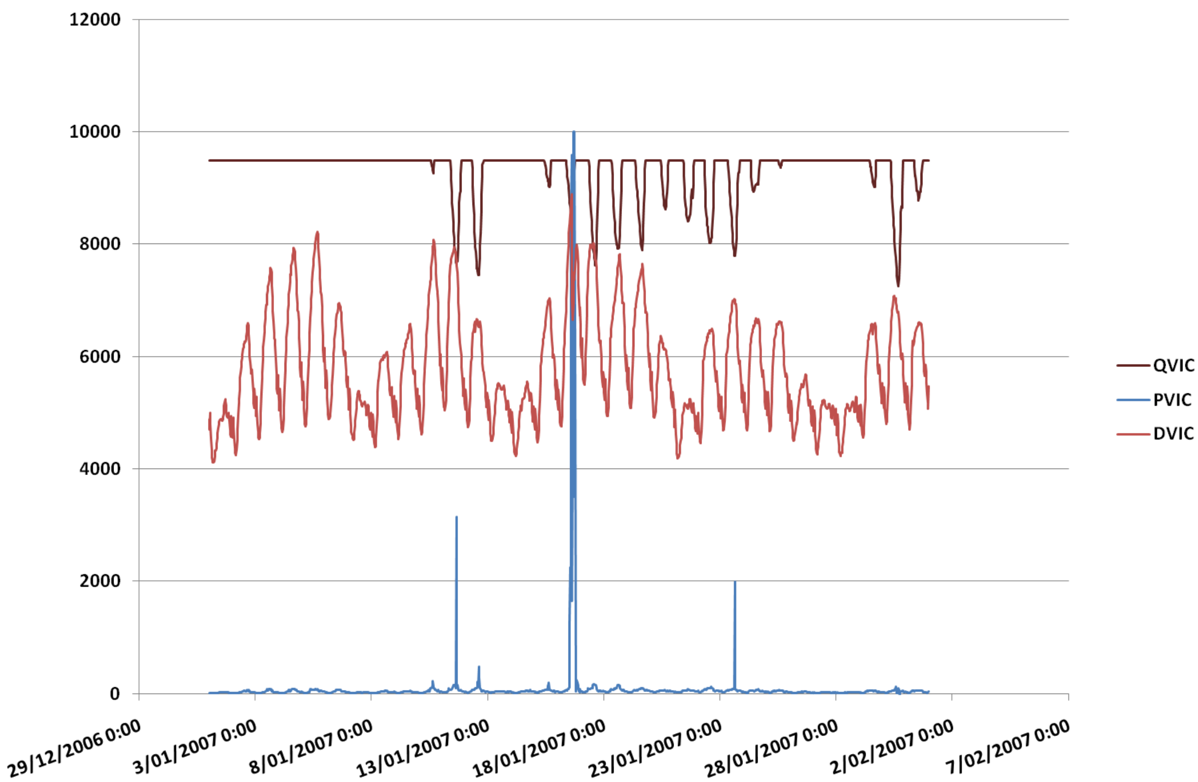

Figure 4.11: Threshold Analysis: NSW and VIC, January 2007

are two such examples. A couple of notable exceptions to our general hypothesis can be found at times when the threshold in NSW is breached, but a contemporaneous spike in price cannot be found. Looking at these days, we can see that abnormal prices do appear - visible as small humps in the time series - but these increases are seemingly incommensurate with the magnitude of the threshold break. Again, our equilibrium analysis suggests that this observation relates closely to the level of uncertainty in demand at the time. Indeed, if we suppose the existence of a mixed strategy equilibrium, we would expect a probabilistic response from price as demand approaches then exceeds the threshold. Of course it is reasonable to expect that there are extra-theoretical factors at play. We will discuss these in §5.

The most logical choice for applying further rigour and generality to our hypotheses would be the use of statistical techniques. In developing a game-theoretic analysis of bidding strategies in the NEM, we have in essence derived a structural model that could be used for econometric modelling. In this case however we are less interested in fitting data to our model than we are in testing the general properties. An important problem in working with National Electricity Market data has traditionally been an imposing degree of nonlinearity [7, 13, 27]. Negotiating these issues in order to conduct a reliable hypothesis test on threshold effects would therefore be a technically involved process. We discuss the modeling options presented by the multi-unit approach in our conclusion. We can however use simple regression techniques to illustrate the importance of threshold effects in estimation. We estimate the following threshold models using every settlement period in 2007:

$$\log(PNSW) = \beta_0^1 + \beta_1^1 DNSW + \beta_2^1 \text{Dummy}_{NSW} + \epsilon_t^1 \tag{4.2}$$

$$PNSW = \gamma_0^1 + \gamma_1^1 DNSW + \gamma_2^1 \text{Dummy}_{NSW} + \mu_t^1 \tag{4.3}$$

$$\log(PVIC) = \beta_0^2 + \beta_1^2 DVIC + \beta_2^2 \text{Dummy}_{VIC} + \epsilon_t^2 \tag{4.4}$$

$$PVIC = \gamma_0^2 + \gamma_1^2 DVIC + \gamma_2^2 \text{Dummy}_{VIC} + \mu_t^2 \tag{4.5}$$

Where the dummy variables are binary indicators that return a value of 1 if demand is above the threshold for that region. We anticipate these models to have very little explanatory power as they exclude influential factors that we have already discussed such as the level of demand in other regions and interconnector constraints.

Table 4.3 shows estimates for the threshold coefficients for each model. We can note that the threshold coefficients are, as predicted by the theory, of significant magnitude. The tendency for

	NSW		VIC	
	β_2^1	γ_2^1	β_2^2	γ_2^2
Value	0.745	585.74	3.432	8894.67
t-stat	29.35	39.82	10.67	68.21

Table 1: Threshold coefficient estimates, NSW and VIC, 2007

price, particularly in New South Wales, to spike only probabilistically when demand is above the threshold has been noted previously. This serves to mute the size of the threshold effect, represented by β_2^1 and γ_2^1, in the estimation for New South Wales.

5 Discussion

The model we have presented here has leant towards simplicity, allowing equilibrium analysis to maintain clarity and precision. As always the cost of this simplicity is the omission of several factors that may influence the accuracy of our theoretical framework. We will address some of these extra-theoretical factors in this section, showing how they may explain the more critical deviations between the model and our observations.

5.1 Hedging

Given the volatility of spot price in the NEM, and electricity markets more generally, both generators and retailers tend to hedge their operations with the use of various financial instruments. The use of such instruments to manage risk points to a gap in our theoretical framework. The use of Contracts for Difference in particular poses a challenge. Commonly, a generator will have a fraction of their capacity hedged under such a contract, wherein they pay or receive the difference between the contract price and the spot price for the settlement period. A hedging strategy such as this would clearly increase the magnitude of the critical threshold. For example suppose that the largest generator faces completely inelastic residual demand and shifts it's capacity to $\overline{p}$. For sufficiently low levels of exceedance, this generator's dispatch would be less than if it had bid competitively and, under a sufficiently extensive contract for difference, would in fact suffer a loss in profit. This intuition is verified by previous work in this area. For example, de Frutos and Fabra [11] have shown that in the multi-unit auction, the introduction of contracts for difference can have pro-competitive effects on price behaviour, but not strictly; they show that such contracts may also assist small firms with anti-competitive behaviour. In what is potentially more relevant work, Wolak [28, 30] has used the same structural estimation method as in [29] to provide empirical evidence suggesting that the use of contracts for difference in the NEM has had a downward effect on prices. He suggests that future contract obligations may in fact help to decrease a generator's cost in production.

5.2 Cost Structure

There are two elements of a typical generation plant cost structure that we have ignored. The first of these is the possibility of increasing marginal costs. It is generally acceptable to assert that costs for generation plants are constant at the margin. However, in general, marginal costs should increase as plants approach their generation capacity.

It is easy to see that the threshold property is still maintained so long as the market maximum $\overline{p}$ is higher than the marginal cost of production, which is a reasonable assumption. We would no longer expect all capacity to be bid in at a particular price. In particular, we would expect equilibria in which some capacity is held at the highest price band given the marginal cost of production at this level of dispatch.

An equally important consideration is the operational constraints faced by generators. To some extent, strategic movements in supply are limited by the ramp-up or ramp-down constraints of a generator. For example, a coal fired plant might be discouraged from sudden shifts in capacity as it cannot afford, either practically or financially, to ramp-down or ramp-up its production as rapidly as is suggested by the model. This also explains why hydroelectric plants, with virtually negligible costs in this regard, are far more likely to rapidly withdraw and supply capacity at different times during the day. Looking back on our analysis, this is an additional possibility when it comes to explaining the gradual movement of capacity to higher price bands, as well as the slow response of spot prices to threshold exceedance. It supports the idea that generators should slowly shift their capacity in anticipation of threshold exceedance.

5.3 Regulatory Factors

A notable omission from our theoretical considerations so far has been the presence of regulatory risk in the auction game. The Australian Energy Regulator, under clause 3.13.17 (d) of the National Electricity Rules (NER), is required to publish a report any time the spot price exceeds \$5000/MWh. It is specified in these rules that this report should assess whether any capacity withdrawal or market rebids contributed to the high spot price, as well as identify the marginal generating units and any units with capacity offered above \$5000/MWh. In addition to this regulatory measure, clauses 3.14.1 and 3.14.2 stipulate that the spot market may be suspended and pricing administered if the cumulative price over the last 336 trading intervals (11.5 hours), exceeds \$150,000. This second measure, in particular, is a significant disincentive to pricing at the market maximum at every opportunity. Given that, as we have seen, demand can often spend multiple hours above the critical threshold, exploiting this opportunity to the fullest possible extent would always lead to a loss of profit due to market suspension.

We can see then that there are dual risks to gaming in the National Electricity Market. There is the risk of regulatory attention and even reproach if a firm's rebids in capacity and high pricing are found to be significantly contributing to spikes in the spot price. Secondly there is the risk of short term losses as a result of market suspension, which places a clear upper bound on the amount of maximum pricing that is profitable. The latter regulatory constraint is a good explanation for the tempered response of prices to threshold exceedance that we have found in the data.

How can we expect rational agents to respond to these risks in the context of a game? Theoretically, because we know little about the degree of firms' risk aversion or the real costs of regulatory action, this is hard to characterise precisely. We can say unambiguously that these new game parameters will temper price spikes in frequency and severity. In addition to this we expect that, in such a setting, it is profitable for generators to begin sharing regulatory risk. That is, price spikes can occur more frequently if all generators partake in the shifting of capacity to higher price bands, something that we observed in our bid analysis for New South Wales on January 12 and February 5.

There is an important consequence for equilibrium analysis if we allow for this behaviour. The possibility of market suspension and regulatory reproach, depending on how we specify the game, renders bids of $\overline{p}$ as suboptimal. Importantly, if multiple generators participate in such action as demand approaches the threshold, we are no longer guaranteed of a single optimal bid. As a result, we no longer have a pure strategy equilibrium for generators facing completely inelastic residual demand. Such a game permits the spot price to spike with variable magnitude when beyond the threshold. Again, this serves as a plausible explanation for why such a pattern is observed in spiking behaviour.

5.4 Tacit Collusion

Given its design as a frequently repeated auction with indefinite horizon, the NEM (and electricity markets more generally) is the type of market where tacit collusion between bidders is a possibility. This issue has received some attention in the more general literature and it has been shown [10, 22, 20] that tacit collusion is generally sustainable in this type of auction, beyond a certain point in demand. A recurring problem with tacit collusion results such as these is that they can only identify a continuum of sustainable collusion paths over time. There is no real theoretical direction for determining which path will actually be followed, which lessens the tractability of such a model.

6 Conclusions

In this paper we have adapted von der Fehr and Harbord's multi-unit auction model [26] for settings applicable to Australia's NEM. In doing so we have shown that, in this model, price spikes are related to a specifiable threshold in demand. We have inspected the data for patterns of behaviour that mirror the stylized facts established by the model, with positive results. Specifically, we have found that price spikes do seem to relate convincingly to times of threshold exceedance. In addition, we observe through specific price events that bid behaviour corresponds to that which is predicted; generators gradually shift capacity to the highest price band as the probability of threshold exceedance increases. Our initial findings indicate that the multi-unit auction may provide the basis of a structural model for econometric analysis. There is definite scope, for example, for the use of threshold regression techniques, such as those explored by Dagenais [9] and more recently Hansen [16].

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