Unconventional Fano resonances in light scattering by small particles

M. I. Tribelsky[1,2,3(a)], A. E. Miroshnichenko[4] and Y. S. Kivshar[4]

[1] *Faculty of Physics, M. V. Lomonosov Moscow State University - 1 Lenin Hills, Bldg. 2, Moscow 119991, Russia*
[2] *Moscow State Institute of Radioengineering, Electronics and Automation - 78 Vernadskiy Ave., Moscow 119454, Russia*
[3] *Max-Planck-Institut für Physik komplexer Systeme - Nöthnitzer Str. 38, Dresden 01187, Germany, EU*
[4] *Nonlinear Physics Centre, Research School of Physical Sciences and Engineering, Australian National University - Canberra ACT 0200, Australia*

Abstract – We introduce a new concept of Fano-like resonances for the extinction cross-section at light scattering by small (relative to the light wavelength) particles. The resonances occur beyond the applicability of the Rayleigh approximation, when the interference of different electromagnetic modes excited in the particle with the same multipole moment is crucial, while the partition of the incident wave bypassing the particle is unimportant. We present two examples of these fundamentally new Fano-like resonances: at the light scattering by a particle with large dielectric permittivity and by a particle with spatial dispersion. In both cases the extinction cross-section as a function of the incident light frequency exhibits a *sequence* of the Fano-like resonances, while each individual resonance is described by the conventional Fano profile.

Being originated in the pioneer paper by Fano [1], the Fano resonances have become one of the most appealing phenomena in the wave scattering. There exist many papers devoted to this topic (see, *e.g.*, the recent review articles [2,3] and references therein). The study of the Fano resonances in light scattering by small particles is an important subfield of this general problem. However, many authors are focused on rather specific problems related to practical applications of the resonances [4,5]. Meanwhile, the original Fano theory deals with a scattering of a quantum particle by a potential with a quasi-discrete level [1], *i.e.*, it is based upon the Schrödinger equation. Its application to the light scattering is not so straightforward, because though the Maxwell equations have a certain similarity with the Schrödinger equation, *they are not identical at all*. On the other hand, the general theory of the Fano resonances in optics does not exist yet.

In this letter, we reveal the fundamental features of the Fano resonances for the extinction cross-section at light scattering by small (relative to the light wavelength) particles. Our analysis is based on the exact Mie solution of the Maxwell equations for a spherical spatially homogeneous nonmagnetic particle and on a generalization of this solution to the case of a particle with spatial dispersion of its dielectric permittivity. We demonstrate that the Fano resonances occur *beyond the applicability of the Rayleigh approximation*, and the fundamental role in the resonances is played by the interference of *different electromagnetic modes* with the *same* multipole moment l (see footnote [1]). We name those resonances *unconventional Fano resonances*. It is remarkable that, despite the drastic differences in the nature, the unconventional resonances exhibit in the leading approximation the conventional Fano profile [1].

(a)E-mail: `tribelsky_at_mirea.ru`

[1]The origin of a Fano profile in [1] is in interference of a background partition of a scattering wave, bypassing a scatterer, and a resonance partition, which first is trapped, exciting a quasi-discrete state, and then reemitted by the scatterer owing to finiteness of lifetime for the quasi-discrete state. In contrast, the discussed optical resonances would correspond to intereference of partitions reemitted from different quasi-discrete levels, while the partition bypassing the scatterer is unimportant.

First, we discuss the physics of these novel Fano-like resonances. In general, the origin of any Fano resonance is in the interference between the resonant and background partitions of the scattered field. In the off-resonant regions the phase difference of the fields in the partitions either vanishes or is equal to π, while the amplitude of the resonant partition is small relative to the background. At the maximum of the resonant line the amplitude of the resonant partition is large relative to the background. Then, at the edges of the resonant line for the resonant partition, there exist two points lying on different sides of the maximum, where amplitudes of the resonant and background partitions coincide. The phase of the background partition is not affected by the resonance, and therefore it is the same at both the points. The phase of the resonant partition at one of these points equal zero while at the other it is π (passage of the resonance adds the phase shift π to the resonant field). Hence, one of the two points should be a point of destructive interference where the resonant and background partition cancel each other.

The study of the field lines for the Poynting vector in the near field of a small light scattering particle indicates that a part of the incident light is scattered passing through the particle, while the other just bypasses it [6,7]. However it is easy to see that, within the framework of the conventional Rayleigh approximation, the lineshapes for the cross-sections are symmetric Lorenzian (Breit-Wigner) [8]. It occurs because for the small particle the resonant dipole mode remains dominant over the background even in off-resonant regions and the destructive interference cannot happen. Therefore, the Fano resonances in optics should take place *beyond the Rayleigh approximation.*

For small scattering particles made of weakly dissipating materials, the destructive interference conditions may be satisfied for two modes with different values of the multipole moment, *e.g.*, for the dipole and quadrupole in the vicinity of the quadrupole resonance [8]. In this case, the off-resonant dipole mode plays the role of the background partition. However, scattering diagrams for modes with different orders have different angular dependences. It means that if at a given light frequency ω the destructive interference conditions are satisfied along a certain direction, they are not satisfied along other ones. Thus, the suppression of the scattering is possible *along the given direction only.* In order to obtain the scattering suppression along *all directions* at once, one should arrange the destructive interference of different modes with the same angular dependences of the scattering diagrams. The latter takes place if and only if the modes have the same value of the multipole moment. These general reasonings allow us to predict *two new classes of Fano-like resonances* at light scattering by small spatially homogeneous particles.

The first class corresponds to the interference of two dipole modes —the off-resonant Rayleigh one and the resonant, excited at a large value of the particle dielectric permittivity ε_p. Though such large values of ε_p hardly could be found in natural materials, they may become meaningful for metamaterials. It should be emphasized also that as long as partial cross-sections are concerned, the obtained results could easily be extended to the case of large particles, where the resonances correspond to much smaller values of the dielectric permittivity. For more details see below.

The second class deals with effects of spatial dispersion, which always exist in any optical material. These effects are usually weak, but they may change the scattering process dramatically and bring about the Fano-like resonances. The resonant modes in this case correspond to longitudinal electromagnetic oscillations realized at vanishing dielectric permittivity of the particle. These modes in light scattering have been discussed in many papers, see, *e.g.*, [9,10], but their connections with the Fano resonances have not been elucidated yet.

The two classes exhibit remarkable similarity in features. It should be also stressed, that while for the sake of simplicity magnetic particles are not discussed here, the Fano resonances for these particles obey essentially the same laws as those inspected in the present letter.

Note also that the pointed out concept of the unconventional Fano resonances, namely, the interference of different electromagnetic modes with the same multipole moment each, while it has not been said explicitly, in fact, has been successfully employed in numerous examples of the constructive (scattering enhancement) or destructive (cloaking, invisibility) interference in light scattering by nanoshells, multilayer nanoparticles and nanocavities, see, *e.g.*, discussion of these effects in review papers [2,3] and references therein. In all these cases the resonant and background modes correspond to modes with the same multipole moments excited in different layers of the particle or cavity.

Now let us consider in detail light scattering by a nonmagnetic ($\mu = 1$) spherical particle with radius R and complex dielectric permittivity $\varepsilon_p(\omega)$ embedded in a transparent medium with purely real dielectric permittivity $\varepsilon_m(\omega)$, $\mathrm{Im}\,\varepsilon_m(\omega) = 0$ [11]. It is convenient to normalize the extinction (σ_{ext}), scattering (σ_{sca}) and absorption (σ_{abs}) cross-sections over the geometrical cross-section of the particle πR^2, introducing the dimensionless scattering efficiencies, so that $\sigma_{ext,\,sca,\,abs} = \pi R^2 Q_{ext,\,sca,\,abs}$. Then, the net efficiencies are presented as sums, of the corresponding partial ones [11],

$$Q_{ext,\,sca} = \sum_{l=1}^{\infty} Q_{ext,\,sca}^{(l)}; \qquad Q_{abs} = Q_{ext} - Q_{sca}, \quad (1)$$

where the partial efficiencies are expressed in terms of the so-called complex scattering electric (a_l) and magnetic (b_l) coefficients,

$$Q_{ext}^{(l)} = \frac{2}{q^2}(2l+1)\mathrm{Re}(a_l+b_l), \;\; Q_{sca}^{(l)} = \frac{2}{q^2}(2l+1)(|a_l|^2+|b_l|^2). \tag{2}$$

Here the size parameter $q = n_m R k = n_m R \omega / c$, k and c designate the incident light wave number and the speed of light in vacuum, respectively, n_m stands for the purely real refractive index of the environmental medium ($n_m = \sqrt{\varepsilon_m}$) and a_l, b_l read as follows:

$$a_l = \frac{F_l^{(a)}}{F_l^{(a)} + i G_l^{(a)}}, \qquad b_l = \frac{F_l^{(b)}}{F_l^{(b)} + i G_l^{(b)}}. \qquad (3)$$

Regarding the general expressions for $F_l^{(a),(b)}$ and $G_l^{(a),(b)}$, they could be found, *e.g.*, in [11]. These expressions are rather cumbersome and will not be presented here.

The points $|a_l|^2 = 1$ and $|b_l|^2 = 1$ correspond to optical resonances, whose positions are defined by equations $G_l^{(a,b)}(q, n) = 0$, where n stands for the relative refractive index $n = \sqrt{\varepsilon_p / \varepsilon_m}$. Through the dispersion relation $n(\omega)$ these equations define the resonant frequencies $\omega_l^{(a,b)}(q)$, which, generally speaking, are complex quantities. It is also convenient to split the partial efficiencies, eq. (2), into the electric $Q^{(l,a)}$ and magnetic $Q^{(l,b)}$ parts, related to the corresponding coefficients, so that $Q_{ext}^{(l,a)} = (2/q^2) \mathrm{Re}\, a_l$, etc.

Smallness of the particle means that $q \ll 1$. As for nq, it should not necessarily be small. Then, expansion of $F_l^{(a)}$ and $G_l^{(a)}$ in powers of small q yields the following expressions:

$$F_l^{(a)} \simeq \frac{q^l}{(2l+1)!!} [(l+1) n \psi_l(nq) - q \psi_l'(nq)], \qquad (4)$$

$$G_l^{(a)} \simeq \frac{(2l-1)!!}{q^l} \left[\frac{nl}{q} \psi_l(nq) + \psi_l'(nq) \right], \qquad (5)$$

$$F_l^{(b)} \simeq \frac{q^l}{(2l+1)!!} [nq \psi_l'(nq) - (l+1) \psi_l(nq)]$$
$$- \frac{q^{l+2}}{2(2l+3)!!} [nq \psi_l'(nq) - (l+3) \psi_l(nq)], \qquad (6)$$

$$G_l^{(b)} \simeq -\frac{(2l-1)!!}{q^l} \left[n \psi_l'(nq) + \frac{l}{q} \psi_l(nq) \right], \qquad (7)$$

where $\psi_l(z) = \sqrt{\frac{\pi z}{2}} J_{l+\frac{1}{2}}(z)$, $J_\nu(z)$ stands for the Bessel function and the stroke indicates differentiation over the entire argument of the corresponding function, so that $\psi_l'(z) = d\psi/dz$, etc. (eq. (6) requires higher accuracy relative to the other expressions because at small nq the leading term there vanishes).

To begin with, we consider the nondissipative limit, *i.e.*, $\mathrm{Im}\, \varepsilon = 0$, $\mathrm{Im}\, n = 0$, $Q_{abs}^{(l)} = 0$, $Q_{ext}^{(l)} = Q_{sca}^{(l)}$. Let us fix q and inspect the scattering coefficients as functions of n. It is seen straightforwardly that the equations $G_l^{(a,b)}(n) = 0$ have *infinite number of roots* $n_{l,r}^{(a,b)}$, $r = 1, 2, 3, \ldots$ associated with oscillations of the Bessel functions and that, $F_l^{(a,b)}(n_{l,r}^{(a,b)}) \neq 0$. At fixed a, b, l the root sequence may be arranged so that $n_{l,1}^{(a,b)} < n_{l,2}^{(a,b)} < n_{l,3}^{(a,b)} < \ldots$. Each $n_{l,r}^{(a,b)}$

corresponds to a resonant point, where a scattering coefficient (a or b, respectively) reaches its maximal value, namely unity. In the vicinity of each root the corresponding resonant electromagnetic mode should be regarded as a resonant partition, while the off-resonant modes play the role of the background ones.

All these resonances are alike. Let us inspect the vicinity of a certain root $n_{l,r}^{(a)} = \tilde{n}$ lying in the range $n \gg 1$. In the leading approximation $\tilde{n}$ is defined as a root of equation $\psi_l(\tilde{n}q) = 0$, see eq. (5). Considering small deviations δn of n from $n = \tilde{n}$ after trivial algebra one obtains the following expression for $F_l^{(a)}$, $G_l^{(a)}$:

$$F_l^{(a)} \simeq \frac{q^{2l+1}}{(2l+1)[(2l-1)!!]^2} [(l+1) \tilde{n} \delta n - 1], \qquad (8)$$

$$G_l^{(a)} \simeq (1 + l \tilde{n} \delta n). \qquad (9)$$

It yields an asymmetric lineshape for $|a_l|^2$ with the peak of the constructive interference ($a_l = 1$) at $\delta n = \delta n_+ \simeq -1/(l\tilde{n})$ and the complete destructive interference ($a_l = 0$) at $\delta n = \delta n_- \simeq 1/[(l+1)\tilde{n}]$ (see footnote [2]). The linewidth for the constructive interference at the level $|a_l|^2 = 1/2$ (FWHM) is given by the following expression:

$$\gamma \simeq \frac{2q^{2l+1}}{\tilde{n} l [l(2l-1)!!]^2} \ll |\delta n_\pm| \ll 1. \qquad (10)$$

Note that linear transformation $\delta n = A\epsilon + B$, where A and B are certain real constants, reduces the obtained line to the conventional Fano profile: $|a_l|^2 \propto (\epsilon + \kappa)^2 / (\epsilon^2 + 1)$; $\kappa = \text{const}$.

As an example, dependences $|a_1|^2$, $|b_1|^2$ and $|a_1|^2 + |b_1|^2$, determining the corresponding partial cross-sections for the dipole modes, are presented in fig. 1(a) as functions of $\mathrm{Re}\, \varepsilon \equiv \mathrm{Re}\, n^2$ (for numerical calculations ε-dependence is more convenient than n-dependence because the dissipation rate is just proportional to $\mathrm{Im}\, \varepsilon$).

Finite dissipation, as usual, makes the resonant peaks less pronounced. To illustrate this feature of the phenomenon, evolution of the fine structure of the resonant line for $|a_1|^2$ with an increase in $\mathrm{Im}\, \varepsilon$ is presented in fig. 1(b). The resonances occur rather robust against the dissipation: the resonant peaks are still quite pronounced at such large values of $\mathrm{Im}\, \varepsilon$ as 2.

To conclude the discussion of this topic note, that a rough estimate of the spacing between two adjacent resonances $\Delta n_{l,r,r+1}^{(a,b)} = n_{l,r+1}^{(a,b)} - n_{l,r}^{(a,b)}$ coincides with that between two adjacent zeros of the Bessel function $J_{l+\frac{1}{2}}(qn)$, see eqs. (5), (7); which, in turn, is defined by the condition $q \Delta n_{l,r,r+1}^{(a,b)} \sim \pi$. Thus, roughly speaking, the spacing is the inverse of the size parameter q. In other words, by increasing the particle size one decreases the spacing and moves the resonances to the range of

---

[2]Despite the perfect vanishing of the given a_l, the net extinction cross-section remains finite owing to the contribution of b_l and other off-resonant modes, see eqs. (1), (2).
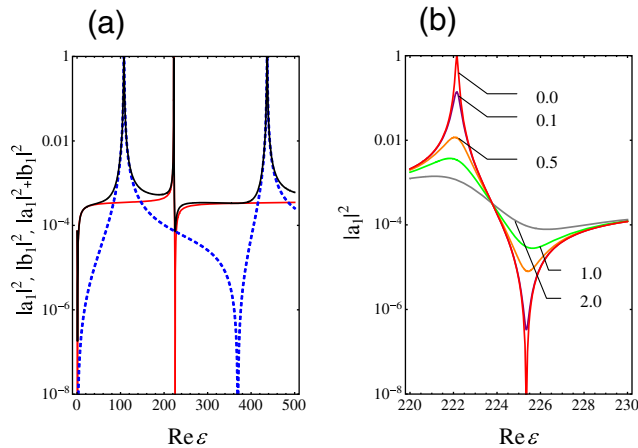
Fig. 1: (Color online) (a) Typical Fano profiles for dependences $|a_1|^2$ (red solid line), $|b_1|^2$ (blue dashed line) and $|a_1|^2 + |b_1|^2$ (black solid line) on $\mathrm{Re}\,\varepsilon$ at $q = 0.3$ and zero dissipation ($\mathrm{Im}\,\varepsilon = 0$). Calculations according to the exact Mie solution. (b) A fine structure of the resonance line for $|a_1|^2$ and its dependence on the value of $\mathrm{Im}\,\varepsilon$ (indicated in the figure).
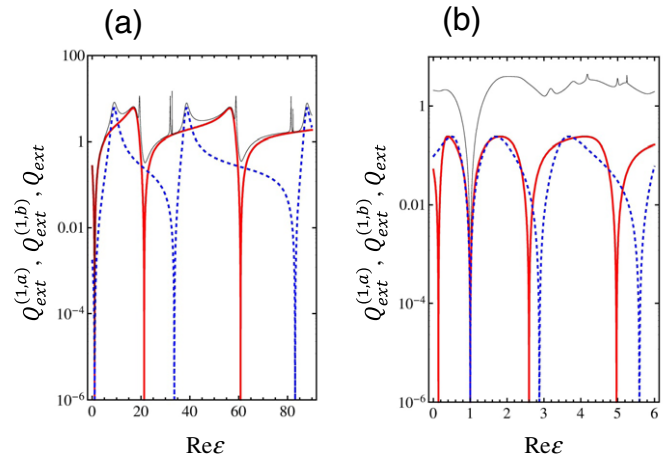

Fig. 2: (Color online) Partial extinction efficiencies for the dipole mode $Q_{ext}^{(1,a)}$ (red solid line), $Q_{ext}^{(1,b)}$ (blue dashed line) and the net extinction efficiency Q_{ext} (thin black solid line) at light scattering by a nondissipating ($\mathrm{Im}\,\varepsilon = 0$) particle as functions of $\mathrm{Re}\,\varepsilon$ at $q = 1$ (a) and $q = 5$ (b). It is seen that an increase in q on the one hand results in a decrease in the spacings between adjacent resonances for the partial efficiencies; on the other hand it makes the influence of peculiarities of a specific mode on the net efficiency less pronounced: if at $q = 1$ the Fano-like resonances of $Q_{ext}^{(1,a)}$ still affect the net efficiency Q_{ext}, at $q = 5$ their effects are already diminished.

moderate values of n. From these arguments it seems that to make experimental observation of the resonances easier one should just select the particle size large enough. Unfortunately, this is not quite the case. An increase in q does decrease the spacing between the adjacent resonances for any given *partial* cross-section, but simultaneously it increases the number of partial cross-sections whose contribution to the net cross-section should be taken into account to provide reasonable accuracy of the calculations. The more partial cross-sections contribute to the net cross-section the less pronounced peculiarities of every individual summand it makes.

To illustrate this issue, plots of the extinction electric ($Q_{ext}^{(1,a)}$) and magnetic ($Q_{ext}^{(1,b)}$) efficiencies for the dipole mode along with the corresponding net efficiency (Q_{ext}) as functions of $\mathrm{Re}\,\varepsilon$ are presented in fig. 2 in the nondissipative limit ($\mathrm{Im}\,\varepsilon = 0$) for spherical particles with $q = 1$ (fig. 2(a)) and $q = 5$ (fig. 2(b)). The calculations are made based upon the exact Mie solution. It is seen clearly that at $q = 5$ the spacings between the adjacent resonant points for the dipole modes are much smaller then those at $q = 1$ (cf. also the spacings at $q = 0.3$ in fig. 1(a)). However, while at $q = 1$ the resonances of $Q_{ext}^{(1,a)}$ are still pronounced in the profile of the net efficiency Q_{ext}, they practically do not affect this profile at $q = 5$.

Now let us discuss the Fano-like resonances in media with spatial dispersion. In isotropic, nonchiral materials the spatial dispersion is a weak effect which occurs when one takes into account that the vector of induction of electromagnetic field $\mathbf{D}$ in addition to its dependence on the vector of electric field $\mathbf{E}$ depends also on spatial derivatives $\partial E_i/\partial x_j$. However, this weak effect bring about a qualitatively new type of electromagnetic modes which may be exited in a medium with the spatial dispersion,

namely longitudinal plasma waves (volume plasmons), which obey the dispersion law

$$\varepsilon_\parallel(k_\parallel, \omega) = 0, \qquad (11)$$

while the transverse electromagnetic waves obey the usual dispersion law

$$k_\perp^2 = \varepsilon_\perp(\omega)\omega^2/c^2. \qquad (12)$$

To understand qualitative features of the phenomenon we employ the simplest heuristic expression for $\varepsilon_\parallel(k_\parallel, \omega)$, namely

$$\varepsilon_\parallel = \varepsilon_\perp(\omega) - \alpha x^2, \quad x \equiv k_\parallel R, \qquad (13)$$

where the dimensionless quantity $\alpha = O(d^2/R^2) \ll 1$. Here d stands for the interatomic distance. In the range of the volume plasmon excitations, according to eqs. (11)–(13) $|\varepsilon_\perp| = |\alpha x^2| \ll 1$, provided $|x| = O(1)$, or smaller. It means, the problem in question has two independent small parameters, namely q and the relative refractive index for the transverse modes n.

Taking into account the spatial dispersion, one increases the order of the Maxwell equations. Therefore, in the case of light scattering by a particle with the spatial dispersion, additional boundary conditions (ABC) are required. A number of possible ABC have been proposed and discussed, see, *e.g.*, [12]. For the problem in question all of them yield similar results. For definiteness we employ the ABC used by Ruppin [10], which correspond to the continuity at the particle boundary of the normal component of the displacement current $(1/4\pi)\partial\mathbf{E}/\partial t$.

Then, the general solution of the diffraction problem yields scattering efficiencies which still keep the structure of eqs. (1)–(3). The explicit expression for b_l remains exactly the same as that in the conventional Mie solution (it is related to the fact that the longitudinal modes do not have magnetic components). Regarding the expression for a_l, in the limit $q \ll 1$ the corresponding quantities $F_l^{(a)}$ and $G_l^{(a)}$ read as follows:

$$F_l^{(a)} \simeq \frac{(1-n^2)(l+1)n^l q^{2l+1}}{[(2l+1)!!]^2}\left(\frac{j_l'(x)}{j_l(x)} - \frac{l}{x}\right), \qquad (14)$$

$$G_l^{(a)} \simeq \frac{ln^l}{2l+1}\left\{(1-n^2)\frac{l+1}{x} - \frac{j_l'(x)}{j_l(x)}\left(n^2 + \frac{l+1}{l}\right)\right\}, \quad (15)$$

where $j_l(x) \equiv \sqrt{\frac{\pi}{2x}}J_{l+\frac{1}{2}}(x)$ stands for the spherical Bessel function.

Once again we begin the analysis from the nondissipative limit. In this case x is a purely real quantity. The extinction and scattering efficiencies vanish when $x = x_{l,r}$, where $x_{l,r}$ are solutions of the transcendental equation $lj_l(x) = xj_l'(x)$, see eqs. (2), (3), (14). We will see that the vanishing corresponds to the destructive Fano resonance and that close to this point there is a point of the constructive resonance, where $a_l = 1$. To this end let us consider the vicinity of a given $x_{l,r}$, introducing $\delta x \equiv x - x_{l,r}$. Then, expanding $F_l^{(a)}$ and $G_l^{(a)}$ in powers of small δx, the expression for a_l may be reduced to the form

$$a_l = \frac{A_{l,r}q^{2l+1}\delta x}{A_{l,r}q^{2l+1}\delta x + in^l\left[B_{l,r}\delta x - (n^2l/x_{l,r})\right]}, \qquad (16)$$

where $A_{l,r}$ and $B_{l,r}$ are cumbersome expressions of order of unity. Equation (16) yields immediately that $\delta x = 0$ corresponds to $a_l = 0$, i.e., it is a point of the destructive interference indeed. Note remarkable similarity of this profile with the one given by eqs. (3), (8), (9).

According to eq. (16), $a_l = 1$ at $\delta x = \delta x_{max} \equiv n^2l/(x_{l,r}B_{l,r}) = O(n^2) \ll 1$. The linewidth of the profile $|a_l|^2$ is given by the solutions $\delta x_{1,2}$ of the equation $|a_l(\delta x)|^2 = 1/2$, which read

$$\delta x_{1,2} = \frac{n^2l}{x_{l,r}}\frac{1}{[n^lB_{l,r} \pm A_{l,r}q^{2l+1}]}. \qquad (17)$$

At $q^{2l+1} \gg n^l$ the profile $|a_l|^2$ has a deep sharp minimum, centered about $\delta x = 0$, while the maximum is bad pronounced (the maximal value is situated close to a plateau of the profile). The linewidth defined at the level $|a_l|^2 = 1/2$ corresponds to the shape of the minimum being of the following order of magnitude: $O(n^2\frac{n^l}{q^{2l+1}}) \ll n^2$.

At $q^{2l+1} \ll n^l$ the profile $|a_l|^2$ has a sharp maximum, centered about $\delta x = \delta x_{max}$, while the minimum is bad pronounced (the plateau height is small). The linewidth at the level $|a_l|^2 = 1/2$ corresponds to the shape of the maximum. Its order of magnitude is $O(\delta x_{max}\frac{q^{2l+1}}{n^l}) \ll n^2$.
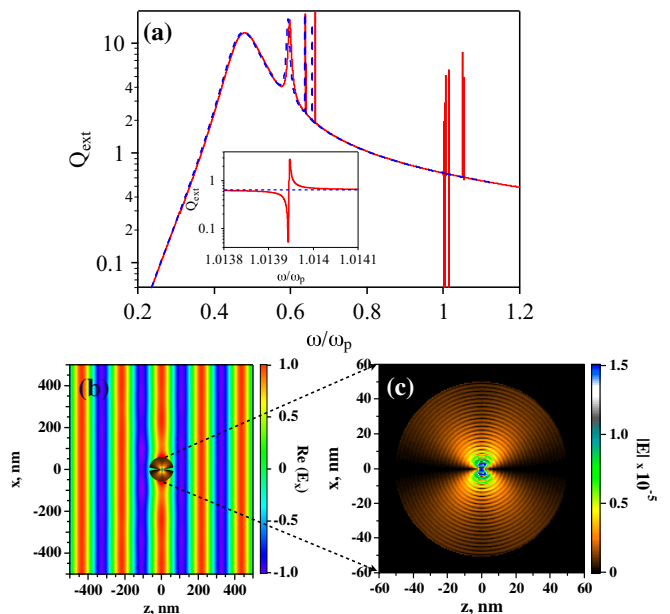

Fig. 3: (Color online) Extinction efficiency Q_{ext} for a nanoparticle of radius $R = 50$ nm (a) with $\varepsilon_\perp(\omega)$ given by eq. (18). The dashed blue line corresponds to the conventional Mie theory, while the solid red line represents the results of its generalization to the spherical particle with spatial dispersion calculated in accord with the Ruppin exact solution [10]. Note a number of Fano-like resonances at $\omega > \omega_p$. A fine structure of the first resonance of that type is shown in the inset with high resolution. A snapshot of the x-component of the total (incident plus scattered) electric field in the particle and its vicinity (b) and modulus of the total electric field inside the particle (c) for the resonant suppression of the scattering efficiency at the first Fano-like resonance. The color scale inside the particle in panel (b) corresponds to that in panel (c). The plotted profiles are normalized over the amplitude of electric field in the incident wave. While the particle does not distort the incident plane wave and remains completely invisible, the field inside it exceeds the one in the incident wave by five orders of magnitude.

The widest line with the linewidth $O(n^2) \ll 1$ is achieved at $q^{2l+1} \sim n^l$. In this case both the minimum and maximum of the resonant line are well pronounced.

To illustrate this general discussion we perform computer simulation of light scattering by a spherical metal particle with spatial dispersion. The transverse permittivity is approximated by the nondissipative version of the Drude formula

$$\varepsilon_\perp(\omega) = 1 - (\omega_p/\omega)^2 \qquad (18)$$

with a constant plasma frequency ω_p. The exact solution of the light scattering problem obtained by Ruppin [10], generalizing the Mie solution to the case of the particle with the spatial dispersion, is employed. In this case α in eq. (13) equals as follows [13]:

$$\alpha = \frac{3}{5}\left(\frac{v_F\omega_p}{\omega^2 R}\right)^2, \qquad (19)$$

where v_F stands for the Fermi velocity of free electrons. To be able to compare our results with previous calculations [10] the material constants are assigned the following values: $\omega_p = 8.65 \times 10^{15}\,\mathrm{s}^{-1}$, $v_F = 1.07 \times 10^8\,\mathrm{cm/s}$, which corresponds to sodium. The results of these calculations for a particle with $R = 50\,\mathrm{nm}$ are presented in fig. 3. The plane incident wave propagates along the z-axis from minus infinity. The electric field shown in fig. 3 is normalized over its amplitude in the incident wave. In addition to the well-known transverse localized surface plasmon resonances below the plasma frequency $\omega < \omega_p$ (the Mie theory), there are a number of asymmetric, very narrow bulk plasmon resonances beyond the plasma frequency $\omega > \omega_p$ (Fano-like resonances), which correspond to excitation of longitudinal modes. Note, that while the destructive Fano-like resonances do not distort the incident wave, which results in complete invisibility of the particle (see fig. 3(b)), the field inside the particle is very large, see fig. 3(c), where the modulus of the total electric field inside the particle $|\mathbf{E}_\perp + \mathbf{E}_\parallel|^2$ is plotted. Here $\mathbf{E}_\perp$ and $\mathbf{E}_\parallel$ stand for the electric fields of the transverse and longitudinal modes, respectively. The high concentration of the electric field inside the particle together with the undistorted field in its vicinity may open new prospects in the design of nanosensors, which do not perturb measured fields.

Finite dissipation diminishes the resonance. Details of this effect require more room for discussion and will be reported elsewhere.

In conclusion, we summarize our results. We have predicted the unconventional Fano resonances in light scattering by small particles. We have shown that i) these resonances are described beyond the applicability of the Rayleigh approximation; ii) the interference of different modes with the same multipole moment each is the key condition for the resonances to come into being; and iii) in the leading approximation the resonances still give rise to the lineshapes described by the conventional Fano profile. We have revealed two new classes of such resonances in light scattering by particles with large dielectric permittivity and the ones with spatial dispersion. We believe our study sheds new light on the Fano resonances in photonics, it may stimulate further study of this important and fascinating phenomenon and might be employed in the design of a new generation of unperturbing nanosensors.

$* * *$

REFERENCES

[1] Fano U., *Phys. Rev.*, **124** (1961) 1866.
[2] Miroshnichenko A. E., Flach S. and Kivshar Yu S., *Rev. Mod. Phys.*, **82** (2010) 2257.
[3] Luk'yanchuk B., Zheludev N. I., Maier S. A., Halas N. J., Nordlander Giessen P. H. and Chong C. T., *Nat. Mater.*, **9** (2010) 707.
[4] Hao F., Sonnefraud Y., Van Dorpe P., Maier S. A., Halas N. J. and Nordlander P., *Nano Lett.*, **8** (2008) 3983.
[5] Mukherjee S., Sobhani H., Lassiter J. B., Bardhan R., Nordlander P. and Halas N. J., *Nano Lett.*, **10** (2010) 2694.
[6] Bohren C. F., *Am. J. Phys.*, **51** (1983) 324.
[7] Tribelsky M. I. and Luk'yanchuk B. S., *Phys. Rev. Lett.*, **97** (2006) 263902.
[8] Tribelsky M. I., Flach S., Miroshnichenko A. E., Gorbach A. V. and Kivshar Yu S., *Phys. Rev. Lett.*, **100** (2008) 043903.
[9] Gil'denburg V. B. and Kondrat'ev I. G., *Radio Eng. Electron. Phys.*, **10** (1965) 560.
[10] Ruppin R., *Phys. Rev. B*, **11** (1975) 2871.
[11] Bohren C. F. and Huffman D. R., *Absorption and Scattering of Light by Small Particles* (Wiley, New York) 1998.
[12] Golubkov A. A. and Makarov V. A., *Phys. Usp.*, **38** (1995) 325.
[13] Ruppin R., *Opt. Commun.*, **190** (2001) 205.