WAVELET TRANSFORM BASED HARMONIC ANALYSIS

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Abstract

A wavelet transform based method is proposed in the paper for harmonic analysis. The research is motivated by the authors' project about power quality monitoring. In the project, a classifier for recognizing power quality disturbances has been developed. The classifier adopts wavelet transform to extract features of various distorted waveforms including those containing harmonics. Obviously, it is no longer necessary or economic to use Fourier transform for evaluating harmonics that are retrievable from the transformed data. Thus a number of algorithms have been developed to estimate the distortion contributions from different sub-band coefficients. By employing techniques of neural networks, furthermore, each harmonic component is directly retrieved. Both the distortion estimation and the harmonics evaluation have achieved a high accuracy.

1. INTRODUCTION

Electric power systems have been perplexed by harmonic distortions due to wide scope applications of power electronic equipment and varieties of other non-linear loads. The problems brought about include malfunction of control devices, data loss of computers, interference to telecommunications, impulsive rotation of induction motors, and extra power dissipation [1]. Before plotting any effective countermeasures to alleviate the impacts of harmonic disturbances, it is necessary to acquire an accurate and reliable evaluation of harmonic components at different frequency ranges. Fourier transform (FT) has long been adopted for this application.

Harmonic distortion is one of the main power quality disturbances frequently encountered in utilities. It would be convenient and economic to use one scheme to monitor all these disturbances as done in the project of the authors. In developing a power quality monitoring system, the wavelet transform (WT) has been utilized for extracting disturbances such as impulsive and oscillatory transients, sag and swell, voltage fluctuation and notching, to name part of a long list [2]. The WT is known as a special type of sub-band decomposition. The transform coefficients thereby obtained contain the information about different sub-band (or scale) harmonic components of the original data [3,4]. Obviously, it is neither necessary nor economic to still use the FT while the harmonics can be directly retrieved form the transformed data, i.e., the approximation and the detail coefficients. In the paper, the WT based harmonic analysis algorithms are presented. The algorithms evaluate distortion contributions from components of different frequency sub-bands and estimate the total harmonic distortion (THD). Furthermore, each harmonic component is to be calculated with the corresponding sub-band coefficients by employing the artificial neural networks in the architecture of Multi-Layer Perceptron (MLP). The structure of the paper is as follows. After a brief introduction to the digital wavelet transform (DWT), the relationship between the transform coefficients and the harmonic components is discussed in Section 2. Section 3 presents the neural networks designed for each harmonic component evaluation and Section 4 provides the simulation results. Section 5 is the conclusion.

2. WAVELETS AND HARMONICS

Discrete wavelet transform (DWT) defines a mapping of $l^2(\mathbf{Z}) \rightarrow l^2(\mathbf{Z}^+,\mathbf{Z})$, i.e. from square summable 1-D sequence to 2-D sequence defined in the half scale-temporal plan with $\mathbf{Z}$ as the set of integers. Stephane Mallat developed a pyramidal algorithm for the implementation of DWT. With the algorithms, a discrete signal can be processed iteratively by a pair of quadrature mirror filters (QMF). While the low-pass filter approximates the signal, the high-pass filter provides the details lost in the approximation. The QMFs can be derived from the adopted wavelet. Figures 1(a) and 1(b) plot the magnitude responses of a pair of filters derived from the cubic spline wavelet. It is seen from that the QMFs will split the processed signal into two sub-bands with the same bandwidth. Should the low- pass and high-pass filtering be applied to a discrete signal repeatedly, the signal would finally be decomposed into a series of octave sub-band components. Let the original signal $v(n)$ be denoted by $CA_0(n)$ (or CA_0 when there is no need to indicate the translation n) and the approximation and detail coefficients at scale-j as $CA_j(n)$ and $CD_j(n)$ (or CA_j and CD_j), the signal can then be expressed by a multi-resolution representation: $\{CA_J,\ \{CD_j\}_{1\le j\le J}\}$ provided that a J-level WT decomposition is adopted. To monitor the power quality, the sampling frequency in capturing the waveforms is chosen as f_s = 12.8 kHz or $256f_1$ in the project, with f_1 as the fundamental frequency. According to Shannon Sampling Theorem, the captured waveform retains harmonic components in the frequency range of $0\sim f_s/2$. Consequently, the transform coefficients from a five-level WT (adopted in the project) should contain the information about the raw data with respect to the following octave sub-bands:

- CD_1: $64f_1\sim128f_1$;
- CD_2: $32f_1\sim64f_1$;
- CD_3: $16f_1\sim32f_1$;
- CD_4: $8f_1\sim16f_1$;
- CD_5: $4f_1\sim8f_1$;
- CA_5: $0\sim4f_1$;

The distortion rate associated with the harmonics in the above frequency ranges can then be estimated from the relevant approximation and/or detail coefficients. According to Stephane Mallat's algorithms, the approximation or the detail signal at each sub-band can be reconstructed with the orthonormal basis (wavelet or scaling function and their dilations and translations) by using the transform coefficients. Thus a simple arithmetic of the transform coefficients at each scale directly provides the measure of the distortion owing to the sub-band harmonic components. In addition, the total harmonic distortion (THD) can be obtained by summating the distortion measures of different scales. It can be easily deduced that the distortion caused by each sub-band harmonics is given by the RMS value of the coefficients $\sqrt{\frac{1}{N_j}\sum_n [CD_j(n)]^2}$ (N_j is the number of the detail coefficients at scale-j) whilst the THD is obtained by including each sub-band contribution $\sqrt{\sum_j (\frac{1}{N_j}\sum_n\ [CD_j(n)]^2)}$.

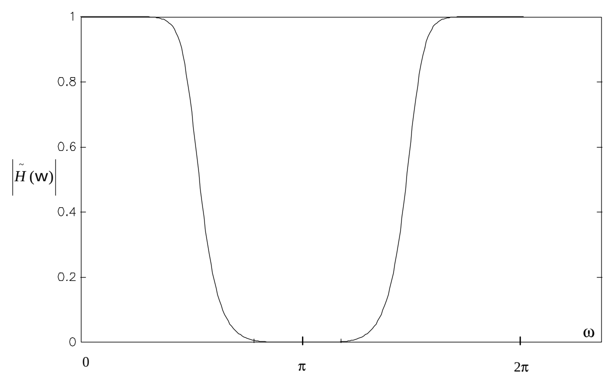

Figure 1 (a) Magnitude response of low-pass filter

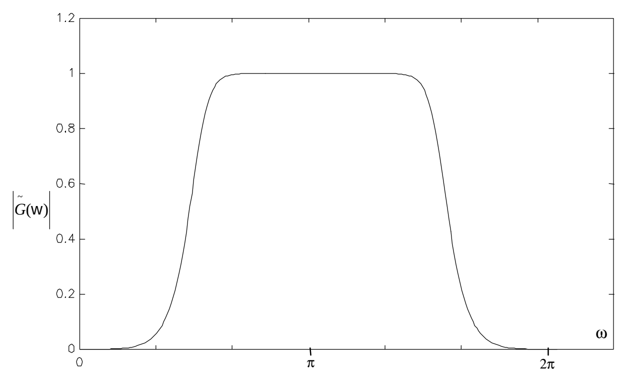

Figure 1 (b) Magnitude response of high-pass filter

In the analysis above, only the detail coefficients have been taken into account. The approximation coefficients CA_5 as described earlier contain the components in the frequency range of $0\sim4f_1$. It is difficult, however, to perform harmonics estimation directly using CA_5 because of the fundamental component it contains. In order to estimate this sub-band harmonics, say the 3rd and the 4th order, the CA_5 can be further split into CA_6 and CD_6 by adding one more level WT decomposition. The CD_6 contains the components in the range of $2\sim4f_1$. Without doing this, the harmonics in the case studies are assumed above the range of $4f_1$ at this stage. As a result, the RMS of CA_5 gives the fundamental component. For harmonics analysis at the low sub-band, neural network based algorithms have been designed. As to be described in Section 3, the neural networks are trained to evaluate each harmonic component from the DC component to the 47th order harmonic.

3. HARMONICS CALCULATION WITH WT COEFFICIENTS

It appears from the discussion of preceding section that harmonics can be retrieved directly from the correspondent transform coefficients. However a detailed study discovers that it is impractical to assume a very sharp splitting through WT decomposition using QMFs and make the harmonics calculation based on this assumption. As a result, the following mapping pairs are made for a more accurate evaluation of harmonics:

- CA_5: → harmonics 0-1;
- CA_5 & CD_5 : → harmonics 2-5;
- CD_5 & CD_4 : → harmonics 6-11;
- CD_4 & CD_3 : → harmonics 12-23;
- CD_3 & CD_2 : → harmonics 24-47;

Except for the first group calculation, transform coefficients at a pair of successive sub-bands are exploited to retrieve a set of harmonics. Altogether, five groups of harmonics are to be calculated, from DC component to the 47th harmonic. The algorithms thereby developed could satisfy most applications since the higher-order harmonics (above the range of the 25th to 50th, depending on system) are negligible. Hence, the highest frequency sub-band CD_1 is discarded here (but used for other applications [2]). The above mapping pairs from the transform coefficients to the harmonics have been arranged by taking into account the overlap in the WT decomposition.

Consequently, five sets of neural networks are needed for evaluating the five groups of harmonics. The neural networks are chosen to take the architecture of MLP. Typically, this kind of networks consists of a sensory units (source nodes) that constitute the input layer, one or more hidden layers of computation nodes, and an output layer. Some preliminary experiments have been carried out to determine the structure of the neural networks in terms of the numbers of hidden layers and the number of neurons at each layer. According to the experiments, the neural networks are determined to adopt two hidden layers in addition to one output layer using linear neurons. The MLP is illustrated by Figure 2, where $\boldsymbol{X}=[x_1, x_2, \ldots x_N]^T$ represents the input vector composed of the transform coefficients. The numbers of neurons at the two hidden layers are determined proportional to the dimensions of the input and output respectively. The linear output layer is used to acquire an appropriate scale.

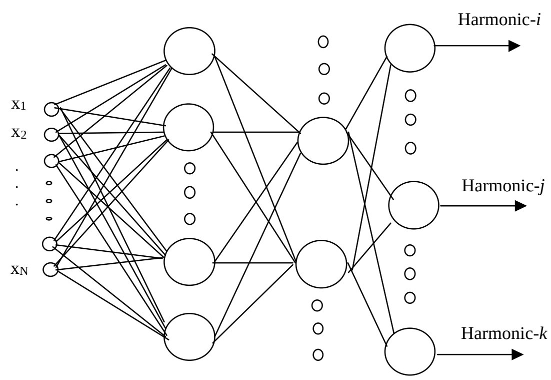

Figure 2 Multi-layer Neural Networks for Harmonics Evaluations
$\boldsymbol{X}=[x_1, x_2, \ldots, x_N]^T$ approximation or detail coefficients

In most of the cases, the inputs comprise the coefficients of two sub-bands. To prevent the predominant influence from the higher valued sub-band, the inputs are normalized into the range in –0.5~0.5, and the targets are also scaled similarly. The activation function of the hidden neurons is chosen as hyperbolic. Experiments reveal that the neural networks learn faster when the sigmoidal activation function built into the neuron model of the network is asymmetric and the data of training sets are scaled in small symmetric vicinity around the zero. By doing these, extremely small gradients can be avoided, at least at early stage of the optimization.

Although the measures mentioned above are adopted, convergence speed has become a problem in the training phase of neural networks for evaluating harmonics of the groups of $12\sim23f_1$ and $24\sim47f_1$. The slow convergence is due to the large size of the neural networks. To speed up the learning process, The above two groups of harmonics are further divided into two and three sub-groups respectively. The neural networks are finally designed for each sub-group following the arrangements below:

- CD_4 & CD_3 : → harmonics 12-17;
- CD_4 & CD_3 : → harmonics 17-23;

- CD_3 & CD_2 : → harmonics 24-31;
- CD_3 & CD_2 : → harmonics 32-39;
- CD_3 & CD_2 : → harmonics 40-47;

4. SIMULATION RESULTS

A testing set is adopted to verify the performance of the developed algorithms and neural networks in distortion estimation and harmonics calculation. The training and testing sets are composed of distorted waveforms containing harmonics up to the 47th order. Figure 3 plots such a distorted waveform used in the case study. By using FT, each harmonic component and the total distortion THD of the waveform are calculated first. The THD so obtained is 25.45%. Then the wavelet transform method is applied to the waveform. By using the transform coefficients, the THD is estimated with the developed algorithms. The result is 24.62%, close to that by using FT method. Table 1 gives the details of the distortions caused by each sub-band harmonics.

Table 1 Distortion estimation using WT transform coefficients

Sub-band	RMS (pu)	Distortion (100%)
CD_1	0.0165	1.64
CD_2	0.0597	5.92
CD_3	0.1134	11.24
CD_4	0.1140	11.31
CD_5	0.1788	17.73
CA_5	1.0087	-
THD	24.62%	

In the waveform plotted in Figure 3, harmonics in the frequency range of 0~4f_1 are of very low magnitudes. As a result, the RMS value calculated from CA_5 gives the fundamental component. The distortion owing to each sub-band is then evaluated with the ratio between the RMS of the sub-band harmonics and that of the CA_5:

$$\sqrt{\frac{1}{N_j}\sum_n [CD_j(n)]^2} \Big/ \sqrt{\frac{1}{N_5}\sum_n [CA_5(n)]^2} .$$

Finally harmonic components are retrieved from the approximation and detail coefficients using the neural networks designed. The simulation results of the case study (using the same waveform) is illustrated by Figure 4, where the left-side bars give the magnitude of the actual harmonics (obtained using WT) whist the right-side bars are the outlets of the neural networks. The accuracy achieved is encouragingly high (5%).

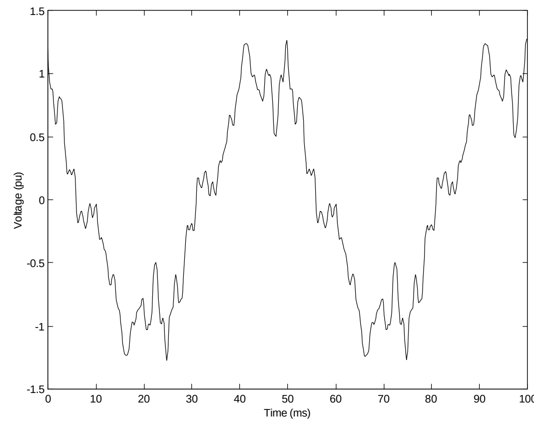

Figure 3 Distorted waveform

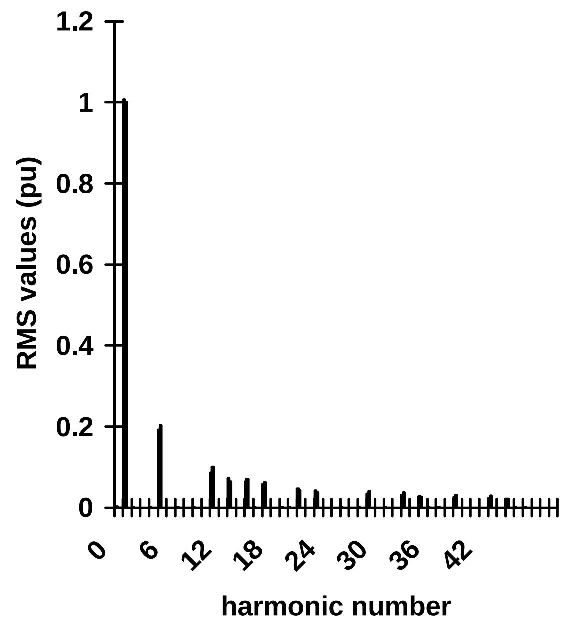

Figure 4 Harmonics evaluation using neural networks
Left bar: actual harmonics;
Right bar: evaluated by neural networks

5. CONCLUSIONS

The proposed wavelet transform based method provides an alternative for harmonic analysis. With this method, distortions due to different sub-band harmonics can be easily estimated and each harmonic component can be evaluated from part of the transformed data. The accuracy achieved is satisfactory with potential of further improvement.

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