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Nonlinear adaptive control using neural networks and multiple models☆

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Abstract

In this paper, adaptive control of a class of nonlinear discrete time dynamical systems with boundedness of all signals is established by using a linear robust adaptive controller and a neural network based nonlinear adaptive controller, and switching between them by a suitably defined switching law. The linear controller, when used alone, assures boundedness of all the signals but not satisfactory performance. The nonlinear controller may result in improved response, but may also result in instability. By using a switching scheme, it is demonstrated that improved performance and stability can be simultaneously achieved.

Keywords: Nonlinear control; Adaptive control; Neural networks; Multiple models; Switching systems; Stability; Robustness; Performance

1. Introduction

Since 1990, nonlinear adaptive control using neural networks has been an active field of research. For complex nonlinear dynamical systems whose mathematical models are not available from first principles, it has been shown that successful identification and control may be possible using neural networks (Narendra, 1996; Juditsky et al., 1995). In such cases, the existence of a dynamical nonlinear map from the input space to the output space is first established, and a neural network is used to approximate the map using the available input–output data. Following this, a second neural network is used to control the system based on its input–output model. At the present time the conditions that the plant must satisfy for a bounded control input to exist in order to achieve asymptotic tracking of any specified bounded signal, are well known (Cabrera & Narendra, 1999). However, due to the complexity of the structure of a multilayer neural network and the nonlinear dependence of its map on the parameter values, stability analysis of the resulting adaptive systems has always been very difficult, and quite often intractable. Although many interesting results have been obtained to further our understanding of the stability problems (Sanner & Slotine, 1992; Jagannathan & Lewis, 1996a,b; Polycarpou, 1996; Chen & Khalil, 1995; Chen & Liu, 1994; Spooner & Passino, 1999; Nam, 1999; Choi & Farrell, 2000; Shukla, Dawson, & Paul, 1999; Yu & Annaswamy, 1997), most of them suffer from one or more of the following drawbacks: (1) The neural networks used are *linearly* parameterized. Even when the structure allows for nonlinear parameters, (e.g., centers of Gaussian functions or weights before the last layer), they are kept fixed. This is done more often in theoretical analysis than in practice, and the potential of a nonlinearly parameterized neural network is not fully realized. (2) The system to be controlled is of a special known structure, e.g., a triangular structure. Such results cannot be directly applied to a system whose first-principle model is unavailable and whose identification has to be carried out by assuming a general structure for the system (e.g., a NARMA model). (3) The result is local, i.e., the initial weights of a neural network have to be "close enough" to the true ones in order for the stability result

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to hold. In practice, the closeness is hard to decide, since it is well known that parameters usually do not converge to their true values even after extensive off-line training. In contrast to the above, the result presented in this paper avoids these drawbacks by establishing a framework in which the stable adaptive control of a fairly general class of systems can be attempted using nonlinearly parameterized neural networks with any parameter adjusting mechanism.

From a practical point of view, the need for nonlinear control often arises when better performance (more accurate tracking, larger domain of operation, etc.) is pursued, *after* the plant is operational with a linear controller. Therefore, the repertoire of well established methods from linear control theory as well as their robustness properties should play a major role in solving a nonlinear control problem. It would be ideal if the linear controller provides a lower bound on the performance, while the nonlinear neural network based controller attempts to improve the performance by identifying and compensating for the nonlinearity.

In his Bode lecture, at the 1995 IEEE Conference on Decision and Control, the second author discussed the use of multiple models for improving the performance of an adaptive system. One of the principal points made in the lecture was that the use of multiple models might provide a convenient framework for obtaining both stability and improved performance simultaneously, or alternately it could be used to combine the advantages of different models. Many attempts have been made (and are being made) to pursue this novel idea, and the result presented in this paper is the first one that substantiates the idea in a mathematically rigorous fashion for a class of problems that are of great importance to the neurocontrol community. The principal contribution described here is concerned with combining linear and nonlinear models to improve the performance of essentially nonlinear dynamical systems even while assuring their stability.

In this paper, a multiple model approach using switching and tuning, proposed initially for continuous time linear systems (Narendra & Balakrishnan, 1997) and later extended to discrete-time stochastic systems (Narendra & Xiang, 2000), is used to control the discrete-time nonlinear system. By making suitable assumptions concerning the nonlinear plant to be controlled, a linear model and a neural network based nonlinear model are used to identify the system simultaneously. Based on their performance as identifiers, a corresponding (linear or neural network based nonlinear) controller is used at every instant to control the system. The main result is that all the signals in the overall system are bounded. Qualitatively speaking, this may be mainly attributed to the linear controller. However, the identification of the nonlinear system by the neural network based model usually improves with time, and in such a case, the system converges to the nonlinear controller and results in improved performance. Thus, by a judicious combination of the two approaches and the choice of a suitable switching law, both stability and performance are achieved. Simulation results are presented in Section 6 to demonstrate the substantial improvement in performance that can result using the proposed method.

The paper is organized as follows: First, the system under consideration is defined and the control problem is stated. Some preliminary comments about neural networks and growth rates of signals are given, which will be central to the proof of stability in Section 4. Following this, well-known results in robust linear adaptive control are reproduced for easy reference. The key result of stability analysis when multiple models are used is presented in Section 4. This is a very general result, and the special case when some models are realized by neural networks follows in Section 5. Simulation studies show the efficacy of the theory, and finally some conclusions are drawn.

2. Mathematical preliminaries

2.1. Statement of the problem

Let a single-input–single-output (SISO) discrete-time nonlinear system be described by

$$\Sigma: x(k+1) = F(x(k), u(k)),$$
$$y(k) = H(x(k)), \tag{1}$$

where $u(k), y(k) \in \mathbb{R}, x(k) \in \mathbb{R}^n$ and F and H are smooth nonlinear functions such that the origin is an equilibrium state. In view of the role played by the linearization around the equilibrium state, (1) is also equivalently represented as (Cabrera & Narendra, 1999; Narendra & Chen, 1998)

$$\Sigma: x(k+1) = Ax(k) + bu(k) + \bar{f}(x(k), u(k)),$$
$$y(k) = cx(k) + \bar{h}(x(k)), \tag{2}$$

where $A \in \mathbb{R}^{n \times n}$, b, $c^{\mathrm{T}} \in \mathbb{R}^n$ and the triple (c, A, b) represents the linearization Σ_{L} of Σ. The nonlinear functions $\bar{f}$ and $\bar{h}$ are consequently obtained by subtracting the linear terms from the functions F and H in Eq. (1), and are said to belong to the class $\mathscr{H}$ of higher order functions. For an observable system of order n, it is well known that the state $x(k+1)$ of (2) can be represented as a function of $y(k), y(k-1), \ldots, y(k-n+1), u(k), u(k-1), \ldots, u(k-n+1)$ so that Eq. (2), in a neighborhood of the origin, can also be represented as

$$\begin{aligned} y(k+d) &= a_0 y(k) + \cdots + a_{n-1} y(k-n+1) \\ &\quad + b_0 u(k) + \cdots + b_{n-1} u(k-n+1) \\ &\quad + f(y(k), \ldots, y(k-n+1), \\ &\qquad u(k), \ldots, u(k-n+1)) \\ &\triangleq \theta^{\mathrm{T}} \omega(k) + f(\omega(k)) \end{aligned} \tag{3}$$

where

$$\omega(k) \triangleq [y(k), \dots, y(k-n+1), u(k), \dots, u(k-n+1)]^{\mathrm{T}}$$

is referred to as the regression vector,

$$\theta \triangleq [a_0, \dots, a_{n-1}, b_0, \dots, b_{n-1}]^{\mathrm{T}}, \quad b_0 \geqslant b_{\min} > 0$$

is the (linear) parameter vector, and f is a smooth nonlinear function that consists of higher order terms of $\omega(k)$ locally. It is assumed that the order of the system n and the relative degree d are specified while the parameter vector θ and the nonlinear function f are unknown. A bounded signal $y^*(k)$ represents the desired output of the system, and the value $y^*(k+d)$ is known to the controller at time k. The objective of adaptive control is to generate a bounded control signal $u(k)$ such that the output $y(k)$ asymptotically approaches the specified bounded signal $y^*(k)$, i.e.,

$$\lim_{k \to \infty} |y(k) - y^*(k)| = 0.$$

When the nonlinearity f is small, it can be treated as a bounded disturbance, and the problem becomes one of linear robust adaptive control, which will be briefly summarized in the next section for easy reference. However, in order to achieve better performance, efforts have to be made to compensate for the nonlinearity f, e.g., by using a neural network. In such a case, stability becomes a central issue. In this paper, a general framework will be set up to deal with this problem. To this end, some assumptions will be made concerning the system Σ:

Assumptions. (i) The system under consideration has a *global* representation (3).

(ii) The nonlinearity $f(\cdot)$ is *globally* bounded, i.e., $|f(\cdot)| \leqslant \Delta$, and the bound Δ is known.

(iii) The system has a globally uniformly asymptotically stable zero dynamics, so that an input sequence never grows faster than the output sequence.

(iv) It is known a priori that the linear parameter vector θ lies in a compact region B.

Comment 1. When these assumptions are satisfied, all signals in the system will be proved to be globally bounded. Since they are bounded, Assumptions (i) and (ii) need only be satisfied within this bound. In other words, if Assumptions (i) and (ii) are not valid globally, but only valid in a neighborhood Ω_{A} of the origin, then by argument of continuity, it could be established that there exists a neighborhood Ω_{I} of the origin such that, as long as the initial conditions and the reference trajectories are within this neighborhood, all signals in the system will be within a bounded neighborhood $\Omega_{\mathrm{B}} \subset \Omega_{\mathrm{A}}$.

Comment 2. It is well known that a necessary condition for proving the stability of the standard linear adaptive control problem is that the system be minimum phase. This ensures that no input signal will grow faster than the output signal. For the nonlinear problem under consideration, a nonlinear counterpart of the same assumption is necessary, i.e., no input signal to the nonlinear system can grow faster than the output signal. While for local results, this follows from the asymptotic stability of the zero dynamics of the system, how to characterize the property globally for general nonlinear systems is still an open question.

2.2. Neural networks as nonlinear identifiers and controllers

Even though much of the analysis of the adaptive systems discussed in this paper is carried out for a general nonlinear function f in Eq. (3), the implementation of the algorithms in practical systems will be carried out using neural networks. From a system theoretic point of view, neural networks are convenient families of nonlinear mappings of the form

$$y = N(u, W) = W_3 \Gamma(W_2 \Gamma(W_1 u + b_1) + b_2) + b_3,$$

where u is the input, y is the output, W represents all the parameters ("weights" and "biases") W_i and b_i, $i = 1,2,3$, and

$$\Gamma([v_1, \dots, v_m]^{\mathrm{T}}) = [\gamma(v_1), \dots, \gamma(v_m)]^{\mathrm{T}},$$

where $\gamma(\cdot)$ is a so-called "activation function", e.g., $\gamma(x) = (1 - \mathrm{e}^{-x})/(1 + \mathrm{e}^{-x})$.

It has been established by several authors, e.g., (Funahashi, 1989), that neural networks are universal approximators. Hence, by a proper choice of the structure and parameters of a neural network, the error $|f(\cdot) - N(\cdot)|$ can be made less than any specified ε over a compact set.

2.3. Growth rates of signals

In Section 3 the stability and robustness of adaptive control systems (using a single model) is discussed, and the results are extended to the multiple model case in Section 4. In all cases, proof by contradiction is used to derive the results. It is first assumed that the signals are unbounded and it is then shown that this leads to a contradiction. Hence, one of the principal tools used in the proof is the relationship between different signals in the system and their rates of growth. Once one set of signals is shown to be bounded, it generally follows, in a relatively straightforward fashion, that all the signals in the system must also be bounded. In view of the above, the study of the growth rates of signals in adaptive systems assumes central importance in the study of their stability (Narendra & Annaswamy, 1989).

Let $x(k)$ and $y(k)$ (scalar or vector) be two discrete time signals defined for all $k \in \mathbb{N}^+$ where $\mathbb{N}^+$ is the set of all nonnegative integers. Let $|\cdot|$ denote a norm.

Definition 1 (*Large order*). We denote $y(k) = O[x(k)]$ if there exist positive constants M_1, M_2 and k_0 such that $|y(k)| \leqslant M_1 \max_{\tau \leqslant k} |x(\tau)| + M_2$, $\forall k \geqslant k_0$.

Definition 2 (*Small order*). We denote $y(k) = o[x(k)]$ if there exists a discrete-time function $\beta(k)$ with the property that $\lim_{k \to \infty} \beta(k) = 0$, and a constant k_0 such that $|y(k)| \leqslant \beta(k) \max_{\tau \leqslant k} |x(\tau)|$, $\forall k \geqslant k_0$.

Definition 3 (*Equivalence*). if $x(k) = O[y(k)]$ and $y(k) = O[x(k)]$, we refer to $x(k)$ and $y(k)$ as being equivalent and denote it as $x(k) \sim y(k)$. If follows directly that this equivalence relation is reflexive, symmetric and transitive, so that the symbol $\sim$ represents an equivalence class.

It is straightforward to verify the following properties about growth rates of signals. (To simplify the notations, time k is dropped.)

(i) If z is bounded and x is unbounded, then

$$y = O[x] \Rightarrow y = O[x + z].$$

(ii) If z is bounded, then

$$y = O[x] \Rightarrow y + z = O[x].$$

(iii) If z_1, z_2 are bounded and x and y are unbounded, then

$$y \sim x \Leftrightarrow y + z_1 \sim x + z_2.$$

(iv) If x is unbounded and y is bounded, then

$$y = o[x].$$

(v) If z is unbounded, then

$$y = o[x],\ x = O[z] \Rightarrow y = o[z].$$

(vi) If y is unbounded, then

$$z = O[y],\ y = o[x] \Rightarrow z = o[x].$$

(vii) $y_1 = o[x]$, $y_2 = o[x] \Rightarrow y_1 + y_2 = o[x]$.

(viii) If y consists of two subsequences y_1 and y_2, and the corresponding subsequences of x is x_1 and x_2, then

$$y_1 = o[x_1],\ y_2 = o[x_2] \Rightarrow y = o[x].$$

(ix) $y = o[y] \Rightarrow \lim_{k \to \infty} y(k) = 0$.

All these properties will be used explicitly or implicitly in the proof of stability of adaptive systems.

2.4. Certainty equivalence control input

When indirect linear adaptive control is extended to the nonlinear case, and a nonlinear model of the plant is obtained in the form

$$\hat{y}(k+1) = f(y(k), \ldots, y(k-n+1), u(k), \ldots, u(k-n+1), \hat{p}),$$

the parameter vector $\hat{p}$ is assumed to be the true parameter of the plant, and the control input $u(k)$ is determined such that

$$\hat{y}(k+1) = y^*(k+1).$$

Since $f(\cdot)$ is now a nonlinear function, the existence of such a $u(k)$, as well as the method for finding it, is no longer straightforward. For reasons which will become clear in Section 4, the following lemma will be very useful.

Lemma 1 (Full range). *Let $b_0 \in \mathbb{R}$ be a nonzero constant, $X_0 \in \mathbb{R}^m$ be any given vector, $f(u, X_0): \mathbb{R} \times \mathbb{R}^m \to \mathbb{R}$ be continuous and bounded. Then the range of the mapping $F(u, X_0) \triangleq b_0 u + f(u, X_0): \mathbb{R} \times \mathbb{R}^m \to \mathbb{R}$ is $\mathbb{R}$.*

Proof. Let $[\alpha, \beta]$ be a finite closed interval on $\mathbb{R}$. For $u \in [\alpha, \beta]$, $F(u, X_0)$ attains every value between the minimum $F_{\min}$ and the maximum $F_{\max}$ on this closed interval, since F is continuous. Further, as $\alpha \to -\infty$ and $\beta \to \infty$,

$$b_0 u|_{u=\alpha} \to -\infty, \quad b_0 u|_{u=\beta} \to \infty,$$

since $b_0 \neq 0$. Finally, since f is bounded,

$$F_{\min} \to -\infty, \quad F_{\max} \to \infty. \qquad \square$$

From this lemma, for a given X and a desired value y^*, there exists at least one u^* to achieve it.

3. Linear robust adaptive control

In this section, results from linear robust adaptive control will be stated, and the proof that all the signals are bounded will be given. This is the basis for using multiple models to improve performance while maintaining stability.

Let the system under consideration be described by

$$y(k+1) = \theta^{\mathrm{T}} \omega(k) + f(\omega(k)), \tag{4}$$

where θ, $\omega(k)$ and f are defined as in Section 2.1, $|f(\cdot)| \leqslant \Delta$, and the component b_0 in θ satisfies

$$b_0 \geqslant b_{\min} > 0. \tag{5}$$

Here only unit delay is considered for the sake of brevity. Modifications that have to be made when the delay is greater than one will be discussed in the following sections.

To solve the adaptive control problem stated in the previous section, the following model is set up to predict the value of the output at time $k+1$:

$$\hat{y}(k+1) = \hat{\theta}^{\mathrm{T}}(k) \omega(k).$$

Using the certainty equivalence principle, the control input $u(k)$ is determined such that $\hat{y}(k+1) = y^*(k+1)$. The parameter $\hat{\theta}(k)$ is updated in two steps. In the first step, an unconstrained value is calculated. In the second step, if the component b_0 of that value is less than $b_{\min}$, it is set to $b_{\min}$ so that (5) always holds. We will first discuss the unconstrained parameter update and its properties, and then show that these properties do not change in the second step.

In the first step, the parameter adaptation law is

$$\hat{\theta}(k) = \hat{\theta}(k-1) - \frac{a(k)e(k)\omega(k-1)}{1+|\omega(k-1)|^2}, \tag{6}$$

where

$$e(k) \triangleq \hat{y}(k) - y(k) = (\hat{\theta}^{\mathrm{T}}(k-1) - \theta^{\mathrm{T}})\omega(k-1) - f(\omega(k-1))$$

and

$$a(k) = \begin{cases} 1 & \text{if } |e(k)| > 2\Delta, \\ 0 & \text{otherwise.} \end{cases}$$

Define $\phi(k) = \hat{\theta}(k) - \theta$ and we have

$$\phi(k) = \phi(k-1) - \frac{a(k)e(k)\omega(k-1)}{1+|\omega(k-1)|^2}$$

and therefore (Goodwin & Sin, 1984)

$$\begin{aligned}
|\phi(k)|^2 &= |\phi(k-1)|^2 - \frac{2a(k)e(k)\phi(k-1)^{\mathrm{T}}\omega(k-1)}{1+|\omega(k-1)|^2} + \frac{a^2(k)e^2(k)|\omega(k-1)|^2}{(1+|\omega(k-1)|^2)^2} \\
&= |\phi(k-1)|^2 - \frac{2a(k)e(k)(e(k)+f(\cdot))}{1+|\omega(k-1)|^2} + \frac{a^2(k)e^2(k)|\omega(k-1)|^2}{(1+|\omega(k-1)|^2)^2} \\
&= |\phi(k-1)|^2 + \frac{a(k)(-2e(k)f(\cdot))}{1+|\omega(k-1)|^2} - \frac{a(k)e^2(k)}{1+|\omega(k-1)|^2}\left(2 - \frac{a(k)|\omega(k-1)|^2}{1+|\omega(k-1)|^2}\right) \\
&\leqslant |\phi(k-1)|^2 + \frac{a(k)(-2e(k)f(\cdot))}{1+|\omega(k-1)|^2} - \frac{a(k)e^2(k)}{1+|\omega(k-1)|^2} \\
&\leqslant |\phi(k-1)|^2 + \frac{a(k)(e^2(k)/2 + 2f^2(\cdot))}{1+|\omega(k-1)|^2} - \frac{a(k)e^2(k)}{1+|\omega(k-1)|^2} \\
&\leqslant |\phi(k-1)|^2 - \frac{a(k)(e^2(k) - 4\Delta^2)}{2(1+|\omega(k-1)|^2)}.
\end{aligned} \tag{7}$$

Two conclusions can be drawn from inequality (7):

(i) Since $a(k) = 1$ for $|e(k)| > 2\Delta$ and is 0 otherwise, $\{|\phi(k)|^2\}$ is a nonincreasing sequence. Hence $\hat{\theta}(k)$ is bounded. Moreover,

(ii) $\lim_{N\to\infty}\sum_{k=1}^{N}(a(k)(e^2(k)-4\Delta^2))/(2(1+|\omega(k-1)|^2)) \leqslant |\phi(0)|^2 - |\phi(N)|^2 < \infty$, and hence,

$$\lim_{k\to\infty} \frac{a(k)(e^2(k)-4\Delta^2)}{2(1+|\omega(k-1)|^2)} \to 0.$$

By using the properties listed in Section 2.3, it can be concluded that

$$e(k) = o[\omega(k-1)]$$

if $\omega(k-1)$ is unbounded.

In the second step of parameter adaptation, if $\hat{\theta}(k) = [\star, \hat{b}_0(k), \star]^{\mathrm{T}}$ and $\hat{b}_0(k) < b_{\min}$, then $\hat{\theta}(k)$ is set to $\hat{\theta}^+(k) = [\star, b_{\min}, \star]^{\mathrm{T}}$.

Note that

$$|\hat{\theta}^+(k) - \theta|^2 - |\hat{\theta}(k) - \theta|^2 = (2(b_{\min} - b_0) + (\hat{b}_0 - b_{\min}))(b_{\min} - \hat{b}_0) < 0$$

or, $|\phi^+(k)|^2 \leqslant |\phi(k)|^2$. Therefore inequality (7) and the conclusions drawn from it still hold.

The proof that all signals are bounded is by contradiction.

Proof. Assume that y is unbounded. Then

1. By certainty equivalence, $u(k)$ is always chosen such that $\hat{y}(k+1) = y^*(k+1)$, and therefore

   $$e(k) = \hat{y}(k) - y(k) = y^*(k) - y(k).$$

   Since $y^*(k)$ is bounded, we have

   $$e(k) \sim y(k) \tag{8}$$

   or, $e(k)$ grows at the same rate as $y(k)$.
2. By the assumption that the system has an asymptotically stable zero dynamics (or minimum phase, for linear systems), i.e., any input sequence $u(k-1)$ cannot grow faster than the output sequence $y(k)$, we have

   $$u(k-1) = O[y(k)].$$

   Since

   $$\omega(k-1) = [y(k-1), \ldots, y(k-n), u(k-1), \ldots, u(k-n)]^{\mathrm{T}}$$

   it follows that

   $$\omega(k-1) = O[y(k)] \tag{9}$$

   or, $\omega(k-1)$ does not grow faster than $y(k)$.

3. By the adaptation law (6),

$$e(k) = o[\omega(k-1)] \tag{10}$$

or, $e(k)$ grows slower than $\omega(k-1)$.

4. Therefore, from (8)–(10),

$$y(k) = o[y(k)]$$

or, $y(k)$ grows slower than itself. This cannot happen if $y(k)$ is assumed to be unbounded. Therefore $y(k)$ is bounded, and the boundedness of other signals follows in a straightforward fashion. □

It is seen from the proof that the property of the identifier, $e(k) = o[\omega(k-1)]$, or, the identification error which grows at a lower rate than the regression vector, plays a central role. When multiple models are used, the objective of the switching scheme is to maintain this relationship, even while the control inputs can be generated by different controllers. This will be demonstrated in the next section.

4. Adaptive control using multiple models

In this section, the problem of adaptive control using multiple models will be considered. To simplify the exposition, results will be stated in terms of two models, and the switching system is illustrated in Fig. 1.

For a general switching system, let I_1 and I_2 be two models which predict the output of the plant in some fashion, with their parameters updated using the input and output data of the plant. Let J_1 and J_2 be the performance indices suitably defined. At every instant k, the I_i with a smaller J_i, $i = 1$ or 2, will be chosen to generate a certainty equivalence control input $u(k)$ to the plant, i.e., to generate a $u(k)$ such that the output of I_i will be equal to the desired reference value. What properties $\{I_i\}$ have to have, and how $\{J_i\}$ are defined, are among the important questions in this multiple model approach.

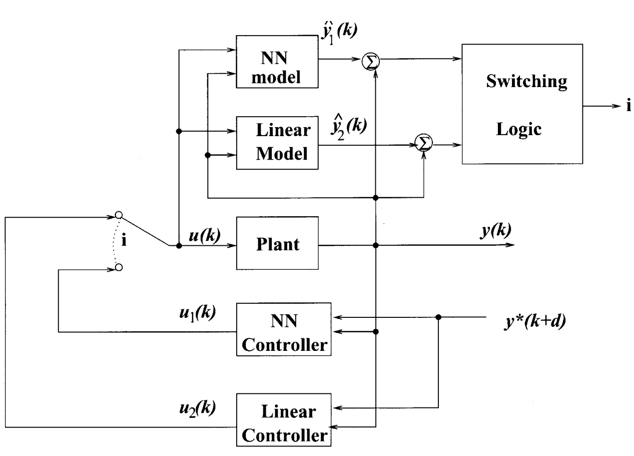

Fig. 1. Multiple models: switching between a linear controller and a neural network controller.

A special case will be first considered below, and a proof of boundedness of all the signals will be given. Following this, modifications that have to be made for the general case will be presented.

4.1. A special case

Let the system be described by

$$\begin{aligned} y(k+1) &= \theta^{\mathrm{T}}[y(k),\ldots,y(k-n+1),u(k),\ldots,u(k-n+1)]^{\mathrm{T}} \\ &\quad + f([y(k),\ldots,y(k-n+1),u(k-1),\ldots, \\ &\qquad u(k-n+1)]^{\mathrm{T}}) \\ &\triangleq \theta^{\mathrm{T}}\omega(k) + f(\bar{\omega}(k)) \\ &\triangleq b_0 u(k) + \bar{\theta}^{\mathrm{T}}\bar{\omega}(k) + f(\bar{\omega}(k)), \end{aligned} \tag{11}$$

where

$$\theta \triangleq [a_0,\ldots,a_{n-1},b_0,\ldots,b_{n-1}]^{\mathrm{T}},$$

$$\bar{\theta} \triangleq [a_0,\ldots,a_{n-1},b_1,\ldots,b_{n-1}]^{\mathrm{T}},$$

$$\omega(k) \triangleq [y(k),\ldots,y(k-n+1),u(k),\ldots,u(k-n+1)]^{\mathrm{T}},$$

and

$$\bar{\omega}(k) \triangleq [y(k),\ldots,y(k-n+1),u(k-1),\ldots, u(k-n+1)]^{\mathrm{T}}.$$

The purpose of these definitions is to isolate terms with $u(k)$. Note that the delay is assumed unity, and the current control input $u(k)$ affects the output $y(k+1)$ only through a linear term $b_0 u(k)$. In such a case, the determination of a certainty equivalence control input $u(k)$ will be straightforward.

Next we will define identifiers. Let identifier I_1 be defined as

$$\hat{y}_1(k+1) = \hat{\theta}_1^{\mathrm{T}}(k)\omega(k), \tag{12}$$

where parameter $\hat{\theta}_1(k)$ is updated as

$$\hat{\theta}_1(k) = \hat{\theta}_1(k-1) - \frac{a(k)e_1(k)\omega(k-1)}{1+|\omega(k-1)|^2}, \tag{13}$$

where

$$e_1(k) \triangleq \hat{y}_1(k) - y(k),$$

$$a(k) = \begin{cases} 1 & \text{if } |e_1(k)| > 2\Delta, \\ 0 & \text{otherwise} \end{cases}$$

and $\hat{b}_0^{(1)}(k)$ in $\hat{\theta}_1(k)$ is always constrained to be greater than $b_{\min} > 0$, as discussed earlier.

Controller C_1 (i.e., control input $u_1(k)$ at time k) is determined as

$$u_1(k) = \frac{1}{\hat{b}_0^{(1)}(k)}(y^*(k+1) - \hat{\bar{\theta}}_1(k)\bar{\omega}(k)).$$

Let identifier I_2 be defined as

$$\hat{y}_2(k+1) = \hat{\theta}_2^{\mathrm{T}}(k)\omega(k) + \hat{f}(\bar{\omega}(k), W(k)), \tag{14}$$

where $\hat{f}(\cdot)$ is a *bounded* continuous nonlinear function parameterized by a vector $W(k)$ (e.g., a neural network with "weights" vector W). No restriction is made on how the parameters $\hat{\theta}_2(k)$ or $W(k)$ are updated except that they always lie inside some pre-defined compact region $\mathscr{S}$:

$$\hat{\theta}(k), \hat{W}(k) \in \mathscr{S}, \quad \forall k \tag{15}$$

and in this region $\hat{b}_0^{(2)}(k)$ is always greater than $b_{\min}$.

Controller C_2 is determined as

$$u_2(k) = \frac{1}{\hat{b}_0^{(2)}(k)}(y^*(k+1) - \hat{\bar{\theta}}_2^{\mathrm{T}}(k)\bar{\omega}(k) - \hat{f}(\bar{\omega}(k), W(k))).$$

4.1.1. Performance criterion and switching rule:

Define

$$J_i(k) = \sum_{l=1}^{k} \frac{a_i(l)(e_i^2(l) - 4\Delta^2)}{2(1 + |\omega(l-1)|^2)} + c \sum_{l=k-N+1}^{k} (1 - a_i(l))e_i^2(l), \quad i = 1, 2, \tag{16}$$

where

$$e_i(k) \triangleq \hat{y}_i(k) - y(k),$$

$$a_i(k) = \begin{cases} 1 & \text{if } |e_i(k)| > 2\Delta, \\ 0 & \text{otherwise,} \end{cases}$$

N is an integer and $c \geqslant 0$ is a constant.

At every instant k, if $J_1(k) \leqslant J_2(k)$, Controller C_1 is used to determine control input $u(k)$ to the system. If $J_1(k) > J_2(k)$, Controller C_2 is used to determine control input $u(k)$.

Theorem 1 (Stable switching). *For system* (11) *with identifiers* (12) *and* (14) *together with their adaptation laws* (13) *and* (15), *and performance index* (16), *all the signals in the closed-loop switching system described above are bounded.*

Proof. We will prove the theorem by contradiction, as in the single model case. Assume that $y(k)$ is unbounded. Note that the second term in (16) is always bounded. By the properties of the linear robust adaptive identifier (13), $J_1(k)$ is bounded, and

$$e_1(k) = o[\omega(k-1)].$$

By the use of certainty equivalence, at every instant k, either

$$e_c(k) = e_1(k) \quad \text{or} \quad e_c(k) = e_2(k),$$

where $e_c(k) \triangleq y^*(k) - y(k)$ is the control error, since we are assuming unit delay. Let $e(k)$ be the "combined" identification error that corresponds to the control error $e_c(k)$. There can be two cases:

(i) $J_2(k)$ is also bounded. It follows that $e_2(k) = o[\omega(k-1)]$. Thus $e(k) = o[\omega(k-1)]$. In fact, this is true regardless of the switching rule used.

(ii) $J_2(k)$ is unbounded. Since $J_1(k)$ is bounded, there exists a constant k_0 such that $J_2(k) > J_1(k)$, $\forall k \geqslant k_0$. By the switching rule, Controller C_1 will be used and therefore $e(k) = e_1(k)$, $\forall k \geqslant k_0 + 1$. Hence $e(k) = o[\omega(k-1)]$.

Since $e(k) = o[\omega(k-1)]$, the rest of the proof follows along the same lines as in the single model case, and is omitted here. □

Comment 3. The performance index (16) consists of two parts. The first part, $\sum_{l=1}^{k} (a_i(l)(e_i^2(l) - 4\Delta^2))/(2(1 + |\omega(l-1)|^2))$, is used to distinguish between signals with different growth rates, so that boundedness of all the signals can be established. The second part, $c\sum_{l=k-N+1}^{k} (1 - a_i(l))e_i^2(l)$, is a measure of the prediction error over a finite window and is included to improve performance. If one of the identifiers predicts the output with a smaller error, this component of the performance index decreases, and hence the controller corresponding to that identifier may be chosen to generate the control input. A suitable choice of c and N can lead to better performance with stability.

Comment 4. The significance of the above theorem is that it *decouples* the stability issue from the performance issue, the issue of how a neural network can learn the nonlinearity and compensate for it. No restriction (except the obvious one that parameters are not allowed to become arbitrarily large) on the architecture, number of nodes, or training method of the neural network is needed for the stability result to hold. This provides a convenient framework in which any type of neural network and training method can be attempted, and it should not come as a surprise that not all of them can actually yield improved performance. The specific neural networks that the authors frequently use will be specified later with the simulation example.

4.2. General case

In the previous section, control of a special class of plants is presented, where the delay d is unity and the control input $u(k)$ does not affect $y(k+1)$ in a nonlinear fashion. In this section we will discuss the modifications that have to be made for more general cases.

4.2.1. Delay $d > 1$

When $d > 1$ in (3), if an identification model

$$\hat{y}(k+1) = \hat{\theta}(k)\omega(k-d+1)$$

is used to predict the output at the next instant of time using the *currently* available parameter estimate $\hat{\theta}(k)$, then the equality

$$e(k) = e_c(k) \tag{17}$$

no longer holds, where $e(k) = \hat{y}(k) - y(k)$ is the identification error, and $e_c(k) = y^*(k) - y(k)$ is the control error. This is because at time k, the measured output $y(k)$ is due to the control input $u(k-d)$, which is computed from the *then* available parameter estimate $\hat{\theta}(k-d)$, while the predicted output $\hat{y}(k)$ is computed from the parameter estimate $\hat{\theta}(k-1)$, which has been updated from $\hat{\theta}(k-d)$ when data become available at time $k-d+1, \ldots, k-1$.

For single model linear adaptive control, change in parameter estimate $\hat{\theta}(k)$ from time $k-d+1$ to time $k-1$ can be taken into account and boundedness of signals can still be established. When multiple models are used, equality (17) simplifies the proof substantially, and therefore we will set up identification models such that (17) will still be valid. To do this, the parameter sequence $\hat{\theta}(k)$ can be divided into d subsequences (the so called "interlacing algorithm"), each one updating itself when data become available.

More specifically, let τ denote a d-time scale where one unit in τ corresponds to d units in k. Then an identification model can be set up using d subsequences of parameter estimates

$$\hat{y}_0(\tau+1) = \hat{\theta}_0(\tau)\omega_0(\tau), \quad k \equiv 0 \bmod d,$$

$$\hat{y}_1(\tau+1) = \hat{\theta}_1(\tau)\omega_1(\tau), \quad k \equiv 1 \bmod d.$$

$$\vdots$$

$$\hat{y}_{d-1}(\tau+1) = \hat{\theta}_{d-1}(\tau)\omega_{d-1}(\tau), \quad k \equiv d-1 \bmod d. \tag{18}$$

Each parameter subsequence $\hat{\theta}_i$ is updated in the same way as is discussed in the special case when $d = 1$.

With such a scheme, the parameter vector used to compute the control input $u(k-d)$, which results in a measured output $y(k)$ at time k, is the same parameter vector used to compute the predicted output $\hat{y}(k)$, and therefore the relationship $e(k) = e_c(k)$ still holds.

Similarly, when $\hat{f}(\cdot, W)$ is used in an identification model as in (14), the parameter sequence $W(k)$ is split into d subsequences. In terms of neural networks used as identifiers, d separate neural networks will be used, each updating its parameters in its own time scale.

4.2.2. Nonlinear dependence on $u(k)$

When the plant is described by

$$\begin{aligned} y(k+d) &= \theta^{T}[y(k), \ldots, y(k-n+1), u(k), \ldots, u(k-n+1)]^{T} \\ &\quad + f([y(k), \ldots, y(k-n+1), u(k), \ldots, \\ &\quad\quad u(k-n+1)]) \\ &\triangleq \theta^{T}\omega(k) + f(\omega(k)) \\ &\triangleq b_0 u(k) + \bar{\theta}^{T}\bar{\omega}(k) + f(u(k), \bar{\omega}(k))\end{aligned}$$

and an identifier is set up accordingly to obtain the predicted value $\hat{y}(k+d)$, the existence of a $u(k)$ such that $\hat{y}(k+d) = y^*(k+d)$, as well as its computation, is no longer straightforward. Existence can be established by Lemma 1 (full range), since the mapping from $u(k)$ to $y(k+d)$ has full range. To compute such a $u(k)$, numerical search method can be used when a closed-form expression is not available. Note that the value of $u(k)$ in each iteration of a searching algorithm is tried on the *identifier*, not the plant. This should not be computationally prohibitive either, since $u(k)$ is only one-dimensional. Work is currently underway to incorporate a second neural network in the above iterative procedure.

5. Neural network based adaptive control

The case when the identification model I_2 is realized using a neural network can be stated as a corollary to Theorem 1:

Corollary 1 (Neuro-control). *Let I_1 be the linear robust adaptive identifier, I_2 be a neural network identifier, and J_1 and J_2 be defined as in Theorem* 1. *Then all the signals in the switching system are bounded. Moreover, if the parameter adjustment mechanism of the neural network reduces the identification error, then the tracking error can go to zero.*

Comment 5. In this paper, the result is stated in terms of two models. This merely serves to bring out the essence of the multiple model approach to neurocontrol. The result can be easily generalized to more than two models.

6. Simulation

6.1. The system and reference

To illustrate the concepts discussed in the preceding sections, simulations were conducted on the following system which corresponds to the general case where delay $d > 1$ and $u(k)$ affects $y(k+2)$ through both linear terms and nonlinear terms:

$$y(k+2) = 2.6y(k) - 1.2y(k-1) + u(k) + 0.5u(k-1) + \sin(u(k) + u(k-1) + y(k) + y(k-1)) - \frac{u(k) + u(k-1) + y(k) + y(k-1)}{1 + u^2(k) + u^2(k-1) + y^2(k) + y^2(k-1)}.$$

A reference trajectory $y^*(k) = 1.5(\sin 2\pi k/10 + \sin 2\pi k/25)$ is to be followed asymptotically, which is shown in dashed lines in the following figures.

6.2. Linear controller only (Fig. 2)

A linear robust adaptive controller was used to control the plant, and its performance is shown in Fig. 2. (The third subplot of the "switching" signal merely serves as a place holder.) Since the two nonlinear terms are both bounded, all the signals in the closed-loop system are bounded. However, the performance does not improve with time, since the linear controller lacks the capacity to compensate for the nonlinearity.

6.3. Nonlinear controller only

With the data collected while the linear controller was in use, a standard feedforward neural network, with two hidden layers and 20 and 10 nodes, respectively, was trained off-line to approximate the input–output map of the plant, using back-propagation with adaptive learning rate in batch mode. Later this neural network was used on-line as an identifier, and its weights were adjusted along the negative gradient of some well-defined performance index. By certainty equivalence principle, at every instant, a control input $u(k)$ is needed such that the output of the identifier matches the desired output of the plant. One usual practice is to train another neural network for this task, and an efficient way to initialize such a neural network through off-line training was proposed in (Chen & Narendra, 1999). In this simulation, $u(k)$ was generated by an iterative procedure which solved a nonlinear equation in $u(k)$ such that $\hat{y}(k+2) = y^*(k+2)$ within a given tolerance (0.001). Note that an exact solution is guaranteed by Lemma 1 (full range).

Due to the complexity of the nonlinear plant and the nonlinearly parameterized neural network controller, the performance of the closed-loop system was hard to predict. For example, it was observed in the simulation that the learning rate, or step size, of the neural network identifier, when its weights were adjusted on-line along some gradient, was critical to the performance or even stability of the closed-loop system. Illustrative figures are omitted here due to space limitations, and interested readers are referred to (Chen, 2001).

6.4. Switching control (Fig. 3)

In this stage the plant was controlled using both the linear robust adaptive controller and the neural network based controller in the framework established earlier.

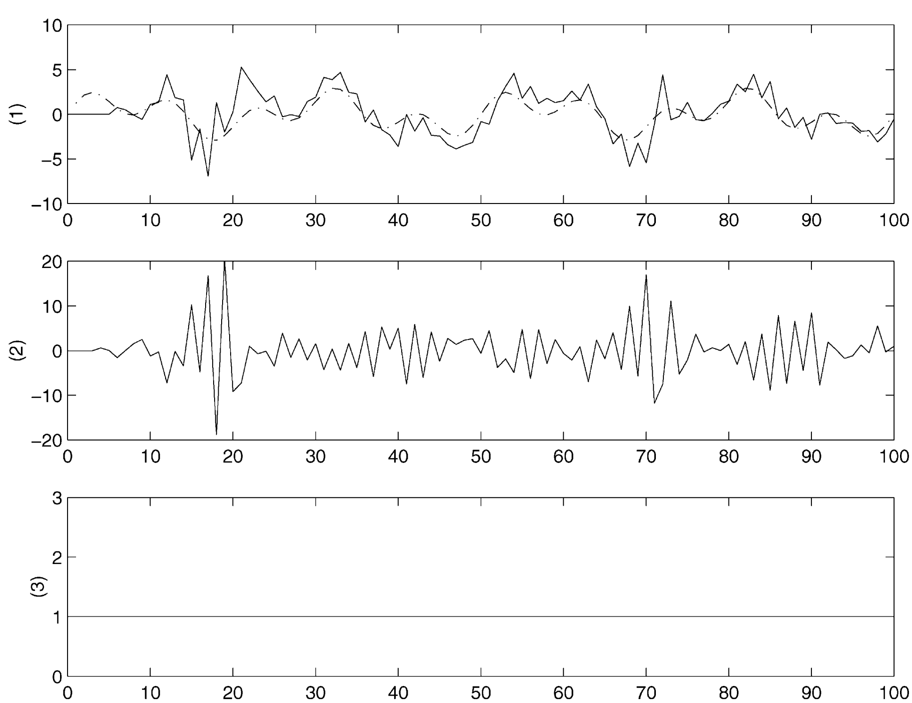

Fig. 2. Performance of the linear controller: (1) output (solid) and reference (dashed); (2) control input; (3) dummy "switching" signal.

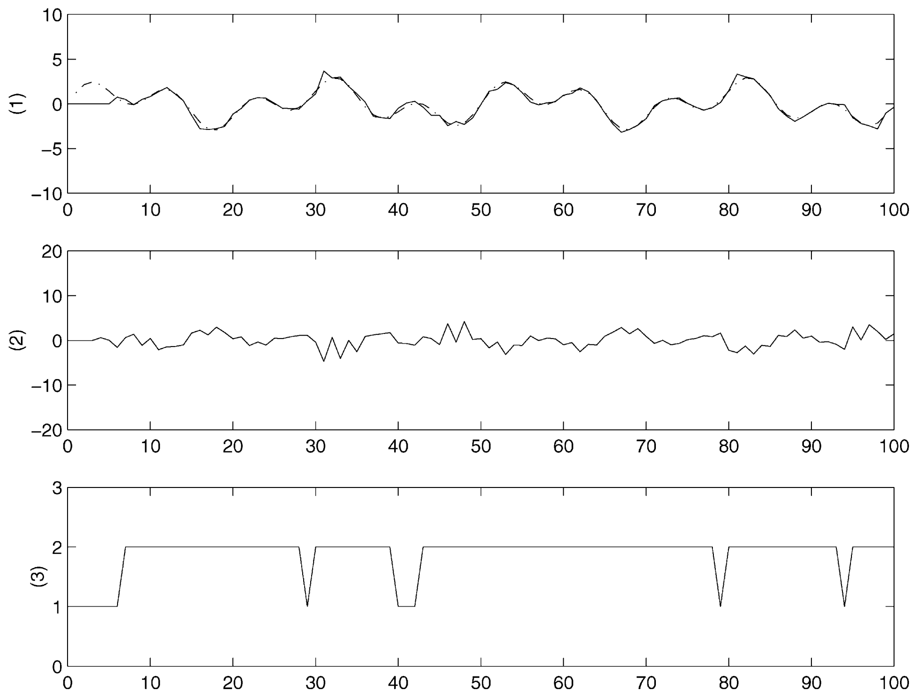

Fig. 3. Performance of the switching system: (1) output (solid) and reference (dashed); (2) control input; (3) switching sequence: (1) linear, (2) nonlinear.

Since the delay is 2, there were actually two neural networks for the nonlinear controller, one operating at time instants $1, 3, 5, \ldots$, and the other operating at time instants $2, 4, 6, \ldots$, as was discussed in Section 4. The tracking performance obtained and the switching sequence of the closed-loop system are shown in Fig. 3, with parameters in (16) chosen to be $c = 1$ and $N = 2$. The learning rate of the neural network was 0.001.

Comment 6. It is seen from Fig. 3 that even though the neural network based nonliner controller worked very well most of the time, occasionally it degraded for some reason, and the linear controller had to take over until the neural network controller recovered. Different sets of parameters of c, N and the learning rate of the neural networks were also simulated, with varying tracking performances, but signals in the system were always bounded.

7. Conclusions

In this paper a new framework is established to adaptively control a class of nonlinear discrete time dynamical systems while assuring boundedness of all the signals. A linear robust adaptive controller and a nonlinear neural network based adaptive controller are used, and a switching law is suitably defined to switch between them, based upon their performance in predicting the plant output. Boundedness of all the signals is established regardless of the parameter adjustment mechanism of the neural network controller, and thus stability and performance are decoupled, and better performance can eventually be achieved in view of the universal approximation capability of neural networks.

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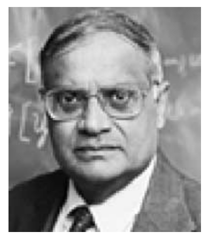

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